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Multi-physics modelings and analysis for magnetized high-energy-density laser driven plasma flows

By

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ABSTRACT

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Laboratory experiments on large laser facilities have been a new way to study astrophysical phenomena and fundamental physics processes, including those related to magnetic fields. Over the past few years, two experiment platforms on OMEGA laser facility, i.e. hollow ring magnetized jet platform and shock-shear platform, have been developed to study the magnetic fields in high-energy-density laser driven plasma flows. This thesis summarizes the multi-year effort on the development of these two platforms.

On the hollow ring magnetized jet platform, highly collimated jets are driven by a ring of 20 OMEGA beams. A bundle of laser beams with given individual intensity, duration and focal spot size, produces a supersonic jet of higher density, temperature and better collimation, if the beams are focused to form a circular ring pattern on a flat target instead of a single focal spot. On the shock-shear platform, counter-propagating shocks in a shock tube induces a shear layer with vortices and mix. The shock-shear configuration allows us to study the shear-induced instabilities and turbulence production under high-energy-density conditions.

For both platforms, three-dimensional FLASH radiation-magnetohydrodynamics modeling is carried out to study the evolution of hydrodynamics and magnetic fields.
While many experiments and observations have been carried out to study the plasmas with external magnetic fields, this thesis focus on the self-generated magnetic fields in high-energy-density plasmas. Magnetic fields are initially self-generated via the Biermann battery term. The accurate prediction and description of the late-time dynamics of magnetic fields are limited by the capability of the simulation code. For the late-time stage, non-ideal terms such as Nernst term, magnetized heat flow and magnetic resistive term may play significant roles.

In the experiments for both platforms, the hydrodynamical evolution of the plasmas was diagnosed by X-ray framing camera. Proton radiography was used to measure the line-integrated strength of magnetic fields. In the magnetized jet experiments, where the density of the plasma is low ($\rho \lesssim 10^{-3}$ g/cc), optical Thomson scattering was used to diagnose the temperature, density and flow velocity of the jets. Synthetic proton radiography simulations, X-ray image simulations, and synthetic Thomson spectrum modelings are carried out and compared with the observable features in the experiments. The results from the simulations and experiments in this thesis have many potential applications to laboratory astrophysics, fundamental plasma physics and other areas.

This thesis also summarizes the development of numerical methods for relativistic particle-in-cell (PIC) simulations and analyses. Those methods significantly improves the reliability of PIC simulations and analyses in high-energy-density physics and high-energy astrophysics.
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Publications

I hereby declare that except where specific reference is made to the work of others, the contents of this thesis are original and have not been submitted in whole or in part for consideration for any other degree or qualification in this, or any other university.

The following journal publications (refereed or under review) and conference proceedings are (co-)authored and extensively reproduced by the Ph.D. candidate, Yingchao Lu, in this dissertation:


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Chapter 1

Introduction

1.1 High-energy-density laser driven plasmas

High-energy-density (HED) physics is a field characterized by extreme states of matter attainable by recent advances in high-energy/high-power lasers, particle beams, and Z-pinch generators. The formal definition of the field first appeared in a report entitled *Frontiers in High-Energy-Density Physics: the X-games of Contemporary Science* [3] published in 2003, which suggested that the definition of “high-energy-density” refers to energy densities exceeding $10^{11}$ J/cm$^3$, or equivalently pressure exceeding Mbar, which is approximately the internal energy density of a hydrogen molecule. Some researchers and literatures, including the graduate texts in physics by Drake [5], suggest revising the definition by lowering the threshold pressure to 0.1 Mbar. Drake [5] suggested the revised definition: High-energy-density physics is the laboratory study of matter that has a pressure of at least 0.1 Mbar (10 GPa) and contains free electrons not present in the solid state, and of lower pressure matter produced using experimental systems that can produce pressures above 0.1 Mbar. The revised definition is also supported in this thesis. As in the two platforms discussed in this thesis, free electrons are present due to the heating by OMEGA [6] laser beams, and the pressure in the regions that the research focuses on excesses 0.1 Mbar.

Figure 1.1 compares the domain of high-energy-density physics with those of other areas of other physical and astrophysical systems. The experiments under such ex-
treme condition (pressure > 0.1 Mbar) span a wide range of physics areas, including plasma physics, laser and particle beam physics, material science and condensed matter physics, nuclear physics, atomic and molecular physics, fluid dynamics, magnetohydrodynamics, and astrophysics. The picture in Figure 1.1 is not complete for HED physics, because it assumes equilibrium and unmagnetized condition while generally dynamical and magnetized processes can be interesting.

The focus of this thesis are HED laboratory astrophysics and magnetic fields. On HED experimental facilities, such as large laser facilities [6] and Z-pinch generators, astrophysical relevant conditions can be created and their properties measured. The OMEGA laser facility [6] at the Laboratory for Laser Energetics (LLE) at the University of Rochester is one of the largest laser facilities in the world. Its 60 beams are capable of delivering 40 kJ at up to 60 TW onto a target less than millimeter in diameter. OMEGA EP [7, 8] is a high-energy short-pulse laser system with petawatt beams in addition to the OMEGA laser system. The National Ignition Facility (NIF) [9] is the largest and highest-energy laser system in the world located at Lawrence Livermore National Laboratory (LLNL). It contains 192 laser beams that can deliver ∼ 2MJ at hundreds of TW in order to achieve inertial confinement fusion (ICF) [10, 11]. The typical peak intensity for OMEGA or NIF lasers is $10^{15}$ to $10^{16}$W/cm$^2$. The processes that can be addressed in the HED regime include hydrodynamic instabilities [12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22] and mixing [23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 22, 36], magnetically collimated jets [37, 38, 39, 40, 41, 42, 43, 44], magnetohydrodynamic turbulence [45, 46, 47, 48, 49, 50, 51, 52, 53] and high-Mach-number jets [54, 55].

HED plasmas are usually difficult to analyze theoretically, due to the high degree
of nonlinearity and complexity of multiple scales. Advances in scientific computation are being utilized in modeling many of the HED systems. The data obtained from HED experiments can be used to verify and validate the theoretical models. The models and computational codes developed with the knowledge from laboratory experiments can be used to understand the fundamental physics and to model astrophysical objects. However, one of the challenges in the numerical modeling of HED systems is that the range of spatial and temporal scale typically needs to be spanned far exceeds what can be computed even using the leading-edge supercomputers. Computational techniques such as sub-grid models (sub-grid models describe the processes not captured by the simulation resolution, such as turbulent viscosity) and implicit diffusion (e.g. magnetic diffusion and thermal diffusion) are developed to overcome the large scale-separations. The research presented in this thesis uses several simulation codes to model the physical processes in the HED experiments related to magnetic fields. FLASH \[56, 57\]† is a publicly available, parallel, multi-physics, adaptive mesh refinement (AMR), finite-volume Eulerian hydrodynamics, and radiation-magnetohydrodynamics code with extensive HED laboratory plasma (HEDLP) capabilities \[58\]. MPRAD \[59\] is a Monte Carlo and ray-tracing code developed by the author of this thesis that can be used for modeling the deflection of proton beams in arbitrary three dimensional electromagnetic fields, as well as the diffusion of the proton beams by Coulomb scattering and stopping power. SPECT3D \[60\] is a collisional-radiative spectral analysis code with ray-tracing capability designed to simulate spatially and spectrally resolved radiative properties of laboratory plasmas.

†FLASH4 is available at https://flash.uchicago.edu/
Figure 1.1: Regimes of high-energy-density experimental range (such as OMEGA, NIF, Z-pinch and short pulse laser plasmas) and their connections to other physical and astrophysical systems. The horizontal axis is logarithmic density and the vertical axis is logarithmic temperature. From *Frontiers in High Energy Density Physics: The X-Games of Contemporary Science* [3].
1.2 Magnetized plasmas in high-energy-density laser driven experiments

Magnetic fields are ubiquitous in the universe. Observations have revealed a broad range of values of magnetic fields, from femtogauss in the voids between galaxy cluster filaments to many teragauss near black holes and neutron stars. A variety of ideas have been proposed in which the magnetic fields could be generated and maintained. It is also highly desirable to recreate magnetic fields in the laboratory to study their physical processes and scaling properties under controllable conditions.

1.2.1 The origin of magnetic fields

One of the ideas is that the seed magnetic field is generated by Biermann battery effect \[61\], which arises from misaligned electron pressure and density gradients, and amplified by dynamo action \[62, 63\], which converts a significant fraction of kinetic energy into magnetic energy. Seed magnetic field generation by Biermann battery mechanism \[61\] has been modeled in a few astrophysical objects, including protogalaxy \[64\], first stars \[65\] and population III stars \[66\]. When an intense laser irradiates a solid target, strong toroidal magnetic fields are created around the laser spot \[67, 68, 69, 70\] by Biermann battery effect and diagnosed using proton radiography \[71, 72\]. In both astrophysical objects and laboratory created plasmas, how are the magnetic fields amplified and maintained is not fully understood, despite rich direct observations and theoretical modelings.

A series of magnetohydrodynamics (MHD) simulations suggest that dynamo action can act in a wide range of the parameter space in astrophysics \[49, 51\]. Magnetic fields can grow in scales larger or smaller than those of the fluid motion, correspond-
ing to large-scale (or mean-field) dynamos and small-scale (or fluctuation) dynamos respectively. A dimensionless parameter to distinguish the different regimes for dynamos is the magnetic Prandtl number $Pr_m$

$$\begin{align*}
Pr_m &\equiv \frac{Rm}{Re} = 2.3 \times 10^{13} \frac{(T_i[eV])^{5/2}(T_e[eV])^{3/2}}{\ln \Lambda_e \ln \Lambda_i A^{1/2} Z^3 (0.33 Z + 0.18) n_e [cm^{-3}]} \\
&= 1.3 \times 10^{-3} \frac{(T_i[K])^{5/2}(T_e[K])^{3/2}}{\ln \Lambda_e \ln \Lambda_i A^{1/2} Z^3 (0.33 Z + 0.18) n_e [cm^{-3}]} 
\end{align*}\tag{1.1}$$

which is the ratio of magnetic Reynolds number

$$\begin{align*}
Rm &\equiv \frac{uL}{\eta_B} = \frac{uL \times 3\sqrt{2\pi} k_B^{3/2} T_e^{3/2}}{(0.33 Z + 0.18) \times 2 \sqrt{m_e e^2 c^2} \ln \Lambda_e} \\
&= 1.2 \times 10^{-6} \frac{u [cm/s] L [cm] (T_e [eV])^{3/2}}{(0.33 + 0.18/Z) Z \ln \Lambda_e} \\
&= 9.7 \times 10^{-13} \frac{u [cm/s] L [cm] (T_e [K])^{3/2}}{(0.33 + 0.18/Z) Z \ln \Lambda_e} 
\end{align*}\tag{1.2}$$

to fluid Reynolds number

$$\begin{align*}
Re &\equiv \frac{uL}{\nu} = \frac{uL \times \sqrt{\pi} \ln \Lambda_i m_p^{1/2} A^{1/2} e^4 Z^3 n_e}{0.72 (k_B T_i)^{5/2}} \\
&= 5.3 \times 10^{-20} \frac{u [cm/s] L [cm] A^{1/2} Z^3 n_e [cm^{-3}] \ln \Lambda_i}{(T_i [eV])^{5/2}} \\
&= 7.5 \times 10^{-19} \frac{u [cm/s] L [cm] A^{1/2} Z^3 n_e [cm^{-3}] \ln \Lambda_i}{(T_i [K])^{5/2}} \tag{1.3}
\end{align*}$$

where $T_i$ is the ion temperature, $T_e$ is the electron temperature, $\ln \Lambda_e$ is the Coulomb logarithm for electrons, $\ln \Lambda_i$ is the Coulomb logarithm for ions, $A$ is the mass number of ions, $Z$ is the charge number of the ions, $n_e$ is the electron density, $u$ is the velocity of the flow, $L$ is the typical scale of the system. Magnetic Prandtl number $Pr_m$ highly depends on temperature, $Pr_m \propto T^4/n$. The value of $Pr_m$ is larger than unity for hot rarefied plasmas such as the warm and hot phases of the interstellar medium or the
intracluster medium, and less than unity for the Sun’s convective zone, planets, and protostellar disks [49].

Most of laboratory plasma experiments for studying magnetic turbulence to-date were in the $\text{Pr}_m < 1$ regime. In the liquid-metal dynamo experiments [73], the amplification of mean square magnetic field is proportional to $\text{Rm}$ for small values of $\text{Rm}$. The dynamo amplification was also observed in laboratory laser-produced shock waves using the Vulcan laser facility at the Rutherford-Appleton Laboratory in the UK, where $\text{Rm}$ is around 3 to 6 and an induction probe was used to resolve the temporal spectrum of the magnetic energy density $M(\omega) \propto \omega^{-11/3}$ [47, 45]. In a recent OMEGA experiment using laser-produced colliding plasma flows to study turbulent dynamo [50, 52], a condition with peak values $\text{Rm}$ being 1300 to 1600 was achieved, the magnetic field was measured using proton radiography [71, 72], and the spatially resolved spectrum for magnetic energy density $M(k) \propto k^{-1}$. Followup experiments on Vulcan laser and LMJ-PETAL facility (LMJ delivers shaped pulses from 0.7ns to 25ns with a maximum energy of 1.5 MJ and a maximum power of 400 TW of UV light on the target. The PETAL project consists of one high-energy multi-petawatt beam to LMJ [74]) were carried out to study magnetic fields in supersonic plasmas and steepened magnetic energy spectra were found [53, 75]. Transport of high-energy charged particles were also studied in the laser driven turbulent magnetic fields [76], providing insights to understand how cosmic rays propagate through space.

While the experiments in $\text{Pr}_m < 1$ regime mentioned above successfully achieve magnetic field probe using different diagnostics and provide excellent verification for numerical MHD simulations [49, 51], the exploration for $\text{Pr}_m > 1$ regime [77] in laboratory still remains limited. On the one hand, creating collisional (i.e. the characteristic length scale $L$ is much larger than the mean free path $\lambda_{\text{mfp}}$, $L \gg \lambda_{\text{mfp}}$) with
high enough temperature that can make $\text{Pr}_m \gg 1$ is challenging. On the other hand, in the kinematic regime magnetic field grows at a length scale that is much smaller than the energy injection scale (for energy injection scale $L$, magnetic field grows at scales $L/\text{Re}^{3/4}/\text{Pr}_m^{1/2} < l < L/\text{Re}^{3/4}$ in the kinematic regime [77]), so that one may need high resolution measurements to resolve the whole process of magnetic field evolution. The first experiment for creating a laboratory $\text{Pr}_m \gtrsim 1$ plasma dynamo was reported recently [78]. Another ongoing campaign has also been focusing on $\text{Pr}_m > 1$ regime, based on a new design [46] for turbulent dynamo experiment on the OMEGA EP laser system.

Another widely studied mechanism for magnetic field generation in both astrophysical objects and laboratory-produced plasmas is the Weibel instability [79] that is relevant to the formation of collisionless shocks. It has been proposed that Weibel instability can generate a turbulent magnetic field and mediate shock formation in both non-relativistic case, such as cosmological shocks [80] and young supernova remnants [81], and relativistic case, such as gamma ray burst afterglows [82, 83, 84, 85, 86]. The instabilities typically grow on the scale of ion skip depth, which is not possible to resolve in astrophysical observations. Thus studying details of the shock structure usually relies on kinetic simulations such as particle-in-cell (PIC) simulations. Laser-driven plasma flow can potentially provide platforms for detail probe for shock structure and evolution, as well as plasma heating and particle accelerations. The development of strong magnetic fields by Weibel-type filamentation instabilities in two symmetric counter-streaming plasma flows was demonstrated in a series of experiments [87, 88, 89, 90, 91, 92] conducted at the OMEGA laser facility. In a recent experiment [92] on NIF that scales up the counter-streaming plasma configuration (with large scale-separation and low collisionality, $L_{\text{m.f.p}} = 80\text{cm}$, $L_{\text{system}} = 2.5\text{cm}$,
ion skin depth $c/\omega_{pi} = 50\mu$m), the measured density compression demonstrates the formation of high-Mach-number collisionless shocks, and electrons are accelerated to relativistic non-thermal energies. Recent work using PIC simulations has studied how Weibel instability is suppressed due to collisions [93].

1.2.2 The effects of magnetic fields on plasma flows

The magnetic fields can also be externally applied to the laser-driven targets, e.g. using MIFEDS (magneto-inertial fusion electrical discharge system, where magnetic field is created by discharging a high-voltage capacitor through a small wire-wound coil) seed-field generator [94, 95] in OMEGA’s target chamber to generate a magnetic field of up to tens of tesla and a magnetization volume of up to $1\text{cm}^3$. The plasmas generated on pulsed-power facilities (laboratory facilities that employ high pulsed currents and voltages) are compressed by strong Lorentz force. Magnetic field is naturally present and controllable in pulsed-power environments. For example, magnetically driven experiments have been conducted on MAGPIE (mega-ampere generator for plasma implosion experiments) pulsed-power facility [96] at Imperial College London to study magnetically driven jets [37, 38, 39, 40, 41, 42, 43], magnetic reconnection [97, 98, 99, 100, 101] and magnetized shocks [102]. In the magnetically driven plasmas, $\beta \lesssim 1$, where $\beta$ is the nominal ratio of the hydrodynamic force to the magnetic force [103]

$$\beta = \frac{n_e k_B T_e + n_i k_B T_i}{B^2/8\pi}$$

$$= 4.0 \times 10^{-11} \times n_e [\text{cm}^{-3}] (T_e [\text{eV}] + T_i [\text{eV}]/Z)/(B [\text{gauss}])^2$$

$$= 3.5 \times 10^{-15} \times n_e [\text{cm}^{-3}] (T_e [\text{K}] + T_i [\text{K}]/Z)/(B [\text{gauss}])^2 \quad (1.4)$$

In the plasmas with $\beta \lesssim 1$, bulk motion of the mass can be accelerated due to the $J \times B$ force.
Besides $J \times B$, magnetic field can also act on the electron heat flow in plasmas, by causing anisotropic and Righi-Leduc heat-flow [104, 105]. Such effects are characterized by electron Hall parameter $\chi_e$, which is the product of the electron gyro-frequency $\omega_{ce} = eB/(m_e c)$ and the mean electron-ion collision time

$$\tau_e = \frac{3\sqrt{m_e k_B^3 T_e^{3/2}}}{4\sqrt{2}\pi e^4 n_e Z \ln \Lambda}$$

$$= 3.4 \times 10^5 s \times \frac{(T_e [eV])^{3/2}}{n_e [cm^{-3}] Z \ln \Lambda}$$

$$= 0.28 s \times \frac{(T_e [K])^{3/2}}{n_e [cm^{-3}] Z \ln \Lambda}$$

Thus the electron Hall parameter is

$$\chi_e \equiv \omega_{ce} \tau_e = \frac{3k_B^3 T_e^{3/2}}{4\sqrt{2}\pi m_e e^3 Z n_e c \ln \Lambda}$$

$$= 6.1 \times 10^{12} \frac{(T_e [eV])^{3/2} B}{n_e [cm^{-3}] Z \ln \Lambda}$$

$$= 4.8 \times 10^6 \frac{(T_e [K])^{3/2} B}{n_e [cm^{-3}] Z \ln \Lambda}$$

In the astrophysics context, the magnetized heat flow affects cooling and instabilities in the intracluster medium of clusters of galaxies [106, 107, 108, 109, 110, 111]. It was shown in simulations that the effects of magnetic field on electron heat flow need to be taken into account for designing pre-magnetized ICF targets [112, 113]. For each term in electron heat transport, there is a corresponding transport of magnetic field. For example, the analog of electron thermal conduction is the Nernst term [114], which transport magnetic flux at the velocity of electron heat transport. The theoretical framework for self-consistently evaluating the transport of electron heat and magnetic flux in a plasma is called the extended-MHD. Simulation studies have found the extended-MHD effects to change plasma properties in both self-generated
field and externally applied field. The examples for self-generated field include the ablation fronts of direct-drive [115] and indirect-drive [116], and the stagnation phase of indirect-drive implosions [117]. For externally applied field to long-pulse laser-gas-jet experiments, simulations predict a significant of cavitation of the field [118]. Extended-MHD effects are also measured in single-spot laser-coil configurations [119, 120, 121].

Despite the excellent measurements of extended-MHD effects in laboratory, individual transport effects still remain unobserved experimentally. To measure driven extended-MHD terms (Nernst, cross-gradient-Nernst, anisotropic thermal conduction, and Righi-Leduc heat-flow) unambiguously, an experimental configuration using a laser-heated pre-magnetized under-dense plasma [122] is explored computationally by the extended-MHD code Gorgon [123]. Kinetic simulations [115, 116, 118, 124] and experimental measurements [119, 120] suggest that nonlocal heat-flow suppression is important. In a recent 2D Vlasov-Fokker-Planck (VFP) simulation work of magnetic field generation by the Biermann battery, field generation rate and Nernst velocity was found to be significantly suppressed even for $\lambda_e/l_T$ being a few percent [125], where $\lambda_e$ is the electron mean-free-path and $l_T$ is the temperature scale length. Applying flux-limiter is a viable method to approximate reductions in the electron heat flux from nonlocal effects [126].

Small values of $\beta$ or large values of $\chi$ characterizes the effect of magnetic field on “global scale”, such as the characteristic scale length for electron temperature and magnetic pressure, or curvature radius of the magnetic field. Magnetic fields are also believed to play roles in micro-scales. For example, in the saturated state for small scale dynamo in $Pr_m > 1$ regime, it occurs below the viscous scale that the magnetic tension force starts to oppose the hydrodynamic terms when the magnetic energy be-
comes comparable to the energy of the viscous-scale eddies (instead of thermal/kinetic energy), i.e. $B^2/(4\pi) \sim \rho u_p^2$ [77]. For an unstructured field, nonlinearity would have set in at much lower field energies $B^2/(4\pi) \sim Pr^{-1/2}_m \rho u_p^2$ [77]. Micro-scales physics have also been studied in collisionless or mildly collisional magnetized plasmas, e.g. ion skin depth in two-spot reconnection configurations [127, 128, 129, 130] and ion gyro-radius in magnetized collisionless shocks [131].

1.3 Overview and organization of this thesis

The rest of this thesis is organized as follows. The models and simulation methods are described in Chapter 2 for collisional plasmas in high-energy-density physics (HEDP), and in Chapter 3 for diagnostics. The progress for simulations, experiments and analyses for the multi-year effort on the development of two HEDP platforms is elaborated in Chapter 4 for the magnetized jet platform, and in Chapter 5 for the shock-shear platform. In Chapter 6 the development of numerical methods for relativistic particle-in-cell (PIC) simulations and analyses is discussed. The summary of this thesis is given in Chapter 7. The variables and equations for electrodynamics are in cgs units.
Chapter 2

Theoretical description and simulation methods for high-energy-density magnetized plasmas

The models for collisional HED laser-drive plasmas are given in this chapter. These models are also applicable to collisional astrophysical plasmas. The collisional HED laser-drive plasmas can be well described by the MHD equations with the extension of electron transport terms, as elaborated in Section 2.1. The radiative transport equation in Section 2.2 is solve for the energy exchange between photons and electrons, as well as the radiation flux in X-ray diagnostics. The transport theory for both electrons and photons can be derived from the equation for phase space density. FLASH code has been used throughout the simulation work in this thesis and is described in Section 2.3. The kinetic simulations introduced in Section 2.4 are complementary to the extended-MHD framework.

2.1 Fluid-MHD equations with collisional transport

This section describes how kinetic theory leads to dissipative plasma hydrodynamics equations. I follow the notation in [132] and [104] starting from Vlasov-Fokker-Planck (VFP) equations but the two ion species $I_1$ and $I_2$ here are arbitrary. More generally, a model that self-consistently combines physics of kinetic electrons and atomic processes is described by Vlasov-Boltzmann-Fokker-Planck (VBFP), where the name Vlasov-Boltzmann-Fokker-Planck comes from the collision model contains Vlasov terms (phase space gradient terms), Fokker-Planck terms (collision terms), and
Boltzmann terms (coupling between free electrons and the atomic state distribution) [133].

For characterizing the collisionality, the scale size $L$ is compared with the electron mean free path

$$\lambda_e = \sqrt{\frac{k_B T_e}{m_e} \tau_e} = \frac{3(k_B T_e)^2}{4\sqrt{2\pi} e^4 n_e Z \ln \Lambda}$$

$$= 1.4 \times 10^{13} \text{cm} \times \frac{(T_e[\text{eV}])^2}{n_e[\text{cm}^{-3}] Z \ln \Lambda}$$

and the ion mean free path

$$\lambda_i = \sqrt{\frac{k_B T_i}{m_i} \tau_i} = \frac{3(k_B T_i)^2}{4\sqrt{\pi} e^4 Z^3 n_e}$$

$$= 2.0 \times 10^{13} \text{cm} \times \frac{(T_i[\text{eV}])^2}{Z^3 n_e[\text{cm}^{-3}] Z \ln \Lambda}$$

and the ion mean free path

$$\lambda_i = \sqrt{\frac{k_B T_i}{m_i} \tau_i} = \frac{3(k_B T_i)^2}{4\sqrt{\pi} e^4 Z^3 n_e}$$

$$= 2.0 \times 10^{13} \text{cm} \times \frac{(T_i[\text{eV}])^2}{Z^3 n_e[\text{cm}^{-3}] Z \ln \Lambda}$$

The complete set of VFP equations consists of the electron kinetic equation (subscript $e$ for electrons)

$$\frac{\partial f_e}{\partial t} + v \cdot \nabla f_e - \frac{e}{m_e} (E + v \times B) \cdot \frac{\partial f_e}{\partial v} = C_{ee}(f_e, f_e) + C_{el_1}(f_e, f_{l_1}) + C_{el_2}(f_e, f_{l_2})$$

(2.3)

and the two ion kinetic equations (subscript $I_1$ for the first ion species with charge $Z_{i_1}$ and $I_2$ for the first ion species with charge $Z_{i_2}$) is

$$\frac{\partial f_{l_1}}{\partial t} + v \cdot \nabla f_{l_1} + \frac{e Z_{i_1}}{m_e} (E + v \times B) \cdot \frac{\partial f_{l_1}}{\partial v} = C_{l_1 l_1}(f_{l_1}, f_{l_1}) + C_{l_1 l_2}(f_{l_1}, f_{l_2}) + C_{el_1}(f_e, f_{l_1})$$

(2.4)

$$\frac{\partial f_{l_2}}{\partial t} + v \cdot \nabla f_{l_2} + \frac{e Z_{i_2}}{m_e} (E + v \times B) \cdot \frac{\partial f_{l_2}}{\partial v} = C_{l_2 l_2}(f_{l_2}, f_{l_2}) + C_{l_2 l_1}(f_{l_2}, f_{l_1}) + C_{el_2}(f_e, f_{l_2})$$

(2.5)
where the Fokker-Planck binary collision operators for test particle species \( j \) colliding with with field particle species \( k \) are

\[
C_{jk} = \frac{1}{2} \Gamma_{jk} \frac{\partial}{\partial v} \int d^3v' \mathcal{U}(v, v') \cdot \left( \frac{\partial}{\partial v} - \frac{m_j}{m_k} \frac{\partial}{\partial v'} \right) f_k(v') f_j(v) \tag{2.6}
\]

\[
\Gamma_{jk} = \frac{4\pi Z_j^2 Z_k^2 e^4 \ln \Lambda_{jk}}{m_j^2} \tag{2.7}
\]

\[
\mathcal{U}(v, v') = \frac{1}{|v - v'|} \left( I - \frac{(v - v')(v - v')}{|v - v'|^2} \right) \tag{2.8}
\]

Note that the Fokker-Planck collision operator is only valid for large values of \( \Lambda \) in the Coulomb logarithm. In partially ionized plasmas, Coulomb logarithm is reduced due to electron-neutral collisions \([134, 135]\). If all three species are in equilibrium with the full collision operators on the right hand side of Eqs (2.3) to (2.5), i.e. in a collisional equilibrium, the unique equilibrium gives local Maxwellian distributions for all three species at a single temperature and fluid velocity. The equilibrium is the zeroth order distribution

\[
f_{j}^{(0)}(x, v, t) = \frac{n_j(x, t)m_j^{3/2}}{(2\pi k_B T(x, t))^{3/2}} \exp \left( -\frac{m_j(v - u(x, t))^2}{2k_B T(x, t)} \right) \tag{2.9}
\]

where \( n_j(x, t) \) is the particle number density for species \( j \), \( T(x, t) \) and \( u(x, t) \) are temperature and fluid velocity for the equilibrium respectively. In the transport theory, different species can have separate mean velocities \( u_j \). Electron and ion temperatures can separate if differential heating and cooling mechanisms are applied, e.g. in laser driven plasmas, electrons are heated through the inverse-bremsstrahlung process and have radiative cooling as discussed in Section 2.2.1. The solution for VFP equations can be of the form \( f_j(x, v, t) = f_{j}^{(0)}(x, v, t) + f_{j}^{(1)}(x, v, t) \) where \( |f^{(1)}| \ll |f^{(0)}| \), if one is interested in changes occurring in time scales much greater than the collision time and length scales much greater than mean free path, i.e. the Knudsen number satisfies \( N_{K_j} = \lambda_j/L \ll 1 \). If one ignores the second order quadratic term in...
In the collision operator, the magnitude of the two terms in $C_{eI_1}$ can be estimated

$$\frac{\partial}{\partial v} f_{i_1}^{(0)}(v') f_e^{(1)}(v) \sim \frac{1}{v_{th,e}} f_{i_1}^{(0)}(v') f_e^{(1)}(v) \sim \sqrt{\frac{m_e}{k_B T}} f_{i_1}^{(0)}(v') f_e^{(1)}(v)$$ (2.10)

and

$$\frac{m_e}{m_{I_1}} \frac{\partial}{\partial v'} f_{i_1}^{(0)}(v') f_e(v) \sim \frac{m_e}{m_{I_1}} \frac{1}{v_{th,I_1}} f_{i_1}(v') f_e(v) \sim \sqrt{\frac{m_e^2/m_{I_1}}{k_B T}} f_{i_1}(v') f_e(v)$$ (2.11)

Since $m_e \ll m_{I_1}$, we have

$$\frac{\partial}{\partial v} f_{i_1}(v') f_e(v) \gg \frac{m_e}{m_{I_1}} \frac{\partial}{\partial v'} f_{i_1}(v') f_e(v)$$ (2.12)

If one assumes that the ions have a small thermal velocity such that in the collisional integrals $f_{i_1}^{(0)}(v) \sim n_{I_1} \delta(v - u)$ and $f_{i_2}^{(0)}(v) \sim n_{I_2} \delta(v - u)$, then

$$C_{eI_1}(f_e, f_{I_1}) + C_{eI_2}(f_e, f_{I_2}) = \frac{14\pi n_{I_1} Z_{I_1}^2 e^4 \ln \Lambda_{eI_1}}{m_e^2} \frac{\partial}{\partial v} \overrightarrow{U}(v, u) \cdot \frac{\partial}{\partial v'} f_e^{(1)}(v)$$

$$+ \frac{14\pi n_{I_2} Z_{I_2}^2 e^4 \ln \Lambda_{eI_2}}{m_e^2} \frac{\partial}{\partial v} \overrightarrow{U}(v, u) \cdot \frac{\partial}{\partial v'} f_e^{(1)}(v)$$

$$= \frac{14\pi (n_{I_1} Z_{I_1}^2 + n_{I_2} Z_{I_2}^2) e^4 \ln \Lambda_{eI_1}}{m_e^2} \frac{\partial}{\partial v} \overrightarrow{U}(v, u) \cdot \frac{\partial}{\partial v'} f_e^{(1)}(v)$$

$$= \frac{14\pi n_{I_1} Z_{eff}^2 e^4 \ln \Lambda_{ee}}{m_e^2} \frac{\partial}{\partial v} \overrightarrow{U}(v, u) \cdot \frac{\partial}{\partial v'} f_e^{(1)}(v)$$ (2.13)

where the effective charge state $Z_{eff}$ is

$$Z_{eff} = \frac{\sum_j Z_j^2 n_j \ln \Lambda_{ej}}{n_e \ln \Lambda_{ee}} \approx \frac{\sum_j Z_j^2 n_j}{n_e} = \frac{Z^2}{Z}$$

Thus for multiple ion species, in the expression for the coefficients of electron transport, e.g. in Eqs (1.2), (1.5), (1.6) and (2.1), $Z \ln \Lambda$ needs to be replaced by $(Z_{eff}^2 \ln \Lambda)/\langle Z \rangle$.

