PPGMotion: A Robust Algorithm for Identification of Motion Artifacts in Photoplethysmography Signals

by

Akash Kumar Maity

A Thesis Submitted
in Partial Fulfillment of the Requirements for the Degree
Master of Science

APPROVED, THESIS COMMITTEE:

[Signatures]

Ashutosh Sabharwal, Chair
Professor of Electrical and Computer Engineering

Ashok Veeraraghavan
Associate Professor of Electrical and Computer Engineering

Reinhard Heckel
Assistant Professor of Electrical and Computer Engineering and Computer Science

Houston, Texas

November, 2018
ABSTRACT

PPGMotion: A Robust Algorithm for Identification of Motion Artifacts in Photoplethysmography Signals

by

Akash Kumar Maity

Photoplethysmography (PPG) is commonly used as a means of continuous health monitoring. Many clinically relevant parameters like heart rate (HR) and blood oxygenation level (SPO2) are derived from the sensor measurements of PPG. Presence of motion artifacts in the measured PPG signal decreases the accuracy of estimating the parameters and therefore reduces the reliability of these sensor devices. Motion artifacts can be both periodic or aperiodic. Existing state-of-the-art methods for motion detection rely on the semi-periodic structure of PPG to distinguish from aperiodic motion artifacts. Periodic motion artifacts that can be introduced by periodic movements like hand tapping, jogging, cannot be reliably detected by current methods.

In this thesis, we propose a novel technique, PPGMotion, for identifying all motion artifacts in PPG signals. PPGMotion relies on the morphological structure of artifact-free PPG signal, which has a fast systolic phase and a slowly decaying diastolic phase. We note that in the presence of motion artifacts, the recorded PPG signals do not exhibit the characteristic PPG shape. Our approach uses this prior information about the PPG morphology to reliably detect periodic motion artifacts, without the need of any additional hardware components like an accelerometer. To evaluate
the proposed method, we use both a simulation and real data based evaluation. For simulation-based evaluation, we use a generative model for motion artifacts to simulate different cases of motion artifacts. For real data, we compare PPGMotion against recent works on motion identification using 3 datasets, where we record the PPG from a pulse-oximeter attached to a finger with subjects making (1) random finger movements, (2) periodic movements like periodic finger tapping, and (3) PPG recordings from Maxim smartwatch with subjects running on a treadmill. Dataset (2) and (3) are expected to introduce periodic motion artifacts in the measured PPG signals. We demonstrate that while our approach is similar in performance to previous methods when random motion artifacts are introduced, the performance is significantly better in the presence of periodic motion artifacts. We show that for simulated dataset, the performance of PPGMotion is significantly better than existing work as the contaminated PPG tends to become periodic, with an increase in sensitivity of atleast 10% over state-of-the-art method. For real data, PPGMotion is successful in identifying the periodic motion artifacts, with mean sensitivity of 95% and accuracy of 95.8%, compared to the state-of-the-art method with mean sensitivity of 66% and accuracy of 89% for dataset (2). For dataset (1), PPGMotion achieves an accuracy of 96.35% with sensitivity of 95.29%, and for dataset (3), PPGMotion achieves an accuracy of 91.89% and sensitivity of 93.03%, compared to the second best method with accuracy 81.23% and sensitivity 74.99%.
# Contents

Abstract .......................... ii  
List of Illustrations ................ vi  
List of Tables ........................ viii  

1 Introduction ........................... 1  
  1.1 Prior Literature .................... 2  
  1.2 Contributions ........................ 5  
  1.3 Organization of Thesis ............ 6  

2 Background ........................... 8  
  2.1 Photoplethysmography .............. 8  
    2.1.1 Heart Rate ..................... 9  
    2.1.2 Heart Rate Variability (HRV) .... 9  
    2.1.3 Shape Morphology .............. 11  
  2.2 Motion Interference ............... 11  

3 Theory of Motion Artifact Detection 14  
  3.1 Characterization of PPG Using Signal Models 14  
    3.1.1 Constraints on Parameters of the Signal Model 15  
    3.1.2 Noise-Motion Model ............ 17  
  3.2 Problem Formulation ............... 20  

4 PPGMotion: Algorithm for Motion Artifact Detection 24  
  4.1 Estimation of PPG signal parameters 25
4.1.1 Estimation of Instantaneous Phase . . . . . . . . . . . . . . . 26
4.1.2 Update Amplitude Envelope . . . . . . . . . . . . . . . . . . 27
4.1.3 Estimation of Shape Function . . . . . . . . . . . . . . . . . . 28
4.2 Estimation of Motion Signal . . . . . . . . . . . . . . . . . . . . . 30
4.2.1 Implementation Details . . . . . . . . . . . . . . . . . . . . . . 31

5 Validation of PPGMotion . . . . . . . . . . . . . . . . . . . . . . 34
5.1 Generating Simulated Dataset . . . . . . . . . . . . . . . . . . . . . 34
5.2 Data Collection Methodology . . . . . . . . . . . . . . . . . . . . . 36
  5.2.1 Experimental Setup . . . . . . . . . . . . . . . . . . . . . . . . 36
  5.2.2 Experimental Protocol . . . . . . . . . . . . . . . . . . . . . . 36
  5.2.3 Performance Comparison . . . . . . . . . . . . . . . . . . . . . 41
5.3 Results . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 42

6 Discussion . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 48
6.1 Conclusion . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 51

Bibliography . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 53
## Illustrations

<table>
<thead>
<tr>
<th>Illustration</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>PPG signal morphology</td>
<td>10</td>
</tr>
<tr>
<td>2.2</td>
<td>Normalized Pulse Shape</td>
<td>10</td>
</tr>
<tr>
<td>2.3</td>
<td>(a) PPG during motion (b) Distorted signal waveform under relative motion between sensor and skin (c) Signal waveform when the sensor is absolutely stationary</td>
<td>12</td>
</tr>
<tr>
<td>3.1</td>
<td>PPG shape estimated on 5 min PPG signal</td>
<td>18</td>
</tr>
<tr>
<td>3.2</td>
<td>PPG shape vs periodic motion contaminated PPG shape overlapped</td>
<td>18</td>
</tr>
<tr>
<td>3.3</td>
<td>PPG signal obtained from a Maxim smart-watch corrupted with motion artifacts</td>
<td>21</td>
</tr>
<tr>
<td>3.4</td>
<td>Indicator function, 0 for clean segment, 1 when corrupted with motion</td>
<td>21</td>
</tr>
<tr>
<td>4.1</td>
<td>PPG signal $y$ in a with motion artifacts in a time window</td>
<td>29</td>
</tr>
<tr>
<td>4.2</td>
<td>Estimated instantaneous frequency $\theta'$</td>
<td>29</td>
</tr>
<tr>
<td>4.3</td>
<td>Estimated motion signal $m$</td>
<td>29</td>
</tr>
<tr>
<td>4.4</td>
<td>Indicator function $i$ given by support vector of $m$</td>
<td>29</td>
</tr>
<tr>
<td>5.1</td>
<td>Synthetic data generation: (a) an example of simulated motion artifact added to PPG signal for $p = 0$, SMR = $-5$ dB (b) frequency spectrum of a real motion corrupted data acquired from finger PPG under random finger movements</td>
<td>37</td>
</tr>
</tbody>
</table>
5.2 Synthetic data generation: (a) an example of simulated motion artifact added to PPG signal for $p = 0.5$, SMR = $-2$ dB (b) frequency spectrum of a real motion corrupted data acquired from finger PPG under arm waving scenario.

5.3 Synthetic data generation: (a) an example of simulated motion artifact added to PPG signal for $p = 0.9$, SMR = $-2$ dB (b) frequency spectrum of a real motion corrupted data acquired from finger PPG under periodically tapping case.

5.4 Validation of PPGMotion on simulated data for varying $p$ and SMR = $-15$ dB.

5.5 Validation of PPGMotion on simulated data for varying $p$ and SMR = $-5$ dB.

5.6 Validation of PPGMotion on simulated data for varying $p$ and SMR = $5$ dB.

5.7 Performance comparison of PPGMotion against TDSVM for identifying motion contamination. (a) Random motion artifact (b) Periodic motion artifact.

6.1 Normalized root mean square error and normalized cross correlation in (%) for clean and contaminated PPG shape. Note that for clean vs clean error plots, two clean PPG segments before and after motion activity for same subject are chosen.

