RICE UNIVERSITY

BENEFIT-COST ANALYSIS IN REHABILITATION PROGRAMS

by

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Abstract

This thesis is concerned with an evaluation process for selection of rehabilitation research projects. An existing several-variable model is studied and extended. The parts of this model that relate to construction of a utility function are discussed in detail. A number of examples are given for illustration.
Acknowledgments

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Appendix 2

Bibliography
I. Introduction

Cost-benefit analysis (CBA) is basically a comparison of the costs and benefits associated with alternative courses of action. It first appeared in the River and Act 1902 which required evaluations of Army Corps of Engineers' river and harbour projects. Then in the thirties thru fifties, the practice of CBA mainly took place in those agencies concerned with water development projects. In the last two decades, due to the fast growth of large investment projects and that of such techniques as operations research, systems analysis, etc., interest among economists in CBA has grown tremendously [34].

One major field in which CBA has been used is medical and health care problems. The public is conscious of the rapidly accelerating costs of medical services. Rates of costs for health care are outstripping all other segments of the national economy. This issue, combined with increased competition for public funds and increased pressure for governmental efficiency, makes it necessary to improve the effectiveness and efficiency of health investments and to demonstrate that improvement has actually occurred [39].

Generally the application of CBA in health investments is in one of three ways. First, CBA has been used to analyze disease-specific programs of intervention. Heart disease [8], spinal cord injury [19], kidney disease [28], mental retardation [5], and alcoholism [21] are among the most commonly analyzed diseases. The analysis has typically focused on the economic impact of the disease, measured in terms of the cost of premature death and disability due to a reduction in the capacity to function. Although most studies have referred to other social benefits, no systematic attempt has been made to estimate their relative magnitude. Secondly, CBA has been used to compare a set of alternative
ways in which a given health or medical care can be produced or distributed \([8,12,15]\). Thirdly, the return to public investment in health-related programs such as medical research and health manpower training has also been estimated using cost-benefit techniques \([13,40]\).

A number of problems have been encountered \([7,8,25]\) in using CBA in evaluating health service policy: (i) How do we measure benefits? A considerable number of benefits such as the value of human survival, reduction of pain, and theoretical research achievement are intangible. Despite some analysts' attempts \([29,36]\), people are reluctant to put dollar values or measurements of any scale on those terms. (ii) The relevant statistical support to estimate the costs and benefits is mostly not available and costly to obtain \([21]\). (iii) Some unsolved issues of the methodology of CBA seriously distort the results of calculation. The first issue is a conceptual one of determining what to count as a benefit and what to count as a cost \([7]\). A second and more difficult problem is the issue of the appropriate discount rate to use in aggregating benefits over time. As discussed in a later section, the choice of discount rate can raise many arguments. Third is the problem of determining the proper criteria in ranking projects. Should we rank the projects on the basis of net present value, benefit/cost ratio or internal rate of return? (iv) The framework of CBA is a partial one and is basically a micro-oriented tool. It may be appropriately used to analyze economic problems with marginal impacts upon the entire economic and social structure of a society. Unfortunately, this basic micro-structural assumption is particularly vulnerable when applied to health programs with a potential for a multiplicity of interactions with other societal institutions \([8]\).

Due to the lack of sufficient resources to promote research in every area, the Rehabilitation Services Adminis-
tration (RSA) of the U.S. Department of Health, Education, and Welfare has long realized the importance of establishing policies and procedures for evaluating research projects. The agency hence has strongly supported the development of analytic methods for identifying research proposals which promise to give the biggest return per dollar spent. Early in 1972, the office of Research and Demonstration of the Social and Rehabilitation Service (SRS) awarded a grant to an interdisciplinary group consisting of members of The Institute for Rehabilitation and Research (TIRR) and Rice University to study the applicability of CBA for evaluating proposed research. This group generated a series of progress report which were labeled Analytic Aids for Research Proposal Evaluation (AARPS) [3,4,43].

The problem that the AARPS model addressed is essentially a multiple-objective Knapsack problem [3,9]. It involves the use of value/utility functions as evaluation devices and the selection of alternatives so that maximal value or utility is achieved within the limits of a scarce resource. The project ranking model was based on the premise that the expected benefits of proposed research are a function of the proposal's relevance to the explicit research goals of the granting agency. The model contained four principal variables: (i) the importance of the research goal to which the proposal was addressed; (ii) the extent to which the project would contribute to the achievement of that research goal; (iii) the likelihood that the project would be successful; and (iv) the likelihood that the results of the project would be utilized. The derivation of the utility of a project is based on a series of procedures including the measurement of these four variables.

The scope of this dissertation is twofold. Chapter II reviews the net benefit-cost difference, benefit/cost ratio, internal rate of return, and cost-effectiveness models. It
also discusses time-related problems such as choosing a proper discount rate and counting inflation. Chapter III generates a cost-benefit model for evaluating rehabilitation research programs. The model is an extension and modification of the previous AARPS model and forms the major portion of this presentation.
II. Cost-Benefit Analysis (CBA): A Review

There is mixed use of the terms "cost-benefit analysis" and "cost-effectiveness analysis" in the literature. Both refer to the attempt to apply a systematic, analytical approach to problems of choice. CBA typically look at the benefits derived by making an investment in relation to its costs. Cost-effectiveness, on the other hand, is usually employed to compare two or more methods of attacking the same problem. If method A accomplishes the same ends as method B but costs less, then A is said to be more cost-effective than B. While we only address the term CBA here, a great part of the theoretical consideration can also be applied to cost effectiveness.

II.A. Cost-Benefit Models for Selection of Alternative Actions

There are four major steps in applying CBA to decision making:

(i) Identification of objectives. The desired goal (or goals) to be obtained by the use of resources, should be defined as explicitly as possible. Misunderstanding of objectives can lead to solution of wrong problem. For example, reduction of traffic fatalities represents a different objective from a reduction of traffic accidents. Wearing seat belts has some effectiveness toward meeting the first objective, but none toward meeting the latter.

(ii) Identification of alternatives, e.g., to reduce traffic fatalities, the law may want to take action to enforce wearing seat belts, using car seats for children, or adding a built-in safety device to each car by the automobile industry. This step requires discarding all alternatives unacceptable for political or other reasons, and determining
acceptable levels of performance for the remaining alternatives. It is worth noting that it is frequently impossible or too costly to study every conceivable alternative. In many cases the limitation to a small number of understood options is a must although this implies a risk of missing the best possible solution.