The definition of the effective charge state for ion transport is not generally available, unless for the case where the masses of ion species are very different \[132\]. In Section \[2.1.1\], the electron transport equations are discussed. In general, ion transport coefficients can also be calculated \[104\] but are not discussed in this thesis.
2.1.1 Electron transport equations

The first order perturbation for electron distribution function $f_e^{(1)}$ is proportional to effects that disturb the distribution, i.e. gradients, electric fields, etc. Two categories of notations have been used for describing the transport as studied in Reference [105]. In Notation I, the current $\vec{J}$ and the electron heat flow $\vec{q}_T$ are expressed as a linear combinations of $\nabla n_e$ (or $\nabla p_e$), $\nabla T_e$ and $\vec{E}$. In Notation II, electric field $\vec{E}$ is expressed in terms of $\nabla p_e$, $\nabla T_e$, and $\vec{J}$, and the intrinsic heat flow $\vec{q}$ ($\vec{q}$ is defined relative to the local reference frame of the electrons, while $\vec{q}_T$ is defined relative to the ion center of mass frame, $\vec{q}_T$ and $\vec{q}$ are related by $\vec{q}_T = \vec{q} - (5k_B T_e/2e) \vec{J}$) is expressed in terms of $\nabla T_e$ and $\vec{J}$. Combining Notation II and the relation between $\vec{q}_T$ and $\vec{q}$, $\vec{E}$ and $\vec{q}_T$ have the following transport relations:

\begin{align}
\vec{E} &= -\frac{\nabla p_e}{n_e e} + \frac{\vec{J} \times \vec{B}}{n_e e c} + \frac{m_e \vec{\alpha} \cdot \vec{J}}{n_e e^2 \tau_e} - \frac{k_B \vec{\beta} \cdot \nabla T_e}{e} \\
\vec{q}_T &= -\frac{n_e k_B^2 T_e \tau_e}{m_e} \kappa \cdot \nabla T_e - \frac{k_B T_e}{e} \vec{\beta} \cdot \vec{J} - \frac{5k_B T_e}{2e} \vec{J}
\end{align}

(2.14) (2.15)

where $\vec{\alpha}$, $\vec{\beta}$ and $\kappa$ are the dimensionless electrical resistivity and thermoelectric and thermal conductivity tensors, respectively, and obey the relation

\begin{align}
\vec{\varphi} \cdot \vec{s} &= \varphi \parallel \hat{b} (\hat{b} \cdot \vec{s}) + \varphi \perp \hat{b} \times (\vec{s} \times \hat{b}) \pm \varphi \land \hat{b} \times \vec{s} \\
&= \varphi \perp \vec{s} + (\varphi \parallel - \varphi \perp) \hat{b} (\hat{b} \cdot \vec{s}) \pm \varphi \land \hat{b} \times \vec{s}
\end{align}

(2.16) (2.17)

where the negative sign applies only to the case $\vec{\varphi} = \vec{\alpha}$. The second term and third term on the right hand side of Eq (2.16) are known as the cross-field term and Righi-Leduc term, respectively. Eq (2.14) is also known as the generalized Ohm’s law.
The numerical coefficients have been tabulated in References [105] and [105]. Reference [104] used an expansion of $f_1$ in Laguerre polynomials with a truncation after the first three terms. References [105] used a finite difference representation of the equation for computation and also reviewed the general numerical methods for computing transport coefficients. The parallel components $\alpha_{\parallel}, \beta_{\parallel}, \kappa_{\parallel}$ only depend on $Z_{\text{eff}}$, while other components generally depend on both $Z_{\text{eff}}$ and $\chi_e$. For $\chi_e \to \infty$, the asymptotic expressions for $\alpha_{\wedge}$ and $\beta_{\perp}$ only depend on $\chi_e$, i.e. $\alpha_{\wedge} \to (8/9)\pi^{4/3}(3/4)^{5/3}\chi_e^{-2/3} \simeq 2.53\chi_e^{-2/3}$, $\beta_{\perp} \to (20/9)\pi^{4/3}(3/4)^{5/3}\chi_e^{-5/3} \simeq 6.33\chi_e^{-5/3}$. The definitions $\delta_{\perp} = \alpha_{\wedge}/\chi_e$, $\delta_{\wedge} = (\alpha_{\perp} - \alpha_{\parallel})$, $\gamma_{\perp} = \beta_{\wedge}/\chi_e$ and $\gamma_{\wedge} = (\beta_{\parallel} - \beta_{\perp})/\chi_e$ have been introduced to identify electric-current-driven magnetic field advection velocity and energy advection velocity [122, 136]. For $\chi_e \to 0$, the fit functions in Reference [105] have the correct asymptotic behavior $\delta_{\perp} \to 0$ and $\gamma_{\perp} \to 0$. However, Reference [136] pointed out that $\gamma_{\wedge} \to 0$ (this is required for recovering the induction equation for resistive MHD) and $\delta_{\wedge} \to 0$ for $\chi_e \to 0$, which cannot be reproduced by the fit functions in Reference [105]. Reference [105] has also provided more accurate fit functions for $\chi_e \to 0$.

The four terms on right hand side of Eq (2.14) are Biermann battery term, Hall term, resistive term and thermoelectric term, respectively. The nominal ratio of Hall term to Biermann battery term is $2/\beta_e$, where $\beta_e = p_e/[B^2/(8\pi)]$ (similar to Eq (1.4) but with electron pressure only). The thermoelectric term has the same order of magnitude as the Biermann battery term. The nominal ratio of resistive term to Biermann battery term is

$$\frac{m_e c B}{4\pi n_e e \tau_e k_B T_e} = \frac{\sqrt{2} m_e^{1/2} c B e^3 Z \ln \Lambda}{3 \sqrt{\pi} (k_B T_e)^{5/2}} = 8.2 \times 10^{-3} \frac{B[\text{gauss}]}{(T_e[\text{eV}])^{5/2}}$$

and is also equal to $2/(\beta_e \chi_e)$.

In the case where $\beta_e \gg 1$ and $\beta_e \chi_e \gg 1$ but $\chi_e \gtrsim 1$, we have
\[ \vec{E} = -\frac{\nabla p_e}{n_e e} - \frac{k_B \beta \cdot \nabla T_e}{e} \quad (2.18) \]
\[ \vec{q}_T = -\frac{n_e k_B^2 T_e \tau_e}{m_e} \kappa \cdot \nabla T_e \quad (2.19) \]

In the case where $\beta_e \gg 1$ and $\chi_e \ll 1$ but $\beta_e \chi_e \lesssim 1$, we have
\[ \vec{E} = -\frac{\nabla p_e}{n_e e} + \frac{m_e \alpha \parallel \vec{J}}{n_e^2 \tau_e} - \frac{k_B \beta \parallel \nabla T_e}{e} \quad (2.20) \]
\[ \vec{q}_T = -\frac{n_e k_B^2 T_e \tau_e \kappa}{m_e} \nabla T_e - \frac{k_B T_e (\beta \parallel + 5/2)}{e} \vec{J} \quad (2.21) \]

For plasmas with single $Z$, the $\beta \parallel \nabla T_e$ term does not produce any magnetic field. However, in the mix region for different material where $\nabla Z$ is present, $\beta \parallel \nabla T_e$ can generate additional thermoelectric magnetic source term [137]. Magnetic field generation in double ablation fronts with different ionization state has also been observed in a recent experiment [138].

### 2.2 Radiative transport

For the equation of radiative transport, I follow the notation in Reference [139]. The Boltzmann transport equation describes the flow of photons in six-dimensional phase space $(\mathbf{x}, \mathbf{p})$.

\[ \frac{\partial f}{\partial t} + \sum_i \left( \dot{x}_i \frac{\partial f}{\partial x_i} + \dot{p}_i \frac{\partial f}{\partial p_i} \right) = S \quad (2.22) \]

where $f(\mathbf{x}, \mathbf{p}, t)$ is the density of photons in phase space, and $S$ describes the creation and destruction at a local point in the phase space. The vector form of Eq (2.22) is

\[ \frac{\partial f}{\partial t} + \dot{\mathbf{x}} \cdot \nabla f + \dot{\mathbf{p}} \cdot \nabla_p f = S \quad (2.23) \]
Let the energy carried by photons with momentum $p = h\nu/c$, moving in a particular direction $\hat{n}$ through a differential area $dA$, into a differential solid angle $d\Omega$, in a differential time $dt$ and a frequency $d\nu$ be $dE_\nu$. Then the specific intensity $I_\nu$ is defined as

$$I_\nu(x, \hat{n}, t) = \frac{dE_\nu}{dA \cos \theta d\Omega d\nu dt},$$

where $\theta$ is the angle between the normal direction of the area $dA$ and $\hat{n}$. The specific intensity has dimensions erg s$^{-1}$cm$^{-2}$ster$^{-1}$Hz$^{-1}$. The specific intensity generally depends on location $x$ in space and the momentum $p$ of photons (or $p = \hat{n}h\nu/c$). The photons traveling in $\hat{n}$ crossing $dA$ in $dt$ comes from a real space volume $dV = cdAdt \cos \theta$. The photons with momentum around $p$ and interval $dp$ traveling into differential solid angle $d\Omega$ have a momentum space volume $dV_p = p^2 dp d\Omega$. Therefore, the differential energy in our definition of specific intensity is

$$dE_\nu = h\nu f(x, p, t)dV_p dV = h\nu p^2 dp d\Omega cdAdt \cos \theta = \frac{h^4 \nu^3}{c^2} f(x, p, t) dA \cos \theta dv dt$$

Combining Eqs (2.24) and (2.25), the specific intensity and the phase space density of photons are related

$$I_\nu(x, \hat{n}, t) = \frac{h^4 \nu^3}{c^2} f(x, p, t), \quad f(x, p, t) = \frac{c^2}{h^4 \nu^3} I_\nu(x, \hat{n}, t)$$

Assume that the gravitational potential gradient is negligible and the medium is dispersionless, using $\dot{x} = c\hat{n}$ and Eqs (2.23) and (2.26), then

$$1 \frac{\partial I_\nu}{c \partial t} + \hat{n} \cdot \nabla I_\nu = \frac{h^4 \nu^3 S}{c^3}$$

The creation rate $S$ includes the contributions from the absorption processes, the scattering processes and the thermal emission processes. The number of photons lost to the differential volume is

$$dn_l = \alpha f dV_p dV$$
where $\alpha$ is the fraction of particles present that are lost due to scattering and absorption. Assume that the thermal emission processes are isotropic so that the number of photon gained in the volume and radiated into a unit solid angle is

$$dn_{gt} = \frac{\epsilon dV_p dV}{4\pi}$$

(2.29)

where $\epsilon$ is the thermal emission per unit phase space volume. The scattering processes are characterized by the redistribution function $R(p, p', \Omega, \Omega')$, which is the probability that a photon with momentum $p'$ from solid angle $\Omega'$ being scattered into a solid angle $\Omega$ with momentum $p$. The redistribution function is normalized so that

$$\int_0^{\infty} dp \int_0^{\infty} dp' \int_0^{4\pi} d\Omega \int_0^{4\pi} d\Omega' R(p, p', \Omega, \Omega') = 1$$

(2.30)

The number of particles gained from the scattering processes is

$$dn_{gs} = \frac{\sigma'}{4\pi} dV_p dV \int_0^{\infty} dp' \int_0^{4\pi} d\Omega' R(p, p', \Omega, \Omega') f(p', \Omega')$$

(2.31)

where $\sigma'(p, \Omega)$ is the fraction of photons entering the volume $dV_p dV$ that undergo the scattering. The net change of particles $dn$ in volume $dV_p dV$ is obtained by Eqs (2.28), (2.29) and (2.31), with phase space density replaced by specific intensity using Eq (2.26). Thus

$$dn = \frac{SdV_p dV}{c} = dn_{gt} + dn_{gs} - dn_l$$

$$= \frac{c^2}{h^4 \nu^3} \left[ \frac{h^4 \nu^3}{c^2} \frac{\epsilon}{4\pi} + \frac{\sigma' h}{4\pi c} \int_0^{\infty} dp' \int_0^{4\pi} d\Omega' \left( \frac{\nu}{\nu'} \right)^3 R(p, p', \Omega, \Omega') I_{\nu'}(\Omega') - \alpha I_{\nu}(\Omega) \right] dV_p dV$$

(2.32)

The volume emissivity $j_\nu$, the mass scattering coefficient $\sigma_\nu$, and the mass absorption coefficient $\kappa_\nu$ and the replaced $R$ are defined such that

$$j_\nu = \frac{h^4 \nu^3 \epsilon}{4\pi c^2 \rho}, \quad \sigma_\nu = \frac{\sigma'}{\rho}, \quad R(\nu, \nu', \Omega, \Omega') = \frac{h(\nu/\nu')^3}{c} R(p, p', \Omega, \Omega')$$

$$\kappa_\nu + \sigma_\nu = \frac{\alpha}{\rho}$$

(2.33)
Thus Eq (2.27) becomes

\[
\frac{1}{c} \frac{\partial I_\nu}{\partial t} + \hat{n} \cdot \nabla I_\nu = \rho \left[ j_\nu - (\kappa_\nu + \sigma_\nu) I_\nu(\Omega) + \frac{\sigma_\nu}{4\pi} \int_0^\infty d\nu' \oint_{4\pi} d\Omega' R(\nu, \nu', \Omega, \Omega') I_{\nu'}(\Omega') \right]
\]

(2.34)

Assume that the gas/plasma is in thermal equilibrium with its surroundings (LTE), so that one can use Kirchhoff’s law for the relationship between the emissivity and absorptivity, namely,

\[ j_\nu = \kappa_\nu B_\nu(T) \]

where \( B_\nu(T) \) is the Planck function and only depends on the local temperature of the gas. Then Eq (2.34) becomes

\[
\frac{1}{c} \frac{\partial I_\nu}{\partial t} + \hat{n} \cdot \nabla I_\nu = \rho \left[ \kappa_\nu B_\nu(T) - (\kappa_\nu + \sigma_\nu) I_\nu(\Omega) + \frac{\sigma_\nu}{4\pi} \int_0^\infty d\nu' \oint_{4\pi} d\Omega' R(\nu, \nu', \Omega, \Omega') I_{\nu'}(\Omega') \right]
\]

(2.35)

If one focuses on the case of no scattering, i.e. \( \sigma_\nu = 0 \), then Eq (2.35) becomes

\[
\frac{1}{c} \frac{\partial I_\nu}{\partial t} + \hat{n} \cdot \nabla I_\nu = \rho \kappa_\nu (B_\nu - I_\nu)
\]

(2.36)

In the case of a steady state and plane-parallel approximation Eq (2.36) becomes

\[
\frac{\partial I_\nu}{\partial x} = \rho \kappa_\nu (B_\nu - I_\nu)
\]

(2.37)

The optical depth \( \tau_\nu \) is defined as \( d\tau_\nu = \kappa_\nu \rho dx \). With this definition Eq (2.37) becomes

\[
\frac{\partial I_\nu}{\partial \tau_\nu} = -I_\nu + B_\nu
\]

(2.38)

Eq (2.38) is used for solving the X-ray as shown in Section 3.3.

2.2.1 Multi-group Diffusion

Solving Eq (2.36) for intensity evolution requires a grid on six-dimensional phase space, which is computationally expensive and usually impossible. Diffusion approxi-
imation has been widely used to reduce the equation to three-dimensional real space. By integrating Eq (2.36) over the solid angle, we get

\[
\frac{1}{c} \frac{\partial u_\nu}{\partial t} + \nabla \cdot \vec{F}_\nu = \rho \kappa_\nu (4\pi B_\nu - u_\nu) \tag{2.39}
\]

where \( u_\nu = \int d\Omega I_\nu \) is the specific radiative energy density and \( F_\nu = \int d\Omega \hat{n} I_\nu \) is the specific radiative flux. By multiplying \( \hat{n} \) on Eq (2.36) and integrating over the solid angle, we get

\[
\frac{1}{c} \frac{\partial F_\nu}{\partial t} + \nabla \cdot \vec{K} = -\rho \kappa_\nu F_\nu \tag{2.40}
\]

where \( K_\nu = \int d\Omega \hat{n} \hat{n} I_\nu \) is the specific radiative pressure tensor. If the material is sufficiently optically thick that radiation diffuses down the radiative temperature gradient through repeated absorption and emission, i.e the photo mean free path \((\rho \kappa_\nu)^{-1}\) satisfies \( L \gg (\rho \kappa_\nu)^{-1} \) (where \( L \) is the characteristic length scale over which the macroscopic properties of the plasma change), then \( F_\nu = -(\rho \kappa_\nu)^{-1} \nabla \cdot K_\nu \). If the specific intensity is close to isotropic, then the specific radiative pressure tensor can be expressed explicitly in terms of the energy density \( K_\nu \approx u_\nu / 3 \). Thus, the equation for specific radiative energy density is

\[
\frac{1}{c} \frac{\partial u_\nu}{\partial t} - \nabla \cdot \left( \frac{1}{3 \rho \kappa_\nu} \nabla u_\nu \right) = \rho \kappa_\nu (4\pi B_\nu - u_\nu) \tag{2.41}
\]

The specific radiative energy density can be integrated over finite number of frequency groups

\[
\frac{1}{c} \frac{\partial u_g}{\partial t} - \nabla \cdot \left( \frac{1}{3 \sigma_{t,g}} \nabla u_g \right) + \sigma_{a,g} u_g = \sigma_{e,g} a T_e ^{15} \frac{15}{\pi} \left[ P(x_{g+1}) - P(x_g) \right] \tag{2.42}
\]

where \( u_g = \int_{\nu_g}^{\nu_{g+1}} d\nu u_\nu, x = h\nu / (k_B T_e) \), and the Planck integral is \( P(x) = \int_0^x dx' (x')^3 / [\exp(x') - 1] \). The transport opacity \( \sigma_{t,g} \) is \( \sigma_{t,g}^{-1} = \int_{\nu_g}^{\nu_{g+1}} d\nu \sigma_{t,\nu} B_\nu / \int_{\nu_g}^{\nu_{g+1}} B_\nu \), the emission opacity is \( \sigma_{e,g} = \int_{\nu_g}^{\nu_{g+1}} d\nu \sigma_{e,\nu} B_\nu / \int_{\nu_g}^{\nu_{g+1}} B_\nu \), and absorption opacity is \( \sigma_{a,g} = \int_{\nu_g}^{\nu_{g+1}} d\nu \sigma_{a,\nu} B_\nu / \int_{\nu_g}^{\nu_{g+1}} B_\nu \).
2.3 FLASH code

FLASH [57, 56] is a publicly available, multi-physics, highly scalable parallel, finite-volume Eulerian code and framework whose capabilities include: adaptive mesh refinement (AMR), multiple hydrodynamic and MHD solvers, implicit solvers for diffusion using the HYPRE library and laser energy deposition. FLASH is capable of using multi-temperature equation of states and multi-group opacity. Extensive HED laboratory plasma (HEDLP) capabilities [140] including magnetic field generation via the Biermann battery term has been implemented and studied in FLASH recently [58].

To simulate laser-driven HEDP experiments, a 3T treatment, i.e. $T_{\text{rad}} \neq T_{\text{ion}} \neq T_{\text{ele}}$, is usually required. The equations which FLASH solves to describe the evolution of the 3T magnetized plasma are

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 \quad (2.43)
\]

\[
\frac{\partial \rho \mathbf{v}}{\partial t} + \nabla (\rho \mathbf{v} \mathbf{v}) - \frac{1}{4\pi} \mathbf{B} \mathbf{B} + \nabla P_{\text{tot}} = 0 \quad (2.44)
\]

\[
\frac{\partial \rho E_{\text{tot}}}{\partial t} + \nabla \cdot (\mathbf{v} (\rho E_{\text{tot}} + P_{\text{tot}}) - \frac{1}{4\pi} \mathbf{B} (\mathbf{v} \cdot \mathbf{B})) - \frac{1}{4\pi} \nabla \cdot (\mathbf{B} \times (\eta \nabla \times \mathbf{B})) - \frac{1}{4\pi} \nabla \cdot (\mathbf{B} \times \frac{c \nabla P_e}{e n_e}) = -\nabla \cdot \mathbf{q} + S \quad (2.45)
\]

\[
\frac{\partial \mathbf{B}}{\partial t} + \nabla \cdot (\mathbf{v} \mathbf{B} - \mathbf{B} \mathbf{v}) = -\nabla \times (\eta \nabla \times \mathbf{B}) + \frac{c}{e} \nabla \times \frac{\nabla P_e}{n_e} \quad (2.46)
\]

where the total pressure is given by $P_{\text{tot}} = P_{\text{ion}} + P_{\text{ele}} + P_{\text{rad}} + B^2/(8\pi)$, and the total specific (per-mass) internal energy $E_{\text{tot}} = e_{\text{ion}} + e_{\text{ele}} + e_{\text{rad}} + B^2/(8\pi) + \mathbf{v} \cdot \mathbf{v}/2$. The total heat flux $\mathbf{q}$ is the summation of electron heat flux $\mathbf{q}_e = -\kappa_e \nabla T_{\text{ele}}$ where

\[
\kappa_e = \left(\frac{8}{\pi}\right)^{3/2} \frac{h_{\mathbf{B}}^{7/2}}{e^4 \sqrt{m_e}} \left(\frac{1}{1 + 3.3/\mathcal{Z}}\right) \frac{T_{\text{e}}^{5/2}}{\mathcal{Z} \ln \mathcal{A}_e} \quad (2.47)
\]

and radiation flux $\mathbf{q}_r$, where $\kappa$ is the Spitzer [141] electron heat conductivity. The
first term on the right hand side of Eq (2.46) contains the Spitzer magnetic resistivity $\eta$. The second term on the right hand side of Eq (2.46) is the Biermann Battery term, which generates the magnetic field even if there is no seed magnetic field initially. The energy equations for the three components are

$$\frac{\partial}{\partial t} (\rho c_{\text{ion}}) + \nabla \cdot (\rho c_{\text{ion}} \mathbf{v}) + P_{\text{ion}} \nabla \cdot \mathbf{v} = \rho \frac{c_{v,\text{ele}}}{\tau_{ei}} (T_{\text{ele}} - T_{\text{ion}})$$

(2.48)

$$\frac{\partial}{\partial t} (\rho c_{\text{ele}}) + \nabla \cdot (\rho c_{\text{ele}} \mathbf{v}) + P_{\text{ele}} \nabla \cdot \mathbf{v} = \rho \frac{c_{v,\text{ele}}}{\tau_{ei}} (T_{\text{ion}} - T_{\text{ele}}) - \nabla \cdot \mathbf{q}_{\text{ele}}$$

$$+ Q_{\text{abs}} - Q_{\text{emis}} + Q_{\text{las}} + Q_{\text{ohm}}$$

(2.49)

$$\frac{\partial}{\partial t} (\rho c_{\text{rad}}) + \nabla \cdot (\rho c_{\text{rad}} \mathbf{v}) + P_{\text{rad}} \nabla \cdot \mathbf{v} = \nabla \cdot \mathbf{q}_{\text{rad}} - Q_{\text{abs}} + Q_{\text{emis}}$$

(2.50)

where $c_{v,\text{ele}}$ is the electron specific heat, $\tau_{ei}$ the ion-electron Coulomb collision time. The $Q_{\text{abs}}$ (absorption) and $Q_{\text{emis}}$ (emission) describe the energy transfer between electrons and radiation, which is modeled using the multi-group flux-limited radiation diffusion. Laser absorption term $Q_{\text{las}}$ is computed using the ray-tracing in the geometric optics approximation via the inverse-Bremsstrahlung process. $Q_{\text{ohm}}$ is the rate of electron energy increase due to Ohmic heating. In addition to inverse-bremsstrahlung, there are other coronal processes in the target ablation, such as Raman and Brillouin scattering, and resonance absorption [142, 143]. This work ignores those anomalous absorption processes and only consider the inverse-bremsstrahlung process. The auxiliary equations Eqs (2.48)-(2.50) are advanced in time such that the distribution of energy change due to the work and the total shock-heating is based on the pressure ratio of the components, which is a method implemented in FLASH inspired by the radiation-hydrodynamics code RAGE [144]. The opacity and EoS tables are computed with PROPACEOS [\,*]. Only the terms in the $\chi_e = 0$ case are included in the FLASH simulations in this work.

*PROPACEOS is available at [http://www.prism-cs.com](http://www.prism-cs.com)
Nonlocal effects in electron heat flux and radiative flux are incorporated using the flux limiter. Using the flux limiter is able to avoid the numerical discontinuity problem and the physics problem of reduced collisionality when gradients are large. In the flux limiter, the transport coefficients are reduced when the mean free path $L_{\text{mpf}}$ is comparable to the scale height $|\nabla \log A|^{-1}$ of the corresponding variable $A$. One physical interpretation of such reduction is that the advection speed corresponding to the transport cannot be faster than a fraction of the thermal speed of microscopic particles.

The initialization and non-ideal MHD in FLASH were modified by the author and collaborators to facilitate the simulations for HED experiments in this thesis. The modification is not part of the publicly available version of FLASH.

### 2.3.1 Non-ideal terms in generalized Ohm’s law

The right hand side of Eq (2.46) contains non-ideal terms in generalized Ohm’s law. These terms are usually computed using the explicit solver, where the flux of the corresponding term is calculated and used to update the magnetic field. If the magnetic resistivity is large, one might use the implicit solver $^{145}$ in HYPRE library $^{146}$.

A straightforward implementation of the Biermann term leads to non-convergent results near shocks $^{147}$. In the simulations for the two experiments studied in this thesis in Chapter 4 and 5, the Biermann term is turned off in the cells near the shocks. A converging scheme has been proposed but requires solving the electron entropy equation $^{58}$.

Magnetic precursors can be generated near shocks $^{58}$. The length scale for the

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$^1$Documentation for implicit solver for curl-curl formulation of the Maxwell equation: https://hypre.readthedocs.io/en/latest/solvers-maxwell.html
resistive magnetic precursor, in which a Biermann-generated field in the shock leaks resistively upstream, is

\[ \lambda_B = 8.2 \times 10^5 \text{cm} \times \frac{(0.33Z + 0.18) \ln \Lambda_e}{(T\text{[eV]})^{3/2}u[\text{cm/s}]} \] (2.51)

The length scale for the thermal magnetic precursor, in which a field is generated by the Biermann effect ahead of the shock front owing to gradients created by the shock’s electron thermal conduction precursor, is

\[ \lambda_T = 2.2 \times 10^{21} \text{cm} \times \frac{(T_e[\text{eV}])^{5/2}}{(Z + 3.3) \ln \Lambda_e n_e[\text{cm}^{-3}]u[\text{cm/s}]} \] (2.52)

2.4 Kinetic simulations

The electron transport theory as discussed in Section 2.1.1 and FLASH code as discussed in Section 2.3 are good approximation when the time scale is much greater than the collision time and length scales much greater than mean free path. In addition, the time scale for reaching ionization-recombination balance and the time scale for the distribution created by laser heating to thermalize has to be much smaller than the collision time. If these scale ordering conditions are not satisfied, kinetic simulations should be performed. For example, kinetic physics has the potential to impact the performance of indirect-drive inertial confinement fusion (ICF) experiments \[148\].

One way to solve the VFP system is to incorporate a spherical harmonic expansion of the electron distribution function \[149\] \[150\] \[151\]. Combining physics of kinetic electrons and atomic processes leads to the VBFP framework \[133\]. The inverse-bremsstrahlung process has action on the electron distribution, known as the Langdon operator \[152\].

Alternatively, PIC codes such as EPOCH \[153\] and PSC \[154\] have been developed with the binary collision operator. PIC codes resolve the distribution function in
momentum space by using sample pseudo-particles, which eliminates the momentum space grid. Thus PIC simulations are suitable for performance scaling on large supercomputers. However, PIC codes are subject to numerical instabilities such as the finite grid instability [155]. In order to overcome the large computational cost due to the separation between electron/ion skin depth due to the large mass ratio \( m_i/m_e = 1836 \), a compressed electron-ion mass ratio can be employed [130, 156]. In the simulations using compressed mass ratio, the substitution \( m_e \rightarrow m^* m_e, \ln \Lambda_e \rightarrow (m^*)^{-1/2} \ln \Lambda_e \) is used and the electron temperature is initialized with the physical value. With this substitution, the energy partition such as \( \beta \) in Eq (1.4), the magnetic Reynolds number as in Eq (1.3), the electron Hall parameter as in Eq (1.6), and the electron thermal conductivity as in Eq (2.47) remain unchanged. However, the electron mean free path \( \lambda_e \) as in Eq (2.1) is scaled \( \lambda_e \rightarrow \lambda_e \sqrt{m^*} \). Thus the compressed mass ratio should work well for electron transport as long as \( N_{Ke} \ll 1 \).

The PIC method is also widely used in the simulations of collisionless plasmas in laser plasma accelerators and astrophysical problems. To improve the reliability of PIC simulations and analyses especially in the relativistic case, I have developed new numerical methods, as discussed in Chapter 6.
Chapter 3

Diagnostics for high-energy-density magnetized plasmas

Several diagnostics for HED plasmas and the simulation techniques are studied in this chapter, including proton radiography in Section 3.1, Thomson scattering in Section 3.2, and x-ray framing camera in Section 3.3.

3.1 Proton radiography

Proton radiography \[71, 72\] is a diagnostic tool for temporal-resolved and spatial-resolved studies of the electromagnetic field structures in ICF and HED plasmas. The deflection of protons in electromagnetic fields can be calculated by solving the Newton-Lorentz equation

\[
\frac{d(m_p v)}{dt} = e \left( E + \frac{v}{c} \times B \right) \tag{3.1}
\]

In a typical MHD fluid, \( E \approx (v_h/c)B \), where \( v_h \) is the characteristic hydrodynamical velocity of the fluid. The ratio of electric force to magnetic force is \( E/[(v_p/c)B] \approx v_h/v_p \). For a proton with energy larger than \( \sim \) MeV, the proton speed \( v_p \) is much larger than \( v_h \), so electric field \( E = 0 \) approximation is used in the modeling.

