Microwave Spectroscopy on Two Dimensional Electron/Hole Gases

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To my husband Nick
ABSTRACT

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We develop new techniques to explore how electrons and holes in 2D semiconductors behave under microwave radiation in a low temperature regime. To observe cyclotron resonance (CR) in GaAs/AlGaAs quantum wells, thermal methods with the sample and thermometer sealed in a vacuum can in a 3He environment and reflection spectroscopy via power sensing are developed. This configuration has high sensitivity and can detect CR with only 10 nW of microwave power. It obviates the need for sampling thinning/wedging which is required in conventional transmission spectroscopy and hence preserves sample quality. With these advantages, we are able to detect narrow CR peaks with a small full width at half maximum (FWHM). Carrier effective masses are measured in various two dimensional electron gases (2DEGs) and two dimensional hole gases (2DHGs). Transport scattering time and single particle relaxation time are two important time scales that can be extracted from this measurement. The ratio of these times can be used as an indication of scattering angle scale limited by carrier mobility. The most exciting feature of this measurement is that a multi-photon phenomenon is observed for the first time in this system other than when using photoresistance measurements. This is made possible due to the use of millimeter waves, as opposed to the more traditional terahertz and infrared regimes used in earlier configurations. We further investigate the density dependence of the observable order of the multi-photon transitions and find that it is more prominent in
higher density samples. By simultaneously measuring microwave reflectance and electrical resistance, we find that the plasmon-coupled CR mode is only present in the optical signal, not in the electrical. We attribute this discrepancy to the difference in the ability to pick up the corresponding signal in the scattering process. Though experimental data is very convincing, theoretical explanation is still needed to account for these phenomena.

We also analyzed the photoresistance of the InAs/GaSb inverted bilayer. This system is known for its marvelous helical edge state in the quantum spin Hall effect (QSHE). By tuning the back gate, the Fermi level can be positioned in the bulk gap where the conductivity only comes from the edge state. This is verified by taking measurements on a Corbino disk where the edge state is shunted. We find that the photocurrent is prohibited in the charge neutral point (CNP) due to the lack of spin flipping mechanisms. A finite magnetic field provides a source for this process from which we see an enhancement of the photocurrent. Further effort in observing gap opening and resonance with the Zeeman energy needs to be made to understand this interesting system. Therefore, better wafer quality is strongly desired.
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Chapter 1

Introduction

Microwave spectroscopy is a powerful tool to study the interaction of electromagnetic fields with two dimensional charge carriers in semiconductor quantum wells due to their similar energy scales. It is of significance to fundamental research such as the origins of microwave-induced resistance oscillations (MIRO) [1] and zero-resistance states (ZRS) [2] discovered decades ago. It also has potential applications in semiconductor band engineering.

The invention of molecular beam epitaxy (MBE) by Arthur, Cho and Foxon in the late 1960s at Bell Telephone Laboratories has led to precise control of chemical compositions and doping profiles. The ability to grow high-purity monolayer semiconductor crystals was a prerequisite for the discovery of the integer quantum Hall effect (IQHE) by von Klitzing and the fractional quantum Hall effect (FQHE) by Stormer and Tsui; while the ability to grow metal layer superstructures led to the discovery of giant magnetoresistance by Fert and Grunberg, both of whom were awarded the Nobel Prize.

Since its observation in Ge crystals in 1953 [3], CR has been used to study band structure, anisotropy of the effective mass tensor and Fermi surfaces. Far-infrared CR was first observed in silicon [4, 5], where it was found that charge carrier effective mass was independent of electron density. Nevertheless, one still has to account for many-body effects arising from impurity scattering if electron-electron interaction is neglected [6]. Recently, the fine structure of CR was discovered by standard transmission spectroscopy, revealing a series of peaks with spacing inversely proportional to the cyclotron frequency lying under the dominant CR absorption peak [7].

The multi-photon phenomenon in quantum wells has practical applications such as realizing all-optical switches in a non-linear photonic crystal [8], optical limiting with
semiconductors [9], [10] and four-wave mixing [11]. It was also predicted that emission of entangled photons can be generated by two-photon absorption. Multi-photon CR absorption of laser radiation has been predicted to dominate over single-photon absorption [12]. Indirect evidence of multi-photon CR subharmonics has been seen in 2DEGs [13] and photoresistance measurements also show the signature of this phenomenon. However, direct evidence in terms of power absorption has not been previously reported.

This thesis presents a new reflection method developed by the author for studying the multi-photon transitions in ultra-clean high mobility GaAs/AlGaAs quantum wells and discusses the underlying physics of this system. We also apply microwave radiation on another type of double quantum wells which has a topological effect in the quantum spin Hall regime. On the way to this goal, we also developed a thermal method to observe CR at 3He temperatures, which has high sensitivity and works for crystals with strong electron-phonon coupling. We also developed a coaxial cable-based reflection method which later we found to suffer from unwanted multiple reflections and resonances. But using on a similar idea, we devised a microwave reflection interferometer with a rectangular waveguide which has superior performance in detecting CR.

In the background chapter of this thesis, we will provide an overview of low temperature quantum transport where mesoscopic physics is involved. The ability to fabricate devices down to micron or sub-micron size using lithographic techniques allows us to probe quantum mechanical motion of charge carriers in condensed matter systems. The advantage of using high mobility samples is that electrons can travel across the sample before experiencing any scattering events—the so-called ‘ballistic transport’. When the energy spacing of the Landau levels matches the energy of incident microwave photons, we see resonant absorption. These resonances manifest themselves as peaks in either optical absorption or electrical resistance measurements, which we discuss in the following chapters.

We find that by using a microwave interferometer where the sample is regarded as a perturbation, the reflection signal has high sensitivity to the sample absorption. With
this device, CR peaks can be resolved with as little as 10 nW of microwave power. It also obviates the need for sample thinning and polishing usually required in standard transmission spectroscopy, and affords the possibility to use a back gate to tune the Fermi level. We find that peaks related to multi-photon absorption are more prominent on samples with higher density. Therefore, we performed measurements on high mobility samples with high carrier density, and found that peaks are well resolved due to their small FWHM. We confirm the hypothesis that more orders of multi-photon transitions are observed on higher density wafers. We also find that the plasmon-coupled CR mode which is well observed in optical probing is always absent in electric transport measurement. We will present a tentative hypothesis for why this mode is not observed in the sequel.

In the next chapter, we present the results of photocurrent measurements generated by the helical quantum spin Hall edge states which are topologically distinguished from the chiral quantum Hall edge. The unique feature of this conducting edge in the background of insulating bulk states when the Fermi level is positioned in the gap includes the locking of spin and momentum which is characteristic of a Luttinger liquid. By applying a back gate in the InAs/GaSb double quantum wells, photocurrent and photovoltage can be scanned by Fermi level with respect to the band structure. The TRS protected edge state would generate a negligible photocurrent at the CNP due to the lack of spin flipping mechanisms. The application of a magnetic field can provide this mechanism from where we observe photocurrent enhancement.

We then summarize the results along with the discussion of possible research directions in this area. Rapid progress of theoretical topological materials research together with experimental techniques using low temperature and high magnetic fields provide platforms where new physics arises.
Chapter 2

Quantum Transport in Two Dimensional Electron Gases

2.1 Two Dimensional Electron Gases and Landau Levels

2.1.1 2D Electron Realization

A microscopic description of the motion of electrons in semiconductor crystals requires innumerable degrees of freedom in the phase space. However, macroscopically, they are well described by just a few parameters. When we reduce the electronic degrees of freedom (say, by constraining dimension), diverse and amazing phenomena arise since more subtle effects like the interplay among many-body Coulomb, Zeeman and exchange effects begin to dominate the interactions.

Low dimensional quantum materials are usually realized in semiconductors by confining charge carriers with potential barriers in certain directions. This drives the interest in fabrication of structures like the two dimensional plane, one dimensional wire or zero dimensional dot. This confinement for two dimensional electron/hole gases can be provided in a couple of ways. The inversion layer in silicon metal-oxide-semiconductor field-effect transistor (MOSFET) is a widely used example where electrons under the oxide gate are confined to the semiconductor-oxide interface. Rectangular or triangular quantum wells in semiconductor heterojunction takes advantages of the difference in band alignment of materials with similar lattice constants. By shrinking the well width so that only the lowest sub-band is occupied, electrons are only allowed to move in the $x$ and $y$ directions. Other methods, for example free electrons floating on the surface of liquid helium, surface states in 3D topological insulators (TI), or the atomically thin 2D materials like graphene
and molybdenum disulfide are of great interest to research scientists. The discovery of 2DEG at the LaAlO$_3$/SrTiO$_3$ interface in 2004 which has relatively high superconducting temperature also draws large attention.

GaAs/AlGaAs quantum well plays a special role in the quantum Hall transport area due to its high mobility up to $10 \times 10^7$ cm$^2$/V s. Since the lattice constants of GaAs and AlGaAs are similar, it is possible to grow arbitrary layer thickness without worrying about the strain. Intrinsic GaAs crystal has a zinc-blende structure (2.1) and is insulating at low temperatures (so is AlGaAs), thus either doping or gating must be performed to introduce charge carriers in this system. Electrostatic gating is usually done by applying a different gate voltage to change the Fermi level, i.e. electron density. But the application of external electric fields sometimes will not only affect the band structure, but also broaden the wavefunctions.

Figure 2.1: GaAs crystal structure. Courtesy of bandgap.io.
2.1.2 Doping Scheme

Directly doping the GaAs QW is unwise because of the large scattering from the ionized donors. Usually the doping layer is about 50–100 nm away from the 2DEG so that only small angle scattering is seen by the free electrons. This scheme is called *modulation doping*, and provides free carriers without degrading the electron mobility.

However, there are still residual impurities and random disorder potentials limit the mobility. Increasing charge density is one way to improve the situation because the electron density situates itself to screen the remote disorder potential. There are several ways to increase electron density, such as increasing doping or decreasing the spacer thickness. The former introduce more random ionized donors (assuming doping level is not high enough to form impurity band and causes parallel conduction) and the latter increases the strength of the scattering, both of which are unfavored. In addition, due to the limit of the magnetic field, the electron density of the wafer desired has a upper bound in most of the cases.

In order to tackle this problem, a more dedicated doping scheme was proposed where short-period superlattice structure with very thin GaAs layers separated by AlAs layers is used [14,15]. Here Si doping is only placed in the GaAs layer. Note that AlAs is an indirect semiconductor where the $X$ point energy is lower than the $\Gamma$ point. Therefore, for $\Gamma$ point electrons, AlAs serves as a barrier for GaAs and for $X$ point electrons, GaAs serves as a barrier for AlAs.

Overall, this superlattice structure raises the energy and improves the charge transfer for the 2DEG. In addition, since the effective mass of the electrons in AlAs layer is larger than that in GaAs layer, the carriers don’t contribute to the parallel conduction strongly but are still mobile enough to provide screening effect for the remote ionized donors. Here Si doping is chosen to be placed in the GaAs layer because creation of DX centers when happens in the GaAs layer, thus charge transfer efficiency is compromised because of the persistent photoconductivity effect. This is due to the charge trapping in the DX centers when illuminated with a light emitting diode.
Figure 2.2: Band structure of GaAs. Calculated using bandgap.io.

Figure 2.3: Band structure of AlAs. Calculated using bandgap.io.
Therefore, designing the structure of MBE grown samples requires a balance among all the parameters besides the precise control of atomic ingredients and temperature. Usually simulations are done before actual growth to avoid repeated parameter adjustments. Appendix C shows the result of band structure and wavefunction simulated a 1D Poisson-Schrödinger solver.

2.1.3 Landau Gauge

Without a magnetic field, the mean free path (MFP) of an electron with mobility \( \mu \) and transport scattering time \( \tau \) is

\[
l = \nu_F \tau = \sqrt{2\pi n} \frac{\hbar \mu}{e} \approx 5.2 \mu m \times \mu [10^6 \text{cm}^2/\text{Vs}] \sqrt{n[10^{11}/\text{cm}^2]} \tag{2.1}
\]

and this is the distance the electron travels before encountering a scattering event.

When positioned in a magnetic field, energy spectrum and eigenstate can be obtained by solving the time independent Schrödinger equation (assuming Landau gauge \( \mathbf{A} = (-B_y, 0, 0) \))

\[
\hat{H} \psi(r) = \frac{1}{2m} [\hat{p}_x + qB \hat{y}]^2 + \hat{p}_y^2 + \hat{p}_z^2 \psi(r) = E \psi(r) \tag{2.2}
\]

Since \( \hat{H} \) commutes with both \( \hat{p}_y \) and \( \hat{p}_z \), \( \hat{p}_y \) and \( \hat{p}_z \) are conserved by the dynamics. The wavefunction therefore must take the form \( \psi(r) = e^{i(px + ipz)/\hbar} \chi(y) \) with \( \chi \) determined by

\[
\left( \frac{\hat{p}_y^2}{2m} + \frac{1}{2} m \omega^2 (y - y_0)^2 \right) \chi(y) = \left( E - \frac{\hat{p}_z^2}{2m} \right) \chi(y) \tag{2.3}
\]

where \( y_0 = -p_x/qB \) and \( \omega = |q|B/m \) (cyclotron frequency). The wavefunction is

\[
\psi_{m,n}(x, y) = \frac{1}{\sqrt{2^nn!\sqrt{\pi}l_B}} H_n \left( \frac{x - l_B^2 k_m}{l_B} \right) \exp \left( -\frac{(x - l_B^2 k_m)^2}{2l_B^2} + ik_m y \right), \tag{2.4}
\]

where \( H_n \) is the \( n \)th Hermite polynomial, \( k_m = 2\pi m/L_y, m \in \mathbb{N} \) is the boundary condition.
and the magnetic length $l_B \approx 25.7 \text{nm}/\sqrt{B[\text{T}]}$ is a commonly used characteristic length scale.

It is obvious that the conserved canonical momentum $p_x$ is in fact the center of a simple harmonic oscillator in the $y$ direction with a frequency $\omega$. Therefore, the eigenvalues of the Hamiltonian consist of the free particle component associated with the parallel motion and the energy spectrum of a harmonic oscillator

$$E_{n,p_z} = \left( n + \frac{1}{2} \right) \hbar \omega + \frac{p_z^2}{2m}, \quad n \in \mathbb{N}. \quad (2.5)$$

These are known as Landau levels (LLs).

To simplify the problem, let’s consider the case where only the lowest subband is occupied ($p_z = 0$). For a rectangular sample with area $A = L_xL_y$, boundary condition $e^{ip_xL_x/\hbar} = 1$ sets $p_x = 2\pi n\hbar/L_x$ and the center of the oscillator $y_0 = -p_x/qB$ must lie between 0 and $L_y$. This indicates that $y_0$ would take a series of evenly-spaced values separated by $\Delta y_0 = \hbar/qBL_x$. As a result, the total number of states $N = L_y/|\Delta y_0|$, i.e.

$$\nu_{\text{max}} = \frac{L_xL_y}{\hbar/(eB)} = \frac{A}{2\Phi_0} \quad (2.6)$$

where $\Phi_0 := \frac{\hbar}{2e}$ is the so-called ‘quantum flux’.