6.2 Simulated periodic motion contaminated PPG signal.

6.3 Normalized residual power without shape constraint.

6.4 Least square error between estimated shape and priori shape.
Tables

5.1 Performance Metrics (Mean+-Std) of our proposed approach vs existing methods
Chapter 1

Introduction

With the emergence of wellness-promoting wearables, such as portable chest heart rate monitor or smart-watches, continuous health monitoring has become more feasible. Continuous vital signs monitoring has been explored extensively in the past decade with the purpose of improving ambulatory care, leading to better utilization of resources. Vital signs monitoring has been shown to be feasible with photoplethysmography, commonly known as PPG, a non-invasive, optical technique for measuring blood volume change in arteries and microvasculature. The principle in which it works is that light is shone across the skin and a sensor detects the light after being reflected or transmitted through the skin tissue. The modulated light is detected and converted into a signal, commonly known as the PPG signal, which reflects the blood volume change in synchrony with the cardiac cycle. PPG has mostly been preferred due to its low cost, convenient, and easy-to-use. A pulse oximeter, which uses the principle of PPG, has been long used for estimating Heart Rate (HR), and oxygenation level (SPO2) in clinical environments. Further research has shown that not only HR and SPO2, but several other clinically relevant parameters related to cardiovascular health like Heart Rate Variability (HRV), Breathing Rate (BR), and arterial stiffness [1, 2, 3], can be estimated from PPG signals.

In most of the cases, the sensor measurements either from a clinical grade pulse-oximeter or a smart-watch are contaminated with noise. The presence of noise decreases the accuracy of estimating the clinical parameters from the sensor signals. In
a typical PPG measurement, there are two significant sources of noise - sensor noise and artifacts introduced by relative motion between the sensor system and the measured body location. Motion artifacts can either arise from body movements while performing daily tasks or while performing physical exercise. The size of the relative motion determines the intensity of motion artifacts and in many cases, motion artifacts, when present, dominate as the primary source of noise. While past research has been primarily focused on estimating these parameters from noisy PPG signals with clinical accuracy, motion artifacts still pose a significant problem for reliable estimates of vital signs. Motion artifacts are difficult to extract out from clean signal due to the fact that the frequency spectrum of motion artifact has a substantial overlap with the information-bearing part of the PPG spectrum. Thus, the development of algorithms to extract PPG signal from a motion contaminated segment continues to be challenging.

While it is difficult to address the challenges involved with the reconstruction of a corrupted PPG signal, an alternative relevant solution can be towards identifying the time and duration when the motion event occurred. A “motion detection unit” therefore may act as an indicator of a motion event, and may provide a signal quality index. A review of prior literature on identifying motion contaminated PPG is briefly described in the next section.

1.1 Prior Literature

There have been several efforts in the past to understand how motion artifacts affect the estimation of HR and SPO2 from the PPG signal [4, 5]. However, most recent approaches towards tackling motion are from the perspective of reducing error in SPO2 measurement, and the dynamic characteristics are often ignored in this process [6].
Time-sensitive clinical parameters like pulse transit time and heart rate variability have been shown to be important biomarkers of cardiovascular health. The presence of motion artifacts in PPG signals limits its usability towards estimating these vital parameters as well [7, 8, 9, 10, 6].

As a first step towards getting rid of any unwanted interference, one of the most common approaches taken is to filter the PPG signal within some frequency range in the heart rate bandwidth [11]. The noise component that lies outside this frequency spectrum gets filtered out, resulting in a de-noised PPG signal. Often, the motion artifact spectrum overlaps in the heart rate bandwidth [12], thereby making it more difficult to separate out motion artifacts. Special types of filters, like the adaptive filters, have been proposed to cancel out the motion artifacts in the signal [13, 14]. The adaptive filters use the contaminated PPG signal along with a reference signal which is either highly correlated with the motion signal, or the clean PPG signal. Using the reference signal, the motion artifact can be canceled out resulting in a clean PPG signal. In most of the prior works, the reference signal is obtained from an additional hardware setup like an accelerometer. The reference signals have to be time-synchronized with the PPG sensor to cancel out the motion artifacts. This is a big caveat of using adaptive filters since many pulse-oximeter may not be equipped with additional hardware. Additionally, it has been reported that there may be no direct correlation between movements reflected in accelerometer data and motion artifacts in PPG signals [15, 16].

To avoid the problem of obtaining a separate reference signal, the reference signal is computed from a filtered version of the PPG signal itself [17, 18]. Although these methods perform well for small duration artifacts, they fail to perform for more extended duration artifacts. The reconstructed signal in contaminated segments usu-
ally misses out on dynamic features in the PPG signal [19, 3, 10]. Moreover, the adaptive filters also operate on the clean section of the PPG signals, thereby introducing some distortions. It has also been reported that the reconstructed signal using these algorithms introduces some delay in the waveform, thus resulting in false interpretation [20].

Considering the above-mentioned factors, a simpler strategy will be to aim for detecting the presence of motion corruption. A motion artifact detection block not only provides a Signal-Quality-Index (SQI) but can also activate reconstruction algorithms to estimate parameters from the corrupted PPG segments instead of operating even on clean segments [21]. There have been several efforts in the past to identify motion corrupted PPG segments in a PPG waveform. The basic idea behind these approaches is that the clean PPG signal has a distinct quasi-periodic structure, and in a motion corrupted segment, this structure is entirely violated [22, 23]. In [24], the quasi-periodicity of PPG signal is quantified in the form of statistical metrics such as skew and kurtosis. Skew, which is a measure of symmetry, is expected to be high in clean PPG segments, and low where the segment is contaminated with motion. In [25], the authors have considered Hjorth parameters- mobility and complexity, to identify the motion corrupted PPG signal. However, the PPG waveform variability among subjects along with the time-varying dynamic properties of the PPG signal is not captured by these measures. Therefore these techniques have low accuracy in identifying a motion-corrupted segment in real-world scenarios. Time-frequency techniques like Wigner Ville distribution [26], Fourier Transform [27], and variable frequency complex demodulation (VFCDM) [19] have also been explored in the past for motion artifact detection. The overall idea behind this approach is that the time-frequency representation of a clean signal will largely differ from that of a motion
artifact signal.

The techniques discussed above are often ad-hoc in their development of identifying motion corrupted signal. An in-depth analysis of the effects of motion artifacts on PPG morphology remains lacking. For instance, most of the prior algorithms have been shown to fail under periodic motion scenarios. Often, motion activities like tapping a finger, shaking, jogging and running induce periodic motion interference in the PPG signal. Therefore a better approach is needed for these scenarios. Concomitantly, a more robust technique is necessary to deal with all types of motion artifact without depending on additional hardware setup.

1.2 Contributions

In this thesis, we take a different route by understanding the characteristic of motion artifacts. Our approach towards motion detection is based on two insights,

1. Clean PPG signal possesses a quasi-periodic structure and can be represented with the help of a signal model. Motion signals can be thought of as additive time-localized noise taking non-zero values during the motion event and is zero otherwise.

2. Even if the motion-induced artifacts tend to be periodic due to periodic movements, the shape or morphology of these artifacts differs from the characteristic PPG shape, which can be learned from the clean portion of the signal.

Using these two key insights, we propose an optimization framework to identify motion artifacts in PPG signals. The framework is derived by describing the clean PPG morphology by a quasi-periodic signal model with prior shape constraint and
modeling motion artifacts as structured sparse outliers to the PPG signal model. The main contributions of the thesis are as follows.

1. We develop a novel robust algorithm called PPGMotion that detects motion corrupted PPG segments. The algorithm leverages the contiguous sparse structure of motion outliers in PPG signal model. We also propose an automatic online method for determining the prior shape template based only on the PPG waveform recorded from the subject.

2. We propose the use of a generative model to simulate different real-life motion scenarios. The proposed generative model is characterized using two parameters: periodicity factor and Signal-to-Motion Ratio. Since the experimental dataset provides only limited data points, the generative model can be used to quantify the performance of our proposed approach on a wide range of motion scenarios.

3. We achieve high accuracy and sensitivity in detecting corrupted PPG segments. We show that for both simulated and real motion corrupted PPG data, our proposed algorithm, PPGMotion matches or significantly exceeds the performance of state-of-the-art motion detection algorithms.

1.3 Organization of Thesis

The thesis is organized as follows. In Chapter 2, we discuss the origin of photoplethysmography (PPG) and several clinical metrics that are obtained from the PPG waveform. We also discuss the origin of motion artifacts and how various activities or movements corrupt the PPG signal.
In Chapter 3, we characterize both the clean and the motion corrupted PPG signal with the help of signal models. We describe certain structural constraints of these models by relating them to the actual morphology or structure exhibited by PPG signals. Next, using this model, we formulate a penalized least squares problem to solve for the PPG signal parameters.