The information about the morphology and strengths of electric and magnetic field is coded in the deflection angle of the proton beams and alters the proton flux after interaction with electromagnetic field. The proton flux is then recorded on a detector. For small angle deflection, the deflection angle of protons by magnetic field
is given by Eq (1) in Reference [157]

$$\alpha_B = 1.80 \times 10^{-2} \text{rad} \times \left( \frac{E_p}{14.7 \text{MeV}} \right)^{-1/2} \times \left( \frac{B}{10^5 \text{G}} \right) \times \left( \frac{l_i}{0.1 \text{cm}} \right) \tag{3.2}$$

where $E_p$ is the energy of proton, $B$ is the strength of magnetic field, and $l_i$ is the longitudinal size of the interaction region. For the case where strong electric field exists in small scale, the small deflection angle of protons by transverse electric field is

$$\alpha_E = 2.04 \times 10^{-1} \text{rad} \times \left( \frac{E_p}{14.7 \text{MeV}} \right)^{-1} \times \left( \frac{E}{10^5 \text{StatV/cm}} \right) \times \left( \frac{l_i}{0.1 \text{cm}} \right) \tag{3.3}$$

The contrast field is defined in the following equation [157]

$$\Lambda(x_\perp) = \frac{\Psi(x_\perp) - \psi_0}{\psi_0} \tag{3.4}$$

where $\psi_0$ is the unperturbed proton flux, which is uniform by assumption, $\Psi(x_\perp)$ is the perturbed proton flux by both deflection and diffusion, and $x_\perp$ is the position vector on the image plane. Eq (19) in Reference [157] gives the expression for the contrast field as a map of MHD current

$$\Lambda(x_\perp) = \frac{e r_i (r_s - r_i)}{r_s \sqrt{2 m_p c^2 E_p}} \hat{z} \cdot \int dz \nabla \times B \tag{3.5}$$

where $r_s$ is the distance between the interaction region and the screen, $r_i$ is the distance between the source and the image plate, and $\hat{z}$ is the unit vector long the line of sight.

Proton radiography has been used to characterize the electromagnetic fields and carry out measurements in a variety of experiments, including ICF implosion capsules [158, 159, 160, 161, 162, 163, 164, 165, 166, 167], magnetic reconnection [127, 168, 169, 170, 171, 172, 173], self-generated magnetic fields through Biermann battery term [67]
In ICF and HED experiments, two distinct types of proton sources have been developed for high performance diagnostics. First, in a capsule implosion, DD (3MeV) and D\textsuperscript{3}He (14.7MeV) protons from fusion reaction driven by multiple laser beams.

\[ \text{D} + \text{D} \rightarrow \text{T} + p \ (3.0\text{MeV}) \]

\[ \text{D} + \text{\textsuperscript{3}He} \rightarrow \text{\textsuperscript{4}He} + p \ (14.7\text{MeV}) \]

The actual spectrum is typically an up-shifted gaussian distribution, FWHM=320keV centered at 3.6MeV for DD protons, FWHM=670keV centered at 15.3MeV for D\textsuperscript{3}He protons. Typically, the emitting position of protons follows a 3D gaussian distribution with e-fold radius equal to 20\( \mu \)m, and the burn time is 150ps. The protons leave tracks in CR-39 which is etched and scanned to get the absolute location and track characteristics of each proton. Second, broadband proton beams up to 60MeV are driven by ultra-intense (> 10\textsuperscript{18}W/cm\textsuperscript{2}) short pulse laser beam through Target Normal Sheath Acceleration (TNSA) mechanism, and the proton flux is recorded on the radiochromic film pack with a sequence of proton energies. In general, the TNSA proton backlighter offers better spatial and temporal resolution, while the D\textsuperscript{3}He fusion-based techniques offers better spatial uniformity and energy resolution.

The proton images are smeared by a few factors: (1) Spatial smearing: the finite size of the proton source, which is \( \sim 45\mu \text{m} \) for the fusion protons and \( \sim 5\mu \text{m} \) for the TNSA protons; (2) Temporal smearing: the pulse duration of the proton source, which is \( \sim 150\text{ps} \) for fusion protons and 1ps for TNSA protons. The pulse duration \( \Delta t \) causes the smearing at length scale \( \Delta l \sim v\Delta t \), where \( v \) is the characteristic speed of the proton.
of the plasma; (3) Spectrum smearing: the energy variation $\Delta E$ of the source proton. Derived from Eq (16) in Reference [157], the variation of deflection angle cause by $\Delta E$ is $\Delta E/(2E)$ times the deflection angle.

The understanding of field structure from proton images is limited by the fact that the images are a two dimensional mapping of the three dimensional field distribution. The general mapping can be nonlinear, degenerate and diffusive. Direct interpretation of the proton images is achievable only under the assumptions of simple field geometries. Some inverse-problem type of general techniques [184, 185, 157] have been developed to infer the integral quantities over the line of sight, e.g. magnetic field perpendicular to line of sight ($\int d\mathbf{z} \times \mathbf{B}$) or MHD current along the line of sight ($\int d\mathbf{z} \cdot \nabla \times \mathbf{B}$). The comprehensive description of the inverse-problem type of techniques for proton images of stochastic magnetic fields has been developed [185]. However, it is still challenging to infer the fields for caustic and diffusive regimes.

There are some general-purpose Monte Carlo toolkits, e.g. MCNP [186] and GEANT4 [157], and tools specifically for HED applications [188, 189], for forward modeling of proton radiography. MCNP and GEANT4 have the features to model the energy lost and collisional scattering of protons in cold matter, but corrections are needed for calculations of plasma stopping power and scattering angle [181]. A more accurate Monte Carlo and ray-tracing tool called MPRAD (multi-MeV proton radiography) for forward modeling of proton radiography is developed by the author to take those corrections related to plasma into account [59]. The feature of MPRAD code [59] is described in Section 3.1.1.
3.1.1 Forward modeling using MPRAD

In MPRAD code [59], relativistic equation of proton motion is solved, i.e. the evolution of position and velocity of the protons in the beam, in electromagnetic field as in Eq (3.1), similar to the features in the existing tools [187, 186, 189, 184]. In addition, the stopping power and Coulomb Scattering is implemented, both in cold matter approximation and weakly interacting plasmas. Pre-calculated quantities are used to speed up the large scale simulations.

Some approximations in the models for Coulomb scattering and stopping power are made. Those are good approximations under the condition that proton energy $E_p > 1\text{MeV}$, electron temperature $(kT_e)/(545\text{eV}) < E_p[\text{MeV}]$, density $\rho/A \ll 10^4\text{g/cc}$ ($A$ is the mass number of the matter), and the transition layer between cold matter and fully ionized plasma is thin compared to the rest of the system. This condition covers the conditions of a range of ICF and HED experiments. MPRAD is written in Python with MPI+OpenMPI parallelization among particles or rays, using Cython and MPI4py package. Cython compilation for the core part of the code is used to improve the computing performance. The output data from plasma-dynamical modeling such as radiation-MHD or PIC can be imported into MPRAD. The Python package from Yt-project [190] is used to read the data from FLASH [56] simulations. Each MPI process gets the whole data set of pre-calculated quantities. Each thread solves the Monte Carlo transport for each particle (or ray transport for each ray) independently. And the binned data (or final quantities for the rays) is collected after each process and thread completes the calculation for the targeted number of particles (or rays).
3.1.1.1 Stopping power calculation

In MPRAD code, the models for stopping power and energy-loss straggling in References [191, 192, 193, 194, 195] are used. The relativistic effects of protons are taken into account to accurately calculate the motion of non-relativistic to highly relativistic protons with $\beta = \frac{v}{c}$, where $v$ is the velocity of the proton, and $c$ is the speed of light. The velocity for the proton beam is assumed to be much larger than the electron thermal speed $v_p > v_{e,th}$, which implies $(E_p/m_p)/(kT_e/m_e) > 1$, i.e.

$$\frac{kT_e}{545\text{eV}} < \frac{E_p}{1\text{MeV}}$$  \hspace{1cm} (3.6)

Under the $v_p > v_{e,th}$ assumption, we further assume that the beam–plasma coupling strength [194] $\gamma_c$ is small, i.e.

$$\gamma_c = 6.8 \times 10^{-3} \left(\frac{\rho}{1\text{g/cc}}\right)^{1/2} \left(\frac{E_p}{1\text{MeV}}\right)^{3/2} A^{1/2} \ll 1$$  \hspace{1cm} (3.7)

where $A$ is the mass number of the matter. For $E_p > 1\text{MeV}$ and $\rho/A \ll 10^4\text{g/cc}$, $\gamma_c$ is always much less than unity, so that the beam–plasma coupling effect can be neglected [194].

For room temperature, the stopping power for cold matter is used. For proton energy $E_p > 1\text{MeV}$, the stopping power in cold matter can be written as [191]

$$\frac{d\langle E_p \rangle}{dx} = -\frac{4\pi e^4 n_e}{\beta^2 m_e c^2} [f(\beta) + a] = -\frac{0.31\text{MeV/cm} \times Z \frac{\rho}{1\text{g/cc}}}{A\beta^2} [f(\beta) + a]$$

where $m_e$ is the mass of electron, $n_e$ is the total electron number density (including both free electrons and bound electrons, which is different from $n^f_e$ in Eq (3.14)), $\rho$ is the density of the matter, $A$ is the mass number of the matter, and $Z$ is the charge number of the matter. The bracket $\langle E_p \rangle$ represents the average energy lost, and $x$ is the path length of the proton. Due to the fact that the collision between the protons
Table 3.1: The values of $f(\beta)$ in Eq (3.8) for several typical values of proton energy $E_p$.

<table>
<thead>
<tr>
<th>$E_p$[MeV]</th>
<th>$f(\beta)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>-5.04342133</td>
</tr>
<tr>
<td>10</td>
<td>-3.83570418</td>
</tr>
<tr>
<td>15</td>
<td>-3.42757309</td>
</tr>
<tr>
<td>20</td>
<td>-3.13723214</td>
</tr>
<tr>
<td>30</td>
<td>-2.72647037</td>
</tr>
<tr>
<td>40</td>
<td>-2.43351955</td>
</tr>
</tbody>
</table>

and the particles in the matter is random, the energy lost follows a distribution deviating from the average energy lost, which is described as the straggling function as given in Eq (3.17). The quantity $a$ is related to the material property and can be found in stopping power database such as PSTAR [196]∗ and SRIM [197]†. The function $f(\beta)$ is

$$f(\beta) = \ln[\beta^2/(1 - \beta^2)]$$  \hspace{1cm} (3.8)

The values of $f(\beta)$ for several typical values of proton energy are shown in Table 3.1. The value of $f(\beta)$ is always negative and the absolute value is large for low energy protons and small for high-energy protons. For mixture, compound or isotopes, i.e. different A’s and Z’s, the stopping power is

$$\frac{d\langle E_p \rangle}{dx} = -\frac{0.31\text{MeV/cm} \times Z_1 \rho_1}{A_1 \beta^2} \left[ f(\beta) + a_{CM} \right]$$  \hspace{1cm} (3.9)

∗PSTAR table is available at [https://physics.nist.gov/PhysRefData/Star/Text/PSTAR.html](https://physics.nist.gov/PhysRefData/Star/Text/PSTAR.html)
†SRIM is available at [http://www.srim.org/](http://www.srim.org/)
Table 3.2: The values of $a_{CM}$ and $a_{plasma}$ for several commonly used materials.

<table>
<thead>
<tr>
<th>Material</th>
<th>$a_{CM}$</th>
<th>$a_{plasma}$ (same density as the cold matter)</th>
</tr>
</thead>
<tbody>
<tr>
<td>He</td>
<td>10.104989408685167</td>
<td>11.55785557540658</td>
</tr>
<tr>
<td>Be</td>
<td>9.593259355116766</td>
<td>10.343643487408874</td>
</tr>
<tr>
<td>C</td>
<td>9.330823405798414</td>
<td>10.245771198845826</td>
</tr>
<tr>
<td>Al</td>
<td>8.62518972831195</td>
<td>10.114589066612082</td>
</tr>
<tr>
<td>CH</td>
<td>9.51012517092332</td>
<td>10.52322391965202</td>
</tr>
<tr>
<td>Au</td>
<td>7.10268247891251</td>
<td>9.22259658902589</td>
</tr>
<tr>
<td>Cu</td>
<td>7.91279323379868</td>
<td>9.541258123308493</td>
</tr>
</tbody>
</table>

where $f_i$ is the atomic number fraction of $i$th element, $A_1 = \sum_i f_i A_i$, $Z_1 = \sum_i f_i Z_i$, and

$$a_{CM} = \frac{\sum_i Z_i f_i a_i}{Z_1}$$  \hspace{1cm} (3.10)

where the subscript "CM" denotes "cold matter".

For the calculations of plasma stopping power, only electron contribution is considered, because the contribution of the plasma ions to stopping power is negligible due to the fact that $m_i/m_e = 1836A \gg 1$. The expression of stopping power in plasma is simply the Bethe formula under our assumptions in Eqs (3.6) and (3.7)

$$\frac{d\langle E_p \rangle}{dx} = -\frac{4\pi e^4 n_e}{\beta^2 m_e c^2} \ln \left[ 1.123 \sqrt{\frac{1}{2\pi} \frac{m_e^{3/2} c^2 \beta^2}{\hbar^2 \sqrt{n_e}}} \right]$$

$$= -0.31 \text{MeV/cm} \times \frac{Z \rho}{A \beta^2} \times \left[ f(\beta) + 10.2 + 0.5 \ln A - 0.5 \ln Z - 0.5 \ln \left( \frac{\rho}{1\text{g/cc}} \right) \right]$$  \hspace{1cm} (3.11)

where $\hbar$ is the reduced Planck constant. Eq (3.11) is consistent with the results from
For the plasma composed of multiple ion species

\[
\frac{d\langle E_p \rangle}{dx} = -\frac{0.31 \text{MeV/cm} \times Z_1 \rho_{1\text{g/cc}}}{A_1 \beta^2} \left[ f(\beta) + a_{\text{plasma}} \right] \quad (3.12)
\]

where \( A_1 \) and \( Z_1 \) has the same definitions as in the cold matter case but \( f_i \)'s are replaced by the number fractions of ions. And

\[
a_{\text{plasma}} = \sum_i f_i \left( 10.2 + 0.5 \ln A_i - 0.5 \ln Z_i \right) \frac{Z_1}{Z_1} - 0.5 \ln \left( \frac{\rho_{1\text{g/cc}}}{\text{cc}} \right) \quad (3.13)
\]

The difference between cold matter stopping power and plasma stopping power is only in the expressions for \( a_{\text{CM}} \) and \( a_{\text{plasma}} \), i.e. Eqs (3.10) and (3.13), while other parts of the two equations are identical. The typical values of \( a_{\text{CM}} \) or \( a_{\text{plasma}} \) are around 10, as shown in Table 3.2. The value of \( a_{\text{CM}} \) is given by fitting the stopping power in PSTAR table for proton energy range \( 1 \text{MeV} < E_p < 150 \text{MeV} \). The fitting error is less than 5% for elements lighter than Cu. For Carbon, “CARBON (amorphous, density 2g/cc)” in PSTAR table is used. The calculation for \( a_{\text{plasma}} \) assumes the same density as the cold matter and neglects the partial ionization [198]. From Eq (3.13) for \( a_{\text{plasma}} \), increasing density will reduce the value of \( a_{\text{plasma}} \), 4x compression will reduce \( a_{\text{plasma}} \) by 0.7, and 10x compression will reduce \( a_{\text{plasma}} \) by 1.15. In a typical HED target system, the ratio of Debye length \( \lambda_D \) to Fermi radius \( a_Z \) is used for quantifying the partition between cold matter and fully ionized plasma

\[
\frac{\lambda_D}{a_Z} = \frac{\sqrt{\frac{kT_e}{4\pi n_e^f e^2}}}{0.885 a_0 Z^{-1/3}} = \frac{0.5 Z^{1/3}}{\sqrt{\frac{T_e}{1 \text{eV}}}} \quad \frac{n_e^f}{10^{24} \text{cc}} \quad (3.14)
\]

where \( a_0 \) is the Bohr radius and \( n_e^f \) is the free electron density (not including the bond electrons, which is different from total electron density \( n_e \)). For matter composed of multiple elements, \( Z \) is replace by the logarithm averaged charge number \( Z_{\text{lg}} = \exp(\sum_i f_i \log Z_i) \) in Eq (3.14). The total stopping power with combined cold matter
and plasma is

\[
\frac{d\langle E_p \rangle}{dx}_{\text{total}} = -\frac{0.31 \text{MeV/cm} \times Z_{\text{ig/cc}}}{A_1 \beta^2} \left[ f(\beta) + a_{\text{total}} \right]
\]

(3.15)

where

\[
a_{\text{total}} = \frac{\lambda_D^2}{\lambda_D^2 + a_{\text{CM}}^2} a_{\text{CM}} + \frac{a_{\text{Z}}^2}{\lambda_D^2 + a_{\text{Z}}^2} a_{\text{plasma}}
\]

(3.16)

The combination using Eq (3.16) is a good approximation if the transition layer between cold matter and plasma is thin compared to the fully cold matter or fully plasma regions, i.e. most regions in the modeling has \( a_Z \ll \lambda_D \) or \( a_Z \gg \lambda_D \). If \( a_Z \approx \lambda_D \) dominates, then one need more precise combination model in the transition region. Figure 3.1 shows the ratio between the total stopping power with combined cold matter and plasma given by Eq (3.15) and cold matter approximation given by Eq (3.9) for plastic (CH with number fraction C:H=1) and copper, and for proton energy 14.7MeV and 3MeV. The mean ionization state from PROPACEOS equation of state table is used to calculate the free electron density. The cold matter approximation is good for the matter near or above solid density, i.e. 1g/cc for CH and 8.9g/cc for Cu. For low densities, the correction from Eq (3.11) has significant contribution to total stopping power, especially for Cu. For high temperatures, i.e. \( T > 50 \text{eV} \), the correction from Eq (3.11) has larger contribution for CH than for Cu.

The straggling function of the proton energy, i.e. the variation of stopping power along the path of motion follows a Gaussian distribution

\[
F(\Delta, s) = \frac{1}{\sqrt{2\pi} \Omega} \exp \left( -\frac{(\Delta - \Delta_{av})^2}{2\Omega^2} \right)
\]

(3.17)

with a variance \( \Omega^2 \), and a mean value \( \Delta_{av} \) equal to the product of path length \( s \) and the stopping power. The expression for the variance [191, 195] is

\[
\Omega = \sqrt{4\pi e^4 n_e \frac{1 - \beta^2/2}{1 - \beta^2} s} = 0.16 \text{MeV} \sqrt{\frac{n_e}{10^{23}/\text{cc} \ \text{1cm}}} \frac{s}{1 - \beta^2/2} \]

(3.18)
Figure 3.1: Comparison between stopping power for cold matter and plasma. (a) plastic (C:H=1), proton energy $E = 14.7\text{MeV}$, (b) plastic (C:H=1), $E = 3.0\text{MeV}$, (c) copper, $E = 14.7\text{MeV}$, (d) copper, $E = 3.0\text{MeV}$. 
Figure 3.1: (Continued) Contours of the ratio between the scattering angle using the full characteristic small scattering angle given by Eq (3.25) and cold matter approximation, for four different case (e) CH with proton energy $E = 14.7\text{MeV}$ and $E = 3\text{MeV}$, (f) Cu with proton energy $E = 14.7\text{MeV}$ and $E = 3\text{MeV}$. There is slight difference between $E = 14.7\text{MeV}$ and $E = 3\text{MeV}$. The column density is $\rho L = 0.005\text{g/cm}^2$ for (e) and (f). The horizontal axes of all the subfigures are densities, and the vertical axes of all the subfigures are electron temperatures.
which only depends on the electron density \( n_e \), the path length \( s \) and the normalized velocity \( \beta \) of the proton.

In both Monte Carlo and ray-tracing calculations, three quantities are pre-calculated, (1) the coefficient for stopping power

\[
\frac{0.31 \text{MeV/cm} \times Z_{1g/cc} e}{A_1} \tag{3.19}
\]

(2) \( a_{\text{total}} \); (3) \( 4\pi e^4 n_e \). For each time step of the particle motion, in both Monte Carlo and ray-tracing, the energy lost can be calculated from the pre-calculated quantities and the current value of \( \beta \) of the particle or ray, as given by Eq (3.15). In the Monte Carlo calculation, the straggling of proton energy is sampled at each time step using Eq (3.17). In the ray-tracing calculation, the total variation of proton energy is calculated by the numerical integration

\[
\Omega_{\text{total}}^2 = \sum_{\Delta s} 4\pi e^4 n_e \frac{1 - \beta^2/2}{1 - \beta^2} \Delta s \tag{3.20}
\]

where the value of \( \beta \) is the current value for the particle and \( n_e \) is at the particle location.

### 3.1.1.2 Coulomb Scattering calculation

The cross-section of single and multiple Coulomb scattering in thick foil was studied in References [199] and [200], which has been used in GEANT4 [187] and MCNP [186]. The cross-section for large angle scattering remains unchanged from cold matter to plasma

\[
N s \sigma(\chi) d\chi = 2\chi^2 \chi d\chi q(\chi)/\chi^4 \tag{3.21}
\]

where \( \sigma(\chi) d\chi \) is the differential scattering cross section into the angular interval \( d\chi \) by each atom (or ion), \( s \) is thickness of the material, \( N \) is the number of scattering
atoms (or ions) per volume, \( q \) is the ratio of actual to Rutherford scattering and approaches unity for large angle scattering, and

\[
\chi_c^2 = 4\pi N s e^4 Z(Z + 1)/(pv)^2 \tag{3.22}
\]

where \( p \) is the proton momentum and \( v \) the velocity of the proton beam. The physical meaning of \( \chi_c \), is that the total probability of single scattering through an angle greater than \( \chi_c \), is exactly one. For mixture, compound or isotopes

\[
\chi_c^2 = 4\pi N s e^4 (Z_2^2 + Z_1)/(pv)^2 \tag{3.23}
\]

where \( Z_2 = \sqrt{\sum_i Z_i^2 f_i} \) and \( Z_1 \) is the same as that in Eq (3.10). The expression for the numerical value of \( \chi_c^2 \) in terms of density \( \rho \) is

\[
\chi_c^2 = 1.8 \times 10^{-7} \times \frac{\rho s}{g/cm^2} \frac{(Z_2^2 + Z_1)}{A} \frac{1 - \beta^2}{\beta^4} \tag{3.24}
\]

For Coulomb scattering in plasmas, we replace the Fermi radius \( a_Z \) of the atom with the Debye length \( \lambda_D \) of the plasma in the calculation of characteristic small scattering angle where \( q(\chi) \) approaches zero. In general, for the regions with both cold matter and plasma, the characteristic small scattering angle is

\[
\lambda_0 = \lambda \sqrt{\frac{1}{a_Z^2} + \frac{1}{\lambda_D^2}} \tag{3.25}
\]

where \( \lambda \) is the De Broglie wavelength of the proton. For cold matter approximation, \( \lambda_D/a_Z \to \infty \), Eq (3.25) recovers the characteristic small scattering angle in cold matter, which is identical to Eq (8) in Reference [199].

For thick target where many scattering events occur, \( B_c > 5 \) for the variable \( B_c \) given by the following equations (I use \( B_c \) instead of \( B \) as in Reference [199] to avoid
confusion with magnetic fields)

\[ B_c - \ln B_c = b = \ln \frac{\chi_a^2}{1.167 \chi_a^2} \]  \hspace{1cm} (3.26)

\[ \chi_a^2 = \chi_0^2 (1.13 + 3.76 (Z_2 e^2)^2 / (\hbar v)^2) \]  \hspace{1cm} (3.27)

where \(3.76 (Z_2 e^2)^2 / (\hbar v)^2\) is the second order term in the Born approximation. The parameter \(\chi_a\) is defined to be

\[- \ln \chi_a = \lim_{k \to \infty} [\int_0^k q(\chi) d\chi / \chi + \frac{1}{2} - \ln k] \]  \hspace{1cm} (3.28)

In \(\lambda_D / a_Z \to \infty\) limit, the expression for the numerical value of \(e^b\) without second order term in Born approximation is

\[ e^b \approx 6680 \frac{\rho_s g}{\text{cm}^2} (Z_2^2 + Z_1) / \beta^2 A Z_{\text{lg}}^{2/3} \]  \hspace{1cm} (3.29)

which is consistent with Eq (22) in Reference [199]. \(Z_{\text{lg}}\) is the logarithm averaged charge number \(Z_{\text{lg}} = \exp(\sum_i f_i \log Z_i)\). The distribution of the scattering angle \(\theta\) is expanded in a series of \(B_c\) [199], i.e. Moliere-Bethe distribution

\[ f(z)dz = zdz[f^{(0)}(z) + \frac{1}{B_c} f^{(1)}(z) + \frac{1}{B_c^2} f^{(2)}(z) + \cdots] \]  \hspace{1cm} (3.30)

where \(z = \theta / (\chi_0 B^{1/2})\) and

\[ f^{(n)}(z) = \frac{1}{n!} \int_0^\infty u du J_0(zu) \exp\left(-\frac{u^2}{4}\right) [\frac{u^2}{4} \ln(\frac{u^2}{4})]^n \]  \hspace{1cm} (3.31)

where \(J_0\) is Bessel functions of the first kind. I keep the first three terms in Eq (3.30), i.e the Gaussian distribution (zeroth order term) with

\[ \sigma_{\text{Gauss}} = \chi_c (B_c / 2)^{1/2} \]  \hspace{1cm} (3.32)

and the terms in \(1/B_c\) and \(1/B_c^2\). The distribution is closer to Gaussian distribution when \(B_c\) becomes larger. At each time step of proton motion, \(p = \chi_c^2 / \chi_m^2\) and \(B_c\) is
calculated, using the length step $\Delta s$ as the target thickness $s$ in Eq (3.23). The final distribution of protons does not depend on the detail form of $\sigma(\chi)$ or $q(\chi)$, as long as the number of effective scattering is sufficiently large and the parameter $\chi_a$ given by Eq (3.28) keeps unchanged. So we can use the simple form for $q(\chi)$

$$q(\chi) = \begin{cases} 
0 & \chi < \chi_m \\
1 & \chi \geq \chi_m 
\end{cases} \quad (3.33)$$

where the minimal scattering angle $\chi_m$ is related to $\chi_a$ by $\ln \chi_a = \ln \chi_m - 0.5$ or $\chi_m = e^{0.5} \chi_a$. The algorithm for calculating Coulomb scattering for each time/length step is given in Algorithm 3.1, where $p_2$ is a given parameter for adjusting the threshold to use Moliere-Bethe distribution. The test in this section uses $p_2 = 50$. A few parameters are pre-calculated for Moliere-Bethe distribution

$$a_0 = \int_0^{1.8} zf^{(0)}(z)dz, \quad a_1 = \int_0^{1.8} zf^{(1)}(z)dz, \quad a_2 = \int_0^{1.8} zf^{(2)}(z)dz,$$

$$b_0 = \int_0^{10} zf^{(0)}(z)dz, \quad b_1 = \int_0^{10} zf^{(1)}(z)dz, \quad b_2 = \int_0^{10} zf^{(2)}(z)dz \quad (3.34)$$

and the following functions are calculated numerically by interpolation

$$f^{(1)}(z)/f^{(0)}(z), \quad f^{(2)}(z)/f^{(0)}(z) \quad \text{for } z \in [0, 1.8]$$

$$zf^{(0)}(z), \quad zf^{(1)}(z), \quad zf^{(2)}(z) \quad \text{for } z \in [1.8, 10] \quad (3.35)$$

The method implemented in this thesis for Coulomb scattering as a random process has been used in other Monte Carlo codes such as MCNP [201] and GEANT4 [187]. Two quantities in each cell are pre-calculated before the Monte Carlo or ray-tracing calculations: (1) the coefficient for large angle scattering cross section, i.e. $4\pi Ne^4(Z_2^2 + Z_1)$, where $Z_2 = \sqrt{\sum_i Z_i^2 f_i}$, (2) the characteristic small scattering angle $\chi_0$. 
**Algorithm 3.1** The algorithm for calculating Coulomb scattering angle for each time/length step

initialize $p = \chi_c^2/\chi_m^2$

while $p > 0$

if $p > p_2$, then

use Moliere-Bethe distribution (Algorithm 3.3) to sample a multiple scattering event

$p \leftarrow 0$

else

get a random number $x_1$ with uniform distribution $x_1 \in [0, 1]$

if $p > -\ln x_1$, then

sample a number single scattering event (Algorithm 3.2)

$p \leftarrow p - (-\ln x_1)$

else

get a random number $x_2$ with uniform distribution $x_2 \in [0, 1]$

if $x_2 < \min(p_2, p)$ then

sample a single scattering event (Algorithm 3.2)

else

pass

$p \leftarrow p - \min(p_2, p)$

**Algorithm 3.2** The algorithm for calculating scattering angle for a single scattering event

get a random number $x_1$ with uniform distribution $x_1 \in [0, 1]$

$\theta = \chi_m/\sqrt{x_1}$
**Algorithm 3.3** The algorithm for calculating scattering angle for a multiple scattering event

get a random $x_1$ with uniform distribution $x_1 \in [0, 1]$

if $x_1 < (a_0 + a_1/B_c + a_2/B_c^2)$ then

    do

    get a random number $x_2$ with uniform distribution $x_2 \in [0, 1]$, let $z_1 = \sqrt{-\ln(1 - \xi_0 x_2)}$

    get a random number $x_3$ with uniform distribution $x_3 \in [0, 1]$

    until $1 + (1/B_c)f^{(1)}(z_1)/f^{(0)}(z_1) + (1/B_c^2)f^{(2)}(z_1)/f^{(0)}(z_1) > x_3 (1 + (1/B_c)f^{(1)}(0)/f^{(0)}(0) + (1/B_c^2)f^{(2)}(0)/f^{(0)}(0))$

else if $x_1 < (b_0 + b_1/B_c + b_2/B_c^2)$

    do

    get a random number $x_2$ with uniform distribution $x_2 \in [0, 1]$, let $z_1 = 1.8 + 8.2x_2$

    get a random number $x_3$ with uniform distribution $x_3 \in [0, 1]$

    until $z_1(f^{(0)}(z_1) + (1/B_c)f^{(1)}(z_1) + (1/B_c^2)f^{(2)}(z_1)) > x_3 1.8(f^{(0)}(1.8) + (1/B_c)f^{(1)}(1.8) + (1/B_c^2)f^{(2)}(1.8))$

else

    $z_1 \leftarrow 10.0$

$z \leftarrow z_1$
For ray tracing calculation, I use the numerical integration of the right hand side of Eq (3.23) and the ion+atom density weighted value of $\ln \chi_0$

$$\ln \chi_0 = \frac{\sum \Delta s (N \ln \chi_0)_{\text{local}} \Delta s}{\sum \Delta s N_{\text{local}} \Delta s}$$  \hspace{1cm} (3.36)

which is an analog of Eq (16) in Reference [190]. The correction of the path length by taking into account the lowest order distribution in Eq (3.30) is $\langle 1/\cos \theta \rangle \approx 1 + (1/2) \langle \theta^2 \rangle \approx 1 + \chi_c^2 B_c/2$. Figure 3.1(e) and (f) show the ratio between the scattering angle using the full characteristic small scattering angle given by Eq (3.25) and using cold matter approximation. The scattering angle is calculated using Eq (3.32), and $B_c, \chi_c$ are calculated by ray-tracing of proton beam through a material with column density $\rho L = 0.005 \text{g/cm}^2$. For low densities or high temperatures, the correction from finite $\lambda_D$ has significant contribution to the total scattering angle as shown in the top left corner of Figure 3.1(e) and (f). The difference between 3MeV and 14.7MeV protons is more prominent in CH than in Cu, which can be explained by the sensitivity of $b$ to the proton energy or proton velocity given Eqs (3.26) and (3.27). For CH, $3.76(Z_2 e^2)^2/(\hbar v)^2$ is 0.6 for $E_p = 3\text{MeV}$ and 0.1 for $E_p = 14.7\text{MeV}$, both less than 1.13, thus $e^b \sim 1/[v^2(1.13 + 3.76(Z_2 e^2)^2/(\hbar v)^2)]$ is sensitive to proton energy. For Cu, $3.76(Z_2 e^2)^2/(\hbar v)^2$ is 26 for $E_p = 3\text{MeV}$ and 5 for $E_p = 14.7\text{MeV}$, both much larger than 1.13, thus $e^b \sim 1/[v^2(1.13 + 3.76(Z_2 e^2)^2/(\hbar v)^2)] \sim 1/[v^2 \times 3.76(Z_2 e^2)^2/(\hbar v)^2] \sim$ constant.