From this we see that the LL degeneracy is field-dependent and the larger the field is, the more electrons can be fit into each LL.

Here we regard the LL as a simple $\delta$-function, but in real materials background impurities and remote ionized donors contribute to the energetic broadening of the LL. These types of disorder also separate the LLs into extended states and localized states where electrons are not able to move across the sample.

### 2.1.4 Symmetric Gauge

We knew that some properties of a physical system does not depend on the gauge we choose. In Landau gauge, we manually select a direction to be $x$ which is different than $y$
direction electron motion. This seems unnatural because there is nothing special about the \( x \) direction. In the symmetric gauge

\[
A := \frac{1}{2} \mathbf{B} \times \mathbf{r}
\]

(2.7)

the vector potential is rotational invariant. Using this gauge, the wavefunction can be rewritten in polar coordinates

\[
\psi_{m,n}(r, \theta) = \frac{1}{L} \sqrt{\frac{n_r!}{(n_r + |m|)!}} \left( \frac{r}{\sqrt{2}l_B} \right)^{|m|} e^{-\frac{r^2}{2l_B^2}} L_{n_r}^{(|m|)} \left( \frac{r^2}{2l_B^2} \right) e^{im\theta},
\]

(2.8)

where \( L_{n_r}^{(|m|)} \) are the associated Laguerre polynomials and the radial quantum number \( n_r \) is the zeros of the radial term of the wavefunction and relates to the Landau level \( n \) by \( n = n_r + (|m| - m)/2 \). The angular quantum number \( m \) is restricted to \( m \geq -n \). The wavefunction extends to the area of rings with width \( l_B \) and radius of \( r_{|m|} = \sqrt{2|m|l_B} \).

The symmetric gauge does not change anything other than the appearance of the wavefunction and indeed the energy spectrum is the same. Therefore, we normally use the Landau gauge because it is easier to handle but for systems that are more naturally described in polar coordinates like quantum dots, it is good to know we have other options.

So far in our calculation, we have not taken spin degree of freedom into consideration. In real physical systems, spin degeneracy can not be avoided and the degeneracy is lifted by Zeeman splitting or spin-orbit coupling. In bulk GaAs crystals, the electrons have an effective \( g \)-factor of -0.44 and an effective mass of 0.067\( m_e \), therefore the Zeeman energy is about 70 times smaller than the cyclotron energy in typical QWs. However, in lower filling factor states, spin polarization plays an important role in the many-particle interactions.

### 2.2 Semiclassical Transport and the Drude Model

Transport is the study of the motions of charge carriers in terms of current generation under electromagnetic field. For a generic 2DEG, the current density is related to the
external electric field by

\[ J := \sigma E \]  \hspace{1cm} (2.9)

where the conductivity tensor is defined by

\[ \sigma := \begin{bmatrix} \sigma_{xx} & \sigma_{xy} \\ -\sigma_{xy} & \sigma_{yy} \end{bmatrix}. \]  \hspace{1cm} (2.10)

The corresponding resistivity tensor is given by

\[ \rho = \frac{1}{\sigma_{xx}\sigma_{yy} + \sigma_{xy}^2} \begin{bmatrix} \sigma_{xx} & -\sigma_{xy} \\ \sigma_{xy} & \sigma_{yy} \end{bmatrix}. \]  \hspace{1cm} (2.11)

In the semiclassical approximation, the transport properties of homogeneous electrons in a weak magnetic field can be obtained by solving the Boltzmann equation

\[ -a(E + v \times B)\frac{1}{\hbar}\nabla_k f = -\frac{f - f_0}{\tau}, \]  \hspace{1cm} (2.12)

where \( f \) and \( f_0 \) are the electron distribution functions with and without the external field respectively, \( \tau \) denotes the relaxation time and drift velocity

\[ v = \frac{1}{\hbar}\nabla_k \epsilon(k) \]  \hspace{1cm} (2.13)

By keeping the first order in \( E \) and second order in \( B \), the current density for 2D case

\[ J = \frac{1}{2\pi^2} \int_{BZ} e v f d^2k \]  \hspace{1cm} (2.14)

can be rewritten as

\[ J = \sigma_0[E - \mu E \times B + \mu^2(E \times B) \times B] \]  \hspace{1cm} (2.15)
where
\[
\sigma_0 = \frac{n_e e^2 \tau}{m}, \mu = \frac{e \tau}{m^*}
\] (2.16)

This is usually known as the semiclassical Drude model where the spherical Fermi surface drift under an electric field leads to a displacement of \(v_F \tau\) before encountering a scattering event which shifts the Fermi surface back to origin.

Since \(E\) lies in the plane while \(B\) is aligned perpendicular to the plane, then the 2D conductivity tensor becomes
\[
\sigma = \sigma_0 \begin{bmatrix}
1 - (\mu B)^2 & -\mu B \\
\mu B & 1 - (\mu B)^2
\end{bmatrix}.
\] (2.17)

To second order in \(\mu B\), the resistivity tensor is
\[
\rho = \begin{bmatrix}
1/\sigma_0 & B/n_e e \\
-B/n_e e & 1/\sigma_0
\end{bmatrix}.
\] (2.18)

It is obvious that the magnetic field does not affect the diagonal resistance while the off-diagonal component is the classical Hall effect terms.

Here the calculation is based on the assumption that the magnetic field is weak \(\mu B = \omega_c \tau \ll 1\) and LLs are not well-resolved.

2.3 Shubnikov-de Haas Oscillations

As shown in the last section, when a relatively strong magnetic field is applied to an ideal 2DEG \(\mu B = \omega_c \tau \gg 1\), the constant density of states breaks into isolated delta functions with a spacing of \(\hbar \omega_c\)

\[
D(\epsilon) = 1 - 2\lambda \cos \left(\frac{2\pi \epsilon}{\hbar \omega_c}\right)
\] (2.19)
where \( \lambda := \exp(-\pi/\hbar\omega_c) \) is the Dingle factor. In a real 2DEG, these delta functions are broadened (but not shifted) into Lorentzians

\[
L(E) := \frac{1}{\pi} \left( \frac{\Gamma/2}{(E - E_{LL})^2 + (\Gamma/2)^2} \right)
\]

(2.20)
due to scattering mechanisms. The peaks \( E_{LL} \) of these Lorentzians are called Landau levels and have a FWHM of \( \Gamma = \hbar/\tau_q \) where \( \tau_q \) is the quantum lifetime. In order to resolve the peak, the magnetic field must be strong enough that \( \hbar\omega_c \gg \Gamma \) or \( \omega_c \tau_q \gg 1 \). Heuristically we understand this condition as meaning that an average electron is be able to undergo at least one cyclotron orbit before scattering. The areal density of states in each Landau level is denoted by \( n_B := eB/\hbar \) giving a filling factor of \( \nu = n_B/2D \). Therefore, when the Fermi energy \( E_F \) lies precisely between two separate Landau levels, longitudinal conductance reaches a local minimum and attains a local maximum when \( E_F \) lies on a peak. As the magnetic field pushes the Fermi energy through adjacent Landau levels, the longitudinal resistance \( R_{xx} \) rises and falls, giving rise to the well-known Shubnikov-de Haas oscillations [Fig. 2.4].

### 2.4 Integer Quantum Hall Effect

The IQHE was discovered by von Klitzing in 1980 where zeros in longitudinal resistance and plateus in Hall resistance were found in low temperature transport measurement on a silicon metal-oxide-semiconductor field-effect transistor [16].

Since electrons going on circular orbits are localized to their cyclotron centers, those in the bulk are not able to conduct current while the ones along the edge scatter off the barrier and form conducting channels for the edge current. This is known as the skipping orbit picture of the IQHE with the number of conducting channels equal to the filling factor \( \nu \), which is also known as the Chern number. This relates to the topological nature of the quantum Hall insulator.

Due to the insulating bulk, the only conducting channel is provided by the quantum
Figure 2.4: Shubnikov-de Haas oscillations

Hall edge state. We know from the last chapter that the energy states are equal-spaced LLs inside the sample and vacuum outside the sample which has infinitely high energy levels. The DOS is continuous, so at the boundary the LLs are forced upwards. Therefore, when the Fermi level cuts across the sample, the number of conducting states is always an integer which depends on the LL degeneracy, i.e. the magnetic field itself.

According to Landauer-Buttiker formalism, the conductance is always equal to the conductance quanta $e^2/h$ times the transmission probability times the number of conducting channels. This is the reason why the Hall resistance is precisely $h/ve^2 = 25.8128$ kΩ. Of course, the detailed resistance between each pair of contacts needs to be calculated in the matrix formulation.
2.5 Fractional Quantum Hall Effect

Experimentally, the FQHE was first observed for $\nu = 1/3$ by Tsui et. al. in 1982 after the discovery of IQHE in 1980 [17]. Since then, more than 80 different FQHS were identified with the improvement of sample quality [18].

The IHQE can be understood based on the non-interacting particles in the framework of strongly interaction-driven phenomena where all electrons in the same LL have identical kinetic energy and all excitations to higher LLs are frozen out. As magnetic field increases, the filling factor $\nu$ decreases to partial fractions where many-body Coulomb, Zeeman and exchange interactions begin to rule giving evidence to the far superior FQHE.

In the attempt to explain the FQHE, many brilliant theoretical formalisms are put forward. We’ll talk about some of the most important concepts.

2.5.1 Laughlin’s Wavefunction

Laughlin’s wavefunction has been successful at describing the ground state for filling factor $\nu = 1/q$ FQHS:

$$\psi_{1/q} = \prod_{i<j} (z_i - z_j)^q \exp \left( -\frac{1}{4\ell_B^2} \sum_i |z_i|^2 \right)$$

where $z_i$ the complex number position of the $i$th electron.

Assuming the electrons in the lowest LL are spin polarized, then this wavefunction must be anti-symmetric which forces $q$ to be odd numbers. From this wavefunction, we can see that if two electrons are at the same position, then the wavefunction or the probability amplitude has a $q$-fold zero which indicates the electrons are well-separated because of Coulomb interaction.

The zeros can be understood as vortices with which the electrons carry to reduce Coulomb energy. Due to the correlation between the electrons and vortices, two electrons are not occurred in the same position. For a 2DEG at $\nu = 1/q$, $q$ vortices are attached to each electrons which is also equal to the number of magnetic flux quanta.
This trial wavefunction provides the basic idea for a more sophisticated idea known as ‘composite Fermions’. The quasiparticle arising from this wavefunction obeys neither Fermi-Dirac nor Bose-Einstein statistics, but a rather exotic anyonic statistics.

Merely explaining the $\nu = 1/q$ state is far from enough. Some other techniques could be used to make generalizations such as particle-hole symmetry can be applied to extend to filling factor $\nu = 1 - 1/q$. Halperin and Haldane proposed the hierarchical approach to construct wavefunction for other states based on Laughlin’s quasi-particle.

This formalism is a very elegant way attempting to explain the fascinating FQHE, yet discrepancies between theory and experiment still exists. Therefore, we need a more advanced model to attack this problem.

### 2.5.2 Composite Fermions

Though sharing the same appearance, FQHE stems from a different origin. J.K. Jain et al. [19] introduced the theory of composite fermions (CF) where an electron is attached with even number of flux quanta (Chern-Simons fluxes) which cancels out the applied magnetic field so that the mean field felt by the CFs is zero at the corresponding even-denominator filling factors. This is a collective state to lower the many-body Coulomb energy, a counterintuitive state called an “anyon” which has fractional charge and obey exchange statistics that are different from fermions and bosons.

The new quasiparticle composite fermion, treated as an independent entity enables us to relate the FQHE as the IQHE of the composite fermions. If plotted the longitudinal resistance near $\nu = 1/2$ right below $B = 0 T$, it can be noted that the resistance trace resembles itself elegantly. Within the composite fermion theory, at $\nu = 1/2$ where electrons in the sample have twice as many flux quanta penetrating making the effective magnetic field zero considering the flux quanta already taken when constructing the composite fermion.
By going away from $\nu = 1/2$, the effective magnetic field increases:

$$B^* = B - B(\nu = 1/2).$$  \hfill (2.22)

Therefore, the quantum Hall states corresponding to the filling factor $\nu^* = n_e h / (eB^*)$ start to develop. And the FQHS with $\nu = 2/5, 3/7, \ldots$ could be identified as the IQHS with $\nu^* = 1, 2, \ldots$. 

Figure 2.5: Integer and fractional quantum Hall effect shown by the diagonal resistance and Hall resistance. Taken from [20].
In general, the FQHS at filling factor

$$\nu = \frac{\nu^*}{p \cdot \nu^* \pm 1}$$  \hspace{1cm} (2.23)

of the composite fermion related to the IQHS with $\nu^*$ where $p$ is an even integer equal to the number of flux quanta attached to the electrons. The corresponding LL separation for composite fermions is

$$E_c = \frac{\hbar e B^*}{m_{CF}^*}$$  \hspace{1cm} (2.24)

where $m_{CF}^*$ is the effective mass of composite fermion which has a magnetic field dependence of $\sqrt{B}$ due to the reason that the energy gap comes from the Coulomb interaction.

The powerful theory of composite fermion has been successfully explaining experimental result like microwave absorption of composite fermion cyclotron resonance [21]. And the fractionally charge excitation is not merely something theorists proposed to account for experiments, but has been confirmed by a number of different measurements such as short-noise probing [22] or experiments involving single electron transistors [23].

Apart from the above theories mentioned, another two types of phases are experimentally confirmed at higher filling factors i.e. lower magnetic field. At partial fillings, the density modulated phases known as “stripe phases” and “bubble phases” were discovered where spontaneous ordering of electrons in spatial patterns with either one-dimensional stripe order or two-dimensional crystalline order. These fantastic phases emerges because of the competition between attractive Coulomb interaction and repulsive exchange interaction. Therefore, a minute change in the magnetic field usually results in the shapes and extent of the wavefunctions inducing the transitions between these phases.

### 2.6 Microwave-Induced Resistance Oscillations

When strong magnetic fields and microwave radiation are applied to high mobility 2D systems, Fig. 2.4 changes quite dramatically. Additional maxima appear with different
intensity at different peaks, which are known as microwave-induced resistance oscillations (MIROs). MIRO is a fascinating nonequilibrium transport phenomenon found in both n-type or p-type ultrahigh mobility 2D systems. It was discovered by Zudov et al. [24], occurring where the magnetotransport features oscillation with $1/B$ period as illustrated in Fig. 2.6. Soon after that, zero-resistance states (ZRS) were discovered between $j = 1$ and $j = 2$, where resistance goes to zero within experimental tolerance [25]. MIRO can be explained by electrons absorbing the energy of the microwave photons and hopping between LLs assisted by disorder. Resistance peaks happen at $\omega/\omega_c$ where $\omega$ is the microwave frequency and $\omega_c$ is the cyclotron frequency. The electrons show momentum change at the peaks by $\Delta q = 2k_F$ where $k_F$ is the Fermi vector [26].

