RICE UNIVERSITY

Applications of Statistical Learning and Stochastic Filtering for Damage Detection in Structural Systems

by

Debarshi Sen

A Thesis Submitted
in Partial Fulfillment of the
Requirements for the Degree

Doctor of Philosophy

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ABSTRACT

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Structural health monitoring (SHM) is necessary for online maintenance of infrastructure systems. It entails deployment of an array of sensors on a structure of interest for data acquisition, based on which, damage detection, quantification and localization may be performed (collectively known as condition assessment), followed by prognosis and estimation of remaining useful life that constitutes a comprehensive decision making framework. This dissertation specifically focuses on damage detection which traditionally follows a model-based approach. This implies construction of high-fidelity models resembling the actual system in its undamaged state followed by a quantitative or qualitative comparison of response from the model and the real infrastructure system. This sheds light on the changes in the system due to advent of damage. However, development of such high-fidelity models is extremely challenging and in many cases renders the damage detection problem computationally prohibitive.

The alternative is using statistical learning and stochastic filtering algorithms that rely on construction of parametric models. Stochastic filtering methods are not necessarily model-free, however, they utilize data acquired from a real system for damage detection. Statistical learning approaches on the other hand either involve construction of a meta-model or no model at all. The meta-models do not necessarily simulate system behavior in
its full complexity, instead they are designed for the specific task of damage detection. If a sizable data is available from deployed sensors, these class of methods becomes far more suitable compared to the traditional model-based regimes.

This dissertation proposes applications of statistical learning and stochastic filtering techniques for the task of damage detection, with an eventual goal of developing efficient SHM systems. The damage detection problems considered involve:

- acoustic emission source localization in plates
- active damage detection of beam, plate and pipe structures using Lamb waves
- decoupling of effects of varying environmental conditions and damage on vibration testing data acquired from bridge structures

The proposed algorithms perform damage detection efficiently. Additionally, in the high frequency regime, the number of sensors necessary for damage detection is reduced.
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Chapter 1

Introduction

1.1 Motivation

Reliable and accurate structural damage detection (DD) is of prime importance to evaluate the current state of engineering systems, encompassing civil infrastructure, aircrafts and pipeline networks. Some recent failures demonstrate the catastrophic impact of damage on the economy, environment and society. Figure 1.1 shows failures in various infrastructure systems. To avoid such catastrophic failures, development of efficient damage detection schemes are necessary.

![Figure 1.1](image)

(a) I35 W bridge [1], $62m (b) Enbridge pipeline [2], $1bn (c) Fuselage damage [3], ~$50m.

To this end, the research community has made significant advances in the field of structural health monitoring (SHM) in recent decades [9]. Farrar et al. [9] define SHM as the process of implementing a damage identification strategy for aerospace, civil and mechanical engineering infrastructure. This entails deployment of an array of sensors on a structure
for continuous data acquisition. Using the acquired data, it is possible to make inferences about structural integrity (damage detection) and estimate the remaining functional life (damage prognosis) in real time. Hence, SHM also ensures continuity of functionality of engineering systems \[10, 11\]. Rytter \[12\] defines four levels of damage detection. They are in order of increasing complexity of the task:

- **Level I**: Damage detection, i.e., detect the presence of damage only.
- **Level II**: Damage localization.
- **Level III**: Damage quantification.
- **Level IV**: Estimation of remaining useful life.

The premise on which most damage detection algorithms is based upon is the effect of physical changes caused by damage in the dynamic characteristics of structures, that in turn reflect on response measurements and damage sensitive features \[13, 14, 15\]. Traditional SHM involves development of high-fidelity structural models for studying the behavior of structural systems in both pristine and damaged conditions \[16\]. One performs damage detection using features, extracted from the models, that best characterize the changes in the behavior of systems from a pristine to damaged condition. However, modeling complicated systems is challenging. Capturing the physics of a system accurately in terms of behavior of all degrees of freedom involved and modeling of specialized boundary conditions pose a significant obstacle to a designer. Although, creating such high fidelity models is possible using advanced simulation softwares, analyzing them becomes computationally prohibitive. For example, Gopalakrishnan \[17\] discusses the various computational issues involved in modeling wave propagation problems using the finite element method. In addition to the computational complexity, the high-fidelity modeling also do not necessarily capture the inherent uncertainties of a problem. To address these issues, the SHM
research community is developing algorithms that help circumvent the need for construction of such models. To this end, this dissertation proposes statistical learning and stochastic filtering-based algorithms for damage detection. These algorithms utilize data acquired from real systems efficiently for the purposes of damage detection.

In particular we look at the following damage detection problems:

- **Acoustic emission (AE) source localization in plates**: This dissertation proposes a Bayesian estimation approach to AE source localization that results in minimizing the number of necessary sensors for the task to three sensors, which is the minimum number possible.

- **Guided ultrasonic wave (GUW)-based damage detection on isotropic plate structures using active sensing**: This dissertation proposes a sparsity-based approach to GUW-based active sensing for plate structures. The key idea is to avoid construction of models and to use only experimental data for the tasks of DD.

- **GUW-based damage detection on hollow steel pipes using active sensing**: This dissertation proposes a semi-supervised algorithm for the purposes of DD on pipes by minimizing both preprocessing of raw signals and the number of sensors necessary for DD.

- **Damage detection on bridge structures, using dynamic characteristics, in the presence of temperature variations**: This dissertation proposes an approach for segregating the effects of temperature changes and damage on modal frequencies of existing bridges for performing efficient DD.

As can be seen, the above problems encompass a wide spectrum of damage detection problems, spanning active and passive problems as well as low and high frequency regime
of signals. The following section provides a brief discussion on the various aspects associated with these classes of damage detection problems.

1.2 Damage detection

Damage detection problems are typically classified as passive or active approaches. As the names suggest, a passive approach involves acquiring response signals generated in a system due to ambient excitations. On the other hand, active approaches involve actuating the structure of interest to acquire response signals. Both types of approaches have been extensively employed by the research community. Damage detection algorithms can also be classified depending on the frequency content of response signals of interest, namely, low frequency and high frequency regimes. Low frequency response signals that are encountered in SHM are typically of the order of 1 to 100 Hz. High frequency response signals, on the other hands, are in the order of 10 to 500 kHz.

Theoretically, these two different regimes are treated distinctly by the vibrations community, although in principle, all vibrations can be defined by a wave equation irrespective of the frequency regime. In the low frequency regime, problems that one associates with structural dynamics, one attempts to approximate the wave solution in terms of a linear combination of the first few orders of possible standing waves. These standing waves are also known as mode shapes in the structural dynamics community, each associated with a modal frequency. This is typically achieved by performing an eigenanalysis. The approximation aids in representing the structure of interest using much fewer degrees of freedom.

For the higher frequency regime, this however becomes computationally inefficient as one would require a large number of modal frequencies for representing the behavior of a system. Instead, the wave equation approach a with D’Alembert like solution is employed. Rather than representing motion in terms of standing waves, the key parameters for this re-
representation are the wave velocity and frequency dependent wave numbers. The frequency dependence of wavenumbers and wave velocities are characterized by dispersion curves. Such curves especially aid in active approaches to damage detection. Specifically for Lamb waves (discussed in chapter 4), the dispersion curves also elucidate the notion of wave modes in the high-frequency regime. These wave modes are distinct from the mode shapes, popularly used in structural dynamics. These wave modes are propagating modes. From dispersion curves one can infer whether a certain wave mode is propagating in a system given the frequency content of acquired or actuation signals. Hence, these curves are used to decide on the frequency content of the actuation signals as one may control the number of propagating wave modes.

From the point of view of damage detection, these two different frequency regimes have their own utilities. Typically for civil infrastructure, it is required that one performs system identification of global parameters associated with the system at hand. For such problems a low frequency regime is typically utilized. However, for detecting small cracks and small changes to systems, a global approach typically fails. A local approach using high-frequency waves is far more suitable owing to their sensitivity to small changes.

1.3 Key objectives

The key objectives of this dissertation are as follows:

1. Develop damage detection algorithms that help circumvent high-fidelity modeling of engineering systems.

2. Utilize sensor data acquired from real systems for performing damage detection.

3. Develop damage detection algorithms based on statistical learning and stochastic filtering.
4. Minimize the number of low-cost transducers necessary for damage detection in the high frequency regime

5. Address the impact of temperature variations on damage detection in bridges in the low frequency regime.

1.4 Thesis organization

This dissertation is organized as follows: Chapter 2 provides a brief primer on statistical learning and their applications in damage detection and SHM. It also describes the various statistical learning and stochastic filtering algorithms used in this dissertation. Chapter 3 introduces the acoustic emission source localization problem followed by the proposed Bayesian parameter estimation-based scheme to account for the underlying uncertainties of the problem and perform damage detection and localization using just three piezo-transducers. Chapter 4 introduces the sparsity-based framework proposed for damage detection in beams. The efficacy of the proposed algorithm is demonstrated using simulation results for beams. Chapter 5 extends the sparsity-based framework for plates. The proposed approach is tested using experimental data and demonstrates that accurate damage detection and localization is possible using only two piezo-transducers. Chapter 6 introduces the GUW-based damage detection in pipes and demonstrates the efficacy of the hierarchical clustering-based algorithm for level I damage detection using experimental cast iron pipes. It is shown that using four low cost piezo-transducers a 100% accuracy in level I damage detection is achieved. Chapter 7 introduces the impact of temperature variations on damage detection. A principal component analysis-based approach is proposed for decoupling the two phenomena for performing damage detection efficiently. The efficacy of the proposed algorithm is tested using both simulation data as well as experimental data acquired from
the Z24 bridge. Chapter 8 summarizes the findings of this dissertation and proposes future research directions to extend this work.
Chapter 2

Statistical Learning and Stochastic Filtering

In this thesis, statistical learning and stochastic filtering algorithms have been used extensively for developing damage detection schemes. This chapter serves as a primer for both classes of techniques. First, a few nuances of statistical learning and their applications in the areas of structural system identification and structural health monitoring are discussed. This is followed by descriptions of each method that have been employed for the proposed algorithms in this dissertation. Lastly, a primer on stochastic filtering is provided along with the details of the algorithms that have been employed in this thesis.

2.1 Statistical Learning

In this section, statistical learning is discussed briefly and the applicability of such algorithms for damage detection is elucidated. For a detailed analysis and understanding of the subject, the readers may refer to textbooks [18, 19] available on the subject. Bousquet et al. [20] states *The main goal of statistical learning theory is to provide a framework for studying the problem of inference, that is of gaining knowledge, making predictions, making decisions or constructing models from a set of data.* This resonates with the ideas of SHM and damage detection discussed in Chapter 1. As discussed earlier, for data-driven approaches to SHM, acquired data is used for model construction and decisions are made based on predictions from these models.

In the statistical learning community, data are generally of two types, one with labels
and the other unlabeled. Data labels signify the origin of the data from a specific source or source condition. For example, a time history data that is known to be from a structure that is damaged may carry a label $\text{damaged}$. A supervised learning scheme uses the labeled data for model construction and label predictions for future data. An unsupervised learning scheme on the other hand does not use such labels. Such schemes are not designed for making predictions, but, are capable of revealing inherent structures and patterns in a data that may be crucial for the problem at hand.

Supervised learning can again be of two types, namely regression and classification. The key distinction between the two approaches are in the output variables obtained that are continuous for the former and discrete for the latter. All supervised learning algorithms are comprised of two steps. The first is training. This entails using of a part of the data, referred to as the $\text{training data}$, for model construction. This model is then used for making predictions using the $\text{test data}$ (the remaining data not used for training). The test data is used as an input to the constructed model for obtaining the desired predictions or outputs. For selection of algorithmic parameters, a process known as $\text{cross validation}$ is typically employed to optimize the parameters to the given data, using the training data set only. For further details the reader should refer to the book by Hastie et al. \cite{18} or Witten et al. \cite{19}.

Based on the above discussion, it is clear that statistical learning algorithms are best suited for data-driven SHM. A primer on applicability of various approaches discussed above can be found in Worden and Manson \cite{14}. The following sections will provide a few examples where statistical learning algorithms have been applied for SHM. These examples are by no means exhaustive, however a range of applications of a variety of learning algorithms are touched upon.
2.1.1 Applications of statistical learning algorithms

This section discusses a few applications of statistical learning algorithms in the structural system identification and structural health monitoring community. First supervised techniques are covered, both regression and classification forms, followed by unsupervised schemes.

Supervised learning: Classification techniques

Naïve-Bayes (NB) classifier classifies data based on conditional probability density functions constructed from training data. Let there be $K$ classes (data labels) for a training data set $X$, with $Y$ being the class variable. Based on Bayes theorem, the classification rule (probability that variable $Y = k$) for a NB classifier is:

$$P(Y = k|X) = \frac{P(X|Y = k)P(Y = k)}{\sum_{i=1}^{K} P(X_i|Y = k)P(Y = k)}$$

(2.1)

Addin et al. [21] use a NB classifier for damage detection in composite materials using Lamb waves. They utilized the underlying distinction in Lamb wave signals generated by different kinds of damages. Muralidharan and Sugumaran [22] also use an NB classifier for fault detection in centrifugal pumps, however, they also demonstrate that a Bayes net classifier produces a better accuracy for their experiments. An extension of the NB classifier is the linear discriminant analysis (LDA) with Fisher’s discriminant. LDA aims at evaluating optimum discriminant functions that maximize the difference between the various classes of data. To this end, LDA assumes a Gaussian distribution for the random variable $X|Y = k$ (a random variable that characterizes data points in the training set belonging to class $k$) with the same covariance for all classes. Farrar et al. [23] use LDA on acceleration data acquired from a concrete column for damage detection. Guadenzi et al. [24] also use LDA
for low velocity impact damage detection in laminated composite plates.

Support vector machine (SVM) is another classifier that constructs linear or nonlinear functions that separate various classes of data in the sample space. It is based on maximizing the distance of data points, belonging to distinct classes, to these separating hyperplanes. SVM solves the following optimization problem in the sample space:

\[
\min_{\beta_0, \beta} \| \beta \|^2_2 \\
\text{subject to } y_i(x_i^T \beta + \beta_0) \geq 1 \quad \forall i = 1, \ldots, n.
\]

(2.2)

where \( x_i \) is a data point in the sample space, \( \beta \) is the slope of a normal to the optimal separating hyperplane, \( \beta_0 \) is an intercept and \( y_i \) is a class variable that can take values of either +1 or −1. This variable essentially aids in the binary classification of the data.

Typically, SVMs are designed for solving two class classification problems. However, they can be extended to solve multi-class problems as well. He and Yan [25] use a SVM-based damage detection scheme for a spherical lattice dome, where feature extraction was performed using wavelet transform of the ambient vibration data of the structure. Shimada et al. [26] use SVM for damage detection in power distribution poles. Bulut et al. [27] propose a SHM system for the Humboldt Bay Bridge using SVM. They use data acquired from a numerical impact test on the bridge for damage detection. Chattopadhyay et al. [28] use a nonlinear version of SVM for damage detection in composite laminates. A nonlinear version implies that the optimal separating hyperplane is a nonlinear function. Bornn et al. [29] use SVM in conjunction with an autoregressive model (AR) for damage detection as well as localization by signal reconstruction and residual estimation. They reconstruct the signal using an AR model and use SVM for damage classification. For nonlinear SVM they use a Gaussian kernel. Worden and Lane [30] demonstrate the efficacy of SVM, in general, as a statistical learning algorithm for classification problems in engineering.
Trees are another kind of popular base learner for classification. They are top down recursive partitions of the feature or data space via binary splits. Although they are poor predictors, they are adept at making interpretations from data sets that eventually aid in decision making. Kilundu et al. [31] use decision trees for development of a early detection system for bearing damage. They use features extracted from vibration data of bearings to train the trees. Vitola et al. [32] also demonstrate the use of trees for damage detection in aluminum plates using high frequency waves. They also show the efficacy of bagging and boosting trees. Bagging, which stands for bootstrap aggregation, as the name suggests involves bootstrapping (re-sampling of data) of training data and application of multiple trees to the enlarged data set. The key idea is to reduce the variance of estimates based on the law of large numbers. Boosting on the other hand does not involve data duplication. It is a weighted congregation of results from weak base learners trained using randomly allocated portion of the training data. One of the most popular boosting algorithm is adaboost [18].

**Regression techniques**

Regression algorithms are also used for the purposes of damage detection. As discussed earlier, regression algorithms are generally used to build models consisting of continuous variables only using available data. The output of such regression models is hence not discrete. The most classical form of regression is the linear regression or least squares that attempts at solving the following:

$$Y_{tr} = X_{tr} \beta$$  (2.3)

where $Y_{tr} \in \mathbb{R}^{n \times 1}$ is the vector of training labels, $X_{tr} \in \mathbb{R}^{n \times p}$ is the training data matrix and $\beta \in \mathbb{R}^{p \times 1}$ are the coefficients to be estimated by the following optimization problem:
where $\| \cdot \|_2$ is the $\ell_2$-norm. The optimal $\beta$ obtained from the above equation is:

$$\hat{\beta} = (X_{tr}^T X_{tr})^{-1} X_{tr}^T Y_{tr}$$ (2.5)

The $\hat{\beta}$ obtained above is then used for making label predictions for a test data set:

$$Y_{tst} = X_{tst} \hat{\beta}$$ (2.6)

where the subscript $tst$ stands for test data sets. Pan [33] and Shahidi et al. [34] use linear regression for damage detection in structures. They formulate the modal expansion of a dynamic response as a linear regression problem. The deviation of slope between the damaged and undamaged models is used as a damage feature. They demonstrate the use of both single and multi variable linear regression.

However, least squares regression suffers from drawbacks related to collinearity, high dimensionality of data and computational issues associated to matrix inversion. To overcome these, regularization terms are added to the optimization problem. Two popular regression techniques can be found in the literature that can overcome the above mentioned issues. They are, Tikhonov Regularization or Ridge Regression and Sparse Regression.

Ridge Regression is based on constraining the elements of vector $\beta$ evaluated using least squares. It is a $\ell_2$-norm regularization. The updated optimization problem is as follows:

$$\min_{\beta} \| Y_{tr} - X_{tr} \beta \|_2^2$$ (2.7)

subject to $\| \beta \|_2^2 \leq t$

The more popular Lagrange equivalent form of Ridge Regression is:

$$\min_{\beta} \| Y_{tr} - X_{tr} \beta \|_2^2 + \lambda \| \beta \|_2^2, \quad \lambda \geq 0$$ (2.8)
where $\lambda$ is the regularization parameter. The solution the Ridge Regression optimization problem is:

$$\hat{\beta}_{\text{ridge}} = \left( X_{tr}^T X_{tr} + 2\lambda I \right)^{-1} X_{tr}^T Y_{tr}$$  \hfill (2.9)

Sparse regression, also known as the LASSO method [35], is another approach to overcome the drawbacks of least squares regression. A vector of length $n$ is $k$-sparse, if and only if $k \ll n$. If the requirement of a problem statement is a sparse $\beta$, the above optimization problem can be further modified by replacing the $\ell_2$-norm by an $\ell_1$-norm. Although the $\ell_1$-norm is a weak definition of sparsity with the $\ell_0$-norm truly defining sparsity, it allows for the optimization problem to remain convex, guaranteeing a global optimum. The optimization problem associated with sparse regression is:

$$\min_{\beta} ~ \| Y_{tr} - X_{tr}\beta \|_2^2 + \lambda \| \beta \|_1, \quad \lambda \geq 0$$  \hfill (2.10)

where $\lambda$ again is a regularization parameter. The choice of the parameter $\lambda$ for both Ridge and Sparse regression can be made by performing cross-validation studies on the training data set.

Zhang and Xu [36] compare both the regularized algorithms for applications to vibration based damage detection and show that sparse regularization performs better. Yang and Nagarajaiah [37] use sparse regression to solve the damage detection problem in a sparse representation framework. The key idea was to construct a dictionary of features for a variety of possible damage scenarios. A sparse vector is then used to point to a specific element of the dictionary which best defines a given test signal. Yang and Nagarajaiah [38] also use sparse regression in a novel sparse component analysis technique that performs blind identification using limited sensor data.
2.1.2 Unsupervised learning

This section discusses the various applications of unsupervised learning in damage detection. As discussed earlier, unsupervised learning does not use data labels, rather they are employed to deduce inherent patterns and structures useful for damage detection in the data. The most common unsupervised algorithms are principal component analysis (PCA), independent component analysis (ICA) and clustering algorithms. Clustering algorithms can range from \( k \)-means clustering to hierarchical clustering.