From Eq (3.32) and Eq (3.2), the ratio between the deflection angle by magnetic field and the Coulomb scattering angle is

$$\frac{\alpha}{\sigma_{\text{Gauss}}} = \frac{2\sqrt{A}}{\sqrt{B_c(Z_2^2 + Z_1)}} \frac{E_p}{m_p c^2} \frac{v_A}{c} \sqrt{\frac{l_i m_p c^2}{e^2}}$$

$$= \frac{6\sqrt{A}}{\sqrt{B_c(Z_2^2 + Z_1)}} \left( \frac{E_p}{14.7\text{MeV}} \right)^{1/2} \times \frac{v_A}{2.8 \times 10^4 \text{cm/s}} \left( \frac{l_i}{0.1\text{cm}} \right)$$  \hspace{1cm} (3.37)
where \( v_A = B/\sqrt{4\pi \rho} \) is the Alfvén speed, \( A \) is the mass number of the matter, \( Z_1 = \sum_i f_i Z_i \) and \( Z_2 = \sqrt{\sum_i f_i Z_i^2} \), where \( f_i \) is the atomic number fraction of \( i \)-th element.

### 3.1.1.3 Benchmark for MPRAD code against MCNP code for cold matter

Under cold matter approximation, the Monte Carlo calculation in MPRAD code is tested by the setup as shown in Figure 3.2. The monoenergetic (\( \Delta E_p = 0 \)) and collimated proton source with \( E_p = 15 \text{MeV} \) is placed 1cm from the center of the slab of matter with given material and thickness. I use \( 10^6 \) particles in the simulations and the proton velocity is perpendicular to the detector plane. The detector plane is 20cm from the center of the slab of matter, and the particles reaching the detector plane are binned by spacial grid with \( \Delta x = \Delta y = 0.01 \text{cm} \) and energy grid with \( \Delta E = 0.05 \text{MeV} \). The detector size is \( 2 \text{cm} \times 2 \text{cm} \). The simulation with the same setup is also carried out using MCNP code [186].

For all the test cases, both the spatially binned proton image and the proton spectrum are consistent between MPRAD and MCNP. An example is shown in Figure 3.3. The protons in the narrow beam are isotropically scattered by colliding with the matter in the slab, so a circular spot on the detector plane is produced as shown in Figure 3.3(b) and (c). The unit of the color code in Figure 3.3(b) and (c) is \( \text{(number of particles)/cm}^2 \) normalized by total particle number \( N = 10^6 \). The protons lose energy and have a finite width in the spectrum at the detector as shown in Figure 3.3(a), because different protons have different path length in the matter due to scattering. For a given composition of the slab material, different density \( \rho \) but same column density \( \rho t \) produces similar image and spectrum. Quantitative comparison between the results from MPRAD and MCNP is shown in Table 3.3 and Table 3.4. The slight
difference between results from MPRAD and MCNP is tolerable for typical proton radiography setup in HED experiments, where the spectrum width of the source is a few keV to MeV [181, 71, 183].

3.1.1.4 Example applications for MPRAD

For the example applications, the setup is as shown in Figure 3.2 with a magnetic field in the slab. The z axis is along the line of sight, and the detector plane is $x - y$ plane. The interaction region is filled with plastic (C:H=1). The thickness of the interaction region is $l_i = 1000\mu m$ with uniform tunable density $\rho$ and fixed temperature $T_e = 100eV$. And the field is centered at $(0,0,0)$. The source is at $(0,0,-1cm)$, monoenergetic $E_p = 15MeV$ and the beam collimated along the z axis.
Figure 3.3: Results of proton radiography test simulation for a slab of 8.96g/cc and 16µm copper. (a) (number of particles)/MeV normalized by total particle number $N = 10^6$. (b) The image generated using MPRAD, (c) same as (b) but using MCNP.
Table 3.3 : Comparison of average energy and energy variation from MPRAD and MCNP for different materials, densities and thicknesses

<table>
<thead>
<tr>
<th>Material, density, thickness</th>
<th>$\mathcal{E}$ (MeV), MPRAD</th>
<th>$\mathcal{E}$ (MeV), MCNP</th>
<th>$\sqrt{E^2 - \langle E \rangle^2}$ (MeV), MPRAD</th>
<th>$\sqrt{E^2 - \langle E \rangle^2}$ (MeV), MCNP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Be, 1.85g/cc, 200µm</td>
<td>13.96</td>
<td>13.96</td>
<td>0.0546</td>
<td>0.0565</td>
</tr>
<tr>
<td>Be, 7.40g/cc, 50µm</td>
<td>13.96</td>
<td>13.96</td>
<td>0.0546</td>
<td>0.0565</td>
</tr>
<tr>
<td>Mg, 1.74g/cc, 16µm</td>
<td>14.93</td>
<td>14.92</td>
<td>0.0157</td>
<td>0.0138</td>
</tr>
<tr>
<td>Mg, 6.96g/cc, 4µm</td>
<td>14.93</td>
<td>14.92</td>
<td>0.0157</td>
<td>0.0138</td>
</tr>
<tr>
<td>Cu, 8.96g/cc, 16µm</td>
<td>14.70</td>
<td>14.71</td>
<td>0.0355</td>
<td>0.0350</td>
</tr>
<tr>
<td>Cu, 17.92g/cc, 8µm</td>
<td>14.70</td>
<td>14.71</td>
<td>0.0355</td>
<td>0.0350</td>
</tr>
</tbody>
</table>

Table 3.4 : Comparison of weighted (by proton flux) average of $x^2 + y^2$ from MPRAD and MCNP for different materials, densities and thicknesses

<table>
<thead>
<tr>
<th>Material, density, thickness</th>
<th>$\sqrt{x^2 + y^2}$ (cm), MPRAD</th>
<th>$\sqrt{x^2 + y^2}$ (cm), MCNP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Be, 1.85g/cc, 200µm</td>
<td>0.381</td>
<td>0.369</td>
</tr>
<tr>
<td>Be, 7.40g/cc, 50µm</td>
<td>0.381</td>
<td>0.369</td>
</tr>
<tr>
<td>Mg, 1.74g/cc, 16µm</td>
<td>0.161</td>
<td>0.161</td>
</tr>
<tr>
<td>Mg, 6.96g/cc, 4µm</td>
<td>0.161</td>
<td>0.161</td>
</tr>
<tr>
<td>Cu, 8.96g/cc, 16µm</td>
<td>0.535</td>
<td>0.536</td>
</tr>
<tr>
<td>Cu, 17.92g/cc, 8µm</td>
<td>0.535</td>
<td>0.536</td>
</tr>
</tbody>
</table>
For the parameters used in the example applications, the contrast field given by Eq (3.5) is
\[
\Lambda(\mathbf{x}_\perp) = 1.7 \times 10^{-6} \text{G}^{-1} \times \left( \frac{E_p}{14.7 \text{MeV}} \right)^{-1/2} \times \mathbf{\hat{z}} \cdot \int dz \nabla \times \mathbf{B}
\] (3.38)

Localized toroidal magnetic fields — The characteristic distribution of a localized toroidal field \[157, 189, 184\] is
\[
\mathbf{\overline{B}} = \frac{B_0}{a} \exp \left( -\frac{x^2 + y^2 + z^2}{a^2} \right) (-y, x, 0)
\] (3.39)

The parameters used are \(B_0 = 1 \times 10^5 \text{G}\) and \(a = 200 \mu\text{m}\). Toroidal magnetic fields have been observed and measured in some HED experiments \[68, 70, 69, 72, 119, 120, 171, 168, 169, 172, 173, 170\], typically for the reconnection geometry. The typical geometry of self-generated magnetic field in the plasma plume produced by single laser spot is toroidal. As shown in Figure 3.4(a), the quasi-monoenergetic proton beam has become a beam with a broad energy distribution as it goes through the target region, and the mean energy becomes lower than the source energy. In the analysis for real data from the experiments, one also has to take the spectrum width of the source into account. The diffusion can affect the interpretation of proton image. As the density increases, the theoretical peak value of the contrast drops as \(E_p^{-1/2}\) given by Eq (3.38) if no diffusion is considered. However, the peak value of the contrast in the simulation drops faster than the theoretical trend \(E_p^{-1/2}\), as shown in Figure 3.4(c). One deduces smaller field or MHD current from the image for large density case. For large densities, the variation level of proton number in each pixel can potentially become comparable or even smaller than the Poisson noise for the CR-39 image, and the variation level of the proton flux can become smaller than the sensitivity of radiochromic film.
Figure 3.4: The results for the example applications. The colors of the curves in subplots are consistent among all panels, red for $\rho = 10^{-7}$ g/cc, blue for $\rho = 0.385$ g/cc, green for $\rho = 0.852$ g/cc, and black otherwise. Subfigure (a) shows the spectrum of the protons in the detector plane for different densities. Subfigure (b) is the contrast field of the proton image for the test case for localized toroidal magnetic field, the feature is coaxial as we can see from the symmetry of the field. A line-out cross the center is shown in (c), for different densities. The dashed lines are the theoretical value of contrast given by Eq (3.38), and the solid lines are from the MPRAD simulations. The results for the localized poloidal magnetic field is in (d) and (e).
Localized poloidal magnetic fields— The characteristic distribution of a localized poloidal field \cite{189,184} is

\[ B_y = B_0 \exp \left( -\frac{x^2 + z^2}{a^2} \right) \]  

(3.40)

The parameters used are \( B_0 = 4 \times 10^5 \) G and \( a = 200 \) µm. A few HED experiments have generated and characterized poloidal magnetic field, such as supersonic jets with mega-gauss self-generated magnetic fields localized in the interaction region \cite{175,176}. The results are shown in Figure 3.4(d) and (e). Similar to the case for the toroidal magnetic fields, the diffusion of the beam affect the final spectrum of the protons and the peak value of proton flux contrast, and thus special care is needed for interpreting the proton images.

Power law energy spectrum in magnetic turbulence — Magnetic turbulence and dynamo have been studied in HED experiments \cite{50,52}. In turbulent magnetic fields, magnetic energy cascades to small scales, and the magnetic energy spectrum follows a power law distribution. The power law spectrum can be inferred using the technique \cite{157,50,52} described in Section 3.1.2. As an example for using MPRAD to study how diffusion affect the inferred spectrum, I use a power law where the magnetic energy spectrum follows \( E(k) \propto k^{-2.3} \) \cite{46}. The method for generating the magnetic field by random numbers for numerical tests is discussed in Reference \cite{185}, and the vector potential is multiplied by \( \exp\left(-\frac{x^2 + y^2 + z^2}{a^2}\right) \) where \( a = 200 \) µm to get the localized field. The RMS value of the magnetic field in the \( l_t^3 \) box is \( B_{\text{rms}} = 1 \times 10^4 \) G, and the maximum field strength is \( B_{\text{max}} = 1 \times 10^5 \) G, same as the test problem for localized toroidal and poloidal magnetic fields. As shown in Figure 3.5, the diffusion affects the cutoff length scale \( \pi/k_c \) of the spectrum given by the reconstruction algorithm. The results for \( \rho = 10^{-7} \) g/cc show little scattering and the spectrum around \( k \sim 10^3 \) cm\(^{-1}\) agrees with the theoretical spectrum (the
Figure 3.5: Inferred turbulence magnetic energy spectrum using for different densities.
spectrum calculated by using the 3D magnetic field), and \( k > 3 \times 10^3 \text{cm}^{-1} \) is beyond the resolution limit. For densities from \( \rho = 3.0 \times 10^{-4} \) and above, there is a critical wavevector \( k_c \) that the diffusion affect damps the small scale feature for \( k > k_c \) but retains the large scale feature for \( k < k_c \). For \( \rho = 7.3 \times 10^{-3} \text{g/cc} \) and \( \rho = 3.6 \times 10^{-2} \text{g/cc} \), the energy density at low \( k \), i.e. \( k < 50 \text{cm}^{-1} \) becomes higher than other densities. The inverse of the cutoff scale \( k_c \) is roughly the scattering angle multiplied by \( r_i \), thus \( k_c \approx \pi/(10r_i\sigma_{\text{Gauss}}) \approx 20\text{cm}^{-1}(\rho/(\text{g/cc}))^{-1/2} \) where the factor 10 in the denominator is an estimate of the scattering angle in \( 1/B_c \) term. \( B_c \) is roughly 6 for \( \rho = 1.5 \times 10^{-3} \text{g/cc} \) so that \( 1/B_c \) term is not negligible. The estimate for \( k_c \) is in good agreement with the results in Figure 3.5.

### 3.1.2 Inferring line-integrated magnetic field and magnetic energy spectrum from proton radiography

**Inferring line-integrated perpendicular magnetic field in 2D or 1D** — From the proton flux on the image, the projection integral of the perpendicular magnetic field can be solved by Eq (25) in Reference [157]:

\[
\nabla \cdot (e^{\Lambda(x_\perp)}\nabla \phi(x_\perp)) = \Lambda(x_\perp)e^{\Lambda(x_\perp)} \tag{3.41}
\]

where \( \Lambda(x_\perp) \) is given by Eq (3.4) and the projection integral of the perpendicular magnetic field may then be obtained by taking the gradient of the scalar field \( \phi(x_\perp) \)

\[
\int_0^{r_s} dz B_\perp = \frac{mcv}{e(r_s - r_i)} \nabla \phi(x_\perp) \times \hat{z} \tag{3.42}
\]

Earlier work on the reconstruction [184] used Poisson equation for the projected potential \( \phi(x_\perp) \), i.e. \( \nabla^2 \phi = \Lambda \), which ignores the effect of lateral displacement. In the case where \( \Lambda \) changes substantially on the displacement scale, Eq (3.41) is more accurate than Poisson equation. However, the accuracy of the magnetic field
reconstruction using Eq (3.41) is limited by a few sources of error, such as Poisson noise, discretization noise, edge effects and obstruction effects. If the feature of the proton image is extensible in one direction, such as the example shown in Section 4.3, then one only needs to solve 1D version of Eqs (3.41) and (3.42)

\[
(e^\Lambda(x)\phi'(x))' = \Lambda(x)e^\Lambda(x)
\]

\[
\int_0^{r_s} dz B_y = \frac{mc}{e(r_s - r_i)}\phi'(x)
\]

Inferring magnetic energy spectrum for isotropic and homogeneous stochastic magnetic field— For isotropic and homogeneous stochastic magnetic field, the energy spectrum of the magnetic field can be inferred from the contrast map, as derived in Reference [157]. The expression for the inferred magnetic energy spectrum is

\[
E_B(r_s/r_i) = \frac{2\pi m^2 c^2 v^2}{e^2(r_s - r_i)^2 l_i L^2} \langle \tilde{\Lambda}(p_\perp) \tilde{\Lambda}(p_\perp)^* \rangle, \quad |p_\perp| < \frac{2\pi}{\Delta x_\perp}
\]

where \( l_i \) is the longitudinal size of the interaction region, \( L \) is the size of the image plate, \( \Delta x_\perp \) is the size of pixel on the image plate, and \( \tilde{\Lambda}(p_\perp) \) is the Fourier transform of \( \Lambda(x_\perp) \)

\[
\tilde{\Lambda}(p_\perp) = \left(\frac{1}{2\pi}\right)^2 \int d^2 x_\perp e^{-ip_\perp \cdot x_\perp} \Lambda(x_\perp)
\]

The ensemble average in Eq (3.45) may be replaced by a rotational average, thus

\[
E_B(r_s/r_i) = \frac{m^2 c^4 [1 - (mc^2/E_k)^2]}{e^2(r_s - r_i)^2 l_i L^2} \int_0^{2\pi} d\theta \Delta x_\perp \sum_{kr_i} \langle |\text{FFT}_\Lambda^{(p_\perp)}| \rangle^2
\]

where \( E_k \) is the kinetic energy of the proton beam. The fast Fourier transform (FFT) function in Python is defined as

\[
y[k] = \sum_{n=0}^{N-1} e^{-2\pi i j k X} x[n],
\]

thus

\[
E_B(k) = \frac{m^2 c^4 [1 - (mc^2/E_k)^2]}{e^2(r_s - r_i)^2 l_i L^2} \left(\frac{\Delta x_\perp^4}{2\pi}\right) \sum_{(kr_i/r_s, \theta) \to (p_x, p_y)} |\text{FFT}_\Lambda^{(p_\perp)}|^2
\]

\[
= \frac{m^2 c^4 [1 - (mc^2/E_k)^2]}{(2\pi)^4 e^2(r_s - r_i)^2 l_i L^2} \sum_{\theta} \Delta \theta |\text{FFT}_\Lambda^{(kr_i \cos \theta / r_s, kr_i \sin \theta / r_s)}|^2
\]
where the range for $k$ is $|k| < (2\pi/\Delta_x \Delta_y \Delta_z)/(r_s/r_i)$.

The energy spectrum of the magnetic field for a given field configuration is

$$E_B(k) = \frac{\pi^2}{V} \int d\Omega k^2 \langle |\tilde{B}(k)|^2 \rangle$$

(3.50)

where $V$ is the volume and $\tilde{B}(k)$ is the Fourier transform of $B(x)$

$$\tilde{B}(k) = \left(\frac{1}{2\pi}\right)^3 \int d^3x e^{-ik \cdot x} B(x)$$

(3.51)

The magnetic energy density is the integral of $E_B(k)$, i.e. $\langle B^2/(8\pi) \rangle = \int_0^\infty E_B(k)dk$.

Using the FFT function in Python, Eq (3.50) can be written as

$$E_B(k) = \frac{(\Delta_x \Delta_y \Delta_z)^2}{64\pi^4 V} \sum_{\mu,\phi} \Delta \mu \Delta \phi k^2 |\text{FFT}_B(k \sin \theta \cos \phi, k \sin \theta \sin \phi, k \cos \theta)|^2$$

(3.52)

where $\mu = \cos \theta$.

### 3.2 Thomson scattering

Thomson scattering [202, 203] is a valuable technique for measuring temporal-resolved local temperature, density, and flow velocity in laser produced plasmas. The measurement uses an optical laser source with given frequency and direction that is scattered by collective fluctuations in plasmas, such as electron plasma fluctuations and ion-acoustic fluctuations. Figure 3.6 shows a typical Thomson-scattering system. Scattered light from a small volume (typically $\sim (50 \mu m)^3$) is collected by a telescope and transported to a spectrometer pair to obtain spectral and temporal resolution [4].

For the calculating Thomson spectrum, the spatial profiles of electron density, electron/ion temperature, flow velocity, and fraction of species are taken as the input for the spectroscopy code [204]. The heating by the probe beam is modeled by laser absorption, i.e. $Q_{\text{las}}$ term as in Eqs (2.49) and (2.50). The dispersion relations for
ion acoustic wave (IAW) and electron plasma wave (EPW) are used to calculate the power output [204]. The instrument broadening [4] is taken into account in the modeling spectrum. The scattering spectrum depends on the velocity distribution of electrons/ions in the small volume. For example, in an experiment for gas jet in the presence of inverse-Bremsstrahlung heating and collisional ionization, super-Gaussian distribution for electrons was found to be more accurate than Maxwellian to fit for the scattering spectrum [205].

### 3.3 X-ray framing camera

X-ray radiography provides temporally and spatially resolved profile of the density and temperature of the plasmas. The X-ray radiation with backlighter or self-emission
is recorded on the X-ray framing camera (XRFC) [206, 207, 208]. SPECT3D [60] is used to generate the synthetic ray-tracing X-ray image.

In the configuration without a backlighter but with a pinhole, as shown in the upper panel in Figure 3.7, the intensity for self-emission is calculated by solving the radiative transport equation Eq (2.38). The integration starts at point $D$ with $I_\nu(\tau = 0) = 0$ and ends at point $B$. The flux on point $A$ on the image plate is

$$F_\nu = A \cos \theta_1 \cos \theta_2 I_\nu(B)/r^2,$$

where $A$ is the area of the pinhole, $\theta_1$ is the angle between the ray and the normal direction of the image plate, $\theta_2$ is the angle between the ray and the normal direction of the pinhole, and $r$ is the distance from pinhole to point $A$.

In the configuration with a backlighter but without a pinhole, as shown in the lower panel in Figure 3.7, the radiative transport equation is also integrated from point $D$ with $I_\nu(\tau = 0)$ equal to the intensity of the source to point $B$. The flux on point $A$ on the image plate is

$$F_\nu = I_\nu(B) \cos \theta_1.$$

### 3.4 Summary

In this chapter, several diagnostics for HED plasmas and the synthetic simulation techniques for these diagnostics are discussed. These diagnostics have been used in the experiments discussed in Chapter 4 and 5. The synthetic simulations for diagnostics take the results of FLASH simulations as input data and can be compared with the experimental data.

---

‡Prism Computational Sciences SPECT3D Overview [Prism Computational Sciences SPECT3D Overview](http://www.prism-cs.com/Software/Spect3D/overview.html)
Figure 3.7: The simulation setup for x-ray images. Upper: without backlighter but with a pinhole. Lower: with a backlighter but without a pinhole. Point $D$ and $B$ are the intersection of light-of-sight with the boundary of the FLASH domain.
Chapter 4

Magnetized jet experiments

Supersonic, well collimated outflows are ubiquitous in the universe, such as those in protostellar disks [209], X-ray binaries [210], young stellar objects (YSO) [211], active galactic nucleus (AGN) [212] and gamma-ray bursts (GRB) [213]. Those outflows exhibit a wide range of size and radiation power. Despite various astronomical observations, theoretical studies, and numerical modelings of astrophysical jets, many fundamental questions remain, e.g. launching mechanism, composition, collimation mechanism, morphology of the magnetic field, and interaction with ambient medium. While traditional approaches to address these questions are mainly direct observation and/or theoretical modeling, laboratory produced jets with proper scaling relations provide an alternative platform to study jets on astrophysical scales.

A novel way of launching high density and high temperature plasma jets using multiple intense laser beams is to utilize the hollow ring configuration as proposed in Reference [214] and [215]. It was demonstrated in two dimensional FLASH simulations that a bundle of laser beams of given individual intensity, duration and focal spot size, produces a supersonic jet of higher density, temperature and better collimation, if the beams are focused to form a circular ring pattern on a flat target instead of a single focal spot [214]. The Biermann Battery ($\nabla n_e \times \nabla T_e$) term [61] can generate and sustain strong toroidal fields downstream in the collimated jet outflow far from the target surface [215]. However, those simulations were carried out in two dimensional cylindrical geometry, where the intensity variation along the ring due to
individual laser beams were neglected. Three dimensional simulations are necessary to understand the formation and evolution of the jet in the actual experiments. The ring jet experiments were designed and carried out on the OMEGA laser facility \cite{6} in 2015 and 2016 \cite{175}. 20 OMEGA beams are used to simultaneously irradiate the target forming a ring pattern. Each beam delivers 500J of energy in 1ns.

4.1 Simulation setup

The FLASH code \cite{56, 57} is used to carry out the detailed physics simulations of our laser experiments to study the formation and dynamics of the jet, as well as the origin of magnetic fields. The detailed description of the FLASH code is in Section \ref{sec:2.3} A Cartesian grid with \( (256 \times 256 \times 512) \) zones is used to resolve a \( (3\text{mm} \times 3\text{mm} \times 6\text{mm}) \) domain, corresponding to \( \sim 11\mu\text{m} \) per cell width. This resolution is sufficient to resolve the spatial distribution of all the quantities that the plasma diagnostics are able to resolve. The test runs with lower resolutions show that the simulation converges at a cell with of \( 11\mu\text{m} \). The plasma has zero initial magnetic field. The laser target is modeled as a 3mm diameter and 0.5mm thick disk with the composition listed in Table \ref{table:4.1}. The equation of state of helium is used in the chamber with initial density equal to \( 2 \times 10^{-7}\text{g/cc} \), which should have been vacuum. The helium does not affect the simulation significantly, as the typical fraction for mass, momentum and energy budget in the modeled helium is much less than 1%. To suppress the magnetic field from numerical artefact, Biermann battery term is turned off and the largest allowed magnetic resistivity in the explicit solver is used for each time step in the regions with density lower than \( 2 \times 10^{-5}\text{g/cm}^3 \). The electron heat conduction is calculated using Braginskii model \cite{104} in weak magnetic field limit.

To model the laser driven blowoffs, the spatial and temporal specifications of each
Table 4.1: Target characteristics used in the magnetized jet experiments

<table>
<thead>
<tr>
<th>Composition (atomic number fraction)</th>
<th>Density</th>
<th>Laser target ring radius</th>
</tr>
</thead>
<tbody>
<tr>
<td>C(50%) H(50%)</td>
<td>1.04g/cc</td>
<td>0, 400, 800, 1200µm</td>
</tr>
<tr>
<td>C(49%) H(49%) Fe(2%)</td>
<td>1.21g/cc</td>
<td>800, 1200µm</td>
</tr>
</tbody>
</table>

Figure 4.1: The illuminated area on the target by 20 OMEGA beams. The transverse section of the beams are circles, but the spots on the target surface are ellipses due to inclination. The incident angle is 59° for the blue spot, 42° for the green spot, and 21° for the red spot. (a) \(d = 400\mu m\) ring radius; (b) \(d = 800\mu m\) ring radius; (c) \(d = 1200\mu m\) ring radius. The \(d = 0\) case is not shown. Note that the red and green spots form a 5-fold symmetry and the blue spots form a 10-fold symmetry.
Figure 4.2: Schematics of diagnostics setup of OMEGA magnetized jet experiments. For DD and D\textsuperscript{3}He protons, the source stands 1cm from TCC, while the image plate CR39 is located 17cm from TCC on the other side. For the TNSA protons, the source stands 0.8cm from TCC, while the radiochromic film pack is located 16.5cm from TCC on the other side. The X-ray framing camera (XRFC) images the jet at 38° from the vertical axis.
of the twenty OMEGA driver beams are used. The 20 driver beams are turned on and turned off simultaneously with a 1ns pulse duration. Each delivers 500J of energy on a target flat-top. The radius of each beam is 125µm. The laser spots are arranged to form a ring pattern of radius $d$, as shown in Figure 4.1. Due to the inclination of the beams, the illuminated areas on the target are actually ellipses. The irradiation is face-on. The target is 0.5mm thick to prevent the burn-through.

The setup of the diagnostics is sketched in Figure 4.2. Optical Thomson scattering (See Section 3.2) is used to probe the electron/ion temperatures, electron density and flow velocity at TCC (0.25cm above the target). The probe beam is a 1ns pulse, 25 ~ 50J energy and 532nm wavelength (2ω) backlifter. The intensity distribution of the probe beam is 70µm FWHM 2D gaussian. Thomson scattering spectra are modeled by taking the 3D FLASH simulation results. The final power output is weight averaged by the spatial intensity distribution of the probe laser. The heating by the probe beam is also calculated for comparison. The experimental data are also fitted using the model to compare with the plasma quantities averaged over a (200µm)$^3$ cube centered at TCC. For the DD and D³He protons, the source stands 1cm from TCC, while the image plate CR39 is located 17cm from TCC on the other side. For the TNSA protons, the source stands 0.8cm from TCC, while the radiochromic film pack is located 16.5cm from TCC on the other side. The energy deposition by a TNSA proton can be approximated by \( \frac{\text{const}}{\sqrt{E_p - E_{p,0}}} \), where \( E_{p,0} \) is the proton energy of maximum sensitivity [71] [216].

For convention, \( t = 0 \) is the time for laser turn on. The \( z \) direction is perpendicular to the surface of the target plane. The jet is formed in the \( z > 0 \) region. Cylindrical coordinates are also used where \( r = 0 \) is the central axis of the target. The target surface is located at \( z = 0 \). Axial direction is along \( z \) axis. Toroidal or azimuthal
direction is the $\varphi$ direction in the cylindrical coordinate system. Target chamber center (TCC) is at $x = y = r = 0$, $z = 0.25\text{cm}$.

### 4.2 Hydrodynamics, x-ray images and Thomson spectrum

*Jet formation and evolution* — As shown in Figure 4.3, the jet is formed by the merging of the plasma plumes produced by 20 individual OMEGA beams through a strong cylindrical shock. By using a large ring radius, the flows will not collide immediately while the lasers irradiate the target. For the collision at later time with more available room, the flows develop larger radial velocities which become more supersonic. Thus a stronger cylindrical shock is generated near the $z$ axis. For the cylindrical shock, the surrounding is in the upstream region and the central core is in the downstream region. The jet is supersonic and well collimated. The jets with different ring radii all travel several millimeters by $t = 3\text{ns}$. The jet keeps traveling and expanding so that the length $L$ and the radius $R$ keep growing even after 3ns. The width and the length of the jet are much larger than the laser spot size ($\sim 250\mu\text{m}$), as shown in Table 4.2.

*Comparison for different ring radius* — Figure 4.4 shows the shape of the jet for different laser ring radii at $t = 3\text{ns}$. Figure 4.5 shows the evolution of electron/ion temperature, electron density and flow velocity at TCC for different runs in the FLASH simulations. The quantities are calculated by averaging over a $(200\mu\text{m})^3$ cubic around TCC ($r = 0$, $z = 0.25\text{cm}$). The peak electron/ion temperature on-axis is higher for larger ring radius. Comparing with the case where $d = 0$, the temperature for $d = 800\mu\text{m}$ or $d = 1200\mu\text{m}$ is about one order of magnitude higher. The peak electron density is highest for $d = 800\mu\text{m}$, which is one order of magnitude higher than the $d = 0$ case. The ratio $L/R$ of the jet becomes larger (see Table 4.2) as $d$
Figure 4.3: Three-slice plots for electron density at $x = 0$, $y = 0$ and $z = 0.01\text{cm}$ planes in FLASH simulations for the run with ring radius $d = 800\mu\text{m}$, and CH target. (a) $t = 0.6\text{ns}$ (when laser is still on), (b) $t = 1.4\text{ns}$ (0.4ns after the laser is turned off). The unit for electron density is $\text{cm}^{-3}$. The $z = 0.01\text{cm}$ plane in the simulation is $(0.3\text{cm})^2$ rectangle.
Figure 4.4: Three-slice plots for electron density at $x = 0$, $y = 0$ and $z = 0.01\text{cm}$ planes at $t = 3\text{ns}$ for four different ring radii $d$ in FLASH simulations. The targets are CH. (a) $d = 0$, (b) $d = 400\mu\text{m}$, (c) $d = 800\mu\text{m}$, (d) $d = 1200\mu\text{m}$. The unit for electron density is $\text{cm}^{-3}$. The $z = 0.01\text{cm}$ plane in the simulation is $(0.3\text{cm})^2$ rectangle.
Figure 4.5: The evolution of plasma variables at TCC for the six different runs in FLASH simulations. The quantities are calculated by averaging over a $(200\mu m)^3$ cubic around TCC.
Table 4.2: Comparison of plasma properties for different ring radii and targets at $t = 3\text{ns}$ and $r = 0$, $z = 2.5\text{mm}$. The $n_e$, $\rho$, $T_e$, $T_i$ and $B$ are calculated by averaging over a $(200\mu\text{m})^3$ cubic around TCC. The jet length $L$ is defined by the point on the $z$ axis where electron density drops to $3 \times 10^{18}\text{cm}^{-3}$. The radius $R$ is defined by reading the position in $z = 2.5\text{mm}$ plane where the density scale height $|\nabla \log \rho|^{-1}$ reaches minimum.

