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Spacecraft Parachute Fluid Mechanics Computation Based on Space–Time Isogeometric Analysis With T-splines

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Abstract

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This thesis is on fluid and structural mechanics computation of spacecraft parachutes based on isogeometric discretization with T-splines. Contrary to the non-uniform rational B-splines (NURBS), T-spline representation does not require a regular control grid, allowing a set of control points to end without traversing the whole domain. We generate a T-spline mesh directly from a block-structured NURBS mesh. In T-spline representation, we use knot removal at the unnecessarily refined regions of the NURBS mesh and reduce the number of control points. We use knot insertion at interfaces between nonmatching meshes. Then, having the matching pairs of control points on each side of the interface, we use knot removal to remove the other knots on the interface sides and enhance the continuity in the whole domain. This gives continuity, or increased continuity, to the volume mesh, improving the solution accuracy and reducing the total number of unknowns compared to NURBS computations.

We first present, for a 2D parachute model, fluid–structure interaction computation based on the Space–Time Isogeometric Analysis (ST-IGA) with NURBS and T-splines. Then, we present spacecraft parachute incompressible- and compressible-flow computations based on the ST-IGA with T-splines.
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Chapter 1

Introduction

1.1 Spacecraft Parachutes

Parachutes have been extensively used for safe and relatively cheap landing in various applications including but not limited to skydiving and cargo delivery. They are usually made of porous materials for better flight stability and supported by several cables (e.g. the suspension cables, tapes and riser) for structural integrity. Parachutes’ fabric surface, their canopy, might have different shape and design, varying greatly depending on their area of application and the payload size. Spacecraft parachutes are among the largest parachutes and they have very complex canopy designs. They have been used for many years in space missions and offered safe landing after the reentry for both manned and unmanned missions. Figure 1.1 shows a National Aeronautics and Space Administration (NASA) spacecraft parachute used for the safe landing of a spacecraft capsule with astronauts. Spacecraft parachutes have gained more popularity after NASA abandoned the space shuttle program due to high cost of maintenance and limited reusability. The new program, named Orion, was a successor to former Apollo programs and utilized spacecraft capsules instead of shuttles. In a similar manner with Apollo programs, the spacecraft parachutes were deployed at the
Figure 1.1: Orion spacecraft parachutes land the manned capsule safely in 2018 [1].

final stages of reentry and used till landing. This offered much more affordable landing
for both manned and unmanned vehicles, and since then the spacecraft parachutes and
capsules have been widely used together for various types of space missions instead
of space shuttles. Figure 1.2 shows the space shuttle Atlantis and Orion spacecraft
capsule crew module. A spacecraft during its reentry goes through various types

Figure 1.2: Atlantis space shuttle (left) and Orion spacecraft capsule crew module (right). Pictures are from [2] and [3].

of flight conditions in the atmosphere, so usually more than one type of spacecraft
parachutes are deployed till the safe landing is achieved. The initial stages of the
reentry take place in extreme conditions like very high speed and temperature. For
this reason, at these stages parachutes are generally not used and the spacecraft is relatively slowed down by the atmospheric resistance itself. In later stages of the reentry, on the other hand, different types of parachutes are deployed in a specific sequence with specific purposes. In Orion spacecraft reentry, a total of 11 spacecraft parachutes are used. The first ones to be deployed are a set of three forward bay cover (FBC) parachutes used to separate the FBC from the crew module. Then, a set of two Drogue parachutes are deployed which stabilize and slow down the spacecraft, attaining optimal conditions for deploying the next sets of parachutes. These next sets of parachutes are the Pilot and Main parachutes. The former is used to lift and deploy the latter set of parachutes. Main parachutes are very large and they are used to decelerate the spacecraft to a safe landing speed and ensure stability. Figure 1.3 shows the stages of Orion spacecraft parachutes deployment. Spacecraft parachutes

![Spacecraft Parachutes Diagram](image)

Figure 1.3: Orion spacecraft parachutes deployment sequence. Picture is from [4].

are also used in space exploration missions. Disk-Gap-Band (DGB) parachute is planned and tested for Mars exploration mission by both NASA and Japan Aerospace
Exploration Agency (JAXA). As the name suggests, it has a disk and a band part for the canopy and a gap in between them. Mars atmosphere has a much lower density than the Earth, and the DGB parachute is deployed in supersonic flow conditions of Mars atmosphere to slow the spacecraft down to a subsonic descent speed level. Figure 1.4 shows the DGB parachute in a test flight that has been performed by NASA for Mars exploration missions. In this thesis, we use two types of spacecraft parachutes, namely the Orion main parachutes and the DGB parachutes. Orion’s main parachutes are actually ring-sail parachutes with modified geometric porosity, so they are sometimes referred to as modified porosity (MP) ring-sail parachutes as well. More details about these two parachute structures and their flight conditions are given in the next two subsections.

1.1.1 MP: Modified porosity ring-sail parachute

The MP parachutes used in Orion missions are very large with nominal diameter $D_0$ of 35 m and they have several added geometric porosity components (e.g. gaps, windows, slits, vent and a wider gap) on the canopy. Their canopy has 80 radial
cables and thus 80 gores. Figures 1.5 and 1.6 show the actual three-cluster formation of the MP parachutes in a Orion flight test and the configurations for a single MP parachute and its single gore, respectively. The MP parachutes are deployed after

Figure 1.5: A three cluster formation of the MP parachute in flight test [6].

the spacecraft is already slowed down and stabilized by the Drogue parachutes, so they operate in incompressible-flow regime, the Mach number is about 0.18.
Figure 1.6: Configuration of an MP parachute (left) and its single gore (right).
1.1.2 Disk-gap-band (DGB) parachute

DGB parachutes have relatively smaller size and simpler canopy shape than the MP, but their area of application is wider. In recent years, they have been used by NASA for Mars exploration missions in the supersonic compressible-flow regime. The actual DGB parachute used in Mars exploration has a $D_0$ of about 20 m, but in the wind tunnel tests usually it is scaled down to a much smaller size; for example a $D_0$ of 171 mm is used in [9]. This smaller, wind tunnel test version of the parachute is called a “subscale” parachute. Figure 1.7 shows the configuration of a subscale DGB parachute.

Figure 1.7: Configuration of a DGB parachute.
1.2 Challenges

1.2.1 Flight and wind tunnel tests

Experiments are essential stages of a spacecraft parachutes design development, specifically the flight and wind tunnel tests offer great insight into a parachute’s performance and behavior during the flight. However, the flight tests for spacecraft parachutes are very expensive as they require full-scale parachutes and a drop test from high altitudes. Also, the flight tests depend on the weather conditions at the day of the test. It is possible that a flight test, which takes large amounts of time and money even for just scheduling and organizing, might get canceled due to unexpected weather conditions. On the other hand, the wind tunnel tests are relatively cheaper and allow the use of subscale models of the parachutes instead of the full-scale. Also, there are options for full-scale wind tunnel tests, but they might cost much higher due to the larger setup. Nevertheless, the wind tunnel tests of spacecraft parachutes have been performed previously for both the MP in [4] and DGB parachutes in [10] for supersonic and in [9] for subsonic flows. Despite these researches, the two main drawbacks of the wind tunnel tests for spacecraft parachutes remain unaddressed; (1) the scaling issue of parachute’s thickness compared to its diameter, and (2) the fact that the gravity acts in the flow direction in real flight while it acts perpendicular to the flow in the wind tunnel tests because of their horizontal setup. The spacecraft parachutes are very large in size, but they are made of very thin materials, so in a wind tunnel test, it is nearly impossible to scale down a parachute’s thickness using the same ratio with the scaling of its diameter. The effect of the direction of the gravitational force in the wind tunnel tests is expected to be low compared to the forces due to the air flow, but it might still affect the parachute’s stability.
1.2.2 Computational studies

The computational research of the spacecraft parachutes does not have the limitations of the flight and wind tunnel tests. However, there are some other challenges involved in the spacecraft parachute computational studies. First of all, the spacecraft parachutes operate in very extreme flight conditions like high Reynolds number and high Mach number. Also, the geometry of the spacecraft parachutes is in general very complex due to the added geometric porosity components like gaps, windows or slits. In addition, the parachute fabric is a porous medium which needs to be taken into account in the computational models. Finally, the spacecraft parachutes are usually very light compared to the forces from the flow passing through them. This means that the fluid-structure interaction (FSI) will be significant and the parachute will go through large deformations during its flight, creating a need for well-developed FSI coupling and mesh moving methods. In conclusion, addressing all these computational challenges will require large scale models and advanced techniques, which might increase the cost of the spacecraft parachute computational research.

1.3 Research Motivation

Here we explain how we address each of the computational challenges described in Section 1.2 using advanced techniques developed by the Team for Advanced Flow Simulation and Modeling (T•AFSM) and other advanced modeling techniques such as the non-uniform rational B-splines (NURBS) and T-splines.

1.3.1 Space–time (ST) methods

The Deforming-Spatial-Domain/Stabilized Space–Time (DSD/SST) method [11, 12, 13, 14, 15, 7] together with the special ST methods have been successfully used to address the challenges of spacecraft parachute computational analysis (see [16, 7]
and references therein, and [17, 18, 19, 20, 21, 22, 23, 24, 25]). These methods were first applied to Orion spacecraft parachutes simulations in [16, 7] and later to various aspects of parachute studies including “disreefing” [17, 20], spacecraft cover separation [18], parachute designs with modified geometric porosity [17, 19], gore curvature calculation [21], and aerodynamic-moment calculation [22]. To address the challenges introduced by the geometric porosity, the Homogenized Modeling of Geometric Porosity (HMGP) was introduced in [26, 17] and later used in various studies of the Orion spacecraft parachutes (see [16, 7] and references therein, and [17, 18, 19, 20, 21, 22, 23]).

In the incompressible-flow regime, the core form of DSD/SST method used Streamline-Upwind/Petrov-Galerkin (SUPG) [27] and Pressure-Stabilizing/Petrov-Galerkin (PSPG) [11] as stabilization terms, and later by combining the first two letters of each stabilization term the method was called “ST-SUPS.” The variational multiscale version, ST Variational Multiscale (ST-VMS) method, was introduced in [14, 15]. This method separates the unknowns into fine- and coarse-scale components, and it represents the fine-scale as residual-based. Other ST methods were later developed used in conjunction with ST-VMS, that is ST Slip Interface (ST-SI) [28, 29], ST-TC [30, 31], and ST Isogeometric Analysis (ST-IGA) [14, 32, 33], to address the challenges in the computational analysis.