(iii) Measuring benefits and costs. This step involves two issues: what to measure? and how to measure? The benefits (costs) to be considered may be direct or indirect, internal or external, monetary or intangible. To identify the benefits and to develop acceptable measurement scales usually needs great effort. In the above example, the benefits are in terms of the percentages reduced for traffic fatality. The costs include the expenditures on the necessary equipment and on the enforcement of law.

(iv) Comparing alternatives by a decision criterion. Three types of valid criterion by which all alternatives can be evaluated are
1. maximize benefits at given costs,
2. minimize costs while attaining a given effectiveness,
3. some combinations of these two.
Here we review several criteria that are commonly used in decision making. They are all based on the net present values of future profits and expenditures.

II.A.a. The Net Present Value

The net present value is given by calculating the present value of all cash benefits and subtracting from it the present value of all the cash outlays required by the project. Let B and C be the present value of all benefits and costs generated by an investment project respectively, i.e.,
\[ B = \sum_{t=1}^{n} \frac{b_t}{(1+r)^t} \]
\[ C = \sum_{t=1}^{n} \frac{c_t}{(1+r)^t} \]

(1)

where \( b_t \) = all benefits obtained in period \( t \),
\( c_t \) = all costs incurred in period \( t \),
\( r \) = the discount rate,
\( n \) = the life of the project.

The net present value, \( NPV \), of an investment is then given as

\[ NPV = B - C \]

(2)

Obviously with an appropriately chosen discount rate, projects with a positive net present value are acceptable and larger net present values are preferred. The issue of choosing a discount rate will be discussed in the next section.

The \( NPV \) approach is considered best by many analysts. However, there are circumstances where this method does not apply well. First, the net-benefit criterion supposes that the levels of output under all alternatives have been established for comparison. In practice this information is often not available or not complete. Second, it fails to account for the differential size of programs. For example, if there are three programs whose estimated benefits and costs are
<table>
<thead>
<tr>
<th>Project 1</th>
<th>Project 2</th>
<th>Project 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benefit</td>
<td>2,000</td>
<td>900</td>
</tr>
<tr>
<td>Cost</td>
<td>1,200</td>
<td>500</td>
</tr>
<tr>
<td>NPV</td>
<td>800</td>
<td>400</td>
</tr>
</tbody>
</table>

It is easy to see that although Project 1 has the largest NPV, we can carry out Project 2 and Project 3 at the same cost (1,200) and obtain an even larger profit (400+500=900).

II.A.b. Benefit/Cost Ratio

The benefit/cost ratio, $R$, is simply the ratio of the present value of cash inflows and outflows as defined in (1). That is

$$ R = \frac{B}{C} $$

Projects with ratios greater than one are acceptable, and the larger ratios are obviously favored.

Some analysts consider only the initial capital expenditure as a cash outlay, while benefits then become net cash inflows, i.e., income minus total outlays. In other words, if the life of an investment is $n$ periods, and we assume that initial financing is required during the first $m$ periods. Define

$$ C_1 = \sum_{t=1}^{m} \frac{c_t}{(1+r)^t} $$

$$ C_2 = \sum_{t=m+1}^{n} \frac{c_t}{(1+r)^t} $$

(4)
and

\[ C = C_1 + C_2 \]

Then the ratio becomes

\[ R = \frac{B - C}{C_1} = \frac{\text{NPV}}{C_1} \] (5)

A variety of contradictory positions is found in the literature regarding these two definitions, (3) and (5), often in a very casual manner. We prefer (5) to (3) for the reason that, by placing \( C = C_1 + C_2 \) in the denominator, the benefit/cost ratio does not recognize the essential difference between costs which the investor has to provide from his scarce capital resources as an input to the system (\( C_1 \)), and costs which may be covered by benefits generated within the system (\( C_2 \)). The partial cost \( C_2 \) "does not exist" if this investment is not undertaken and hence it may not be treated the same as \( C_1 \) which can be consumed to invest other projects.

Still the question remains: which costs should be subtracted from the numerator and which should be added to the denominator? Concerning vocational rehabilitation programs, Conley [7] suggests that "all program effects on well-being should be counted as a benefit even if negative". It is widely believed that whatever procedure is used for analyzing one project, the same procedure must be used for all projects analyzed and compared.

There are circumstances in which the benefit/cost ratio approach is the most useful model. That is when several independent projects are to be chosen, given some capital constraint. Then it is appropriate to rank the projects by their respective B/C ratios, implementing successively lower projects until the capital budget is exhausted or until the ratio of the marginal project reaches unity.
II.A.c. Internal Rate of Return (IRR)

The internal rate of return of a project is defined to be some $\mu$ such that

$$\sum_{t=0}^{n} \left\{ \frac{b_t-c_t}{(1+\mu)^t} \right\} = 0 \quad (6)$$

where $b_t$, $c_t$, and $n$ are those terms defined in (1), hence it is assumed that $b_0-c_0<0$. Alternatively, IRR is the rate which would make the NPV of the project equal to zero. The greater the value of IRR, the higher the net benefit.

One purpose of IRR is to simplify the benefit stream. If a man invests $100 for five years, and suppose that the stream of profit is 30, 35, 45, 60, 48.4. The IRR is computed to be 10%. Then the man is justified in thinking of the investment as equivalent to one in which his initial outlay is compounded forward at the rate of 10% for 5 years. An equivalent investment stream would be -100, 0, 0, 0, 0, 100·(1+0.1)^5.

The advantage of the IRR approach lies in the fact that it can be calculated on the basis of project data alone. For one thing, it does not require data on the opportunity cost of capital which is often difficult to estimate [20]. Thus, when a decision maker has several projects to be evaluated, he may independently calculate the IRR on each, and use the resulting figures as a basis of comparison.

A severe disadvantage of the internal rate of return is that it may not be determined uniquely [24]. For example, if in a project major items of equipment need to be replaced after a period of time, giving rise to negative net benefits. Assume that its investment stream is -500, 1200, -700.
Obviously one solution for the internal rate of return is zero, for at a zero discount rate NPV=0. Another solution is 40%:

\[-500 + \frac{1200}{1+\mu} - \frac{700}{(1+\mu)^2} = 0\]

Solving the equation

\[-500(1+\mu)^2 + 1200(1+\mu) - 700 = 0\]

we have \(\mu=0\), or \(\mu=0.4\). When multiple solutions occur, the choice of IRR presents a further problem.