Theoretical explanations of MIRO involve both a displacement mechanism and an inelastic mechanism. Both these mechanisms modify the scattering of electrons due to
microwave assistance. This theory predicts that the photoresistance oscillates as

$$\delta R_{\omega}/R_0 = -2\pi \eta P \lambda^2 \epsilon \sin(2\pi \epsilon)$$  \hspace{1cm} (2.25)$$

where $\eta$ is the dimensionless scattering rate, $P$ is the dimensionless microwave power, $\lambda$ is the Dingle factor which is related to the quantum lifetime, as well as cyclotron frequency and $\epsilon := \omega/\omega_c$.

Durst et al. [27] proposed the displacement mechanism in which the overlapping LLs contribute to the photoresistance takes the form of

$$\delta \rho_{\text{dis}} = 4\pi \rho_0 P_\omega \frac{3\tau_{qim}^i}{\tau_{tr}} \epsilon_{ac} \lambda^2 \epsilon^2 \sin(2\pi \epsilon_{ac})$$ \hspace{1cm} (2.26)$$

where $\tau_{qim}^i$ is the long range impurity contribution to the single particle lifetime,

$$P_\omega^0 = \frac{(eE_{ac}^e v_F \tau_{em})^2}{2\epsilon_{\text{eff}} (\hbar \omega)^2} E_{ac}$$ \hspace{1cm} (2.27)$$

is the dimensionless parameter proportional to the microwave power, $E_{ac}$ is the microwave electric field, $\tau_{em} = 2\epsilon_0 \sqrt{\epsilon_{\text{eff}} m^* c/n_e e^2}$ relates to the coupling of the microwave to the 2DES [28]. Current is then induced by photoexcited disorder-scattered electrons oscillating in DOS. Kohn’s theorem said that in the absence of disorder, conductivity is independent of the distribution of the electrons over LLs, in which case photoexcited electrons can make no additional contributions to the DC current. However, in the presence of disorder, electrons are scattered by impurities as they are excited and make contribution to the conductivity as

$$\Delta \sigma_{xx} \propto -\sin(2\pi \omega/\omega_c)$$ \hspace{1cm} (2.28)$$

where the period and phase are consistent with experiments. This mechanism describes MIRO efficiently but there are flaws such as: it neglects the difference between transport
lifetime $\tau_{tr}$ and single particle relaxation time $\tau_r$ which overestimates $\rho_{xx}^0$ by a factor of $\tau_{tr}/\tau_r$. It also needs to introduce localization or electron-electron interaction in order to understand ZRS. This temperature independence also disagrees with experiments.

The proposition of inelastic mechanism by Dmitriev et al. solved some of the problems. In LLs overlapping regimes, inelastic modification to the Fermi distribution function is $f(\epsilon) = f_0(\epsilon) + f_{osc}(\epsilon)$, where

$$f_{osc}(\epsilon) = \frac{\lambda \omega_c}{2\pi} \frac{\partial f_T}{\partial \epsilon} \sin \left( \frac{2\pi \epsilon}{\omega_c} \right) \frac{2\pi P_\omega^0 \epsilon_{ac} \sin(2\pi \epsilon_{ac})}{1 + P_\omega^0 \sin(\pi \epsilon_{ac})^2}$$

(2.29)

and $f_T$ is the smooth part of the Fermi distribution function $f_0(\epsilon)$ at temperature $T$. Therefore, for $\epsilon_{ac} \geq 1$, the correction due to the inelastic mechanism is:

$$\delta \rho_{in} = -4\pi \rho_0 P_\omega^0 \frac{\tau_{in}}{\tau_{tr}} \epsilon_{ac}^2 \frac{\lambda^2}{2\pi} \sin(2\pi \epsilon_{ac})$$

(2.30)

where $\tau_{in} \propto E_F T^{-2}$ is the inelastic scattering time. In this case, the dominant contribution comes from the electron-electron inelastic scattering, which is strongly temperature dependent. The inelastic mechanism predicts ZRS at high microwave powers but cannot explain the oscillatory Hall resistivity.

In order to account for Hall-Induced resistance oscillations (HIROs), Dmitriev et al. came up with the photovoltaic and quadruple mechanisms in 2007 which calculate the off-diagonal parts of the photoconductivity tensor [29].

Additional to the resonance phenomenon due to cyclotron motion, other resonance features have been discovered such as geometry resonance [30, 31], magneto-acoustic-phonon resonance [32], and magneto-Zener-tunneling resonance [33].
Chapter 3

Cyclotron Resonance Thermal Detection in 2DEG

3.1 Cyclotron Resonance in 2D Systems via Different Methods

Resonance transition by pump solid state matter with electromagnetic field is an efficient way to probe energy levels if the off-diagonal component of the Hamiltonian is non-zero. Electron spin resonance (ESR) and CR are two of the most significant features of electrons under magnetic field and they can be detected via different ways such as conventional transmission spectroscopy, reflection spectroscopy, resistive detection and thermal detection.

In order to detect the ESR of a single nano-object, a highly sensitive tool is required. So far, the best commercial ESR detector can detect around 1000 spins. Aiming at resolving the ESR of one nano-object, we develop an ultra-sensitive calorimeter and a reflective spectroscopy. Both of them can operate at 300 mK and the precision is improved to tens of micro-Kelvins, thereby increasing the sensitivity to several nanowatts. As a proof of concept, we show that CR can be measured via heat generated by resonant absorption of photons using this setup.

3.2 Thermal Detection of Behavior of Electrons under the Illumination of Microwaves

An increase in the lattice temperature can be detected when the energy of the incident microwave photons matches the energy difference between adjacent Landau levels.
3.2.1 Theoretical Support and Preparation in 4K

The density of states of a 2DEG forms Landau levels when a magnetic field is applied. Therefore, sweeping either the microwave frequency or magnetic field will result in the case where energy of the incident photons is equal to the energy gap of the Landau levels, therefore causing resonance. However, in AlGaAs/GaAs material, electron relaxation can be non-radiative by passing the extra energy to phonons and raising the lattice temperature. This temperature change can be measured by a nano-calorimeter. We constructed such a device using a special Cernox thermometer, which has a huge resistance difference for a tiny temperature change at low temperature.

This resonance-measuring method has been tested in our group with liquid helium for detecting geometric resonance, which is the coupling between cyclotron resonance and plasma resonance.

For comparison, on a bare 2DEG quantum well, only one CR peak is observed (Fig. 3.3); while on a triangular patterned dot sample (with a feature size on the order of \( \mu m \), one can observe two obvious resonances at

\[
\omega_{\pm} = \frac{\omega_c}{2} \pm \sqrt{\omega_0^2 + \left( \frac{\omega_c}{2} \right)^2},
\]

where \( \omega_0^2 = \frac{N_t e^2}{2m^* \epsilon_{\text{eff}} r} \), effective mass \( m^* = 0.067 m_e \), \( \epsilon_{\text{eff}} = 6.5 \) and \( r \) is the radius of the dots [34].

The bulk magnetoplasmon has been eliminated by choosing irregularly shaped samples and the frequency is limited by the size of our waveguide.

3.2.2 Setup Construction

In order to improve the sensitivity, we constructed a setup which works in 300 mK because the lower the temperature becomes, the sharper the resistance response the Cernox thermometer will show.

However, there are problems to overcome by going from liquid \(^4\)He temperature to
\(^3\text{He}\) temperature. We constructed a vacuum can and sealed it in room temperature. When the can is cooled down to low temperature, the molecules are absorbed by the wall and make no contribution to the thermal conductivity. The most significant issue is sealing the vacuum can in the small \(^3\text{He}\) molecule environment. We tap the inner wall of the cap and outer wall of the base (vacuum can design can be found in appendix), therefore compressing the flat surfaces of those two parts which are polished mechanically to seal.
Before that we also dissolve soap in glycerin and scrub the surfaces to fill up any possible tiny gaps. Validation of sealing can only be tested in low temperature measurement.

Since the amount of heat generated by the non-radiative relaxation is relatively small, the sample needs to be in a vacuum to avoid heat being carried away by the mass of $^3$He liquid, and a bridge with high thermal conductivity is required to pass the temperature change to the sensor. Meanwhile, to keep the temperature around base value, a heat sink to the exterior environment with high thermal conductivity and a link between the heat sink and the bridge with low thermal conductivity are most effective. In this case, we use 200\,\mu m thick sapphire crystals ($\kappa \sim 3\,\text{W/m} \cdot \text{K}$) as the bridges between samples and thermometers, two copper pillars ($\kappa \sim 100\,\text{W/m} \cdot \text{K}$) positioned symmetrically for the heat sink and thin magnin wires ($\kappa \sim 0.05\,\text{W/m} \cdot \text{K}$) to link between the bridge and the heat sink pillars.

The whole vacuum can is immersed in liquid $^3$He in a top-loaded refrigerator. The
microwave is sent through a coaxial cable in the probe and a properly cut antenna.

### 3.3 Thermal Cyclotron Resonance

We use a differential circuit to filter the background noise, and an amplitude modulation technique to quench the asymmetry left in the differential geometry. The voltage signal difference from the two arms can be calculated by

\[
\frac{|\tilde{E}|}{|\tilde{V}|} = \frac{j\omega_0 (M + L) \delta R}{R^2 + 2j\omega_0 RL + \omega_0^2 (M^2 - L^2)} \approx \frac{\delta R}{2R}
\]  

(3.2)

This indicates that the output signal \( V_{out} \) is linearly proportional to the difference between the resistance of those two Cernox thermometers \( \delta R \), which reflects the amount of heat generated by the sample with the application of microwave. The differential signal is fed to a first lock-in amplifier, the output of which then is fed to a second lock-in amplifier.
Figure 3.4: Trace of GR measured on a triangular anti-dot patterned sample via thermal setup.

Figure 3.5: Cernox Thermometer for temperature down to 300 mK. Datasheet can be found in appendix.

The amplitude of the input microwave is modulated internally by the microwave generator controlled by the frequency of the second lock-in amplifier, which is oscillating much more
slowly than the rapid oscillating signal from the first lock-in amplifier. Therefore, after integrating the signal twice, the output is a clean response of 2DEG CR signal like (Fig. 3.10).

When microwave is applied, obvious resonance peaks appear when sweeping the magnetic field. And the signal amplitude increases with increasing microwave power. By applying power dependence, the signal can still be resolved even with power as low as -20dBm (0.01 mW). Compared to the setup without these two measuring techniques, sensitivity is at least thirty times higher in terms of input microwave power.

Figure 3.6: Temperature calibration of the Cernox thermometer
3.4 Thermal Spin Resonance on DPPH

ESR or electron paramagnetic resonance (EPR) spectroscopy is a technique for studying materials with unpaired electrons using the Zeeman splitting

\[ E = m_s g \mu_B B_0 \]  

(3.3)

where \( g \) is the electron’s Landé g-factor and \( \mu_B \) is the Bohr magneton. Therefore, resonance happens when

\[ \Delta E = g \mu_B B_0. \]  

(3.4)

The conventional method of measuring ESR is transmission spectroscopy. There are other ways, such as magnetic torque detection, optically detected ESR, STM-ESR and resistively detected ESR, which measures the longitudinal resistance of 2DEG. Our thermal
detection method is another alternative; measuring the temperature change induced by the non-radiative relaxation of the photon absorption.

DPPH is the abbreviation for the organic chemical compound 2,2-diphenyl-1-picrylhydrazyl which has two major applications. It can be used for monitoring chemical reactions involving radicals or determining the position and intensity of ESR signals. We use DPPH as a test to see if the ESR signal can be resolved by our setup. Raw data is shown in Fig. 3.11; a clear resonance dip appears around 1.42T with a line width of 150 Gauss at the microwave frequency of 40 GHz. Therefore, the g-factor can be estimated as

$$g = \frac{\hbar \omega}{\mu_B B} = 1.9934$$  \hspace{1cm} (3.5)

as compared to the free electron value of $g = 2.0023$. 

Figure 3.8: Thermal detection setup schema
Figure D.1: Pictures of the probe stages: 4K, 1K, and the sample holder.

Figure 3.9: $^3$He top-loaded refrigerator coaxial probe. Taken from [35].

By subtracting the background and fitting with a Lorentzian function:

$$L(B) = \frac{A}{\left[1 + \left(\frac{\mu}{\Delta B}\right)(B - B_r)\right]^2}$$  \hspace{1cm} (3.6)$$

we extract a decoherence time of $\tau = 7.78 \text{ ns}$. 
To conclude, we demonstrate that the nano-calorimeter constructed has the ability to resolve both CR and ESR signals for materials with non-radiative relaxation processes. However, for those materials without good electron-phonon coupling, in which the electron energy can not be transferred to lattice heat, this method would be very limited.
Chapter 4

Microwave Reflection Spectroscopy via Coaxial Cable

In searching for a more universal method for microwave resonance spectroscopy, we develop a reflective setup via coaxial cable.

4.1 Measurement via Reflection

In order to fully understand the physical processes involved in the setup, it is important to consider the problem from the very beginning.

4.1.1 Maxwell Equations and Boundary Conditions

Microwave, as a certain frequency range of electromagnetic waves, has its own unique properties. While visible light or even short wavelength types like X-ray and gamma rays travel straight as a ray and radio waves are able to bypass large objects, microwave interacts with experimental configurations as their sizes are comparable with its wavelengths. When there is no source, electromagnetic waves spread obeying the Maxwell equations:

\[
\nabla \cdot E = 0, \quad \nabla \times E = -\frac{\partial B}{\partial t} \tag{4.1}
\]

\[
\nabla \cdot B = 0, \quad \nabla \times B = \frac{1}{c^2} \frac{\partial E}{\partial t} \tag{4.2}
\]

By the curl of the curl identity

\[
\nabla \times (\nabla \times X) = \nabla (\nabla \cdot X) - \nabla^2 X \tag{4.3}
\]
we get the wave equations:

\[ \frac{1}{c^2} \frac{\partial^2 E}{\partial t^2} - \nabla^2 E = 0 \]  
\[ \frac{1}{c^2} \frac{\partial^2 B}{\partial t^2} - \nabla^2 B = 0. \]  (4.4) (4.5)

Assuming the plane wave form of \( E = E_0 e^{ik \cdot x} \), where \( k = \beta + i\alpha \), the wave equation is rewritten to:

\[ \nabla^2 E_0 + k^2 E_0 = 0, k = \omega \sqrt{\varepsilon' \mu}, \varepsilon' = \varepsilon + i \frac{\sigma}{\omega}. \]  (4.6)

Therefore, the electric field has the form of:

\[ E(x, t) = E_0 e^{\alpha \cdot x} e^{i(\beta \cdot x - \omega t)}, \]  (4.7)

coefficients \( \alpha \) and \( \beta \) correspond to the attenuation and phase shift of the wave and relate to the other parameters:

\[ \beta^2 - \alpha^2 = \omega^2 \mu \varepsilon \]  (4.8)
\[ \alpha \cdot \beta = \frac{1}{2} \omega \mu \varepsilon. \]  (4.9)

When electromagnetic field encounters a medium, for the non-absorption case, we have

\[ E + E' = E^* \]  (4.10)
\[ H + H' = H^*. \]  (4.11)
for nonmagnetic material and normal incident conditions, the second condition becomes
(assume $\mu = \mu_0$)

$$\frac{E'}{E} = -\frac{1 + i - \sqrt{\frac{2\omega \varepsilon_0}{\sigma}}}{1 + i + \sqrt{\frac{2\omega \varepsilon_0}{\sigma}}}. \quad (4.12)$$

Define reflectivity and we have

$$R = \left| \frac{E'}{E} \right|^2 = \frac{(1 - \sqrt{\frac{2\omega \varepsilon_0}{\sigma}})^2 + 1}{(1 + \sqrt{\frac{2\omega \varepsilon_0}{\sigma}})^2 + 1} \approx 1 - 2\sqrt{\frac{2\omega \varepsilon_0}{\sigma}}. \quad (4.13)$$

For a good metal ($\sqrt{\frac{2\omega \varepsilon_0}{\sigma}} \ll 1$), $R \approx 1$. However, for a typical GaAs semiconductor quantum well where carrier density $n_e = 2.5 \times 10^{11} / \text{cm}^{-2}$, and mobility $\mu = 1.3 \times 10^7 \text{ cm}^2 / \text{V} \cdot \text{s}$, Drude conductivity $\sigma_0 = n_e e \mu = 0.52$ while $\varepsilon_0 \omega = 2.136$ for 40 GHz, good metal condition is not satisfied, meaning transmission is allowed.