In the compressible-flow regime, the first 3D computation with the compressible-flow DSD/SST method was presented in [34]. Then, compressible-flow SUPG method was first introduced in [35, 36, 37] and later developed, with addition of new shock-capturing and stabilization terms, into its current version as presented in [24, 25]. ST-SI and HMGP methods were also incorporated into compressible-flow computation in [24] for finite element analysis and later in [25] for IGA.

For handling the large mesh deformation during the spacecraft parachute’s flight, a mesh moving method based on a linear elasticity model was first introduced in [38]
and later presented in [39, 40] with addition of the solid-extension technique offering better control and robustness during the mesh update procedures. More recently, a mesh moving method based on fiber-reinforced hyperelasticity and optimized zero-stress state (ZSS) was introduced in [8] which not only offered unique mesh solution for each step of the mesh deformation but also made improvements to the initial mesh quality possible.

### 1.3.2 NURBS and T-splines

**NURBS**

NURBS is a special form of B-splines that can exactly represent simple geometrical shapes like conic sections through a projective transformation and use of weights. In NURBS representation, similar to the B-splines, the basis functions are constructed from a knot vector, which is a non-decreasing sequence of parameters showing the coordinates of each element in the parametric space. In general, a knot vector is written in the form $\Xi = \{\xi_1, \xi_2, \ldots, \xi_{n+p+1}\}$ where $\xi_i$ is the $i$th knot, $n$ is the number of basis functions (or control points) and $p$ is the polynomial degree. The B-spline basis functions are then calculated using the Cox-de Boor recursion formula [41] as shown in Eq. (1.2).

$$
N_{i,0}(\xi) = \begin{cases} 
1 & \text{if } \xi_i \leq \xi < \xi_{i+1}, \\
0 & \text{otherwise}.
\end{cases}
$$

(1.1)

$$
N_{i,p}(\xi) = \frac{\xi - \xi_i}{\xi_{i+p} - \xi_i} N_{i,p-1}(\xi) + \frac{\xi_{i+p+1} - \xi}{\xi_{i+p+1} - \xi_{i+1}} N_{i+1,p-1}(\xi).
$$

(1.2)

A knot vector is defined for each parametric direction and used throughout the whole domain in NURBS. This means that a NURBS topology requires a regular control grid, meaning regular rectangular elements in the whole domain. An example of a
NURBS mesh is shown in Figure 1.8 in both the physical and the parametric space and its knot vectors for each parametric direction are given as well. The basis functions for this NURBS mesh are shown in Figure 1.9 for both horizontal and vertical parametric directions. NURBS are very efficient in accurately representing the geometry and they allow refinement without changing the geometry. For these features, they are very favorable and recently have been used extensively in the IGA for fluid and structural mechanics. Also, their incorporation into existing finite element
codes is made possible by the use of Bézier extraction operator and the introduction of Bézier decomposition in [42] where the numerical integration is done on $C^0$ (i.e. continuous) Bézier elements instead of $C^1$ or higher order (i.e. smooth, continuous derivatives) B-spline or NURBS elements while preserving the continuity of the original model. The NURBS basis functions $N_i$ are represented as tensor product of the Bézier extraction operator and the Bernstein basis functions $B_i$. We use the algorithm given in [42] and calculate the Bézier extraction operators for each parametric direction of the NURBS mesh shown in Figure 1.8. In the horizontal direction, it is calculated as

\[
\begin{pmatrix}
N_1 \\
N_2 \\
N_3 \\
N_4 \\
N_5 \\
N_6
\end{pmatrix} =
\begin{bmatrix}
1.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 1.0 & 0.5 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.5 & 1.0 & 0.5 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.5 & 1.0 & 0.5 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.5 & 1.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 1.0
\end{bmatrix}
\begin{pmatrix}
B_1 \\
B_2 \\
B_3 \\
B_4 \\
B_5 \\
B_6 \\
B_7 \\
B_8 \\
B_9
\end{pmatrix},
\] (1.3)
and in the vertical direction as

\[
\begin{align*}
\{ N_1 \} & = \begin{bmatrix} 1.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \end{bmatrix} \begin{bmatrix} B_1 \end{bmatrix} \\
\{ N_2 \} & = \begin{bmatrix} 0.0 & 1.0 & 0.5 & 0.0 & 0.0 & 0.0 \end{bmatrix} \begin{bmatrix} B_2 \end{bmatrix} \\
\{ N_3 \} & = \begin{bmatrix} 0.0 & 0.0 & 0.5 & 1.0 & 0.5 & 0.0 \end{bmatrix} \begin{bmatrix} B_3 \end{bmatrix} \\
\{ N_4 \} & = \begin{bmatrix} 0.0 & 0.0 & 0.0 & 0.0 & 0.5 & 1.0 \end{bmatrix} \begin{bmatrix} B_4 \end{bmatrix} \\
\{ N_5 \} & = \begin{bmatrix} 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 1.0 \end{bmatrix} \begin{bmatrix} B_5 \end{bmatrix} \\
\end{align*}
\]

(1.4)

**T-splines**

NURBS are widely used in computer designs and computational analysis. They offer several benefits over the more traditional finite element method; for example, more accurate geometry representation, nonnegative basis functions in higher order representations, easier control of continuity, mesh refinement without changing the geometry and so on. However, they also have some limitations. For example, they only achieve $C^0$ continuity across patch boundaries, which might adversely affect the solution stability and accuracy, and they only allow regular control grid and global refinement, which might greatly increase the computational cost in case of refinement studies. On the other hand, T-splines don’t have these limitations while still having the benefits of NURBS. T-splines were first used in the area of computer graphics design and introduced in [43]. Later, T-splines were, in more generalized forms, applied to the IGA in [44] and an algorithm for calculating the Bézier extraction operator for T-splines were introduced in [45]. Unlike NURBS, T-splines elements are not defined by global knot vectors for each parametric direction, instead local knot vectors are defined for each element. Thus, T-splines don’t require regular elements and a set of control points can end without traversing the whole domain contrary to NURBS. Figure 1.10 shows a T-spline representation of the same geometry from
Figure 1.8.

Figure 1.10: A quadratic T-spline mesh in the physical (left) and parametric space (right). Red spheres indicate the control points. The checkerboard coloring is for differentiating between the elements. The knot vector in the vertical direction is global and defined as \( \{0, 0, 0, 1, 2, 3, 3\} \), but in the horizontal direction it is defined locally.
Since the knot vectors are locally defined in T-splines and T-junctions only occur in the first layer of elements in the horizontal direction of the mesh in Figure 1.10, here we present the shape functions and Bézier extraction operators only for this first layer of elements and only for the horizontal direction. For convenience, we will label the first layer of elements in the horizontal direction as “E1”, and their sets of control points as “CP1”, “CP2” and “CP3” based on their order in the horizontal direction. Figure 1.11 shows the T-spline basis functions in horizontal direction for each set of control points, and Eqs. (1.5) and (1.6) show the Bézier extraction operators for these sets. We use the algorithm given in [45] in calculating the Bézier extraction operator for T-splines.

Figure 1.11: T-spline mesh and basis functions. E1 and its control points (left) and the basis functions in the horizontal direction (right). Basis functions are defined for each set of control points in the horizontal direction. Number of basis functions in a set corresponds to the number of control points in that set. From bottom to top, the basis functions are shown for CP1, CP2 and CP3.
The Bézier extraction operator for CP1, shown in Figure 1.11, is calculated as

\[
\begin{bmatrix}
N_1 \\
N_2 \\
N_3 \\
N_4 \\
N_5 \\
N_6
\end{bmatrix} = \begin{bmatrix}
1.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 1.0 & 0.5 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.5 & 1.0 & 0.5 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 1.0
\end{bmatrix}
\begin{bmatrix}
B_1 \\
B_2 \\
B_3 \\
B_4 \\
B_5 \\
B_6 \\
B_7 \\
B_8 \\
B_9
\end{bmatrix}, \quad (1.5)
\]

and is the same as the Bézier extraction operator in the horizontal direction for NURBS given in Eq. (1.3). On the other hand, for CP2 and CP3 we can see in Figure 1.11 that they both have the same missing knots, which are \(\{1, 3\}\), compared to the knot vector for CP1. So for both of these sets, using the same algorithm given in [45] and inserting the missing knots, the Bézier extraction operator is calculated as

\[
\begin{bmatrix}
N_1 \\
N_2 \\
N_3 \\
N_4
\end{bmatrix} = \begin{bmatrix}
1.0 & 0.5 & 0.25 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.5 & 0.625 & 0.75 & 0.5 & 0.25 & 0.125 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.125 & 0.25 & 0.5 & 0.75 & 0.625 & 0.5 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.25 & 0.5 & 1.0
\end{bmatrix}
\begin{bmatrix}
B_1 \\
B_2 \\
B_3 \\
B_4 \\
B_5 \\
B_6 \\
B_7 \\
B_8 \\
B_9
\end{bmatrix}. \quad (1.6)
\]
1.4 Overview

In this dissertation, we first introduce our target spacecraft parachutes, their operating regimes and conditions, and the importance in understanding their flight behaviors well. We then talk about the challenges involved in spacecraft parachute computational research and describe the advanced techniques we use to address them. We then describe limitations of some of these techniques and explain the new method and special techniques we developed to make enhancements to them. Chapters 2 and 3 explain the governing equations and ST formulations we use. In Chapter 4, we explain the special techniques we developed to reduce the computational cost and enhance the solution accuracy and robustness. We make use of T-splines to address some of the deficiencies of the NURBS models especially the matching and regular elements requirements. We also introduce a method to improve the mesh quality and utilize it mainly in improving our T-spline meshes. In Chapters 5 and 6, these techniques are first applied to a 2D test model for validation purposes, and then they are applied to actual spacecraft parachutes fluid and structural mechanics analysis. We first validate our techniques on FSI computation for a 2D parachute model, and then apply our techniques to the spacecraft parachutes described in Section 1.1 and perform both incompressible-flow analysis with high Reynolds number and compressible-flow analysis with a bow shock as well as structural mechanics analysis. We compare the computational results from the method (ST-IGA with T-splines) and special techniques we introduce in this thesis to the results from more traditional ST-IGA with NURBS and ST-SI methods. We focus on the enhancements made to the results by using our method and techniques. Finally, we present our conclusion and remarks in Chapter 7.
Chapter 2

Governing Equations

For the structural mechanics computations presented in this thesis, we use the governing equations for thin structures (i.e. shell, membrane and cables) as described in [7].

For the fluid mechanics computations, we explain the governing equations of compressible- and incompressible-flow in the following sections.