<table>
<thead>
<tr>
<th>Plasma property</th>
<th>$d = 0$</th>
<th>$d = 400\mu\text{m}$</th>
<th>$d = 800\mu\text{m}$</th>
<th>$d = 1200\mu\text{m}$</th>
<th>$d = 800\mu\text{m}$ \hspace{1cm} 2% Fe</th>
<th>$d = 1200\mu\text{m}$ \hspace{1cm} 2% Fe</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electron density $n_e (\text{cm}^{-3})$</td>
<td>$1.7 \times 10^{19}$</td>
<td>$1.2 \times 10^{20}$</td>
<td>$2.0 \times 10^{20}$</td>
<td>$1.5 \times 10^{20}$</td>
<td>$2.7 \times 10^{20}$</td>
<td>$1.6 \times 10^{20}$</td>
</tr>
<tr>
<td>Electron temperature $T_e (eV)$</td>
<td>81</td>
<td>$2.1 \times 10^2$</td>
<td>$5.1 \times 10^2$</td>
<td>$1.0 \times 10^3$</td>
<td>$3.9 \times 10^2$</td>
<td>$8.5 \times 10^2$</td>
</tr>
<tr>
<td>Ion temperature $T_i (eV)$</td>
<td>76</td>
<td>$2.2 \times 10^2$</td>
<td>$5.7 \times 10^2$</td>
<td>$1.4 \times 10^3$</td>
<td>$5.9 \times 10^2$</td>
<td>$2.4 \times 10^3$</td>
</tr>
<tr>
<td>Magnetic field $B (\text{gauss})$</td>
<td>$2.4 \times 10^4$</td>
<td>$1.4 \times 10^5$</td>
<td>$3.3 \times 10^5$</td>
<td>$3.1 \times 10^5$</td>
<td>$3.5 \times 10^5$</td>
<td>$3.7 \times 10^5$</td>
</tr>
<tr>
<td>Jet width $R (\text{cm})$</td>
<td>$&gt;0.15$</td>
<td>0.091</td>
<td>0.049</td>
<td>0.039</td>
<td>0.047</td>
<td>0.042</td>
</tr>
<tr>
<td>Jet length $L (\text{cm})$</td>
<td>0.46</td>
<td>0.52</td>
<td>0.54</td>
<td>0.53</td>
<td>0.55</td>
<td>0.50</td>
</tr>
<tr>
<td>$L/R$</td>
<td>$&lt;3.1$</td>
<td>5.7</td>
<td>11</td>
<td>13.6</td>
<td>11.7</td>
<td>11.9</td>
</tr>
</tbody>
</table>
gets larger. Large ring radius also reduces the opening angle of the jet. The flow velocity is hardly affected by increasing $d$. These results are in good agreement with the previous 2D cylindrical hydrodynamics simulations by Fu et. al. [214] using 2D FLASH. The simulations in this thesis are in 3D Cartesian geometry. The full details of the laser configuration are taken into account. In 3D simulations, even though there is the azimuthal asymmetry of the laser intensity on the target as shown in Figure 4.1, the jet is still well collimated and has similar hydrodynamical properties as in the 2D cylindrical case. The azimuthal asymmetry level for electron density can exceed 10%, and the pattern of density distribution in z-slice resembles a “sun flower” as shown in Figure 4.11.

Comparison for non-dopant and dopant targets — As shown in Figure 4.6, the jet for the 2% Fe dopant shot is slightly different than the one without dopant. As shown in Table 4.4, the jet in a dopant shot radiates several times more than that in a non-dopant shot. However, the radiative cooling time at $t = 3$ ns for the jet is much large than nanosecond even in the dopant shot. Thus, the radiation cooling has little to do with the shape of the jet after it has grown to millimeter size. For an earlier time, however, cooling rate is large enough to play a role. As a result, the electron temperature at TCC for doped jets is always lower than that in the non-doped jets with the same ring radius $d$. The reduction in electron temperature relaxes the cylindrical shock. Thus, more electrons flow into the core, which causes the jets in the dopant shots to have higher electron density than the non-dopant ones. In both doped and non-doped case, the jets are always optically thin, making a good view for X-ray images. As shown in Figure 4.7, the simulated x-ray images are consistent with the images in the experiments. In both the simulated x-ray images and the experimental images, the size of jet grows and the center of jet moves forward.
Figure 4.6: Three-slice plots for electron density (unit: \( \text{cm}^{-3} \)) at \( x = 0, y = 0 \) and \( z = 0.01\text{cm} \) planes at \( t = 3\text{ns} \) for two different ring radii \( d \) and for two different types of targets in FLASH simulations. (a) \( d = 800\mu\text{m}, 2\% \text{Fe-doped target} \), (b) \( d = 1200\mu\text{m}, 2\% \text{Fe-doped target} \) (c) \( d = 800\mu\text{m}, \text{CH target} \), (d) \( d = 1200\mu\text{m}, \text{CH target} \). The \( z = 0.01\text{cm} \) plane in the simulation is \((0.3\text{cm})^2\) rectangle.
Table 4.3: Simulated plasma properties for case $d = 800\mu m$, $t = 3\text{ns}$ at $r = 0$ and $z = 0.25\text{cm}$. All quantities are in Gauss cgs units except temperature expressed in eV. The length scale $L$ is approximately the width of the jet at $z = 2.5\text{mm}$, which is $L \approx 1\text{mm}$. The quantities $\rho$, $n_e$, $T_e$, $T_i$, $B$, $Z$, $A$ and $u$ are calculated by averaging over a $(200\mu m)^3$ cubic around TCC. The variation of $n_e$ is $\Delta n_e = \sqrt{n_e^2 - \overline{n_e^2}}$, similar for other variables.

<table>
<thead>
<tr>
<th>Plasma property</th>
<th>Formula</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electron density $n_e$ (cm$^{-3}$)</td>
<td>⋯</td>
<td>$2.0 \times 10^{20}$</td>
</tr>
<tr>
<td>$\Delta n_e$ (cm$^{-3}$)</td>
<td>⋯</td>
<td>$1.5 \times 10^{19}$</td>
</tr>
<tr>
<td>Mass density $\rho$ (g/cm$^3$)</td>
<td>⋯</td>
<td>$6.3 \times 10^{-4}$</td>
</tr>
<tr>
<td>$\Delta \rho$ (g/cm$^3$)</td>
<td>⋯</td>
<td>$4.7 \times 10^{-5}$</td>
</tr>
<tr>
<td>Electron temperature $T_e$ (eV)</td>
<td>⋯</td>
<td>$5.1 \times 10^{2}$</td>
</tr>
<tr>
<td>$\Delta T_e$ (eV)</td>
<td>⋯</td>
<td>1.2</td>
</tr>
<tr>
<td>Ion temperature $T_i$ (eV)</td>
<td>⋯</td>
<td>$5.7 \times 10^{2}$</td>
</tr>
<tr>
<td>$\Delta T_i$ (eV)</td>
<td>⋯</td>
<td>$1.2 \times 10^{2}$</td>
</tr>
<tr>
<td>Magnetic field $B$ (gauss)</td>
<td>⋯</td>
<td>$3.3 \times 10^{5}$</td>
</tr>
<tr>
<td>$\Delta B$ (gauss)</td>
<td>⋯</td>
<td>$1.5 \times 10^{5}$</td>
</tr>
<tr>
<td>Average ionization $Z$</td>
<td>⋯</td>
<td>3.5</td>
</tr>
<tr>
<td>Average atomic weight $A$</td>
<td>⋯</td>
<td>6.5</td>
</tr>
<tr>
<td>Flow velocity $u \approx u_z$ (cm/s)</td>
<td>⋯</td>
<td>$1.1 \times 10^{8}$</td>
</tr>
<tr>
<td>$\Delta u$ (cm/s)</td>
<td>⋯</td>
<td>$3.2 \times 10^{6}$</td>
</tr>
<tr>
<td>Perpendicular velocity $\sqrt{u_x^2 + u_y^2}$ (cm/s)</td>
<td>⋯</td>
<td>$2.7 \times 10^{6}$</td>
</tr>
<tr>
<td>Plasma property</td>
<td>Formula</td>
<td>Value</td>
</tr>
<tr>
<td>---------------------------------</td>
<td>-------------------------------------------------------------------------</td>
<td>------------</td>
</tr>
<tr>
<td>Electron thermal velocity $v_T$ (cm/s)</td>
<td>$4.2 \times 10^7 T_e^{1/2}$</td>
<td>$9.5 \times 10^8$</td>
</tr>
<tr>
<td>Sound speed $c_s$ (cm/s)</td>
<td>$9.8 \times 10^5 \times [Z T_{ele} + 1.67 T_{ion}]^{1/2}/A^{1/2}$</td>
<td>$2.0 \times 10^7$</td>
</tr>
<tr>
<td>Mach number $M$</td>
<td>$u/c_s$</td>
<td>5.5</td>
</tr>
<tr>
<td>Electron plasma frequency $\omega_{pe}$ (rad/s)</td>
<td>$5.6 \times 10^4 n_e^{1/2}$</td>
<td>$7.9 \times 10^{14}$</td>
</tr>
<tr>
<td>Coulomb logarithm $\ln \Lambda$</td>
<td>$\min(23.5 + \ln(T_e^{1.5}/n_e^{0.5}/Z), 25.3 + \ln(T_e/n_e^{0.5}))$</td>
<td>8.1</td>
</tr>
<tr>
<td>Electron mean free path $\lambda_e$ (cm)</td>
<td>$1.4 \times 10^{13} cm \times T_e^{2}/(Z n_e \ln \Lambda)$</td>
<td>$6.4 \times 10^{-4}$</td>
</tr>
<tr>
<td>Electron Hall parameter $\chi_e$</td>
<td>$6.1 \times 10^{12} \times T_e^{3/2} B/(Z n_e \ln \Lambda)$</td>
<td>4.1</td>
</tr>
<tr>
<td>Plasma $\beta$</td>
<td>$4.0 \times 10^{-11} n_e(T_e + T_i/Z)/B^2$</td>
<td>49</td>
</tr>
<tr>
<td>Reynolds number $Re$</td>
<td>$uL/\nu$ $(\nu = 1.9 \times 10^{19} \times T_i^{5/2}/(A^{1/2} Z^{3} n_e \ln \Lambda))$</td>
<td>$1.3 \times 10^4$</td>
</tr>
<tr>
<td>Magnetic Reynolds number $Rm$</td>
<td>$uL/\eta$ $(\eta = 8.2 \times 10^{5} \times (0.33Z + 0.18) \ln \Lambda/T_e^{3/2})$</td>
<td>$1.4 \times 10^4$</td>
</tr>
<tr>
<td>Biermann number $Bi$</td>
<td>$1.0 \times 10^{-8} \times uBL/T_e$</td>
<td>71</td>
</tr>
<tr>
<td>Hall number $\Omega_H$</td>
<td>$2.0 \times 10^{-19} \times n_e uL/B$</td>
<td>$1.3 \times 10^3$</td>
</tr>
</tbody>
</table>
Table 4.4: Radiation properties of the jet at TCC for $d = 800 \mu m$ ring radius. The temperature and density are in Table 4.2.

<table>
<thead>
<tr>
<th>Plasma property</th>
<th>Formula</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Planck opacity $\kappa_P$ (cm$^2$/g) for CH target</td>
<td>from PROPACEOS</td>
<td>$1.8 \times 10^{-2}$</td>
</tr>
<tr>
<td>Optical depth $\tau$ for CH target</td>
<td>$\kappa_P \rho L$</td>
<td>$1.1 \times 10^{-6}$</td>
</tr>
<tr>
<td>Cooling rate (1/s) for CH target</td>
<td>$0.72AZ^{-1}\kappa_P T_e^3$</td>
<td>$3.2 \times 10^6$</td>
</tr>
<tr>
<td>Planck opacity $\kappa_P$ (cm$^2$/g) for 2% Fe dopant target</td>
<td>from PROPACEOS</td>
<td>$7.2 \times 10^{-1}$</td>
</tr>
<tr>
<td>Optical depth $\tau$ for 2% Fe dopant target</td>
<td>$\kappa_P \rho L$</td>
<td>$6.1 \times 10^{-5}$</td>
</tr>
<tr>
<td>Cooling rate (1/s) for 2% Fe dopant target</td>
<td>$0.72AZ^{-1}\kappa_P T_e^3$</td>
<td>$5.8 \times 10^7$</td>
</tr>
</tbody>
</table>

However, the experimental images have narrow jets in early time and more collimated jets in late time. This difference between the simulation and experiment might be due to the fact that we do not include the anisotropic electron heat flow (see Section 2.1.1) in the simulation. The magnetic field is mostly axial near the $z$ axis as discussed in Section 4.3, thus the heat flow is reduced perpendicular to the jet. The reduced heat flow causes the early time growth of the diameter of jet slower and might be a collimation mechanism at late time.

**On-axis quantiles from simulations and Thomson scattering** — A list of on-axis plasma properties from a snapshot in FLASH simulation results is listed in Table 4.3 using the snapshot for $d = 800 \mu m$ case at $t = 3$ns. Other relevant physical terms can be deduced from the scales and dimensionless numbers in Table 4.3. The plasma in the jet is fully ionized, i.e $A = 6.5$ and $Z = 3.5$ for non-doped shots, $A = 7.49$ and $Z = 3.95$ for doped shots. The evolution of electron/ion temperature, electron density and flow velocity at TCC for different runs is shown in Figure 4.5. The optical Thomson-
Figure 4.7: Comparison of XRFC images for the simulations and experiments. The red line is the location of TCC.
Figure 4.8: Quantities deduced from optical Thomson spectrum and comparison with FLASH simulations (200µm)$^3$ averaged at TCC. (a) Electron density deduced form EPW spectrum. (b) Flow velocity deduced from IAW spectrum.
Figure 4.8: (Continued) (c) Electron temperature deduced from EPW spectrum. (d) Electron temperature deduced from IAW spectrum. (e) Ion temperature deduced from IAW spectrum.
scattering spectrum is fitted to infer the temperature, density, and flow velocity as shown in Figure 4.8. As shown in Figure 4.8(a) and (b), the evolution of electron density and flow velocity agrees well between the simulation and the experiment data, indicating that the FLASH code can well capture the zeroth and first moments of the distribution function. However, there are a few issues in the data for the temperature, which is related to the second moments of the distribution function: (1) The electron temperature deduced from the EPW spectrum has large error bar, indicating that EPW are not accurate for measuring electron temperature in this experiment. (2) The electron temperature deduced from the IAW spectrum is always higher than the temperature in FLASH simulations, except the data for early time for the $d = 1200\mu m$ case. (3) The ion temperature deduced from the IAW spectrum is also always higher than the temperature in FLASH simulations. Fitting the Thomson spectrum for quantities near TCC provides good measurement for electron density and flow velocity, but may mislead the interpretation for electron/ion temperatures if one directly compares the deduced quantities with those predicted in Figure 4.5. As shown in Table 4.3, the variation of some quantities can exceed 10% and thus significantly alters the spectrum. Moreover, the $2\omega$ probe beam can potentially heat the plasma near TCC. The heating effect and all the gradients are taken into account for the improved simulations discussed in the next paragraph.

*Details of Thomson scattering spectrum* — By including laser energy deposition from the probe beam, the hydrodynamical variables in a small region of around $(100\mu m)^3$ will change significantly. Although the heating effect is not of the main interest in the dynamical evolution of the jet, it is significant for the analysis of diagnostics. Figure 4.9 and 4.10 shows that the heating from the probe beam has a significant impact on the measured spectra. Although the energy in the probe
Figure 4.9: Comparison between the synthetic optical Thomson-scattering spectra based on FLASH simulations and the experimental data. The red solid line is the experimental data, the blue dotted line is the synthetic spectrum without probe beam heating, and the black dashed line is the synthetic spectrum with probe beam heating. (a) EPW spectrum at 3.6ns. (b) IAW spectrum at 3.9ns.
Figure 4.10: Comparison between the synthetic optical Thomson-scattering spectra based on FLASH simulations and the experimental data for one shot.
beam (25 ∼ 50J) is low compared to the drive beams, the 70µm diameter focal spot results in an intensity of 10^{15}W/cm^2. The locations of the peaks in the simulated spectra that includes the probe beam are in much better agreement with the measured spectra. The effect is more pronounced for smaller ring radii because the electron temperature is lower, which leads to higher collisional absorption. The background of the measured EPW spectrum comes from the bremsstrahlung radiation, which is not calculated in the simulation. By studying the Thomson spectrum from different shots, I conclude that the bremsstrahlung shape is apparent when the electron density is larger than ∼ 10^{20}cm^{-3}. The agreement for IAW spectrum is excellent for d = 0 when the heating is included, as shown in the first plot in Figure 4.9(b). However, for finite d, the simulation always underestimates the width of the broadened line. The depth of the valley in the middle of the shape is corrected by including the heating effect, which can be explained by the increasing of the electron temperature from probe heating. The evolution trend of the spectrum in one shot is shown in Figure 4.10. The under-predicted width of the IAW spectrum indicates the under-predicted electron/ion temperature. The under-predicted temperature might be due to the reduced electron heat flow in the presence of strong magnetic field, which makes the temperature distribution near the axis of jet less diffusive than the simulation predicts.

4.3 Magnetic field and proton radiography

*Magnetic field generation and evolution* — The simulation is initialized with zero magnetic field, i.e. B = 0. The seed magnetic filed is generated via the Biermann battery term caused by the individual beam heating. On the one hand, the Hall number Ω_H is much larger than the Biermann number Bi as shown in Table 5.2. On
the other hand, the Hall term \((-\frac{c}{e})\nabla \times \left((\nabla B) \times B\right)/(4\pi n_e)\) is second order in \(B\) and does not generate seed fields. Thus Hall term is negligible in this experiment. The Biermann battery term is the only source term in the generalized Ohm’s law that is calculated in FLASH simulation for this experiment. Although magnetic resistivity is also included in the computation, magnetic Reynolds number is very large that the dissipation of the magnetic field is negligible. The azimuthal asymmetry in the system is significant for the generation of seed fields. The generation and evolution of the axial dominant magnetic field is demonstrated in Figure 4.11. Because of the radial temperature gradient as shown in Figure 4.11(a) and the azimuthal density gradient as shown in Figure 4.11(b), the Biermann battery term \((\frac{c}{e})\nabla \times (\nabla P_e/n_e) = (ck_B/n_e)\nabla T_e \times \nabla n_e\) is mainly in axial direction. Toroidal dominated magnetic fields are only generated near the surface of the target to fulfill the closure of the magnetic field line, where there is little azimuthal density gradient but large axial density gradient. At a millimeter above the target surface, the magnetic field is generated in the surrounding, as shown by the ring near \(r \approx 0.08\) cm in Figure 4.11(c), and advected into the core, as shown by the central part of Figure 4.11(d). The shock amplifies the axial magnetic field by a factor of \(\sim 4\) due to the flux conservation as the plasma flows from the surrounding to the core. The cylindrical shock makes the magnetic field highly concentrated as shown in Figure 4.11(e). Because the gradient of density alternates several times azimuthally, the generated axial field also alternates. The 5-fold symmetry in the field comes from the 5-fold symmetry in arranging the laser spots as shown in Figure 4.1. The symmetry is slightly broken in the simulation due to the cubic cells and finite resolution. By using a larger ring of laser spots, the magnetic energy is more concentrated in the core of the jet as shown in Figure 4.13 and 4.14.
Figure 4.11: Demonstration of the generation and evolution of the axial dominant magnetic field with alternating polarity and 5-fold symmetry, using slice plot of several quantities at $z = 0.1 \text{cm}$ for $t = 1.6 \text{ns}$. (a) Electron temperature (eV). The pattern is concentric circles. (b) Electron density ($\text{cm}^{-3}$). The “sunflower-like” pattern has 5-fold symmetry due to the laser pattern. The symmetry is slightly broken due to finite number of cells in the simulation (c) $z$ component of Biermann battery term ($\text{kG/s}$) (d) $z$ component of advection term $\nabla \times (\mathbf{v} \times \mathbf{B})$ ($\text{kG/s}$) (e) $z$ component of magnetic field ($\text{kG}$) (f) $\varphi$ component of the magnetic field ($\text{kG}$).
Figure 4.12: Sample magnetic field lines (color scale unit: kG) for $d = 800\text{cm}$, CH target. (a) at $t=1.6\text{ns}$ (b) at $t=3.6\text{ns}$. The field far way from the target is mainly axial and the field close the target is toroidal.
Figure 4.13: Three slice plot for magnetic field amplitude (unit:kG) at $x = 0$, $y = 0$ and $z = 0.1$cm for different laser ring radii $d$ at $t = 1.6$ns. The disk slice is at $z = 0.1$cm with 0.3mm diameter. (a) $d = 0$ (b) $d = 400\,\mu$m (c) $d = 800\,\mu$m (d) $d = 1200\,\mu$m
Figure 4.14 : Three slice plot for magnetic field amplitude (unit:kG) at $x = 0$, $y = 0$ and $z = 0.25\text{cm}$ for different laser ring radii $d$ at $t = 3.6\text{ns}$. The disk slice is at $z = 0.25\text{cm}$ with 0.3mm diameter. (a) $d = 0$ (b) $d = 400\text{µm}$ (c) $d = 800\text{µm}$ (d) $d = 1200\text{µm}$.
Figure 4.15: The dimensionless transport coefficients for $Z_{\text{eff}} = 5$.

The feature of magnetic field in 3D — The magnetic field is mostly axial near the $z$ axis and mostly toroidal near the surface of the target, as shown in Figure 4.11(f) and Figure 4.12. The width and the length of the field bundles grow with the jet. The maximum field strength reaches several hundred kilo-gauss. The maximum magnitude of magnetic field at $t = 3.6\text{ns}$ increases with the radius $d$ of the laser ring, as shown in Figure 4.14. This is consistent with the 2D cylindrical simulation [215]. However, the full three dimensional simulation predicts a magnetic field axial polarized and much stronger than those in the two dimensional cylindrical simulation. In the 2D cylindrical simulation, the laser intensity is azimuthally uniform, thus the Biermann battery term only has the toroidal component.

How does magnetic affect the flow? — The value of the plasma $\beta$ is around 50 in the core of the jet, as shown in Table 5.2. Thus the hydrodynamical properties are not
significantly affected by the magnetic pressure. However, the electron Hall parameter \( \chi_e \sim 4 \) and is even higher for stronger local magnetic fields. Thus the electron heat conduction can be suppressed perpendicular to the field, since the cross field electron conductivity scales as \( 1/\chi_e^2 \) as \( \chi_e \gg 1 \). The suppressed cross field electron heat conduction can potentially make the jet narrower and the electron energy more concentrated. The underestimated temperature and IAW spectrum width can be explained by the suppressed heat conduction. As \( \chi_e \) becomes larger, the electron component is more collisional, and thus there are less nonlocal effects. The Righi-Leduc conductivity scales as \( 1/\chi_e \) as \( \chi_e \gg 1 \) and drops slower than the cross field conductivity as shown in Figure 4.15(b). The Righi-Leduc heat flow is in the \(-b \times \nabla T_e\) direction, dominated by the azimuthal direction, and can cause the non-uniform distribution of \( T_e \) in azimuthal direction, in contrast to Figure 4.11(a).

The resolution of proton radiography — The proton source size is \( \sim 45 \mu m \) for the fusion protons and \( \sim 5 \mu m \) for the TNSA protons. The smearing caused by the pulse duration has a length scale \( \Delta l \sim v \Delta t \), where \( \Delta t \sim 150 \text{ps} \) for fusion protons and \( \Delta t \sim 1 \text{ps} \) for TNSA protons, thus using the velocities in Table 4.3 for fusion protons, \( \Delta l_z \sim 160 \mu m \), \( \Delta l_{x,y} \sim 13 \mu m \), and for TNSA protons with \( E_p = 10 \text{MeV} \), \( \Delta l_z \sim 1 \mu m \), \( \Delta l_{x,y} \sim 0.15 \mu m \). Assuming the proton is shifted by \( 200 \mu m \) (which is typical), then for \( E_p = 10 \text{MeV} \) TNSA protons, \( \Delta E/(2E) = 3.79 \text{MeV}/2/2 \times 10.2 \text{MeV} \) (\( \Delta E \) is half of the effective temperature), the maximum possible resolution for magnetic field \( \sim 20 \mu m \). DD protons with \( \Delta E/(2E) = 0.32 \text{MeV}/(2 \times 3 \text{MeV}) \) resolve magnetic field at \( \sim 11 \mu m \), and D\(^3\)He protons with \( \Delta E/(2E) = 0.67 \text{MeV}/(2 \times 14.7 \text{MeV}) \) resolve magnetic field at \( \sim 5 \mu m \). The energy gain or lost from the electric field is estimated to be less than \( 0.1 \text{MeV} \), which is negligible compared to the \( \Delta E \) of the beam itself. The overall estimation shows that FLASH simulation is able to resolve a smaller spatial scale.
than the experiment.

**Inferring magnetic field strength from proton images** — Figure 4.16 shows an example for inferring magnetic field strength from proton images. Since the feature is extensive in the flow direction (45 degree to the upright in the image), Eqs (3.43) and (3.44) are used to infer the line integral of the magnetic field perpendicular to the line of sight. There are two peaks in the inferred $\int B_\perp dl$, $-1.3$MG $\cdot$ mm around $r = -0.8$cm and $-1.7$MG $\cdot$ mm around $r = 0.5$cm. Combining the restriction for the diameter of the jet from Figure 4.7, which is smaller than 1.1mm, the maximum field in the jet should be larger than 1MG [175].

**Features of the proton images** — The data shown in Appendix A compares the simulation synthetic and experimental proton images. To the lowest order the light and dark patterns correspond to the averaged MHD current $(\nabla \times B)$ projected along the light of sight, as given by Eq (3.5). In the data for $d = 400\mu$m, $800\mu$m and $1200\mu$m, it is common to have vertical dark and bright strips extended along the axial direction and a curved horizontal strip close to the surface of the target. The presence of the vertical stripes is a result of alternating axial field. The curved horizontal strip is produced by the large loop of surface toroidal field near the surface of the target. In the $d = 800\mu$m and $d = 1200\mu$m cases, the good qualitative agreement between the synthetic images and the ones from experiment on the general trend of large scale features suggests that the magnetic field structures we predict using FLASH simulation are consistent with the structures in the experiments. The agreement for $d = 0$ and $d = 400\mu$m cases is not as good as the $d = 800\mu$m and $d = 1200\mu$m cases.

What causes the difference between simulations and experiments? — In the FLASH simulations in this work, the effects of magnetization as in Eqs (2.18) and (2.19), i.e. $\chi_e \gtrsim 1$, are not computed. On the one hand, strong magnetic field reduces
From XRFC data we know the size of the jet < 1.1mm
So B(max) > 1MG

Figure 4.16: Inferring magnetic field from the proton image.
the cross-field head conduction \((\kappa_\perp \propto \chi_e^{-2})\) and causes Righi-Leduc heat conduction \((\kappa_\wedge \propto \chi_e^{-1})\), thus alters the electron temperature distribution. The magnitude of Biermann battery effect \(((ck_B/e)\nabla T_e \times \nabla n_e)\) is also altered due to the altered electron temperature profile. On the other hand, the coefficients for the \(\vec{\beta} \cdot \nabla T_e\) term in Eq (2.18) depends on \(\chi_e\). As \(\chi_e\) becomes large, \(\beta_\perp\) drops faster than \(\beta_\wedge\) as discussed in Section 2.1.1 and shown in Figure 4.15(a), and the field generated by Biermann battery term is perpendicular to \(\nabla T_e\). Thus, the dominant term in \(\vec{\beta} \cdot \nabla T_e\) is the \(\hat{b} \times \nabla T_e\) term, although \(\beta_\wedge\) also drops \((\propto \chi_e^{-1})\) as \(\chi_e\) becomes larger. Using the values in Table 4.2 the estimated value for \(\chi_e\) is 0.2 for \(d = 0\), 0.8 for \(d = 400\mu m\), 4 for \(d = 800\mu m\), and 14 for \(d = 1200\mu m\). It is a good assumption for large \(\chi_e\), i.e. \(d \geq 800\mu m\), that only the \(-\nabla p_e/(n_e e)\) term has significant contribution in Eq (2.18) so that the equation is as same as the \(\chi_e = 0\) case. This explains why the agreement between the synthetic images and the ones from experiment is better for \(d \geq 800\mu m\) than for \(d \leq 400\mu m\). If one assumes the balance between \(-\nabla p_e/(n_e e)\) and \(- (k_B/e) \beta_\wedge \hat{b} \times \nabla T_e\), then by calculating the cross product of \(\vec{E}\) and \(\nabla T_e\), \(\beta_\wedge = |\nabla p_e \times \nabla T_e|/(n_e k_B |\nabla T_e|^2)\), which may be used to estimated the saturation value for \(\chi_e\).

### 4.4 Discussions

**The geometry of magnetic fields** — The geometry of magnetic fields in the laboratory created jets using the hollow ring of laser beams is dominated by the poloidal component. In many astrophysical context, it is generally believed that toroidal field supposedly dominates along the jet axis, e.g. in the jet of accreting black hole [217]. It has been proposed that the stability of bipolar protostellar jets may be caused or enhanced by strong poloidal magnetic fields [218, 219, 220]. Large-scale poloidal magnetic flux is also a key jet-making ingredient for accreting black holes [221]. Under
controllable condition, much can be learned about the magnetic effect on jet collimation, stability and structure in the laboratory. The characteristics of $B_z/B_\phi$ or the magnetic pitch angle in the jet can be well controlled by tuning the ring radius.

**Nonlocal effects** — By using the electron heat conductivity in Eq (2.47) and the values in Table 4.2, the ratio of heat flux $\kappa_e T_e/L$ to the free-streaming heat flux $n_e k_B T_e \sqrt{k_B T_e/m_e}$ can be estimated. For $d = 0$ and $d = 400 \mu m$, the ratio is 0.014, thus nonlocal effects are negligible. The ratio is 0.046 for $d = 800 \mu m$ and 0.24 for $d = 1200 \mu m$, which is comparable to the coefficient 0.06 used in the flux-limiter. However, the electron temperature gradient is mostly perpendicular to the magnetic field, thus the heat flux is reduced, making the nonlocal effects less significant. In conclusion, the local terms in Eq (2.19) in the presence of magnetic field can accurately describe the heat flow for different ring radii if one assumes a coefficient 0.06 in the flux-limiter.

**Dynamo** — The values for fluid Reynolds number Re, magnetic Reynolds number Rm, and magnetic Prandtl number Pr$_m$ are listed in Table 4.5. It is worth noting that as $d$ increases, Pr$_m$ transits from Pr$_m \ll 1$ to Pr$_m \gg 1$. While the experiments in the Pr$_m < 1$ regime [49][51] have been carried out extensively to study dynamo

<table>
<thead>
<tr>
<th>d(\mu m)</th>
<th>0</th>
<th>400</th>
<th>800</th>
<th>1200</th>
</tr>
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<tr>
<td>Re</td>
<td>$1.7 \times 10^5$</td>
<td>$8.6 \times 10^4$</td>
<td>$1.3 \times 10^4$</td>
<td>$1.1 \times 10^3$</td>
</tr>
<tr>
<td>Rm</td>
<td>$8.9 \times 10^2$</td>
<td>$3.7 \times 10^3$</td>
<td>$1.4 \times 10^4$</td>
<td>$3.9 \times 10^4$</td>
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<tr>
<td>Pr$_m$</td>
<td>$5.2 \times 10^{-3}$</td>
<td>$4.4 \times 10^{-2}$</td>
<td>1.1</td>
<td>37</td>
</tr>
</tbody>
</table>

Table 4.5 : List of fluid Reynolds number Re, magnetic Reynolds number Rm, and magnetic Prandtl number Pr$_m$. 