### 4.1.2 Reflectometry

Though transmission spectroscopy is widely used and relatively easy to interpret, for thick samples, generally few transmission signals are obtained when $d \gg \lambda$. Reflection spectroscopy as an alternative suffers from disadvantages as well, such as the data obtained usually very complex due to multiple reflections and phase shifts.

However, many tools are being developed to deal with this problem. For example, in seismic modeling, tomography has been invented to get an idea of the subsurface area using arrival time or amplitude inversion. In optics, scientists use Kramers-Kronig relations to analyze the optical index.

Reflectometries can be obtained by sweeping different physical quantities such as time-domain reflectometry or frequency-domain-reflectometry (ultrasound). It can be also applied to different substance such as neutron or X-ray. Reflectometries are widely used in radar systems and medical imaging. They can also perform non-destructive testing such
as cable diagnostics or material characterization like viscosity, shear thinning and shear thickening.

Though performing reflection spectroscopy using infrared source has great advantages - for example, no requirement for liquid helium temperature or high quality sample - since the constraint of $\omega \tau \gg 1$ can be easily satisfied, small reflection minimum offset can result in big effective mass error. The assumption that scattering time is not frequency dependent, $\varepsilon_{\text{inf}}$ is known and sample thickness is infinite need to be made. Therefore, reflectivity can be obtained [36]

$$R = \frac{(n - 1)^2 + k^2}{(n + 1)^2 + k^2}$$

(4.14)

by solving the set of equations:

$$n^2 - k^2 - \varepsilon_{\text{inf}} = -\frac{Ne^2}{m\varepsilon_0} \frac{\tau^2}{1 + \omega^2 \tau^2}$$

(4.15)

$$2nk\omega = \frac{Ne^2}{m\varepsilon_0} \frac{\tau}{1 + \omega^2 \tau^2}$$

(4.16)

where $n$ is the refractive index and $k$ is the absorption index.

Note that this scenario does not apply to us since the microwave wavelength $\lambda$ is much larger than the sample thickness, which is generally 0.5mm for a typical GaAs quantum well sample. Conservation law of energy tells that $1 = R + A + T$ and when absorption $A$ increases, transmission $T$ usually decreases, but then it’s difficult to tell what happens to the reflection $R$.

For our case (see Fig. 4.1) which is the model of multilayer semi-infinite backed by free space (Fig. 4.2), attempts to retrieve $\kappa$ have been failed, which is related to absorption coefficient $\alpha$ from admittance [37].

Let’s consider it from a different perspective then. Since the electromagnetic field wavelength is much greater than the sample thickness, the sample can be regarded as a thin film, thus we should have a valid case of Fresnel’s law. If we assume that the CR
absorption in the 2DEG layer is homogenous, we could have an effective refractive index \( \tilde{n} \), which relates to the dielectric constant \( \varepsilon' \):

\[
\tilde{n} = \sqrt{\varepsilon'} = n' + i\kappa'.
\]  

(4.17)
In optics, when a ray is normally incident from medium \( n_0 \) to absorbing medium \( n_1 \), the intensity of which decreases exponentially:

\[
I = I_0 \exp \left( -\frac{4\pi n_0 ks}{\lambda_0} \right)
\]  

(4.18)

where \( s \) is the traveled distance and \( \lambda_0 \) is the incident wavelength, then the reflectivity can be obtained just by substituting the refractive index in the non-absorbing media with its complex counterpart:

\[
R_{\text{reg}} \equiv \frac{|E_{r0}|^2}{E_0^2} = \frac{(n_1 - n_0 - in_1\kappa_1)(n_1 - n_0 + in_1\kappa_1)}{(n_1 + n_0 - in_1\kappa_1)(n_1 + n_0 + in_1\kappa_1)} = \frac{(n_1 - n_0)^2 + (n_1\kappa_1)^2}{(n_1 + n_0)^2 + (n_1\kappa_1)^2}.
\]  

(4.19)

In our case, the microwave is incident from the coaxial cable where the insulation material is low density polytetrafluoroethylene, which has a refractive index of 1.35 to GaAs \((n = 3.3)\) backed with vacuum \((n = 1)\) where the effective refractive index \( n' \approx 1.7 \). Therefore, the reflectivity coefficient is \( R_{\text{reg}} = 1 - \frac{9.2}{9.3 + (1.7\kappa)^2} \), which indicates that a absorption peak would result in a reflection peak as well.

### 4.1.3 Microwave Electronics

Microwave electronics measurement can be applied to analyze dielectric constant, magnetic moment, surface impedance and band structures in semiconductors. It can even measure superconductor complex conductivity when combining transmission with reflection. The circuit can be operated with a resonant or non-resonant mode, open or short circuit, at free space or confined structure like waveguides or coaxial cable—see Fig. 4.3 and 4.4.

There are different approaches to analyze the data, e.g., field theory studies the distribution of the electromagnetic field, while microwave network method regards everything as equivalent lumped elements and only measures their response to the external microwave signals.

Understanding the concept of transmission lines is very important in microwave electronics. All the elements in the circuits are considered part of a transmission line
that can be equivalent to combinations of RC circuits. Relations between the reflection and transmission coefficients can be calculated specifically by impedance and admittance. Consider the device under test as a black box, and recording the response by sending microwave input and waiting for the reflection is a simple and effective procedure for obtaining the reflection coefficient magnitude, but it also depends on the availability of a good coupler and the matching of impedance load (Fig. 4.5).

However, researchers have come up with different methods for how this should be realized. Directional coupler (DC) and Magic Tee are two of the typical ones for coaxial cable and waveguide, see Fig. 4.6 and 4.8.
The perfect coupler should completely separate the input and output in the same device and this is generally very difficult. Some technical terms can be defined to describe the level of coupling and isolation:

**Coupler factor**: \( C_{3,1} = 10 \log\left(\frac{P_3}{P_1}\right)\) dB \hspace{1cm} (4.20)

**Insertion loss**: \( L_{I,2,1} = 10 \log\left(\frac{P_2}{P_1}\right)\) dB \hspace{1cm} (4.21)

**Coupled loss**: \( L_{C,3,1} = -10 \log\left(1 - \frac{P_3}{P_1}\right)\) dB \hspace{1cm} (4.22)

**Isolation**: \( I_{4,1} = -10 \log\left(\frac{P_4}{P_1}\right)\) dB \hspace{1cm} (4.23)

**Directivity**: \( D_{3,4} = -10 \log\left(\frac{P_4}{P_3}\right) = -10 \log\left(\frac{P_4}{P_1}\right) + 10 \log\left(\frac{P_3}{P_1}\right)\) dB \( = I_{4,1} + C_{3,1}\) \hspace{1cm} (4.24)

Assuming a perfect match meaning no reflection and perfect isolation, the S-matrix of a
A directional coupler is:

\[
S = \begin{bmatrix}
0 & \tau & \kappa & 0 \\
\tau & 0 & 0 & \kappa \\
\kappa & 0 & 0 & \tau \\
0 & \kappa & \tau & 0 
\end{bmatrix}
\] (4.25)

where \( \tau \) is the transmission coefficient and \( \kappa \) is the coupling coefficient. There are different ways to build this device physically; Bethe-hole DC and Riblet short-slot coupler are the most commonly used ones.

DC can be modified in order to achieve more complex goals. For example, dual DC (Fig. 4.7) is a combination of two 3-port couplers with the main line cascaded and internally terminated ports facing each other. It’s intended to allow forward and reflected signals to be sampled simultaneously and it’s different from a bidirectional coupler which is a 4-port coupler with no internal termination.

### 4.2 Setup Construction and Calibration

#### 4.2.1 Detector Calibration

In order to measure the reflection power signal, it is important to have a detector with high sensitivity within the microwave frequency range. Zero-bias Schottky diodes are widely used in high performance microwave power detection circuits. It utilizes the simple fact that photons incident on semiconductor diodes can produce photocurrent and build photovoltage, the magnitude of which relies on the number and frequency of the photons.

Fig. 4.9 shows the negative voltage of a Anritsu 560-7K50 detector with a sampling frequency step of 1 GHz. Note that the larger the frequency becomes, the larger the absolute value of the voltage signal below 12 GHz. But beyond this range, there are fluctuations in frequency response. Fig. 4.10 shows the power dependence (sampling with 1dbm step) of the detector at different frequencies. As expected, more power yields a more negative
Figure 4.7: Dual Directional Coupler: From Krytar.com

Figure 4.8: Magic Tee structure. From wikipedia.
voltage which can be fitted by polynomials up to three orders (Fig. 4.11) and this is more prominent with lower frequencies.

![Graph showing frequency calibration of Anritsu 560-7K50 detector.](image)

**Figure 4.9:** Frequency calibration of Anritsu 560-7K50 detector.

Subsequently we realized that increasing the detector sensitivity can greatly improve the measurement performance. Model DZR400KB from Herotek provides a working frequency range of 10 MHz-40 GHz and an acceptable price. More information can be found in the appendix.

From Fig. 4.13, it can seen that the new Herotek sensor has a much more stable performance than the old Anritsu detector and additionally has larger response at higher frequencies. This makes it favorable for detecting CR, which is more distinguishable at higher frequencies when the peak are separated in the magnetic field. Figure 4.14 shows that the two sensors purchased together behave very similarly, which could later be used to advantage in the control experiment.

Linear regression on the power dependence curve converts the negative voltage, which
could be precisely measured by a lock-in amplifier to the actual microwave power it detects:

$$P = \left( -\frac{U(\text{mV})}{344.3} \right)^{\frac{1}{0.775}} \text{ mW}. \quad (4.26)$$

This results in a response of 0.4 mV/μW which is consistent with the value provided in
the manufacture data sheet.

Figure 4.13: Herotek sensor frequency dependence.

Figure 4.14: Comparison between two Herotek sensor power dependences.
4.2.2 Coaxial Cable Reflection Setup Calibration

Figure 4.16 shows the configuration for calibrating the coaxial cable reflection setup. Microwave source is sent through port 1 of the dual DC and output port 2 is connected to the coaxial cable of the probe. The microwave then gets reflected by the sample at the end of the cable and back to the dual DC. Since most of the reflection power is only coupled to port 4, which feeds to sensor 1 and the input power is coupled to port 3, which feeds to sensor 2, the reflection and original microwave power can be detected simultaneously by measuring their output voltage via lock-in amplifier. The amplitude of the microwave is modulated at 17Hz by the lock-in amplifier which effectively filters out the background noise.

We calibrated the system by monitoring the reading of the lock-in amp while sweeping input microwave frequency and power intensity. Fig. 4.19 shows a linear relation between the voltage of the sensor and the power of the microwave input at different frequencies. Fig. 4.18 shows the relation between $U_{\text{out}}$ and $f_{\text{in}}$ with a constant power of 1 mW. Many fluctuations can be found, which indicates possible standing waves in the coaxial cable.
Fig. 4.16: Configuration of calibration on the coaxial cable setup.

Fig. 4.17 shows a typical coaxial resonator with inner diameter of $2a$ and outer diameter of $2b$. Working at TEM mode

\begin{align*}
E_r &= -j \frac{2E_0}{r} \sin \beta z \quad \text{(4.27)} \\
H_{\phi} &= Y_0 \frac{2E_0}{r} \cos \beta z \quad \text{(4.28)}
\end{align*}

normally to avoid possible resonance at $\phi$ direction, $\pi(a + b) < \lambda_{\text{min}}$ needs to be satisfied.

When it is at resonance, the length of the resonator and the wavelength are related:

$$l = n \frac{\lambda_0}{2}. \quad \text{(4.29)}$$

Fig. 4.20 and 4.21 shows frequency sweeping steps of 0.01 GHz and 0.001 GHz. They have very similar forms which indicates that the sweeping step is fine enough to show resonance features. Frequency peaks are found separated with 0.1 GHz at high frequency.
Assuming at the adjacent resonant peaks, we have $n\frac{\lambda_1}{2} = (n + 1)\frac{\lambda_2}{2} = (n + 2)\frac{\lambda_3}{2}$, then

$$\frac{1 - \frac{f_2}{f_1}}{\frac{f_2}{f_3} - 1} = \frac{1}{2\frac{f_2}{f_3} - 1} \quad (4.30)$$

If we use $f_1 = 40$ GHz, $f_2 = 39.9$ GHz, $f_3 = 39.8$ GHz, then LHS=0.99499 and RHS=0.995. This shows that the coaxial resonance is responsible for the frequency fluctuation behaviors.
4.3 Cyclotron Resonance Data

Applying this reflection method to high mobility quantum wells, we expect clear CR traces.
4.3.1 Electron Sample Results

We use GaAs/AlGaAs 2DEG with moderate density $n_e \sim 2 \times 10^{11} / \text{cm}^2$ and high mobility $\mu \sim 1 \times 10^7 \text{ cm}^2/\text{V} \cdot \text{s}$ as a test. Fig. 4.22 shows CR as peaks around 0.1 T with 40 GHz microwave radiation. Note that the background is relatively large and there are multiple unwanted reflection peaks that cannot be eradicated since no tuning mechanism is allowed in the coaxial resonance here. Despite this problem, sensitivity is already much better than the thermal measurement setup, with resolvable microwave power input being an order of magnitude smaller (Fig. 4.23).

Another problem with this setup is that the performance is highly frequency dependent (Fig. 4.24). This makes applying measurements at different frequencies to extract the effect mass and magnetoplasmon frequency extremely hard.
4.3.2 Hole Sample Results

Good quality hole samples sometimes provide a better platform than electron samples because the resonant peaks are more separated due to higher effective mass (Fig. 4.25). Though mobility of the hole sample $\mu \sim 2 \times 10^6 \text{cm}^2/\text{V} \cdot \text{s}$ is smaller than the electron sample, the linewidth stays narrow because of the large single particle relaxation time.