Moreover, the IRR approach assumes that the investment is depreciated over time and reinvested which is not true for investment in people. Thus the internal rate of return is usually inappropriate for evaluating health investments [7].
II.B. Time-Related Problems in CBA

II.B.a. Choice of Discount Rate

Most often, the benefits and costs of a project are spread over a number of years. In order to evaluate projects or compare alternatives, one must reduce the time stream of benefits and costs to a single number. The NPV approach defined in II.A.a. is commonly used for this purpose.

The computation procedure for NPV is simple. However, choosing a discount rate can be difficult in the sense that it might be hard to make people agree on a certain rate. This is because discount rates can be critical in evaluating investments. For example, if one has a choice of two investments, A and B. The estimated annual profits are

<table>
<thead>
<tr>
<th>Year</th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-100</td>
<td>-100</td>
</tr>
<tr>
<td>1</td>
<td>60</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>80</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>80</td>
<td>230.15</td>
</tr>
</tbody>
</table>

Their NPV's at rates 1%, 5%, and 9% are

<table>
<thead>
<tr>
<th>Discount Rate</th>
<th>1%</th>
<th>5%</th>
<th>9%</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>115.5</td>
<td>98.8</td>
<td>84.2</td>
</tr>
<tr>
<td>B</td>
<td>123.4</td>
<td>98.8</td>
<td>77.7</td>
</tr>
</tbody>
</table>

Thus B is superior to A in one case, equivalent to A in another, and inferior to A in the third case.

It is commonly assumed that the correct rate is the rate at which society as a whole is willing to trade off present for future. The rate is called the social rate of
time preference. If, for instance, society is taken to be indifferent between having $1 million today and $1.07 million next year, then the social rate of time preference is 7%.

A number of concepts on which the selection of discount rates can be based have been raised in the literature [38]. The controversy concerns which of the conceptual rates actually measures the social rate of time preference. The selection usually depends upon the decision maker's interest and on who offers the monetary support. The following are two of the commonly considered conceptual rates:

(i) The marginal productivity of capital in the private sector. This is the return of the economy's marginal investment projects yield. For instance, suppose the least profitable investment in the economy yields an annual profit of $.1 for a $1 initial investment, then the marginal productivity is 10%. If extra investment funds are available, and if they are invested in the private sector, a 10% return will be realized by the economy.

In case a public project is to be financed by borrowing from the private sector, it is to be presumed that the funds could have been used to finance private investments. Thus there is a direct sense in which the private marginal return can be considered as the relevant factor. When, on the other hand, the funds to be used are part of the savings of the public sector, the connection between this project and the private sector may not be clear.

(ii) Market interest rates. These are the interest rates of government bonds, accounts in commercial banks, debt to loan associations, etc. For example, if a corporation wishes to undertake a large investment, and intends to collect the necessary money by selling bonds. The bond buyers are promised $146.4 four years from now for a $100
bond. Then the interest is 10%, for \(146.4/(1+.1)^4 = 100\).

Generally, government financed projects can be financed at a lower rate than private industry. This is because the loan to the government is relatively risk free and the overall degree of security for the loan is relatively high. It is recommended that the average government bond rate to be used as the discount rate for calculating public investments and that higher rates be used for private investments. In the above example, the corporate bonds might have 4% risk. They hence offer 6% + 4% = 10% interest as opposed to the 6% interest of federal bonds to attract buyers.

II.B.b. Simultaneous Use of Different Discount Rates

It can be verified that the simultaneous use of different discount rates in evaluating federal projects is inefficient for resource allocation [38].

Let us consider a hypothetical situation. Assume that the government wants to invest in two broad categories of projects: 1. energy exploration, and 2. transportation development. Let \(r_1\) and \(r_2\) be the discount rate used for Category 1 and Category 2, respectively. A project in Category 1 with an internal rate of return greater than \(r_1\) promises a positive net benefit and shall be undertaken. The smaller the value \(r_1\), the more money be put in Category 1. Hence if we let \(W_1\) be the number of dollars invested in Category 1, \(W_1\) is clearly a monotonically decreasing function of \(r_1\): \(W_1 = W_1(r_1)\). The last dollar invested (the marginal investment) will have a return of \((1+r_1)\) dollars. Similarly, for Category 2 we have another decreasing function \(W_2 = W_2(r_2)\). For simplicity let us assume that \(W_1(r_1)\) and \(W_2(r_2)\) are continuous functions and their curves are shown in the following figure.
Moreover, let $B_i(W_i)$ be a function relating the dollar value of benefits to the dollars invested for $i = 1, 2$. Also let $W$ be the total budget available. Then the efficiency requirement can be formulated as the following problem

$$\text{maximize } B_1[W_1(r_1)] + B_2[W_2(r_2)]$$

subject to $W_1(r_1) + W_2(r_2) = W$

Using a Lagrangian multiplier, we get

$$L = B_1[W_1(r_1)] + B_2[W_2(r_2)] + \lambda [W - W_1(r_1) - W_2(r_2)]$$

The first order conditions are

$$\frac{\partial B_1}{\partial W_1} \frac{\partial W_1}{\partial r_1} - \lambda \frac{\partial W_1}{\partial r_1} = 0$$

$$\frac{\partial B_2}{\partial W_2} \frac{\partial W_2}{\partial r_2} - \lambda \frac{\partial W_2}{\partial r_2} = 0$$
Solving each equation for \( \lambda \) yields

\[
\frac{\partial B_1}{\partial w_1} = \frac{\partial B_2}{\partial w_2}
\]

\( \frac{\partial B_i}{\partial w_i} \) is the benefit from the marginal dollar invested in projects in Category \( i \), which was identified above as \( 1 + r_i \). Thus

\[ 1 + r_1 = 1 + r_2 \]

and

\[ r_1 = r_2. \]

Therefore a necessary condition for maximizing benefits of investments in different categories is that the same discount rate be used for all programs.

Cretin [8] claims that it is improper to discount the costs at a higher rate than that for the benefits of a project. Let \( r_b = \) discount rate for benefits, and \( r_c = \) discount rate for costs. Then

\[
B = \sum \left\{ \frac{b_t}{(1 + r_b)^t} \right\}
\]

\[
C = \sum \left\{ \frac{c_t}{(1 + r_c)^t} \right\}
\]

and

\[
\text{benefit/cost ratio} = \frac{B}{C}.
\]

Assume that \( r_b < r_c \). If we postpone the action by one year then the benefit/cost ratio becomes
\[
B'/(C') = \sum_{t} \frac{b_t}{(1+r_b)^{t+1}} \div \sum_{t} \frac{c_t}{(1+r_c)^{t+1}}
\]

\[
= \frac{B}{C} \cdot \frac{1+r_c}{1+r_b}
\]

\[
= B/C.
\]

Repeating the postponement it may be concluded that this action should be deferred for infinitely long time to be most profitable.