PCA is well equipped for performing two tasks, namely, data compression and data visualization. The notion of principal components is mathematically defined as the eigenvectors of the covariance matrix of a data set. Physically, they represent the directions of maximum variances. The data when projected on to these eigenvectors help observe patterns in data, which in particular aids the SHM community in damage detection. For a data matrix \( \mathbf{X} \in \mathbb{R}^{n \times p} \), the covariance matrix is defined as \( \mathbf{X}^T \mathbf{X} \). It can be shown that the eigenvectors of the covariance matrix are equivalent to the right singular vectors \( \mathbf{V} \) of \( \mathbf{X} \) that is defined as \( \mathbf{X} = \mathbf{U} \mathbf{D} \mathbf{V}^T \). The projection of the data in the eigenvector space is then defined by \( \hat{\mathbf{X}} = \mathbf{XV} = \mathbf{UD} \).

As discussed earlier, PCA is used in most damage detection studies for reducing the dimension of the data. This helps increase numerical efficiency of most algorithms. The idea is to reconstruct the data matrix using eigenvectors with significant singular values only. This is akin to data compression using singular value decomposition (SVD). Zang and Imregun [39] use PCA for frequency response function (FRF) data reduction for application to an Artificial Neural Network (ANN) based damage detection scheme. Yan et al. [40, 8] use PCA for for damage detection in bridges under environmental variations. They use a portion of the undamaged data for obtaining the principal component directions. The Tibaduiza et al. [41] includes a brief review of applications of PCA for SHM.
Similar to PCA, ICA too is a matrix factorization algorithm. The algorithm was first developed with regards to solving the blind source separation (BSS) problem. It is defined as the retrieval of a set of \( k \) independent signals of length \( n \), \( S \in \mathbb{R}^{k \times n} \) and a corresponding mixing matrix \( A \in \mathbb{R}^{k \times k} \) from a set of recorded signals \( X \in \mathbb{R}^{k \times n} \) that are assumed to be a mixture of the independent signals. Mathematically this can be expressed as:

\[
X = AS
\]  

(2.11)

The key distinction between PCA and ICA is that, PCA evaluates projection directions for maximum variance, whereas ICA evaluates most statistically independent vectors that can construct the given data set. Tibaduiza et al. [42] compares the performance of PCA with ICA for damage detection by active sensing in plates using high frequency waves. They report that ICA does not necessarily hold an advantage over PCA for their application. Similar to PCA, Zang et al. [43] use ICA for data dimension reduction for use in FRF damage classification using an ANN.

Another class of unsupervised learning algorithms are the clustering algorithms. Although they cannot perform data reduction like PCA and ICA, they are effective in recognizing different patterns in a data that might be useful for damage detection. Da Silva et al. [44] compare the performance of two fuzzy clustering algorithms for damage classification from vibration based data. Park et al. [45] use a \( k \)-means clustering algorithm for damage detection using electro-mechanical impedance. The clustering algorithm is used on principal component projections of the impedance data acquired from a experimental beam. Santos et al. [46] use a mean shift clustering algorithm for damage detection on the data from the Z-24 bridge. They also compare the performance of the proposed algorithm to that of \( k \)-means clustering, fuzzy \( c \)-means clustering and Gaussian Mixture Model (GMM) based techniques. Nair and Kiremidjian [47] demonstrate the use of GMMs for damage
detection using vibration data from a benchmark building.

2.2 Statistical learning algorithms

2.2.1 Principal component analysis

Principal component analysis (PCA) is an unsupervised statistical learning technique. It typically aids in dimensionality reduction and visualization. If \( X \in \mathbb{R}^{n \times p} \) is a data set (where each column is centered, i.e., each column has zero mean) such that, \( n \) is the number of observations and \( p \) is the number of features, the covariance matrix of the data is defined as \( X^T X \). The eigenvectors of the covariance matrix are the principal component directions, defined as,

\[
X^T X = \Phi \Lambda \Phi^T, \tag{2.12}
\]

where \( \Phi \in \mathbb{R}^{p \times p} \) is the matrix whose columns are the eigenvectors of the covariance matrix and \( \Lambda \in \mathbb{R}^{p \times p} \) is a diagonal matrix, whose diagonal elements are the corresponding eigenvalues. The projected version of the data in principal component space is

\[
\hat{X} = X\Phi. \tag{2.13}
\]

The above computation can also be performed using a Singular Value Decomposition (SVD) approach. The SVD of the data matrix is defined as:

\[
X = UDV^T, \tag{2.14}
\]

where \( U \in \mathbb{R}^{n \times n} \) and \( V \in \mathbb{R}^{p \times p} \) are unitary matrices and \( D \in \mathbb{R}^{n \times p} \) is a diagonal matrix whose diagonal elements are the corresponding singular values sorted in descending order. On applying this decomposition, the covariance matrix will be:

\[
X^T X = VD^T U^T UDV^T = VD^T DV^T. \tag{2.15}
\]
By comparing equations 2.12 and 2.15, one can conclude that the matrix consisting the right singular vectors as columns, $V$ is equivalent to the matrix of eigenvectors $\Phi$ of the covariance matrix, $X^T X$ and the singular values of $X$ are square roots of the eigenvalues of $X^T X$. This implies that the right singular vectors are the principal components and hence, the projected data on the principal component space is

$$\hat{X} = XV = UD^TV = UD.$$  \hfill (2.16)

Each column of the matrix $\hat{X}$ represents the projection of the data set $X$ on to each of the principal components. The projection of the data on the $i^{th}$ principal component can then be evaluated as $u_i \sigma_i$, where $u_i$ is the $i^{th}$ left singular vector ($i^{th}$ column of $U$) and $\sigma_i$ is the $i^{th}$ singular value.

In this dissertation, PCA is used for:

- Dimensionality reduction. This allows for reducing the number of variables involved with any data and aids in focusing on the low rank information of the data that are of interest to us.

- Data visualization. The data sets acquired in this dissertation are typically high-dimensional. For ease of visualization, two principal components are used as basis vector of a two dimensional plane, where the dataset is projected for ease of visualization.

### 2.2.2 Hierarchical clustering

Clustering techniques are another unsupervised statistical learning approach. As the name suggests these algorithms congregate similar data into clusters. Various distance measures define the similarity of data. The number of clusters is an input parameter for these algorithms. In chapter 6, for level I damage detection, this parameter is set to two, representing
either a damaged or an undamaged state. The most basic algorithm for clustering is the k-means clustering. However, it is inefficient with nonuniform datasets and when within cluster distributions are not spherical [18]. Hence, hierarchical clustering (HC) is used in this dissertation.

HC typically constructs a hierarchy of data clusters. HC uses a dissimilarity function for allocating data points to clusters. Euclidean distance and Mahalanobis distance, to name a few, are popular choices for a dissimilarity function. The complete linkage variant of HC uses a minimax formulation for constructing the clusters based on the choice of dissimilarity function (other variants being single and median linkage).

Let there be two clusters \(s_j\) and \(s'_j\). Then the dissimilarity function for a complete linkage and the ensuing optimization problem for HC is:

\[
d_{\text{complete}}(s_j, s'_j) = \max_{i \in s_j, i' \in s'_j} d(i, i')
\]

\[
\min_{s_j, s'_j} d_{\text{complete}}(s_j, s'_j)
\]

The above equation essentially implies in order to construct clusters, HC with complete linkage minimizes the maximum dissimilarity between two data points in the feature space for all clusters. In this dissertation, the Euclidean distance is used as a dissimilarity function. Given the number of required clusters, the algorithm reports the clusters based on the number of branches from the dendrogram associated with HC.

### 2.3 Stochastic Filtering: Bayesian Parameter Estimation

The previous section discussed statistical learning algorithms. This section provides a discussion on another class of algorithms employed in this dissertation, stochastic filtering algorithms, in particular Bayesian parameter estimation. In Bayesian estimation the pa-
Parameters to be estimated, $\Phi \in \mathbb{R}^n$, are treated as random variables, and their estimate is given by a probability density function (pdf) conditional on the available information. The estimated pdf (also known as the posterior) is denoted as $p(\Phi|Y)$ where $Y$ is the gathered data. In principle, the posterior is given by Bayes’ theorem

$$p(\Phi|Y) = \frac{p(Y|\Phi)p(\Phi)}{p(Y)}$$

(2.18)

where $p(\Phi)$ denotes the prior distribution, $p(Y|\Phi)$ as a function of $\Phi$ is the likelihood function, and $p(Y)$ is a normalizing constant. The prior distribution $p(\Phi)$ is used to incorporate information available about the parameters before the data is available. The choice of distribution is governed by the problem at hand. For instance, in the acoustic emissions source localization problem, described later in chapter 3, the location of the emissions source is unknown and it is desirable that the weight of the prior is reduced in the estimation as much as possible. For example, the use of a Gaussian distribution for the prior induces a strong bias around the mean, requiring an increased amount of information (data) to shift the prior towards the actual source location, in the case the mean is not in close proximity to the actual source location. For this reason in this thesis a non-informative prior is adopted such that its effect in the posterior is minimized, resulting in a Bayesian data-driven approach where the information contained in the data is maximized [48].

From the posterior distribution a point value can be selected (usually the mean or the mode). The mode of the posterior (or maximum a posteriori estimate) is

$$\Phi_{MAP} = \arg \max_{\Phi} p(\Phi|Y)$$

(2.19)

The marginal distributions of parameter $\Phi_i$ are also of interest, and given by

$$p(\Phi_i|Y) = \int p(\Phi|Y)d(\Phi \setminus \Phi_i)$$

(2.20)
where the integral is about a measure that excludes the subspace of \( \Phi_i \) (\( \Phi \setminus \Phi_i \) denotes a set where subspace \( \Phi_i \) is excluded from the set \( \Phi \)). To quantify the uncertainty in the estimates confidence intervals (CI) are used. Bayesian confidence intervals provide the probability that the true parameter lies in a region of the parameter space, given the observed data. For parameter \( \Phi_i \) the CI is defined as

\[
CI_i = [\phi_i^a, \phi_i^b]
\]

such that

\[
P(\Phi_i \in CI_i | Y) = \int_{\phi_i^b}^{\phi_i^a} p(\Phi_i | Y) d\Phi_i
\]

(2.21)

where \( P(\Phi_i \in CI_i | Y) \) is a target probability. In practice the 95\% mode-centered confidence interval is typically employed. To formulate the emission source estimation problem in the Bayesian probabilistic setting the residual error vector is defined as

\[
e(\Phi) = Y - \hat{Y}(\Phi) = Y - h(\Phi)
\]

(2.22)

where \( \hat{Y}(\Phi) \) is the estimate of the response (time of flight), obtained using the map \( h \) defined by Eq. 3.1. The residual vector is the estimation error resulting mainly from modeling errors and measurement noise. The residual is generally modeled as a white noise sequence, and when only the mean and covariance are available a Gaussian distribution yields the maximum entropy distribution \[49\]. Under such conditions the likelihood function is given by

\[
p(Y | \Phi) = \frac{1}{\sqrt{(2\pi)^n|R|}} e^{-\frac{1}{2}((Y - h(\Phi))^T R^{-1} (Y - h(\Phi)))}
\]

(2.23)

where \(|R|\) is the determinant of \( R \), and the posterior becomes

\[
p(\Phi | Y) = p(\Phi) \frac{1}{p(Y) \sqrt{(2\pi)^n|R|}} e^{-\frac{1}{2}((Y - h(\Phi))^T R^{-1} (Y - h(\Phi)))}
\]

(2.24)

Although straightforward in principle, the estimation poses significant implementation, numerical and practical difficulties, mainly because in general the posterior PDF \( p(\Phi | Y) \) cannot be computed in closed-form, especially when nonlinear models are used \[48\]. For
instance, the computation of the normalization factor $p(Y)$ involves a multidimensional integral over the parameter space, which in general cannot be computed in closed-form. Moreover, to obtain the marginal distributions of the parameters similar integrals over subspaces of the parameter space are required.

Under special conditions, such as when the number of measurements or data size is large, asymptotic (Laplace) approximations may be employed. The asymptotic posterior approaches a Gaussian distribution in the limit of increasing data, a result known as Bernstein-Von Mises theorem \[48\]. A popular alternative that has gained attention in recent time is a family of methods known as Markov chain Monte Carlo (MCMC) simulation, where a sample of the posterior is obtained by constructing a Markov chain with certain asymptotic properties \[50\]. Specifically, a Markov chain that converges to the posterior as the limit of samples increases is constructed to explore the parameter space. The points in the chain are then treated as samples from the posterior, used to compute sampling estimates of the parameters of interest. MCMC allows to overcome the issues related to computing the integrals, at the expense of a high computational cost.

Several theoretical and practical difficulties of the previously discussed approaches can be pointed out. Although in many applications the posterior distribution tends to be unimodal, the limited amount of data available in practice makes the use of an asymptotic approximation suspicious. In particular the posterior distribution tends to have heavy tails that cannot be characterized by the Gaussian distribution obtained from the asymptotic approximation. Typically the solution to this issue is the use of MCMC simulation. The main drawback of MCMC simulation is the computational effort required by the algorithms \[48\]. In structural health monitoring applications an efficient approach where the computational cost is minimum is needed in order to evaluate the state of damage of engineering systems in a timely manner. To overcome these difficulties the use of a stochastic simulation algo-
rithms is adopted typically. Specifically the ensemble Kalman filter and the particle filter are adopted to approximate the posterior distribution \( p(\Phi|Y) \). In the case of the particle filter the posterior is approximated by a set of weighted samples that define a probability mass function. The advantage of this non-parametric estimate is that no assumptions regarding the functional form of the prior (e.g., the typically used Gaussian distribution) or the posterior are made. The particle filter and ensemble Kalman filter are discussed in the following section.

### 2.3.1 Particle Filter

The particle filter (PF) is a Bayesian filtering algorithm used to obtain a discrete approximation to the joint probability distribution of a state-space model conditioned on response measurements [51]. The PF is suitable for applications where a Gaussian approximation cannot represent the conditional distribution accurately resulting in errors in the estimates and/or divergence of the filter; this can potentially arise when the distributions involved are multi-modal, skewed, or have heavy tails.

The objective is to compute the posterior PDF of the state, \( p(\theta_k|Y_k) \), as the data is gathered. The posterior is given by

\[
p(\theta_k|Y_k) = \frac{p(y_k|\theta_k)p(\theta_k|Y_{k-1})}{p(y_k|Y_{k-1})}
\]  
(2.25)

where \( p(y_k|\theta_k) \) is obtained from a measurements model, with an additive measurement noise model. In the particle filter \( p(\theta_k|Y_k) \) is approximated by a discrete set of samples \( \{\theta^i_k\} \) as

\[
\hat{p}(\theta_k|Y_k) = \frac{1}{N} \sum_{i=1}^{N} \delta_{\theta^i_k}(\theta_k)
\]  
(2.26)

where \( \delta_{\theta_0}(\theta) \) is the Dirac measure at \( \theta_0 \). Statistical moments of \( p(\theta_k|Y_k) \) are then readily estimated from the sample. Since \( p(\theta_k|Y_k) \) is unknown (and thus cannot be directly
sampled), an importance sampling strategy is typically employed.

In order to arrive to a recursive implementation (i.e., a method where samples from previous steps are used in the current step) an alternative formulation is useful. Let $\Theta_k$ denote the set of states $\Theta_k = \{\theta_0, \theta_1, \ldots, \theta_k\}$, then from Bayes theorem and using the structure of the output and dynamic models, it follows that [52]

$$p(\Theta_k | Y_k) = \frac{p(\theta_0)}{p(Y_k)} \prod_{i=1}^{k} p(y_i | \theta_i) p(\theta_i | \theta_{i-1})$$

(2.27)

To obtain features of the joint posterior distribution in Eq. 2.27, such as marginal distributions, high-dimensional integrals need to be evaluated. Moreover, the distribution of the data $p(Y_k)$ typically cannot be computed analytically. An alternative is to recur to stochastic simulation, where samples are used to represent and evolve the probability distributions on time [51].

![Particle filter](image)

Figure 2.1: Particle filter. A set of weighted samples is used to approximate a conditional probability density function by a probability mass function.

The traditional approach used to obtain a recursive algorithm is based in importance sampling (IS), originally a Monte-Carlo integration method. With IS expectations of func-
tions of the state given by

\[ \mathbb{E}[g(\Theta_k)] = \int g(\Theta_k) p(\Theta_k | Y_k) d\Theta_k \]  

(2.28)

are estimated as

\[ \hat{\mathbb{E}}[g(\Theta_k)] = \frac{\sum_{i=1}^{N} g(\Theta_k^{(i)}) \tilde{w}_n^{(i)}}{\sum_{j=1}^{N} w_n^{(j)}} \]  

(2.29)

\[ \tilde{w}_n^{(i)} = \frac{w_n^{(i)}}{\sum_{j=1}^{N} w_n^{(j)}} \quad w_n^{(i)} = \frac{p(Y_k | \Theta_k^{(i)}) p(\Theta_k^{(i)})}{\pi(\Theta_k^{(i)} | Y_k)} \]  

(2.30)

where \( \{\Theta_k^{(i)}\}_{i=1}^{N} \) is a set of independent samples drawn form the importance distribution \( \pi(\cdot) \). In this formulation the distribution of the data \( p(Y_k) \) is also estimated using \( \pi(\cdot) \), so that it does not have to be computed analytically. If the support of \( \pi(\cdot) \) includes the support of \( p(\Theta_k | Y_k) \) and \( \mathbb{E}[g(\Theta_k)] \) is finite, the estimator \( \hat{\mathbb{E}}[g(\Theta_k)] \) converges almost surely to \( \mathbb{E}[g(\Theta_k)] \), and the rate of convergence depends on the choice of \( \pi(\cdot) \) [51]. When a recursive algorithm is desired, the typical choice is a function of the form [52]

\[ \pi(\Theta_k | Y_k) = \pi(\theta_0) \prod_{i=0}^{k} \pi(\theta_i | \Theta_{i-1}, Y_i) \]  

(2.31)

and a convenient choice for the marginal proposals is the transition PDF of the model \( \pi(x_i | \Theta_{i-1}, Y_i) = p(\theta_i | \theta_{i-1}) \); it follows that the weights can be calculated recursively as

\[ w_k^{(i)} = w_{k-1}^{(i)} p(y_k | \theta_k^{(i)}) \]  

(2.32)

Using the updated weights the probability mass function approximating the posterior is given by

\[ \hat{p}(\theta_k | Y_k) = \sum_{i=1}^{N} w_k^{(i)} \delta_{\theta_k^{(i)}}(\theta_k) \]  

(2.33)

This approach is illustrated in Figure 2.1, which shows the updating of a prior distribution using the above discussed approach.
2.3.2 Ensemble Kalman Filter

The Ensemble Kalman filter (EnKF) is a Monte-Carlo estimation based method used to obtain a Gaussian approximation of the posterior distribution using the Kalman filter equations [53]. The EnKF uses the conditional mean estimate as the optimal (Bayes) estimate of the state, and the estimate of the covariance matrix as a measure of uncertainty.