...
amplification and the structure of the magnetic fields, the experiments for the \( \text{Pr}_m > 1 \) regime \cite{77} remains limited in laboratory. The hollow ring configuration with \( d = 1200 \mu m \), where the fluid dissipation scale is \( l_\nu = 5 \mu m \) and the magnetic dissipation scale is \( l_\eta \approx 0.9 \mu m \), is suitable for exploring for the small scale dynamo in \( \text{Pr}_m > 1 \) regime. Resolving the magnetic dissipation scale is a major challenge in both experiments and simulations. The magnetized terms in \( \vec{\beta} \cdot \nabla T_e \) interplay with the dynamo, especially when \( \chi_e \approx 1 \), i.e. when the plasma is not sufficiently magnetized.

**Using colliding jets to create shocks** — One of the original motivations for using the hollow ring configuration was to optimize the conditions for achieving a laboratory collisionless shock from the plume collisions \cite{214}, which can be used to study astrophysical collisionless shocks. The configuration for colliding involves two contour-propagating jets, each of them created by using a hollow ring of beams. To evaluate the viability of shock formation, three length scales need to be considered \cite{91,214}. They are the inter-stream mean free path

\[
\lambda_{\text{mfp}} = 5 \times 10^{-13} \text{cm} \times \frac{A^2(u[\text{cm/s}])^4}{Z^3n_e[\text{cm}^{-3}]} \tag{4.1}
\]
and the characteristic electrostatic instability length scale

\[
l_{ES} = 2.4 \times 10^{-14} \text{cm} \times \frac{A^{3/2}(u[\text{cm/s}])^3}{Z^{1/2}(n_e[\text{cm}^{-3}])^{1/2}T_e[\text{eV}]}
\]  
(4.2)

and the characteristic electromagnetic instability length scale (100 times ion skin depth)

\[
l_{EM} = 2.3 \times 10^9 \text{cm} \times \frac{A^{1/2}}{Z^{1/2}(n_e[\text{cm}^{-3}])^{1/2}}
\]  
(4.3)

A necessary but insufficient condition for shock formation is that \(\lambda_{mfp}\) has to be greater than both \(l_{ES}\) and \(l_{EM}\). As shown in Table 4.6 for \(d \geq 400\mu m\), \(l_{EM}\) is much larger than \(l_{ES}\) but comparable to \(\lambda_{mfp}\) and the characteristic system size \(L \approx 0.1\text{cm}\). Such a scale ordering precludes shock formation as observed in a few previous experiments for interpenetrating plasma flows [87, 90] at the OMEGA laser facility. The formation of collisionless shocks mediated by the Weibel instability has been observed in recent experiments [92] at NIF, where the scale separation is much larger (ion–ion collisional mean free path is \(\sim 30\) times characteristic system size, and characteristic system size is \(\sim 500\) times ion skin depth). In the jets created using the hollow ring configuration in this thesis, the electron mean free path after jet collision can be estimated using the strong shock limit, where the temperature is given by Eq (4.20) in Reference [5]

\[
k_B T_d \sim (3/16) Am_p u_s^2 / 16(1+Z) = (1/3) Am_p u^2 / (1+Z) = 6.0 \times 10^3 \text{eV}
\]

(shock velocity \(u_s\) is related to the velocity \(u\) of one stream in the laboratory frame by \(u - u/4 = u_s\)) and the electron density is \(n_{e,d} = 4n_e\). Thus, \(\lambda_e = 0.02\text{cm for } d = 800\mu m\) and \(\lambda_e = 0.03\text{cm for } d = 1200\mu m\) given by Eq (2.1). The ion mean free path given by Eq (2.2) is one order of magnitude smaller than the electron mean free path. This estimation indicates that the shock formed after the counter-streaming jets get thermalized is collisional instead of collisionless. The electron Hall parameter given by Eq (1.6) after shock formation remains large or even becomes larger, since \(T_e\) increase
more than $n_e$ and $B$ either remains the same (for field parallel to the shock normal direction) or gets compressed (for field perpendicular to the shock normal direction). An interesting question is: how does the electron transport effects as discussed in Section 2.1.1 interplay with the shock?

Scaling up the platform to larger laser facility — The hollow ring laser platform is suited to scale up to NIF with 192 beams and more energy per beam, creating centimeter-sized magnetized jets. The jets produced with the NIF platform will have several distinctive properties from OMEGA experiments, but are of key importance for astrophysical jet modeling. The higher temperature, density, flow velocity, and magnetic field will lead to large dimensionless parameters. It is worth noting that several dimensionless parameters have strong dependency on temperature, e.g. magnetic Prandtl number $\Pr_m \propto T^4/n_e$, electron Hall parameter $\chi_e \propto T^{3/2}B/n_e$, electron Knudsen number $N_{Ke} \propto T^2/(n_eL)$. While the jet-jet collision can potentially achieve large temperature increase, the temperature that is achievable on NIF is even larger, making $\Pr_m \gg 1$ and $\chi_e \gg 1$. The collisionality can stay strong due to the large magnetization. Under such condition, the magnetic dissipation scale is much smaller than the fluid dissipation scale so that detail features for small-scale dynamo might be observed. The heat flow is well aligned with the magnetic field, similar to the situation in the intracluster medium of clusters of galaxies [106, 107, 108, 109, 110, 111].

4.5 Summary

The creation of the supersonic collimated jets and strong magnetic fields using the ring laser pattern is discussed in this chapter. The FLASH simulation results were validated against a subset of experimental data from the OMEGA experiments. The effects of magnetization with $\chi_e \gtrsim 1$ are not computed in the to-date simulations.
Those effects cause the quantitative difference between simulations and experiments, and will be implemented in future simulations. The applications of the experiment platforms are also discussed.
Chapter 5

Shock shear experiments

In ICF capsule implosion, the material turns into dense plasmas and recent simulations have shown that such plasmas tend to be unstable and turbulence can develop. Turbulence has been suggested as a mechanism for degrading the hot-spot conditions by altering transport properties, introducing mix, or reducing the conversion of kinetic energy to hot-spot heating [222]. It has been debated whether turbulence is damped by the viscosity in the hot spot [223, 224, 225, 226, 227, 228, 229]. However, the shocked interfaces as well as the interface between the shell and the hot spot can have very different dynamics and can indeed be unstable. It is believed that turbulence and the associated mixing process can be crucial for understanding ICF [230].

The Biermann battery effect [61] is known to generate seed magnetic fields in laser driven plasmas and has been studied extensively in HED laser-driven experiments [72, 68, 175, 70, 174, 69, 231] such as the one discussed in Chapter 4. In low density laser driven plasmas, the magnetic field can be amplified by turbulence and measured using temporal diagnostics by B-dot probe [17] and spatial diagnostics by proton radiography [52]. The magnetic frequency spectrum in supersonic plasma turbulence has been measured in a recent experiment [53] on the Vulcan laser. However, the strength and importance of these fields in the close to or higher than solid density plasmas such as an ICF implosion are still unknown. To address the dynamics of magnetic field in high density targets, 3D extended-MHD simulations of
the stagnation phase of ICF including Biermann battery term \[61\], Nernst term \[114\] and anisotropic heat conduction in the magnetic field, indicate that self-generated magnetic fields can reach over \(10^4\) tesla and can affect the electron heat flow \[117\]. The simulations with pre-magnetization for ICF implosions show the significance of Lorentz force and \(\alpha\)-particle trapping \[113\].

The shock-shear platform \[35, 31\], as a platform to study the shear-induced instabilities and turbulence production under HED conditions, has been used to investigate the turbulent mixing \[30, 29\] at material interfaces when subject to multiple shocks and reshocks or high-speed shears \[35, 32\]. The experiments \[25, 26, 33, 28, 27\] using the shock-shear platform has been carried out on the OMEGA laser facility and NIF. These experiments provide quantitative measurements to assist in validation efforts \[23, 24, 34\] for mix models, such as Besnard-Harlow-Rauenzahn (BHR) model \[232, 233\]. The experimental data and the validation efforts constrain models relevant to integrated HED experiments such as ICF or astrophysical problems. In the shock-shear targets, the Biermann Battery \((\nabla n_e \times \nabla T_e)\) term \[61\] can generate and sustain strong magnetic fields in the vortices due to the misalignment of the density gradient and temperature gradient caused by electron heat conduction.

In this chapter, I discuss the effort using the shock-shear platform \[35, 31\] developed at LANL to quantify the dynamics of magnetic fields in HED plasmas with instabilities and turbulence. The shock compression can achieve a regime where the density is hundreds of \(\text{mg/cc}\) to \(1\text{g/cc}\). One of the major concerns is that the targets with large density can diffuse the proton beam and affect the interpretation of the proton image \[59\]. However, the simulations for the synthetic proton image including the stopping power and Coulomb scattering show that the deflection of proton beam by magnetic fields is still detectable. Further improvements are still needed to make
Table 5.1: The parameters and the maximum values of magnetic field and electron temperature for the three different targets/runs. $T_e$ and $B$ are calculated by averaging over a $(200\mu m)^2$ around the center of the target in the $x-z$ plane. PPS stands for pepper-pot screen.

<table>
<thead>
<tr>
<th>Target/Run label</th>
<th>Slanted slots</th>
<th>Layer thickness</th>
<th>Layer material</th>
<th>Wall thickness</th>
<th>Wall material</th>
<th>$T_e$(eV) at 10ns</th>
<th>$B$(kGauss) at 10ns</th>
</tr>
</thead>
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<tr>
<td>A</td>
<td>Yes</td>
<td>15$\mu$m</td>
<td>Mg</td>
<td>100$\mu$m</td>
<td>Be</td>
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<td>158</td>
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<tr>
<td>B</td>
<td>No</td>
<td>6$\mu$m</td>
<td>Cu</td>
<td>150$\mu$m</td>
<td>CH</td>
<td>26</td>
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<td>C</td>
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<td>CH</td>
<td>150$\mu$m</td>
<td>CH</td>
<td>28</td>
<td>86</td>
</tr>
</tbody>
</table>

the fields high enough to change the dynamics of the small-scale evolution of vortices like those in a turbulent cascade, and affect our understanding of turbulence.

5.1 Simulation setup

FLASH code is used to model the evolution of the shock-shear system on OMEGA. The simulation is initialized using the geometry and parameters of targets used for OMEGA experiments.

The target system is composed of the shock tube, the gold cone for minimizing stray laser light, the foam filling the shock tube and a plastic cap covering the end of the tube, as shown in Figure 5.1 and 5.2. As shown in Figure 5.1(a), a window is opened in the middle of the tube and along the path of the proton beam to make the proton beam less diffusive, i.e. less energy lost and scattering. However, the opened window can make the plasma squirt outwardly. The foam density is 62mg/cc, and the foam is divided by a layer with slanted or non-slanted slots, as shown in Figure
Figure 5.1: The experiment setup. The shapes and dimensions of different parts of the target is used to initialize the FLASH simulations. (a) The far-view of the target system, including the shock tube, the gold cone for shielding and the plastic end cap. The foam and the layer are not shown. The $x$ axis extends through the window. (b) The target with a pepper-pot screen (PPS) for a narrow view proton radiography. The screen has five large holes with $200\mu m$ diameter and four small holes. The screen is at $x = -1.3$mm plane, attached to the edge of the gold cone. (c) The dimension of the shock tube, the window and the end cap. The beryllium shock tube has a oval-shape window in the middle. The end cap is plastic. The foam and the layer is not in this figure. The inner radius of the tube is $250\mu m$, and the outer radius of the tube is $350\mu m$. (d) Same as (a) but the shock tube is plastic and thicker. The inner radius of the tube is $250\mu m$, and the outer radius of the tube is $400\mu m$. 
Figure 5.2: (a) The magnetism layer with 45 degree slanted slots. The wavelength of the slots is $150\mu$m. (b) The plastic or copper layer with straight slots. The wavelength of the slots is $150\mu$m. (c) A layer divides the low density foam into two half-cylinders to collimate the shock flow. The gold plugs hold back the shock at the end of each half-cylinder of foam.
The end cap is 1g/cc plastic. For each type of the target, the shape of the slots, the material and the thickness of the layer, and the material of the wall are listed in Table 5.1. Some targets are built with a pepper-pot screen (PPS) [234], as shown in Figure 5.1(b). The PPS is used for a narrow view of the proton deflection signal in proton radiography, reducing the signal contamination from off-center line-of-sight. The 200µm diameter hole in the middle allows proton beams to go through the central part of the target. Other holes are used as references to register the position of protons. The PPS is a 40 µm thick tantalum foil.

In the initialization, the pressure of all the solid regions is $5 \times 10^9$bar ($= 5 \times 10^9$erg/cm$^3$), and the temperature is calculated self-consistently from the equation of state table. Using the same pressure instead of the same temperature among all the solid regions can prevent one solid region from expanding into another solid region and launching artificial shocks before the HED condition is reached. Under HED condition, the pressure is larger than $10^5$bar ($= 10^{11}$erg/cm$^3$), thus the initial pressure is low enough to have negligible effect on the simulations. The vacuum region is initially filled with $10^{-6}$g/cc helium to avoid numerical problems in hydrodynamics or MHD solvers. The density is low enough that the effect of helium on the simulations is negligible.

A 3D Cartesian grid with $(240 \times 240 \times 464)$ zones is used to resolve a $(1440\mu m \times 1440\mu m \times 2784\mu m)$ domain, corresponding to $6\mu m$ per cell width. Using AMR, each zone is adaptively refined to one leaf level, i.e. a resolution of $3\mu m$ or $2^3 = 8$ zones, if the mass fraction of the layer material is larger than 10%. The refinement allows us to efficiently resolve the dynamics near the layer and reduce the computing time spent on the zones far away from the layer. Although we cannot resolve the turbulence dissipation scale with the current computing capability and neither do we
use Reynolds-averaging Navier-Stokes (RANS) models such as BHR model to resolve the small scale dissipation processes of the fluid, FLASH is still a suitable tool for designing these experiments because the fabricated layers have low surface roughness.

To model the laser driven energy deposition, the spatial and temporal specifications of each of the 16 OMEGA driver beams are input into the code. Ray tracing by solving the geometric optics and the inverse-bremsstrahlung absorption is used. The 16 driver beams are turned on and off simultaneously with a 1ns pulse duration and 8 beams on each side of the target. Each delivers about 500J of energy on a target. The radius of each beam is $283\mu m$ and the intensity distribution we use is gaussian.

For convention, $t = 0$ is the time for laser turn on. The axis of the shock tube is the $z$ axis. The layer dividing the foam is in the $y-z$ plane, i.e. the plane with $x = 0$. The center of the target is at $x = y = z = 0$. The $x$ axis extends through the window.

The primary diagnostic for temporally and spatially resolved profile of the density and shock propagation in the experiments is the point projection X-ray radiography with a vanadium backlighter at $23\times$ magnification. The backlighter source emits 5180eV and 5205eV helium like lines $[235]$. The images are recorded on the X-ray framing camera (XRFC) $[206, 207, 208]$. SPECT3D $[60]$ is used to generate the synthetic ray-tracing X-ray image. The line of sight of XRFC is along the $y$ axis, which captures the distortion of the layer.

Proton radiography $[72]$, using $D^3He$ (14.7 MeV) protons from fusion, measures magnetic fields. The diffusion of the proton beam caused by Coulomb scattering $[200, 199]$ and stopping power $[192, 195, 193, 194, 236]$ is significant for the targets we use. MPRAD code (see Section 3.1.1) is used to model the synthetic proton radiography, including the Lorentz force and the effects from Coulomb scattering.
and stopping power. The proton source stands at (−0.75cm, 0, 0), while the image plate CR39 is located 27cm from the center on the other side. The line of sight of the proton radiography is perpendicular to the line of sight of the X-ray image. The energy distribution of the proton source used in the simulation is a gaussian distribution with FWHM = 0.25MeV centered at 14.7MeV. The typical size 45µm for proton source is used in the MPRAD simulations.

The evolution of different quantities for the three runs is shown in Figure 5.3 to 5.5. The size of all plots is 1200µm × 1200µm. From first to fourth row are: density at the \( y = 0 \) plane, electron temperature at the \( y = 0 \) plane, X-ray flux normalized by the purely transparent flux, magnetic field \( B_y \) in kilo-gauss at \( y = 0 \) plane (positive for into the plane). The plots in the second and the fourth rows are overlaid with magenta contours for the density of the wall material equal to 0.5g/cc. From fifth to the last rows are proton images for four different cases as labeled.

5.2 Hydrodynamics

A list of on-axis plasma properties from a snapshot in FLASH simulation results is listed in Table 5.2.

The evolution of density, electron temperature and X-ray flux as shown in Figure 5.3 to 5.5 shares some common features. The gold plugs hold back the shock at one end of each half-cylinder of foam. Two shocks of roughly same strength in the same material propagate from opposite directions towards the center of the tube. The layer placed in the middle between the two regions collimates the shocked flows and introduces a length scale through its thickness which will influence the dominant modes of the resulting shear instability. The cut slots in the layer introduce alternating density gradients and causes magnetic field generation by Biermann battery term, which is
Figure 5.3: Spatial distribution of different quantities at different time for runA.
Figure 5.4: Same as Figure 5.3 but for runB
Figure 5.5: Same as Figure 5.3 but for runC
Figure 5.6: Spatial distribution of different quantities for runA with or without window at 10ns. The size of all plots is $1200\,\mu\text{m} \times 1200\,\mu\text{m}$. From first to last row are: density at the $y = 0$ plane, electron temperature at the $y = 0$ plane, magnetic field $B_y$ in kGauss at $y = 0$ plane (positive for into the plane). The plots in the third row are overlaid with magenta contours for the density of the wall material equal to $0.5\,\text{g/cc}$. 
Table 5.2: Simulated plasma properties for runA. All quantities are in cgs units except temperature, which is expressed in eV. The length scale, $L$ is approximately the diameter of the tube ($\approx 500\mu$m). The $n_e$, $\rho$, $T_e$ and $T_i$ are calculated by averaging over a $(200\mu m)^2$ square around the center of the target in the $x-z$ plane, at $t = 10$ns. The flow speed is $u = 7 \times 10^6$cm/s for each counter propagating flow.

<table>
<thead>
<tr>
<th>Plasma property</th>
<th>Formula</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electron density $n_e$(cm$^{-3}$)</td>
<td>...</td>
<td>$5.6 \times 10^{22}$</td>
</tr>
<tr>
<td>Mass density $\rho$(g/cm$^3$)</td>
<td>...</td>
<td>$0.36$</td>
</tr>
<tr>
<td>Electron temperature $T_e$(eV)</td>
<td>...</td>
<td>$25$</td>
</tr>
<tr>
<td>Ion temperature $T_i$(eV)</td>
<td>...</td>
<td>$25$</td>
</tr>
<tr>
<td>Magnetic field $B$ (gauss)</td>
<td>...</td>
<td>$1.6 \times 10^5$</td>
</tr>
<tr>
<td>Average ionization $Z$</td>
<td>...</td>
<td>$1.9$</td>
</tr>
<tr>
<td>Average atomic weight $A$</td>
<td>...</td>
<td>$7.3$</td>
</tr>
<tr>
<td>Flow speed $u$(cm/s)</td>
<td>...</td>
<td>$7 \times 10^6$</td>
</tr>
<tr>
<td>Sound speed $c_s$ (cm/s)</td>
<td>$9.8 \times 10^5 \times [ZT_{e}\text{le} + 1.67T_{i}\text{ion}]^{1/2}/A^{1/2}$</td>
<td>$3.4 \times 10^6$</td>
</tr>
<tr>
<td>Mach number $M$</td>
<td>$u/c_s$</td>
<td>$2$</td>
</tr>
<tr>
<td>Coulomb logarithm $\ln \Lambda$</td>
<td>min$(23.5 + \ln(T_e^{1.5}/n_e^{0.5}/Z), 25.3 + \ln(T_e/n_e^{0.5}))$</td>
<td>$1.4$</td>
</tr>
<tr>
<td>Electron mean free path $\lambda_e$ (cm)</td>
<td>$1.4 \times 10^{13} \text{cm} \times T_e^2/(Zn_e \ln \Lambda)$</td>
<td>$5.9 \times 10^{-8}$</td>
</tr>
<tr>
<td>Hall parameter $\chi$</td>
<td>$6.1 \times 10^{12} \times T_e^{3/2}B/(Zn_e \ln \Lambda)$</td>
<td>$8.2 \times 10^{-4}$</td>
</tr>
<tr>
<td>Plasma $\beta$</td>
<td>$4.0 \times 10^{-11} n_e(T_e + T_i/Z)/B^2$</td>
<td>$3.3 \times 10^3$</td>
</tr>
<tr>
<td>Magnetic Reynolds number $Rm$</td>
<td>$uL/\eta \left(\eta = 8.2 \times 10^5 \times (0.33Z + 0.18) \ln \Lambda/T_e^{3/2}\right)$</td>
<td>$47$</td>
</tr>
<tr>
<td>Reynolds number $Re$</td>
<td>$uL/\nu \left(\nu = 1.9 \times 10^{19} \times T_i^{5/2}/(A^{1/2}Z^2n_e \ln \Lambda)\right)$</td>
<td>$8.6 \times 10^6$</td>
</tr>
</tbody>
</table>
discussed in Section 5.3. Because the layer does not fully collimate the shocks, oblique shocks are launched into the opposite volumes of the tube. The shock front near the end of the tube travels further transversely. It takes roughly 8.5 ns for the shocks to cross and create the pressure-balanced shear mixing region. The pressure in the two regions is roughly equal and the shocked material is the same on each side of the mixing layer, so that the mixing region does not experience a net translation away from the center of the shock tube. After 8.5 ns, the oblique shock on either end of the tube gradually crosses the primary shock from the other direction. An oblique region of high density is developed by the reverse shock.

The ideally constructed target should be symmetric about a rotation of 180 degrees. However, the different effective laser intensities on two ends of the target due to different laser incident angles cause the two shocks to move at slightly different speeds. The shock from the right side in Figure 5.3 to 5.5 moves slightly faster. This asymmetry does not affect the overall picture of the hydrodynamical and magnetic field evolution, but the asymmetry of the density distribution can affect the proton radiography which is discussed in Section 5.3.

Because of the opened window on the wall, there are plasma plumes traveling outside the window. As shown in Figure 5.6, the overall picture of hydrodynamical evolution is still similar to previous shock-shear experiments without a window [23, 24, 32, 26], although the plasma plume carries mass and energy away from the tube. At later times, the shock can penetrate through the wall. This results in plumes outside the wall, which can then interact with the plume from the window.

The transmitted X-ray flux is shown in the third rows in Figure 5.3 to 5.5. In the X-ray flux, the location and the shape of the shock front is consistent with the density distribution and can be easily identified. The shocks in the wall can also be
seen in the X-ray image. The plume launched from the wall or the window has low
density and is not visible in the X-ray flux. The layer has high density and low X-ray
transmission, leading to the low flux on the X-ray image. For runA and runB, where
the layer material is magnesium and copper respectively, the contrast of X-ray flux
between the layer and the foam is high, while for runC where the layer material is
CH, the X-ray contrast is low.

5.3 Magnetic field generation and evolution

When the shock from one end of the tube passes, the temperature is high near the
center of the half-cylinder as shown in the second rows in Figure 5.3 to 5.5. A cold
region is left behind the shock. The temperature gradient near the layer is mainly
perpendicular to the layer and pointing towards the shocked region, due to electron
heat conduction. The density gradient is alternating, caused by the cut slots on the
layer. Thus the Biermann battery term generates the alternating magnetic field in
the $\pm y$ direction, as shown in Figure 5.7(a). However, the cold region left behind
the shock has low electron temperature and thus high resistivity. The magnetic fields
behind the shock diffuse very quickly. In the end, the only significant field remains
near the center of the tube is in the $-y$ direction, because near the center of the tube,
the layer is at high density instead of at a cut slot. On both sides of that high density
layer, the field generation is in the $-y$ direction. Two shocks from two ends of the
tube cross, amplify the magnetic field and create a doubly shocked, high temperature
region, which has low resistivity and the field is less diffusive.

The magnetic field in the plume traveling outside the window is generated in a
similar way to the magnetic field generated in the ablation plume of a laser interaction
with a solid target [72, 68, 67, 70]. The plume is continuously launched by the flow
Figure 5.7: Schematics of the magnetic field generation by Biermann battery term ($\nabla n_e \times \nabla T_e$). (a) Near the layer, the temperature gradient is perpendicular to the layer due to thermal conduction, the density gradient is alternating and along the layer due to the cut slots on the layer, so that the Biermann generated field is alternating into and out of the plane. (b) Outside the window, the density gradient points to the dense part of the plume, the temperature gradient along the outflow direction is small due to conduction, but the temperature gradient perpendicular to the outflow direction survives due to continuous launching of the plume from the shock tube, thus the field is into the plane on the right side and out of the plane on the left side.
inside the shock tube and expands in all directions, with the density gradient to point towards the dense part of the plume, as shown in Figure 5.7(b). The temperature gradient along the outflow direction is reduced due to electron thermal conduction, but the temperature gradient perpendicular to the outflow direction survives due to continuous launching of the plume from the shock tube. Thus the magnetic field generated by the Biermann battery term is into the plane on the right side and out of the plane on the left side in Figure 5.7(b).

The magnetic field evolution is shown in the fourth row in Figure 5.3 to 5.5. In the center of the tube, a field pointing in $-y$ direction dominates. Outside the window, the field pointing in $+y$ direction survives, while the field pointing in $-y$ direction diffuses quickly due to low temperature and high resistivity. The total magnetic flux in the $y = 0$ plane is conserved and vanishes. We are interested in the magnetic field near the center of the tube which can potentially affect the mix. The magnetic field outside the window plays a role in the proton radiography as discussed in Section 5.4, but we are not interested in its dynamical importance because it is far away from the mix region. As shown in Figure 5.6, the magnetic field near the center of the tube is similar between the runs with and without the window.

### 5.4 Synthetic proton radiography

The features of the proton images are most prominent in 14.3MeV to 14.5MeV band, i.e. protons losing between 0.2MeV and 0.4MeV of kinetic energy. Obtaining the proton images for different energy bands put challenges on the etching process of CR-39 [182].

The proton images with/without field, and with/without pepper pot screen (PPS) are compared in the fifth to the last rows in Figure 5.3 to 5.5. To quantify the
Figure 5.8: The evolution of the averaged position of protons in the blob in final energy range 14.3MeV to 14.5MeV. The scale is divided by the magnification to align with the scales on the target system. The red curves are for runA, the black curves are for runB, and the blue curves are for runC. The dashed curves are for the MPRAD runs with magnetic field turned off, and the solid curves are for MPRAD runs with magnetic field turned on. (a) is for no PPS case and (b) is for with PPS case.
asymmetry of the proton image, the averaged horizontal proton position in the blob at the center of the proton image is plotted in Figure 5.8. The ideally constructed target should be symmetric about a rotation of 180 degrees and the proton image should also be symmetric in the absence of magnetic field. The asymmetry of the proton image about the vertical axis can be interpret as the existence of magnetic field. However, even in the no PPS case, i.e. the fifth rows in Figure 5.3 to 5.5, the blob in the middle of the image can be slightly asymmetric even without magnetic field. This asymmetry is not as large as the asymmetry in the images where there is field but no PPS, i.e. the six rows, which means the proton deflection by magnetic field causes more asymmetry than by the density asymmetry due to the fact that the shock from the right side in Figure 5.3 to 5.5 moves slightly faster. This slight difference is caused by the different effective laser intensities on two ends of the target due to different laser incident angles. In the simulations, the unevenness of the foam and the power imbalance on two ends of the tube has not been taken into account, which can also cause asymmetry on the proton image.

One advantage of using PPS is that the viewing of the surrounding holes is through the regions without the field and the viewing of the hole in the middle is only thorough the region with magnetic field, so that the net deflection caused by the magnetic field can be determined without another control shot using same target. With PPS, the asymmetry in the no field case, i.e. the seventh row in Figure 5.3 to 5.5 is significantly less than the without field and without PPS case, i.e. the fifth rows. The PPS is very efficient in reducing the asymmetry of the proton image cause by the intensity imbalance on two ends and the unevenness of the foam. As shown in Figure 5.8(b), the asymmetry caused by the proton deflection is significantly larger than that caused by the nonuniform density. The blob has a positive net shift at early time, because of
the field pointing in \(+y\) direction in the plume outside the window. At about 8.5ns, the proton deflection caused by the field pointing in \(+y\) direction in the plume outside the window and by the field in near the center of the tube pointing in \(-y\) direction cancel each other, resulting in zero net shift of the blob on the proton image. At a late time \(t > 10\)ns, the field pointing in \(+y\) direction moves away from the \(z = 0\) plane, but the field near the center of the tube has no net advection, and the net shift of the blob is negative. The shift value on the image plate divided by the magnification can reach 50 to 70\(\mu\)m. The difference between the early time shift and late time shift can reach 70 to 90\(\mu\)m.

5.5 Analysis of proton radiography

The analysis of the proton radiography from the OMEGA shot days is still ongoing by the author and collaborators. In the analysis for the proton image for the targets without PPS, one important process is to determine the edges in the image. Figure 5.9(a) and (b) show the following steps for determining the edges: (1) Pick 12 populations (a1, a2, b1, b2, c1, c2, d1, d2, e1, e2, f1, f2) of points on two sides of each of the 6 different edges as shown in Figure 5.9(a); (2) Pick one point from population a1, and another point from population a2, using the bisect method to determine the point on the edge. For \(N_1\) points from a1 and \(N_2\) points from a2, we can get \(N_1N_2\) points on the edge and do a linear fit, and get the equation for edge, i.e. \(a0\), similarly we can get \(b0\), \(c0\), \(d0\), \(e0\), \(f0\) as shown in Figure 5.9(b).

Figure 5.9(c) shows the following steps for determining the shift of the blob in the center for the images for the target with PPS: (1) Pick the center of two blobs S1 (top), S2 (bottom); (2) randomly shift the location of S1 and S2, iteratively calculate the max-flux-location of tracks within 100um radius until it converges, repeat the
Figure 5.9: The analysis of proton images.
random shift and iteration multiple times; (3) calculate the average and variance from step 2; (4) calculate the center-of-weight of the blob $S_0$ (in the middle) with respect to $(S_1+S_2)/2$.

5.6 Discussions

*Using TNSA high-energy proton beam for probing magnetic fields* — The predicted proton images only show the contribution from the mean magnetic fields from different columns along the line of sight without the detail of the field structure. The signal from small scale fields always gets damped by the diffusion of the proton beam. High-energy proton beam accelerated by TNSA mechanism using OMEGA EP beam experiences less diffusion through the target \[71\]. The ratio of the deflection angle by magnetic field to the Coulomb scattering angle is roughly proportional to $E_p^{1/2}$ as shown in Eq (3.37).

*The effects of magnetic fields on plasma flows* — As shown in Table \[5.2\], the plasma beta is $\beta \approx 3.3 \times 10^3$ and the electron Hall parameter is $\chi_e \approx 8.2 \times 10^{-4}$. Thus the $J \times B$ force and the magnetized heat flow and induction is negligible. However, $2/(\beta \chi_e) \approx 0.74$ is close to unity and the effective charge state $Z_{\text{eff}}$ has spatial variation, so that one needs to use Eqs \[2.20\] and \[2.21\] to compute the evolution of heat flow and magnetic field more precisely. How the extra terms in Eqs \[2.20\] and \[2.21\] affect the magnetic field and plasma flow is discussed in the next two paragraphs. As shown in Table \[5.2\], the Reynold number $\text{Re}$ is high enough to ensure turbulence, and the magnetic Reynolds number $\text{Rm}$ is around 50. Under this condition, the magnetic field remains dynamically unimportant. The magnetic energy density from Table \[5.2\] is $10^9\text{erg/s}$, which is only 0.3% of the turbulent kinetic energy reported in the simulation in Reference \[24\] for a previous mix modeling for shock-
shear targets under similar condition to this work. Thus the magnetic field is also negligible for mix modeling in the shock-shear targets. It is desirable to optimize the measurable magnetic fields and improve the dynamical importance of the magnetic fields.