Let $\Omega_t \subset \mathbb{R}^{n_{sd}}$ be the spatial domain with boundary $\Gamma_t$ at time $t \in (0, T)$, $n_{sd}$ is the number of spatial dimensions and the subscript $t$ indicates the time-dependence of the domain.

2.1 Compressible Flow

This section is mostly from [25]. The Navier–Stokes equations of compressible flows can be written on $\Omega_t$ and $\forall t \in (0, T)$ as

$$\frac{\partial U}{\partial t} + \frac{\partial F_i}{\partial x_i} - \frac{\partial E_i}{\partial x_i} - R = 0, \quad (2.1)$$

where $U = (\rho, \rho u_1, \rho u_2, \rho u_3, \rho e)$ is the vector of conservation variables; $\rho$ is density, $u_i$ are the components of the velocity and $e$ is the total energy per unit volume. Euler
and viscous flux vectors are $F_i$ and $E_i$, respectively. The term $R$ represents all other sources. $F_i$ and $E_i$ are given as

$$F_i = \begin{pmatrix} u_i \rho \\ u_i \rho u_1 + \delta_{i1} P \\ u_i \rho u_2 + \delta_{i2} P \\ u_i \rho u_3 + \delta_{i3} P \\ u_i (\rho e + p) \end{pmatrix}, \quad E_i = \begin{pmatrix} 0 \\ T_{i1} \\ T_{i2} \\ T_{i3} \\ -q_i + T_{ik} u_k \end{pmatrix}. \quad (2.2)$$

Here $\delta_{ij}$ are the components of $I$, $q_i$ are the components of the heat flux vector, and $T_{ij}$ are the components of $T$ which is the Newtonian viscous tensor. Eq. (2.1) can further be written in the form

$$\frac{\partial \mathbf{U}}{\partial t} + \mathbf{A}_i \frac{\partial \mathbf{U}}{\partial x_i} - \frac{\partial}{\partial x_j} \left( \mathbf{K}_{ij} \frac{\partial \mathbf{U}}{\partial x_j} \right) - \mathbf{R} = \mathbf{0}, \quad (2.3)$$

where

$$\mathbf{A}_i = \frac{\partial F_i}{\partial \mathbf{U}}, \quad \mathbf{K}_{ij} \frac{\partial \mathbf{U}}{\partial x_j} = \mathbf{E}_i. \quad (2.4)$$

The essential and natural boundary conditions for Eq. (2.1) are represented as $\mathbf{U} = \mathbf{G}$ on $(\Gamma)_G$ and $n_i \left( \mathbf{K}_{ij} \frac{\partial \mathbf{U}}{\partial x_j} \right) = \mathbf{H}$ on $(\Gamma)_H$. In general, the ideal gas assumption is used together with Eq. (2.1) as the equation of state.
2.2 Incompressible Flow

This section is mostly from [46]. The Navier–Stokes equations of incompressible flows can be written on $\Omega_t$ and $\forall t \in (0, T)$ as

$$
\rho \left( \frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} - \mathbf{f} \right) - \nabla \cdot \sigma = 0,
$$

(2.5)

$$
\nabla \cdot \mathbf{u} = 0,
$$

(2.6)

where $\rho$, $\mathbf{u}$, $p$, $\mathbf{f}$ and $\sigma$ are the density, velocity, pressure, external forces and stress tensor, respectively. $\sigma$ is defined as

$$
\sigma(\mathbf{u}, p) = -p\mathbf{I} + 2\mu \varepsilon(\mathbf{u}),
$$

(2.7)

where $\mathbf{I}$ is the identity tensor, $\mu$ is the absolute viscosity, and $\varepsilon(\mathbf{u})$ is given as

$$
\varepsilon(\mathbf{u}) = \frac{1}{2} \left( (\nabla \mathbf{u}) + (\nabla \mathbf{u})^T \right).
$$

(2.8)

The essential and natural boundary conditions are $\mathbf{u} = \mathbf{g}$ on $(\Gamma_t)_g$ and $\mathbf{n} \cdot \sigma = \mathbf{h}$ on $(\Gamma_t)_h$, where $(\Gamma_t)_g$ and $(\Gamma_t)_h$ are complementary subsets of the boundary $\Gamma_t$, $\mathbf{n}$ is the unit normal vector, and $\mathbf{g}$ and $\mathbf{h}$ are given functions.
Chapter 3

ST Discretization

This chapter is partly from [7]. An abstract representation of ST concept is shown in Figure 3.1. Here the formulations are written over a sequence \( n = \{0, 1, \ldots, N\} \) ST slabs \( Q_n \), where \( Q_n \) is the slice of the ST domain between the time levels \( t_n \) and \( t_{n+1} \). The essential and natural boundary conditions are enforced on the complementary subsets of lateral boundary \( P_n \). At each time step \( n \), the integrations are performed over \( Q_n \) using ST interpolation functions, which are continuous within a ST slab but discontinuous across two ST slabs. The superscripts “−” and “+” indicate the values just below and just above the time level. Each \( Q_n \) is decomposed into elements \( Q^e_n \), where \( e = 1, 2, \ldots, (n_{el})_n \).

Figure 3.1: Abstract representation of a ST slab [7].
3.1 ST-VMS

This section is mostly from [28], with the method originating from [14]. The convective form of ST-VMS method is given as

\[
\int_{Q_n} \mathbf{w}^h \cdot \rho \left( \frac{\partial \mathbf{u}^h}{\partial t} + \mathbf{u}^h \cdot \nabla \mathbf{u}^h - \mathbf{f}^h \right) \, dQ
+ \int_{Q_n} \varepsilon(w^h) : \sigma(u^h, p^h) \, dQ - \int_{(P_n)_h} \mathbf{w}^h \cdot h^h \, dP
+ \int_{Q_n} q^h \nabla \cdot \mathbf{u}^h \, dQ + \int_{\Omega_n} (w^h)^+ \cdot \rho \left( \left( u^h \right)^+ - \left( u^h \right)^- \right) \, d\Omega
+ \sum_{e=1}^{(n_e)_n} \int_{Q_n^e} \tau_{\text{SUPS}} \rho \left( \rho \left( \frac{\partial \mathbf{w}^h}{\partial t} + \mathbf{u}^h \cdot \nabla \mathbf{w}^h \right) + \nabla q^h \right) \cdot \mathbf{r}_M(u^h, p^h) \, dQ
+ \sum_{e=1}^{(n_e)_n} \int_{Q_n^e} \nu_{\text{LSIC}} \nabla \cdot \mathbf{w}^h \rho r_C(u^h) \, dQ
- \sum_{e=1}^{(n_e)_n} \int_{Q_n^e} \tau_{\text{SUPS}} \mathbf{w}^h \cdot \left( \mathbf{r}_M(u^h, p^h) \cdot \nabla \mathbf{u}^h \right) \, dQ
- \sum_{e=1}^{(n_e)_n} \int_{Q_n^e} \tau_{\text{SUPS}}^2 \rho \mathbf{r}_M(u^h, p^h) \cdot \left( \nabla \mathbf{w}^h \right) \cdot \mathbf{r}_M(u^h, p^h) \, dQ
= 0, \tag{3.1}
\]

where

\[
\mathbf{r}_M(u^h, p^h) = \rho \left( \frac{\partial \mathbf{u}^h}{\partial t} + \mathbf{u}^h \cdot \nabla \mathbf{u}^h - \mathbf{f}^h \right) - \nabla \cdot \sigma(u^h, p^h), \tag{3.2}
\]

\[
r_C(u^h) = \nabla \cdot \mathbf{u}^h \tag{3.3}
\]

are the residuals of the momentum equation and incompressibility constraint. The velocity and pressure test functions are \( \mathbf{w} \) and \( q \), respectively. The superscript “\( h \)” indicates that the function is “coarse-scale” or “grid-scale”.

There are various ways of defining the stabilization parameters \( \tau_{\text{SUPS}} \) and \( \nu_{\text{LSIC}} \) (see [12, 13, 47, 48, 28]), and the stabilization parameters definitions used in this
thesis is from [49]. For more ways of calculating the stabilization parameters in finite element computation of flow problems, see [50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71].

The ST-SI method was introduced in [28], where the interface terms were added to the ST-VMS method to accurately connect the two sides of the SI, especially in the simulations with spinning solid surfaces. The ST-SI version for modeling porous medium was also included in [28].

3.2 Compressible-Flow ST SUPG

This section is mostly from [24], with the method originating from [34, 55]. With suitably-defined finite-dimensional trial solution \((S^h)\) and test function \((V^h)\), the compressible-flow DSD/SST formulation [72, 73, 74, 52, 53, 54, 55, 71] of Eq. (2.1) can be written as

given \((U^h)^-\), find \(U^h \in (S^h)\), such that \(\forall W^h \in (V^h)\):

\[
\begin{align*}
\int_{Q_n} W^h \cdot \left( \frac{\partial U^h}{\partial t} + A^h_i \frac{\partial U^h}{\partial x_i} - R^h \right) dQ \\
+ \int_{Q_n} \frac{\partial W^h}{\partial x_i} \cdot K^h_{ij} \frac{\partial U^h}{\partial x_j} dQ - \int_{\Omega_n} W^h \cdot H^h dP \\
+ \int_{\Omega_n} (W^h)^+ \cdot ((U^h)^- - (U^h)^-) d\Omega \\
+ \sum_{e=1}^{(n_e)_n} \int_{Q^e_n} \tau^{\text{SUPG}} \left( \frac{\partial W^h}{\partial t} + \frac{\partial W^h}{\partial x_k} A^h_k \right) \cdot R_A(U^h) dQ \\
+ \sum_{e=1}^{(n_e)_n} \int_{Q^e_n} \nu^{\text{SHOC}} \frac{\partial W^h}{\partial x_i} \cdot \frac{\partial U^h}{\partial x_i} dQ = 0,
\end{align*}
\]  

(3.4)

where

\[
R_A(U^h) = \frac{\partial U^h}{\partial t} + A^h_i \frac{\partial U^h}{\partial x_i} - \frac{\partial}{\partial x_i} \left( K^h_{ij} \frac{\partial U^h}{\partial x_j} \right) - R^h,
\]  

(3.5)
\( \tau_{\text{SUPG}} \) is the SUPG stabilization matrix, and \( \nu_{\text{SHOC}} \) is the shock-capturing parameter. The definitions for \( \tau_{\text{SUPG}} \) and \( \nu_{\text{SHOC}} \) are given in [25]. Both the stabilization and the shock-capturing terms are expressed as based on the residual of the compressible-flow equations.