Let \( p(\theta_k | Y_k) \) denote the estimate of the parameters based on step \( t = t_{k-1} \), with mean and covariance \( \hat{\theta}_k \) and \( \hat{\theta}_{\theta_k} \) respectively. To estimate the statistics of propagating this distribution and the measurement prediction, an ensemble of vectors \( \{\hat{\theta}^i_k\}_{i=1}^N \) obtained by random sampling is used to compute the following Monte-Carlo based estimates

\[
\hat{\theta}_{k+1}^- = \frac{1}{N} \sum_{i=1}^N g(\hat{\theta}^i_k) \tag{2.34}
\]

\[
\hat{P}_{\theta_{k+1} \theta_{k+1}}^- = \frac{1}{N-1} \sum_{i=1}^N (g(\hat{\theta}^i_k) - \hat{\theta}_{k+1}^-) (g(\hat{\theta}^i_k) - \hat{\theta}_{k+1}^-)^T \tag{2.35}
\]

\[
\hat{y}_{k+1}^- = \frac{1}{N} \sum_{i=1}^N h(g(\hat{\theta}^i_k)) \tag{2.36}
\]

\[
\hat{P}_{y_{k+1}y_{k+1}}^- = \frac{1}{N-1} \sum_{i=1}^N (h(g(\hat{\theta}^i_k)) - \hat{y}_{k+1}^-) (h(g(\hat{\theta}^i_k)) - \hat{y}_{k+1}^-)^T + R \tag{2.37}
\]

\[
\hat{P}_{\theta_{k+1}y_{k+1}}^- = \frac{1}{N-1} \sum_{i=1}^N (g(\hat{\theta}^i_k) - \hat{\theta}_{k+1}^-) (h(g(\hat{\theta}^i_k)) - \hat{y}_{k+1}^-)^T \tag{2.38}
\]

where the function \( h \) is given by Eq. 1 and \( g \) is the identity map. The posterior ensemble is then obtained using the Kalman filtering equations

\[
\hat{\theta}^i_{k+1} = g(\hat{\theta}^i_k) + \hat{P}_{\theta_{k+1}y_{k+1}}^- \left( \hat{P}^-_{y_{k+1}y_{k+1}} \right)^{-1} [y_{k+1} - h(g(\hat{\theta}^i_k))] \tag{2.39}
\]

and the posterior is approximated by a Gaussian distribution with mean and covariance given by

\[
\hat{\theta}_{k+1} = \frac{1}{N} \sum_{i=1}^N \hat{\theta}^i_{k+1} \quad \hat{P}_{\theta_{k+1} \theta_{k+1}} = \frac{1}{N-1} \sum_{i=1}^N \left( \hat{\theta}^i_{k+1} - \hat{\theta}_{k+1} \right) \left( \hat{\theta}^i_{k+1} - \hat{\theta}_{k+1} \right)^T \tag{2.40}
\]
Chapter 3

Particle Filter-based Acoustic Emission Source Localization

In this chapter, a stochastic filtering-based algorithm is formulated to solve a passive damage detection problem in plates that involves the use of acoustic emissions (AE). Failure of structural and mechanical systems have promoted the development of structural health monitoring (SHM) in different applications. The main tasks of SHM can be broadly classified as damage diagnosis (detection, localization and quantification) and damage prognosis (damage prediction and estimation of remaining useful life). Traditional vibration-based SHM algorithms rely on the use of dynamic response measurements to detect changes in damage-sensitive features, usually stiffness, vibration frequencies, mode shapes, among others \[54, 55\]. Recently the use of ultrasonic guided waves (UGW) measurements has been a popular approach for SHM in aerospace and structural engineering applications \[56, 57, 58, 59, 60\]. This generally entails one of the following schemes: i) active monitoring using artificial and external high frequency excitations; ii) passive monitoring by sensing high frequency waves due to ambient processes affecting the system at hand. This chapter focuses on passive approaches using UGW, bypassing the need to excite the system of interest.
3.1 Acoustic Emissions Source Localization

When a structure is impacted by foreign bodies or crack propagation commences, high-frequency waves are generated that propagate through the system. The high frequency nature of these waves gives them the name acoustic emissions (AE). The acoustic emission source localization problem involves acquiring such vibration signatures to localize their source. To acquire such signals multiple sensors are typically deployed on systems. The time-of-flight (TOF) of each wave signal is then computed for performing triangulation, to estimate the location (in a Cartesian coordinate system) of the source. This is very similar to approaches used by geologists in estimating the location of the epicenter of an earthquake. If the wave propagation velocity is known a priori, the distance from the source to a receiver can be estimated using the TOF data. Although several approaches for source location estimation are available in the literature, the TOF-based triangulation scheme still remains the most popular choice owing to both its simplicity and accuracy.

In a deterministic setting the geometric and material properties of the system, and the TOF measurement are assumed to be perfectly known. The group velocity of acoustic waves ($V_g$) is obtained from dispersion curves of Lamb waves (the thickness of the plate is assumed to be small enough such that it is amenable for generation of Lamb Waves). If the TOF from source to sensor $i$ is $t_i$, the following relationship is used to relate the source location to the TOF

$$t_i = \frac{1}{V_g} \sqrt{(x_i - x_s)^2 + (y_i - y_s)^2} \quad i = 1, 2, \ldots, n$$

(3.1)

where the coordinates $(x_i, y_i)$ and $(x_s, y_s)$ are, respectively, the Cartesian coordinates of sensor $i$ and the acoustic emission source. The total number of available sensors is $n$. The geometric interpretation of this approach is to simply draw circles of radius $r_i = V_g t_i$.
around each sensor, and to find the point of intersection of these circles. This is shown schematically in Figure 3.1.

A drawback of the deterministic formulation is that it does not account for the uncertainties associated with this problem, such as material properties of the plate, the frequency content of the load applied and the noise in measurements. All these uncertainties collude and lead to a situation shown in Figure 3.1. The bands of color show the uncertainty that propagates to the estimated acoustic source location. In the Bayesian probabilistic approach discussed in a following section, the AE source location and the wave propagation velocity are treated as random variables described by a probability distribution that incorporates the uncertainties involved.

Although recent research in the subject has considered some of the sources of uncertainty, the inverse problem of finding the source location with only three sensors (which is the minimum number of sensors required for triangulation on a 2-D plane) has proven to be a challenging task [61]. Clearly, a large number of sensors will effectively reduce the effects of uncertainties on the performance of any algorithm. However, it is of interest to be able to restrict the number of sensors to a minimum. This is important to reduce economic costs of the monitoring system, and also because the number and location of the sensors might be restricted by the physical configuration of the system.

### 3.2 Prior work on AE source localization

The objective of passive approaches using UGW is to be able to sense elastic waves propagating in the structure and localize the source of the elastic wave. As discussed earlier, these are known as acoustic emissions and are generated when a material undergoes changes in its internal structure, such as crack formation and propagation. AE have been employed for damage diagnosis in systems ranging from civil structures to mechanical components
Figure 3.1: Source estimation from triangulation. The yellow disks denote the sensors, the red patch is the source (in this case structural damage in the form of a crack), the gray circles are the AE waves propagating outward from the damage, the black circles along with the blurred colors are the circles with radius obtained from TOF data along with the associated uncertainties.

and tools [62, 47, 63, 64]. The AE research community has actively developed efficient schemes for performing AE source localization; a brief review can be found in Kundu [65].

In the context of SHM applications ultrasonic waves have been employed for identifying structural damage using time-of-flight (TOF) and attenuated amplitude [66, 67, 68, 69]. The time-of-flight is defined as the time lag from the moment when a sensor catches the damage-reflected signal to the moment when the same sensor catches the incident signal [70]. Various approaches are used in the literature to calculate the TOF, including thresholding procedures or envelope algorithms based on a Hilbert or Wavelet transform [67]. Algorithms developed for TOF-based source localization range in complexity from
triangulation to application of numerical schemes, such as genetic algorithms and neural networks [71, 72]. Akaike Information Criterion (AIC) based arrival time estimation is another alternative [73].

Although these algorithms have their own advantages and limitations, a common inherent issue is the effect of uncertainties in the results obtained by the different approaches, which may result in overconfidence in the estimates and lack of robustness [61]. Sources of uncertainty include variability in mechanical properties of materials, measurement noise, and geometry of the system of interest. The uncertainties involved can be incorporated in the analysis by adopting a probabilistic model. In particular, Bayesian inference can be adopted as a consistent and rigorous theory that can be applied to a large class of estimation problems [48, 49]. In Bayesian inference the unknown quantities are treated as random variables, and their estimate is given by a probability density function (pdf) conditional on the available information known as the posterior distribution, obtained in principle using Bayes’ theorem. When using complex nonlinear models, the functions needed to evaluate the posterior are difficult to compute in closed-form, and thus numerical approximations are sought.

The application of Bayesian inference to the AE source localization problem has gained significant attention in recent years, mainly due to its capability to quantify the uncertainty in the estimates, accuracy and robustness to modeling errors [74, 75]. In this chapter particle-based stochastic filters are applied to estimate acoustic emission sources in plate type structures in a Bayesian framework; stochastic filters are typically used in online state-estimation and damage estimation, and in particular particle-based methods are useful when the models involved are highly nonlinear [76, 52, 15, 77]. The use of particle-based filters results in an increased accuracy in the estimates of the parameters and their uncertainty due to their ability to accurately characterize the posterior distribution. In this setting a set of
particles is used to approximate the posterior and to compute its statistics using sampling estimates. The use of two particle-based filters is explored: the ensemble Kalman filter and the particle filter. The ensemble Kalman filter results from the application of the Kalman filter equations, with the projection of the mean and the covariance achieved by Monte Carlo estimation; the posterior is approximated by a Gaussian distribution. On the other hand, the particle filter results from formal use of Monte Carlo integration to estimate the posterior directly, making no assumptions about its functional form, and thus resulting in a non-parametric estimate of the posterior [51, 52]. An advantage of the particle filter is that it imposes no restriction on the functional form in the prior distribution, allowing to use any probability density function. Of particular interest is to use a non-informative prior in order to maximize the weight of the data (Bayesian data-driven) [48].

The Bayesian approach for AE source localization is experimentally validated in a laboratory environment using an aluminum plate. A pencil lead break on the plate surface is used to simulate an acoustic emission source (also known as the Hsu-Nielsen source [78]). It is shown that the particle-based filtering approach has the capability to locate the source accurately, with the minimum number of sensors necessary for triangulation, providing confidence intervals that describe the uncertainty in the estimated source location.

### 3.3 Wavelet Transform based Time of Flight Estimation

To estimate the location of AE sources wave travel time measurements are necessary. In this work a continuous wavelet transform (CWT) is performed on the signals to isolate the primary frequency and estimate the time of arrival of that content. The use of CWT is justified by its efficacy in time frequency analysis of signals. Although other time-frequency decomposition methods like short time Fourier transform (STFT) or Hilbert-Huang transform (HT) may suffice for TOF estimation, the frequency content information is also criti-
cal in the proposed algorithm. The algorithm not only takes into account the uncertainties in the AE source location, but also the uncertainty involved in the group velocity.

An initial estimate of the group wave velocity is obtained using the frequency content of the signal and the dispersion curves. The specific wavelet used for this task is the complex Morlet wavelet, which is defined by

\[ f(x) = \frac{1}{\sqrt{\pi f_b}} \exp(i2\pi f_c x - x^2 / f_b) \]  

(3.2)

where, \( f_b \) and \( f_c \) are the frequency bandwidth and the central frequency respectively. A typical complex Morlet Wavelet is shown in Figure 3.2. Figure 3.3 shows a typical response signal acquired from the experimental setup (details of the setup are discussed later). The CWT of the signal is shown in Figure 3.4. As it can be seen, wavelet level 43 contains the majority of the frequency content of the signal. The arrival of the peak frequency at level 43 is hence treated as the TOF. The initial estimate obtained by this approach will be employed to define a prior distribution for Bayesian estimation. This is further discussed in the following section.

### 3.4 Stochastic Filtering Acoustic Emission Source Identification Algorithms

Based on the discussion in chapter 2, the problem is formulated as a stochastic filtering problem. To estimate the source location using a particle filter a state-space model needs to be defined. This state-space model constitutes only time-invariant parameters, namely the Cartesian coordinates of the source and the group wave velocity. Hence, defining the parameter state as \( \theta = \{x_S, y_S, V_g\} \), a time-invariant state-model is given by

\[ \theta_{k+1} = \theta_k \]  

(3.3)
Figure 3.2: Absolute value, Real and Imaginary parts of the complex Morlet wavelet with $f_b = 1$ and $f_c = 1$

Figure 3.3: A typical set of signals acquired from AE experiments on a square aluminum plate (edge length 91 cm) from three sensors. The experimental setup will be described later in the chapter.

The algorithms resulting from the application of the particle filter and ensemble Kalman filter to the AE source identification problem are summarized next.
Figure 3.4: CWT of signal from Sensor 1 in Figure 3.3. The dominant frequency level is marked by the horizontal red dashed line. The TOF can be estimated by the arrival time of this dominant frequency as shown by the vertical red dashed line.

### 3.4.1 Particle filter

The algorithm resulting from employing the particle filter to estimate the acoustic emissions source location is given as follows:

- Acquire AE signals using available sensors
- Apply CWT to the acquired signals and compute the (noise contaminated) TOF data $y = \{t_i\}$
- Based on the available information select the parameters prior distribution $p(\Phi)$
- Select the number of samples $N$ and set the initial weights to $w_0^{(i)} = 1/N$
- Obtain $N$ random samples $\phi_i$ from the prior distribution

For $i = 1$ to $N$
• For each sample φᵢ estimate the time of flights {\(\hat{t}_i\)} using Eq. 3.1

• Using a Gaussian noise model the updated weights (Eq. 2.32) are given by
\[
  w_{1}^{(i)} = w_{0}^{(i)} N(y, \hat{t}_i, r)
\]

\(r\) is the measurement noise standard deviation

\(N(z, \mu, \sigma)\) is a Gaussian distribution evaluated at \(z\), with mean \(\mu\) and standard deviation \(\sigma\)

A flowchart describing the proposed algorithm is shown in Figure 3.5.

![Flowchart showing the proposed Particle Filtering based source identification algorithm](image)

Figure 3.5: Flowchart showing the proposed Particle Filtering based source identification algorithm

### 3.4.2 Ensemble Kalman filter

The algorithm resulting from employing the ensemble Kalman filter to estimate the acoustic emissions source location is given as follows:
• Acquire AE signals using available sensors

• Apply CWT to the acquired signals and compute the (noise contaminated) TOF data $y = \{t_i\}$

• Based on the available information select the parameters prior distribution $p(\Phi)$, which is approximated by a Gaussian distribution

• Select the number of samples $N$

• Obtain $N$ random samples $\phi_i$ from the prior distribution

• For each sample $\phi_i$ estimate the time of flights $\{\hat{t}_i\}$ using Eq. 3.1

• Compute the projected mean, auto-covariance and cross-covariance using Eqs. 20-24

• Estimate the mean and covariance of the posterior distribution using Eq. 26

3.5 Experimental Validation

In this section the validation of the use of particle-based stochastic filters for AE source localization in a laboratory environment is presented. For this purpose an aluminum plate was subjected to loads produced by pencil lead breaks at various locations. The pencil lead break, popularly known as the Hsu-Nielsen source, simulates waves produced typically by AE sources [78]. The experimental setup consisted of a 3.51 cm thick aluminum square plate of side 91 cm. Three piezo-transducers were used to record the signals of the traveling waves (this is the minimum number of sensors necessary for triangulation). As discussed earlier, the inverse problem of source localization using three sensors only, with parameter uncertainties incorporated has proven to be a challenging task [61]. The piezo-transducers used are 10mm diameter and 3mm thick lead zirconate titanate (PZT) sensors
manufactured by StemInc Piezo, with a dominant R mode vibration at a resonant frequency of 215kHz. The sensors were attached on the plate with 3M transparent double sided tape. The electrical signals generated at the PZT transducers are amplified and acquired through a TDS 2024C oscilloscope at a sampling rate of 10 MHz.

Figure 3.6: Experimental setup showing the location of the sensors and the location of the AE sources.

The experimental setup including the sensor locations are depicted in Figure 3.6. The
locations of the three sensors used are summarized in Table 3.1. The green crosses on the plate, along with the corresponding numbers, are the different simulated AE sources used for the experiment. The locations of all the possible AE sources are listed in Table 3.5. Six different locations of the AE source was considered. The vector of parameters to be estimated is $\theta = \{x_S, y_S, V_g\}$. For the particle filter application the prior distribution of the source location was selected as a uniform (flat) two dimensional distribution covering the plate. The use of this non-informative distribution is one of the main features of this approach, since it allows the data to control the posterior. The Gaussian prior used by Kalman filter based methods, on the other hand, impose a significant bias when the amount of data is limited.

<table>
<thead>
<tr>
<th>Sensor</th>
<th>X (m)</th>
<th>Y (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.265</td>
<td>0.46</td>
</tr>
<tr>
<td>2</td>
<td>0.53</td>
<td>0.305</td>
</tr>
<tr>
<td>3</td>
<td>0.61</td>
<td>0.53</td>
</tr>
</tbody>
</table>

Table 3.1 : Locations of PZT sensors on the plate

To highlight this feature the use of the particle filter will be compared to an unscented Kalman filter (UKF)-based algorithm, where Gaussian prior and posteriors for the AE source coordinates are used [61]. This is the basis for dividing the plate into two different regions as shown in Figure 3.6. The region inside the red rectangle is where the actual AE source is in the vicinity of the prior mean of the source coordinates. Hence, inside this region, the UKF and EnKF based algorithms might still perform satisfactorily as the likelihood distribution will be high due to a large pre-assigned weight, owing to the shape of a Gaussian distribution. For points further away the results will be affected as the information contained in the minimum number of sensors used (three sensors) is not enough to
Table 3.2: AE source locations on the experimental plate along with the corresponding labels

<table>
<thead>
<tr>
<th>AE Source</th>
<th>X (m)</th>
<th>Y (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.46</td>
<td>0.445</td>
</tr>
<tr>
<td>2</td>
<td>0.395</td>
<td>0.46</td>
</tr>
<tr>
<td>3</td>
<td>0.39</td>
<td>0.39</td>
</tr>
<tr>
<td>4</td>
<td>0.47</td>
<td>0.39</td>
</tr>
<tr>
<td>5</td>
<td>0.47</td>
<td>0.235</td>
</tr>
<tr>
<td>6</td>
<td>0.18</td>
<td>0.455</td>
</tr>
</tbody>
</table>

overcome the strong bias in the prior. It should be pointed out that effect of the prior bias is alleviated by increasing the number of sensors, however, for practical considerations it is desired to keep the number of sensors at a minimum. The estimation accuracy of the stochastic filters has been assessed in validation studies where it has been shown that under limited instrumentation and increased nonlinearity the PF outperforms the UKF and EnKF [15, 79].

Due to the uncertainty in the group velocity, the frequency content of signals acquired at each sensor was used in conjunction with available dispersion curves for Lamb waves for selecting the mean of the prior for the group velocity. Due to the existing dispersion curves, and lower uncertainties associated with material properties, a Gaussian prior is effective in capturing the uncertainties involved in group velocities.

In the particle filtering based approach, the prior distributions for the source coordinates are uniformly sampled. The prior distribution for the group wave velocity was selected as a Gaussian distribution with mean $\mu V_g = 2600 m/s$ and a coefficient of variation of 10%. The joint distributions are then updated using weights that reflect how well the samples
Figure 3.7: Prior and Posterior distributions for the coordinates. (a) Uniform PDF used as prior. (b) Posterior PDF sample; the size of each point is representative of the associated weight assigned to it by the PF.
reproduce the measurements. In this example 30,000 random samples of the prior were used. The number of samples was selected based on a convergence analysis increasing the number of samples until no significant variation in the estimates was observed. In terms of computational cost the approach is very efficient, taking only around 5 seconds to run the algorithm on a desktop computer. As mentioned before, the efficiency of stochastic simulation algorithms is of importance for practical applications, such as structural health monitoring applications where a decision about the state of damage of an engineering system needs to be taken in a short period of time after a potentially damaging event.

The prior distribution for the coordinates are depicted in Figure 3.7(a). Figure 3.7(b) shows a typical randomly drawn sample for the coordinates using the PF algorithm, which was used to construct the posterior probability density function, with the diameter of each point representative of the associated weight assigned by the algorithm. Figure 3.8 shows the posterior distributions obtained for two different locations, 1 and 5 as described in Figure 3.6. As can be seen in Figures 3.8(a) and(c), the estimates obtained are in agreement with the actual source locations. Moreover, the distribution is highly peaked, and thus the uncertainty in the estimates is small. The posterior distribution of the group wave velocity is also depicted in this figure. Figure 3.8(b) and Figure 3.8(d) shows the prior and the posterior distribution of the wave group velocity.

The estimation results are summarized in Table 3.3, where the 95% mean-centered confidence intervals (C.I.) are shown as a quantitative measure of the uncertainty in the parameters.