The frequency dependence of CR on the hole sample (Fig. 4.26) shows better behavior but still suffers from relatively large background noise, possibly due to the multiple unwanted reflections at the coaxial cable connectors.

Therefore, a setup with better frequency performance and less background noise is still needed.
Figure 4.23: CR power dependence zoom in data from coaxial cable reflection
Figure 4.24: CR frequency dependence data from coaxial cable reflection

Figure 4.25: CR signal on a hole sample from the coaxial cable setup.
Figure 4.26: CR signals on a hole sample from the coaxial cable setup at different frequencies.
Chapter 5

Multi-photon Phenomenon in High Mobility 2DEG

Inspired by the idea of reflection spectroscopy via coaxial cable, we construct a low-temperature microwave waveguide interferometer for measuring high-frequency properties of two-dimensional electron gases (2DEGs). With much better frequency performance we are able to extract effective mass, bulk plasmon frequency, and carrier relaxation times from coupled plasmon-cyclotron resonance (PCR) spectra. With better resolution and sensitivity, multi-photon transitions are observed, which previously had been discovered only by microwave-induced resistance oscillations. Subsequently we apply this method together with electrical transport on different density samples and find that multi-photon phenomenon is more prominent on higher density wafers.

5.1 Background on Multi-photon CR in 2DEG

Semiconductor interband/intraband transitions are traditionally studied via optical absorption and transmission. Nonlinear processes involving multiple photons attract tremendous attention not only for elucidating band structure beyond the experimentally attainable frequency range, but also for understanding the kinetic properties of solids in the presence of strong radiation. With millimeter wave irradiation, non-equilibrium phenomenon such as MIRO [38, 39] and ZRS [25, 40] were observed in 2DEGs over a decade ago (see Chapter 2).

Multi-photon CR corresponding to the rational values of

\[ j = \frac{\omega}{\omega_c} = \frac{p}{q} = \frac{1}{2}, \frac{1}{3}, \frac{2}{3}... \]  \hspace{1cm} (5.1)
where $p$ and $q$ are integers is equivalent to sub-harmonic transitions where the energy of $q$ photons matches the spacing of $p$ Landau levels (LLs) (see Fig. 5.6 (d)). Experiments have been performed to study this phenomenon, but only using transport measurements. Dorozhkin et al. [41] proposed that sub-harmonic MIROs were suppressed below 30GHz due to single photon inelastic mechanisms when LLs start to overlap, yet the experiments of Zudov et al. [42] found no frequency threshold. Wiedmann et al. observed high order MIRO at temperatures up to 6.5K [43].

Although multi-photon processes usually happen at high intensities in the terahertz and infrared regime, they also occur when large DC fields are applied [44]. We access this process without high AC or DC fields in a much lower frequency range i.e. milli-meter wave, where multi-photon transitions can be clearly observed in high mobility 2DEGs. A simultaneous measurement of PCR absorption and MIRO are performed on high mobility samples with different carrier densities to study the multi-photon phenomenon in different electron-electron interaction strength. The results show that multi-photon transitions are facilitated by higher density [45] via both optical absorption and electrical transport measurement, thus more orders of transitions can be observed in high density samples. We analyze the power, frequency and temperature dependence on a specific wafer with medium density. The visibility of sharp signals from devices fabricated at varying widths also enables us to tune the bulk plasmon frequency, which can be used as a potential tool for coupling other fundamental excitations, like composite fermions in the FQH states to photons in a detectable frequency range.

### 5.2 Demonstration of Reflection via Rectangular Waveguide

Waveguide as an alternative substitution to coaxial cable for microwave conductions generally has smaller transmission loss and the electromagnetic field radiating on the sample is more well-defined compared to the field from an antenna. Some of the resonance problems that we could not solve in the coaxial cable setup can be mitigated in the waveguide setup.
5.2.1 Setup Construction

We use GaAs/Al$_{1-x}$Ga$_x$As QWs grown by MBE with carrier densities ranging from $0.8 - 4 \times 10^{11}$ cm$^{-2}$ and mobility above $1 \times 10^7$ cm$^2$/V·s after illumination under red LED for 30 minutes. The sample is inserted into a microwave reflection interferometer (Fig. 5.1(a)) where microwave is transmitted and reflected through a WG28 waveguide with TE$_{01}$ mode in the range of 26-40 GHz. This is to make sure only one mode is allowed so that the interference between the two reflection beams are properly configured. The incident microwave field is firstly divided by a four-terminal device called magic tee introduced in the last chapter which isolates the two collinear ports. The sample is mounted on a homemade thin copper plate to increase reflectivity thus resolving the reflectometry problem by eliminating transmission part. An adjustable short which totally reflects the microwave is used to tune this interference sinusoidally at a fixed frequency (Fig. 5.2). Note that it is because of the ability to tune at resonance that the performance is much superior than the coaxial cable setup.

A more sensitive Herotek sensor is mounted to the output port for power detection, the voltage of which is then further fed into a lock-in amplifier. We attach 8 indium contacts for standard transport measurement (Fig. 5.6) which are annealed at 420 °C (procedure can be found in Appendix). The amplitude modulation of the microwave is provided by the first lock-in amplifier at 100 Hz, which is much larger than 7 Hz used for photoresistance measurement controlled by the second lock-in amplifier. A small current of 1 μA is applied to avoid non-linear effects which can be potentially caused by strong DC bias field [44]. The probe is top-loaded into a $^3$He cryostat, which operates at a base temperature of 300 mK and is equipped with a 12 T superconducting magnetic coil. In this way we are able to perform simultaneous measurement of photoresistance and reflective absorption by fixing microwave frequency and sweeping the external magnetic field.
Figure 5.1: (a) A schematic of a waveguide interferometer is shown (b,c) Power dependence of PCR absorption on hole sample "a" with a fixed frequency of 40 GHz. For clarity, each data trace is shifted vertically by subtracting its value at -10 kG. The vertical axis is the amplitude of the detector signal.

5.2.2 Cyclotron Resonance Data and Fitting

Fig. 5.1 (b) shows that even with a source power as low as 10 nW (-50dbm), the cyclotron resonance signal still can be observed, attesting to the high sensitivity and signal/noise ratio. Note that this is another two orders of magnitude smaller than the minimum power solvable for the coaxial cable setup.

We have measured a variety of electron and hole samples (see Table 5.1) in GaAs/Al-
GaAs heterostructures. Carrier effective mass, plasmon frequency, transport lifetime and relaxation time are extracted from the data. Remarkably, we observed multi-photon absorption in all the electron samples, and also on the hole sample with higher density. Effective mass and bulk plasmon frequency are extracted by least squares fitting (Fig. 5.3) the data with different microwave frequencies. Different-sized samples cut from the same wafer have very close effective masses but different plasmon frequencies. We found the effective masses of hole samples are $\sim 0.25m_e$ which is in the reasonable range of $0.2m_e$ to $0.5m_e$, depending on QW width [44]. Electron samples generally have higher plasmon frequencies than hole samples due to their smaller effective mass (see Fig. 5.5).

### 5.2.3 Linewidth Analysis

We then analyzed the PCR line-shape, the sharpness of which can be quantified by its FWHM. This value corresponds to single particle relaxation time $\tau_s$, which is related to the imaginary part $\Gamma_s$ of the self-energy function [46] by $\Gamma_s = \hbar/2\tau_s$. Broadening of the
Table 5.1: Parameters of hole samples a, b and electron sample c. Subscripts t and s refer to transport scattering time and single particle relaxation time, respectively. Line shape fitting for samples c and d is difficult because of the peak overlapping and shortage of samples.

<table>
<thead>
<tr>
<th>Sample</th>
<th>Carrier</th>
<th>Density ( (1 \times 10^{11}/\text{cm}^2) )</th>
<th>Mobility ( (1 \times 10^6 \text{ cm/V}\cdot\text{s}) )</th>
<th>( m^* )</th>
<th>( \tau_{tr} ) (ps)</th>
<th>( \tau_s ) (ps)</th>
<th>( \tau_{tr}/\tau_s )</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>hole</td>
<td>1.45</td>
<td>2.25</td>
<td>0.27</td>
<td>346</td>
<td>18</td>
<td>19</td>
</tr>
<tr>
<td>b</td>
<td>hole</td>
<td>1.16</td>
<td>0.25</td>
<td>0.25</td>
<td>36</td>
<td>10</td>
<td>2.6</td>
</tr>
<tr>
<td>c</td>
<td>electron</td>
<td>1.45</td>
<td>1.3</td>
<td>0.07</td>
<td>52</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>d</td>
<td>hole</td>
<td>2.4</td>
<td>1.3</td>
<td>0.26</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
</tbody>
</table>

LLs due to electron-impurity interaction in a Coulomb potential is generally considered responsible for broadening characterized by \( \tau_s \). The transport scattering time \( \tau_{tr} \) from the semi-classical Drude model, on the other hand, is extracted from the DC conductivity using

\[
\sigma_0 = \frac{n_se^2\tau_{tr}}{m^*}
\]  

(5.2)

The CR relaxation time, often estimated by the single particle quantum lifetime \( \tau_s \), can be up to two orders of magnitude smaller than \( \tau_{tr} \) when long-range impurity scattering is dominating in high mobility 2DEG. Since our sample sizes are much smaller than electromagnetic wavelength, contribution of radiative decay for cyclotron linewidth can usually be neglected [47]. Therefore the single particle relaxation time \( \tau_s \) could be extracted from fitting the absorption trace with a Lorentzian function. Note that for a low mobility sample where scattering is mostly short-range and its cross-section is independent of scattering angle, these two time scales can be equal [46]. But in the high mobility case, scattering is strongly peaked in the forward direction, where transport scattering time \( \tau_{tr} \) can be up to two orders of magnitude greater than the relaxation time \( \tau_s \).

A comparison between these two time scales is given in Table 5.1 where scattering time and relaxation time are extracted from transport measurement (Fig. 5.4 (a, b)) and PCR Lorentzian fitting (Fig. 5.4 (c)) respectively. We confirmed that \( \tau_{tr}/\tau_s \) increases with sample
Figure 5.3: Frequency dependence of PCR absorption on hole sample "a" under a fixed power of 1 mW. Inset: Each data trace is first shifted vertically by subtracting its value at -10 kG, and then shifted upward consecutively by 0.025 unit. Effective mass and plasmon frequency are extracted using least square fitting.

\[ \omega^2 = \omega_c^2 + \omega_p^2 \]
\[ m^* = 0.26m_0 \]
\[ f_p = 5.1\text{GHz} \]

mobility, based on the data in Table 5.1. It should be emphasized that in order to use this method to extract relaxation time, we assume that small absorption approximation is valid, meaning saturation is not relevant (cyclotron absorption will saturate when carrier density and mobility increases). In the saturation case, the 2DEG behaves like a metallic sheet and reflects most of the incident microwave.
5.2.4 Multi-photon Preliminary Data

Now let’s examine the multi-photon phenomenon we observed as a test. Classically this is a non-linear process characterized by higher-order terms of the macroscopic polarization. High order contributions are usually several orders of magnitude weaker than the linear contribution, but can dominate over the linear term at high intensities. It is shown that multi-photon processes are associated with fractional ratios

\[
j \equiv \frac{\omega}{\omega_c} = \frac{p}{q}
\]

where \(p\) and \(q\) are integers. We term the \(p > q\) case as high-harmonics while the case \(p < q\) sub-harmonics. In our case, when the sample has relatively high mobility and density, positions of PCR peaks split into values that could be interpreted as multi-photon processes.
associated with sub-harmonic transitions where the resonant frequency is

\[ \omega = \sqrt{\left( j e B/m^* \right)^2 + \omega_p^2}, \quad (j = p/q) \]  

(5.4)

Figure 5.5: Multi-photon PCR on GaAs hole sample "d" and GaAs electron sample "c" data (scattered points) and fitted with the dispersion relation (solid line). (a) Recorded data of hole sample "d" with MW of 31 GHz and 1\( \mu \)W. Sub-harmonic orders are labeled. (b) Recorded data of electron sample "c" with WM of 34.8 GHz and 0.1 mW. Sub-harmonic orders are labeled. (c) 2nd derivative of data in (b).

For hole samples where the resonance peaks are widely separated because of their large effective masses, multiple-peak features are more visible (Fig. 5.5 (a)). Electron samples with not high enough mobility, however, are generally more difficult to analyze since the
peak separations are smaller than their FWHM (Fig. 5.5 (b)) due to their smaller effective mass. It can be seen from Fig. 5.5 (c) that by taking a second derivative of the recorded data to filter out the background contribution, the absorption peaks can be revealed. We note that intensities of the sub-harmonic peaks generally do not show monotonic order, for example sample "d" in Fig. 5.5 (a). Phenomenologically, we found that multi-photon process is favored with increasing carrier density as the 2DHG wafer used in Fig. 5.5 has a hole carrier density of $2.4 \times 10^{11} / \text{cm}^2$. We will study this phenomenon in detail in the following sections.

5.3 Results and Discussions

5.3.1 Multi-photon CR in High Mobility 2DEGs

Electrical and optical methods are two of the widely-used ways to probe cyclotron transitions in a 2D system with LL spectrum. A resonance near the Fermi energy occurs from the highest LL $|N\rangle \rightarrow |N + 1\rangle$ transition (Fig. 5.6(b)) resulting in a peak in both the optical and photoresistance spectrum. The effective mass of charge carriers can be extracted by least squares fitting of the MIRO peaks to this linear dispersion. It is well known that electrons in GaAs/AlGaAs QWs usually have an effective mass around 0.067 $m_e$, while hole effect mass varies between 0.2 $m_e$ and 0.5 $m_e$ depending on other parameters like well width and doping. The effective mass from all the wafers we measured are around the well-known value of 0.067$m_e$ for GaAs 2DEGs (Table 5.2).