The compressible-flow ST-SI formulations are given in [25].
Chapter 4

Special Techniques

In this chapter, we introduce three new special techniques to use in conjunction with ST-IGA with T-splines method. First two of these techniques are developed to make enhancements to the ST-IGA with NURBS in terms of solution accuracy, robustness and computational cost. These techniques address the deficiencies of NURBS, specifically the regular rectangular grid requirement, by utilizing T-splines which allow local refinement and coarsening. In the last special technique introduced in this chapter, we present a low-distortion mesh-moving method based on fiber-reinforced hyperelasticity [8], which can also be used for improving the initial mesh quality and orthogonality. We call this process “mesh relaxation” as it was done in [8].

4.1 Reducing the Number of Control Points by Knot Removal

The matching and regular elements requirements of the block-structured NURBS might cause bad aspect ratio elements or unnecessarily high mesh resolution especially in a computational domain that is irregular and narrower in some parts than the rest (e.g. the shorter edge of trapezoids or smaller radius of a hollow cylinder).
Also, the matching requirement limits the connection between two NURBS patches to only the patches with the same number of elements. In addition, due to the regular elements requirement, only the global refinement is allowed in a NURBS patch and, together with the matching requirement, they require all the subsequent NURBS patches that are connected to it to be refined in the same manner as well. This issue not only causes extra computational cost but also might adversely affect the solution stability due to causing bad aspect ratio regions in the domain.

This issue has previously been addressed by the use of ST-SI together with NURBS. In this method, a nonmatching interface is placed in the computational domain as needed or desired for the mesh-generation convenience (see [28]). ST-IGA with NURBS together with ST-SI offered great flexibility in mesh generation and reduced computational cost by removing the matching requirement between NURBS patches. However, even with the addition of ST-SI, the regular elements of a NURBS patch could not be avoided. Also, the domain has become discontinuous across the interface possibly adversely effecting the solution stability and accuracy. An example of this issue where bad aspect ratio elements occur in narrower regions of the domain due to the regular elements requirement of NURBS and how NURBS with ST-SI method can be used to address this issue is shown in Figure 4.1 for a geometry that is quarter of a hollow cylinder.
Figure 4.1: A quadratic NURBS (left) and NURBS with SI (right) meshes for a quarter of a hollow cylinder domain. Bad aspect ratio elements appear near the inner circular edge of NURBS representation, while better aspect ratios and reduced number of control points achieved in NURBS with SI. SI is located between the two NURBS patches. Spheres indicate control points and checkerboard coloring is for differentiating between elements.
Here we present a new way to address this issue by use of T-splines and knot removal technique. Firstly, we show a T-spline representation for the same geometry given in Figure 4.1 and also show its representation in the parametric space in Figure 4.2. We then start from the NURBS representation in the parametric space and decide what knot values to be removed in order to reach our target T-spline representation. Figures 4.3–4.6 shows the sets of elements in the horizontal direction and their desired local knot vectors with red color indicating the knot values to be removed. We calculate the new control point locations after the knot removals in the parametric space and calculate the Bezier extraction operator for T-splines using the algorithm given in [45] as we described in Chapter 1. Finally, we project the physical coordinates data from NURBS to T-spline representation in the parametric space and then map the T-spline representation to the physical space, obtaining the desired final T-spline mesh.

Figure 4.2: T-spline mesh in the physical (left) and parametric (right) space for the same geometry given in Figure 4.1. Spheres indicate control points and checkerboard coloring is for differentiating between elements.

In conclusion, using this special technique we introduced, we achieve better aspect ratio elements in certain regions of the computational domain and decrease the total number of control points used. Compared to the ST-IGA with NURBS and ST-SI methods, ST-IGA with T-splines and reducing the number of control points by
Figure 4.3: First layer of elements in the horizontal direction for NURBS (left) and T-spline (right) meshes in the parametric space. Highlighted elements indicated the layer of elements. Red spheres indicate the control points in the support of these elements. Local knot vectors for each set of these control points (bottom-to-top) with red color indicating knots to be removed are as follows: \{0, 0, 0, 1, 2, 3, 4, 5, 6, 7, 8, 8, 8\}, \{0, 0, 1, 2, 3, 4, 5, 6, 7, 8, 8, 8\} and \{0, 0, 1, 2, 3, 4, 5, 6, 7, 8, 8, 8\}.

knot removal technique enables better solution stability and accuracy thanks to better mesh quality and also reduces computational cost thanks to the decreased total number of control points.
Figure 4.4: Second layer of elements in the horizontal direction for NURBS (left) and T-spline (right) meshes in the parametric space. Highlighted elements indicated the layer of elements. Red spheres indicate the control points in the support of these elements. Local knot vectors for each set of these control points (bottom-to-top) with red color indicating knots to be removed are as follows: \{0, 0, 0, 1, 2, 3, 4, 5, 6, 7, 8, 8, 8\}, \{0, 0, 0, 1, 2, 3, 4, 5, 6, 7, 8, 8, 8\} and \{0, 0, 0, 1, 2, 3, 4, 5, 6, 7, 8, 8, 8\}.

Figure 4.5: Third layer of elements in the horizontal direction for NURBS (left) and T-spline (right) meshes in the parametric space. Highlighted elements indicated the layer of elements. Red spheres indicate the control points in the support of these elements. Local knot vectors for each set of these control points (bottom-to-top) with red color indicating knots to be removed are as follows: \{0, 0, 0, 1, 2, 3, 4, 5, 6, 7, 8, 8, 8\}, \{0, 0, 0, 1, 2, 3, 4, 5, 6, 7, 8, 8, 8\} and \{0, 0, 0, 1, 2, 3, 4, 5, 6, 7, 8, 8, 8\}.
Figure 4.6: Fourth layer of elements in the horizontal direction for NURBS (left) and T-spline (right) meshes in the parametric space. Highlighted elements indicated the layer of elements. Red spheres indicate the control points in the support of these elements. Local knot vectors for each set of these control points (bottom-to-top) with red color indicating knots to be removed are as follows: \{0, 0, 0, 1, 2, 3, 4, 5, 6, 7, 8, 8, 8\}, \{0, 0, 0, 1, 2, 3, 4, 5, 6, 7, 8, 8, 8\} and \{0, 0, 0, 1, 2, 3, 4, 5, 6, 7, 8, 8, 8\}. 
4.2 Giving Continuity to a Slip Interface (SI)

As mentioned in the previous section Section 4.1, ST-IGA with NURBS and ST-SI have been used together in many studies to remove the matching requirement of the block-structured NURBS meshes and make the mesh generation more convenient. This method also helped reducing the total number of control points, reducing the computational cost. An SI is located between two, or more, NURBS patches, and the connection across this interface is only $C^{-1}$ (i.e. discontinuous), even lower than $C^0$ connection between two matching NURBS patches. This might affect the solution stability and accuracy adversely in some cases especially if the mesh resolution between the two sides of the SI is not similar and if the SI is located very close to a boundary layer. Figure 4.7 shows, in the parametric space, two nonmatching NURBS patches and an SI in between them as well as the basis functions in the horizontal direction for each NURBS patches. Here we introduce a special technique to remove

![Figure 4.7: Two NURBS patches with an SI in between (left) and the horizontal basis functions (right) for the above (top) and the below (bottom) patches. Colors represent separate NURBS patches, and checkerboard coloring is for differentiating between elements. Spheres indicate the control points.](image)
the SI from the computational domain and give $C^1$ or higher continuity to the interface by using T-splines and knot insertion technique. As we can see in Figure 4.7, the knot vectors in the horizontal direction for the above patch can be written as \{0, 0, 0, 4, 8, 12, 12, 12\} and for the below patch as \{0, 0, 0, 3, 6, 9, 12, 12, 12\}. We first start by inserting the missing knots to the knot vectors in each side of the interface using the algorithm given in [45] and as we described in Chapter 1, and when the matching sets of knot vectors obtained in each side, we use knot removal and connect the patches continuously in the vertical direction. For better explanation of the process, we first show the original NURBS and the desired final T-spline meshes in Figure 4.8, and then show more detailed explanations for the layers of elements in each side of the interface in Figures 4.9 and 4.10.

Figure 4.8: A square domain as represented by two NURBS patches with an SI in between (left) and by a T-spline (right) mesh. Colors represent separate NURBS patches, and checkerboard coloring is for differentiating between elements. Spheres indicate the control points.
Figure 4.9: Above layer of elements at the interface for NURBS (left) and T-spline (right) meshes. Highlighted elements indicated the layer of elements. Highlighted spheres indicate the control points in the support of these elements. Local knot vectors for each set of these control points (bottom-to-top) with blue color indicating knots to be inserted are as follows: \( \{0, 0, 3, 4, 6, 8, 9, 12, 12\} \), \( \{0, 0, 3, 4, 6, 8, 9, 12, 12\} \), and \( \{0, 0, 3, 4, 6, 8, 9, 12, 12\} \).

Figure 4.10: Below layer of elements at the interface for NURBS (left) and T-spline (right) meshes. Highlighted elements indicated the layer of elements. Highlighted spheres indicate the control points in the support of these elements. Local knot vectors for each set of these control points (bottom-to-top) with blue color indicating knots to be inserted are as follows: \( \{0, 0, 3, 4, 6, 8, 9, 12, 12\} \), \( \{0, 0, 3, 4, 6, 8, 9, 12, 12\} \), and \( \{0, 0, 3, 4, 6, 8, 9, 12, 12\} \).
4.3 Mesh Moving Method and Mesh Relaxation

This section is mostly from [8]. Due to the nature of block-structured mesh generation process or the complex topology of the computational domain, it is not always that we have an initial mesh that is in equilibrium with its optimized zero-stress state (ZSS), meaning some of the elements are not close to orthogonality as possible. For such cases, to improve the mesh quality and reach an equilibrium with its optimized ZSS, a mesh relaxation method was also introduced in [8]. The process first starts with determining a metric tensor for the ZSS, and then creating a modified ZSS by removing the off-diagonal terms as described in [8]. Fibers, reinforcing each parametric direction for optimal orthogonality, are generated from this modified ZSS and added to the neo-Hookean constitutive model. Then, a steady-state structural mechanics computation is performed by gradually changing the ZSS to its modified form till the steady-state solution is obtained. An example of a rectangular domain in its initial state and after mesh relaxation process is shown in Figure 4.11. The overall mesh orthogonality given as the deviation from $\pi/2$ for the initial mesh is 8.8° and for after the mesh relaxation is 2.3°. We see that the overall element orthogonality in the domain is improved thanks to the mesh relaxation method.
Figure 4.11: A rectangular domain in its initial state (top) and after mesh relaxation (bottom). This figure is from [8].
After obtaining an equilibrium with the optimized ZSS by performing mesh relaxation, the application of the fiber-reinforced hyperelasticity model to the mesh update process is also shown in [8] for various types of deformations. In Figure 4.12, we can see the same rectangular domain from Figure 4.11 going through expansion and compression. We use optimized ZSS and fibers to strengthen the elements against losing their orthogonality throughout whole duration of the deformation. The overall mesh orthogonality given as the deviation from $\pi/2$ at the maximum compression is $8.6^\circ$ and at the maximum expansion is $1.6^\circ$. These values are very reasonable for such large deformation validating the effectiveness of fiber-reinforced hyperelasticity model in problems with large mesh deformations.
Figure 4.12: A rectangular domain at its maximum compression (top) and maximum expansion (bottom). This figure is from [8].
Chapter 5

2D Test Problem

In this chapter, the method and special techniques described in the previous chapter are applied to a 2D test problem. Firstly, to obtain a reference solution, FSI computation for a 2D parachute model using ST-IGA with NURBS and ST-SI are performed. Then, using the same parachute model and computational parameters, FSI computation using ST-IGA with T-splines are performed and the results obtained are compared to the reference solution for validation of the method and special techniques explained in this thesis.