In Chapter 4, we explain the pipeline of extracting the parameters related to the signal model. We present an algorithm—PPGMotion to accurately identify the motion artifacts in the PPG signal.

Chapter 5 lays out the evaluation methodology for validating our proposed algorithm. First, we detail the simulation studies conducted, generating different instances of motion artifacts and validating our approach on this simulated dataset. Secondly, we explain the experimental details to acquire real motion scenarios. We quantify the performance of our algorithm against prior methods on both simulated and real motion dataset. We show that our proposed approach improves the overall accuracy of detecting motion artifacts.

Chapter 6 concludes our work and discusses the possible implications of our work.
Chapter 2

Background

In this chapter, we discuss the origin of PPG and some of the clinical relevant metrics derived from the PPG signal like Heart Rate (HR) and Heart Rate Variability (HRV). A detailed understanding of these PPG signal derived metrics help characterize the signals mathematically and will be discussed in details in the next chapter.

2.1 Photoplethysmography

Photoplethysmography (PPG) is an optical technique to record blood volume change in arteries; the blood volume change occurs due to the regular pumping action of the heart. A pulse oximeter, which is based on PPG, consists of a simple LED and photo-diode detector setup, where the LED flashes light through the exposed skin surface into the tissue, and it interacts with the blood flow in the arteries. Since the amount of blood in the arteries changes in synchrony with the cardiac cycle, the amount of incident light absorbed also changes accordingly. After the interaction, the remaining light is reflected back to the sensor. The PPG signal is recorded by measuring the remaining light intensity.

The PPG waveform is commonly analyzed to obtain information related to the cardiac well-being of a person. The blood volume waveform recorded from anywhere in the body shows temporal variations due to two factors: (i) inter-beat-variations are due to the variations in pumping action of the heart that causes nearly periodic
variations of the blood volume at the measurement site, and (ii) intra-beat variations are due to the actual morphology of each blood volume pulse, caused by systole and diastole. Physiologically, the inter-beat-variations are due to brain-mediated mechanism such as the sympathetic and parasympathetic tone that controls the heart-rate-variability (HRV), whereas intra-beat variations, i.e., the blood volume pulse shape, is a function of the elastic properties of the arteries that deliver blood to the measurement site and usually varies from one body location to the other.

The features in the PPG waveform can be quantified in the form of clinical metrics and provide useful information about cardiac health. We briefly describe these metrics which characterize the PPG waveform.

2.1.1 Heart Rate

Heart rate (HR) is the number of times the heart beats in one minute and is an indicator of cardiovascular health since it is directly related to the functioning of the heart. Cardiac output, which is the total amount of blood pumped out in each minute is also a function of the heart rate.

For each beat, the sudden surge in blood volume in the arteries produces a peak in the PPG signal waveform. Therefore, counting the number of these peaks appearing in one minute is a (naive) estimate of the heart rate. Since the HR measurement is taken from the pulse of blood waveform in the artery, it is also commonly known as pulse rate.

2.1.2 Heart Rate Variability (HRV)

As discussed before, the inter-beat variation arises from the fact that the heart does not beat at a constant rate, but slowly fluctuates around the average heart rate. This
Figure 2.1: PPG signal morphology

Figure 2.2: Normalized Pulse Shape
fluctuation is termed as heart rate variability and is a function of the parasympathetic nervous system. It is measured by the variation in the beat-to-beat interval in the PPG waveform. The presence of HRV makes the PPG waveform a quasi-periodic signal.

2.1.3 Shape Morphology

The PPG shape is a function of arterial properties and can be used as a biomarker for determining the cardiovascular well-being of a person [3]. In one pulse, as shown in Figure 2.1, the PPG waveform is characterized by a sharp rising systolic phase, followed by a slowly decaying diastolic phase. The PPG shape mostly consists of a distinct systolic and a diastolic peak. The systolic peak arises due to the pressure wave traveling from the heart to the peripheral organs. The diastolic peak is mainly due to the reflection of this forward traveling wave whenever there is a change in arterial impedance, such as a bifurcation of the artery. As the change in arterial properties is relatively slow compared to the temporal scale of pulsatile blood volume change, the PPG shape has been seen to be constant or slowly varying within a time period of recording.

2.2 Motion Interference

In PPG, the intensity of light sensed by the detector is dependent on the amount of light absorbed by the tissue and the blood flowing in the arteries. When the sensor is perfectly stationary, the only dynamic component is the variation of blood volume in the arteries. As a result, the intensity waveform over time has a distinct morphology, and it correlates with the change in blood volume. In the presence of motion when there is a relative displacement between the sensor and the skin, the path of light
Figure 2.3: (a) PPG during motion (b) Distorted signal waveform under relative motion between sensor and skin (c) Signal waveform when the sensor is absolutely stationary

from LED to the detector changes. Owing to the inhomogeneities in the skin tissue, the intensity of light at the detector also gets modulated accordingly, resulting in a distorted waveform shown in Figure 2.3(b). Any clinical metric obtained from the corrupted waveform is often not clinically relevant.

Motion artifacts can create challenging clinical situations, e.g., while monitor-
ing patients in ICU, or continuous monitoring of vital signs using wearables while conducting daily activities. Motion contamination can be classified by two types of movements: involuntary movements like sneezing, or some random movements of arms, and voluntary actions like walking or doing some exercise. Contamination arising from involuntary movements are generally shorter in duration as compared to the voluntary movements. In hospital settings, most of the artifacts are due to involuntary movement of the subjects. It has been reported in [28], that 20% of the patients in clinical settings exhibited short duration movements, which affected the vital signs measurements. However, most of these motion events recorded in the study occurred for less than 30 seconds and therefore can be easily accounted for.

On the other hand, motion events can be a serious problem in the context of wearable devices- either in smart-watches used for self-monitoring, or for ambulatory care settings, where data is recorded from the user in the comfort of their homes or offices. Motion events in the context of wearables can be due to various reasons, mainly arising from performing daily activities. Movements like walking, doing exercise induces long duration artifacts in the PPG signal, and are more difficult to handle. Often periodic hand or limb movements induce periodic motion interference, and this interference strongly overlaps with the heart rate signal. Wearable devices are very sensitive towards these type of motion activities, and their presence seriously degrades the accuracy of measuring vital signs from these devices [29].

If the reliability of the PPG sensors is to be improved, then the motion corrupted PPG segments need to be identified and accounted for.
Chapter 3

Theory of Motion Artifact Detection

In this chapter, we describe the morphological characteristics of the PPG waveform mathematically. Using some prior information related to the signal morphology, we pose the motion detection problem as an estimation problem by defining an optimization problem with respect to the signal parameters.

3.1 Characterization of PPG Using Signal Models

Since the PPG signal associated with the cardiac cycle is quasi-periodic in nature, it is modeled by an adaptive non-harmonic model [30] that is a combination of amplitude modulated and frequency modulated signal. The term adaptive means the properties or frequency are not constant over time, and non-harmonic implies the signal cannot be represented by a single sinusoid but a summation of multiple sinusoid waves. In a time analysis window or a time epoch, the quasi-periodic component of the PPG signal is modeled as,

\[ y(t) = a(t)s(\theta(t)) + r(t), \quad s \text{ is } 2\pi \text{ periodic,} \]

where \( y(t) \) is the observed PPG signal, \( a(t) \) is the amplitude envelope, \( s \) is the wave-shape function which is used to model how a signal oscillates over the period and the pulse shape is given \( s(\theta(t)) \) in \( \theta(t) \) from 0 to \( 2\pi \). The shape function is assumed to be constant within the recording time span. The instantaneous phase of a signal \( \theta(t) \)
is given by
\[ \theta(t) = 2\pi f_{pr} t + 2\pi \int_0^t (f_{\text{inst}}(t) - f_{pr}) dt, \] (3.2)
where \( f_{pr} \) is the average pulse rate in the recording duration, and \( f_{\text{inst}}(t) \) is the instantaneous frequency, which varies according to the heart rate variability. We also assume that the instantaneous phase is a monotonically increasing function such that \( \theta'(t) > 0 \forall t \in \mathbb{R} \). Since \( s \) is periodic in \( 2\pi \), it can be represented as
\[ s(\tau) = \sum_{k=-\infty}^{\infty} z_k e^{j2\pi k\tau/T} \]
\[ = \sum_{k=-K}^{K} z_k e^{jk\tau}. \] (3.3)

We assume that shape function \( s \) is \( K \)-band limited, which is a good approximation as long as \( s \) is smooth enough and \( K \) is large enough. It is also assumed that all the Fourier modes of \( s \) are dominated by Fourier coefficient corresponding to the fundamental frequency.