<table>
<thead>
<tr>
<th>Wafer</th>
<th>Density (1 x 10^{11} / \text{cm}^2)</th>
<th>Mobility (1 x 10^6 \text{ cm/V \cdot s})</th>
<th>$m^*$</th>
<th>$\tau_{tr} (\text{ps})$</th>
<th>$\tau_s (\text{ps})$</th>
<th>$\tau_{tr}/\tau_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>0.8</td>
<td>14</td>
<td>0.069</td>
<td>560</td>
<td>162</td>
<td>3.5</td>
</tr>
<tr>
<td>b</td>
<td>1.3</td>
<td>13</td>
<td>0.075</td>
<td>520</td>
<td>115</td>
<td>4.5</td>
</tr>
<tr>
<td>c</td>
<td>2.7</td>
<td>33</td>
<td>0.070</td>
<td>1310</td>
<td>300</td>
<td>4.4</td>
</tr>
<tr>
<td>d</td>
<td>3.6</td>
<td>4.7</td>
<td>0.069</td>
<td>187</td>
<td>53.6</td>
<td>3.5</td>
</tr>
<tr>
<td>e</td>
<td>4.1</td>
<td>11</td>
<td>0.072</td>
<td>450</td>
<td>114</td>
<td>3.9</td>
</tr>
</tbody>
</table>

Table 5.2: Description of electron samples a, b, c, d, e. $\tau_{tr}$ and $\tau_s$ refer to transport scattering time and single particle relaxation time.
Due to the high mobility of our samples, multi-photon transitions are clearly observable. A zero-resistance state is shown in Fig. 5.8 (f) which is identified with other sub-harmonic peaks corresponding to 2- and 3-photon absorption. The reflection absorption signal in Fig. 5.6 (c) also has multiple peaks related to fractional mode transitions. Transport parameters such as carrier density, mobility and scattering time $\tau$, are extracted from the
Figure 5.7: Transport data shows integer quantum Hall effect (IQHE) and fractional quantum Hall effect (FQHE) on sample "a", "b" and "c" from which carrier density and mobility could be extracted. Symbol "x" denotes electron density in the unit of $1 \times 10^{11} / \text{cm}^2$. Wafer parameters can be found in Fig. 5.2

magnetoresistance data in Fig. 5.6 (d) with single particle relaxation time $\tau_s$ obtained by fitting the absorption trace with a Lorentzian. The ratio $\frac{\tau}{\tau_s}$ is greater than unity for each wafer and this indicates that small angle scattering [48] is dominating.

5.3.2 Role of Metal Contacts

Multi-photon transitions are higher in strong electric fields and they can be observed under intense radiation or large DC electric fields. In our measurement, a small current is used in the photoresistance probing, so the existence of a strong electric field near the metal contacts is possible. When the large electric field of a surface plasmon at an interface interacts with photons, non-linear optical effects like second harmonics are observed [49]. In order to rule out the possibility that the multi-photon phenomenon is caused by the electric field of the surface plasmon near the contacts, which is relevant in many photovoltage measurements, we prepared different sample geometries for comparison.
As shown in the bottom of Fig. 5.6 (a), a large sample whose metal contacts are positioned outside of the region of MW radiation is used, compared to a small sample cut from the same wafer whose contacts are within the radiation. Here no qualitative difference in terms of the number of observable sub-harmonic orders in MIRO is found except additional peaks originating from the CR of electrons in the metals are observed for small samples. We further analyze the difference of cyclotron absorption signals with or without metal contacts, yet only slight modification of the peak strength is found.

Therefore, we conclude that metal contacts on a sample perimeter are not vital in generating high harmonics or multi-photon processes in MIRO and cyclotron reflection experiments. The phenomenon of multi-photon transitions in high mobility 2DEGs is a general result of interaction between 2D electrons from the highest LL and MW field.

### 5.3.3 Plasmon Coupling

Coupling between $\omega_c$ and the bulk magnetoplasmon frequency is widely observed in CR measurements. The bulk magnetoplasmon frequency is

$$\omega_p = \sqrt{\frac{n_s e^2}{2m^* \epsilon_{\text{eff}}} N}$$

(5.5)

where $N \in \mathbb{N}$ is the plasmon harmonic, $n_s$ is the sheet carrier density, and $\epsilon_{\text{eff}} = (\epsilon_0 + \epsilon_1)/2$ is the effective dielectric constant, which is the average of the dielectric constant $\epsilon_1$ of GaAs and $\epsilon_0$ the dielectric constant of the vacuum) [47]. The effective resonance transition manifests itself as the plasmon-cyclotron modes

$$\omega = \sqrt{\omega_c^2 + \omega_p^2}$$

(5.6)

In addition to the well-known PCR mode with $N = 1$, we observe the odd plasmon harmonic modes where $N = 1, 3, 5$ in low density samples (Fig. 5.9 (a)) using microwave reflection.
However, the MIRO spectrum only shows the non-coupled cyclotron mode (Fig. 5.8) and there have been no theories proposed to explain the absence of the plasmon-coupled modes in MIRO. We speculate that the different phenomena observed here in electric transport and cyclotron absorption measurement in essentially the same process is due to the fact that different probing methods are used. Therefore, even with the existence of all the physical processes, only those that could be coupled to the probing device can appear in certain measurements.

The difference between an optical probing and an electrical measurement under microwave radiation is that the latter involves a biased DC electric field in addition to the rapidly oscillating AC field. And only those scattering processes that respond to the applied DC field would participate in the transport measurement. Note that though the microwave has a large electric field ($< 1 \times 10^4 \text{ V m}^{-1}$ due to screening) compared to the DC field ($\sim 1 \times 10^{-3} \text{ V m}^{-1}$), it’s oscillating with a very high frequency. Therefore as an accumulative multiplication of field and effective time, the Fermi sphere displacement in the Drude model caused by DC field, dominates over AC field which could be regarded as unbiased when it interacts with electrons. This phenomenon is the same with bulk magnetoplasmon whose rapidly oscillating field is biased in a certain direction, whereas the DC electric fields have well-defined positive/negative ends. We think the lack of direct correlation between the magnetoplasmon and transport scattering process might be a candidate to explain the failure to observe the PCR modes in MIRO.

5.3.4 Density Differences

From the linear dependence of the CR frequency on the applied magnetic field in the MIRO spectra (Fig. 5.8), transition modes with different orders can be identified. Apart from the obvious fact that the number of visible high order harmonic peaks depends on the mobility, or more precisely the single particle quantum lifetime (without peak overlapping), the carrier density also plays an important role. In Fig.2, we show that for wafers with carrier density lower than $1.3 \times 10^{11} \text{ cm}^2$, no transitions with $j < 1$ can be observed, whereas for
densities greater than $2.7 \times 10^{11} \text{ cm}^2$, two photon transitions ($j = 1/2$) are always seen, and even other fractional transitions like $j = 3/4$ are visible in our highest electron density sample. Therefore, we can confidently conclude that higher carrier densities increase the observable orders of the multi-photon transitions in high mobility 2DEGs.

Surprisingly, our reflection interferometer reveals more orders of multi-photon cyclotron resonance signals compared to photoresistance. Limited by the fundamental mode of the waveguide to 2-40 GHz, the highest PCR frequency we are able to observe is restricted. So a smaller magnetoplasmon frequency would leave us a wider frequency range to work with. Compared to 2DHGs, the small effective mass in 2DEGs yields a higher plasmon frequency (usually around 20GHz) for millimeter size samples [45]. From equation 5.5, we can see that this frequency is determined by carrier density and the width of the sample given the same type of QWs. Therefore, a lower density wafer (e.g. wafer a, b) is preferred when smaller size stripe patterns are fabricated. In fact, for wafer b, resonance peaks are observed in the entire frequency range of 26-40 GHz on the 200 μm wide stripe sample, while when the plasmon frequency is pushed to even higher frequency by narrowing the stripe width to 100 μm, only 3 data points close to 40 GHz can be collected (red dot in Fig. 5.9 (d)). Therefore, a wafer with lower density allows us to observe more coupling modes. For high density samples (Fig. 5.9 (c-f)), a single magnetoplasmon frequency with different cyclotron harmonics is found, while for low density samples (Fig. 5.9 (a)), higher odd plasmon harmonics ($N = 1, 3, 5$) with single cyclotron modes branch out in the dispersion. The different behavior of the coupling mode still needs a theoretical explanation.

However, $j < 1$ transitions are found in high-density samples with reflection method, and much richer dispersion relations are apparent. Unlike transport data where the major peaks always relate the $j = 1$ mode, PCR data shows a rather different pattern, where some fractional $j < 1$ transitions could well dominate over the fundamental cyclotron peaks. Table 5.3 shows a typical reflection line at a fixed frequency and power on a sample cleaved from wafer e. Since the mobility is so high that peaks corresponding to different harmonic orders are very well separated, Lorentzian shape fitting becomes possible for
major signals. Single particle relaxation time with each transition can be extracted from the FWHM of all the peaks. We think the wide $j = 1/3$ peak may associate to the process of subsequent absorption of two photons after the first one before the system relaxes to the ground state, therefore relaxation time is almost 3 times that of $j = 1$ (assuming the same broadening for those processes). The even smaller $\tau_s$ for $j = 3/4$ or $4/5$ indicates overlapping LLs, which is reasonable at low magnetic field.

The fact that multi-photon processes and MIRO strength favor higher carrier density has been widely observed experimentally. The simplest explanation is that more electrons per unit area generate relatively higher signals. However, if we assume this nonlinearity originates from the electron-electron interaction, the ratio of Coulomb energy and kinetic energy is actually higher at low densities. Therefore, a proper theoretical framework is needed to address the density dependence when multi-photon transitions are included in the whole picture.

### 5.3.5 Frequency, Power, and Temperature Dependence

On wafer c where two-photon transitions are significant, we study microwave power, frequency and temperature dependence of PCR and MIRO signals. At a fixed frequency, naturally both signals increase with microwave power (Fig. 5.10 (a, b)) but in a rather different manner. For MIRO, the major peak height ($j = 1$) saturates at high microwave power which is consistent with the findings of [50], while the $j = 1/2$ peak is hardly visible with low radiation power. However, in the case of the reflection signal observed here, the dominant peak always corresponds to the two-photon absorption, which has no onset power threshold, and the peak heights depend linearly on the input power.

Fig. 5.10 (c) and (d) illustrate magnetoresistance and cyclotron reflection measurements at different microwave frequencies with a fixed power. Two-photon MIRO peaks are more prominent at lower frequencies. This is a reasonable result because the number of photons is inversely proportional to the frequency with the same power. However, this phenomenon is not observed in the reflection spectra, where the major peak is always
related to the two-photon transition in the frequency range. In spite of the fact that there is no frequency threshold for two-photon MIRO observed in our experiment, we cannot exclude the possibility that for separated LLs, there might exists a higher frequency upper threshold of the form \( \omega_c \left[ 1 - \arccos(1 - \alpha) - \sqrt{(2 - \alpha)\alpha} \right], (\alpha < 2) \).

For a fixed microwave frequency and power, increasing temperature will smear out the MIRO peaks but the reflection line shape remains the same within the experimental temperature range we utilized (Fig. 5.10 (e, f)). SdHO, MIRO and cyclotron reflection are visible at different temperatures because their mechanisms are different. Within the investigated temperatures, the CR line shape remains unchanged as long as the temperature is low enough for electrons to undergo circular motions (\( \omega_c \tau_q \gg 1 \)). However, SdHO can only be resolved at a lower temperature and a higher field because the LL separation (\( \hbar \omega_c \)) needs to be larger than smearing of the Fermi surface (\( k_B T \)) in order for the DOS to vary. It has a form

\[
\delta \rho = 4 \rho X_T \lambda \cos(2\pi \nu) / \sinh(X_T)
\]

where \( X_T = 2\pi^2 k_B \hbar \omega_x \). In order to see MIRO, in addition to all the conditions above, strong microwave radiation is required. The contribution to the photoresistance is

\[
\delta \rho_i = -4\pi \rho_0 P_0 \frac{\tau_i}{\tau_{tr}} \epsilon_{ac} \lambda^2 \text{sinc}(2\pi \epsilon_{ac})
\]

in the overlapping LL regime (\( \omega_c \tau_q < \pi / 2 \)), where \( \tau_i = 3\tau_q^{im} \) in the displacement model and \( \tau_i = \tau_{in} \) in the inelastic model. Here \( \tau_q^{im} \) is the long range impurity scattering time and \( \tau_{in} \) is the electron-electron inelastic scattering time contribution to the single particle quantum lifetime \( \tau_q \).

5.3.6 Theory of Simultaneous/Step-wise Transition in MIRO

When we consider multi-photon processes, simultaneous or stepwise absorption are both possible mechanisms. Dmitriev et al. [51] proposed that for well-separated LLs,
sub-harmonic processes in MIRO are dominated by multi-photon inelastic mechanisms. However, microwave-induced spectral reconstruction of density of states shows single photon transitions are possible as well when Landau levels start to overlap. Though the process seems the same, the underlying physics of these mechanisms are rather different.

For a simultaneous absorption of $m$ ($m \geq 2$) photons, the corresponding contribution to the photoresistance is $W_\pm$, where

$$W_\pm = \frac{\tau_q}{\tau_{tr}} \left[ \frac{e\epsilon v_F}{\hbar \omega (\omega - \omega_c)} \right]$$

(5.9)

(+ and - correspond to left and right circularly polarized radiation, respectively) [52]. While in the framework of stepwise transitions of $m$ single photons, the ratio of inelastic photoresistance and DC resistance has the form

$$\frac{\delta \rho_{in}}{\rho_D} \propto \lambda^{2m}$$

(5.10)

where $\lambda = \exp \left(-\frac{\pi}{\omega_c \tau_q}\right)$ is the Dingle factor [53]. Some parameters of the contribution above can be estimated for wafer c. Both transport scattering $\tau_{tr} \approx 1.3$ ns and quantum relaxation time $\tau_q \approx 0.7$ ps could be extracted from SdHO profiles. We notice that the ratio $\frac{\tau_q}{\tau_{tr}}$ is almost 100 times smaller than the value presented in [43]. The Fermi velocity is estimated at $2 \times 10^5$ m s$^{-1}$. Due to the screening effect, the magnitude of the electric field that the electrons experience is difficult to estimate directly, though damping effect of SdHO could be used as a reference. Since we have a small quantum lifetime, the Dingle factor is vanishingly small ($\sim 10^{-31}$), meaning a step-wise absorption is unlikely. Therefore, multi-photon inelastic mechanisms seem to be more probable for sub-harmonic processes in MIRO.

In addition, the quasi-linear dependence of the two-photon peak amplitude in MIRO at larger power for high frequencies indicates the significant effect of microwaves on the electron distribution function [28]. This is an important feature of the inelastic mechanism, further confirming our conjecture that sub-harmonic MIRO is dominated by multi-photon
inelastic processes. Note this is consistent with one-photon CR except it saturates at a lower microwave power [28,43].

For wafer c, we also extract the single particle relaxation time $\tau_s \approx 0.3$ ns responsible for LL broadening from Lorentzian fitting of the cyclotron reflection line shape. This is considered as the counterpart of, but much larger than the quantum lifetime in MIRO. Within the experimental frequency range, the Dingle factor is estimated to be 0.96. Therefore, a model of stepwise absorptions of single photons before the electron system is completely relaxed would be a more suitable model—accounting for the two-photon peak in reflection spectra due to the long relaxation time $\tau_s$ in the strongly overlapped LL regime. And the fact that two-photon peaks are taller than one-photon peaks also supports the argument for long quantum relaxation time.