5.1 Problem Setup

A parachute-like 2D model is prepared with an unstressed half-circle “canopy” shape and “suspension lines” similar to the one in [40]. In its initial configuration, the canopy section has a diameter of 1.0 m, and the suspension lines have a length of 0.90 m. The material properties for both the canopy and suspension lines are also the same as the ones described in [40] and here given in Table 5.1. However, to increase geometric complexity and to better resemble an actual spacecraft parachute’s geometric porosity, some parts of the canopy section are removed from the computational domain, and “vent,” “gaps” and “windows” are created as shown in Figure 5.1.
To obtain an inflated shape for FSI computation, a structure-alone computation is performed by applying a uniform pressure difference of 0.5 N/m² to the canopy section and keeping the “payload” fixed. We use bending-stabilized cable model [75] for both the canopy and suspension lines. To obtain a stable equilibrium for the inflated shape, a mass-proportional damping is used in this computation. The initial, unstressed configuration and the inflated shape from the structure-alone computation can be seen in Figure 5.2. The number of cubic NURBS elements used to represent the canopy is 90 and the suspension lines is 25, and the number of total control points is 141. Both the NURBS and T-spline fluid mechanics meshes for FSI computation are generated around the inflated shape obtained here. The initial flow distribution

Table 5.1: Material properties for the 2D parachute model. Density ($\rho$), the cross-sectional area (A) and Young’s modulus (E), with unit length in the third dimension.

<table>
<thead>
<tr>
<th></th>
<th>$\rho$ (kg/m³)</th>
<th>A (m²)</th>
<th>E (Pa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>canopy</td>
<td>$2.5 \times 10^4$</td>
<td>$2 \times 10^{-4}$</td>
<td>$2.5 \times 10^6$</td>
</tr>
<tr>
<td>suspension lines</td>
<td>$5.6 \times 10^2$</td>
<td>$2 \times 10^{-4}$</td>
<td>$5.0 \times 10^6$</td>
</tr>
</tbody>
</table>

Figure 5.1: Problem setup for FSI computation of a 2D parachute model.
for FSI is obtained by first performing fluid-alone simulations with both NURBS and T-splines static meshes. For these stand-alone fluid dynamics computations and for the subsequent FSI, the free-stream velocity is $0.35 \text{ m/s}$, the Reynolds number is 1,000, and the fluid density is $1.0 \text{ kg/m}^3$ as in [40].

![Figure 5.2: Unstressed (left) and inflated (right) configurations for the 2D parachute model. Purple and gray represent the “canopy” and the “suspension lines.”](image)

### 5.2 NURBS and T-Spline Fluid Mechanics Meshes

First, the block-structured NURBS mesh is generated around the inflated shape by extending three layers of quadratic NURBS elements outward from the upper and lower surfaces of the canopy. Then, an SI is used for mesh-generation convenience and the matching requirement of block-structured NURBS is avoided for the rest of the fluid mechanics volume mesh. The rest of the volume mesh is also generated using quadratic NURBS elements. T-spline mesh is then generated directly from the NURBS mesh by giving continuity to the SI using the special techniques described in Chapter 4, removing the SI and increasing the overall continuity in the fluid mechanics
volume mesh. The whole computational domain for the NURBS and T-spline fluid mechanics meshes and zoomed-views to the region near the canopy can be seen in Figure 5.3. The total number of elements and the control points for the NURBS and T-spline meshes are shown in Table 5.2. We make note here that even though the number of elements are increased in the T-spline mesh, the number of total control points are decreased, reducing the computational cost. Figure 5.4 shows the NURBS mesh and T-spline mesh after giving continuity to the SI with zoomed-views to the canopy and the vent regions.

Table 5.2: NURBS and T-spline fluid mechanics meshes for 2D parachute model. Number of control points ($nc$) and elements ($ne$).

<table>
<thead>
<tr>
<th></th>
<th>$nc$</th>
<th>$ne$</th>
</tr>
</thead>
<tbody>
<tr>
<td>NURBS</td>
<td>3,737</td>
<td>2,720</td>
</tr>
<tr>
<td>T-splines</td>
<td>3,090</td>
<td>2,846</td>
</tr>
</tbody>
</table>
Figure 5.3: NURBS (left) and T-spline (right) fluid mechanics meshes for the 2D parachute model. Full views (top) and zoomed views (bottom) near the “canopy” section. Purple and green represent the “canopy” and the SI.
Figure 5.4: NURBS (left) and T-spline meshes after giving continuity to the SI (right) with zoomed views of the “canopy” (top) and “vent” (bottom). Purple and green represent the “canopy” and the SI.
A mesh relaxation, as also described in Chapter 4, is performed on the T-spline volume mesh to improve element orthogonality especially near the region where the SI was removed and elements were connected continuously. The material properties for the mesh relaxation are given in Table 5.3 in nondimensional units. To protect the boundary layer mesh against too much deformation, we use the material properties “M1” for the first two layers of elements from the upper and lower surfaces of the canopy and use “M2” for the rest of the mesh. Figure 5.5 shows the T-spline volume mesh before and after mesh relaxation and zoomed-views to the canopy and the vent sections.

Table 5.3: Material properties for the mesh relaxation of the 2D parachute model T-spline mesh. Bulk modulus ($\kappa_B$) and Shear modulus ($\mu$). $\beta_B$, $C_1$ and $C_2$ are model parameters and coefficients as described in [8].

<table>
<thead>
<tr>
<th></th>
<th>$\kappa_B$</th>
<th>$\beta_B$</th>
<th>$\mu$</th>
<th>$C_1$</th>
<th>$C_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1</td>
<td>1×10^{-1}</td>
<td>4</td>
<td>2×10^{-1}</td>
<td>1.0</td>
<td>1×10^{3}</td>
</tr>
<tr>
<td>M2</td>
<td>1×10^{-3}</td>
<td>4</td>
<td>2×10^{-3}</td>
<td>1.0</td>
<td>1×10^{3}</td>
</tr>
</tbody>
</table>
Figure 5.5: T-spline mesh before (left) and after (right) mesh relaxation. Full views (top), and zoomed views of the “canopy” (middle) and “vent” (bottom) sections. Purple represents the “canopy.”
5.3 Computational Settings for FSI

The fluid-alone computations are carried out for 2,500 time steps with a time step size of 0.03 s, and then FSI computation is carried out for 1,000 time steps with a time step size of 0.01 s. Both fluid-alone and FSI computations have 3 nonlinear iterations and the number of GMRES [76] iterations per nonlinear iteration is 300. We use ST-VMS method with stabilization parameters and element length definitions described in [49] for both fluid-alone and FSI computation to accurately represent the turbulent behavior of the flow. For the structural mechanics part, mass-proportional damping is not used during FSI.

5.4 Results and Discussion

Drag and moment aerodynamic coefficients from ST-IGA with NURBS and T-splines results are given in Figure 5.6. Equations used for calculating these aerodynamics coefficients are shown in Eq. (5.1). We use the per-unit-length values of the drag force \( F_D \) and the moment about the payload \( M_P \) together with the free-stream values of the density \( \rho_\infty \) and the velocity \( u_\infty \). We select the reference length \( D_0 \) as the undeformed shape’s diameter and calculate the reference area \( S_0 \) by multiplying this diameter with the unit length.

\[
C_D = \frac{F_D}{\frac{1}{2} \rho_\infty ||u_\infty||^2 S_0}, \tag{5.1}
\]

\[
C_M = \frac{M_P}{\frac{1}{2} \rho_\infty ||u_\infty||^2 S_0 D_0}. \tag{5.2}
\]

Figures 5.7 and 5.8 show the velocity magnitude and pressure results from uniform time intervals of FSI computation with NURBS and T-spline meshes. As FSI computation progresses, the resulting parachute shape and the flow field differ between NURBS and T-spline cases due to the slight differences in their initial flow conditions.
Figure 5.6: Drag (left) and moment (right) coefficients from the NURBS and T-spline results. *Blue* highlighted regions indicate the FSI computation.

However, the overall flow distribution and the aerodynamics coefficients from both NURBS and T-spline results look reasonable and they are in good agreement with each other, validating the ST-IGA with T-splines method for computational problems with moving boundaries and FSI.
Figure 5.7: Velocity magnitude (m/s) from NURBS (left) and T-spline (right) meshes. The results from 75 to 85 s are shown at 2.5 s intervals. Purple represents the “canopy.”
Figure 5.8: Pressure (Pa) from NURBS (left) and T-spline (right) meshes. The results from 75 to 85 s are shown at 2.5 s intervals. Purple represents the “canopy.”
Chapter 6

Spacecraft Parachute Computation

Spacecraft parachutes are used in various altitudes and flow conditions as well as different atmospheres like the ones used in Mars exploration missions. They also vary in size and geometric complexity substantially. Among the spacecraft parachutes described in Chapter 1, the MP parachute operates in incompressible-flow regime and has a very complex canopy shape with added geometric porosity. On the other hand, the DGB parachute operates in, computationally more challenging, supersonic compressible-flow regime but has a relatively simpler canopy shape. In this chapter, we apply the ST-IGA with T-splines method and the special techniques introduced in Chapter 4 to these spacecraft parachutes and perform both incompressible- and compressible-flow analysis as well as structural mechanics analysis. We compare results from IGA with NURBS and T-splines to show the effectiveness of our method and special techniques.
6.1 Incompressible-Flow Computation of the MP Parachute

We start with the MP parachute incompressible-flow simulation. We generate a NURBS representation of the undeformed shape of the MP parachute directly from the previous finite element mesh shown in [22]. We obtain the inflated parachute shape from structure-alone computation using IGA with NURBS. Then, we perform MP parachute 5-gore flow computation with rotational periodicity using ST-IGA with NURBS, and use HMGP-FG technique to calculate fabric and geometric porosity coefficients and obtain the MP canopy shape with HMGP. The details of the MP structure-alone and 5-gore flow computation with rotational periodicity are given in Appendix A. Using this parachute with HMGP, we perform full-canopy fluid mechanics computation first using ST-IGA with NURBS and consider this as our reference solution. Then, we apply the special techniques and give continuity to all SIs in the computational domain and perform simulations using ST-IGA with T-splines. We compare the results from both cases and present the enhancements to the NURBS solutions made possible by the special techniques we introduced and T-splines.