Resistive term for the magnetic field — Some experiments [237] and theories [238, 239] show that around 10eV the value of electrical resistivity is different from the Spitzer resistivity. However, the electrical resistivity with temperature and density dependency under the condition of our experiment design is not well constrained. If the modeling in this work is correct in terms of electrical resistivity, then this would indicate that the magnetic field may not be dynamically important. However, if the electrical resistivity is significantly lower than the Spitzer resistivity that we use in this work, then the code underestimates the magnetic fields, and the mix model could potentially cover up the magnetic field effects by the choice of the initial input conditions for the model. Future experiments executed at higher temperatures can potentially make magnetic fields start to play a more important role. The implementation of implicit method for the magnetic diffusion equation is desirable for the case of large resistivity where fully explicit method requires small time step.

Thermoelectric induction and heat flow associated with current — The thermoelectric term, i.e. third term in the right hand side of Eq (2.20), is an additional source for magnetic field. By substituting Eq (2.20) into the induction equation \( \frac{\partial \mathbf{B}}{\partial t} = -e \nabla \times \mathbf{E} \), the thermoelectric magnetic source term is \( (ck_B/\beta_\parallel/e)\nabla Z_{\text{eff}} \times \nabla T_e \), versus \( (ck_B/en_e)\nabla T_e \times \nabla n_e \) for Biermann battery. \( \beta_\parallel \) is maximal for high \( Z_{\text{eff}} \) and its derivative is maximal for \( Z_{\text{eff}} = 1 \), giving \( \beta'_\parallel(Z_{\text{eff}}) \approx 0.3 \). If \( \nabla Z_{\text{eff}} \) is quasi-parallel to \( \nabla n_e \), the thermoelectric term reduces the generation rate of the magnetic field. If \( \nabla Z_{\text{eff}} \) is quasi-antiparallel to \( \nabla n_e \), the thermoelectric term increases the generation...
rate of the magnetic field. The second term in Eq (2.21) is antiparallel to the current \( \mathbf{J} \). If \( \mathbf{J} \) is parallel to the temperature gradient, then the additional heat flow is antiparallel to the temperature gradient, thus \( \nabla T_e \) will be flattened and the source terms in the induction equation will be reduced. If \( \mathbf{J} \) is antiparallel to the temperature gradient, then the additional heat flow is parallel to the temperature gradient, thus \( \nabla T_e \) will be steepened and the source terms in the induction equation will be increased.

*Increasing the magnetic field in the experiments* — The Biermann battery generated magnetic field is roughly \( c k_B T_e/(e Lu) \) by balancing the Biermann battery term with the advection term. The plasma beta \( \beta \) is then proportional to \( n_e T_e/(T_e/Lu)^2 \propto n_e u^2/(L^2 T_e) \). If we keep the size of the target and the laser power, then \( n_e u^2 \) and \( L \) are roughly constants, then \( \beta \propto 1/T_e \). Thus increasing \( T_e \) can reduce \( \beta \) and make the Lorentz force more important. The electron Hall parameter \( \chi_e \) is proportional to \( T_e^{3/2}/n_e \) and the magnetic Reynolds number \( R_m \) is proportional to \( T_e^{3/2} \). Both \( \chi \) and \( R_m \) increase with temperature. For low \( R_m \) and low magnetic Prandtl number \( Pr_m \), i.e. \( Pr_m = R_m/Re \ll 1 \), the power spectrum of the kinetic energy \( E(k) \) and the power spectrum of the magnetic energy \( M(k) \) are related by \( M(k) \propto k^{-2} E(k) \), and \( M(k) \) is always softer than \( E(k) \), and the magnetic field remains dynamically unimportant even in small scales \([51, 48, 47]\). High \( R_m \) is favorable for the amplification of magnetic fields and a hard power law for magnetic energy spectrum \([51, 50, 52]\). One way to achieve a higher temperature is to lower the density of the foam. However, making a low density foam in the target is challenging for target fabrication. It causes the unevenness in the foam, leads to the unevenness of the proton image, and makes it difficult to interpret the experimental data from proton radiography. In a low density foam, the flow may move too fast so that the time window for diagnostics is narrow.
Electromagnetic fields near shocks — The length scale of the magnetic precursors in the shock can be estimated using Eqs (2.51) and (2.52), giving $\lambda_B = 11\mu m \gg \lambda_T = 0.024\mu m$. Thus the length scale of the field in the shock is around $\lambda = 11\mu m$, which is below the resolution of proton radiography. The electric field near the shock can be estimated as $E \approx | - \nabla p_e/(n_e e) | \approx k_B T_e/(e\lambda) = 80\text{stat} V/cm$, which is not sufficient to have visible signal according Eq (3.3) ($\alpha_E$ is two orders of magnitude smaller than $\alpha_B$).

5.7 Summary

In this chapter, radiation-MHD modeling and synthetic radiographs predicted the X-ray and proton images for the experiments on OMEGA using the modified shock-shear targets. The hydrodynamical evolution can be measured using XRFC and compared with the simulation results. The predicted proton radiography shows the direction and the amount of the shift of the proton beam going through the window and/or PPS. Although the target can diffuse the proton beam significantly, the evolution of the shift in the synthetic proton radiography is still consistent with the evolution of the magnetic fields in the target system and shows change between early time. However, the prediction for the magnetic field is subject to the uncertainty for the resistivity and the thermoelectric terms.
Chapter 6

Relativistic particle-in-cell method

The PIC method [240] is widely used for the simulations of plasma dynamics ranging from Laser Plasma Accelerators (LPAs) to collisionless astrophysical problems. In the PIC method, quasi-particles are used to sample the phase-space distribution of physical charged particles. The equations of motion of quasi-particles are solved using a particle-push algorithm, e.g. Boris algorithm [240]. The electromagnetic field is defined on a grid, usually the staggered Yee grid [241]. The Lorentz force acting on a quasi-particle is calculated by interpolating the electromagnetic field from nearby grid points to the quasi-particle location using a force interpolation scheme. The on-grid current density is calculated using a current deposition scheme according to the quasi-particle motion and is used to update the on-grid electromagnetic field. The PIC method can be implemented without solving a Poisson equation for the electric potential if one uses an exact charge conservation scheme. Although the exact charge conservation current deposition scheme [242] allows an arbitrary form-factor for quasi-particle, the most commonly used form-factor is a B-spline function. Using B-splines has a few advantages [243], including the easiness of computation due to their polynomial nature, the smoothness of the charge assigned to the grid as the particles move across the grid, and the negligible fluctuations at long-range. If one requires momentum to be conserved, then the force interpolation function should be identical to the charge assignment function. Higher order B-spline functions have better smoothness and long-range properties, but are more computationally
Relativistic PIC simulations with drifting plasma beams are vulnerable to an electromagnetic numerical instability known as the Numerical Cherenkov instability (NCI) [244]. This numerical instability is caused by the resonance between two modes in the numerical method: (1) the vacuum electromagnetic mode, which has a deviation of the dispersion relation from the physical one, i.e. $\omega = ck$, due to the discretization of Maxwell equations, (2) the drifting plasma beam mode, which is dispersionless but has its aliasing beam modes [155]. This resonance is a numerical artifact and unphysical. It is desirable to have an efficient numerical method which significantly suppresses the NCI in order to improve the quality of relativistic PIC simulations. An analytical expression for lowest order NCI growth rate was derived [245, 246]. The numerically most unstable mode and its growth rate can be calculated from the analytical expression without carrying out any numerical experiments. It was found that in the momentum conserving scheme, if one uses time step $\Delta t = \Delta x_1/(2c)$ for a drifting plasma in $x_1$ direction where $\Delta x_1$ is the grid spacing in $x_1$ direction, the lowest order NCI growth rate vanishes [247].

I propose a time-step dependent force interpolation scheme in Section 6.1, which removes the lowest order NCI growth for a drifting plasma in $x_1$ direction for arbitrary time step allowed by the Courant condition, not just for $\Delta t = \Delta x_1/(2c)$. The “WT scheme” [248] stands for “weighting with time-step dependency”, or for the form of multidimensional interpolation function having $W$’s and $T$’s as in Eq (6.19). In section 6.2, the Maxwell solvers are discussed and a fully explicit Maxwell solver with fourth order dispersion error is emphasized. Loading pseudo-particles with relativistic drifting distribution usually requires the sampling of the distribution function in the co-moving frame and transformation into the simulation frame.
transform between two frames into account is significant, which is discussed in Section 6.3. Ultra-relativistic PIC simulations are usually scalable for large bulk Lorentz factor and/or magnetization parameter, which is discussed in Section 6.4. In the analysis for generalized Fermi acceleration in relativistic case, it is important to be consistent with Lorentz transform in performing the decomposition of electromagnetic fields, which is discussed in Section 6.5.

6.1 WT interpolation scheme

For a 3D relativistic electromagnetic PIC code with momentum conserving (MC) and exact charge conservation scheme in Cartesian coordinate, the electromagnetic field that is spatially interpolated from grid point \( x_g = (x_{g,1}, x_{g,2}, x_{g,3}) = (n_1 \Delta x_1, n_2 \Delta x_2, n_3 \Delta x_3) \) (\( n_1, n_2, n_3 \) can be half-integer or integer depending whether the component of electromagnetic field has a half-grid offset in the \( i \)-th direction) to a particle position \( x = (x_1, x_2, x_3) \) can be expressed as

\[
E_i(x) = \sum_{n_1, n_2, n_3} W_l(x - x_g) E_i(x_g) \\
B_i(x) = \sum_{n_1, n_2, n_3} W_l(x - x_g) B_i(x_g)
\]

(6.1)

and the on-grid charge density of a quasi-particle is calculated from the form-factor

\[
\rho(x_g) = \frac{q}{V_c} W_l(x - x_g)
\]

(6.2)

where

\[
W_l(x - x_g) = W_l^{(1)}(x_1 - x_{g,1}) W_l^{(2)}(x_2 - x_{g,2}) W_l^{(3)}(x_3 - x_{g,3})
\]

(6.3)
and $W_l^{(i)}$ is the $l$-th order B-spline with width $(l+1)\Delta x_i$ in $i$-th direction, and $V_c$ is the volume of a mesh cell, for one-dimensional schemes, $V_c = \Delta x_1$, for two dimensions, $V_c = \Delta x_1\Delta x_2$, and for three, $V_c = \Delta x_1\Delta x_2\Delta x_3$. In the exact charge conservation scheme [242], the current density associated with the motion of a single quasi-particle is the unique linear combination of the form-factor differences in consistency with the discrete continuity equation. The Fourier transform of the interpolation tensor in Eq (6.1) is [245]

\[ S_{E1} = s_{l,1}s_{l,2}s_{l,3}\eta_1 \quad S_{B1} = \cos(\omega'\Delta t/2)s_{l,1}s_{l,2}s_{l,3}\eta_2\eta_3 \]
\[ S_{E2} = s_{l,1}s_{l,2}s_{l,3}\eta_2 \quad S_{B2} = \cos(\omega'\Delta t/2)s_{l,1}s_{l,2}s_{l,3}\eta_1\eta_3 \]
\[ S_{E3} = s_{l,1}s_{l,2}s_{l,3}\eta_3 \quad S_{B3} = \cos(\omega'\Delta t/2)s_{l,1}s_{l,2}s_{l,3}\eta_1\eta_2 \] (6.4)

where the factor $\eta_i = (-1)^{\nu_i}$ is multiplied when the electromagnetic field has a half-grid offset in the $i$-th direction, and

\[ s_{l,i} = \left( \frac{\sin(k'_i\Delta x_i/2)}{k'_i\Delta x_i/2} \right)^{l+1} \] (6.5)

and the aliasing frequency and wave vectors with aliasing orders $(\mu, \nu_1, \nu_2, \nu_3)$ are

\[ \omega' = \omega + \mu\frac{2\pi}{\Delta t}, \quad \mu = 0, \pm 1, \pm 2, \ldots \quad k'_i = k_i + \nu_i\frac{2\pi}{\Delta x_i}, \quad \nu_i = 0, \pm 1, \pm 2, \ldots \] (6.6)

There is one momentum conserving interpolation tensor for each $l$, and we call it MCI.

Derived in Reference [245], the asymptotic expression for NCI growth rate for a cold drifting plasma beam traveling in $x_1$ direction with an ultra-relativistic speed $v_1 \to c$ is

\[ \Gamma = \frac{\sqrt{3}}{2} \left| \omega_p^2 c^2 S_{J1}\{ (S_{B3}\xi_0 - S_{E2}[k]B_1c)[k]E_2k_2 + (S_{B2}\xi_0 - S_{E3}[k]B_1c)[k]E_3k_3 \} \right|^{1/3} \] (6.7)
where \( S_{ji} \) is the interpolation tensor for the current density after Fourier transformation \([245]\), and \( \omega_p = \sqrt{4\pi q^2 n_e / (\gamma_0 m_e)} \) is the relativistic plasma frequency, and the bulk Lorentz factor is \( \gamma_0 = 1/\sqrt{1 - \beta^2} = 1/\sqrt{1 - v_1^2/c^2} \), and the finite difference operators are

\[
[\omega] = \frac{\sin(\omega \Delta t/2)}{\Delta t/2}, \quad [k]_{Ei} = A_i \frac{\sin(k_i \Delta x_i/2)}{\Delta x_i/2}, \quad [k]_{Bi} = \frac{\sin(k_i \Delta x_i/2)}{\Delta x_i/2} \quad (6.8)
\]

where \([k]_{Ei}\) or \( A_i \) depends on the spatial derivative stencil in Faraday’s equation, and \([k]_{Bi}\) is related to the spatial derivative stencil in Ampere’s equation which is unmodified from the standard Yee scheme, and

\[
\xi_0 = \frac{\sin(k'_i c \Delta t/2)}{\Delta t/2}, \quad \xi_1 = \cos(k'_i c \Delta t/2) \quad (6.9)
\]

For MC scheme where \( S_E \) and \( S_B \) are given by Eq \((6.4)\), the lowest order NCI growth rate given by Eq \((6.7)\) depends on the time step and only vanishes for \( \Delta t = \Delta x_1/(2c) \). In order to remove the time-step dependency of the NCI growth rate given by Eq \((6.7)\), I propose the WT scheme \([248]\), where Eq \((6.4)\) is modified to the following form

\[
S_{E1} = s_{l,1} \tau_{l,2} \tau_{l,3} \eta_1 \quad S_{B1} = \cos(\omega' \Delta t/2) \tau_{l,1} s_{l,2} s_{l,3} \eta_1 \eta_3
\]
\[
S_{E2} = \tau_{l,1} s_{l,2} \tau_{l,3} \eta_2 \quad S_{B2} = \cos(\omega' \Delta t/2) \tau_{l,1} s_{l,2} s_{l,3} \eta_1 \eta_3 \quad (6.10)
\]
\[
S_{E3} = \tau_{l,1} \tau_{l,2} s_{l,3} \eta_3 \quad S_{B3} = \cos(\omega' \Delta t/2) s_{l,1} s_{l,2} \tau_{l,3} \eta_1 \eta_2
\]

where

\[
\tau_{l,i} = \left( \frac{\sin(k'_i \Delta x_i/2)}{k'_i \Delta x_i/2} \right)^l \left( \frac{\sin(k'_i c \Delta t)}{k'_i c \Delta t} \right) \quad (6.11)
\]

For the interpolation tensor in Eq \((6.10)\), the growth rate in Eq \((6.7)\) in the WT scheme can be calculated by substituting Eqs \((6.10)\) and \((6.11)\) into Eq \((6.7)\). The
common factor in \( S_{B3}\xi_0 - S_{E2}[k]B_1 \) is

\[
\frac{S_{B3}\xi_0 - S_{E2}[k]B_1}{s_{l-1,ls,2,l,3}} = \cos(\omega' \Delta t/2) \left( \frac{\sin(k'_1 \Delta x_1/2)}{k'_1 \Delta x_1/2} \right) (-1)^{\nu_1} \left( \frac{\sin(k'_1 \Delta t/2)}{\Delta t/2} \right) \\
- \left( \frac{\sin(k'_1 \Delta t)}{k'_1 \Delta t} \right) \left( \frac{\sin(k_1 \Delta x_1/2)}{\Delta x_1/2} \right)
\]

(6.12)

Using the fact that the NCI resonance satisfies the dispersion relation of the beam \( \omega' = ck'_1 \), we have

\[
\frac{S_{B3}\xi_0 - S_{E2}[k]B_1}{s_{l-1,ls,2,l,3}} = \cos(k'_1 c \Delta t/2) \left( \frac{\sin(k'_1 \Delta x_1/2)}{k'_1 \Delta x_1/2} \right) (-1)^{\nu_1} \left( \frac{\sin(k'_1 \Delta t/2)}{\Delta t/2} \right) \\
- \left( \frac{\sin(k'_1 \Delta t)}{k'_1 \Delta t} \right) \left( \frac{\sin(k_1 \Delta x_1/2)}{\Delta x_1/2} \right)
\]

\[
= \left( \frac{\sin(k'_1 \Delta x_1/2)}{k'_1 \Delta x_1/2} \right) (-1)^{\nu_1} \left( \frac{\sin(k'_1 \Delta t)}{k'_1 \Delta t} \right) \left( \frac{\sin(k_1 \Delta x_1/2)}{\Delta x_1/2} \right)
\]

Using \( k'_1 = k_1 + \nu_1 (2\pi)/\Delta x_1 \) we have \( \sin(k'_1 \Delta x_1/2) \times (-1)^{\nu_1} = \sin(k_1 \Delta x_1/2) \), thus

\[
S_{B3}\xi_0 - S_{E2}[k]B_1 = 0 \tag{6.13}
\]

In the same way, we have

\[
S_{B2}\xi_0 - S_{E3}[k]B_1 = 0 \tag{6.14}
\]

Thus the NCI growth rate in Eq (6.7) is zero for arbitrary time step \( \Delta t \). Thus, the WT scheme allows flexibility in the choice of the time step, because the asymptotic expression for NCI growth rate vanishes for arbitrary time step \( \Delta t \), not just for \( \Delta t = \Delta x_1/(2c) \) as found in previous studies [245, 247]. The above derivation is valid for arbitrary aliasing beam and arbitrary spatial derivative stencil in Faraday’s equation, as long as the ultra-relativistic beam is moving along the axis of the grid, and the spatial derivative stencil in Ampère’s equation is not modified from the standard Yee stencil. For \( \Delta t = \Delta x_i/(2c) \), we have \( \tau_{l,i} = s_{l,i} \), which recovers the MC scheme.
To get the interpolation function in real space, I calculate the inverse Fourier transform of \( s_{l,i} \) and \( \tau_{l,i} \). I constrain the discussion to \( \Delta t \leq \Delta x_i/(2c) \), because for \( \Delta t > \Delta x_i/(2c) \) the width of the interpolation function becomes large and more grid points are needed for interpolation. The inverse Fourier transform of \( s_{l,i} \) is simply the \((l + 1)\)-th order B-spline function \( W_l^{(i)} \). The interpolation function corresponding to \( \tau_{1,i} \) is

\[
T_1^{(i)}(\tilde{x}_i) = \mathcal{F}^{-1}(\tau_{1,i}) = \begin{cases} 
\frac{1 + 2\Delta \tilde{t}_i - 2|\tilde{x}_i|}{4\Delta \tilde{t}_i} & \text{if } \frac{1}{2} - \Delta \tilde{t}_i < |\tilde{x}_i| \leq \frac{1}{2} + \Delta \tilde{t}_i \\
1 & \text{if } |\tilde{x}_i| \leq \frac{1}{2} - \Delta \tilde{t}_i \\
0 & \text{otherwise}
\end{cases}
\]

where \( \mathcal{F}^{-1} \) is the inverse Fourier transformation, \( \Delta \tilde{t}_i = c\Delta t/\Delta x_i \), and \( \tilde{x} = (x_i - x_{g,i})/\Delta x_i \) is the normalized coordinate difference between the particle and the grid point. The interpolation function corresponding to \( \tau_{2,i} \) is

\[
T_2^{(i)}(\tilde{x}_i) = \mathcal{F}^{-1}(\tau_{2,i}) = \begin{cases} 
\frac{(\Delta \tilde{t}_i + 1 - |\tilde{x}_i|)^2}{4\Delta \tilde{t}_i} & \text{if } 1 - \Delta \tilde{t}_i < |\tilde{x}_i| \leq 1 + \Delta \tilde{t}_i \\
1 - |\tilde{x}_i| & \text{if } \Delta \tilde{t}_i < |\tilde{x}_i| \leq 1 - \Delta \tilde{t}_i \\
\frac{2\Delta \tilde{t}_i - (\Delta \tilde{t}_i - |\tilde{x}_i|)^2}{2\Delta \tilde{t}_i} & \text{if } |\tilde{x}_i| \leq \Delta \tilde{t}_i \\
0 & \text{otherwise}
\end{cases}
\]
The interpolation function corresponding to $\tau_{3,i}$ is

$$T_3^{(i)}(\tilde{x}_i) = \mathcal{F}^{-1}(\tau_{3,i}) = \begin{cases} \frac{(3+2\Delta \tilde{t}_i-2|\tilde{x}_i|)^4}{96\Delta t_i} & \text{if } \frac{3}{2} - \Delta \tilde{t}_i < |\tilde{x}_i| \leq \frac{3}{2} + \Delta \tilde{t}_i \\ \frac{4\Delta t_i^2+3(3-2|\tilde{x}_i|)^2}{24} & \text{if } 1 + \Delta \tilde{t}_i < |\tilde{x}_i| \leq 2 + \Delta \tilde{t}_i \\ \frac{-8\Delta t_i^3-36\Delta t_i^2(1-2|\tilde{x}_i|)-3(1-2|\tilde{x}_i|)^3+6\Delta t_i(15-12|\tilde{x}_i|-4\tilde{x}_i^2)}{96\Delta t_i} & \text{if } \frac{1}{2} - \Delta \tilde{t}_i < |\tilde{x}_i| \leq \frac{1}{2} + \Delta \tilde{t}_i \\ \frac{9-4\Delta t_i^2-12\tilde{x}_i^2}{12} & \text{if } |\tilde{x}_i| \leq \frac{1}{2} - \Delta \tilde{t}_i \\ 0 & \text{otherwise} \end{cases}$$

(6.17)

The interpolation function corresponding to $\tau_{4,i}$ is

$$T_4^{(i)}(\tilde{x}_i) = \mathcal{F}^{-1}(\tau_{4,i}) = \begin{cases} \frac{(\Delta \tilde{t}_i+2-|\tilde{x}_i|)^4}{48\Delta t_i} & \text{if } 2 - \Delta \tilde{t}_i < |\tilde{x}_i| \leq 2 + \Delta \tilde{t}_i \\ \frac{(2-|\tilde{x}_i|)(2-|\tilde{x}_i|)^2+\Delta t_i^2}{6} & \text{if } 1 + \Delta \tilde{t}_i < |\tilde{x}_i| \leq 2 - \Delta \tilde{t}_i \\ \frac{-1(|\tilde{x}_i|)^4+2\Delta \tilde{t}_i(6-6|\tilde{x}_i|+|\tilde{x}_i|^3)-6\Delta t_i^2(1-|\tilde{x}_i|)^2+2\Delta t_i^2|\tilde{x}_i|-\Delta \tilde{t}_i^4}{12\Delta t_i} & \text{if } 1 - \Delta \tilde{t}_i < |\tilde{x}_i| \leq 1 + \Delta \tilde{t}_i \\ \frac{4-6\tilde{x}_i^2+3|\tilde{x}_i|^3-\Delta t_i^2(2-3|\tilde{x}_i|)}{6} & \text{if } \Delta \tilde{t}_i < |\tilde{x}_i| \leq 1 - \Delta \tilde{t}_i \\ \frac{3\tilde{x}_i^4+\Delta \tilde{t}_i(16-24\tilde{x}_i^2)+18\Delta t_i^2\tilde{x}_i^2-8\Delta t_i^3+3\Delta \tilde{t}_i^4}{24\Delta t_i} & \text{if } |\tilde{x}_i| \leq \Delta \tilde{t}_i \\ 0 & \text{otherwise} \end{cases}$$

(6.18)
The width of $T_l^{(i)}$ is $l \Delta x_i + 2c \Delta t$, which decreases as the time step decreases. There is one WT scheme interpolation tensor for each $l$, and I call it WT$l$. The full interpolation form for electromagnetic field in WT$l$ scheme is

\[
E_1(\mathbf{x}) = \sum_{n_1,n_2,n_3} W_l^{(1)}(x_1 - x_{g,1}) T_l^{(2)}(x_2 - x_{g,2}) T_l^{(3)}(x_3 - x_{g,3}) E_1(x_g)
\]

\[
E_2(\mathbf{x}) = \sum_{n_1,n_2,n_3} T_l^{(1)}(x_1 - x_{g,1}) W_l^{(2)}(x_2 - x_{g,2}) T_l^{(3)}(x_3 - x_{g,3}) E_2(x_g)
\]

\[
E_3(\mathbf{x}) = \sum_{n_1,n_2,n_3} T_l^{(1)}(x_1 - x_{g,1}) T_l^{(2)}(x_2 - x_{g,2}) W_l^{(3)}(x_3 - x_{g,3}) E_3(x_g)
\]

\[
B_1(\mathbf{x}) = \sum_{n_1,n_2,n_3} T_l^{(1)}(x_1 - x_{g,1}) W_l^{(2)}(x_2 - x_{g,2}) W_l^{(3)}(x_3 - x_{g,3}) B_1(x_g)
\]

\[
B_2(\mathbf{x}) = \sum_{n_1,n_2,n_3} W_l^{(1)}(x_1 - x_{g,1}) T_l^{(2)}(x_2 - x_{g,2}) W_l^{(3)}(x_3 - x_{g,3}) B_2(x_g)
\]

\[
B_3(\mathbf{x}) = \sum_{n_1,n_2,n_3} W_l^{(1)}(x_1 - x_{g,1}) W_l^{(2)}(x_2 - x_{g,2}) T_l^{(3)}(x_3 - x_{g,3}) B_3(x_g)
\]

And the on-grid charge density of a quasi-particle in the WT scheme is still given by Eq (6.2), which can be inserted in the derivation of the current deposition in an exact charge conserving scheme [242].

The combination of Eq (6.2) with Eq (6.19) has zero self-force under certain condition. As long as one uses a charge conserving deposition scheme for calculating current density, the following Gauss’s equation is conserved [251]

\[
\hat{D} \cdot \mathbf{E} = 4\pi \rho \tag{6.20}
\]

where the difference operator $\hat{D}_l(X_{n_1,n_2,n_3}) = (X_{n_1+\frac{1}{2},n_2,n_3} - X_{n_1-\frac{1}{2},n_2,n_3})/\Delta x_1$ and similarly for the remaining spatial coordinates. Eq (6.20) is conserved automatically, if it is fulfilled in the initial moment. Following Reference [243], the approximate equations used to solve the grid-defined electric fields can be formally expressed in
the form
\[ \mathbf{E}(\mathbf{x}_g) = V_c \sum_{g'} G(\mathbf{x}_g; \mathbf{x}_{g'}) \rho(\mathbf{x}_{g'}) \]  

(6.21)

I assume that the components of the Green’s function \( G \) satisfies symmetry under the interchange of one coordinate

\[ G_1(\mathbf{x}_g; \mathbf{x}_{g'}) = -G_1(x_{g,1}, x_{g,2}, x_{g,3}; x_{g,1}, x_{g',2}, x_{g',3}) \]

\[ G_2(\mathbf{x}_g; \mathbf{x}_{g'}) = -G_2(x_{g,1}, x_{g',2}, x_{g,3}; x_{g',1}, x_{g,2}, x_{g',3}) \]  

(6.22)

\[ G_3(\mathbf{x}_g; \mathbf{x}_{g'}) = -G_3(x_{g,1}, x_{g,2}, x_{g',3}; x_{g',1}, x_{g',2}, x_{g,3}) \]

The symmetry can be inherited from the symmetry in the boundary condition, e.g. periodic boundary condition in each direction. Using the form-factor in Eq (6.2) and the force interpolation in Eq (6.19), the self-electric-force in \( x_1 \) direction for a particle of charge \( q \) at position \( \mathbf{x} = (x_1, x_2, x_3) \) gives

\[
F_{\text{self},1}(\mathbf{x}) = F_{\text{self},1}(x_1, x_2, x_3) = q^2 \sum_{g,g'} G_1(\mathbf{x}_g; \mathbf{x}_{g'}) W_l(x_1 - x_{g,1}) T_l(x_2 - x_{g,2}) T_l(x_3 - x_{g,3}) \times W_l(x_1 - x_{g',1}) W_l(x_2 - x_{g',2}) W_l(x_3 - x_{g',3}) 
\]

(6.23)

Using Eq (6.22) and interchanging \( x_{g,1} \) and \( x_{g',1} \) we have \( F_{\text{self},1}(\mathbf{x}) = -F_{\text{self},1}(\mathbf{x}) \), thus \( F_{\text{self},1}(\mathbf{x}) = 0 \). Similarly \( F_{\text{self},1}(\mathbf{x}) = F_{\text{self},2}(\mathbf{x}) = 0 \). The analysis for self-force here only applies to the electrostatic part of the field. In full electromagnetic PIC, a more comprehensive analysis for the self-force is desirable.

In the rest of this section, I show the results of two test problems for relativistic drifting plasmas, a drifting cold pair plasma and a relativistic striped wind. The simulations are carried out using EPOCH 2D [153] with modified force interpolation scheme by the author.
Table 6.1 : Parameters for the test problem of drifting cold pair plasma.

<table>
<thead>
<tr>
<th>domain size</th>
<th>$L_x = 16d_e, L_y = 8d_e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>boundary condition</td>
<td>periodic in both $x$ and $y$</td>
</tr>
<tr>
<td>number of cells</td>
<td>$N_x = 256, N_y = 128$</td>
</tr>
<tr>
<td>pseudo-particles per cell</td>
<td>$N_{PPC} = 64$ (32 for each species)</td>
</tr>
<tr>
<td>drift Lorentz factor</td>
<td>$\gamma_0 = 1000$</td>
</tr>
<tr>
<td>temperature</td>
<td>$k_B T_e = k_B T_i = 0.01 m_e c^2$</td>
</tr>
<tr>
<td>time step</td>
<td>$\Delta t / \Delta t_{CFL} = \Delta t / [\Delta x / (\sqrt{2}c)] = 0.1, 0.3, 0.5, 0.7, 1 / \sqrt{2}$</td>
</tr>
</tbody>
</table>

Drifting cold pair plasma — The parameters for the test problem of drifting cold pair plasma are listed in Table 6.1. The time and the spatial coordinates of the simulations are normalized by the inverse relativistic electron plasma frequency $\omega_{pe}^{-1} = 1 / \sqrt{4\pi n_e e^2 / (\gamma_0 m_e)}$ and the relativistic electron skin depth $d_e = c / \omega_{pe} = c / \sqrt{4\pi n_e e^2 / (\gamma_0 m_e)}$, respectively. The plasma is initially unmagnetized and should have no physical instabilities. The instabilities in the simulations are always numerical artifacts. The simulations for the unmagnetized uniform drifting pair plasma have been extensively used in literatures [245, 247] for testing NCI in PIC codes. I use the WT4 scheme and the regular momentum-conserving MC4 scheme with $\Delta t \leq \Delta x / (2c)$ and $\Delta x_1 = \Delta x_2 = \Delta x$. For Maxwell solvers, I use the one with fourth order dispersion accuracy (we call it M4), i.e. $\beta_{12} = \beta_{21} = \delta_1 + 1/12 = \delta_2 + 1/12 = (c\Delta t)^2 / [12(\Delta x)^2]$, as derived in 6.2 and the Yee solver, i.e. $\beta_{12} = \beta_{21} = \delta_1 = \delta_2 = 0$.