Cretin's arguments fail in the case \(r_b \geq r_c\). Also if we consider the net benefit-cost difference of a deferred project

\[
B' - C' = \frac{B}{(1+r_b)} - \frac{C}{(1+r_c)}
\]

Postponement is favorable if \((B' - C') - (B - C) > 0\). Now

\[
(B' - C') - (B - C) = \frac{B}{(1+r_b)} - \frac{C}{(1+r_c)} - (B - C)
\]

\[
\Rightarrow C \left[ 1 - \frac{1}{(1+r_c)} \right] - B \left[ 1 - \frac{1}{(1+r_b)} \right]
\]

Suppose \(B > C\) which is true most often. Then the above expression shows that the NPV will increase with postponement only if \(\left[ 1 - 1/(1+r_c) \right]\) is sufficiently greater than \(\left[ 1 - 1/(1+r_b) \right]\), alternatively, if \(r_c\) is sufficiently greater than \(r_b\).
II.B.c. Inflation

In estimating benefits (costs) of projects, very often an analyst may ignore the problem of inflation. For instance, to assess earnings of several vocational rehabilitation programs, we may assume the future wages will be raised to meet inflation and hence may only compare real values of the alternatives. These unadjusted rates represent an estimate in "constant dollars" -- that is, in dollars that are all equivalent in purchasing power. Only if one wants to estimate actual dollar expenditures, for financial planning purposes, will he need to become concerned about general price changes. As a simple example, suppose a person is considering buying a condominium or renting an apartment, and suppose he is only concerned about regular expenditures of these two plans. For the time being, the monthly mortgage payment might be much more than the rent, but ten years from now, when inflation is counted in, the opposite can well be true.
III. An Application of CBA in Rehabilitation Research*

III.A. Introduction to the AARPS Model

The allocation of funds for rehabilitation research occurs in two contexts: one in more general, relating to the choice of various types of research RSA wishes to support (referred to as "project concepts"); the other is the choice of soliciting research proposals from researchers, which are ordinary replies to the "requests for proposals" describing the chosen concepts. These two situations are not distinguished in this presentation.

In applying CBA to the problem of evaluating rehabilitation research proposals, a number of questions must be considered. Some of them are: (i) What benefits should be measured? The effects of all proposed programs could be wide-ranged in nature, could be either explicit or implicit, and could have any type of expected time profile. To provide an effective, sound description of those effects needs a great deal of effort. (ii) How should the benefits be measured? Apparently a large proportion of the benefits are intangible, e.g. improvement of a client's social life, or enhancement of human knowledge. Is it conceivable to put dollar values on such diverse items? (iii) How do we construct an acceptable model which makes direct comparisons among research objectives of considerable diversity?

* This work was partially sponsored by the ongoing study Analytic Aids for Research Proposal Selection. The AARPS group is an interdisciplinary team composed of Dr. David Cardus, Dr. Marcus J. Fuhrer, Dr. Robert M. Thrall, Ms. Shou C. Chan and Mr. Robert S. Kelley.
The AARPS study is an attempt to attack these questions. Although the results may not be comprehensive or definitive, the study has actually accomplished several of its objectives. First, it has identified some of the types of information needed for benefit-cost estimation. Second, it has built a foundation on which more complete models can be constructed. Third, it has uncovered some of the limitations and difficulties of CBA in medical service decision making.

The AARPS model comprises several subjects. Among them the target population estimation is a basic one.

III.A.a. Target Population Assessment

The target population of a research proposal is defined to be the set of individuals who are the intended or potential beneficiaries of the outcomes of this proposed research. The quantification of this variable has usually a great impact on the computation of total benefits. An approach is suggested here to identify a target population as a union of subsets which are better described and easier to quantify.

The individuals being addressed by different rehabilitation projects can be highly diverse in nature. A system of classification on three levels was proposed [35] in this connection: (i) Impairment is the residual limitation resulting from congenital defect, disease or injury, e.g., amputated limb, paralysis after polio, myocardial infarction, epilepsy, and mental retardation. (ii) Functional limitation is the inability to perform some key life functions, such as walking, carrying weights, stooping, using stairs, reading, handling and fingerling. (iii) Disability is caused by impairment and/or functional limitation as the environment imposes impediments to an individual's essential components of daily living, such as self-care and economic activity. The following is a model for assessment of target population
based on the level of impairment

Let \( I' = \{I_1, I_2, \ldots, I_m\} \) be the set of all impairments (or functional limitations), and let \( S_i \) denote the population of \( I_i, 1 \leq i \leq m \). Suppose, in evaluating a research proposal, it is observed that \( J' \subset I' \) consists of all the impairments of which populations include intended beneficiaries of this proposal. The parent population \( PP \) is defined to be the union

\[
PP = \bigcup_{I_i \in J'} S_i
\]

In general, there are three limitations for a research project to be applicable to an individual. First, the person may have to possess one of \( r \) specific combinations, \( C_1, C_2, \ldots, C_r \), of impairments (\( C_j \subset J' \) for \( 1 \leq j \leq r \)). Second, in the presence of \( C_j \), there may be a set \( D_j \) of impairments which must be absent. Third, there may also be an environmental condition \( E_j \) which must be met. We denote the population satisfying \( E_j \) by \( T_j \) (\( D_j \) and \( E_j \) could be empty).

The target population \( TP \) is then a subset of the parent population \( PP \) and can be represented as a union of \( r \) subclasses \( R_1, R_2, \ldots, R_r \):

\[
TP = \bigcup_{j=1}^{r} R_j
\]

where

\[
R_j = \bigcap_{I_i \in C_j} S_i \bigcap_{I_i \in D_j} \overline{S_i} \bigcap_{T_j}
\]

(here \( \overline{S_i} \) is the complement of the set \( S_i \)). Note that we also have the following equality
\[ PP = \bigcup_{i \in J'} S_i = \bigcup_{j=1}^{r} \bigcup_{i \in C_j} S_i \]

Example 1

A project to improve electrical stimulators for helping persons with drop-foot to work.