The performance of the algorithms was compared to the previously proposed UKF-based scheme, with the prior mean located at the center of the plate [61]. Figure 3.9 compares the results from the UKF, EnKF and PF based approaches. As can be seen the performance of the UKF and EnKF based approaches decrease as the source location moves
Figure 3.8: Posterior Distributions of the coordinates and the group velocity obtained by the proposed algorithm. Figures (a) and (b) are for location 1 and (c) and (d) are for location 5.
<table>
<thead>
<tr>
<th>AE Source</th>
<th>Actual</th>
<th>Estimate</th>
<th>95% C.I.</th>
<th>95% C.I.</th>
<th>Estimate</th>
<th>95% C.I.</th>
</tr>
</thead>
<tbody>
<tr>
<td>x-coordinate</td>
<td>1</td>
<td>0.46</td>
<td>0.47</td>
<td>[0.40, 0.54]</td>
<td>0.48</td>
<td>[0.43, 0.52]</td>
</tr>
<tr>
<td>y-coordinate</td>
<td>1</td>
<td>0.445</td>
<td>0.44</td>
<td>[0.36, 0.53]</td>
<td>0.46</td>
<td>[0.41, 0.52]</td>
</tr>
<tr>
<td>x-coordinate</td>
<td>2</td>
<td>0.395</td>
<td>0.40</td>
<td>[0.34, 0.46]</td>
<td>0.42</td>
<td>[0.38, 0.47]</td>
</tr>
<tr>
<td>y-coordinate</td>
<td>2</td>
<td>0.46</td>
<td>0.46</td>
<td>[0.36, 0.57]</td>
<td>0.49</td>
<td>[0.43, 0.53]</td>
</tr>
<tr>
<td>x-coordinate</td>
<td>3</td>
<td>0.39</td>
<td>0.37</td>
<td>[0.30, 0.44]</td>
<td>0.40</td>
<td>[0.36, 0.44]</td>
</tr>
<tr>
<td>y-coordinate</td>
<td>3</td>
<td>0.39</td>
<td>0.41</td>
<td>[0.29, 0.53]</td>
<td>0.47</td>
<td>[0.41, 0.53]</td>
</tr>
<tr>
<td>x-coordinate</td>
<td>4</td>
<td>0.47</td>
<td>0.47</td>
<td>[0.39, 0.54]</td>
<td>0.47</td>
<td>[0.43, 0.52]</td>
</tr>
<tr>
<td>y-coordinate</td>
<td>4</td>
<td>0.39</td>
<td>0.38</td>
<td>[0.28, 0.47]</td>
<td>0.42</td>
<td>[0.37, 0.48]</td>
</tr>
<tr>
<td>x-coordinate</td>
<td>5</td>
<td>0.47</td>
<td>0.47</td>
<td>[0.40, 0.55]</td>
<td>0.46</td>
<td>[0.42, 0.51]</td>
</tr>
<tr>
<td>y-coordinate</td>
<td>5</td>
<td>0.235</td>
<td>0.27</td>
<td>[0.18, 0.36]</td>
<td>0.38</td>
<td>[0.33, 0.44]</td>
</tr>
<tr>
<td>x-coordinate</td>
<td>6</td>
<td>0.18</td>
<td>0.22</td>
<td>[0.13, 0.30]</td>
<td>0.30</td>
<td>[0.26, 0.35]</td>
</tr>
<tr>
<td>y-coordinate</td>
<td>6</td>
<td>0.455</td>
<td>0.49</td>
<td>[0.38, 0.60]</td>
<td>0.56</td>
<td>[0.51, 0.62]</td>
</tr>
</tbody>
</table>

Table 3.3: AE source estimates statistics summary.
away from the center of the plate. Since, similar to the UKF, the EnKF assumes Gaussian priors and posteriors the result from these filters is close, and for this reason no distinction can be seen in the figure. In order to assess the results quantitatively the following relative \( \ell_2 \) error norm was defined as an error metric

\[
PM = \frac{\|\hat{x} - x_s\|^2}{\|x_s\|^2}
\]

(3.4)

where \( \hat{x} \) is the estimated AE source \((\hat{x}_S, \hat{y}_S)\), \( X_S \) is the actual AE source coordinates \((x_S, y_S)\), and \( \|\cdot\|_2 \) denotes the \( \ell_2 \) norm. The error metric results are presented in Table 3.4. As can be seen the particle filter shows an increased accuracy with respect to the other filters. This is chiefly due to the use of a flat prior distribution, which shifts the sensitivity of the posterior to the likelihood, and thus to the measured data. The Gaussian prior assumed by the UKF and EnKF results in a strong bias which requires an increased number of sensors to obtain more accurate results. Indeed, as the amount of information increases the estimates provided by all Bayesian methods converge to the same value given by the asymptotic approximation [48]. In this chapter applications with minimum instrumentation are of interest, and thus the number of sensors was kept at the minimum of three needed for triangulation.

### 3.6 Conclusion

The application of particle-based stochastic filters to the acoustic emissions source localization problem was presented. The approach relies on a wavelet-based time-of-flight measurements triangulation model, combined with particle-based stochastic filters to estimate the location of the acoustic emissions source. The estimate of the source location is given by a conditional probability density function known as the posterior distribution which embodies all the information about the parameters contained in the data. The application
The approach incorporates in the analysis inherent uncertainties resulting from measurement noise, geometry of the system and modeling errors. The ensemble Kalman filter approximates the posterior by a Gaussian distribution, while the particle filter provides a non-parametric estimate as a weighted set of samples. Moreover, the particle filter allows to employ any prior distribution for the estimation; in particular a non-informative prior was adopted, eliminating the bias in the source location when there is no additional information about the source location. Minimizing the effect of the prior results in a Bayesian data-driven approach where the weight of the data is maximized.

The approach was experimentally validated in a laboratory environment. For this purpose an aluminum plate was subjected to pencil lead break at different locations. It was shown that the algorithms have the capability to estimate the acoustic emission source accurately, with the minimum number of sensors needed for triangulation (three sensors),
<table>
<thead>
<tr>
<th>AE Source</th>
<th>PM for PF</th>
<th>PM for UKF</th>
<th>PM for EnKF</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0207</td>
<td>0.0390</td>
<td>0.0397</td>
</tr>
<tr>
<td>2</td>
<td>0.0115</td>
<td>0.0679</td>
<td>0.0650</td>
</tr>
<tr>
<td>3</td>
<td>0.0508</td>
<td>0.1458</td>
<td>0.1456</td>
</tr>
<tr>
<td>4</td>
<td>0.0218</td>
<td>0.0569</td>
<td>0.0612</td>
</tr>
<tr>
<td>5</td>
<td>0.0709</td>
<td>0.2705</td>
<td>0.2854</td>
</tr>
<tr>
<td>6</td>
<td>0.1152</td>
<td>0.3404</td>
<td>0.3324</td>
</tr>
</tbody>
</table>

Table 3.4: Error metric $PM$ comparison. The metric $PM$ is the $\ell_2$ norm of the estimated parameters error vector.

with the particle filter providing the most accurate results. In addition to point estimates, confidence intervals were used to describe the uncertainty in the source location estimate. For all the cases considered, the actual source location was close to the posterior distribution mode, and based on the tight confidence intervals obtained the uncertainty in the estimates was small. The approach also proved to be efficient in terms of the computational cost, with the algorithms requiring only a few seconds to run in a desktop computer. This is desirable in structural health monitoring applications, where an estimate of the state of damage of a system needs to be provided in a timely manner after a potentially damaging event takes place.
Chapter 4

Sparse damage detection in beams

In this chapter, the concept of harnessing the inherent sparsity associated with active-sensing-based damage detection in beams using Lamb waves is introduced. To accomplish this, the damage detection problem is formulated in a sparse representation framework. Mathematically, this framework is represented by a set of linear equations. Sparse regression is employed to obtain a solution that aids in damage detection and localization

4.1 Introduction

The increase in popularity of the use of guided waves for damage detection owes to their sensitivity to small damages over a considerable distance [80]. Guided waves are high frequency waves (frequency in the order of 100 to 1000 kHz), that propagate through a continuous medium. Some of the most popular theoretical treatise of the subject can be found in books by Graff [81], Rose [58] and Doyle [82]. Damage detection using such waves typically entails the understanding of the signal features that a damage may induce [83, 84].

Use of guided ultrasonic waves is prevalent in many structural and aerospace applications. They are typically used for defect detection in plate [85, 86, 87] and pipe [88, 89, 90] structures. A summary of all such applications may be found in Liu and Kleiner[91]. However, all these methods are typically model-based. This implies they require construction of high fidelity models or development of theoretical models for comparing with acquired
data for the purposes of damage detection. Such high fidelity models may fail to capture all the physics involved in system. In addition, high-fidelity models are typically computationally prohibitive. For example, the application of finite element methods require a large number of elements in order to obtain reliable response estimates in the high frequency domain [17]. An alternative is to use signal-based or data-driven techniques.

Data-driven techniques do not require assumptions about the system and capture all the physics involved in the system. Such an approach typically constructs a parametric meta-model from data, that does not necessarily mimic system behavior, but performs damage detection efficiently. This idea has led to the applications of statistical/machine learning algorithms in SHM [92, 93]. As elucidated in chapter 2, statistical learning algorithms are a potent tool for development of parametric models from data [18]. They are widely used for SHM and damage detection applications. For example, [94] use independent component analysis and sparse regression-based approaches [38, 37] to damage detection involving vibration-based techniques. Recently, statistical learning algorithms have gained popularity in the guided wave-based SHM and damage detection community too [95, 96, 97, 98, 99, 100, 101].

In this chapter a statistical learning-based approach is proposed that harnesses the inherent sparsity in a beam damage detection problem using guided-ultrasonic waves. A sparse representation framework is proposed for damage detection which is then tested for various scenarios that may be encountered in the field. To test the proposed framework, numerical simulations of Lamb waves propagating in beams are used. Prior to defining the proposed framework and describing approaches for numerical simulations, an understanding of Lamb waves is necessary. Hence, the following section provides a brief discussion on Lamb waves.
4.2 Lamb waves

When an elastic medium is excited using high frequency waves, one typically observes various types of waves propagating through the media, for example body waves and surface waves to name a few [81]. Lamb waves are a special type of waves generated in thin elastic bodies. The inherent assumption is that the medium is bounded only in one direction. The thin system then enforces interferences between multiple reflections of body waves propagating through the system producing Lamb waves. Lamb waves are of two kinds, symmetric and antisymmetric, based on the nature of the displacements of the particles of the waveguide. Figure 4.1 shows the two types of waves in a semi-infinite waveguide. They are generally referred to as A and S waves. The figure only depicts the zeroth order modes, hence they are called the A0 and S0 modes. Higher order modes namely, A1 and S1, A2 and S2 and so on can be generated when a waveguide is excited using a higher frequency. The choice of frequency can be made from dispersion curves that relate the wavenumber or wave velocities with excitation frequency. It is observed that higher order wave modes do not propagate below a certain frequency, referred to as the cut-off frequency for that wave mode.

![Figure 4.1: Anti-symmetric and Symmetric wave modes in an semi-infinite waveguide.](image-url)
These waves have a relationship with vibration types from engineering mechanics. For example, for a rod, the S0 mode represents an axial mode of vibration, whereas for a Euler-Bernoulli beam, A0 wave mode represents flexural vibrations. These relationships are important when solving the governing partial differential equations of these models. The solution methodology for wave propagation in structures is discussed in section 4.4.

4.3 Sparse representation framework for damage detection

In this section the sparse representation (SR) framework for damage detection is developed. The SR framework is based on a dictionary matrix $A$, for a given beam. Each column of the matrix $A$ represents a signal acquired from the beam with damage at a single location, for a specific actuation signal. Additionally, each signal is baseline subtracted or background subtracted (subtraction of an a priori acquired undamaged signal from the given signal) within an appropriate time window, prior to inclusion in the dictionary. The time window is selected based on the a priori knowledge of the geometry and wave velocity in the material, in order to better focus on damage signatures.

For $p$ damage locations and time-windowed signals of length $n$, the dictionary matrix $A \in \mathbb{R}^{n \times p}$. This implies that the first column of $A$, $a_1 \in \mathbb{R}^{n \times 1}$ is a signal acquired from the beam when there is damage at location 1. Similarly the second column of $A$, $a_2 \in \mathbb{R}^{n \times 1}$ is a signal acquired from the beam when there is a damage at location 2 and so on. Hence, $A = [a_1 \ a_2 \ ... \ a_p]$. As discussed earlier, the dictionary matrix is a representative of the beam in terms of its damaged and undamaged states. Additionally, the readers should note that each element of $A$ is both time windowed and background subtracted.

Now, let us assume $y \in \mathbb{R}^{n \times 1}$ is a background subtracted and time-windowed signal acquired from the beam, from a case when there is damage at location $m$, where $1 \leq m \leq p$. This signal can then be represented as
\[ y = Ax \quad (4.1) \]

where \( x \in \mathbb{R}^{n \times 1} \) is a vector such that \( x_i = 0 \ \forall \ i \neq m \) and \( x_m = 1 \). A vector like \( x \) where most of the elements are zero is referred to as a sparse vector. Mathematically, a vector of length \( n \) is defined as \( k \)-sparse \( i f f \) there are only \( k \) nonzero elements such that \( k \ll n \).

For equation 4.1 to hold, it is assumed that the number of damages in the system is low, which is a fair assumption for a system. This leads to vector \( x \) being sparse. In addition, if \( y \) is acquired from an undamaged case, it will be a vector of zeros due to the background subtraction, implying that \( x \) will also be a vector of zeros. Equation 4.1 is the SR framework for damage detection in beams. Since the non-zero element of the vector \( x \) provides us with the damage location, henceforth, it will be referred to as the damage pointer vector. One should note that, using equation 4.1 both level I and level II damage detection (as defined in chapter 1) is performed. If vector \( x \) has non-zero entries, it establishes the presence of damage. The location of the non-zero element helps carry out damage localization. For multiple damages, there will be multiple nonzero elements in \( x \). However, this assumption holds only when the damages are small and the effects of multiple damages can be linearly combined [102]. This means one can neglect the inter-damage reflections that may be acquired through a sensor if the damages are large enough. When damages are small, such waves attenuate swiftly and do not affect the waveforms acquired. This formulation also requires that the number of damages should be such that the sparsity condition on vector \( x \) holds.

Given this framework, one needs to solve for \( x \) from the linear system of equations \( y = Ax \), under the assumption that \( x \) is sparse. This problem is formulated as an optimization problem as follows [103].
\[
\begin{align*}
\min_x \| \mathbf{x} \|_0 \quad \text{such that} \quad \mathbf{y} &= \mathbf{A}\mathbf{x}, \quad (4.2)
\end{align*}
\]
where \( \| \cdot \|_0 \) is the \( \ell_0 \)-norm that counts the number of nonzero elements in a vector. In addition to being non-convex, equation 4.2 is also numerically unstable and NP complete. Solving it entails enumeration of all \( \binom{p}{k} \) possible combinations of nonzero entries of \( \mathbf{x} \), if \( \mathbf{x} \) is \( k \)-sparse [104]. To overcome these issues, equation (4.2) is typically reformulated in a \( \ell_1 \)-norm minimization framework (also popularly referred to as the relaxation of \( \ell_0 \) norm to \( \ell_1 \) norm), generally referred to as a *basis pursuit* problem [103]

\[
\begin{align*}
\min_x \| \mathbf{x} \|_1 \quad \text{such that} \quad \mathbf{y} &= \mathbf{A}\mathbf{x}, \quad (4.3)
\end{align*}
\]
where \( \| \cdot \|_1 \) is the \( \ell_1 \)-norm defined as \( \| \mathbf{x} \|_1 = \sum_{i=1}^{n} |x_i| \) for any vector \( \mathbf{x} \in \mathbb{R}^{n \times 1} \), where \( | \cdot | \) computes the absolute value of its argument. The computational complexity of the above problem is \( \mathcal{O}(p^3) \). To account for the effects of noise, equation 4.3 is further modified as:

\[
\begin{align*}
\min_x \| \mathbf{x} \|_1 \quad \text{such that} \quad \| \mathbf{y} - \mathbf{A}\mathbf{x} \|_2 &\leq \varepsilon, \quad (4.4)
\end{align*}
\]
where \( \varepsilon \) is a measure of the noise. A framework similar to equation 4.4 has been used earlier for various classes of problems over a wide spectrum of engineering fields like face recognition [105] and medical imaging [106]. As discussed in chapter 2, the \( \ell_1 \) minimization or sparse regression framework has also been used for vibration-based damage detection purposes earlier [38, 37, 107]. This chapter extends the application of this framework to high frequency ultrasonic wave-based damage detection.

To obtain solutions to equations 4.3 and 4.4 pursuit algorithms are typically used. Alternatively, one may reformulate the problem as a sparse regression problem and use the LASSO (a regularized linear regression technique used for sparse regression) [18] technique as follows
\[
\min_{\mathbf{x}} \| \mathbf{y} - \mathbf{A}\mathbf{x} \|_2^2 + \lambda \| \mathbf{x} \|_1 , \lambda > 0, \tag{4.5}
\]

where \( \| . \|_2 \) is the \( \ell_2 \)-norm (for any vector \( \mathbf{x} \in \mathbb{R}^{n \times 1} \), \( \| \mathbf{x} \|_2 = \sqrt{\sum_{i=1}^{n} |x_i|^2} \)) and \( \lambda \) is the regularization parameter governing the degree of sparsity in vector \( \mathbf{x} \). This can be looked at as a Lagrange multiplier form of equation (4.4). A detailed description of the LASSO can be found in [35]. In this chapter, the \texttt{l1ls} solver developed by Kim et al. [108] is used for LASSO implementation.

Based on the above discussion, it becomes clear that, for an effective implementation of the proposed algorithm an inherent assumption about the system is made: the damage locations used for dictionary construction are the most probable locations of damage and hence, characterizes the damaged system. Under this assumption, the efficacy of the proposed algorithm is numerically demonstrated when the test vector \( \mathbf{y} \) is obtained from scenarios of single damage but different extents to that used for dictionary construction as well as scenarios involving multiple damages.

### 4.4 Simulations

In this chapter damage detection is performed on a homogeneous and isotropic cantilever Timoshenko beam. In particular, numerical simulations are used for generating signals to demonstrate the efficacy of the proposed approach. Many simulation techniques for wave propagation problems are available in the literature. Traditional finite element method (FEM) is one of the most popular approaches owing to its ubiquity and ability of modeling complicated geometry in systems. However, it has its limitation when applied to this class of problems [17]. In order to accurately capture propagation of high frequency waves, the element size for FEM simulations need to be at least 10 times smaller than the wavelength of the waves expected to propagate in the system. This makes FEM extremely computation-
ally demanding and inefficient. To overcome this drawback, other simulations techniques have been developed. For example, instead of the traditional FEM, the Boundary Element Method (BEM) has been employed to tackle these problems [58]. The Spectral Element Method in time domain proposed by Patera [109] is another popular means of simulation. Extensions of this method have been used for both two dimensional and three dimensional wave propagation simulations [110]. These techniques use a special class of shape functions belonging to the orthogonal Legendre polynomials and use Gauss-Lobatto-Legendre points for numerical integration. Consequently, this improves the computational efficiency compared to traditional FEM as it reduces the number of elements necessary for simulations.

The class of Spectral Element Methods (SEM) developed by Doyle [82], improves the computational efficiency further for a limited class of problems. This is possible by formulating the problem in the frequency domain, that drastically reduces the number of elements necessary. However, these methods do not have the capabilities of traditional FEM in modeling complicated geometries. The spectral finite element method (SFEM) is an extension of the SEM developed by Doyle with improvements in terms of numerical computations involved for solving eigenvalue problems inherent in this method [111]. These class of spectral element methods are summarized in figure 4.2.

Other methods worthy of mention, used for solving these problems are the Finite Difference Method (FDM) and the Local Interaction Simulation Approach (LISA). LISA was developed for ease of computation using supercomputers. Applications of LISA for solving Lamb wave propagation problems are available in the literature using GPU-based systems [112].