6.1.1 Problem setup

The MP parachute has a nominal diameter about 35 m and the descent speed is about 7.8 m/s as it was used in [22]. We use the air properties at sea level, so the density is 1.225 kg/m$^3$ and the kinematic viscosity is $1.45 \times 10^{-5}$ m$^2$/s. The Reynolds number is about 18 million. The flow at this speed and conditions is considered incompressible as the Mach number is below 0.3, at about 0.18. The surface with HMGP is porous and we use the fabric and geometric porosity coefficients obtained from Appendix A. We perform full-canopy fluid mechanics computation using these flow properties for both ST-IGA with NURBS and T-splines.
6.1.2 NURBS and T-spline meshes

The computational domain is a cylinder with a diameter of 283 m and a height of 442 m. Similar to the 2D test problem in Chapter 5, we first generate block-structured NURBS mesh and then generate the T-spline mesh directly from the NURBS mesh by giving continuity to the SIs. In generating the NURBS mesh, we start by extending outward three quadratic NURBS elements from both lower and upper sides of the surface mesh of the canopy with HMGP. After that, we use SI to avoid the matching requirement of the NURBS for the rest of the computational domain. We also use a second SI to avoid the matching requirement in the center region of the cylindrical domain where the elements might have become otherwise very thin and small resulting in large aspect ratios. The locations of the parachute and the SIs inside the computational domain are shown in Figure 6.1. To generate the T-spline mesh, we first give continuity to both SIs improving continuity in the volume mesh. We also improve continuity on the parachute canopy by connecting the NURBS patches that

Figure 6.1: Positions of the MP parachute with HMGP (left) and the two SIs (right) in the cylindrical computational domain. Purple shows the SI around the canopy and green shows the SI around the center region of the domain.
were prepared separate due to matching requirement of the block-structured NURBS mesh. Then, we perform mesh relaxation on the T-spline mesh to improve mesh orthogonality. The materials properties and approach used in the mesh relaxation computation here are similar to the one described in Chapter 5. The whole meshes and zoomed-views to the parachute canopy are shown in Figure 6.2 for both NURBS and T-splines. The number of elements and the control points for each mesh are given in Table 6.1. In Figures 6.3 and 6.4, we include close-ups views to a gap and skirt region to show how the continuity were given to the SIs in the T-spline mesh and how the mesh was changed after the mesh relaxation. And, Figure 6.5 shows the block-structured NURBS patches on the parachute surface and how they were connected continuously, where possible, in the T-spline mesh.

Table 6.1: NURBS and T-spline fluid mechanics meshes for the MP parachute. Number of control points and elements.

<table>
<thead>
<tr>
<th></th>
<th>nc</th>
<th>ne</th>
</tr>
</thead>
<tbody>
<tr>
<td>NURBS</td>
<td>552,544</td>
<td>378,408</td>
</tr>
<tr>
<td>T-splines</td>
<td>422,872</td>
<td>411,584</td>
</tr>
</tbody>
</table>

Figure 6.2: NURBS (left) and T-spline (right) fluid mechanics meshes for the MP parachute. Full views (top) and zoomed views (bottom) near the canopy section. Orange indicates the surface with HMGP, and green indicates both SIs in the NURBS mesh.
Figure 6.3: Giving continuity to the SI near a gap for the MP parachute. NURBS mesh (top), T-spline mesh before (middle) and after (bottom) mesh relaxation. Purple and green represent the canopy and the SIs.
Figure 6.4: Giving continuity to the SI near the skirt for the MP parachute. NURBS mesh (top), T-spline mesh before (middle) and after (bottom) mesh relaxation. Purple and green represent the canopy and the SIs.
Figure 6.5: MP parachute canopy with HMGP NURBS (left) and T-spline (right) meshes. The colors are for differentiating between the patches, and the checkerboard coloring is for differentiating between the elements.
6.1.3 Computational settings

The computations using both NURBS and T-spline meshes are carried out for about 30 s using a time step size of 0.023 s. For both computations, the number of nonlinear iterations is 3 and the number of GMRES [76] iterations per nonlinear iteration is 300. For more accurate representation of the turbulent nature of the flow, we use ST-VMS [14] method with stabilization parameters and element length definitions described in [49].

6.1.4 Results and discussion

Drag and moment aerodynamic coefficients for the NURBS and T-spline meshes are given in Figure 5.6 after the flow field settles. The equations used for calculating the drag and moment coefficients are the same as Eq. (5.1), and we again use the free-stream values of the velocity and the density. However, for the reference length and area, we use the nominal diameter $D_0$ and the nominal area $S_0$ of the MP parachute. The nominal area is calculated directly from the nominal diameter using $S_0 = \frac{\pi D_0^2}{4}$. In Figure 6.7, the velocity magnitude from uniform intervals of fluid

![Figure 6.6: Drag (left) and moment (right) coefficients for the MP parachute from the ST-IGA with NURBS and T-splines.](image)

mechanics computation with NURBS and T-spline meshes are shown for the flow
field near the canopy. The results shown in these figures represent a global flow distribution, and we can see that they are similar between the ST-IGA with NURBS and T-splines. We can also see in the aerodynamic coefficient plots a close agreement between the results from ST-IGA with NURBS and T-splines. In Figure 6.8, the same results are shown with a more zoomed-view near the vent. In these more localized representations of the velocity magnitude, we can see a smoother solution in the ST-IGA with T-splines results especially at the regions where the SIs were removed and continuity in the domain were established. Given this region’s close proximity to the parachute canopy, this indicates a better solution accuracy and stability in the results from ST-IGA with T-splines compared to the results with NURBS. Together with its reduced computational cost due to the decreased number of control points, ST-IGA with T-splines has shown great enhancements to more traditional ST-IGA with NURBS and ST-SI method in problems with very complex geometry and high Reynolds number.
Figure 6.7: Velocity magnitude (m/s) near the canopy from ST-IGA with NURBS (left) and T-splines (right). The results from 25 to 30 s are shown at 1.25 s intervals.
Figure 6.8: Velocity magnitude (m/s) results near the vent from ST-IGA with NURBS (left) and T-splines (right). The results from 25 to 30 s are shown at 1.25 s intervals.
6.2 Structure Computation of a Subscale DGB Parachute

Here we perform structural mechanics analysis for a subscale DGB parachute using both IGA with NURBS and T-splines. The aim for the computation is to obtain an inflated parachute structure shape from the initial configuration, which is referred to as undeformed or unstressed state. The inflated parachute canopy shape from this computation will be used later for performing fluid mechanics computation. In the computation with NURBS, we represent both the parachute canopy and the cables with cubic NURBS basis functions. In T-spline computation, we establish continuity between the parachute canopy and the cables on the canopy, and we also present the option to represent these cables as one dimensional cables or two dimensional tapes.

We finally compare the resulting inflated parachute structures and gore formations on the canopy from IGA with NURBS and T-splines. We will label our subscale DGB parachute NURBS model as “DGB-N”, T-spline model with 1D radial cables as “DGB-T1”, and T-spline model with 2D radial tapes as “DGB-T2.”

6.2.1 Problem setup

The nominal diameter of the subscale DGB parachute is 171 mm, and the length of the suspension lines is 294 mm. A uniform pressure difference distribution, 66 kPa, corresponding to the stagnation pressure at Mach number 1.4 is applied to the disk and 0.27 times this value is applied to the band as it was done in [77]. The undeformed shape of the subscale DGB parachute is shown in Figure 6.9. Material properties for the parachute canopy and the cables are the same as [77] and were obtained from Fujikura Parachute Co., Ltd.
We first generate DGB-N for the undeformed shape from scratch. We use the subscale DGB parachute configuration and dimensions provided to us by Fujikura Parachute Co., Ltd. We make use of cubic NURBS elements for both the canopy and the cables. The connection between the gores of the canopy as well as the connection between the cables and canopy are $C^0$. Figure 6.10 shows a zoomed view of the elements on the canopy for DGB-N.

Figure 6.9: Undeformed shape of the subscale DGB parachute. Black lines indicate the cables.

6.2.2 NURBS and T-spline meshes

Figure 6.10: DGB-N canopy (left) and zoomed view of the cables (right). Red indicates the cables.
In T-spline representation, we start from DGB-N and first connect the gores to each other continuously. Then, we establish continuous connection between the cables on the canopy and the canopy itself using T-splines. With this, we have DGB-T1 representation as shown in Figure 6.11. We then generate DGB-T2 representation by first removing the one dimensional radial cables from the domain and then adding the tapes by duplicating the canopy elements at these locations. Figure 6.12 shows DGB-T2. The total number of elements and the control points for all meshes are shown in Table 6.2.

![Figure 6.11: DGB-T1 canopy (left) and zoomed view of the cables (right). Red indicates the cables.](image)

![Figure 6.12: DGB-T2 canopy (left) and zoomed view of the tapes (right). Red and black indicate the cables and tapes.](image)

### 6.2.3 Computational settings

For all structural mechanics computations with NURBS and T-splines, the Kirchhoff-Love shell model for IGA [78, 79] is used for representing the canopy, and the bending-
Table 6.2: DGB-N, DGB-T1 and DGB-T2 structural mechanics meshes for the sub-scale DGB parachute. Number of control points and number of shell and cable elements.