Drop foot can be caused by stroke, spine injury, or neural disease. Hence

\[ J' = \{ I_1, I_2, I_3 \} \]

where \( I_1 = \text{stroke}, \ I_2 = \text{spine injury}, \ \text{and} \ I_3 = \text{neural disease}. \ PP = S_1 \cup S_2 \cup S_3. \) However, a stroke-impaired individual might also have brain damage (\( I_4 \)), and thus be incapable of understanding instructions for use of the device. Also, a spinal-cord injured person might suffer perceptual impairment (\( I_5 \)) and thus be unable to see on the level, and thus be unable to use the device. Supposing no other restrictions upon usage of electrical stimulators, then

\[ TP = R_1 \cup R_2 \cup R_3 \]

where

\[ R_1 = S_1 \cap \overline{S_4}, \]

\[ R_2 = S_2 \cap \overline{S_5}, \]

\[ R_3 = S_3. \]

Example 2

A project to develop a motorized wheelchair.

The target population is made up of persons who need a wheelchair but cannot propel one normally. Thus to be a
beneficiary, an individual should have his lower limbs missing or paralyzed and at least one arm impaired. So, $I_1 =$ paralysis of legs, $I_2 =$ amputation of legs, $I_3 =$ impairment of at least one arm.

$$PP = S_1 \cup S_2 \cup S_3$$

$$TP = R_1 \cup R_2$$

where

$$R_1 = S_1 \cap S_3,$$

$$R_2 = S_2 \cap S_3.$$ 

III.A.b. A Cardinal Model

A survey of Social Rehabilitation Services (SRS) activities disclosed three classes of benefits for these activities. (i) Benefits received by a certain group of individuals, given that a project is successful, e.g. the increase of earnings if a vocational training program for blind people is carried out. (ii) Non-personal benefits of a successful project, e.g. the advancement of human knowledge if a basic research has successfully proved a new theory. (iii) Benefits that are independent of the success of research, e.g. the political impact of an immediate action when an accidental chemical pollution has caused disastrous physical damage.

The aggregation of these three classes of benefits is described by the following formula [43]

$$B = P_S (P_{U, NB} I + B_S) + B_P$$

where
\( B = \text{expected benefit of project} \)
\( P_s = \text{probability of success} \)
\( P_y = \text{probability of utilization if successful} \)
\( N = \text{number of individuals in the target population} \)
\( B_i = \text{expected benefit per individual (or unit) impacted if successful} \)
\( B_s = \text{indirect benefits if successful} \)
\( B_f = \text{benefits of funding research whether or not it is successful} \)

**Probability of success** \((P_s)\) is defined as the perceived likelihood that the objectives of research will be fulfilled. Model (7) assumed that a project is either a success or a failure. However, quite often this definition and its use in model (7) oversimplify the situation. For one thing, many intermediate outcomes of a research might occur. The treatment of different levels of success in using (7) can complicate the estimation considerably. For example, a project proposes an approach of producing medical treatment for deaf. The outcomes might be one of the following:

1. \( O_1 \): the deaf are cured completely,
2. \( O_2 \): the impairment is reduced enough that a hearing aid is usable,
3. \( O_3 \): cure a certain type of deafness, but no effects on the others,
4. \( O_4 \): no useful treatment can be produced using this approach,
5. \( O_5 \): no useful results of research.

The various outcomes are sometimes unpredictable, not mention the difficulties in evaluating their likelihood of occurrence.

**Probability of utilization** \((P_y)\) is the likelihood, from certain to impossible, that the research results will be utilized. It may be estimated by considering a number of factors such as payoff, ease of implementation, agency pri-
orities, and past experience in using similar facilities.

Target population \( (N) \) may be assessed using the model in III.A.a. In general, the total target population may be partitioned into subgroups over each of which the expected net benefit and the probability of utilization per individual are approximately constant.

Individual benefits \( (B_I) \) include all benefits that are proportional to the size of some impacted population which may be persons, families or administrative agencies. On different kinds of populations the impacts of \( B_I \) might have very much different characteristics. To identify variables for characterizing the full range of individual benefits and to scale the variables has been a major task in the AARPS study.

Indirect benefits if successful \( (B_S) \) measure benefits that are not in any way proportional to the number of units impacted. For instance, the effort by the AARPS group may have the impact "expanding R&D potential" which is conditioned only by the success of the research.

Benefits of funding \( (B_F) \) are related to factors that are independent of the population and the success of a project. This term is commonly neglected by economists. It is left in model (7) to help indicate the role of administrative action in the evaluation process.

III.A.c. Cluster Analysis

To identify relevant benefit factors, a modified Delphi procedure was carried out. A group of persons inside SRS was asked to provide terms they found important in describing aspects of rehabilitation work in general and rehabilitation
research in particular. These terms with semantic equivalents combined, produced a list of 246 items which were called the 0-th order benefits.

Clearly the number in the list was much too large to incorporate into a cost-benefit model directly. The same group of respondents was then asked to judge which items were "similar", and an analysis of their responses yielded a collection of 46 groups or "clusters", divided into two lists. One list contained 23 items which will benefit certain group of individuals, and was called "personal benefits". The other consisted of the 23 "non-personal benefits". These clusters were called the 1-st order benefits and are shown in Appendix 1.

Later, these two lists were further categorized by representatives of interested role groups (e.g. handicapped persons, RSA managers). The effort resulted in a 2-nd order list of 18 benefits, and finally a 3-rd order list of 5 benefits was produced (Appendix 2). This presentation will mainly use the 3-rd order clusters:

1. enhancing service quality,
2. containing costs of rehabilitation services,
3. improving individual client outcomes,
4. improving administrative bases for service provision,
5. improving public policy bases for rehabilitation.

Each of these five factors needs to be considered when assessing the $B_f$ term in (7), since they all involves benefits that are conditioned by some target population and probability of utilization. On the other hand, only a few of the five factors will have an impact on the class $B_g$ or $B_p$. In Table 1 the symbol "x" indicates the relevant benefit factors with respect to the three terms in model (7).
III.A.d. A Utility Function

Let the set of proposed projects be \( A \). Then the ranking process involves a utility function

\[ u : A \rightarrow R \tag{8} \]

So that for every \( PC \in A \), \( u(PC) \) is the value or utility that evaluates \( PC \). The selection of projects for funding is to maximize the utility achieved within the limits of a scarce resource, in this case, the total funds available.