### 3.1.1 Constraints on Parameters of the Signal Model

Clearly, there can be infinite solutions of \( a(t), s \) and \( \theta(t) \) from equation (3.1). Some prior information is needed to constrain the solution space for the unknown parameters. In fact, the identifiability issue can be solved under some certain assumptions [30]. We assume that the amplitude envelope, \( a(t) \) and the instantaneous heart rate , i.e. the first derivative of instantaneous phase, \( \theta'(t) \) does not change too fast. To ensure that the \( a(t) \) and \( \theta'(t) \) are not rapidly varying in time, the following conditions must hold,
\[ \left| \frac{a'(t)}{a(t)} \right| < \epsilon \quad \left| \frac{\theta''(t)}{(\theta_0')^2} \right| < \epsilon \quad \forall t \in \mathbb{R}, \] (3.4)
where $\theta_0$ corresponds to the average instantaneous phase, corresponding to the average heart rate frequency, and $\epsilon > 0$ is known as the scale separation factor.

It has been proved that if the signal satisfies the scale separation property, then the error in the estimates of $a(t)$ and the phase function $\theta(t)$ is bounded by $|\epsilon|$ [31]. To ensure that $a(t)$ and $\theta'(t)$ are slow varying functions or less oscillatory, a linear space $V(\theta)$ is constructed for a given phase function $\theta_0(t)$ belonging to heart rate frequency, and is spanned by the following basis,

$$V(\theta_0, \lambda) = \text{span} \left\{ \cos \left( \frac{k\theta_0}{2L_\theta} \right), \sin \left( \frac{k\theta_0}{2L_\theta} \right) \right\}_{0 \leq k \leq 2\lambda L_\theta}, \quad (3.5)$$

where $\lambda < 1/2$ is a parameter to control the smoothness of functions in $V(\theta_0, \lambda)$, $L_\theta = (\theta(T) - \theta(0))/2\pi$, and the signal is recorded from $[0, T]$ in time. The linear space is a collection of all the functions that are smoother than $\cos \theta_0(t)$.

Since the PPG waveform has a distinct characteristic shape, a natural question to ask is if we can utilize any prior information on PPG shape. The apparent difficulty is how do we obtain the prior shape information. Since the PPG shape varies across subjects, one approach can be using a database containing PPG shape from a large number of subjects, and the average shape of the dataset can be used as the prior shape information. The drawback with the discussed approach is that it will introduce a bias in the estimated shape, and may not reflect the exact morphological parameters. However, it has been observed the change in PPG shape or morphology is negligible over time [32]. Therefore, a constraint can be imposed which constrains the change in PPG shape over a consecutive time window. In other words, if the clean PPG shape is $s_0$ in a time window $t_0$, the PPG shape in the next clean time window $t_1$ should not deviate too much from $s_0$. The slow temporal fluctuation of PPG shape is highlighted in Figure 3.1, which shows the PPG shape from consecutive time window in a 5 min recording overlapped in a single frame. A slight modification to the prior
shape update plan is to consider shape estimates from all the previous time windows instead of a single previous time window. The proposed methodology is discussed in more details in the next chapter.

We also consider the case when the contaminated PPG signal itself is periodic in nature. Contaminated PPG signal tends to be periodic when a large repeated arm or wrist movements are involved, and the periodicity is not due to the cardiac cycle. In Figure 3.2, we show the shape of a contaminated PPG segment is significantly different from that of a clean PPG signal. Leveraging the prior shape information may help towards a more accurate detection of periodic motion contaminated PPG signal.

If the additive noise term $r(t)$ in equation (3.1) is known to have a Gaussian distribution, we can formulate the problem as a least squares problem, given by

$$\text{Minimize} ||s - s_0||_2,$$

such that:

$$y(t) - a(t)s(\theta(t)) \leq \epsilon$$

$$s \text{ is } 2\pi \text{ periodic and } a, \theta \in V(\theta_0, \lambda),$$

where $s_0$ is the prior shape information and $\theta_0$ is the phase corresponding to the prior heart rate, and $\epsilon$ is some positive number determined by the amount of noise in the observed signal.

### 3.1.2 Noise-Motion Model

Obviously, the noise $r(t)$ in the previous formulation is not Gaussian. In equation (3.1), the residual $r(t)$ which can be either sensor noise, or some motion artifact that may have arisen due to movement of the subject. The residual therefore can be broken down into two components - Gaussian sensor noise component, and a non-Gaussian
Figure 3.1: PPG shape estimated on 5 min PPG signal

Figure 3.2: PPG shape vs periodic motion contaminated PPG shape overlapped
motion component. Basically, \( r(t) \) is given by,

\[
r(t) = m(t) + n(t),
\]

(3.7)

where \( m(t) \) is the motion event, and \( n(t) \) is the additive Gaussian noise with a fixed variance.

In the past, motion artifacts in PPG signal have been modeled or simulated as a Gaussian white noise [21] for a particular time duration, with some variance depending on Signal-to-Motion Ratio (SMR). In [33], it has been characterized in terms of statistical metrics like skew, kurtosis, standard deviation correlation coefficient, and mutual information. But these metrics are not evaluated for all types of motion, for example, periodic motion interference.

Looking at multiple instances of motion events, we note that these events are localized in time, i.e., they occur for some fixed time duration. Therefore, in any time duration of PPG signal recording in \([0, T]\), the motion artifacts affect some duration \([T_1, T_2]\) in between, where \(0 < T_1, T_2 < T\). We represent the motion artifact in a time duration using the following expression,

\[
m(t) = p.q(t) + (1 - p)g(t),
\]

(3.8)

where \( q \) is a periodic or a quasi-periodic signal, and \( g \) is random aperiodic band-limited noise. The motion signal is characterized by a periodicity factor \( p \) which controls how much periodic component exists in the motion signal, and the overall amplitude is determined by the Signal-to-Motion Ratio (SMR). For \( p = 0 \), the motion signal is represented by a random noise component, and for \( p \) close to 1, the motion signal is represented by a quasi-periodic signal component. An important point to note here is that even though there is a periodic component \( q \) in the motion signal, the shape of the periodic signal component is assumed to be significantly different
from that of the clean PPG signal. The motion signal, when active, can, therefore, be thought of an outlier to the clean PPG signal model, since the morphological characteristics of clean PPG are not observed in the motion contaminated segments. We show one instance of motion contaminated PPG in Figure 3.3. To compensate for the under-determinacy, we need to impose some structure on the motion signal. A generalized and simple model of motion signal can be that of contiguous sparsity or block sparsity. The contiguous sparsity constraint is due to the fact that the motion interference in the PPG waveform occurs in groups, and the motion time duration is assumed to be less compared to the total PPG signal recording duration.

3.2 Problem Formulation

As discussed in the previous section, we can describe the motion artifacts as contiguous sparse outliers in the PPG signal model. To recover sparse models, sparsity-inducing norms have been widely used in the application of compressive sensing [34]. The common formulation is to penalize the $l_2$ norm criterion by the $l_0$ norm of the coefficient vector. But solving for the $l_0$ formulation is np-hard since the $l_0$ norm is non-differentiable. Researchers have proposed the use of the closest convex relaxation of the $l_0$ norm, which is the $l_1$ norm, and is popularly known as the lasso problem [35]. While the lasso formulation is shown to recover the sparse models successfully under certain conditions, one drawback is that it does not implement any prior knowledge [36]. An alternative of the lasso formulation is group lasso, which induces block sparsity or group sparsity [37, 38] and is implemented by penalizing the $l_2$ criterion with $l_{2,1}$ norm [39]. The $l_{2,1}$ norm of a vector $w$ is given by,

$$
||w||_{2,1} = \sum_{g \in G} ||w_g||_2,
$$

(3.9)
Figure 3.3: PPG signal obtained from a Maxim smart-watch corrupted with motion artifacts.

Figure 3.4: Indicator function, 0 for clean segment, 1 when corrupted with motion.
where $g$ indicates groups, $w_g$ represents the coefficients belonging to the $g^{th}$ group and $w = [w_1, w_2, \ldots w_G]$. The group lasso formulation has been explored a lot with applications in the field of bio-informatics; the problem statement is to select a few relevant genes out of thousands of genes contributing towards a specific disease or biological function [40]. Using a similar idea, an $l_{2,1}$ penalty can be imposed on the motion signal $m(t)$ to induce contiguous sparse or block sparse structure on the motion signal.