<table>
<thead>
<tr>
<th></th>
<th>nc</th>
<th>$n_{\text{shell}}$</th>
<th>$n_{\text{cable}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>DGB-N</td>
<td>8,335</td>
<td>5,616</td>
<td>2,036</td>
</tr>
<tr>
<td>DGB-T1</td>
<td>7,903</td>
<td>5,616</td>
<td>2,036</td>
</tr>
<tr>
<td>DGB-T2</td>
<td>7,903</td>
<td>6,048</td>
<td>1,604</td>
</tr>
</tbody>
</table>

stabilized cable [75] model is used for all cables. We use a time step size of $2.5 \times 10^{-6}$ s and carry out the simulations for $1.2 \times 10^{-2}$ s till the shape settles with a mass-proportional damping. We then continue the computations without the damping for another $2.5 \times 10^{-3}$ s and obtain the final inflated shapes. The number of nonlinear iterations for all computations is 4, and the number of GMRES iterations per nonlinear iteration is 300.
6.2.4 Results and discussion

The inflated parachute configurations obtained from the IGA with NURBS and T-splines are shown in Figure 6.13. The elongation of the subscale DGB parachute is similar in all cases. In addition, Figure 6.14 show the inflated parachute canopy shapes and zoomed-views to a gore formation on the canopy from the DGB-N, DGB-T1 and DGB-T2 results. We can see in these figures that DGB-N result does not represent a smooth gore intersection and is not favorable especially for the purpose of mesh generation in the subsequent fluid mechanics computation. On the other hand, both DGB-T1 and DGB-T2 represent the gore formations smoothly. Since we resolve the tapes’ width in DGB-T2 mesh, the gore intersections in DGB-T2 are flatter and wider than the ones in DGB-T1 as expected.

Figure 6.13: Inflated parachute structures for DGB-N (top), DGB-T1 (middle) and DGB-T2 (bottom.) Red and black represent the cables and radial tapes.
Figure 6.14: Inflated canopy shapes (left) and zoomed views of a gore formation (right). DGB-N (top), DGB-T1 (middle), and DGB-T2 (bottom.)
6.3 Compressible-Flow Computation of a Subscale DGB Parachute

In the compressible-flow computation of the subscale DGB parachute, we first generate a block-structured NURBS mesh around the parachute shape obtained from the structural mechanics computation with DGB-T2 described in Section 6.2. We represent the fluid mechanics parachute surface mesh continuous in the circumferential as well. In the NURBS volume mesh, we use enough number of elements and control points to capture the shock and complex flow patterns in the near wake of the parachute. Then using the special techniques introduced in this thesis, we generate a T-spline mesh directly from this NURBS mesh. With the aim of reducing computational cost, we keep the mesh resolution near the parachute the same while reducing the number of control points in the rest of the domain using the reducing the number of control points by knot removal special technique in Chapter 4.

6.3.1 Problem setup

The flow properties for the supersonic compressible-flow are the same as in [77] and here given in Table 6.3. The parachute surface is porous and the porosity model using ST-SI for compressible-flow [24] is applied with fabric-porosity coefficient of 24.92 CFM. The computational domain is a cylinder with a diameter of 1.35 m and a height of 2.5 m. The parachute is located 0.5 m downstream of the inlet.

Table 6.3: Flight conditions for the subscale DGB parachute. Free-stream values of the Mach number $M_{\infty}$, the temperature $\theta_{\infty}$ and the density $\rho_{\infty}$.

<table>
<thead>
<tr>
<th>$M_{\infty}$</th>
<th>$\theta_{\infty}$ (K)</th>
<th>$\rho_{\infty}$ (kg/m$^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.4</td>
<td>216</td>
<td>0.76</td>
</tr>
</tbody>
</table>
6.3.2 NURBS and T-spline meshes

We use quadratic NURBS basis functions for generating the NURBS mesh. T-spline mesh is generated directly from this NURBS mesh by first connecting the elements on the parachute surface continuously and then reducing the number of control points in the far regions of the volume mesh. Figure 6.15 shows the whole computational domain and a focused-view to the canopy as well as the continuous fluid mechanics surface for the T-spline mesh. We show how the reducing of number of control points technique was utilized in Figure 6.16 by comparing the NURBS and T-spline meshes on a cut-plane that goes through the middle point of the band. The number of elements and control points for both meshes are shown in Table 6.4.

Figure 6.15: Subscale DGB parachute T-spline fluid mechanics mesh. Full view (left), parachute surface (top-right) and zoomed view near surface (bottom-right.) The checkerboard coloring for differentiating between the elements.
Figure 6.16: Subscale DGB parachute NURBS (top) and T-spline (bottom) fluid mechanics meshes. Zoomed view on a cut-plane where the special technique for reducing the number of control points is utilized.

Table 6.4: Subscale DGB parachute NURBS and T-spline fluid mechanics meshes. Number of control points and elements.

<table>
<thead>
<tr>
<th></th>
<th>nc</th>
<th>ne</th>
</tr>
</thead>
<tbody>
<tr>
<td>NURBS</td>
<td>775,658</td>
<td>650,304</td>
</tr>
<tr>
<td>T-splines</td>
<td>562,507</td>
<td>485,694</td>
</tr>
</tbody>
</table>
Figure 6.17: Subscale DGB parachute NURBS (top) and T-spline (bottom) fluid mechanics meshes. Further zoomed view on a cut-plane where the special technique for reducing the number of control points is utilized.
6.3.3 Computational settings

We obtain initial flow conditions by performing incompressible-flow computation using ST-VMS and the T-spline mesh. For this computation, the time step size is $2.5 \times 10^{-6}$ s and we carry out the computation till the flow settles. For this computation, the number of nonlinear iterations is 3 and GMRES iterations is 300. Then, we calculate the initial conditions for the compressible-flow computation from these results, and continue the simulations using compressible-flow ST SUPG [24] with the T-spline mesh. For this part of the computations, we first use a smaller time step size of $1.25 \times 10^{-6}$ s till initial oscillations in the flow disappears and then increase the time step size four times to $5.0 \times 10^{-6}$ s to reach the steady-state solution faster. For all compressible-flow computations, the number of nonlinear iterations is 4 and number of GMRES iterations per nonlinear iteration is 30. We select the stabilization parameters and element length definitions as described in [49].

6.3.4 Results and discussion

Figure 6.18 shows the density isosurfaces obtained from compressible-flow computation using ST-IGA with T-splines. As it can be seen in these figures, the developed flow field is complex due to vortex shedding in the far-wake of the parachute; however, it is symmetric in the near-wake and upstream of the parachute. Given this, in the upcoming figures we present the development of the other flow properties on a cut-plane for clearer representations. In Figures 6.19–6.21, the density, Mach number and temperature results from ST-IGA with T-splines are shown for uniform time intervals. In the density figures, we can see the bow shock forming below the parachute. In Mach number figures, the shocks and expansion waves in the flow are indicated with white color. In temperature figures, we see hotter regions where the flow stagnates and we see discontinuity in the shock region. These results show that ST-IGA with T-splines can capture the shock and discontinuities in the solution. In Figure 6.22, we also
show a comparison between the Courant numbers based on the element lengths from
the NURBS and T-spline meshes, with the flow solution coming from the T-spline
computation, by projection in the NURBS-mesh case. In T-splines results, we can see
how reducing the number of control points in certain regions and having better aspect
ratios, especially at the center of the domain, helps with the stability of the solution
indicated by lower Courant numbers. Thanks to the ST-IGA with T-splines and the
special technique for reducing the number of control points introduced in this thesis,
the supersonic compressible-flow simulations of a subscale DGB parachute has been
performed and the bow shock has been represented in the solution while reducing
the computational cost compared to the more traditional ST-IGA with NURBS and
ST-SI methods.
Figure 6.18: Density (kg/m³) isosurfaces from ST-IGA with T-splines. The results from 0 to 3.3×10⁻³ s are shown at 4.5×10⁻⁴ s intervals. The order is first left to right then top to bottom.
Figure 6.19: Density (kg/m$^3$) from ST-IGA with T-splines on a cut-plane. The results from 0 to 3.3×10$^{-3}$ s are shown at 4.5×10$^{-4}$ s intervals. The order is first left to right then top to bottom.
Figure 6.20: Mach number from ST-IGA with T-splines on a cut-plane. The results from 0 to $3.3 \times 10^{-3}$ s are shown at $4.5 \times 10^{-4}$ s intervals. The order is first left to right then top to bottom.
Figure 6.21: Temperature (K) from ST-IGA with T-splines on a cut-plane. The results from 0 to $3.3 \times 10^{-3}$ s are shown at $4.5 \times 10^{-4}$ s intervals. The order is first left to right then top to bottom.
Figure 6.22: Courant numbers based on the element lengths from the NURBS (top) and T-spline (bottom) meshes, with the flow solution coming from the T-spline computation.
Chapter 7

Concluding Remarks

Spacecraft parachutes have been an integral part of the space exploration missions for several decades now. They provide safe and cheap landing for all kind of reentry spacecrafts for both manned and unmanned missions. However, due to their light weight and large size as well as the extreme flight conditions they operate in, their flight behavior and interaction with the flow surrounding them is very complex. In this dissertation, we have presented multiple advanced computational techniques to address the challenges in the spacecraft parachute computation, and we have developed a new method and three new special techniques to further enhance the solution accuracy and stability as well as reduce the computational cost.

In Chapter 1, we have introduced the spacecraft parachutes we used in this thesis and explained the challenges involved in studying them both experimentally and computationally. We then listed the ST methods and advanced geometric modeling techniques we use to address these challenges. In Chapters 2 and 3, we have presented the governing equations of fluid and structure mechanics as well as ST discretization formulations we used.

After that, we presented three new special techniques we developed in this thesis for addressing some deficiencies of the more traditional methods. The first and second
of these techniques were developed to improve upon the ST-IGA with NURBS and ST-SI methods by use of T-splines. In the first special technique, we presented a way to reduce the number of control points in the computational domain especially where the matching-requirement of the block-structured NURBS caused either bad aspect ratio elements or unnecessarily high mesh resolution. This deficiency of the NURBS had already been previously addressed by the use of NURBS together with an SI. While NURBS with an SI did address the bad element quality and unneeded refinement zones, it also introduced a discontinuity in the computational domain. On the other hand, utilizing T-splines with our special technique address the same issue in a more robust way, without discontinuities in the domain. In the second special technique, we presented a way to give continuity to a nonmatching interface, SI, in NURBS models. This is accomplished by using T-splines together with knot insertion and knot removal techniques in order to connect elements at each side of the SI continuously, offering better solution stability. Finally, we introduced our third special technique that not only handles the large mesh deformations but also improves the initial mesh for both NURBS and T-splines.

In Chapter 5, we have applied our method and special techniques to a FSI computation for a 2D parachute model with the purpose of validating our techniques. We have first performed ST-IGA with NURBS and ST-SI computation, and considered these results as our reference solution. We generated a T-spline representation of the same computational domain by using our special techniques, and performed the FSI analysis using the same conditions and with the T-spline mesh. We then compared the results from ST-IGA with NURBS and ST-SI to our method, ST-IGA with T-splines. The results have shown good agreement between both solutions, validating ST-IGA with T-splines for the problems with moving boundaries or FSI.