SFEM however, has it’s own drawbacks. For example, using SFEM, damage can only be modeled approximately, whereas in traditional FEM it can be captured much more ac-
Excitation signal $f(t)$

$F$ represents the Fourier transform. This may be replaced by either the wavelet or Laplace transforms depending on the requirements of the problem at hand.

Figure 4.2: Description of the SFEM algorithm. $F$ represents the Fourier transform. This may be replaced by either the wavelet or Laplace transforms depending on the requirements of the problem at hand.

Figure 4.3: A schematic showing the ideology behind HFEM. Kinematic relationships are developed between the nodes of SFEM and FEM elements.

accurately. Although, Chakraborty and Gopalakrishnan [113] and Kumar et al. [114] model damages in beams and plates using SFEM, without taking aid of traditional FEM, calibrations with traditional FEM is necessary to characterize such approximate damages. As discussed earlier for SEM, SFEM also fails when it comes to modeling complex geometries [82, 111]. A combination of traditional FEM and SFEM can overcome such design issues with satisfactory results [115, 116]. This hybrid SFEM is used, which will be referred to as HFEM henceforth, for all simulations in this chapter. Figure 4.3 shows the idea behind
HFEM. The key idea is to model the damaged regions using traditional FEM in order to capture the complicated geometries involved. To ensure lower number of elements, the undamaged portions are modeled using SFEM elements. Kinematic constraints are used in between the SFEM and traditional FEM boundaries. In this chapter, the damage in a beam is modeled as a rectangular notch crack.

The original SFEM utilizes a Fourier transform-based frequency domain analysis. A drawback of Fourier transform-based SFEM is the fact that it suffers from *wrap around* effect [82]. Wrap around is the phenomena that occurs when the time vector considered for the Fourier Transform is not long enough and low frequency components wrap around the signal and corrupt the initial portion. One way of overcoming this is to take longer time windows. However, that will only increase the computational effort involved. The other issue involved with Fourier Transform based SFEM is the fact that a fixed end (no displacement and rotations at the boundary) boundary condition cannot be simulated [82, 117]. It is normally approximated by adding a semi infinite element (also known as a throw off element) with very high stiffness [117]. Wavelet Transform based SFEM has been proposed to overcome the wrap around issue [118], however the accuracy of the frequency domain is lost using the wavelet based SFEM. A Laplace Transform based SFEM has been shown to be the most efficient in overcoming all these [119, 120] drawbacks, and hence has been used for this work.

### 4.5 Numerical studies

#### 4.5.1 Setup

For simulations, an isotropic cantilever beam with Young’s modulus $E = 210 \text{ GPa}$, shear modulus $G = 79.3 \text{ GPa}$, mass density $\rho = 7860 \text{ kg/m}^3$, span 2 m and a square cross section
of side 20 $mm$ is considered. Damage is modeled as cracks that are 5 $mm$ deep and 2 $mm$ wide. Throughout this work, the width of the crack is assumed to be constant. To vary damage extent, only the crack depth is varied. As discussed earlier, HFEM is employed for modeling a damaged beam. This involves modeling a FEM-based crack system in conjunction with SFEM formulation for the undamaged portion, as described in section 4.4. For the dictionary, 9 possible locations for damage are assumed, and they are located at every 200 $mm$ along the length of the beam, as shown in figure 4.4. The damage closest to the fixed end is damage location 1 and the one closest to the free end is damage location 9. Figure 4.4 shows all the possible damage locations used to construct the dictionary, $A$.

Both the transmitter and the receiver are attached at the free end of the beam, as shown in figure 4.4. The beam is excited using a 5 cycle modulated tone burst with a central frequency of 50 $kHz$. Figure 4.5 shows the time and frequency domain representation of this load. The central frequency is chosen such that the frequency-thickness value is less than the cut-off value for shear modes to be activated. This ensures propagation of only one wave mode in the system.

Figure 4.6 shows a comparison between a damaged and an undamaged signal obtained from simulation. The wave packet generated due to the presence of damage at location

Figure 4.4: A schematic of the simulated beam. The red lines are the possible damage locations. Signals from these locations are used to construct the dictionary $A$ from equation 4.1.
Figure 4.5: The time and frequency domain representation of the actuation load used in this study. This is a 5 cycle tone burst with a central frequency of 50 kHz.

Figure 4.6: A comparison of damaged and undamaged signals. A time windowed version of the simulated signals is shown herein. The second figure shows the background subtracted signal, that enhances the damage feature.
8 is clearly visible. The clarity of this damage feature is further enhanced by performing background subtraction.

### 4.5.2 Results: Noise free

The first step is to create a dictionary, $A$, of signals containing information of various damage signatures. A dictionary is constructed with a damage size of 2 mm depth. Figure 4.7 shows the acquired signals from the grid locations, namely damage locations 1 through 9.

![Signals for dictionary](image)

**Figure 4.7**: The time signals acquired at the various grid locations for the dictionary. DL: Damage location.

Figure 4.7 shows the signals acquired at the grid locations defined earlier. The image at the top shows the full time history. The first wave packet at around 0.001 sec is the incident signal. The subsequent wave packets are the reflections from the fixed end. The lower image shows a time-windowed magnified version of the full time history. This allows us to...
observe the damage signatures for each of the nine grid locations. These damage signatures are crucial for the functioning of the proposed algorithm.

As discussed earlier, background subtracted signals are used for the dictionary. This helps enhance the damage features. Figure 4.8 shows the dictionary elements after background subtraction. Clearly it can be observed that the damage features are two orders of magnitude smaller than the incident signal. This fact will be used later when dealing with multiple damages.

![Diagram showing the elements of the constructed dictionary. Each signal is a time windowed background subtracted version of the original time history. DE: Dictionary element.](image)

Figure 4.8: The elements of the constructed dictionary. Each signal is a time windowed background subtracted version of the original time history. DE: Dictionary element.

**Case 1: Test signals have the same damage extent as in the dictionary.**

In this case, the test signal is from a scenario where the damage extent is the same as that used to construct the dictionary. The purpose of this case is to demonstrate the working of the proposed approach.
Figure 4.9 : Absolute values of the elements of the vector $x$. As discussed earlier, they point to the location of the damage.

Figure 4.9 shows the absolute values of the elements of the damage pointer vector, $x$. Figure 4.9(a) shows the results when the test signal is acquired from a scenario where there is a damage at location 8. So for a single damage scenario the vector $x$ is 1-sparse. Figure 4.9(b) shows the results when the test signal is from an undamaged scenario. As expected, all the elements of the vector $x$ are zero. Figure 4.10 shows the absolute values of the vector $x$ for all possible damage locations. It can be observed that the proposed approach accurately points to the damage location.

Case 2: Test signals have multiple damages

This case study demonstrates the performance of the proposed algorithm when the test signals are acquired from a scenario when there are multiple damages. For this case, it is assumed that the effects of multiple damage may be linearly combined. This implies, if damage at locations 1 and 2 yields the background subtracted signals $a_1$ and $a_1$, respectively, then the background subtracted test signal, $y$, maybe approximated as $y \approx a_1 + a_2$. This only holds for small damages. This assumption implies that the scattering of scattered
waves from damages is neglected. For larger damage, such scattering will have significant energy. As discussed earlier, based on figure 4.7, the damage feature wave packet amplitudes are two orders of magnitude lower than the incident wave packet. By extrapolating this idea, it can be concluded that the scattering of the scattered will be at least another order of magnitude lower compared to damage feature wave packet. Typically, such low amplitudes will be shrouded by noise and hence can be neglected.

Figure 4.11(a) shows a comparison of a test signal acquired from a case when the beam is damaged at locations 8 and 9 and the dictionary elements $a_8$ and $a_9$. Figure 4.11(b) shows the damage pointer vector, $x$, for this case. Clearly, the assumption of linearity holds, and the proposed approach localizes damage accurately.
Figure 4.11: (a) A comparison of the test signal acquired from a scenario where the beam is damaged at locations 8 and 9 with the dictionary elements 8 and 9. DE: Dictionary element, DL: Damage location (b) Absolute value of the vector $\mathbf{x}$ for the test signal shown in (a). Clearly the damage pointer vector points to locations 8 and 9.

Figure 4.12: The damage pointer vector elements for all possible combinations of two damages. The proposed approach accurately identifies all the multiple damages.
Case 3: Test signals with single damage of different extents

This case study demonstrates the efficacy of the proposed approach when the test signals are acquired from scenarios when the damage extent is different from the one used for dictionary construction. Figure 4.13(a) compares the damage features for different damage extents at location 7 with that of the dictionary element and the undamaged signal. One can observe that the phase information of the wave packet is very similar to the corresponding wave packet of the dictionary element. The only difference lies in the amplitude. Hence, higher magnitudes associated with the corresponding element of the damage pointer vector, $x$, are expected. Figure 4.13(b) demonstrates this. With an increase in the damage extent the magnitude of element 7 of vector $x$ also increases. The expected magnitude of the damage pointer vector may be calibrated to damage extent for successful damage quantification.

Figure 4.13 : (a) A comparison of the damage features produced by different intensities of damage at location 7. The crack depth captures the variation in damage intensity. The dictionary has a crack depth of 5mm. DE7: Dictionary element 7, (b) The elements of the damage pointer vector $x$ for the signals shown in (a).
4.5.3 Impact of noise

The previous subsection demonstrates the power of harnessing the inherent sparsity of the damage detection problem. This section studies the impact of noise on the proposed approach, hence, showcasing the robustness. This is achieved by adding Gaussian white noise to the signals in the following manner

\[ \tilde{s} = s + \sigma_{\text{noise}} \varepsilon \]  (4.6)

where \( \tilde{s} \) and \( s \) are noisy and noise-free signals. \( \varepsilon \) is a Gaussian white noise process with each time element being a Gaussian random variable with zero mean and unit variance. \( \sigma_{\text{noise}} \) is the intensity of noise corrupting the signal. The noise intensity is measured in terms of \( SNR_{\text{dB}} \):

\[ SNR_{\text{dB}} = 20 \log_{10} \left( \frac{s_{\text{RMS}}}{\varepsilon_{\text{RMS}}} \right) \]  (4.7)

where \( s_{\text{RMS}} \) is the root mean square (RMS) value of the noise-free signal. \( \varepsilon_{\text{RMS}} \) is the RMS value of the added noise. For a zero mean stationary Gaussian process \( \varepsilon_{\text{RMS}} = \sigma_{\text{noise}} \).

Figure 4.14 shows the effect of noise on each case defined earlier. For cases 1, 2 and 3 there are 9, 36 and 45 test signals respectively. For each \( SNR_{\text{dB}} \) level the sparse representation problem is solved 50 times. For each trial, the number of correct localizations are counted. The accuracy is defined as the ratio of number of correct classifications to the total number of test signals for the case. This ensures accuracy is between 0 and 1. Taking all the 50 trials the mean and standard deviation of the accuracies are estimated. The standard deviations for case 1 is greater than that in cases 2 and 3 because of the lower number of test signals. In general, one can observe a very high accuracy beyond a noise level of 25 dB in all cases.
Figure 4.14: Impact of noise on the performance of the proposed algorithm for (a) Case 1, (b) Case 2, (c) Case 3

4.6 Conclusions

An approach that harnesses the inherent sparsity in the damage detection problem in beams using Lamb waves is proposed in this chapter. The damage detection problem is formulated in a sparse representation framework. This involves the construction of a dictionary consisting of signals acquired from various damaged scenarios of the beam, that characterizes the damaged behavior. Subsequently, this dictionary is used for performing damage detection and localization for a signal acquired from an unknown damaged scenario, by
solving a sparse regression problem. Numerical simulations are conducted to demonstrate the ubiquity of the proposed framework in dealing with scenarios of multiple damage as well as various damage extents. In addition, it is shown that the proposed approach achieves accuracy even with noisy signals of $SNR_{dB}$ of 25 dB.
Chapter 5

Sparse damage detection in plates

This chapter extends the idea presented in chapter 4 to plate structures. The inherent sparsity is harnessed again to perform damage detection. The efficacy of this approach is demonstrated using simulations first and followed by experiments. The key idea is to avoid modeling systems and perform damage detection based on only data acquired from a system.

5.1 SDD-ON: Sparse Regression for on the grid Damage Detection

This section extends the sparse representation framework developed in chapter 4 for damage detection in plates. To that end, SDD-ON, a sparsity-based algorithm for damage detection is proposed. Figure 5.1 shows the basic idea behind formulating damage detection as a sparsity-based problem. Although the framework is similar, the description of the framework is described in detail for the sake of completeness. The matrix $A$ is the dictionary, or damage characterization matrix (DCM), for the given system. The columns of matrix $A$ are signals retrieved from the experiments for different damage locations in the system when subjected to a specific actuation signal. Baseline or background subtraction (subtraction of an undamaged signal from the acquired signal) on all these signals before they are added to the DCM. Assuming that each signal is of length $n$, and that there are $p$ possible damage locations, the matrix $A \in \mathbb{R}^{n \times p}$. The vector $y$ is a test vector, that is a background subtracted response retrieved from the system for an unknown damage lo-
cation(s) when subjected to the same actuation signal used for construction of the DCM, hence $y \in \mathbb{R}^{n \times 1}$, $x \in \mathbb{R}^{p \times 1}$ is a damage location pointer vector. Assuming that the DCM contains signals from all possible damage locations and the number of damages is low, the vector $x$ will be sparse, i.e. only a few non-zero elements. As discussed earlier, mathematically, a $k$-sparse vector of length $n$ is one with $k$ non-zero elements such that $k \ll n$. To be more precise, if the test vector was obtained from a single damage scenario, only one element in $x$ will be nonzero. The location of the nonzero element will correspond to the column of the DCM that is similar to the damage signal of the test vector $y$. When $y$ is from an undamaged signal, post background subtraction $y = 0$, that would render $x = 0$. For a multiple damage case, there will be multiple non-zero elements in $x$. As discussed in chapter 4, this assumption holds only when the damages are small and the effects of multiple damages can be linearly combined. In addition, the number of damages must be limited such that the vector $x$ is still sparse.

In the present formulation, elements of the DCM characterize different damage locations. In reality, damages in a system may be of various sizes and orientations. These may affect the acquired signals in very different ways. Hence, this framework is limited to the task of damage detection and localization. Another issue that needs a discussion is the case for multiple damages. For example, if the test signal is a signal acquired from a scenario where there is damage at locations 1 and 2. In that case it is assumed that $y = a_1 + a_2$, where $a_1$ and $a_2$ are the first and second columns of the DCM $A$. This assumption holds only for small damages [102]. This works because the scattered waves from small damages are typically an order of magnitude lower than the incident signals. These scattered waves, when scattered by other damages produce signals that are another order of magnitude lower than the incident signal. Such low amplitude signals are typically lost in noise and may be safely neglected. For larger damages the amplitudes may not be insignificant.
Figure 5.1: Illustration of SDD-ON for damage detection. Each element of the dictionary is constructed using processed acquired signals from different damage locations as shown. In this case the unknown damage location, from the test vector \( y \), is location 3. Hence, the third element of the sparse vector \( x \), evaluated by sparse regression, points to location 3 in the dictionary \( A \) (DCM).

and the assumption of linearity will not hold.

Given this framework, one needs to solve for \( x \) from the linear system of equations \( y = Ax \), under the assumption that \( x \) is sparse. Based on the discussion in chapter 4, \( x \) can be obtained by solving the following sparse regression using the LASSO (a regularized linear regression technique used for sparse regression) \([18]\) technique:

\[
\min_{x} \| y - Ax \|_2^2 + \lambda \| x \|_1, \quad \lambda > 0,
\]

(5.1)

where \( \| . \|_2 \) is the \( \ell_2 \)-norm and \( \lambda \) is the regularization parameter governing how sparse vector \( x \) should be. A detailed description of the LASSO can be found in \([35]\). The \texttt{111s} solver developed by \([108]\) is used for LASSO implementation.
Based on the above discussion, it becomes clear that, for an effective implementation of the algorithm described in figure 5.1, an inherent assumption about the system is made: the damage locations used for dictionary construction are sufficient for effectively detecting and localizing damage of the system. Since the test vector \( y \) is from a damage location already in the DCM, the nomenclature on-the-grid problem is used, which implies that the test vectors are already on the DCM grid. SDD-ON addresses this problem. An extension of the on-the-grid problem is the off-the-grid problem. In that case, it is not necessary for test signals to be obtained from dictionary grid points. More details about off-the-grid problems can be found in [121].

5.2 Results and discussions

5.2.1 SDD-ON

Finite Element Method based Simulations

This work studies active SHM in systems using high frequency GUWs as a means of damage detection. The word *active* implies that the system at hand is subjected to an excitation using a piezo-electric actuator. The response of the system, subjected to the excitation is then sensed for the purposes of damage detection. Guided waves are generally multi-modal in nature, resulting in complicated waveforms traversing through systems. It becomes necessary to limit the frequency content of the signal to a narrow band, below a cut-off frequency (to minimize number of participating modes), to ensure effective damage detection. To achieve this, a 5 cycle tone burst signal with an appropriate central frequency \( f_0 \) is used. Dispersion curves, associated with each material, govern the choice of the central frequency based on the number of modes that will be allowed to propagate. Each wave mode is generally associated with a cut-off frequency and is not triggered until that frequency is reached.
Figure 5.2 shows a typical five cycle tone burst signal with $f_0 = 50$ kHz.

![5 cycle tone burst signal used](image)

Figure 5.2: A typical 5 cycle tone burst signal used in this chapter. The narrow band in the frequency domain ensures minimal dispersion of propagating waves. The central frequency, $f_0$ for this signal is 50 kHz. The dispersion curves of the system governs the choice of the central frequency.

A FEM software, developed by Doyle [?] and named QED, is used for simulating response signals in plates. The FEM model constitutes a square aluminum plate of side 12 in and 0.1 in thick with fixed boundary condition at all the edges. This alternate boundary condition is for the purpose of demonstration of efficacy of the proposed algorithms only and does not reflect the exact boundary conditions of the experimental setup. In addition, the size of the simulated plate is much smaller than the experimental plate discussed in the following section. This is to reduce the computational effort involved in the simulations. For guided wave response simulation the element size used in FEM has to be of the order of the wavelength of propagating elastic wave, to ensure numerical accuracy [17]. Typically a very fine mesh is used to realize this, which leads to heavy computational effort. A circular through-hole of diameter 0.5 in is used for modeling damage. A 5 cycle tone burst signal
with a central frequency of 100 kHz is used for excitation. This ensures that only the S0 and A0 Lamb modes propagate. The excitation is modeled as a point load applied on the plate. Responses are measured at a single sensor location. Figure 5.3 shows the FEM setup of the system. The simulations are based on a single damage. As discussed earlier, the algorithms can easily be extended to multiple damages, under the assumption that the size of the damage is small compared to the actual size of the system.

Velocity response signals from nodes of the finite element mesh, at the sensor location are observed. The signals are then filtered using a bandpass filter with frequency cutoffs 50 kHz and 100 kHz. Figure 5.4 shows the location of the actuator, sensor and twenty five possible locations of damage. For each simulation, one of the possible locations is damaged and a response signal is observed at the sensor. As described in Figure 5.1 this is repeated for all the twenty five different locations producing a set of response signals. The coordinates of the possible damage locations constituting the dictionary, actuator and sensor are shown in figure 5.4. Locations 1, 3, 5, 11, 13, 15, 21, 23 and 25 are selected for constituting the grid. The rest of the locations are used to test the prowess of this algorithm for off-the-grid cases. The length of each response signal acquired is 8192, with a sampling frequency of 1 MHz. A typical signal acquired from the simulation is shown in Figure 5.5.