<table>
<thead>
<tr>
<th>Harmonic order $j$</th>
<th>Peak width $\Delta B(T)$</th>
<th>$\tau_s$ (ps)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/3</td>
<td>0.0184</td>
<td>22.3</td>
</tr>
<tr>
<td>3/4</td>
<td>0.0036</td>
<td>113.8</td>
</tr>
<tr>
<td>4/5</td>
<td>0.0040</td>
<td>102.4</td>
</tr>
<tr>
<td>1</td>
<td>0.0059</td>
<td>69.4</td>
</tr>
</tbody>
</table>

Table 5.3: Fitting parameters of reflection absorption peaks on sample cleaved from wafer e

5.3.7 Multi-photon Cyclotron Absorption Probability Calculation

Theoretically, it is important to know the probability of cyclotron absorption in the manner of quantum mechanics. With the constraint of energy conservation and high radiation intensity $I$, coherent multi-photon transitions between real initial and final states are possible with the assistance of virtual intermediate states. Based on the lowest-order perturbation theory, the $n$-photon absorption coefficient is $K_n \propto I^n$. When intensities become high enough, multi-photon absorption becomes comparable with single photon
absorption. A nonlinearity parameter $\eta_n$ to characterize this feature could be introduced in this case,

$$\eta_n = \frac{K_n}{K_{n-1}}$$  \hfill (5.11)

where are $K_n$ and $K_{n-1}$ photon absorption coefficients and the total absorption coefficient is

$$K = \sum K_n$$  \hfill (5.12)

Unlike the lowest order perturbation theory where $K_n$ is determined by the absorption of $n$ photons, to fully develop high-order nonlinearity, virtual transitions that involve the absorption of $(n + m)$ photons and simultaneous emission of $m$ photons should also be considered. These absorption and emission channels would interfere and suppress each other contributing to $K_n$ substantially and this would make the transition probability more complex [54]:

$$W_n = \frac{nK_n I}{\hbar \omega} = \frac{m^3}{\pi \hbar^4} f(n\varepsilon_0)(2n\hbar\omega)^{1/2}(M_n)^2$$  \hfill (5.13)

Here the high order processes are contained in the matrix element

$$M_n \approx \frac{1}{4} \sqrt{\frac{2}{5}} \left(\frac{eE}{\omega}\right)^2 \frac{n + 1}{n - 1} \sum_{m \in \mathbb{Z}} J_m(\rho_2^n) J_{n-2-2m}(\rho_1^n)$$  \hfill (5.14)

with

$$\rho_1^n = \left(\frac{8n}{3m_\ast \hbar \omega}\right)^{1/2} \frac{eE}{\omega},$$  \hfill (5.15)

$$\rho_2^n = \left(\frac{eE}{\omega}\right)^2 \frac{1}{2m_\ast \hbar \omega} \frac{n^2}{n^2 - 1}$$  \hfill (5.16)
where \( f(\varepsilon) \) is the distribution function, \( J_m \) is the \( m^{th} \) order Bessel function, \( m^* \) is the effective mass and \( E \) is the electric field. This is under the assumption that \( |M_n| \ll \hbar/\tau \) where \( \tau \) is the lifetime of the final state. In order to achieve \( \eta_n \approx 1 \), intensities of tens of GW/cm\(^2\) are needed, which could go beyond the destructive threshold. Whereas if we use a lower frequency regime, a much lower power intensity can allow this transition.

We believe it is the gigahertz frequency range and high sample mobility combined that makes the multi-photon processes visible in our measurement. Increasing radiation intensity obviously is not preferred since heating becomes an issue and might alter the dielectric properties. Therefore, general CR in high mobility GaAs 2DEG quantum wells is a powerful approach to study multi-photon transitions as the effective mass of the carriers has a stable value and the peaks are narrow.

### 5.4 Conclusion

Using the reflection method in the waveguide setup, we finally overcame many other obstacles and improved the sensitivity significantly. In this way, we are able to discern some interesting physics in this system. From a simultaneous measurement of photoresistance and reflection in the gigahertz regime, we observed multi-photon processes in high mobility 2DEGs where sub-harmonic transitions are identified by their magnetic field dispersion. We found that the plasmon coupling CR mode is only relevant in cyclotron reflection but not MIRO. And more multi-photon orders are revealed in higher carrier density samples; and this is confirmed by both transport and optical experiments. We discussed the possible explanations and presented results of frequency, power and temperature dependences for a specific type of wafer. Then we further discuss the different theories regarding the multi-photon transition mechanisms with respect to the observations in the low frequency range.
Figure 5.8: (a, b, c, d, e) MIRO peaks fitting showing linear dependence of cyclotron frequency with respect to the magnetic field on samples cleaved from wafer "a", "b", "c", "d" and "e". (f) MIRO signal on a sample cleaved from wafer c, when irradiated with a fixed frequency of 40 GHz and power of 5 dbm. Insets are MIRO and SdH signals when irradiated with a fixed microwave frequency. Symbol x denotes electron density in the unit of $1 \times 10^{11} \text{ cm}^{-2}$.
Figure 5.9: Frequency dependence of the reflection signals on the magnetic field on (a) a sample with width of 0.76 mm (unpatterned) cleaved from wafer a; (b) a sample cleaved from wafer a patterned with 200 µm wide stripes; (c) samples cleaved from wafer b patterned with 100 µm wide stripes (red line) and 200 µm wide stripes (other colors); (d) a sample with width of 1.7 mm (unpatterned) cleaved from wafer b; (e) a sample with width of 4.5 mm (unpatterned) cleaved from wafer c; (f) a sample with width of 2.1 mm (unpatterned) cleaved from wafer e. Insets are the reflection signal line shape at a fixed frequency and 5 dbm microwave power.
Figure 5.10: (a) Power dependence of MIRO with a fixed frequency of 33.5 GHz. All data traces are shifted by 0.5 Ω for clarity. (b) Power dependence of reflection signal with a fixed frequency of 33.5 GHz. All data traces are shifted by subtracting their value at -0.3 T. (c) MIRO traces with selected frequencies, shifted with 100 Ω. (d) Reflection traces with selected frequencies, shifted with 2 mV. (e, f) Temperature dependence of MIRO and reflection traces with a fixed frequency of 36 GHz and power of 5 dbm. All data traces are shifted by 1 in (e) and 10 mV in (f).

Figure 5.11: A typical reflection trace (red curve) with a fixed frequency of 35.8 GHz and power of 5 dbm on a sample cleaved from wafer e. Peaks with different harmonic order are labeled by fractional j and fitted by a Lorentzian.
Chapter 6

Photocurrent in InAs/GaSb Quantum Spin Hall Edges

6.1 Topology in Physics

When Avogadro numbers of atoms and electrons are combined due to their interaction, a condensed type of matter tends to be formed such as solid or liquid. The existence of many-body effects makes it impossible to describe this system by solving single-particle Schrödinger equation. However, the symmetries of the solid state crystal system greatly simplify the problem. Translational symmetry in crystals led to the invention of band theory, and most of the condensed matter phase transitions can be explained by Landau’s spontaneous symmetry breaking by introducing a local order parameter.

IQHE and FQHE were discovered in the 1980s when researches applied a strong perpendicular magnetic field to a 2DEG in low temperature. Quantized Hall plateaus are explained by the only conducting edge channels while the bulk state is completely insulating, known as the first type of topological insulator (TI) called quantum Hall state. This universal conductance quantization is independent from the sample sizes, impurities or materials and has unprecedented precision ($10^{-9}$). Scientists soon realized that this quantum Hall state only violates TRS, and therefore cannot be described by a local order parameter; instead a global topological order (later known as a Chern number) is needed. The interesting thing about this topological state is that the conducting edge is dissipationless, making it perfect for building new computing blocks, which usually suffer from massive heating.

Ever since then, people have been rigorously searching for a new type of topological material. Quantum Spin Hall Insulator (QSHI), three dimensional topological insulators and Weyl semimetals have been confirmed experimentally, subsequently facilitating the
development of areas like Majorana fermions and topological quantum computing.

6.2 Quantum Spin Hall Effect in InAs/GaSb Broken Gap Double Quantum Wells

In 2005, QSHE was first proposed by Kane and Mele using spin-orbital coupling without breaking 2D mirror symmetry [55–57] in graphene. The linear dispersion Dirac cone in graphene would be gapped out due to spin-orbital interaction, resulting in a spin-polarized edge current which is protected by TRS. However, experimentally it is very hard to measure this phenomenon because of the small spin-orbital coupling ($10^{-3}$ meV) in carbon-based materials.

In 2006, Bernevig, Hughes and Zhang (BHZ) proposed that in the inverted HgTe/CdTe quantum wells, due to strong spin-orbital coupling, QSHE could exist with the top of the valence band higher than the bottom of the conduction band [58]. Molenkamp et al. confirmed this prediction experimentally using MBE grown HgTe/CdTe quantum well by measuring the longitudinal conductance of a Hall bar [59]. Since single electron elastic backscattering between the two counter-propagating edges is prohibited due to spin-momentum locking, the edge conductance quantized to the predicted value of $2e^2/h$ (Fig. 6.1) which can be calculated by Landauer-Buttiker formula.

In 2008, C.X. Liu et al. proposed that QSHE also exists in InAs/GaSb double quantum wells (DQWs) where the bottom of the conduction band in InAs is tens of meV lower than the top of the valence band in GaSb [60]. A hybridization gap is opened in the bulk due to the spatial overlap between the electron and hole wavefunctions and different regimes in the phase diagram (Fig. 6.3) can be accessed by tuning of the front/back gate [61, 62].

Compared to HgTe, which could only be tuned discretely by the well width, this system offers a more flexible alternative to continuously tunable experimental parameters in and out of the topological phase. The rich physics in this system is manifested in the quantum spin Hall effect (QSHE) [60], exciton insulator (EI) [63] and in transitions between semiconductor and semimetal phases. Application of strain increases the bulk gap and also
Figure 6.1: Longitudinal resistances of four-terminal devices with normal, inverted and different lengths produced in HgTe/CdTe QW structures. From [59].

Figure 6.2: Energy dispersion of InAs/GaSb bilayers in the normal and inverted regime. From [60]

pushes the band structure towards deeper inversion [64] without significantly sacrificing coherent length.

Terahertz (THz) [65] and infrared (IR) [66] spectroscopies have been powerful tools to probe 2D and 3D topological materials. Recently, the photovoltaic effect was used to study electron spin imbalance on the helical edges in HgTe QWs induced by circular photogalvanic effects in unbiased devices by selectively exciting spins [67] or utilizing the chiral properties of unique matter like Weyl semimetals [68].
In this chapter, we observe bias-current-polarized microwave (MW) photocurrent in strain layer InAs/GaSb DQWs and demonstrate that this signal comes from helical edge states. The miniscule photocurrent near the CNP marked the spin-momentum locking which is a feature of a strongly interacting Luttinger liquid [69]. The Rashba-type spin-orbit coupling is believed to be the main reason for spin flipping induced by broken TRS; therefore, this phenomenon disappears under the application of perpendicular/parallel magnetic fields.

Majorana-bound states were predicated in the system [70–72] and there are possible approaches for manipulating them with electromagnetic wave which could be used for topological quantum computing.

### 6.3 Experimental Techniques

The experiments are done on strained InAs/In$_{0.32}$Ga$_{0.68}$Sb DQWs grown by MBE on n-doped GaSb substrates. The electron/hole QWs have well widths of 8nm/4nm respectively (Fig. 6.4 (a)). Transport properties of this wafer can also be found in [64]. Hall bar geometry
with a length of 75 μm and width of 25 μm is produced with photolithography as shown in Fig. 6.4 (d). Ohmic contacts are made by Indium soldering at relatively low temperature to avoid possible damage, and a back gate from the conducting substrate is used to tune the Fermi level in the device. Usually the band structure and Fermi level can be tune using both front and back gate but the ability to reach the bulk gap without a front gate (Fig. 6.4 (a)) enables the MW to interact with the charge carriers in the device.

The sample is immersed in $^3$He liquid in a $^3$He cryostat with a base temperature of 300mK and equipped with a superconducting magnetic coil up to 12 T. The microwave transmits via a semi-rigid coaxial cable with an antenna in the end. The magneto-transport is measured by a standard low frequency lock-in amplifier which also provides the amplitude modulation of the MW in the photocurrent probing. A sufficient large DC bias current ($\sim 10nA$) is used to provide directional selection for the excited carriers.

In order to rule out the possibility that the observed photocurrent comes from the bulk contribution, a Corbino sample is fabricated on the same wafer using the same procedure as shown in Fig. 6.4 (b). Photocurrent is measured by the photovoltage drop across a load resistor with a resistance much smaller than the resistance of the sample itself.

### 6.4 Result Discussion

Compared to HgTe/CdTe QWs, InAs/GaSb DQWs have unique properties as type II QWs in which the electron subband and hole subband are separated in two different layers. In addition to a quantitative hybridization correction, bulk inversion asymmetry (BIA) and structural inversion asymmetry (SIA) also enter nontrivially into the effective Hamiltonian in the BHZ model [60]:

$$H = H_0 + H_{BIA} + H_{SIA}$$

Projected to the effective 4-band basis, $H_{BIA}$ could be recognized as a k-linear Rashba term by neglecting the high-order k-cubic term responsible for deviation from the perfect
Figure 6.4: (a) Structure of strain layer InAs/GaSb wafer used in the experiment, which hosts robust quantum spin Hall helical edge states. (b) Experimental setup of the photovoltage and photocurrent measurement on a typical Corbino disk with a backgate. (c) Resistance measured for Hall bar sample and conductance measured for the Corbino sample. (d) Photoresponse measurement setup on a Hall bar sample fabricated the same way with the Corbino sample shown in (b). Arrows along the edges illustrating spin-momentum locking and selection of current direction by DC bias current.

Despite these factors, a QSH regime can still be reached in this system with the help of gate tuning, and resistance quantization can be observed in samples with sizes smaller than the phase coherent length [75]. Both the application of perpendicular and parallel magnetic fields destroy QSH states by breaking TRS and cause backscattering between the alignment of spin away from the CNP [73] (Fig. 6.5).
counter-propagating edges, thus increasing resistance near the CNP (Fig. 6.6 (b,c)). Here resistance saturation happens at a smaller magnetic field (consistent with the saturation of photocurrent) and a minimal excitation current is used to reduce heating. Large magnetic fields might also form Landau levels in the bulk and turn the conducting edges from helical to chiral ($B_\perp$), or shift the subbands to the semi-metal regime ($B_\parallel$) [76]. Neither of those effects is relevant to our case here.

### 6.4.1 Exclusion of the Bulk Contribution

In order to study the photoresponse of the helical edge, contribution to photocurrent from the bulk needs to be excluded. In a Corbino device the edges are shunted and make no contribution to transport properties. Therefore, we fabricate a Corbino disk with the same wafer and procedure and show that it has a truly insulating bulk (Fig. 6.6 (a)). We also demonstrate in 6.4 (c) that when excited by MW radiation, the bulk still contributes no photoconduction up to 8 T. Therefore, it is reasonable to conclude that the photoresponse signal comes from the sample edges when the Fermi level is tuned in the gap.
6.4.2 Understanding the Photoresponse

Generally when a semiconductor device is illuminated with MW radiation, charge carriers below the Fermi level absorb photons and become excited to higher density of states (DOS), thus creating free electron/hole pairs. These mobile charge carriers will drift under a
bias electric field induced by the application of external current (Fig. 6.9). Therefore, the photocurrent formed always has the same polarity with the external bias. However, the voltage drop caused by this mechanism is due to the fact that electrons and holes are spatially separated, meaning the voltage drop always tries to compensate for the external field. As a result shown in Fig. 6.8, when a negative bias current is applied, the current shows a negative sign while the voltage is positive. Note that this case is different from photocurrents excited by circularly polarized radiation in unbiased devices [77], where spin is selectively excited.