We then applied our method and special techniques to actual spacecraft parachute computation. We first started with incompressible-flow computation of the MP
parachute, obtaining our reference solution from ST-IGA with NURBS and ST-SI computation. After that, in a similar manner to the 2D test problem, we have applied our special techniques to the NURBS with SI mesh and generated a T-spline representation with reduced number of control points and given continuity to the SI. Again similar to the 2D test problem case, the results from both ST-IGA with NURBS and ST-SI and ST-IGA with T-splines have shown good agreement but only in more global sense. In more localized investigations of the flow field, especially at the regions where we give continuity to the SI, the smoother flow distribution was clearly seen in the results from ST-IGA with T-splines. We concluded that ST-IGA with T-splines enhances the solution stability and accuracy compared to ST-IGA with NURBS and ST-SI. Also, we note that these enhancements were achieved while also decreasing the computational cost thanks to ST-IGA with T-splines.

Next, we used our method and special techniques in structural mechanics analysis of a subscale DGB parachute. We again first obtained a reference solution using NURBS mesh. Then, using T-splines, we have introduced two possible options for representing the parachute canopy and the radial cables connections. The first option represented a continuous parachute canopy and established continuous connection between the canopy and the radial cables. The second option also represented the canopy and its connection to the cables continuously but used radial tapes instead of radial cables. We compared the inflated shape results from all three cases. The elongation of the parachute structure looked similar in all cases, but the inflated surface shapes have shown differences. In NURBS case, a continuous gore formation was not possible due to $C^0$ continuity between NURBS patches that make the gores. In both T-spline results, we have achieved a continuous (i.e. smooth) gore formation on the canopy, which is very desirable for subsequent fluid mechanics volume mesh generation.

Finally, we have applied our special techniques to the same subscale DGB parachute supersonic compressible-flow computation. We started with a NURBS mesh that is
reasonably refined for accurately capturing the bow shock expected to form below the parachute. We then generated a T-spline mesh and used our special technique to reduce the number of control points in the far regions of the domain where we do not expect complex flow patterns. We have carried computations till the bow shock reaches a steady-state solution using compressible-flow ST SUPG together with our method, ST-IGA with T-splines. Looking at the results we presented, we concluded that ST-IGA with T-splines can represent discontinuities in the flow field (e.g. shocks and expansion waves) while allowing us to reduce the number of control points elsewhere in the domain, thus reducing the computational cost. Also, Courant number results comparison between ST-IGA with NURBS and ST-IGA with T-splines results showed that the latter offered better solution stability as well.

In conclusion, ST-IGA with T-splines offers enhancements to ST-IGA with NURBS and ST-SI method in terms of solution accuracy, stability and robustness. We also note that ST-IGA with T-splines and the special techniques we introduced can easily be applied to other spacecraft parachute computation as well as the other engineering applications with complex geometries.
Appendix A

MP NURBS Computations:
Parachute Structure Computation
and 5-Gore Flow Computation
With Rotational Periodicity

A.1 Parachute Structure Computation

A.1.1 Problem setup

To obtain an inflated shape for the fluid mechanics computations, structural mechanics computations were performed by applying the parachute surface a uniform pressure field corresponding to the stagnation pressure at the descent speed of 7.8 m/s. A cubic NURBS mesh for the undeformed parachute shape were generated directly from the previous finite element mesh given in [80]. In total, we used 58,420 control points in the NURBS mesh. Rings and sails were modeled as membranes using membrane-wrinkle model which, in total, consists of 24,240 NURBS elements. All the cables were modeled as cable elements using truss model. The number of cable elements
is 12,561, in total. The parachute is fixed at the payload point. Figure A.1 shows the structural mechanics control mesh used in the computations presented here. A mesh refinement study on the membrane elements, at the parachute surface, was also performed, and the results were compared with the coarse mesh. Table A.1 shows the number control points and number of elements used in both refined and coarse mesh computations. Figure A.2 shows the comparison between the coarse and refined control meshes on parachute surface.

Table A.1: Number of control points and elements for the coarse and refined meshes.

<table>
<thead>
<tr>
<th></th>
<th>Coarse</th>
<th>Refined</th>
</tr>
</thead>
<tbody>
<tr>
<td>$nc$</td>
<td>58,420</td>
<td>157,204</td>
</tr>
<tr>
<td>Membrane $ne$</td>
<td>24,240</td>
<td>96,960</td>
</tr>
<tr>
<td>Cable $ne$</td>
<td>12,561</td>
<td>23,121</td>
</tr>
<tr>
<td>Payload $ne$</td>
<td>1</td>
<td>1</td>
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</table>

**A.1.2 Computational settings**

For both cases, the time dependent computations were carried out using a generalized-$\alpha$ model with spectral radius set to zero ($\rho_\infty = 0$). The time step size is 0.01 s.
Rayleigh damping coefficients were used as $\eta = 12.57\, s^{-1}$ and $\zeta = 0.05\, s$. The number of nonlinear iterations is 4, and the number of linear iterations is 100. The computations were carried out till the parachute shape settles for both coarse and refined mesh computations.

### A.1.3 Results

Figures A.3 and A.4 show the inflated shapes obtained from the coarse and refined mesh structural mechanics computations. The results from both coarse and refined mesh show good agreement in terms of the skirt diameter convergence. Figure A.5 shows the change of percentage of the area for each parachute membrane. It can be clearly seen from the results that there is very small difference between the membrane areas obtained from both coarse and refined mesh computations as well as a tiny difference in the resulting skirt diameters.
Figure A.3: Inflated parachute configurations obtained from the structural mechanics computations with the coarse (left) and refined (right) meshes.

Figure A.4: Inflated parachute canopy shapes obtained from the structural mechanics computations with the coarse (left) and refined (right) meshes.
Figure A.5: Membrane area convergence from the computations with the coarse and refined meshes.
A.2 5-Gore Flow Computation With Rotational Periodicity

A.2.1 Problem setup

To calculate the porosity coefficients to be used in full-canopy computations, fluid mechanics computations with rotational periodicity were performed using a 5-gore slice of the MP parachute. 5-gore parachute geometry including all the gaps, slits and window were modeled using quadratic NURBS basis functions. The computational domain is a 22.5° slice of a 100 m radius cylinder with a very small axial circular hole. The inlet boundary is located 91.4 m from the parachute vent. A uniform inflow condition of 7.8 m/s is assigned at the inlet boundary corresponding to the parachute descent speed. The outlet boundary is located 172 m from the vent, and a stress boundary condition is assigned there corresponding to the atmospheric pressure. A slip boundary condition on the inflow direction is used at the inner and outer sides of the domain. A rotationally-periodic boundary condition is assigned to the sides of the computational domain in the circumferential direction. Fabric porosity coefficient for each parachute membrane is 40 CFM. Figure A.6 shows the control mesh used in these computations. A mesh refinement study on the parachute surface was also performed. Table A.2 shows the number of control points and elements for both coarse and refined mesh. Figure A.7 shows the control mesh on the parachute surface for both coarse and refined mesh. Figure A.8 shows the control points at the slits for both coarse and refined mesh.

Table A.2: Number of control points and elements for the coarse and refined mesh.

<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
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<td></td>
<td>nc</td>
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<td></td>
<td>ne</td>
<td>45,179</td>
</tr>
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<td>Surface</td>
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</tr>
<tr>
<td></td>
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<td>1,606</td>
</tr>
<tr>
<td></td>
<td>ne</td>
<td>768</td>
</tr>
</tbody>
</table>
A.2.2 Computations

5-gore flow computation with rotational periodicity was carried out using ST-IGA with NURBS. Porosity interface was active from the start. The number of nonlinear iterations is 4. For the first two iterations ST-SUPS formulation was used, and for the last two ST-VMS. For the computations with the coarse mesh, the number of GMRES iterations is 300 for each nonlinear iteration. The computations were carried out for 1,500 time steps using the coarse mesh and the time step size of 0.1167 s. Then, the time step size was decreased to 0.0233 s and computations were continued for another 750 time steps. In the mesh refinement study, we refine the mesh using h-refinement for NURBS, which inserts new knots and keeps the geometry the same. Then, the data from 1,500 time step of the coarse mesh computations were projected to the refined mesh, and the computations were continued for 750 time steps. The time step size for the refined mesh computations is 0.0233 s. The number of nonlinear iterations and the methods used in each iterations are the same with the coarse mesh.
computations. The number of GMRES iterations for each nonlinear iteration here are 300, 350, 400, and 450 respectively.

A.2.3 Results

Figure A.9 shows the comparison between gauge pressure fields obtained from the 5-gore flow computations with rotational periodicity with the coarse and refined meshes, and Figure A.10 the velocity magnitudes. The velocity vectors obtained from the refined mesh computation can be seen in Figure A.11.
Figure A.9: Pressure (Pa) obtained from the computations with the coarse and refined meshes.

Figure A.10: Velocity (m/s) magnitudes obtained from the computations with the coarse and refined meshes.

Figure A.11: Velocity vectors near the parachute surface. Pressure on the cutting plane and velocity vectors colored by magnitude.
A.2.4 HMGP-FG

HMGP-FG method [16] was used for calculating geometric porosity coefficients. Parachute was divided into four patches. The first patch consists of all 4 rings and the first sail as well as the gaps between them. The second patch contains the sails from number 2 to 6 and the slits between them. The third one consists of the seventh sail and the slits above and below it. Finally, the fourth patch is formed by the sails numbered 8 and 9 as well as the silt between them. Figure A.12 shows the patch formation used in geometric porosity modeling. It needs to be noted that the third patch is replaced with a window in every fifth gore while all the other patches present in every gore. Table A.3 shows the geometric-porosity values obtained from the computations with the coarse and refined meshes. The porosity coefficients are time averaged using the last 500 time steps for both computations.

Table A.3: Geometric-porosity coefficients for each patch obtained from the computations with the coarse and refined meshes.

<table>
<thead>
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<tbody>
<tr>
<td>Patch 1</td>
<td>0.04659112</td>
<td>0.04525306</td>
</tr>
<tr>
<td>Patch 2</td>
<td>0.02415985</td>
<td>0.02777664</td>
</tr>
<tr>
<td>Patch 3</td>
<td>0.05423271</td>
<td>0.06021918</td>
</tr>
<tr>
<td>Patch 4</td>
<td>0.01793576</td>
<td>0.01886949</td>
</tr>
</tbody>
</table>
Figure A.12: Patch formation for geometric-porosity modeling.
Bibliography


[27] A.N. Brooks and T.J.R. Hughes, “Streamline upwind/Petrov-Galerkin formulations for convection dominated flows with particular emphasis on the incom-