Validation of SDD-ON using FEM simulation results

SDD-ON uses the signals sensed in FEM simulation as inputs. Since, each signal is of length of 8192, the dictionary matrix or DCM, \( A \in \mathbb{R}^{8192 \times 9} \) as only nine of the damage locations constitute the grid for DCM construction. To test SDD-ON, signals acquired from damage locations 5 and 18 are used as test signals. Figure 5.6 shows the results from SDD-ON. It shows the values of each element of the sparse vector \( x \). The labels of each element are such that they represent the corresponding damage location. The element with
Figure 5.3: The FEM setup of the plate. (a) The plate used for simulation, with the mesh of triangular elements used for modeling the plate. (b) The model of damage that is used in the simulations. The through damage is depicted by the big circle on the plate. The cross shows the point of application of the pulse load. As discussed earlier, the actuator is modeled as a point load.

the maximum value points to the specific column in the DCM. For figure 5.6 (a), location 5 is returned as the damage location for the test vector $y$. Location 18 is a damage location
Figure 5.4: Simulation setup with 25 potential damage locations. The filled squircle is the actuator and the filled square is the sensor. The locations of each damage is shown as a hollow disc. The coordinates of the actuator, sensor and the potential damage locations are shown. The potential damage locations are labeled with blue digits. All the coordinates are in inches.

Figure 5.5: A typical signal acquired from the FEM simulation.
that is off-the-grid. In such cases, SDD-ON predicts damage locations on-the-grid close to these off-the-grid locations. In this case, since locations 13 and 23 have the highest magnitudes, it is clear that the damage is located somewhere in between these two on-the-grid damage locations.

![Figure 5.6](image)

Figure 5.6: Results of sparse regression for simulation results. This shows the absolute value of the elements of the vector $x$ obtained from SDD-ON for two different test cases. Damage location 5 is a on-the-grid location, whereas location 18 is not. So SDD-ON outputs the exact location if the damage location is on-the-grid. If not it returns a damage location on-the-grid in the vicinity of the actual damage location. In this case damage location 13 is close to damage location 18.
The above demonstration was for a deterministic case. In real field monitoring, noise in the sensors will corrupt the acquired signals and may affect the performance of SDD-ON. To observe the effect of noise, SDD-ON was applied for each on grid location for multiple times with varying levels of noise corrupting the DCM elements and test vectors independently.

\[
\tilde{a}_i = a_i + \varepsilon_a \quad \varepsilon_a \sim \mathcal{N}(0, \sigma^2),
\]
\[
\tilde{y} = y + \varepsilon_y \quad \varepsilon_y \sim \mathcal{N}(0, \sigma^2),
\]

(5.2)

where \(a_i\) and \(y\) are the noise free signals constituting the DCM and test vector respectively. \(\tilde{a}_i\) and \(\tilde{y}\) are the noisy signals constituting the DCM and the noisy test vector, respectively. \(\varepsilon_a\) and \(\varepsilon_y\) are the zero mean Gaussian white noise with a variance of \(\sigma^2\) added to the DCM elements and test vector respectively. The additive noise is measured using signal to noise ratio (SNR) in dB:

\[
\text{SNR}_{dB} = 10 \log_{10} \left( \frac{X_{signal}}{X_{noise}} \right)^2
\]

(5.3)

where \(X_{signal}\) and \(X_{noise}\) are the root mean square amplitudes of the noise-free signal and the noise. To quantify the accuracy, the number of incorrect on grid predictions is used as a measure (maximum being 9). Figure 5.7 shows the results for the sensitivity to noise study. Clearly, SDD-ON is shown to be robust to effects of noise. This is because, only negative values of SNR adversely affect the performance of the platform.

**Experimental validation of SDD-ON**

For experimental verification of SDD-ON, a square aluminum plate of 90 cm edge length and 3.51 mm thickness as shown in figure 5.8 is used. In order to isolate the plate from the bench on which it is placed, the plate is supported at four corners using rubber supports. The actuator and the sensor are circular piezoelectric sensors made of lead zirconium ti-
Figure 5.7: The effect of noise on SDD-ON, where noise is quantified using signal to noise ratio. The number of wrong locations is an average of the number of wrong localizations over all possible damage locations. The maximum number of wrong localizations possible is 9. The above figure reports both the mean and standard deviations for various noise levels.

vanate (PZT) manufactured by Steminc Piezo. These circular piezosensors are 10 mm in diameter and 3 mm thick with a dominant R mode vibration at a resonant frequency of 215 kHz. They are attached to the plate using 3M double sided tape. The plate is subjected to a five cycle tone burst signal of 10 V peak-to-peak with a central frequency of 100 kHz using a Tektronix AFG 3022B function generator. The signals are acquired using a Tektronix TDS2024C oscilloscope. Sixty four signals for each case are acquired at a sampling rate of 5 MHz and then averaged for noise reduction. This is followed by bandpass filtering of the average signal between 50 kHz and 150 kHz. As discussed earlier, masses are placed at the prospective locations for simulating damage (see figure 5.8). Figure 5.9 shows the location of the actuator, sensor and the damage locations used to construct the dictionary. The filtered and baseline subtracted signal from the nine different on grid locations constitute the
DCM. Following the construction of the DCM, test signals are acquired from the system.

Figure 5.8: Experimental setup. The yellow circle shows the mass used for simulation of damage.

Figure 5.9: Locations of actuator, sensor and possible damage locations along with labels for the plate experiment.

Figure 5.11 shows the results for SDD-ON for a typical on-the-grid problem. The test vector is a signal acquired when the damage is at location 3. Again the elements of the
sparse vector $\mathbf{x}$ is shown. The third element has the maximum value again, demonstrating that SDD-ON is an effective tool for solving on the grid damage detection problems. It should be noted that due to presence of noise, all the elements of the vector $\mathbf{x}$ is non-zero. To quantify the performance of SDD-ON platform for on-the-grid points, a receiver operating characteristic (ROC) curve is used. A ROC curve compares the true positive and false positive rates for binary classification problems and helps in quantifying the efficiency of the classifier. This is typically achieved by measuring the area under the curve (AUC) of the ROC curve.

The parameter for the ROC curve is the threshold value for magnitude of each element of $\mathbf{x}$ such that a location is classified as damaged if the value of the location pointer vector at that coordinate is above the specified threshold. Figure 5.10 shows the ROC curve. The AUC associated with this ROC curve is 0.997. This shows that SDD-ON performs efficiently for on-the-grid locations. Figure 5.12 shows the performance of SDD-ON for off-the-grid points. It is expected that for the off-grid points, the on-grid-point close to the actual damage location should carry the maximum weight, as evaluated from SDD-ON. The title of each subplot in the above figure contains two numbers. The first number is the actual location of the damage and the second signifies the predicted damage location. It can be seen that only for damage locations 7, 8, 9, 10 and 20 that SDD-ON makes inaccurate predictions. However, for all the other cases an on grid location in the neighborhood of the actual damage is predicted by the algorithm. SDD-ON is clearly effective for on-the-grid problems.

**Detection and localization of damage on an experimental plate**

In section 5.2.1, the efficacy of SDD-ON is demonstrated using test signals acquired by placing masses at various locations to simulate damage. In this section, the same dictionary
from section 5.2.1 is used, but a test signal acquired by drilling a partial hole at location 8 is used, as shown in figure 5.9. Although an added mass and a partial hole are very different from a physics point of view, the sparsity of the damage detection problem is still preserved. This implies, for a small enough damage, the dictionary constructed using adding masses, can still detect and localize a damage that was induced by drilling a partial hole. For a more comprehensive understanding of correlation between the adding mass and partial damages, one requires to perform a calibration study that relates crack size and added mass.

Damage is induced to the plate at location 8 by drilling a 1 \text{mm} deep circular hole with a diameter of 4.5 \text{mm}. Figure 5.13 (a) shows the damage. Figure 5.13 (b) shows the results after applying SDD-ON. Clearly, SDD-ON localizes damage at location 8.
5.3 Conclusions

A sparsity-based damage detection algorithm for SHM of plates using GUWs called SDD-ON is proposed in this chapter. The key idea behind this algorithm is the construction of a dictionary, or damage characterization matrix (DCM), representing the system at hand as it constitutes damaged response profiles of the system for different locations of damage. This aids in avoiding development of detailed models for damage detection purposes. Sparse damage pointer vectors are used for damage localization. The algorithm is shown to perform damage localization on simulation as well as experimental data from plates with satisfactory accuracy. By avoiding development of high-fidelity models, SDD-ON is a promising for extension to online health monitoring systems for complicated structures.
Figure 5.12: Performance of SDD-ON from plate experiment for all damage locations. The numbers on top of each figure denote the actual location of damage and the predicted damage location, shown by the small and big circles respectively. The black crosses denote the damage locations used as grid points for DCM construction.
Figure 5.13: (a) The circular damage induced at location 8, (b) Results of sparse regression for actual damage scenario. The algorithm correctly localizes damage at location 8.
Chapter 6

Damage detection in pipes

This chapter introduces guided ultrasonic wave-based damage detection in pipes. A hierarchical clustering-based algorithm is proposed that aims at minimizing the number of piezo-transducers necessary for detecting the presence of damage.

6.1 Introduction

SHM of pipes has been a crucial task for engineers for the past quarter of a century owing to the catastrophic nature of failures, not only for the environment but the society as a whole [122]. Guided ultrasonic wave (GUW)-based technology has been in use for SHM of pipelines for many years now [89, 88]. The readers can find a review of these applications in Liu and Kleiner [91] which classifies the application of GUW-based SHM as either passive or active sensing. As the names suggest, active sensing involves actuation of the systems at hand using an external source, whereas, passive sensing involves acquisition of signals resulting from ambient vibrations. In this chapter, the focus is on active sensing.

A survey of the literature on pipeline monitoring using GUW-based active sensing yields ample research. These range from development of sensors to algorithms for crack and corrosion detection. For example, Rose [123, 124] uses GUWs for crack detection in pipelines. Giurgiutiu [85] uses piezoelectric wafer active sensors [86] for corrosion detection on pipelines. Lee et al. [125] propose a baseline-free pipeline monitoring scheme taking aid of optical fiber-guided laser ultrasonics. Na and Kundu [90] study the effects
of varying incident angles of ultrasonic waves for defect detection in underwater pipelines. Lowe et al. [88] provide a comprehensive discussion on this approach to damage detection in pipelines. In addition to the efforts of the research community, major industrial sectors plan on enhancing their capabilities as well with the market size expected to grow from USD 4.13 Billion in 2015 to USD 8.72 Billion by 2026 [126].

Traditional approaches to damage detection in pipelines typically involve the use of model-based approaches wherein a high-fidelity model represents system behavior [83, 84] and hence allows for detecting changes in the system due to advent of damage. However, constructing such models is in many instances physically intractable and computationally prohibitive. Given the advancement in sensors and hardware development, data-driven approaches become far more suitable for damage detection. The aim of data-driven techniques is to alleviate the need of a high-fidelity model for differentiating between damaged and undamaged configurations of the system. The past decade has seen a rise in attention towards data-driven techniques for SHM using GUWs [95, 41, 121]. Some of the popular efforts in pipeline monitoring are from Liu et al. [96], Ying et al. [97, 98] and Eybpoosh et al. [101]. Data-driven approaches based on statistical learning algorithms typically require extensive signal processing of raw data for feature selection. Such an approach helps improve the efficiency of the damage detection algorithms in the literature. In addition, typically the existing methods require a moderately dense to highly dense array of sensors for efficient performance. In this chapter two key issues are addressed; first, the feature selection phase circumvented and it is demonstrated that bandpass filtered signals themselves are efficient for damage detection, second, the number of sensors typically used for performing GUW-based damage detection is minimized. The second point helps in minimizing maintenance costs as the proposed algorithms is shown to perform efficient damage detection using low-cost ($5 per sensor) piezoelectric transducers.
Typically for pipe monitoring, the state-of-the-art either involves a sophisticated sensor that may increase the overall cost of a monitoring scheme, or a large number of regular sensors in an attempt to reduce measurement noise or generate specific wave modes in order to accrue useful signal information necessary for damage detection. Thus, in addition to implementation of a data-driven framework, a key objective of this chapter is to demonstrate the use of small number of low-cost sensors for performing reliable damage detection.

To accomplish these objectives, a semi-supervised statistical learning algorithm is proposed which performs level I damage detection. The efficacy of the proposed algorithm is demonstrated experimentally on cast iron pipes with bituminous anti-corrosion paint layer. It is demonstrated that effective damage detection is possible with just two low-cost piezoelectric transducers using the proposed semi-supervised learning scheme.

The following section describes the theoretical aspects utilized for development of the proposed algorithm.

### 6.2 Helical guided ultrasonic waves

This section discusses the physical basis of the proposed algorithm. Since guided waves are extremely useful for SHM [80], the intricacies involved in the mechanics of wave propagation in a pipe has been addressed in great detail in the literature. The key difference between propagating elastic waves in plates and pipes is the different types of wave modes propagating. For studying wave propagation, pipes are typically treated as a three dimensional structure (either in Cartesian or polar coordinate systems) in contrast to plates which are generally modeled as two dimensional systems. Consequently, for plates two types of wave modes exist, namely, symmetric and anti-symmetric. Correspondingly, for pipes, three kinds of wave modes exist, namely, longitudinal, flexural and torsional. Presence of three kinds of waves, instead of two like in plates, makes wave propagation in pipes far
more complicated. This can be observed when comparing the dispersion curves for plates and pipes. If all wave modes in pipes are allowed to propagate the acquired signals become extremely dense, rendering damage detection an extremely complicated task. To overcome this, multiple actuators are deployed simultaneously for generating specific wave modes to improve the efficacy of damage detection. However, that amounts to a significant rise in maintenance costs.

For hollow pipes, a high diameter to pipe thickness ratio, leads to the propagating Lamb waves in a pipe behaving like those in a plate \cite{127}. This implies that the wave propagation in the pipe can then be represented in terms of symmetric and antisymmetric wave modes similar to a plate. This happens because many wave modes for a pipe overlap under this geometric condition. In such cases, wave propagation along the pipe can be thought of as helical waves along the surface of pipes \cite{4} (see figure 6.1). Between any two points on the pipe, there are infinitely many possible helical paths joining them. If there is a damage in the pipe, few of the helical paths will be impeded. If the acquired waveforms are studied, the changes can be monitored for the purposes of damage detection. Figure 6.2 shows the differences in response between damaged and undamaged scenarios. To generate this image, 100 instances of undamaged and damaged signals each were acquired. The figure shows the mean damaged and undamaged signal along with a measure of the variance. The proposed algorithm utilizes the changes observed in between damaged and undamaged signals for differentiating between damaged and undamaged signals.

### 6.3 Proposed algorithms

This section describes the proposed algorithm. For the purposes of level I damage detection one would want to cluster the data into two clusters only, namely damaged and undamaged. As discussed earlier, HC is an unsupervised algorithm implying the absence of labels ac-
Figure 6.1: Helical guided ultrasonic waves as defined in [4]. A few of the multiple possible wave paths possible from the actuator and sensor are shown on the pipe the corresponding plate representation of the pipe. A damage typically blocks off a small number of such infinite possible paths for waves to reach the sensor. The small change in the acquired signal due to the damage is used for the purpose of damage detection.

Figure 6.2: A comparison envelopes of between mean damaged and mean undamaged response signals, along with a measure of the variance, acquired from a real pipe. Damage is simulated by attaching a mass to the pipe.

companying the data. Hence, without a priori information on the labels, classification of each cluster obtained from HC is not possible. Hence, a single data from the undamaged state of a pipe is required as input to this algorithm with a label. Once, a label is available for one of the undamaged signals, the cluster consisting of the labeled data may be
classified as constituting the undamaged responses, and the other damaged. The proposed algorithm may be summarized as follows:

- Step 1: Acquire response signals (damaged or undamaged) from a pitch-catch set up.
- Step 2: Apply hierarchical clustering with complete linkage and Euclidean distance as the difference measure with number of clusters set to two.
- Step 3: Search for the labeled undamaged data in the clusters.
- Step 4: Assign the cluster with the undamaged label where the labeled data is present. This essentially leads to the knowledge about existence of damage.

6.4 Experimental setup

6.4.1 Description of the system and input excitation

Two cast iron pipes are used for the experimental verification. Table 6.1 lists a brief description of each pipe. Both the pipes have a bituminous anti-corrosive paint on the surface. This paint typically reduces the intensity of the applied excitation. It is unknown whether this layer of paint affects the acquired signals in any other way. This is similar to a real life scenario where the properties, and their effect on acquired signals, of paints on existing pipeline networks may be unknown. However, it is demonstrated that the proposed algorithm is effective even in presence of such a paint layer.

The pipes are supported at the ends on wooden benches with rubber padding to isolate it from external vibrations from the laboratory environment. For acquiring damaged response, the presence of damage is simulated instead of inducing damage on the pipe. In order to do so, a mass is attached on the pipe at the desired damage locations. Grease is
<table>
<thead>
<tr>
<th>Pipe label</th>
<th>Material</th>
<th>Length (m)</th>
<th>Outer diameter (cm)</th>
<th>Thickness (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Cast iron</td>
<td>1.23</td>
<td>16</td>
<td>5.2</td>
</tr>
<tr>
<td>B</td>
<td>Cast iron</td>
<td>3.05</td>
<td>16</td>
<td>6.73</td>
</tr>
</tbody>
</table>

Table 6.1: Description of each of the pipes used for the experiments. The labels for each pipe listed here will be used henceforth in this report to refer to each pipe used to ensure acoustic coupling between the mass and the pipe. The mass scatters the elastic waves propagating in body of the pipe (similar to a real damage).

Figure 6.3: The general setup of the pipes. Based on the position of the actuator and sensor, the pipe is divided into three regions in the vicinity of the actuator-sensor pair. The sectional view A-A shows the possible angular positions of damages with respect to the angular position of the actuator and the sensor.

The placement of the sensors is as shown in figure 6.3. The location of the actuator and sensor divides the length of the pipe into three regions. The actuator and sensor is assumed to be always along the zenith line of the pipe, i.e., at an offset of zero degrees in view A-A in figure 6.3. The damages however may lie at any angular offset with respect to the zenith line. To this end, a set of angles at which the damages may lie is defined, $\theta_S \in \{0^\circ, 30^\circ, 60^\circ, \ldots, 180^\circ\}$. The choice of these angles will shed light on the performance.
of the proposed approach where there is an angular offset between the relative angular positions of the damage with respect to the actuator and sensor. For pipe B a similar setup is used in two different regions. One is away from the boundary and the other near the boundary. The experiments performed near the boundaries helped observe the impact of boundary effects on the proposed approach.

As discussed earlier, the focus is on guided wave-based active damage detection. The external excitation used is a five cycle tone burst signal such that the frequency spectrum of the signal is narrow. This is to ensure minimal wave dispersion. The central frequency of the tone burst signal is selected such that the number of wave modes propagating in the elastic medium is minimized in order to reduce the complexity of the acquired signals. Figure 6.4 shows a typical input excitation that is used, in the time as well as frequency domain.

Figure 6.4: A typical 5-cycle tone burst input excitation with a central frequency of 100 kHz.
6.4.2 Hardware used

Circular disk shaped lead zirconium titanate (PZT) piezoelectric transducers, manufactured by Steminc piezo, are used in all the experiments. These sensors are used as both the actuator (transmitter) as well as sensor (receiver). They are 10 mm in diameter and 3 mm thick with dominant R mode of vibration with a resonant frequency of 215 kHz. To ensure a strong contact, the piezo-transducers are attached on pipes A and B using liquid nails, a glue manufactured by perfect glue. This ensures that the angle of incidence of the pressure waves are 90°, resulting in propagation of both symmetric and anti-symmetric wave modes along the pipe. Tektronix AFG3022B and Tektronix TDS2025C with four channels are used as the function generator and oscilloscope respectively. Texas Instruments (TI) high frequency booster packs (provided by TI) are used for amplification of received signals. Figure 6.5 shows the experimental setup described above.

![Experimental Setup](image)

Figure 6.5: Experimental Setup. The three different regions shown with respect to the position of the actuator and sensor. There is one actuator and four sensors at four angular locations.

6.4.3 Data acquisition

For the setup shown in figure 6.3, data is acquired from all the three regions for both damaged and undamaged cases. To improve the signal to noise (SNR) of the acquired signals, one hundred and twenty eight records of the signals are averaged to produce one acquisition. The acquired signals are further denoised by a bandpass filter with a spread of 100
kHz centered at the central frequency of the actuation signal. As discussed earlier, damage is simulated by using a mass which scatters the high frequency waves that propagate through the structural medium of the pipe. Twenty samples per region and one hundred samples per region are acquired for pipes A and B, respectively.