The elements in Table 1 that need to be considered motivate the generation of a 8-dimensional utility space, so that within each dimension a single scale can be readily used to measure a specific benefit. After an analysis of the nature of the 8 benefit elements, we scoop the monetary part out of each element and use a single scale for the total dollar benefit. Furthermore, we combine all the non-personal benefits \( B_s \) in one dimension, and the benefits of funding \( B_F \) in another. That is, we intend to evaluate each project by assigning to it a seven dimensional evaluation vector

\[
x = \begin{bmatrix}
x_1 \\
\vdots \\
\vdots \\
x_7
\end{bmatrix}
\tag{9}
\]
The components of X are described as

\[ x_1 = \text{Economic benefit} , \]
\[ x_2 = B_I \text{ in cluster 1} , \]
\[ x_3 = B_I \text{ in cluster 3} , \]
\[ x_4 = B_I \text{ in cluster 4} , \]
\[ x_5 = B_I \text{ in cluster 5} , \]
\[ x_6 = B_S , \]
\[ x_7 = B_F . \]

Thus the utility function can be constructed as a composition of two functions:

\[ u = g \circ f \tag{10} \]

where \( f: A \to \mathbb{R}^7 \) maps each project \( PC^j \) to its evaluation vectors \( X^j \) described above; \( g:f(A) \to \mathbb{R} \) relates the seven independent scales and combines the components of each vector \( X^j \) into a single value. The cost-benefit model thus comprises two parts (III.B. and III.C.), the construction of function \( f \) and that of function \( g \). The final selection procedure according to the utility outcomes will also be discussed in III.C.
III.B. Evaluation of Projects in a Seven-Dimensional Utility Space

We now consider the function

$$f : A \rightarrow \mathbb{R}^7$$

(11)

$$f(PC^j) = x^j$$

where $A$ consists of $n$ projects $PC^j$, $j = 1, 2, \ldots, n$. The vector $x^j$ estimates the expected benefits of $PC^j$ with respect to the seven benefit factors stated in (9).

The first component is easily distinguished from the rest by the identification "economic benefits". It is also the only one that has a well-defined measurement unit namely, the dollar. In computation of $x^j_1$, a standard accounting procedure and/or some special techniques for R&D expenditures [14] may be applied. However, the refinement of analysis is very often limited by the costs of collecting data. We shall illustrate this point and the task of accounting by a few hypothetical examples given in III.B.a.

On the other hand, the last six components should be assessed in a different manner. The extreme softness of those variables and the lack of measurement criteria makes highly subjective evaluations inevitable. The AARPS group has suggested to RSA a rating process which will be discussed in III.B.b.

III.B.a. The Economic Submodel

It seems reasonable to drop the terms $B_S$ and $B_F$ in Formula (7) if we are only considering economic benefits. This is because a monetary profit can be obtained only through monetary gain by some individuals or units, who
constitute the beneficiaries (target population) of the project. It is also apparent that for any economic benefit to be realized the research must be successful and the results utilized. Thus, (7) can be simplified to:

\[ B = P_S P_U \sum B I \quad (12) \]

In general, the target population TP must be partitioned into subgroups \( T_1, T_2, \ldots, T_k \) over each of which the expected net benefit and the probability of utilization per individual are approximately constant:

\[
TP = \bigcup_{\alpha=1}^{k} T_\alpha \\
N = \sum_{\alpha=1}^{k} N_\alpha
\]

and

\[
B = P_S ( \sum_{\alpha=1}^{k} P_{U\alpha} N_\alpha B_{I\alpha} ) \quad (13)
\]

Thus in using (7), \( P_{U\alpha} \) and \( B_{I\alpha} \) stand for average figures in the subpopulation \( T_\alpha \). The extent of the refinement should then represent a compromise between precision and costs of analyses.

Note that the benefits mentioned above are all net benefits, that is, they have been aggregated over a number of years, the costs have been subtracted and the discount rates have been counted in.
Example 1

Suppose that for the possible action of developing an improved wheelchair, the estimated target population is 1000 and is composed of the following subgroups differentiated by four levels of functional limitations:

$S_1$: able to walk with some aid (cane, etc.),

$S_2$: partially dependent on wheelchair (during times of fatigue, pain, stress, etc.),

$S_3$: totally dependent on a wheelchair for mobility,

$S_4$: immobile (bedridden).

The economic benefits of this action are attributed to restoration of earning power and savings in self care. Some direct and indirect costs considered are purchase and maintenance of equipment, job training expense and other employment-related expenses such as extra transportation costs.

While assessing earnings in a lifetime, the target population should be further refined into age groups. Table 2 gives the number of people in each $S_i$-Age category.

<table>
<thead>
<tr>
<th>Age</th>
<th>10-20</th>
<th>21-30</th>
<th>31-40</th>
<th>41-50</th>
<th>51-60</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_1$</td>
<td>65</td>
<td>50</td>
<td>41</td>
<td>56</td>
<td>60</td>
</tr>
<tr>
<td>$S_2$</td>
<td>31</td>
<td>45</td>
<td>52</td>
<td>98</td>
<td>126</td>
</tr>
<tr>
<td>$S_3$</td>
<td>20</td>
<td>37</td>
<td>53</td>
<td>40</td>
<td>66</td>
</tr>
<tr>
<td>$S_4$</td>
<td>5</td>
<td>18</td>
<td>29</td>
<td>44</td>
<td>64</td>
</tr>
</tbody>
</table>
To illustrate the estimation of $B_I$, let us take a sample beneficiary aged 35 in group $S_J$. His benefit would be:

$$B_I = \sum_{j=35}^{80} \frac{E_j - C_j}{(1+r)^{j-35}} Q_j$$

where $E_j =$ expected benefit at age $j$
$C_j =$ expected cost at age $j$
$r =$ discount rate $= 10\%$
$Q_j =$ probability of survival* at age $j = (0.98)^{j-35}$
$80 =$ maximal age in consideration

Also assume that

- price of wheelchair $= \$ 2500$
- job training cost $= \$ 1000$
- expected earnings $= \$ 10000$ per year
- savings in self care $= \$ 1000$ per year
- maintenance fee $= \$ 100$ per year
- other employment expenses $= \$ 400$ per year

Then

$$E_j \begin{cases} 10,000 + 1000 & 35 \leq j \leq 65 \\ 1000 & j > 65 \end{cases}$$

$$C_j \begin{cases} 2,500 + 1000 + 100 + 400 & j = 35 \\ 100 + 400 & 35 < j \leq 65 \\ 100 & j > 65 \end{cases}$$

* This formula for $Q_j$ is used for purposes of illustration only. It clearly underestimates survival to intermediate ages and overestimates survival to large ages.
Thus

\[ B_1 \leq -(2500 + 1000) \sum_{j=35}^{65} \frac{10500}{(1.1)^{j-35}} (0.98)^{j-35} \]

\[ + \sum_{j=66}^{80} \frac{900}{(1.1)^{j-35}} (0.98)^{j-35} \]

\[ \leq 90000 \text{ dollars.} \]

If we use this figure as an average for the \( S_3(31-40) \) group, then take another sample aged 25 from \( S_3(21-30) \) group, and so on. The 20 values \( B_{1a} \) are then summarized in Table 3.