The observed PPG signal is represented by a linear addition of clean PPG component, noise and motion signal. Though we note that the observed PPG is given by a much more complex relation rather than a simple linear combination, we use the linear model for simplicity.

$$y(t) = a(t)s(\theta(t)) + m(t) + n(t),$$

such that: $s$ is $2\pi$ periodic and $a, \theta' \in V(\theta_0, \lambda)$.

Since $s$ is a periodic function, it can be represented in terms of Fourier series,

$$a(t)s(\theta(t)) = a(t) \sum_{k=-K}^{K} z_k e^{jk\theta(t)},$$

$$= a(t) \sum_{k=-K}^{K} c_k \cos(k\theta(t)) + d_k \sin(k\theta(t)), \quad (3.11)$$

$$= \sum_{k=-K}^{K} c_k a(t) \cos(k\theta(t)) + d_k a(t) \sin(k\theta(t)).$$

We have discrete points from the time series parameters, and can be written in terms of vector notations, $y = [y(1), y(2), \ldots y(N)]$, where $N$ is the total number of samples in one time analysis window. Similarly, the other vectorized variables are instantaneous phase $\theta \in R^{N \times 1}$, motion signal $m \in R^{N \times 1}$, instantaneous amplitude $a \in R^{N \times 1}$ and the discretized linear space $V(\theta_0, \lambda) \in R^{N \times (4\lambda L + 1)}$. 
The equation (3.11) can be rewritten via matrix multiplication as,

\[ \alpha F_{a,\theta}, \]

where

\[ F_{a,\theta} = [u_0^T, u_1^T u_2^T ... u_K^T, v_1^T ... v_K^T] \in R^{2K \times N}, \]

\[ \alpha = [c_0, c_1, ... c_K, d_1, ... d_K] \in R^{(2K+1) \times 1}, \]

and \( u_l, v_l \in R^{N \times 1} \), whose \( j^{th} \) elements are as follows,

\[ u_l(j) = a(j) \cos(l\theta(j)), \]

\[ v_l(j) = a(j) \sin(l\theta(j)), \quad j = 1, 2, \ldots N. \] (3.12)

Using the linear model and combining the ideas presented the formulation in equation (3.6) and (3.9), we have the following optimization problem,

\[ \hat{m} = \arg \min_{a, \alpha, \theta, m} ||y - \alpha F_{a,\theta} - m||_2^2 + \lambda_1 ||\alpha - \alpha_0||_2^2 + \lambda_2 ||m||_{2,1} \] (3.13)

such that: \( a, \theta' \in V(\theta_0, \lambda). \)

Additionally, we define an indicator function which denotes where the motion signal is active and is given by the support vector of \( m \),

\[ i = \text{supp}(m). \] (3.14)

Specifically, when motion signal groups have non-zero values, the indicator function \( i \) becomes 1 and 0 elsewhere. The details of solving for the formulation in equation (3.14) and (3.11) is highlighted in the next chapter.
Chapter 4

PPGMotion: Algorithm for Motion Artifact Detection

In this chapter, we discuss solving for the optimization problem framed in the previous chapter. We develop the PPGMotion algorithm for detecting motion artifacts and discuss some implementation details related to solving the problem.

The problem in equation (3.13) is to be solved for $\alpha$, $a$, $\theta$ and $m$, and is broken down into smaller subproblems, using the idea of alternate minimization method [41]. Solving for the parameters follows an iterative procedure where in a iteration, the loss function is optimized with respect to one variable while the other variable is fixed. With the updated variable kept fixed, the objective function is then optimized over second parameter. This process is repeated until convergence. Breaking the problem in (3.13) in subproblems gives rise to the following algorithm for a specific time analysis window,
Initialize $\alpha_0, m^0 = 0$

for $k = 1, 2, ..$

$P1: \hat{\alpha}^k, \hat{a}^k, \hat{\theta}^k = \arg\min_{\alpha, a, \theta} \|y - \alpha F_{a, \theta} - \hat{m}^{k-1}\|^2$

$+ \lambda_1 \|\alpha - \alpha_0\|^2$

such that: $a, \theta' \in V(\theta_0, \lambda)$

$P2: \hat{m}^k = \arg\min_{m} \|y - \hat{\alpha}^k F_{\hat{a}^k, \hat{\theta}^k} - m\|^2 + \lambda_2 \|m\|_{2,1}$

Repeat until convergence.

### 4.1 Estimation of PPG signal parameters

Solving for the first problem $P1$ involves estimating the instantaneous phase function $\theta$, the instantaneous amplitude $a$, and the shape coefficient vector $\alpha$.

$P1: \hat{\alpha}^k, \hat{a}^k, \hat{\theta}^k = \arg\min_{\alpha, a, \theta} \|y - \alpha F_{a, \theta} - \hat{m}^{k-1}\|^2$

$+ \lambda_1 \|\alpha - \alpha_0\|^2$

such that: $a, \theta' \in V(\theta_0, \lambda)$.

As the optimization problem $P1$ is highly nonlinear in $a$ and $\theta$, it is very difficult to solve. The envelope $a$, the instantaneous phase $\theta$ and the wave shape function $\alpha$ are all unknown and are adaptive to the data. The optimization problem is tackled in two steps- the instantaneous phase function and the amplitude can be estimated first from the first harmonic of the signal, assuming that the PPG signal satisfies the scale separation property. This property is satisfied when the frequency corresponding to the harmonics are well separated from each other, as stated in [31]. Once $a$ and $\theta$ are estimated, the shape function $\alpha$ becomes a linear optimization function, which can be solved easily. Several time-frequency methods like Continuous Wavelet
Transform, Hilbert Transform, Synchrosqueezing Transform can be used to estimate the instantaneous phase function from the signal. In [42, 30], the instantaneous frequency $\theta'$ and $a$ are estimated first from the signal by Synchrosqueezing Transform (SST) [43]. In this work, we use a data-driven method, as in [31] to estimate the instantaneous phase function and the amplitude, and is discussed next.

### 4.1.1 Estimation of Instantaneous Phase

The instantaneous phase corresponding to the fundamental frequency of the signal $r^k \in \mathbb{R}^{N \times 1}$, where $r^k = y - m^{k-1}$ in equation (4.2) is updated using the nonlinear least solver as developed in [31]. The second part of problem $P1$ does not play a part since the instantaneous phase or amplitude is not involved in the constraint of shape function. The algorithm starts with an initial guess of instantaneous phase $\theta_0$ and is highlighted below.

Step1 : Solve the L1 regularised nonlinear least square problem:

$$(\hat{p}, \hat{q}) \in \underset{\hat{p}, \hat{q}}{\arg \min} \gamma (||\hat{p}|| + ||\hat{q}||) + ||r - \Phi_{\theta_0} \cdot \hat{p} - \Psi_{\theta_0} \cdot \hat{q}||_2^2$$

where each row $j$ in $\Phi_{\theta_0}$ and $\Psi_{\theta_0}$ is given by

$$\Phi_{\theta_0,j} = \cos \theta(j) \cdot V(\theta_0, \lambda)_j \text{ and } \Psi_{\theta_0,j} = \sin \theta(j) \cdot V(\theta_0, \lambda)_j, \quad j = 1, 2, \ldots, N.$$  

The envelope $p$ and $q$ is given by, $p = V(\theta_0, \lambda)_j \cdot \hat{p}$ and $q = V(\theta_0, \lambda)_j \cdot \hat{q}$

Step 2: Update $j^{th}$ element of $\hat{\theta}$ as

$$\Delta \theta(j) = \tan^{-1} \left( \frac{q(j)}{p(j)} \right),$$

$$\hat{\theta}(j) = \theta_0(j) - \Delta \theta(j), \quad j = 1, 2, \ldots, N.$$  

(4.3)
The regularization parameter $\gamma$ is chosen according to the noise level in the signal. A rough initial guess for instantaneous phase is chosen according to the frequency $f_{pr}$ corresponding to the maximum peak in the spectral domain, and is given by,

$$\theta_0(j) = 2\pi f_{pr} j, \quad j = 1, 2, \ldots, N,$$

where $f_{pr}$ is the average heart rate. \hfill (4.4)

Detailed analysis in [31] has shown that the $p$ and $q$ obtained from Step 1 of the algorithm in equation (4.3) can be written as

$$p(j) \approx a(j) \cos(\theta_0(j) - \theta_g(j)),$$

$$q(j) \approx a(j) \sin(\theta_0(j) - \theta_g(j)), \quad j = 1, 2, \ldots, N.$$ \hfill (4.5)

where $\theta_g$ is the actual instantaneous phase corresponding to the fundamental frequency of the signal $r$, $\theta_0$ is the initial instantaneous phase at the start of the algorithm and $a$ is the amplitude envelope.