![Figure 6.7: (a) The MW power-dependent photocurrent traces measured in a Hall bar device with no magnetic field application and a large external bias current. Enhanced photocurrent on both sides of the gap boundary is detected with increasing MW radiation. Miniscule photocurrent at the CNP is maintained up to 10mW. This phenomenon starts to break down at 100 mW MW power due to heating. (b) MW frequency-dependent photocurrent measurement is shown by tuning the MW power to keep temperature at a fixed value of 650 mK. This is chosen instead of using a fixed MW input power in order to rule out the possibility of different attenuations at various frequency ranges. Shifting of the CNP towards lower backgate voltage is observed with increasing MW frequency.](image)

When the Fermi level is tuned from the conduction band towards the bulk gap, the linear dispersed edge states start to contribute to the photocurrent formation. Since the MW photon energy ($\sim 50 \mu$eV) is rather small compared to the hybridization gap ($\sim 10$ meV), once the gate voltage passes the bottom of the electron subband, photocurrent starts
to decrease, thus creating the first photoresponse maximum. For the same reason, we observed another maximum near the top of the valence band. In the bulk gap, since only one of the helical channels is selected on each edge by the external bias current, spins are polarized to a certain direction due to spin-momentum locking. The lack of spin degree of freedom disallows photoexcitation between the two helical edges (Fig. 6.9 (c)). The overlap between the minimum of photocurrent (photovoltage) trace and maximum of the resistance confirms photoexcitation prohibition happens mostly near the CNP.

6.4.3 Other Spin Flipping Mechanisms

Besides the prohibition from spin flipping between the two helical edges, other mechanisms should be considered as well. It has been theoretically proposed that potential inhomogeneities like Kondo impurities in the vicinity of the edge, which can trap bulk electrons won’t destroy the helical property, instead, they completely screen the impurity spins by forming local Kramer singlets [78]. Electron-phonon coupling also does not affect the edge robustness to first order approximation even in the presence of spin-orbital coupling [79]. However, one-particle inelastic scattering or two-particle interactions like umklapp scattering [80] even in the absence of impurities are still considered relevant and might be responsible for the residual photocurrent formation.

In addition, we observed a relatively large photoresponse fluctuation in the gap and we ascribe this phenomenon to the enhancement of universal charge fluctuation which generally exists in mesoscopic devices. The strongly interacting Luttinger liquid enhances universal charge fluctuation and can be observed in resistance measurement with small bias [69]. Here data are collected by repeatedly downwards sweeping gate voltage to rule out temporal drift from the charge relaxation.

6.4.4 Power and Temperature Dependence

The power dependence measurement in Fig. 6.7 (a) shows that photocurrent increases with MW radiation until heating comes into play and thermal energy might also start to
excite carriers.

Figure 6.8: Photocurrent (a) and photovoltage (b) measurement in a Hall bar device with fixed MW frequency and power. Backgate voltage is swept at different perpendicular magnetic fields. A moderate magnetic field is employed to break TRS, thus causing Rashba-type spin flipping between the two branches of helical edge states. This results in an enhancement of the photocurrent and photovoltage in the bulk gap. Comparison between the effect of parallel and perpendicular magnetic fields is displayed in the inset of (a) indicating a larger g factor for perpendicular fields.

The optical transition rate can be estimated by first-order perturbation theory where excited states are

\[ |m\rangle \approx |m_0\rangle + \sum_k \frac{H'_{f m}}{H^{(0)}_m - H^{(0)}_f} |k^0\rangle. \]  

(6.2)

Here the perturbation term mainly comes from the Rashba type which has a linear dependence on wave vector \( k \). Since the wavelength of the microwave photons are about six orders of magnitude larger than that of the edge state quasiparticles, momentum transfer between the photons and charge carriers can be neglected. Near the CNP point where the energy dispersion is also linear:

\[ E = \pm \hbar v_F k \]  

(6.3)
\[ E_m^{(0)} - E_k^{(0)} = 2\hbar v_F k \]  \hspace{1cm} (6.4)

Therefore, the numerator and denominator would cancel out and no frequency dependence is expected to the lowest order. Experimentally, we tune MW power to fix the temperature at 650 mK in order to rule out the attenuation effect in the circuit and vary MW frequency. Except from frequencies below 5 GHz where photon energy is much lower than thermal energy (heating effect dominates), no frequency dependence is observed up to 40 GHz. Interestingly, there is a large shift (~0.1 V) of the photocurrent minima around three orders of magnitude larger than the MW photon energy which known mechanisms do not explain.

### 6.5 Prospects for Future Work

Theoretical [81] and experimental [75] studies have shown that strong backscattering only exists in the system with sufficient disorder, indicating that the application of magnetic field is not enough to gap out the edge states. In InAs/GaSb DQWs, this type of disorder is generally provided by randomly positioned scattering centers, like the bulk potential inhomogeneities near the edge induced by ionized dopant atoms in the QW, disordered nuclear spins or charge puddles formed by gate contacts. As an effect, the non-uniform distribution of disorder on the helical edges blurs the band gap boundary. This might be even more significant with broken TRS, and responsible for the relatively large noise we observed (Fig. 6.8) when helical edge states are destroyed by a magnetic field. For devices the length of which is much larger than the phase coherent length, disorder has a non-uniform effect on the helical edges. Therefore, a smaller device within coherence length is needed in order to observe a clear resonance in the gap opened by a magnetic field.

Despite that, data in Fig. 6.8 shows a significant enhancement in photocurrent/photo-voltage under magnetic field, indicating broken TRS and the destruction of proper spin alignment. Without this constraint, photocurrent can be formed (Fig. 6.9 (d)). Note that
Figure 6.9: Schematic of optical transitions for linearly polarized radiation and photocurrent formation for three positions of the Fermi level $E_F$ in the band structure. $k_x$ is the wave vector along the Hall bar edges and $E_{ex}$ the external electric field induced by the bias current. Region I: $E_F$ lying in the conduction band and electrons contributes to the formation of photocurrent. Region II: $E_F$ lying in the valence band and hole contributes to the photocurrent. Region III: $E_F$ lying in the bulk gap and helical edge states contribute no photocurrent due to the lack of spin flipping mechanism other than the Rashba term from SIA. Region IV: With the application of perpendicular/parallel magnetic field, helicity of linear-dispersed edge states is destroyed and photoexcitation near the CNP becomes possible.

the magnetic field where photocurrent starts to saturate is consistent with the value from resistance saturation. We also found that a perpendicular field has a larger effect than a
parallel field in this case (see Fig. 6.8 (a) inset) indicating a larger out-of-plane g factor. Possible future work also includes producing electron backscattering by locally controlling the spin with coherent electromagnetic wave manipulation or via electrostatic means. Therefore, it may be possible to spatially moving Majorana bound states to achieve non-Abelian statistics via braiding operations for topological quantum computing.
Chapter 7

Conclusion

In this thesis, we’ve collected experimental results of transport and optical phenomenon of 2DEG/2DHG under microwave radiations over a period of years.

Microwave spectroscopy is a very powerful tool to study various resonance features in 2D systems due to its frequency range comparable to the electronic structure in semiconductors. CR, ESR and GR have been observed in different setups using distinct mechanisms. In Chapter 3, we showed that thermal measurement can be applied to materials with good electron-phonon coupling. Therefore, heat generation as an indication of electronic excitation can be used to quantify resonance signals via a setup with a proper heat sink and cryogenically pumped to vacuum by sealing in a He3 environment.

With thermal measurement’s limitation of electron-phonon coupling requirement, we kept exploring more universal methods. We find that optical reflection as compared to conventional transmission has its own advantages, such as no requirement for sample thinning/wedging procedure. This usually reduces its mobility and its ability for back gate application would also allow for more broad use, such as Fermi level tuning. We first constructed a reflection spectroscope via coaxial cable and used a dual directional coupler to monitor input and output power at the same time. By calibrating the detector and setup, we found that though the sensitivity was already an order of magnitude better than the thermal configuration, this system suffers from serious background noise and unwanted multiple reflections at the connector. It is also highly frequency dependent because of the lack of resonance tuning mechanisms.

Realizing these shortcomings, we applied the same idea on a waveguide setup with a Magic Tee replacing the directional coupler and an adjustable short for tuning microwave
phase. This is essentially an interferometer between the incident and reflected waves with the sample inserted as a perturbation. We find that this setup is far superior than the coaxial cable one in terms of sensitivity and frequency dependence. We use this configuration to measure CR signal on various 2DEGs/2DHGs and discovered for the first time that multi-photon phenomenon can be seen using optical reflection instead of using photoresistance measurement. We also combine the optical reflection with MIRO and found that different properties of the same cyclotron process can appear when probed with different methods.

Finally, we present our results on the microwave application to the particular topological state known as the QSHI in InAs/InGaSb double QWs. We show that the photocurrent we observed comes purely from the linearly-dispersed helical edge state by tuning a back gate to position the Fermi level in the hybridization gap. We find that magnetic field as a method to break TRS can introduce a spin-flipping mechanism, which is originally locked to its momentum thus increasing the photoresponse. Though microwave photocurrent has been successful in measuring electromagnetic interaction between narrow gap topological edge states, better samples within coherent length and more sophisticated techniques are still very desirable.

Overall, we’ve developed a set of new tools to better understand the interaction between millimeter wave and two dimensional QWs. The rich physics and possible applications in these systems are expected to be explored by future physicists.
Appendix A

Sample fabrication

A.1 Mask Making

1. Draw desired pattern in AutoCAD or other software and save it in dwg format.
2. Load a blank mask into the machine for exposure.
3. Mask writing generally takes a couple of hours depending on the size of the area.
4. Develop: wash in MS-351:H₂O with 2:5 ratio for 60 s.
5. Etching: CEP-200 for 60 s until Cr is fully etched away and can be seen through the clear region.
6. Stripping: immerse in stripper PRS-100 for 2-5 mins until photoresist is washed away.
7. Clean: Wash with DI water.

A.2 Indium Contact Annealing for GaAs QWs

1. Use wafer cutting pen to slightly open the surface on the perimeter.
2. Put a minute amount of mixture of In/Sn (1:1) for an electron sample or In/Zn (1:1) for a hole sample on the position where it is sketched.
3. Anneal in the forming gas 1:20 H₂:N₂ for 20 mins with temperature between 420 °C and 450 °C.
4. Solder gold wire to the indium contact.
A.3 Photolithography

A.3.1 Sample Cleaning

(a) Acetone 5 mins.
(b) Methanol 5 mins.
(c) DI water 5 mins.

A.3.2 Spin Coating

Thickness of S1813 positive photoresist varies with coating speed.
(a) 1200 rpm 10 s at 100 rpm acceleration.
(b) 5000 rpm 1 min at 1000 rpm acceleration.
(c) Soft bake at 95 °C for 2 mins.

A.3.3 Exposure

Depending on the thickness of the photoresist, dose of the exposure may vary.

A.3.4 Developing

MF-321 for 60 s until pattern is clearly resolved.

A.3.5 Etching

Post bake at 100 °C for 2 mins. Etchant: H₃PO₄:H₂O₂:H₂O (1:1:38). Speed: 1200 Å min⁻¹.

A.3.6 Cleaning

Acetone for 3 mins to completely remove the leftover photoresist, heat slightly or use ultrasound if needed.
Dip in DI water to rinse off and then blow with dry N₂.
A.4 Metal Deposition

A.4.1 2DEG Contacts

High mobility GaAs/AlGaAs 2DEG contacts via e-beam evaporator:

Use standard photolithography to open a window for contacts and then deposit metal with the recipe in A.1:

Lift off in acetone (Use heating or ultrasound if needed.)
Then anneal at 450 °C for 20 mins in forming gas.
Figure A.2: The amount of exposure dose needed to fully effect the photoresist depending on the layer thickness. Interference pattern is observed.

<table>
<thead>
<tr>
<th>Thickness (Å)</th>
<th>Ge</th>
<th>Pd</th>
<th>Au</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rate (Å/min)</td>
<td>1.5</td>
<td>1.5</td>
<td>2.0</td>
</tr>
</tbody>
</table>

Table A.1: E-beam evaporator recipe for making contacts for 2DEG.

A.4.2 2DEG Gates

High mobility GaAs/AlGaAs 2DEG gates via e-beam evaporator:

Use standard photolithography to open a window for gates and then deposit metal with the recipe in A.2:

Lift off in acetone (use heating or ultrasound if needed.)
Table A.2: E-beam evaporator recipe for making gates for 2DEG.

<table>
<thead>
<tr>
<th>Thickness (Å)</th>
<th>Ti</th>
<th>Au</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rate (Å/min)</td>
<td>1.5</td>
<td>2.0</td>
</tr>
</tbody>
</table>

Table A.2: E-beam evaporator recipe for making gates for 2DEG.
Appendix B

Thermo-measurement Setup

B.1 Vacuum can stycast

1. Mix Stycast 1266 part A and B in a ratio of 28:100 in a container, stir slowly until it is clear and homogeneous.
2. Use a mechanical pump to pump the air bubbles out. Make sure the pressure is not so low that mixture is boiling.
3. After all the little bubbles rise to the surface, take it out and pour into a cylinder mold about 1.5 inch in diameter.
4. Wait overnight until it is totally solidified before machining.

B.2 Vacuum Can Design

Vacuum can schematics.

B.3 Vacuum can design

B.4 Vacuum can design

B.5 Herotek Sensor

Herotek Zero-bias Schottky Diode Detectors Data Sheet.
B.6 Lakeshore Sensor

In order to detect very small temperature changes, a sensor with negligible mass and thermocapacitance is needed. Lakeshore Cryotronics model CX-1030-BG-HT satisfies our requirement. B.6 shows the temperature dependence of a typical sensor.
Figure B.2: (b) Vacuum can bottom with outward thread. Grooves are carved to fit the sapphire crystal symmetrically.
Figure B.3: (c) Vacuum can connector to the probe and copper pillars for a heat sink.
Figure B.4: Information about Zero-bias Schottky Detector.

Figure B.5: Data sheet for Zero-bias Schottky Detector.
Figure B.6: Temperature dependence of Cernox sensors from Lakeshore Cryotronics.
Appendix C

Simulation for Quantum Well Design

Semiconductor heterostructures are common host for various types of 2D electron/hole systems. In order to achieve different conditions like carrier density, mobility, well width or even super lattice structures, we need to design according to the specific requirement. In that sense, a numerical simulator is essential for this purpose. 1D Poisson/Schrödinger solver is the most widely used band diagram calculator which is not only able to simulate band structure for various III-V group compound, but also does calculations under bias voltage, determine C-V characteristics. In most of the softwares, you can specify the layer thickness, doping level, then the charge concentrations are calculated using Boltzmann statistics.

Here is a typical package written in Python and the result of simplest simulation for GaAs QW. The source code can be downloaded from http://www.aestimosolver.org/.
Figure C.1: Band structure simulation of GaAs DQW

Figure C.2: Other complementary simulation result of GaAs DQW
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