6.5 Results

6.5.1 Pipe A

Results from pipe A are used to demonstrate the application of the proposed semi-supervised algorithm. For this demonstration, data acquisition from near the boundaries of pipe A is avoided. The effects of boundaries will be explored using the data from pipe B. As discussed earlier, there are six possible angular locations of damage, in each of the three different regions in figure 6.3.

Figure 6.6: Results obtained from the proposed semi-supervised algorithm when the actuator, sensor and the simulated damage are all at an angular position of 0 degrees. The figure on the right shows the ground truth, i.e., the type of each data point, whether they are damaged or undamaged. The figure on the left shows the clusters obtained from HC. Clearly, for any of the undamaged data points treated as the \textit{a priori} data, the proposed semi-supervised algorithm will provide a 100% accuracy in damage detection.
Figure 6.6 shows the results for the case where damage aligns with the straight line joining the actuator and the sensor. Both plots in the figure show a two dimensional representation of the data. The readers should note that each point shown is a time series in itself and is represented on a two-dimensional plane. The first two principal component directions for the data form the basis for the two dimensional representation. The right hand side plot shows the ground truth, namely, it shows the source of each data point, i.e., undamaged or damaged from regions one, two and three. The clustering algorithm efficiently separates damaged and undamaged data into two separate clusters. Assuming one may treat any one of the undamaged points as the a priori data, based on the clusters obtained, one can state that cluster 1 in figure 6.6 is a cluster of undamaged data points.

Figure 6.7 shows the dendrogram obtained from the hierarchical clustering algorithm. It highlights the differences in between various data points based on the dissimilarity function used for hierarchical clustering.

![Dendrogram for Pipe A with 0° mass offset](image)

Figure 6.7 : A dendrogram obtained from hierarchical clustering for the results shown in figure 6.6. One can observe how each data point has been clustered based on the Euclidean dissimilarity function. The labels “Damaged” and “Undamaged” are based on a priori undamaged data knowledge.
Figure 6.8: Results obtained from the proposed algorithm for various damage angular positions. In figure (d) the red circles show damaged data classified as undamaged and in figure (e) the blue circles show undamaged data classified as damaged.

Figure 6.8 shows the clustering results for all the other angular positions of damage. Clearly, for angular damage positions between 0 and 90 degrees, the clustering algorithm is able to distinguish between the damaged and undamaged data points by comparing the
clusters with the ground truth. The 100% efficiency is attributed to the strong scattering signals from the damage and larger number of helical wave paths impeded. For larger angular offsets, these signatures attenuate swiftly or are not captured by the sensor, that leads to reduction of classification accuracy.

In figure 6.8 (d), for angular damage position $120^\circ$, it can be observed that a few damaged data included in cluster 1, which consists of all the undamaged data. Data points with red circles around them in figure 6.8 (d) shows such points. This implies that some damaged data points are classified as undamaged data points. Similarly, figure 6.8 (e) shows the results for angular damage offset of $150^\circ$. The blue circles show undamaged points clustered into cluster 2 that consists of all of the damaged points. This leads to a situation wherein, if any one of the undamaged data from cluster 2 are selected as the a priori undamaged data, this will imply all the damaged data will be classified as undamaged.

![Figure 6.9: Statistical false alarm performance of the proposed semi-supervised algorithm for pipe A. It achieves 100% accuracy up to damage angular positions of $90^\circ$.](image)

To quantitatively define the performance of the proposed algorithm, statistical measures from hypothesis testing are used. A positive event is defined as the detection of damage and no detection as negative. Hence, a true positive (TP) scenario is when a damage is detected
when a damage exists in reality. Similarly false positive (FP), true negative (TN) and false negative (FN) scenarios are defined and summarized in table 6.2.

<table>
<thead>
<tr>
<th>Term</th>
<th>Ground Truth</th>
<th>Classified as</th>
</tr>
</thead>
<tbody>
<tr>
<td>True Positive (TP)</td>
<td>Undamaged</td>
<td>Undamaged</td>
</tr>
<tr>
<td>False Negative (FN)</td>
<td>Undamaged</td>
<td>Damaged</td>
</tr>
<tr>
<td>False Positive (FP)</td>
<td>Damaged</td>
<td>Undamaged</td>
</tr>
<tr>
<td>True Negative (TN)</td>
<td>Damaged</td>
<td>Damaged</td>
</tr>
</tbody>
</table>

Table 6.2 : Definition of statistical measures used for quantifying the performance of the proposed semi-supervised learning algorithm.

To evaluate rates of each statistical measure, for each damage angular offset the number of correct and incorrect classifications is evaluated. This is done by considering each undamaged data as the a priori undamaged data. Figure 6.9 shows the performance of the proposed algorithm for various angular damage offsets. From the above results, it is concluded that using a single actuator sensor pair one can accurately detect the presence of damage up to damage offsets of $120^\circ$. Since the accuracy is 100% up to an offset of $90^\circ$, the deployment of single actuator-sensor pair units at diametrically opposite locations on the pipe surface is proposed to maintain 100% accuracy in damage detection. Figure 6.10 shows the proposed setup. The actuator-sensor pair on the top will cover the area shaded in red whereas the actuator-sensor pair at the bottom covers the area shaded in blue.

### 6.5.2 Pipe B

This section discusses the results from pipe B. Due to the relatively longer length of pipe B with respect to pipe A, the performance of the proposed semi-supervised approach at two different locations is studied, one away from the boundary and one close to the boundary.
Figure 6.10: The proposed deployment of two actuator-sensor pairs for maintaining 100% accuracy in damage detection for all damage offsets. The he pair on the top covers the area shaded in red, while the pair at the bottom covers the area shaded in blue.

Away from the boundary

For pipe B, the setup of actuator and sensor pair is similar to that of pipe A. However, in this case, data is acquired from four sensors at once, to also analyze the effects of sensor locations. Figure 6.5 shows the locations of the actuator, the sensors and various regions, created with respect to actuator-sensor locations, of the pipe. The angular positions of the sensors are 0°, 30°, 60° and 90°.

The same input excitation is used as before. However, one should note that the thickness of pipe B is greater than that of pipe A. Hence, a stronger signal attenuation was expected leading to poorer performance with respect to pipe A. However, the thickness to diameter ratio is still within the limits of helical GUW assumption, hence the proposed semi-supervised approach is still applicable.

Figure 6.11 shows the results when the simulated damage is at an angular position of zero degrees with respect to the actuator. The results for two different angular positions of the sensor, zero and sixty degrees, are demonstrated. Clearly for a damage location of zero degrees, the damaged and undamaged data are well separated. This again, is the best case scenario with minimum signal attenuation, hence produces the best results. The same damage location, but with a sensor that is offset by an angle of sixty degrees, produces
Figure 6.11: Results obtained from pipe B, when the actuator-sensor pair is away from the boundary. Damage is located at an angular position of 0°, in all the regions.

Figure 6.12: Results obtained from pipe B, when the actuator-sensor pair is away from the boundary. Damage is located at an angular position of 60° in all the regions.

worse results. Due to the relative distances in between the damage and the sensor location, it becomes more difficult for the algorithm to classify damages in regions 2 and 3 as damaged situations. This is because, the damage signals from regions 2 and 3 are based on reflected signals obtained from damage.

Similarly, for a damage angular position of sixty degrees, as shown in figure 6.12, the response from the sensor at sixty degrees performs better than the one at zero degrees. For pipe B, it is observed that beyond a damage offset of ninety degrees, the performance of the semi-supervised approach degrades to the point that it becomes unreliable for damage.
detection. However, the key take away from this discussion is that even with very few low quality piezo-transducers as transmitter and receiver, one can perform damage detection up to damage angular offsets of ninety degrees.

**Impact of a boundary on the proposed algorithm**

In this section the effects of a boundary on the performance of the proposed semi-supervised algorithm for damage detection is studied.

![Clustering](image1)

![Ground Truth](image2)

![Clustering](image3)

(a) 0 degrees  
(b) 60 degrees

Figure 6.13 : Results for pipe B when the actuator-sensor pair is near the boundary. Here the results for sensor 3 are shown.

As expected a degradation was observed in the performance of the proposed approach. This is due to the additional reflections from the boundary that corrupt the acquired signals at the sensors. In addition a greater energy dissipation of signals due to interaction with the supports was also expected. Due to additional reflection signals, damage responses from corresponding regions 2 and 3 are adversely affected as they are reflection based. It is demonstrated that damage detection is still possible in region 1.

Figure 6.13 shows the results for damage angular locations of zero and sixty degrees. These results are based on a sensor angular offset of sixty degrees. For damage located at zero degrees, there is no distinction in between undamaged scenarios and damage located
in regions 2 and 3. However, the proposed approach is able to detect damage when located in region 1.

6.6 Conclusions

A semi-supervised learning algorithm for damage detection is proposed for active sensing in steel pipes. The semi-supervised approach requires \textit{a priori} label for just one undamaged data. It then uses a hierarchical clustering scheme for damage detection, efficiently separating damaged and undamaged cases using low-profile piezo-transducers up to damages located up to 90 degrees offset angles from the zenith line of the cylindrical pipe, where the actuators are located. The efficiency of the proposed approach is demonstrated for data acquired from both away from boundaries and close to them. It is envisioned that the data-driven approach can be used for health monitoring of even more complex structural systems such as a network of pipes which is deferred to a future work.
Chapter 7

Temperature PCA

This chapter introduces the effects of temperature variations on bridge monitoring data. A principal component analysis-based algorithm is proposed for decoupling the effects of temperature and damage. The efficacy of the proposed approach is demonstrated using both numerical simulations and experimental data.

7.1 Introduction

The premise of most damage detection algorithms is the effect of physical changes caused by damage in the dynamic characteristics of structures, that in turn reflect on response measurements and damage sensitive features [13, 92, 77]. It is important to note, however, that in addition to structural damage the natural variations in the environmental conditions can also affect the dynamic characteristics of structural systems. In particular temperature fluctuations have an impact on the material properties of the system and the boundary conditions [128]. For example, the approximate relationship between vibration frequencies and elastic modulus given by $f \sim \sqrt{E}$, shows that any dependency of $E$ on temperature will also manifest in the vibration frequency $f$. Consequently, Farrar et al. [129] found that the fundamental vibration frequency of the Alamosa Canyon bridge varies by approximately 5% during a day. Many SHM methods available in the literature neglect variations in operational and environmental conditions (variations in temperature, humidity, wind, among others). However, it has been found that environmental variations can shroud the effects
of damage in many applications leading to inaccurate estimates of the state of integrity of structures [130, 131, 132, 133]. This is a major drawback and questions the reliability of existing methods under various operational conditions typically observed in practice.

Recently, the study of the impact of environmental effects on SHM has garnered a lot of interest from the research community. Subsequently, this has lead to the development of algorithms to decouple environmental effects and structural damage [131]. These methods can be broadly classified as static or dynamic, linear or nonlinear and input-output or output-only. Most of the proposed approaches belong to the class of linear methods, that rely on the assumption of a linear correlation between different damage features (typically vibration frequencies) under variations of temperature. These correlations depend on the temporal length of the analyzed signals. Structures behave like linear time-invariant systems when the time interval considered is small (in the order of minutes). This is due to the fact that the frequency of variation of environmental conditions is much lower than the fundamental period of the structure. Hence, effects of temperature variations may not be observable in such short time intervals. However, when a longer time is considered nonlinearities are observed in the temperature-frequency relations. This is mainly due to the following factors:

1. **Presence of different material properties under non-uniform temperature gradients**, such as steel-concrete composite sections in bridge decks with a layer of asphalt on top. The elastic modulus of these materials increase with temperature in their own unique ways, leading to a differential increase in stiffness of each individual component and discontinuities in the stresses acting on the cross sections.

2. **Changes in boundary conditions**. Environmental variations induce displacements that cause expansion and contraction of joints modifying the boundary conditions.
For example, it has been shown that freezing of moisture induces an increase in the rotational stiffness of the supports of bridges [131].

Yan et al. [40] and Kulla et al. [134] show that linear approaches successfully decouple temperature and damage effects in homogeneous systems. Similarly, Sohn et al. [132] develop a (static) linear regression model for the Alamosa Canyon bridge that relates temperature at different locations to fundamental frequencies of vibration, and use confidence intervals for discriminating between changes from environmental and structural damage effects. In Peeters et al. [6] and Moa et al. [135] regression models are further extended to account for dynamic effects using an auto-regressive exogenous (ARX) model. The model accounts for thermal dynamics, i.e., the dependence of the current dynamic properties' estimates in the past time-history of temperatures. Data from the Z24 bridge in Switzerland and a footbridge under different environmental conditions were used for validating the model. Fritzen et al. [136] propose a linear approach where the response of a possibly damaged structure is compared to a reference undamaged state by computing a residual. The undamaged response is stored by computing singular vectors from vibration measurements at different temperatures. This assumes the orthogonality of the subspaces spanned by the undamaged and damaged data. In addition, the residual norm being non-normalized can take arbitrary values due to its dependence on the length of data set. This potentially leaves difficulties in calibration of the index under different conditions.

The majority of the previously discussed approaches rely on a linear model and an input-output relationship between temperature and fundamental frequencies. More recently researchers have focused on output-only approaches which rely only on response measurements, and do not require explicit measurement of input quantities such as temperature at different locations. In Ref. [40] an output-only linear approach based on principal component analysis (PCA) is proposed to decouple environmental changes from structural
damage. The proposed approach relies on the analysis of residual errors of the PCA prediction. Damage is assessed by novelty detection using the Euclidean norm of the residual. This approach assumes that the variations of temperature and damage are orthogonal. The PCA approach was further extended to account for nonlinearities by using piecewise linear approximations and employing PCA for the linear regions (local PCA) [8]. As mentioned before, nonlinearities arise due to the presence of different temperature gradients and multiple materials (for example, composite beams). An alternative is to use a kernel PCA approach to take into account the nonlinearity in time intervals in the order of hours or days [137].

In this chapter an approach is presented to decouple structural damage and temperature effects based on the application of principal component analysis (PCA). The proposed approach uses data projections on the principal components space directly for damage detection. To this end, it is shown that sudden discontinuities in the principal components are observed at the time instants that damage is induced, and that these discontinuities are observed in the projections of the vibration data on the principal component vectors. The approach has the capability to perform temporal damage identification. A damage index is proposed as a quantitative measure of damage for SHM applications. Considering the above advantages, the proposed algorithm can be implemented as part of a real-time damage identification framework.

The proposed approach is numerically validated on a multi-layer composite section model of a two span bridge, where the results show that the approach is effective in demarcating the effects of structural damage under different environmental conditions. It is shown that the approach is robust to mild nonlinearities caused by the effect of temperature variations on material properties of composite sections and boundary conditions. Finally, the efficacy of the algorithm is studied using data from a full-scale bridge in Switzerland.
The experimental validation shows that the approach effectively decouples damage and temperature effects for different damage states under conditions typically observed in practice.

### 7.2 Decoupling damage and temperature effects using PCA

This section discusses the proposed PCA-based algorithm. A set of identified modal frequencies were used as data for PCA. The key idea is to use data projections on principal component planes directly. Even if environmental effects and damage are not orthogonal, the separation will be apparent in the principal component planes as long as the correlations between the various modal frequencies is not highly nonlinear. Since a damage event can be assumed to be sudden with respect to environmental variations (which are relatively slow), temporal variation of data projections on individual principal components will capture such sudden changes. A damage index as a function of the slope of temporal variations of these projections is defined heuristically, and use them for temporal localization and quantification of damage.

The algorithm is as follows:

- Acquire a set of undamaged data (first \( p \) modal frequencies) \textit{a priori}, \( \mathbf{X}_{ap} \in \mathbb{R}^{n \times p} \), where \( n \) is the number of undamaged data points acquired \textit{a priori}.

- Apply PCA on \( \mathbf{X}_{ap} \) and acquire principal component vectors \( \mathbf{V}_{ap} \), where each column of \( \mathbf{V}_{ap} \) is a principal component vector. In addition store the projected data on these principal component directions \( \hat{\mathbf{X}}_{ap} = \mathbf{X}_{ap} \mathbf{V}_{ap} \).

- Acquire new data set of first four modal frequencies which might contain damaged data as well, \( \mathbf{X}_{new} \in \mathbb{R}^{k \times p} \).
• Project the newly acquired data on the principal directions obtained from the data acquired \textit{a priori}, \( \hat{X}_{\text{new}} = X_{\text{new}} V_{\text{ap}} \).

• Congregate the \textit{a priori} and new projected data sets producing a matrix \( U \in \mathbb{R}^{(k+n) \times p} \). Each column of this matrix is a projection of the data in each principal component direction, \( U = [PC_1(t) \quad PC_2(t) \quad PC_3(t) \ldots \quad PC_p(t)] \), where \( PC_i(t) \in \mathbb{R}^{(k+n) \times 1} \). It should be noted that the minor dimension is still \( p \) because there were only \( p \) variables in the data.

• Use a damage index which is a function of the slope of the projection of the data on individual principal component direction, \( \frac{dPC_i(t)}{dt} \), where \( i \in [1 \quad p] \). The sudden change in damage will be apparent in these damage indices.

The above algorithm has been summarized in the flowchart depicted in figure 7.1.

![Flowchart](image_url)

**Figure 7.1**: The PCA-based algorithm for decoupling the effects of temperature and damage.

In this study, only the first four modal frequencies are acquired for damage detection. Hence, \( p = 4 \). The damage index is based on the idea that if \( PC_i(t) \) has a sudden jump or a
discontinuity, it will be reflected in the slope of $PC_i(t)$. It is worth to point out that this is an heuristic damage index, since rigorously speaking the slope is not defined at discontinuities. Instead, the derivative at the jump is heuristically interpreted as a Dirac delta function.

7.3 Numerical validation

In this section the effectiveness of the approach described in the previous section is studied using a numerical example. The cases studied include the assessment considering the two main sources of nonlinearity in problems involving changing temperature fields, namely composite structures and temperature-dependent boundary conditions.

7.3.1 Structural model

For the numerical validation a continuous two-span composite beam with the dimensions shown in figure 7.2 is considered; this configuration is typical of steel girder highway bridges [138]. The model is discretized into 90 beam elements, each of length 2 feet. The finite element modeling and analysis is performed using the MATLAB-based free-ware CalFEM [139]. The temperature field is modeled as a uniform field along the length and depth of the beam; the robustness to non-uniform fields is assessed in the experimental validation section of this chapter. Structural damage is modeled as a reduction in the stiffness of an element at $2ft$ from the middle support (see Figure 7.2).

Two case studies are considered based on the two different beams shown in figure 7.2. As discussed earlier, the literature lists two main sources of nonlinearity in damage detection due to temperature effects, namely differential change in material properties of different materials, and changes in the boundary condition. In case study 1 (Beam 1), the boundary conditions are temperature-independent. However, three materials constitute the composite cross section: steel, concrete and asphalt. These three materials respond
Figure 7.2: Bridge beam model used for the numerical validation. Beam 1 is a composite beam with a steel-concrete-asphalt cross section. Beam 2 is a steel beam with springs-restrained ends, which is used to model the temperature-dependent boundary conditions. The red elements are the potential damage sites. Damage is simulated by reducing the stiffness in the labeled elements.

differently to temperature changes which leads to a nonlinear relationship between modal frequencies and temperature. In case study 2, a steel beam (Beam 2) is considered with temperature-dependent boundary conditions. The temperature models are described next.

It is well-known that temperature variations affect the microscopic and macroscopic properties of materials. The cause of change in mechanical properties is related to the variation of bond forces holding atoms together; roughly speaking, the macroscopic properties, such as Young’s modulus are an average manifestation of such atomic forces. For example, at high temperatures the strength of structural steel decreases and its ductility increases, while at low temperatures the strength increases and the ductility decreases (eventually becomes brittle). In this work it is of interest to study the effect of temperature changes in the vibration characteristics of a beam. To describe the temperature-elastic modulus relationship several models have been proposed; the following recently proposed model is adopted.
for structural steel [140]

\[ E(T) = e_0 + e_1 T + e_2 T^2 + e_3 T^3 \quad T \geq 0^\circ C \]  

(7.1)

where the temperature \( T \) is given in Celsius degrees

\[ e_0 = 206 \text{GPa} \]
\[ e_1 = -4.326 \times 10^{-2} \text{GPa/}^\circ C \]
\[ e_2 = -3.502 \times 10^{-5} \text{GPa}/(^\circ C)^2 \]
\[ e_3 = -6.592 \times 10^{-8} \text{GPa}/(^\circ C)^3 \]

For temperatures below \( 0^\circ C \), an approximately linear relationship is assumed with slope \( e_1 \) (ASME B31.1-1995). Below freezing, the effects of temperature on modulus of elasticity is significantly less, in contrast to the behavior at high temperatures. However, one may observe a substantial reduction in ductility at such low temperatures. As can be seen, for temperatures up to about \( 200^\circ F \) the relationship is linear. It should be pointed out, however, that the relationship between vibration frequencies of structural systems and the elastic modulus is nonlinear: \( f \sim \sqrt{E} \).