Each element $j$ of the instantaneous phase $\hat{\theta}$ can then be obtained as,

$$\Delta \theta(j) = \tan^{-1} \left( \frac{q(j)}{p(j)} \right),$$

$$\hat{\theta}(j) = \theta_0(j) - \Delta \theta(j), \quad j = 1, 2, \ldots, N.$$ \hfill (4.6)

### 4.1.2 Update Amplitude Envelope

The amplitude envelope $\hat{a}$ is then estimated as

$$\hat{a}(j) = \sqrt{p(j)^2 + q(j)^2}, \quad j = 1, 2, \ldots, N.$$ \hfill (4.7)
4.1.3 Estimation of Shape Function

Once the parameters $a$ and $\theta$ are estimated, the problem $P1$ is written as,

$$
\hat{\alpha}^k = \arg\min_{\alpha} \| r^k - \alpha F_{\hat{a}^k, \hat{\theta}^k} \|^2 + \lambda_1 \| \alpha - \alpha_0 \|^2.
$$

(4.8)

One important parameter that we have to decide on is the prior information about the shape, $\alpha_0$. Using one particular shape function will produce a huge bias in the estimated shape since the PPG shape is highly subject dependent. As discussed before, we note that the PPG signal does not change abruptly across time for a specific subject. To ensure smoothness of the shape function across time, the estimates of shape function in previous time analysis windows can be therefore be used as a prior shape for the next time window. The problem can be reformulated as,

$$
\hat{\alpha}_w^k = \arg\min_{\alpha} \| r^k - \alpha F_{\hat{a}^k, \hat{\theta}^k} \|^2 + \lambda_1 \sum_{i=1}^w \tau^{w-i} \| \alpha - \alpha_{i-1} \|^2,
$$

(4.9)

where $\hat{\alpha}_w^k$ corresponds to the shape function from the $w^{th}$ time analysis window, and $\tau$ is the forgetting factor which can take in values between 0 and 1. The estimation problem in equation (4.9) is a simple ridge regression problem, and the solution is obtained by,

$$
\hat{\alpha}_w^k = (F_{\hat{a}^k, \hat{\theta}^k}^T F_{\hat{a}^k, \hat{\theta}^k} + \lambda_1 I)^{-1} (F_{\hat{a}^k, \hat{\theta}^k}^T r^k + \lambda_1 \bar{\alpha}),
$$

(4.10)

where $\bar{\alpha}$ is given by,

$$
\bar{\alpha} = \frac{\sum_{i=1}^w \hat{\alpha}_{i-1} \tau^{w-i}}{\sum_{i=1}^w \tau^{w-i}},
$$

(4.11)

and $I \in R^{N \times N}$ is the identity matrix.
Figure 4.1: PPG signal $y$ in a with motion artifacts in a time window.

Figure 4.2: Estimated instantaneous frequency $\theta'$.

Figure 4.3: Estimated motion signal $m$.

Figure 4.4: Indicator function $i$ given by support vector of $m$. 
### 4.2 Estimation of Motion Signal

Once the problem $P1$ is solved, the motion signal $m$ needs to be solved in $P2$. The problem can be written as,

$$
P2: \hat{m}^k = \arg\min_m ||z^k - m||^2_2 + \lambda_2 ||m||_2,1, \tag{4.12}
$$

where $z^k = y - \hat{\alpha}F_{\hat{a}^k, \hat{\theta}^k}$.

The problem in equation (4.12) is very similar to a general group lasso problem given by,

$$
\beta = \arg\min_{\beta} ||o - X\beta||^2_2 + \lambda ||\beta||_2,1, \tag{4.13}
$$

where, $o \in \mathbb{R}^n$ is an observed vector and $\beta \in \mathbb{R}^p$ is the coefficient vector which is to be estimated for a given design matrix $X \in \mathbb{R}^{n \times p}$. The groups are known a priori and can either be overlapping or non-overlapping. In the overlapping case, where the support of $\beta$ is given by the union of groups, a simple way for implementation is given in [38]. The design matrix $X \in \mathbb{R}^{n \times p}$ is replaced by $\tilde{X} \in \mathbb{R}^{n \times \sum |g|}$ which is formed by concatenating the copies of design matrix for each groups, i.e $\tilde{X} = [X_{g1}, X_{g2}, ... X_{gG}]$.

The problem in equation (4.13) is then be solved as a classical group lasso approach for non-overlapping groups.

In this work, we have assumed the motion event duration is at-least 4 seconds, therefore the group length for calculating $||m||_{2,1}$ was assigned to be 4 seconds, with 50% overlap between the groups. Comparing equations (4.12) and (4.13), it can be seen that our design matrix $X$ is an identity matrix. After rewriting the design matrix $\tilde{X}$ as discussed above, ADMM is used to solve for the group lasso problem as given in [44]. An indicator function is used to identify if a certain group is a motion artifact or not. According to equation (3.11), this is done by assigning 1 to the groups which
form the support for $\hat{m}$, and 0 elsewhere, as detailed below,

$$i = \bigcup_{g \in G} i_g$$

where $i_g = 1$ when $m_g \neq 0$,

and $g$ denotes the group.

The plots of some of the parameters estimated as a result of the proposed algorithm are shown in Figure 4.1 to 4.4. The pipeline is outlined as the PPGMotion algorithm in the next page.

4.2.1 Implementation Details

The PPGMotion algorithm divides the overall PPG signal recording of total time duration $T$ into smaller overlapping time analysis window $w$. In each of the smaller time window, PPGMotion detects motion corrupted PPG segments. The assumption in the signal model is that the shape function is a periodic function in a small time window. Therefore, the analysis $w$ has to be chosen small enough for the assumption to be valid. On the other hand, too small value for $w$ will result in high variance in the estimated parameters. Based on previous literature on the temporal variation of PPG morphology, we choose $w = 20$ seconds, with overlap duration of 2 seconds between two consecutive time windows.

In equation (4.3), the value of $\lambda$ determines the smoothness of the instantaneous frequency and the amplitude function. As mentioned in [31], we chose $\lambda = 0.5$. Additionally, we note that low pass filtering $p$ and $q$ in equation (4.5) with a cut-off frequency of 0.5 Hz results in a smoother estimate of the instantaneous phase. The cutoff frequency was chosen as 0.5 Hz because it has been reported in several studies [45] that heart rate variability lies within this particular frequency range.
Algorithm 1 PPGMotion algorithm for motion artifact detection

Input: Motion corrupted PPG signal $y$

Output: Detected motion contamination $i$

Initialize: $\bar{\alpha}$, $w = 1$, $i = 0$  \hspace{1cm} (w is the time analysis window)

1: while $w \neq T$ do  \hspace{1cm} (T is the total recording time of the PPG signal)
2: \hspace{1cm} Initialize: $m_w = 0$, $\theta_0$ as in equation (4.4), $\lambda_1$, $\lambda_2$
3: \hspace{1cm} repeat for $k = 1, 2, ...$  \hspace{1cm} (k is the iteration counter)
4: \hspace{1.5cm} $\hat{\alpha}_k, \hat{a}_k, \hat{\theta}_k = \arg\min_{\alpha, a, \theta} ||y - \alpha F_{a, \theta} - \hat{m}_k^{-1}||^2_2 + \lambda_1 ||\alpha - \bar{\alpha}||^2_2$
\hspace{2cm} such that: $a, \theta' \in V(\theta_0, \lambda)$
5: \hspace{1.5cm} $\hat{m}_k = \arg\min_{m} ||y - \hat{\alpha}_k F_{\hat{a}_k, \hat{\theta}_k} - m||^2_2 + \lambda_2 ||m||_{2,1}$
6: \hspace{1cm} until convergence
7: \hspace{1cm} Update: $\bar{\alpha}$ as in equation (4.11)
8: \hspace{1cm} Update $i_w$ as in equation (4.14)
9: \hspace{1cm} Update next time window $w = w + 1$ \hspace{1cm} (Consider next time window)
10: end while
11: $i = \bigcup i_w$
12: return $i$

For equation (3.11), the value of $K$ denotes the number of harmonics present in the PPG signal. Since the cardiac-related peaks are associated with the first 4-6 harmonics of the fundamental heart rate frequency [30, 46], we use a more conservative harmonic range and choose $K = 8$.