<table>
<thead>
<tr>
<th></th>
<th>Age</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>10-20</td>
</tr>
<tr>
<td>( S_1 )</td>
<td>125</td>
</tr>
<tr>
<td>( S_2 )</td>
<td>110</td>
</tr>
<tr>
<td>( S_3 )</td>
<td>98</td>
</tr>
<tr>
<td>( S_4 )</td>
<td>9</td>
</tr>
</tbody>
</table>

Table 4 gives estimated probability of utilization by each subpopulation. The numbers give out information such as the percentage of people who would actually buy these improved wheelchairs, how much they depend on this improvement for the purpose of employment, for self care, etc.
Table 4  \( P_{UA} \)

<table>
<thead>
<tr>
<th></th>
<th>10-20</th>
<th>21-30</th>
<th>31-40</th>
<th>41-50</th>
<th>51-60</th>
</tr>
</thead>
<tbody>
<tr>
<td>( S_1 )</td>
<td>.01</td>
<td>.01</td>
<td>.01</td>
<td>.005</td>
<td>.005</td>
</tr>
<tr>
<td>( S_2 )</td>
<td>.2</td>
<td>.25</td>
<td>.25</td>
<td>.2</td>
<td>.2</td>
</tr>
<tr>
<td>( S_3 )</td>
<td>.4</td>
<td>.45</td>
<td>.45</td>
<td>.4</td>
<td>.3</td>
</tr>
<tr>
<td>( S_4 )</td>
<td>.02</td>
<td>.02</td>
<td>.01</td>
<td>.005</td>
<td>.001</td>
</tr>
</tbody>
</table>

Suppose that \( P_S = .30 \). Then substituting this value for \( P_S \) and values for \( N_a \), \( B_{1a} \), \( P_{UA} \) from tables 2, 3, 4 into formula (13), we get \( B = $4,400,000 \).

Example 2

A project using new computer techniques is proposed to improve management of rehabilitation systems at the state level. If applied to a state agency, it can save 30% of the paperwork and 10% of the staff, and hence have significant impact on the budget of the next ten years. On the other hand, this project needs large amounts of initial investment and may produce side effects by laying off employees. To each agency,

\[
B_i = -C_0 + \sum_{i=1}^{10} \frac{E_i - C_i}{(1+r)^i}
\]

where

- \( C_0 \) : contains expenses due to reorganization of the personnel and a new computer system,
- \( E_i \) : expected budget savings during the \( i \)th year,
- \( C_i \) : maintenance fee of this new system in the \( i \)th year.
In the calculation below we selected for \( C_0 \) and \( E_i \) the values given in Table 5 and \( C_i = 0.05 \) million dollars. Among the 50 states (target population) 5 have already accomplished this improving process. To another 12 states, for political or financial reasons, this project is inapplicable. Assume that \( P_S = 1 \) and that \( B_I \) and \( P_U \) are estimated for each state. The result, within a permitted range of error, falls into one of the following 8 categories.

Table 5

<table>
<thead>
<tr>
<th>( a )</th>
<th>( B_{i\alpha} ) (million dollars)</th>
<th>( P_{U\alpha} )</th>
<th>No. of states ( N_\alpha )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>-</td>
<td>0</td>
<td>12</td>
</tr>
<tr>
<td>3</td>
<td>0.17</td>
<td>0.15</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>0.17</td>
<td>0.15</td>
<td>0.7</td>
</tr>
<tr>
<td>5</td>
<td>0.27</td>
<td>0.20</td>
<td>0.5</td>
</tr>
<tr>
<td>6</td>
<td>0.35</td>
<td>0.25</td>
<td>0.5</td>
</tr>
<tr>
<td>7</td>
<td>0.35</td>
<td>0.25</td>
<td>0.8</td>
</tr>
<tr>
<td>8</td>
<td>0.50</td>
<td>0.42</td>
<td>0.9</td>
</tr>
</tbody>
</table>

Expected benefit \( = \sum_{a=1}^{8} P_{U\alpha} N_\alpha B_{i\alpha} = 15.4 \) million dollars.

The twenty population breakdown in Example 1 is obviously inefficient in practice as it requires twenty assessments of population, probability of utilization, and individual benefit. The work is extremely tedious, if not impossible, when a similar partition technique is applied to a large number of project concepts. One modification is suggested here to reduce the number of subpopulations to four using a single average number for the individual benefit within each subset of the same functional limitation.
Consider, for example, the total dollar benefit \( B_3 \) obtained by the 216 clients in \( S_3 \) group, given that the research is successful. \[
B_3 = .4 \times 20 \times .98000 + .45 \times .37 \times .100000 + .45 \times .37 \times .90000 + .4 \times .4 \times .80000 + .3 \times .66 \times .70000 = 5,981,500.
\]
Now let \( Q_3 \) be the benefit of an individual from the 21-30 age group and let \( P_3 \) be his probability of utilization. If we use these figures for everyone in \( S_3 \), and compute \( P_3 \times 216 \times Q_3 \), then it is necessary to adjust the result by a multiplier \( \tau^3 \). That is,
\[
B_3 = \tau^3 \times P_3 \times Q_3
\]
where \( P_3 = .45 \), \( N_3 = 216 \), \( Q_3 = 100,000 \) and \( \tau^3 = .6154 \). Similarly, we have an equation for each \( S_\alpha \):
\[
B_\alpha = \tau^\alpha \times P_\alpha \times Q_\alpha \times N_\alpha \quad \alpha = 1, 2, 3, 4
\]
where
\[
\tau^1 = .7083,
\tau^2 = .7036,
\tau^3 = .6154,
\tau^4 = .2515.
\]

Let \( \tau = \tau^2 = .7036 \). If we use \( \tau \) instead of \( \tau^\alpha \) for group \( S_\alpha \), then the model (9) implies a new formula for this example
\[
B = P_S \times \tau \left( \sum_{\alpha = 1}^{4} P_\alpha \times N_\alpha \times Q_\alpha \right)
\]
where
\[
N = \text{population of } S_\alpha,
\text{ } P = \text{probability of utilization of the } S_\alpha/21-30 \text{ group},
\text{ } Q = \text{individual benefit of the } S_\alpha/21-30 \text{ group}.
\]
The reason we let \( \tau \) be \( \tau^2 \) is that \( S_2 \) has the largest population (352) of these four subsets.
Substituting data from Example 1 we have $B = $4,474,000 as opposed to the previously calculated $4,400,000. The error is about 1.7%.