For reinforced concrete, the following model is used for the variation of the ultimate strength with temperature [141]

\[ f'_{c,T} = \begin{cases} f'_c[1.0 - 0.003125(T - 20)], & T < 100^\circ C \\ 0.75 f'_c, & 100^\circ C \leq T \leq 400^\circ C \\ f'_c[1.33 - 0.00145T], & 400^\circ C < T \end{cases} \]  

(7.2)

where \( f'_c \) is the 28 day compressive strength of concrete at \( 20^\circ C \) and \( T \) is the temperature in \( ^\circ C \). The AASHTO Load and Resistance Factor Design Manual [142] relates modulus of elasticity of concrete to its strength as

\[ E_c = 1820 \sqrt{f'_c} \]  

(7.3)
where $f'_c$ is in ksi. In addition to steel and concrete, the effect of asphalt on bridge behavior is also studied. It has been shown in previous work that because of the sensitivity of the asphalt elastic modulus to temperature variations, the asphalt is a major contributor to changes in the stiffness of bridges [6, 143]. Chen et al. [144] defines the variation of modulus of elasticity for asphalt as follows:

$$E_T = E_{68} 10^{-0.0153(T-68)}$$

(7.4)

where $E_{68}$ is the modulus of elasticity of asphalt at 68°F. Figure 7.3 shows the variation of modulus of elasticity with temperature.

![Graph showing variation of modulus of elasticity with temperature for steel, concrete, and asphalt](image)

Figure 7.3: Variation of modulus of elasticity with temperature for the three materials

In addition to the temperature-dependent mechanical properties, the boundary conditions are also modeled as temperature-dependent. For this purpose rotational springs with temperature-dependent stiffness $k_\theta$ are added to the two ends of the beam to model freezing of moisture at the abutments (see Figure 7.2). The value of $k_\theta$ governs the degree of restraint in the rotation of the end supports. The spring model that is used is adapted from
Hsu and Loh [145], where the stiffness-temperature relation is given by a linear function. Based on the numerical results for stiffnesses of the order of $1 \times 10^{14}$ the frequencies are close to those of a fixed-end beam. Figure 7.4 shows the model adopted for the variation of $k_{\theta}$ with temperature.

![Variation of Rotational stiffness](image)

Figure 7.4: Variation of rotational stiffness with temperature.

### 7.3.2 Case study 1 results

As discussed earlier, case study 1 involves a study where the effect of nonlinear behavior due to temperature variations in a composite section is considered. The materials involved are asphalt, concrete and steel comprising the cross section of a typical bridge deck, and the beam is subjected to a uniform temperature field. Damage is modeled as a reduction of flexural stiffness of an element close to the middle support where the negative bending moment is maximum. In addition, the possible damage sites are restricted to steel and concrete portion of the cross section of the beam, since in practice this is the portion that provides most of the flexural stiffness. Figure 7.2 depicts the model.

To gain insights to the problem at hand, the sensitivity and influence of vibration fre-
frequencies to both temperature variations and damage is first assessed. This entails the study of the behavior of modal frequencies of the bridge under such temperature variations. Figure 7.5 shows the temporal variation of temperature adopted. The temperature data obtained from measurements made on the Beijing-Hangzhou Grand Canal Bridge [5] was used. The mean of the above data was brought down to 0°F so that the effects of nonlinearities due to asphalt on modal frequencies can be maximized (material nonlinearity of asphalt in temperature range near 0°F is significant as shown in Figure 7.3).

![Daily mean temperature variation](image)

Figure 7.5: Variation of temperature with time measured on a box girder bridge. The mean value is used for our analysis.

Figure 7.6 shows the variation of modal frequencies with temperature, and also the effect of the presence of damage. It is observed that modes 2 and 4 are the most sensitive to damage. This is related to the location of the damage, since the curvature of modes 1 and 3 is small near the damaged region. Figure 7.7 shows the correlation between the first four modal frequencies for all the undamaged and damaged cases with the presence of temperature changes, where an approximately linear correlation is observed. In addition, shifts due to presence of damage can be seen for the second and fourth mode. This linearity is amenable for application of PCA to decouple these effects.
As discussed earlier, PCA is used for decoupling the effects of temperature variations and structural damage. The data matrix $X$ consists of the first four modal frequencies for
undamaged and damaged states at various temperatures. It should be noted that, the data set consists of only four features which are the first four modal frequencies. Figure 7.8 shows the data visualized in the various combination of PC planes. It is observed that PC1 primarily captures the variation in temperature. The lower PCs capture the effects of damage as the separation between various damaged and undamaged cases can be observed. However, PC2 incorporates the separation due to damage linearly. When the data is projected on to PCs 3 and 4, it takes the form of a highly nonlinear function which might not be amenable for damage detection purposes.

Figure 7.8: PCA of the whole data set showing that effects of temperature and damage can be separated in a PC space

The following protocol for damage detection is adopted. It is assumed that vibration data is available in both the undamaged (healthy) and damaged states. To compute the baseline system PC vectors, data of approximately a full-day is employed. The length of data should be such that, undamaged data for most of the temperatures, in the variation
range, aids in construction of the principal component directions. Subsequently, all future data points are projected on to the principal component planes obtained from the initial data. The premise of the approach is that PC projections of the data can detect changes that may not be evident in the modal frequency data. The following six possible damage scenarios for the bridge are studied:

- **Damage Case 1**: A 5% reduction of the concrete stiffness taking place after 36 hours.
- **Damage Case 2**: A 5% reduction of the steel stiffness taking place after 36 hours.
- **Damage Case 3**: A 5% reduction followed by a 20% reduction of the concrete stiffness at 36 and 48 hours respectively.
- **Damage Case 4**: A 5% reduction followed by a 20% reduction of the steel stiffness at 36 and 48 hours respectively.
- **Damage Case 5**: A 5% reduction of the concrete stiffness followed by a 20% reduction of the steel stiffness at 36 and 48 hours respectively.
- **Damage Case 6**: A 5% reduction of the steel stiffness followed by a 20% reduction of the concrete stiffness at 36 and 48 hours respectively.

Figure 7.9 shows the first four modal frequencies, as well as the projections of the data on the first four principal component vectors. The black portion of the curve signifies the data used for obtaining the principal component vectors. This is similar to training in a statistical learning setting. As described earlier, the principal components are then used for obtaining the PC projected data depicted by the blue curve. In the PC projections of the data, it is clear that a sudden change is visible in the PC projections. An abrupt change is however not visible in the frequency plots, showing the utility of observing the data in a
Figure 7.9: The temporal variation of modal frequencies and principal components for damage case 1. Clearly the damage is picked up by PCA projected space. In addition, the time instant of the sudden change corroborates with the instant of advent of damage.

Figure 7.10 shows the modal frequencies and PC projections for damage case 6. As discussed earlier, the structure is inflicted with two types of damage. The first is a 5% stiffness reduction in steel after thirty six hours followed by a 20% stiffness reduction in concrete...
Figure 7.10: The temporal variation of modal frequencies and principal components for damage case 6. Clearly the damage is picked up by PCA after forty eight hours. This is depicted by the red dotted and dashed lines in the figure. Similar to damage case 1, the changes in the modal frequencies are negligible. However when the data is predicted onto the principal component directions the effects of damage are enhanced. It can clearly be observed the discontinuities in the PC projections over time. The time coordinate of the discontinuities allows for this approach to be applicable for real
time structural health monitoring.

It should be noted that the extent of these discontinuities are proportional to the extent of damage as can be seen in Figure 7.10(b). As discussed earlier, for this system, data projected on principal component 2 helps to identify the presence of damage as the discontinuities are most pronounced numerically. For the purposes of damage detection, a heuristic damage index is proposed which is a normalized slope of the projection of the data on second principal component

\[
DI_2 = \left| \frac{dPC_2(t)}{dt} \right| \Delta t
\]  
(7.5)

where \(PC_2(t)\) is the projection of the data over time on principal component 2 and \(\Delta t\) is the time step. The slope of the curve quantifies the presence and extent of discontinuity in the PC projections over time. The use of the time step, \(\Delta t\), for normalization is to make the damage index independent of the sampling frequency of the measurements. If the frequencies are obtained from input/output or output only system identification, this sampling frequency is related to the time window used for identification of the system using these algorithms. It should be pointed out that this damage index is heuristic and not rigorous, since the slope is not defined at points of discontinuity. The heuristic damage index adopts the use of the Dirac delta function as the derivative at a discontinuity.

Figure 7.11 shows the \(DI_2\) for all the damage cases defined earlier. As discussed earlier, the damage index spikes whenever a damage event occurs. This provides information about the damage time. In addition, the magnitude of the spikes are proportional to the extent of damage. A simple correlation study would yield the underlying relationship.
Figure 7.11: The damage index for all the six damage cases showing that the occurrence time of damage can be evaluated using PCA.

### 7.3.3 Case study 2 results

In this section the effect of temperature variations on boundary conditions is assessed. For this purpose the model labeled as “Beam 2” in Figure 7.2 is considered. As mentioned before, due to freezing of moisture in bridge abutments the boundary condition changes
as temperature decreases [145, 132]. This is modeled using rotational springs at the beam ends with temperature-dependent stiffness. A preliminary analysis is performed, similar to case study 1, to study the sensitivity of the beam to both temperature and damage.

![Figure 7.12](image)

Figure 7.12: First mode shape and corresponding frequency for the unrestrained and fully-restrained rotation of member ends.

The most significant impact of a temperature dependent boundary condition is the drastic change in mode shapes and corresponding modal frequencies. Figure 7.12 demonstrates the significant change in modal frequency of the first mode. Such variations are also observed in the higher modes. These temperature variations are capable of shrouding any effect due to damage on the modal frequencies, which make SHM methods based on the direct analysis of identified frequencies perform poorly. To capture the impact of changes in the stiffness of the rotational spring due to temperature variations, the mean daily temperature variation from the previous case study is modified such that the average is set at 32°F. This ensures that the bridge will have both simply supported as well as near fixed boundary conditions. Figure 7.13 shows this modified temperature variation.

For this case study the following damage scenarios are considered

- **Damage Case 1**: A 5% stiffness reduction at 36 hours.
• **Damage Case 2**: A 20% stiffness reduction at 36 hours.

• **Damage Case 3**: A 5% stiffness reduction at 36 hours followed by a 20% stiffness reduction at 48 hours.

Figure 7.14 shows the behavior of the vibration frequencies of the first four vibration modes under both damaged and undamaged conditions. Clearly, the effect of the varying boundary conditions are far more significant than that of damage. In addition, the behavior of the frequencies with respect to temperature shows an increased nonlinearity with respect to the previous case. Figure 7.15 shows the correlation between the various modes, where it can be observed the nonlinear nature of the correlation between the various modal frequencies. These nonlinearities make damage detection a challenging task.

Figure 7.16 shows the projection of the data in two dimensional spaces of various combinations of principal components as basis vectors. Nonlinear relationships are observed between the various projections on principal components. However, principal component 3 of the data projection, is able to discern the undamaged and damaged cases. This fact is used for construction of a damage index for this case study. Figure 7.17 shows the modal
Figure 7.14: Modal frequencies sensitivity to temperature effects.

Figure 7.15: Correlation structure between modal frequencies. The black straight lines aid in observing the nonlinear nature of the correlation.

frequencies and corresponding principal component projections of the data over time for damage case 3. It is observed that the curves are far more complex with respect to the
Figure 7.16: Principal components plots for the complete data set.

previous case. The PC projections of the data allows one to observe the sudden changes due to advent of damage. Using an appropriate damage index, one can perform damage detection. As discussed earlier, the third principal component projection of the data aids in damage detection, hence, the damage index is defined as follows:

$$DI_3 = \left| \frac{dPC_3(t)}{dt} \right| \Delta t$$  \hspace{1cm} (7.6)

where $PC_3(t)$ is the projection of the data over time on principal component 3 and $\Delta t$ is the time step for this data. Figure 7.18 shows the damage index for the three cases. By setting an appropriate threshold, damage detection is possible as can be observed from the temporal concurrence of damage time and the damage index. Similar to the previous case, the extent of damage is correlated to the magnitude of the damage index.
7.4 Experimental validation

In this section the proposed algorithm is employed on the data from the Z24 bridge (shown in figure 7.19) in Switzerland obtained during a SHM program [6]. The monitoring involved both temperature and vibration measurements. The bridge was artificially damaged over time to varying degrees during the course of the year. The readers should refer to [6] for details of the experiments conducted. From the vibration measurements, the first four
modal frequencies were estimated. Similar to case studies 1 and 2, this data is used for our PCA-based damage detection approach.

Figure 7.20(a) shows the variation of the first four modal frequencies of the Z24 bridge over time. The first damage is induced around day 269 [8]. This is associated to foundation settlement. Subsequently, the bridge is progressively damaged experiencing tilt of foundation, concrete spalling, landslides, concrete hinge failure, anchor head failure and finally tendon ruptures. First, the principal component directions are obtained using the undamaged data that will be used for performing damage detection. As in previous cases, this training is crucial as it should consist of all possible temperature changes in the bridge. Finally, the principal directions are used to project all future data and obtain the variations of each principal component over time.
Figure 7.19: A schematic of the Z24 bridge [6, 7].

Figure 7.20: (a) Variation of the first four modal frequencies of the Z24 bridge over time. Damage was induced in day 269, and the effects are seen most clearly in mode 2. The jumps in frequencies prior to day 269 are solely due to temperature variations. Clearly the changes due to temperature are more significant than the induced damages. (b) Variation of air temperature with time.

Figure 7.21 shows the principal component projections of all the complete available data. Based on the spread of the data it is clear that PC 1 captures the spread of the tem-
Figure 7.21: Projections of the data set in the first four principal component direction planes. The red triangles indicate the data in the damaged state.

Figure 7.22 shows the variation of the PC values over time and observe that PC shows the same behavior as the measured temperature curve shown in figure 7.20(b). It can also be observed that principal component PC 3 shows a clear downward trend as the extent of damage increases.

Figure 7.23 shows a zoomed version of PC 3 and 4. The figure depicts the type and instant of damage on the Z24 bridge. A reduction in the mean trend is clearly observed in PC 3 resulting from the series of induced damages. It is clear that the installation of the settlement on day 269 is a major event as it can be clearly observed in both PCs 3 and 4. In addition, it can be seen that the tilt of foundation starts a downward trend in PC 3 and shows as a spike in PC4. The other damages however, fail to show up as major spikes. This
may be due to the severity of the damages discussed above.

7.5 Conclusion

In this chapter a framework is proposed to decouple structural damage and environmental effects. The approach is based on the application of principal component analysis and a change in coordinates of the available data. For this purpose the principal component directions are computed using a data set acquired at a reference (undamaged) state of the structure. All future data from both undamaged and a potentially damaged state is projected on to the principal directions obtained using a reference data set. The projected data sets allows for the decoupling of temperature and damage effects. Subsequently, the slopes of the projected data on each principal component for structural damage diagnosis are used. The use of principal components allows to separate the effects of temperature variations and structural damage.
The proposed algorithm is validated numerically using a two-span beam model of a bridge under various conditions, including composite sections and temperature-dependent boundary conditions. It was shown that the approach has the capability of detecting structural damage when the beam is subjected to varying temperature fields. The approach is further validated experimentally using data from a structural health monitoring program implemented on the Z24 bridge in Switzerland. In this case the method also showed to be effective in the temporal detection of damage. The experimental validation shows that the approach is robust to the typical conditions seen in practice for this type of problems, such as non-uniform temperature fields, temperature-dependent boundary conditions, interaction of various temperature-dependent material properties, among others.
Figure 7.23: Zoomed version of (a) PC 3 and (b) PC 4. The instants of induced damage on the bridge are also shown [8].
Chapter 8

Contributions and Future work

8.1 Contributions

This dissertation proposes statistical learning and stochastic filtering based algorithms for damage detection in a variety of infrastructure systems. The key goal of this work was to circumvent high-fidelity modeling of systems for damage detection by using sensor data primarily for the purposes of damage detection. Additionally, the proposed algorithms also ensure the minimization of actuators and sensors necessary for damage detection. This makes the proposed algorithms suitable for SHM of structural systems. The major contributions of this thesis are as follows:

1. *Sparsity-based damage detection in beams and plates*: Formulated a sparsity-based formulation for damage detection in beams and plates using high frequency wave-based active sensing. This entails construction of a dictionary consisting of damaged signatures obtained from an experimental setup, hence, circumventing the need for development of high-fidelity models. Damage detection and localization is possible using one actuator and one sensor. This work is summarized in [146, 121].

2. *Damage detection in pipes*: A semi-supervised learning scheme is proposed for level I damage detection in cast iron pipes. A hierarchical clustering-based algorithm is developed to distinguish damaged and undamaged signals acquired from a pipe. It is demonstrated that with two actuator and two sensors level I damage detection is
possible with 100% accuracy. In addition to the use of minimal low-cost sensors, the proposed algorithm avoids extensive feature extraction and training. This work is summarized in [147].

3. Acoustic emission source localization: A Bayesian parameter estimation-based algorithm is developed for acoustic emission source localization that accounts for uncertainties associated with the problem. A particle filter-based algorithm is proposed that facilitated the use of a non-informative prior for the Bayesian framework. This ensured the possibility of efficient source localization with only three sensors, the minimum number necessary for triangulation on a two dimensional plane. Additionally, it was also demonstrated that the proposed algorithm reliably localized acoustic sources when they exist outside the region bounded by the sensors. This work is summarized in [148].

4. Decoupling the effects of temperature and damage: A principal component analysis-based approach to decoupling effects of temperature variations and damage was proposed for applications in bridge monitoring. This linear approach was shown to effectively segregate the effects of nonlinear variations in temperature and damage. The efficacy of the proposed algorithm was further demonstrated on simulation data as well as data acquired from the Z24 bridge in Switzerland. This work is summarized in [149, 150].

8.2 Future work

The algorithms proposed in this dissertation have demonstrated their efficacy through both simulation and experimental results. However, prior to implementation for real field monitoring, further investigations are necessary. This section summarizes the future research
directions necessary for further extending the work in this dissertation.

1. **Sparsity-based damage detection**

   - A detailed finite element analysis and experimental study needs to be conducted to fully understand the capabilities of attaching masses on a system to simulate the effects of damage. For real field applications, it is necessary to calibrate different masses to various types of damage that one may encounter.
   
   - The limitations in terms of damage sizes that the proposed algorithm can accurately detect and localize should be studied. This is extremely pertinent when dealing with multiple damages.
   
   - The proposed sparsity-based framework produces a deterministic solution. Extending the framework, to estimate probabilistic solutions, that better account for uncertainties, will make it more amenable for real field applications.

2. **Damage detection in pipes**

   - The impact of temperature and environmental conditions on the proposed algorithm needs to be studied.
   
   - Although the proposed algorithm accurately detects cracks, further field testing is necessary to determine its efficacy when dealing with corrosion like damages.
   
   - The impact of fluid flow on the performance of the proposed framework should be studied.

3. **Decoupling effects of temperature**

   - The proposed algorithm requires testing on more data sets before it can be implemented in real field monitoring.
• The proposed algorithm utilizes the first four modal frequencies for damage detection. The algorithm can be extended for use with more damage-sensitive dynamic characteristics of a structure.

• When dealing with concrete bridges, the effects of humidity needs to be taken into account. A further study should be undertaken to understand the confluence of the three variables namely, temperature, humidity and damage.
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