We choose the groups $g$ for achieving block sparse motion signal $m$ in a sliding window approach. Each window corresponds to 4 second duration with 2 seconds overlap between consecutive groups. The underlying assumption we make is
that any motion event is at least 4 seconds long.

The value of the Lagrangian multipliers, $\lambda_1$ and $\lambda_2$ depend on the prior information about the sensor noise and the extent of temporal smoothness in shape function respectively. The parameter values are often set to some fraction of the signal energy. In this work, we have empirically chosen $\lambda_1 = 600 * ||y_w||_2^2$ and $\lambda_2 = 4 * ||y_w||_2$, based on the dataset.

For calculating $\bar{\alpha}$ from equation (4.11), we choose forgetting factor $\tau = 0.99$.
Chapter 5

Validation of PPGMotion

In this chapter, we characterize and quantify the performance of the PPGMotion algorithm. First, we simulate motion artifacts based on a generative model and quantify the performance of PPGMotion for varying parameters of the model. Second, we conduct some experiments using a pulse-oximeter and a smartwatch, to capture some real-world motion scenarios. The performance of the PPGMotion algorithm on the datasets is evaluated and compared against some of the existing methods.

5.1 Generating Simulated Dataset

To simulate different instances of motion artifacts, we use the generative model,

\[ m(t) = p.q(t) + (1 - p)g(t), \]

where \( p \) governs the amount of periodic interference component \( q(t) \) present in the motion signal \( m(t) \), and \( g(t) \) is a random aperiodic signal. The overall amplitude of the signal \( m(t) \) is governed by the Signal-to-Motion Ratio (SMR). Different instances of motion signals are generated with varying periodicity factor \( p \) and SMR. The periodic component of the motion signal is generated according to the following equation,

\[ q(t) = a_m(t)s_m(\omega t), \]

where \( a_m(t) \) is the amplitude envelope,

\[ s_m(\omega t) \] is a periodic function with fundamental frequency \( \omega \).
The amplitude envelope is generated randomly from a Gaussian distribution with mean $\mu = 1$ and variance $\sigma^2 = 10$, and then passed through a low-pass filter with a cutoff frequency of 0.5 Hz. The fundamental frequency $\omega$ is chosen uniformly in between 0.5 Hz and 3 Hz, because periodic movements like walking or running have a cadence rate of 30 steps per min to 180 steps per min (0.5 Hz – 3 Hz). The function $s_m$ is generated randomly in one period from a Gaussian distribution with mean $\mu = 0$ and variance $\sigma^2 = 1$ and then repeated with a period according to the fundamental frequency $\omega$. To get rid of the high-frequency components, the periodic signal $s_m(\omega t)$ is then passed through a bandpass filter having cutoff frequency 0.5 Hz – 5 Hz, and multiplied with $a_m(t)$ to generate $q(t)$.

Next, we generate the aperiodic random signal component $g(t)$. We collected PPG signals from a pulse oximeter during random arm and finger movements. Analyzing the frequency spectrum of the corrupted PPG signals, we observe that the frequency spectrum is not flat but decays with increasing frequency. If we generate $g(t)$ from a Gaussian distribution, the frequency spectrum will be flat. Instead, we use a colored noise, specifically Brown noise for generating $g(t)$. The power spectral density of Brown noise is proportional to $1/f^2$; hence the magnitude of frequency spectrum decays with increasing frequency.

Although we propose that the observed signal is a linear combination of the motion signal and the clean PPG component, in a practical scenario, the relation is much more complicated. However, for our study, we assumed a linear combination for simplicity. The generative model is used to simulate different motion signals by varying values of $p$ and SMR. The generated motion signals are then added to a clean PPG signal component in a specific time duration. Some instances of simulated motion contaminated PPG signal in the frequency domain are shown in Figure 5.1,
5.2 and 5.3 for comparison against some real-life motion scenarios. The frequency spectrum of simulated PPG contaminated data are visually similar to the frequency spectrum of actual contaminated data collected from a pulse oximeter, but we do not have any metric to quantify the similarity. The performance of PPGMotion compared to a prior method will be discussed in the results section. Since experimental data provides only certain data points for \( p \) and SMR, it is useful to validate our algorithm on simulated data for a wide range of \( p \) and SMR. We also perform experiments to capture real motion scenarios and explain the methodology in the next section.

5.2 Data Collection Methodology

5.2.1 Experimental Setup

We recruited 13 healthy subjects for the study (6 male, 7 female), in the age group between 24-35 years. All the experiments done in this research were approved by the Rice University Institute Review Board (IRB-FY2018-434, Approval Date: 11/20/2018). We used a BIOPAC pulse oximeter (MP150 module with PPG100C amplifier and TSD200 finger probes; sampling rate of 500 Hz) as well as a Maxim Healthband (sampling rate of 25 Hz) to record the PPG waveforms. We resample the signals acquired from BIOPAC pulse oximeter to 30 Hz for ease of computation. MATLAB as used for all the processing required. We highlight the details of experimental protocols in the next section.

5.2.2 Experimental Protocol

To quantify the performance of the proposed PPGMotion algorithm in comparison to existing methods, we conducted 3 experiments. In the first experiment, two probes
Figure 5.1: Synthetic data generation: (a) an example of simulated motion artifact added to PPG signal for $p = 0$, SMR = $-5$ dB (b) frequency spectrum of a real motion corrupted data acquired from finger PPG under random finger movements
Figure 5.2: Synthetic data generation: (a) an example of simulated motion artifact added to PPG signal for $p = 0.5$, SMR = $-2$ dB (b) frequency spectrum of a real motion corrupted data acquired from finger PPG under arm waving scenario
Figure 5.3: Synthetic data generation: (a) an example of simulated motion artifact added to PPG signal for $p = 0.9$, SMR = $-2$ dB (b) frequency spectrum of a real motion corrupted data acquired from finger PPG under periodically tapping case
from BIOPAC pulse oximeter were connected to the index fingers of left and right hands. The subjects were asked to stay still initially for 2 minutes. At the end of 2 minutes, the subjects were asked to perform random small finger movements for 20 seconds in one hand while keeping the other hand still. After the movement phase, the subjects were asked to stay still for 30 seconds. Next, the subjects were asked to perform arm and elbow movements, thus inducing large motion artifacts in the signal. The movement phase lasted for 40 seconds, followed by 30 seconds of no activity. The subjects were then asked to remain still for the next 30 seconds. The duration of PPG waveform acquired from each subject was 4 minutes. We label the dataset acquired from the first experiment as dataset (1).

In the second experiment, the subjects were asked to stay still initially for 2 minutes before the movement phase. Next, they were asked to tap fingers from one hand on a table periodically. Tapping fingers on the table is expected to introduce periodic motion contamination in the recorded PPG signals. The duration of motion activity was 30 seconds, followed by 30 seconds of no activity. The duration of PPG waveform acquired from each subject was 3 minutes. We label the dataset recorded from the second experiment as dataset (2).

In the third experiment, we used a Maxim Health band to record PPG waveform from wrists of subjects while they were running on a treadmill. The subjects were asked to run for 2 minutes with a speed of 1, 3, 5 mph with 2 min resting interval between the activity durations. The duration of PPG signals acquired from each subject was 14 minutes. We discard data from 3 subjects because the whole duration of the PPG waveform was contaminated with noise and artifacts. The corrupt waveform might arise due to the watch being too loosely strapped, resulting in a very low PPG signal strength. We label the dataset from the third experiment as dataset (3).
5.2.3 Performance Comparison

To compare the PPGMotion algorithm, we implement the idea presented in [19], which uses features from the time-frequency representation of a PPG segment to identify motion corrupted PPG segments. We extract features like projected frequency modulation difference and HR frequency difference and then train these features with Support Vector Machine (SVM) to distinguish motion contamination and clean counterparts. In this work, we use a more common time-frequency representation, Synchrosqueezing Transform (SST) to extract these features. We term this method as the SST-SVM method.

Secondly, we implement the idea presented in [21], where the features corresponding to morphological variability are considered. We extract features like standard deviation of PPG shape, amplitude and heart rate variability from a PPG segment window. We use SVM to train the features to differentiate between a clean segment and motion segment. We term this method as TD-SVM.

Third, we also implement the idea presented in [47] which uses statistical metrics like skewness and Shannon’s entropy to identify motion corruption in PPG signals. We term this method as KSE.