The growth history of the fraction of the total electromagnetic energy $E / E_0$ is shown in Figure 6.1 where $E_0$ is the initial total kinetic energy of all particles and $E$ is the total energy of electromagnetic field which is a function of time. The growth
Figure 6.1: Results for the test problem of drifting pair plasma. The interpolation scheme and Maxwell solver are marked in the caption for each subfigure.
of $E/E_0$ is always unphysical after an initial transient that produces electromagnetic fields in thermal equilibrium. In Figure 6.1(a) with MC4+Yee, the case for $\Delta t = \Delta t_{\text{CFL}}/\sqrt{2}$ has slower NCI growth than the cases for $\Delta t \leq 0.7\Delta t_{\text{CFL}}$, where $\Delta t_{\text{CFL}} = \Delta x/(\sqrt{2}c)$. This is consistent with previous studies [245, 247] and can be explained by the fact that the lowest order NCI growth rate given by Eq (6.7) vanishes for $\Delta t = \Delta t_{\text{CFL}}/\sqrt{2}$ but not for other time steps. In Figure 6.1(b) with MC4+M4, the case for $\Delta t = \Delta t_{\text{CFL}}/\sqrt{2}$ also has slower NCI growth than other cases, but the time for NCI to saturate is similar to Yee solver. In Figure 6.1(c) and (d), WT4 scheme is used. Note that for $\Delta t = \Delta t_{\text{CFL}}/\sqrt{2}$ the WT scheme recovers the standard momentum conserving scheme, and the scale of time axes for Figure 6.1(c) and (d) is different from that for Figure 6.1(a) and (b). In Figure 6.1(c) with WT4+Yee, the NCI grows much slower and saturates at a much later time if a smaller time step is used. For $\Delta t = 0.3\Delta t_{\text{CFL}}$ and $\Delta t = 0.1\Delta t_{\text{CFL}}$, $E/E_0$ stays between $10^{-5}$ to $10^{-4}$ for a long time. The results for the tests with WT4 scheme and M4 solver are shown in Figure 6.1(d). In the case for $\Delta t = 0.3\Delta t_{\text{CFL}}$ and $\Delta t = 0.1\Delta t_{\text{CFL}}$, NCI grows slower using M4 solver than using Yee solver. The trend observed in the numerical tests using WT4 scheme is that NCI grows slower if a smaller $\Delta t$ is used, which indicates that the high order growth rate not included in Eq (6.7) depends on $\Delta t$ and decreases as $\Delta t$ decreases. The detailed analysis of the high order growth rate will be subject of future reports.

Relativistic striped wind — Relativistic striped wind [252, 253, 254] is a steady electron-positron flow before it interacts with the termination shock of pulsar wind nebula (PWN). For a flow propagating along $-x$ direction with a bulk Lorentz factor
\( \gamma_0 \), the spatial profile of the electromagnetic field in the simulation frame is

\[
B_y = B_0 \tanh \left\{ \frac{1}{\delta} \left[ \alpha + \cos \left( \frac{2\pi(x + \beta_0 ct)}{\lambda} \right) \right] \right\}
\]

(6.24)

\[
E_z = \beta_0 B_0 \tanh \left\{ \frac{1}{\delta} \left[ \alpha + \cos \left( \frac{2\pi(x + \beta_0 ct)}{\lambda} \right) \right] \right\}
\]

(6.25)

where \( \beta_0 \) is the velocity of the wind normalized by the speed of light \( c \), and \( \lambda \) is the wavelength of the stripes in the wind. The dimensionless parameters \( \delta \) and \( \alpha \) are such that the half thickness of the current sheet is \( \Delta = \lambda \delta / (2\pi) \), and \( B_y \) averaged over one wavelength is \( \langle B_y \rangle_\lambda = B_0 [1 - 2(\arccos \alpha)/\pi] \). The background cold plasma in the wind is uniform, with constant density \( n_{e^{-},e^{+}}^{\text{cold}} = n_{e^{0}}/2 \) and constant temperature \( kT_{e^{-},e^{+}}^{\text{cold}} = 0.04mc^2 \) for both electrons and positrons. A hot electron-positron plasma inside the Harris current sheets balances the magnetic pressure and maintains the steady profile of electromagnetic field. The density of the hot electron-positron plasma in the current sheet in the simulation frame is

\[
n_{e^{-},e^{+}}^{\text{hot}} = \frac{n_{h0}}{2 \cosh^2 \left\{ \frac{1}{\delta} \left[ \alpha + \cos \left( \frac{2\pi(x + \beta_0 ct)}{\lambda} \right) \right] \right\}}
\]

(6.26)

where \( n_{h0}/n_{e^{0}} = \eta \) is the over-density factor relative to the cold particles outside the layer, and is set to be \( \eta = 3 \) [253] [254]. The drift velocity in \( z \) direction of the hot particles is setup to ensure the steady profile of electromagnetic field, i.e. in the rest frame of the wind \( \nabla \times \mathbf{B} = (4\pi/c)\mathbf{J} \) is satisfied everywhere so that the electric field stays zero. The test problem is in 2D and periodic in both directions. The size of the simulation box \( L_x \times L_y = 640d_e \times 400d_e \) The parameters used are \( \alpha = 0.1 \) (i.e. \( \langle B_y \rangle_\lambda = 0.064B_0 \)), \( \Delta = d_e \), \( \lambda = 640d_e \), \( \gamma_0 = 1/\sqrt{1 - \beta_0^2} = 10^4 \) and \( \sigma_0 = B_0^2/(4\pi\gamma_0 m_e n_{e^{0}} c^2) = 10 \), and \( d_e \) is resolved with 7.5 computational cells. Each computational cell is initialized with two electrons and two positrons in the cold wind, and additional two electrons and two positrons in the current sheets. The
Figure 6.2: The results for the test problem for relativistic striped wind interpolation scheme is WT4 and the Maxwell solver is M4.

The results for the test problem are shown in Figure 6.2. According to the growth rate of tearing instability given in a previous study [255, 256], the current sheets in the test problem should not reconnect when $\omega_p t$ is a few thousand. Any current sheets reconnect before that time is caused by numerical problems. For the runs with $\Delta t = 0.7 \Delta t_{CFL}$ and $\Delta t = 0.3 \Delta t_{CFL}$, the reconnection happens before $\omega_p t < 1000$ and the islands have significantly grown their sizes at $\omega_p t = 2000$. For the run with
$\Delta t = 0.2\Delta t_{\text{CFL}}$, the current sheets have not significantly change their morphology from the initial condition until $\omega_p t = 2000$. Since the run with $\Delta t = 0.2\Delta t_{\text{CFL}}$ is the most stable run in Figure 6.2, and it can make sure the current sheet is stable before it interacts with termination, small time step $\Delta t = 0.2\Delta t_{\text{CFL}}$ is used in the recent work for simulating magnetic reconnection at the termination shock of a relativistic striped wind [257].

6.2 Maxwell solvers

One of the approaches to improve the numerical dispersion in relativistic PIC method is to use the spatial Fourier transform based methods, such as Pseudo-Spectral Time Domain (PSTD) algorithms [258, 259, 260]. Those methods can constrain numerical resonance between EM mode and beams to large wave vectors, i.e. no resonance near $k = 0$. The numerical instabilities can then be suppressed by applying low-pass filtering [247]. The nonlocal nature of communication cost and the $O(N \log N)$ computational cost of the spatial Fourier transform algorithms make those methods challenging to scale on massively parallel computers. In the generalized PSTD algorithm [260], the components of the wavevectors can be replaced by the Fourier transforms of finite difference approximations to spatial derivatives on a grid, which reduces the communication cost to local and computational cost to $O(N)$. The mixed FD-FFT solver use 1D FFT only [261] or high order FDTD in one direction only [262]. However, the mixed solver nature in different directions requires correction due to the loss of charge conservation. An alternative Maxwell solver [263] is derived using finite elements in both space and time [264]. This scheme has superluminous numerical dispersion ($\omega > ck$) in a large range of $k$ including $k = 0$ with $O(N)$ computing cost and only local communication.
In a wide range of PIC simulations, the wavevector $k$ of physical significance satisfies $k\Delta x \ll 1$, and the Maxwell solvers with fourth order dispersion error, i.e. $\omega/(ck) = 1 + \mathcal{O}(k\Delta x)^4$ as $k\Delta x \to 0$, might be sufficient. In the cases where the wavevector with $k\Delta x \approx 1$ has physical significance, e.g. high harmonic generation by the plasma mirror [205], pseudo-spectral solvers were required to suppress spurious numerical dispersion of electromagnetic waves. A fully explicit Maxwell solver is usually more computationally efficient than FFT-based or implicit solvers, and thus favorable for large scale simulations. The fully explicit Maxwell solvers in Reference [206], which modify the spatial derivative stencil in Faraday’s equation and keep the spatial derivative stencil in Ampere’s equation, are compatible with the WT scheme.

The dispersion relation of electromagnetic waves for the Maxwell solvers with modified spatial derivative in Faraday’s equation is

$$s^2 - s^2 A_1 - s^2 A_2 - s^2 A_3 = 0$$

with the abbreviations

$$s_\omega = \frac{\sin(\omega \Delta t/2)}{c \Delta t}, \quad s_i = \frac{\sin(k_i \Delta x_i/2)}{\Delta x_i}, \quad i = 1, 2, 3$$

and

$$A_1 = 1 - 2\beta_{12}[1 - \cos(k_2 \Delta x_2)] - 2\beta_{13}[1 - \cos(k_3 \Delta x_3)] - 2\delta_1[1 - \cos(k_1 \Delta x_1)]$$

$$A_2 = 1 - 2\beta_{23}[1 - \cos(k_3 \Delta x_3)] - 2\beta_{21}[1 - \cos(k_1 \Delta x_1)] - 2\delta_2[1 - \cos(k_2 \Delta x_2)]$$

$$A_3 = 1 - 2\beta_{31}[1 - \cos(k_1 \Delta x_1)] - 2\beta_{32}[1 - \cos(k_2 \Delta x_2)] - 2\delta_3[1 - \cos(k_3 \Delta x_3)]$$

where are $\beta_{ij}$ and $\delta_i$ are dimensionless tunable parameters and the six $\beta$ coefficients
depend on $\hat{\beta}_i$ as following

$$\hat{\beta}_1 = \frac{\Delta x_2}{c^2 \Delta t^2} \beta_{12} = \frac{\Delta x_2}{c^2 \Delta t^2} \beta_{13}, \quad \hat{\beta}_2 = \frac{\Delta x_3}{c^2 \Delta t^2} \beta_{23} = \frac{\Delta x_3}{c^2 \Delta t^2} \beta_{21},$$

$$\hat{\beta}_3 = \frac{\Delta x_1}{c^2 \Delta t^2} \beta_{31} = \frac{\Delta x_1}{c^2 \Delta t^2} \beta_{32} \quad (6.30)$$

A systematic approach for minimizing the dispersion error \[266\] can be done in general cases, but the focus of this section is to reduce the dispersion error near $k \Delta x = 0$. By expanding the phase velocity $v_g = \omega/k$ to second order using the spherical coordinates for the wave vectors $(k_1, k_2, k_3) = (k \sin \theta \cos \phi, k \sin \theta \sin \phi, k \cos \theta)$, we have

$$\frac{\omega}{ck} = 1 + \left[ \frac{c^2 \Delta t^2 - \Delta x_1^2(1 + 12 \delta_1)}{24} \sin^4 \theta \cos^4 \phi + \frac{c^2 \Delta t^2 - \Delta x_2^2(1 + 12 \delta_2)}{24} \sin^4 \theta \sin^4 \phi \right. \right.$$

$$+ \frac{c^2 \Delta t^2 - \Delta x_3^2(1 + 12 \delta_3)}{24} \cos^4 \theta + \frac{c^2 \Delta t^2(1 - 12 \hat{\beta}_1)}{48} \sin^2 2\theta \sin^2 \phi \right.$$

$$\left. + \frac{c^2 \Delta t^2(1 - 12 \hat{\beta}_2)}{48} \sin^2 2\theta \cos^2 \phi + \frac{c^2 \Delta t^2(1 - 12 \hat{\beta}_3)}{48} \sin^4 \theta \sin^2 2\phi \right] k^2 + \mathcal{O}(k \Delta x)^4 \quad (6.31)$$

If one requires that the second order term is zero, then

$$c^2 \Delta t^2 - \Delta x_1^2(1 + 12 \delta_1) = 0 \quad 1 - 12 \hat{\beta}_1 = 0$$

$$c^2 \Delta t^2 - \Delta x_2^2(1 + 12 \delta_2) = 0 \quad 1 - 12 \hat{\beta}_2 = 0 \quad (6.32)$$

$$c^2 \Delta t^2 - \Delta x_3^2(1 + 12 \delta_3) = 0 \quad 1 - 12 \hat{\beta}_3 = 0$$

which implies

$$\hat{\beta}_i = \frac{1}{12}, \quad \delta_i = \frac{(c \Delta t/\Delta x_i)^2 - 1}{12}, \quad \text{and} \quad \beta_{ij} = \frac{(c \Delta t/\Delta x_j)^2}{12} \quad (6.33)$$

### 6.3 Loading relativistic pseudo-particle

The method for loading particles with relativistic distributions from Reference \[251\] can be generalized to arbitrary drifting directions. For relativistic PIC simulations, one usually needs to load the pseudo-particles with shifted-Maxwell distribution. The
particles can be loaded in the center-of-mass (CM) frame $S'$ where the distribution function is isotropic and transformed into the simulation frame $S$, assuming that $S'$ is moving at velocity $\vec{\beta}c = \vec{n} \beta c$ with $\beta < 1$ with respect to $S$, and $\gamma = 1/\sqrt{1 - \beta^2}$.

The commonly used momentum distribution in $S'$ frame is usually the Jüttner-Synga distribution, which represents the thermal equilibrium state with relativistic temperature $T \gtrsim mc^2/k_B$

$$f'(\vec{p}')d^3\vec{p}' = \frac{N}{4\pi \frac{k_B T}{mc^2} (mc)^3 K_2\left(\frac{mc^2}{k_B T/c}\right)} \exp\left(-\frac{\sqrt{m^2 c^2 + p'^2}}{k_B T/c}\right)d^3\vec{p}' \quad (6.34)$$

where $\vec{p}'$ is the momentum of the particle, $N$ is the number of particles, $m$ is the mass of one particle, and $K_2(x)$ is the modified Bessel function of the second kind.

In the low temperature limit $k_B T/(mc^2) \to 0$, the distribution recovers the Maxwell-Boltzmann distribution

$$f'(\vec{p}')d^3\vec{p}' = \frac{N'}{(2\pi mk_B T)^{3/2}} \exp\left(-\frac{p'^2}{2mk_B T}\right)d^3\vec{p}' \quad (6.35)$$

The momentum distribution can be initialized in $S'$ using the widely used Box-Muller algorithm [267] for non-relativistic Maxwell-Boltzmann distribution in Eq (6.35), or Sobol algorithm [268] for relativistic Jüttner-Synga distribution function in Eq (6.34).

The momentum $\vec{p}'$ loaded in $S'$ is transformed into momentum $\vec{p}$ in $S$ frame by the Lorentz transform

$$\vec{p} = \left[\vec{p}' - (\vec{p}' \cdot \vec{n}) \vec{n}\right] + \gamma \left(\vec{p}' \cdot \vec{n} + \beta \frac{E'}{c}\right) \vec{n} \quad (6.36)$$

where $E' = \sqrt{p'^2 c^2 + m^2 c^4}$ is the energy of the particle in $S'$ frame. The momentum distribution function $f(\vec{p})$ in $S$ frame is related to $f(\vec{p}')$ by

$$f(\vec{p})d^3\vec{p} = \frac{E'}{E} f(\vec{p}')d^3\vec{p}' = \gamma \left(1 + \beta c \frac{\vec{p}' \cdot \vec{n}}{E'}\right) f(\vec{p}')d^3\vec{p}' \quad (6.37)$$
For the volume transform part $\gamma(1+\beta c \vec{p}' \cdot \vec{n} / E')$, Reference [251] proposed to use the rejection method. Another random number $X_1 \in [0,1]$ is needed to do the rejection. If $(1 + \beta c \vec{p}' \cdot \vec{n} / E')/2 > X_1$, then we need to reject the pseudo-particle. However, if the particle distribution in $S'$ is symmetric in the $\vec{n}$ direction, i.e. $f'(\vec{p}') = f'(\vec{p}' - 2\vec{n} \cdot \vec{p}' / \vec{n} / E')$ (an isotropic distribution in Eq (6.34) or Eq (6.35) is a special case for symmetric distribution), then because $[1 + \beta c (\vec{p}' - 2\vec{n} \cdot \vec{p}' / \vec{n} / E')]/2 = (1 - \beta c \vec{p}' \cdot \vec{n} / E')/2 < 1 - X_1$, we can flip the momentum $\vec{p}' \rightarrow \vec{p}' - 2\vec{n} \cdot \vec{p}' / \vec{n}$ instead of rejecting the pseudo-particle if $(1 + \beta c \vec{p}' \cdot \vec{n} / E')/2 > X_1$. Then for the symmetric distribution in the $\vec{n}$ direction, the acceptance efficiency is 100%.

### 6.4 Scaling for ultra-relativistic plasmas

The equations for relativistic PIC modeling are

$$
m_s \frac{d\vec{u}_s}{dt} = q_s (\vec{E} + \frac{\vec{v}_s}{c} \times \vec{B})$$

$$
d\vec{x}_s/dt = \vec{v}_s$$

$$
\frac{\partial \vec{E}}{\partial t} = c\nabla \times \vec{B} - 4\pi \vec{J}
$$

$$
\frac{\partial \vec{B}}{\partial t} = -c\nabla \times \vec{E}
$$

$$
\vec{J} = \sum_s w_s q_s \vec{v}_s
$$

where $s$ stands for $s$-th quasi-particle and $w_s$ is the weight of $s$-th pseudo-particle.

The following normalization is introduced

$$
t = \sigma_0^{1/2} \frac{1}{\omega_{pe} \tilde{l}}, \quad \vec{x}_s = \sigma_0^{1/2} (c/\omega_p) \tilde{x}, \quad \vec{u}_s = \sigma_0 \gamma_0 c \tilde{u}_s, \quad \vec{v}_s = c \tilde{v}_s,
$$

$$
m_s = m_e \tilde{m}_s, \quad q_s = e \tilde{q}_s, \quad \tilde{w}_s = n_c \tilde{n},
$$

$$
\vec{E} = \sqrt{4\pi \gamma_0 n_c \tilde{m}_e c^2 \sigma_0} \tilde{E}, \quad \vec{B} = \sqrt{4\pi \gamma_0 n_c \tilde{m}_e c^2 \sigma_0} \tilde{B}
$$

(6.39)
where \( \omega_p = \sqrt{4\pi n_0 e^2/(\gamma_0 m_e)} \), then the dimensionless equations become

\[
\frac{d\tilde{u}_s}{dt} = \frac{\tilde{q}_s}{\tilde{m}_s} (\tilde{E} + \tilde{v}_s \times \tilde{B}) \quad \frac{d\tilde{x}_s}{dt} = \frac{\tilde{v}_s}{c} \\
\frac{\partial \tilde{E}}{\partial t} = \nabla \times \tilde{B} - \sum_s \tilde{w}_s \tilde{q}_s \tilde{v}_s \\
\frac{\partial \tilde{B}}{\partial t} = -\nabla \times \tilde{E}
\]

(6.40)

and

\[
\tilde{v} = \frac{1}{\sqrt{1 + 1/(\tilde{u}_s^2 \gamma_0^2 \sigma_0^2)}} \frac{\tilde{u}}{\tilde{u}} = (1 - \frac{1}{2\tilde{u}^2 \gamma_0^2 \sigma_0^2} + \mathcal{O}(\frac{1}{\tilde{u}^4 \gamma_0^4 \sigma_0^4})) \frac{\tilde{u}}{\tilde{u}}
\]

(6.41)

As long as \( \gamma_0 \sigma_0 \tilde{u} \gg 1 \), Eq (6.41) can be well approximated by \( \tilde{v} = \tilde{u}/\tilde{u} \), then the dimensionless equations Eq (6.40) are the same for different \( \gamma_0 \) and \( \sigma_0 \). In unmagnetized plasmas, one can use the argument for the scaling by letting \( \sigma_0 = 1 \).

### 6.5 Fermi acceleration and decomposing electromagnetic fields

The generalized Fermi acceleration \[269\] follows the particle journey through a sequence of local frames where local electric field vanishes. To find such local frames, one can decompose the electric fields based on Lorentz transformation. In this section, I generalize the decomposition in previous studies \[270, 271, 272, 273\] to the relativistic case. For convenience, the complex vector of electromagnetic field is defined as \( \mathbf{F} = \mathbf{E} + i\mathbf{B} \) \[274\]. One obvious decomposition is the regions for \( E > B \) and \( E < B \), because \( E^2 - B^2 \) is a Lorentz invariant. Other two decompositions are described below.

Removing the electric field in the fluid co-moving frame and distinguishing the electric field associated (not associated) with bulk plasma motion — The Lorentz
transformation for $F = E + iB$ in the simulation frame $S$ to $F' = E' + iB'$ in fluid co-moving frame $S_f$ which moves at speed $\beta_f c$ with respect to $S$ is (see Chapter 26 in Reference [275])

$$F' = (1 - \gamma_f)(F \cdot n_f)n_f + \gamma_f(F - i\beta_f \times F)$$

(6.42)

where $n_f = \beta_f/\beta_f$ is the direction of $\beta_f$, and $\gamma_f = 1/\sqrt{1-\beta_f^2}$. If one has a field configuration $F'_m$ in $S_f$ frame such that the electric field $E'_m$ is zero and the magnetic field $B'_m$ is same as $B'$, then

$$F'_m = iB' = i[(1 - \gamma_f)(B \cdot n_f)n_f + \gamma_f(B - \beta_f \times E)]$$

(6.43)

Transform back $F'_m$ into $F_m$ in $S$ frame

$$F_m = (1 - \gamma_f)(F'_m \cdot n_f)n_f + \gamma_f(F'_m + i\beta_f \times F'_m)$$

$$= [-\gamma_f^2\beta_f \times B - (\gamma_f^2 - 1)(E - (n_f \cdot E)n_f)]$$

$$+ i[-\gamma_f^2\beta_f \times E + \gamma_f^2(B - (B \cdot n_f)n_f) + (B \cdot n_f)n_f$$

(6.44)

The real part of $F_m$, i.e. the electric field due to plasma motion, is

$$E_m = -\gamma_f^2(\beta_f \times B + E - (n_f \cdot E)n_f) + (E - (n_f \cdot E)n_f)$$

$$= (\mathcal{P} \cdot E)\mathcal{P} - \mathcal{P}^2 E - (E/c)\mathcal{P} \times B$$

$$= \mathcal{P} \times [\mathcal{P} \times E - (E/c)B]$$

(6.45)

$$= \frac{(\mathcal{E}/c)^2 - \mathcal{P}^2}{(\mathcal{E}/c)^2 - \mathcal{P}^2}$$

where $\mathcal{P}$ is the momentum density of the fluid and $\mathcal{E}$ is the energy density of the fluid, and the fluid velocity is given by $\beta_f c = \mathcal{P}c^2/\mathcal{E}$. In the case where $\beta_f \ll 1$, we have $E_m \approx -\beta_f \times B$.

Removing the electric field in the frame where the electric and magnetic fields are parallel — A frame where the electric and magnetic fields are parallel can be generally
found for given electromagnetic field \((\mathbf{E}, \mathbf{B})\) as long as \(E^2 + B^2 > 0\), even for the \(B = 0\) case. The velocity \(v_g = \beta_g c\) of such frame is given by (see [276] or Section 25 in [274])

\[
\frac{\beta_g}{1 + \beta^2_g} = \frac{\mathbf{E} \times \mathbf{B}}{E^2 + B^2}
\]

(6.46)

where \(\beta_g\) and the corresponding Lorentz factor \(\gamma_g\) can be solved as

\[
\beta_g = \frac{2E \times B}{E^2 + B^2 + \sqrt{(E^2 - B^2)^2 + 4|\mathbf{E} \cdot \mathbf{B}|^2}}
\]

(6.47)

\[
\gamma^2_g = \frac{E^2 + B^2 + \sqrt{(E^2 - B^2)^2 + 4|\mathbf{E} \cdot \mathbf{B}|^2}}{2 \sqrt{(E^2 - B^2)^2 + 4|\mathbf{E} \cdot \mathbf{B}|^2}}
\]

(6.48)

The electromagnetic field \((\mathbf{E}, \mathbf{B})\) in the simulation frame is transformed into the electromagnetic field \((\mathbf{E}', \mathbf{B}')\) in frame \(S_g\) through a Lorentz transformation using \(v_g\). In frame \(S_g\), the electromagnetic field \((\mathbf{E}', \mathbf{B}')\) satisfies \(\mathbf{E}' \parallel \mathbf{B}'\) and the particle motion follows an accelerating motion in \(\mathbf{B}'\) direction and a gyro-motion perpendicular to \(\mathbf{B}'\) direction. The complex scalar

\[
\mathbf{F}^2 = \mathbf{F} \cdot \mathbf{F} = (E^2 - B^2) + 2i \mathbf{E} \cdot \mathbf{B} = C_R + iC_I
\]

(6.49)

is a Lorentz invariant, where \(C_R = E^2 - B^2\) and \(C_I = 2 \mathbf{E} \cdot \mathbf{B}\) [274]. The Lorentz transform for \(\mathbf{F}\) in the simulation frame \(S\) to \(\mathbf{F}' = \mathbf{E}' + i\mathbf{B}'\) in a frame \(S'\) that moves at speed \(\beta_g c\) with respect to \(S\) is

\[
\mathbf{F}' = (1 - \gamma_g)(\mathbf{F} \cdot \mathbf{n}_g)\mathbf{n}_g + \gamma_g(\mathbf{F} - i\beta_g \times \mathbf{F})
\]

(6.50)

where \(\mathbf{n}_g = \beta_g / \beta_g\) is the direction of \(\beta_g\). If one defines a field configuration \(\mathbf{F}'_g\) in \(S'\) frame such that the electric field \(\mathbf{E}'_g\) is zero and the magnetic field \(\mathbf{B}'_g\) is same as \(\mathbf{B}'\), then by transforming back \(\mathbf{F}'_g\) into \(\mathbf{F}_g\) in \(S\) frame, then
\[ F_g = \gamma_g [F'_a + i \beta_g \times F'_a] \]
\[ = -\gamma^2_g \beta \times (B - \beta_g \times E) + i \gamma^2_g (B - \beta_g \times E) \]
\[ = \left( \frac{1}{2} - \frac{C_R}{2 \sqrt{C_R^2 + C_l^2}} + \frac{C_l i}{2 \sqrt{C_R^2 + C_l^2}} \right) (E + i B) \] (6.51)

or
\[ E_g = \left( \frac{1}{2} - \frac{C_R}{2 \sqrt{C_R^2 + C_l^2}} \right) E - \left( \frac{C_l}{2 \sqrt{C_R^2 + C_l^2}} \right) B \]
\[ B_g = \left( \frac{1}{2} - \frac{C_R}{2 \sqrt{C_R^2 + C_l^2}} \right) B + \left( \frac{C_l}{2 \sqrt{C_R^2 + C_l^2}} \right) E \] (6.52)

Since \( E_g \cdot B_g = 0 \) and \((E - E_g) \parallel B_g\), the generalized perpendicular electric field is \( E_{\perp} = E_g \) and generalized parallel electric field is \( E_{\parallel} = E - E_g \). In the case where \( |E| \ll |B| \), we have \( |C_l/C_R| \approx 2 E \cdot B / B^2 \ll 1 \) and \( C_R < 0 \), thus the transformation is equivalent to removing the parallel electric field and keeping the perpendicular electric field
\[ E_{\perp} \approx E - \frac{E \cdot B}{B^2} B \]
\[ E_{\parallel} \approx \frac{E \cdot B}{B^2} B \] (6.53)
\[ B_g \approx B \]

6.6 Summary

In this chapter, WT scheme is proposed to eliminate the lowest order NCI growth rate. The WT scheme is efficient for improving the quality and flexibility of relativistic PIC simulations, although the reason for having small growth rate for small time steps is yet to be understood. The additional considerations including Maxwell solvers, loading relativistic pseudo-particle and scaling for ultra-relativistic plasmas are also generally significant for relativistic PIC simulations and analyses.
Chapter 7

Summary of the thesis

In this thesis, the progress for the experiments on hollow ring magnetized jet platform and shock-shear platform is elaborated. The simulation results are consistent with a subset of experimental data, and further improvements in the simulation are necessary to achieve better consistency. Some analysis for the experimental data is still ongoing.

A common feature for both platforms is that the plasma created is collisional, i.e. the system size is much larger than mean free path. It is suspected that the collisionality of the plasma flow is increased due to magnetization in the magnetized jet experiments. Collisional transport effects are estimated to be significant according to the conditions in the plasma flows. To model the magnetic field dynamics more accurately, Eqs (2.18) and (2.19) need to be implemented in the simulations for magnetized jet experiments, and Eqs (2.20) and (2.21) need to be implemented in the simulations for the shock-shear targets. The data from multiple diagnostics, including Thomson scattering, X-ray framing camera and proton radiography, will be useful for validating the extended-MHD model.

The development of PIC method in this thesis is useful for carrying out ultra-relativistic PIC simulations. One of the ongoing works by the author is the ultra-relativistic PIC simulation for kinetic processes, including instabilities, reconnection and particle acceleration, in the termination shock of pulsar wind nebula.
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Appendix A

Proton radiography data from experiments and simulations for magnetized jet experiments

The next eight pages are the proton images from experiments and simulations for the magnetized jet experiments described in Chapter 4. The title of each page contains the ring radius and the target information. The shot number, the time for the proton probe beam to go through the target, and the proton energy are on the left side of each row. The images in the left column is from the MPRAD simulations by post processing the FLASH simulations at given probe time and proton energy. The right column is from the OMEGA shot day on December 10th 2015 and joint shot day November 29th 2016.
Proton images for d=800μm CH target

Simulation

Experiment

Shot 79654
1.6ns, 14.7MeV

Shot 83661
1.9ns, 10.2MeV

Shot 79657
3.6ns, 14.7MeV
Proton images for d=800μm CH(2%Fe) target

Simulation

Experiment

Shot 83656
1.4ns, 10.2MeV

Shot 83658
2.3ns, 14.7MeV

Shot 83659
2.9ns, 10.2MeV
Proton images for d=800μm CH(2%Fe) target

Simulation

Shot 83660
2.8ns, 14.7MeV

Experiment

Shot 83662
3.3ns, 14.7MeV
Proton images for $d=1200\mu m$ CH target

Simulation

Shot 83650
2.3ns, 14.7MeV

Experiment

Shot 83652
4.3ns, 14.7MeV
Proton images for d=1200μm CH(2%Fe) target

Simulation

Shot 83654
0.9ns, 10.2MeV

Experiment

Shot 83655
4.6ns, 14.7MeV
Proton images for d=400μm CH target

Simulation

Experiment

Shot 79660
1.8ns, 3MeV

Shot 79661
2.6ns, 14.7MeV
Proton images for \( d=0 \) CH target

The artificial white strip near target plane is probably due to thermal conduction.
Proton images for d=0 CH target

Shot 79665
3.6ns, 14.7MeV