Prior works on motion artifact detection use human visual inspection to identify the segments corrupted with motion artifacts which serve as a reference for comparison [19]. In this work, to obtain a reference or ground truth, we also resort to visual inspection to identify corrupted PPG segments.

We compare the PPGMotion algorithm against the three prior works in terms of accuracy, sensitivity, and specificity. Sensitivity, also known as the true positive rate, is the proportion of correctly identified motion artifact segment. Specificity, known as the true negativity rate, is associated with the proportion of correctly classified
clean PPG segments. Accuracy is given by the ratio of the total correctly classified segments.

5.3 Results

The performance of PPGMotion on simulated motion data for SMR of -15 dB, -5 dB and 5 dB is shown in Figure 5.4, 5.5 and 5.6. The sensitivity of past work (TDSVM) falls rapidly for increasing $p$ value. The rapid decrease in sensitivity for TDSVM is expected since high $p$ value denotes the presence of strong periodic motion interference. On the other hand, the decrease in the sensitivity for PPGMotion is more graceful compared to TDSVM. Figure 5.7(a) and (b) corresponds to an instance of random motion corrupted PPG segment from dataset (1) and an instance of periodic motion artifact from dataset (2) respectively. In the case of random motion corruption, we observe that PPGMotion performs slightly better than the TDSVM method. On the other hand, PPGMotion performs significantly better than the TDSVM method in the case of periodic motion contamination.

We quantify the performance of the PPGMotion algorithm against the three existing methods, namely TDSVM, SSTSV, and KSE for dataset (1), (2) and (3) in Table 5.1. PPGMotion has higher accuracy in detecting motion artifacts than all existing methods for all the datasets. For the random motion dataset (1), the improvement over the second best algorithm (TDSVM) is nominal– there is only 2% increase in accuracy and 1% increase in sensitivity. While on the other hand, there is a 25% increase in sensitivity over the TDSVM algorithm for periodic motion dataset (2), and around 4% increase in accuracy. Similarly, for dataset (3), PPGMotion achieves approximately 9% increase in accuracy and around 25% increase in sensitivity over the TDSVM method.
Table 5.1: Performance Metrics (Mean±Std) of our proposed approach vs existing methods

<table>
<thead>
<tr>
<th>Dataset</th>
<th>PPGMotion</th>
<th>TDSVM</th>
<th>KSE</th>
<th>SSTSV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Finger Accuracy</td>
<td>94.35 ± 2.67</td>
<td>92.49 ± 4.436</td>
<td>85.80 ± 8.60</td>
<td>92.77 ± 2.65</td>
</tr>
<tr>
<td>random Sensitivity</td>
<td>92.25 ± 4.57</td>
<td>90.87 ± 8.60</td>
<td>52.67 ± 20.10</td>
<td>89.87 ± 2.60</td>
</tr>
<tr>
<td>Specificity</td>
<td>95.17 ± 3.4</td>
<td>92.91 ± 5.96</td>
<td>93.19 ± 6.99</td>
<td>94.01 ± 2.90</td>
</tr>
<tr>
<td>Finger Accuracy</td>
<td>93.05 ± 2.96</td>
<td>89.23 ± 3.10</td>
<td>73.06 ± 33</td>
<td>75.89 ± 3.23</td>
</tr>
<tr>
<td>periodic Sensitivity</td>
<td>91.86 ± 6.35</td>
<td>66.06 ± 33.03</td>
<td>24.89 ± 33</td>
<td>53.56 ± 27.14</td>
</tr>
<tr>
<td>Specificity</td>
<td>94.14 ± 3.23</td>
<td>97.53 ± 2.10</td>
<td>95.38 ± 3.65</td>
<td>81.89 ± 4.13</td>
</tr>
<tr>
<td>Exercise Accuracy</td>
<td>89.89 ± 1.53</td>
<td>80.66 ± 5.96</td>
<td>60.39 ± 5.16</td>
<td>81.23 ± 2.31</td>
</tr>
<tr>
<td>dataset(3) Sensitivity</td>
<td>91.03 ± 5.87</td>
<td>65.88 ± 32.68</td>
<td>33.84 ± 10.10</td>
<td>74.99 ± 11.29</td>
</tr>
<tr>
<td>Specificity</td>
<td>87.29 ± 5.07</td>
<td>85.08 ± 18.47</td>
<td>94.02 ± 2.25</td>
<td>83.60 ± 14.16</td>
</tr>
</tbody>
</table>
Figure 5.4: Validation of PPGMotion on simulated data for varying p and SMR = -15 dB
Figure 5.5: Validation of PPGMotion on simulated data for varying \( p \) and SMR = -5 dB
Figure 5.6: Validation of PPGMotion on simulated data for varying p and SMR = 5 dB
Figure 5.7: Performance comparison of PPGMotion against TDSVM for identifying motion contamination. (a) Random motion artifact (b) Periodic motion artifact
Chapter 6

Discussion

The robustness of the PPGMotion algorithm lies in the assumption that even if the motion artifact tends to be periodic, the shape is different from the characteristic shape possessed by a clean PPG signal. From our experimental dataset (2) and (3), we estimate the shape from clean PPG segments as well as from periodic motion contaminated segments. To quantify the difference in the two estimated shapes, we calculate two metrics; the Normalized Root Mean Squared Error (NRMSE) and the cross-correlation value between the two estimated shapes. If the two shapes, i.e., the motion corrupted shape and a clean PPG shape are different, the NRMSE between them is expected to be high. On the other hand, the cross-correlation between them is expected to be low, compared to the case when two clean PPG shapes obtained from different instances of time are considered. The computed metrics for all the subjects from two datasets are shown in Figure 6.1, and it demonstrates that the periodic motion contaminated PPG shape is significantly different ($p$ value < 0.001) from that of the clean PPG shape.

Considering the difference in shape morphology makes PPGMotion more robust in identifying motion corrupted PPG segments. PPGMotion extracts a quasiperiodic component from the corrupted PPG signal under some constraints and studies the structure of the signal residuals in each time analysis window.

For random motion artifact, the normalized power of residuals in a single time analysis window is higher compared to the case when periodic motion interference is
Figure 6.1: Normalized root mean square error and normalized cross correlation in (%) for clean and contaminated PPG shape. Note that for clean vs clean error plots, two clean PPG segments before and after motion activity for same subject are chosen.
Figure 6.2: Simulated periodic motion contaminated PPG signal

Figure 6.3: Normalized residual power without shape constraint

Figure 6.4: Least square error between estimated shape and priori shape
present. In the presence of periodic motion contamination, the process extracts most of the periodic component from the signal. The solution we propose is not only to analyze the residuals but also analyze the difference in shape morphology. Figure 6.2 shows a PPG signal, with simulated periodic motion corruption in between 250 and 300 seconds. We show the plot of signal residuals power without the shape constraint in Figure 6.3. The normalized residual power in the motion corrupted time duration is low, because most of the periodic motion interference is misjudged as that arising from the cardiac signal, and is extracted out. On the other hand, the difference in shape estimated in the motion corrupted window and the prior shape information is high which indicates the presence of motion corruption, as shown in Figure 6.4. The essence of PPGMotion is combining information on shape as well as signal residuals to identify motion corrupted PPG segments.

One important point to note is that unlike many artifact reduction algorithms, the performance of PPGMotion does not depend on the accuracy of estimating the initial average heart rate. For a particular time analysis window, even if the initial frequency estimated corresponds to the motion frequency, the segment is still labeled as a corrupt segment owing to the mismatch between the motion signal shape and some pre-learned clean PPG morphology.

6.1 Conclusion

We propose a novel algorithm PPGMotion for detecting motion artifacts in PPG signals. We also validate the PPGMotion algorithm against other motion detecting algorithms for different scenarios of motion. PPGMotion achieves the same accuracy as that of the state-of-the-art methods when the motion induced artifacts are random in nature. Whereas on the other hand, in the case of periodic motion induced artifacts,
PPGMotion shows a significant increase in the accuracy of detecting these artifacts. Periodic motion artifacts may arise during daily activities or during cardiopulmonary exercise training [48], and is difficult to handle since traditional algorithms mistake the periodic artifact to be arising from the cardiac activity.

In the future, we would like to extend our work by considering additional reference source like an accelerometer signal. Incorporating an additional reference signal will likely result in a more robust algorithm to detect as motion contamination well as reconstruct a denoised PPG from motion corrupted PPG waveforms.
Bibliography


[6] J. Y. A. Foo, S. J. Wilson, G. R. Williams, M. Harris, and D. M. Cooper, “Motion artefact reduction of the photoplethysmographic signal in pulse transit...