III.B.b. Evaluation of Non-Economic Benefit Factors

In this section we propose an approach for constructing utility scales for measuring non-economic factors. Along with the analytic description we consider a hypothetical example to illustrate the criteria.

Suppose there are three projects: $PC^1$ is the action of developing an improved wheelchair (Example 1 in III.B.a.): $PC^2$ is the possible usage of a new computer system in the state agencies; $PC^3$ is a service to be provided to the national legislative headquarters. In the judgemental group there are one decision maker and four raters, $k = 1, 2, 3, 4$ who are representatives from different interest groups. On each project $PC^j$, the decision maker has his opinion of the four raters and expresses it as a vector

$$w_j = \begin{bmatrix} w_1^j \\ w_2^j \\ w_3^j \\ w_4^j \end{bmatrix}$$

where $\sum_{k=1}^{4} w_k^j = 1$, e.g.,

$$w^1 = \begin{bmatrix} .4 \\ .2 \\ .2 \\ .2 \end{bmatrix}$$
Here we assume a population breakdown \((S^1_{\alpha}: \alpha=1,2,\ldots,20)\) for \(PC_1\); \((S^2_{\alpha}: \alpha=1,2,3)\) for \(PC_2\); and \((S^3_{\alpha}: \alpha=1)\) for \(PC_3\) such that the use of average values within each is justified. Also let \(N_{S^j}^1\) denote the size of \(S^j_{\alpha}\). Then the 7-vector obtained for \(PC^j\) will be

\[
x^j = \begin{bmatrix}
x^j_1 \\
\vdots \\
\vdots \\
x^j_7
\end{bmatrix}
\]

where

\[
x^j_i = p^j_S \left( \sum_\alpha p^j_{\alpha \alpha} \cdot N_{S^j}^1 \cdot B^j_{1\alpha, i} \right) \quad i = 1,2,3,4,5
\]

\[
x^j_6 = p^j_S B^j_S
\]

\[
x^j_7 = B^j_F
\]

Now, each rater \(k\) is asked to assess all the noneconomic individual benefits \(B^j_{1\alpha, i}(k)\), \(2 \leq i \leq 5\), within each subpopulation, and non-individual benefits \(B^j_S, B^j_F, j = 1,2,3\). It does not matter what scale he is using since his assessment will be normalized so that all raters have equal "importance". One way of doing this is computing

\[
B^j_{1\alpha, i}(k)^* = B^j_{1\alpha, i}(k) \div \sum_\alpha B^j_{1\alpha, i}(k) \quad i = 2,3,4,5
\]

\[
B^j_S(k)^* = B^j_S(k) \div \sum_j B^j_S(k)
\]

\[
B^j_F(k)^* = B^j_F(k) \div \sum_j B^j_F(k)
\]
So that \( \sum_{i,j} B^j_{\alpha,i} (k) = \sum_j B^j_s (k) = \sum_j B^j_F (k) = 1. \)

For example, in Table 6 judge 1 measures the achievability of "enhancing service quality" of the three projects (refer to Table 2 for the population breakdown of PC\(^l\)).

\begin{table}[h]
\centering
\begin{tabular}{|c|cccc|}
\hline
\text{PC}\(^l\) & 1 & 1 & 1 & 2 & 2 \\
1 & 1 & 1 & 2 & 2 \\
2 & 2 & 2 & 3 & 3 \\
3 & 3 & 3 & 4 & 4 \\
\hline
\text{PC}\(^2\) & 55 & 55 & 55 \\
\hline
\text{PC}\(^3\) & 70 \\
\hline
\end{tabular}
\caption{\textbf{B}\(^l\),\(^2\)(1)}
\end{table}

Then \( \sum_j \sum_{i} B^j_{\alpha,i} (1) = 278 \), and the normalized values are in Table 7.

\begin{table}[h]
\centering
\begin{tabular}{|c|cccc|}
\hline
\text{PC}\(^l\) & .0036 & .0036 & .0036 & .0072 & .0072 \\
.0036 & .0036 & .0036 & .0072 & .0072 \\
.0072 & .0072 & .0072 & .0108 & .0108 \\
.0108 & .0108 & .0108 & .0144 & .0144 \\
\hline
\text{PC}\(^2\) & .19784 & .19784 & .19784 \\
\hline
\text{PC}\(^3\) & .2518 \\
\hline
\end{tabular}
\caption{\textbf{B}\(^l\),\(^2\)(1)*}
\end{table}
With the data from all raters, the expected benefits of each project can be computed according to the raters' weights. That is

\[ B_{IA,i}^j = \sum_{k=1}^{4} w_k B_{IA,i}^j (k) \quad i=2,3,4,5 \]

\[ B_S^j = \sum_{k=1}^{4} w_k B_S^j (k) \]

\[ B_F^j = \sum_{k=1}^{4} w_k B_F^j (k) \]

In our example, \( B_{11,2}^1(1) = .0036, B_{11,2}^1(2) = .00712, B_{11,2}^1(3) = .00671, B_{11,2}^1(4) = .01923 \). So \( B_{11,2}^1 = .4 \cdot .0036 + .2 \cdot .00712 + .2 \cdot .00671 + .2 \cdot .01923 = .00805 \). Substituting all \( B_{11,2}^1 \)

values, \( a=1,2,\ldots,20 \), and the information about \( P_S^1, N_a^1, P_{Ua}^1 \) into (1), we thus obtain the value of \( x_2^1 \).

Remark: There is a difficulty in using this model on projects with significantly different target populations. In those cases the 0-10, 0-100, or 0-1000 scale used for \( B_{IA,i}^j \) does not allow enough space to show the difference between population characteristics. For example, if the target population of Project 1 consists of one million people and that of Project 2 consists of 80 agencies. When substituting these numbers and estimated \( B_{IA,i}^j \) (e.g. \( B_{11,2}^1 = 1, B_{11,2}^2 = 99 \) on a 0-100 scale) into formula (14), \( x_2^1 \) will be unreasonably larger than \( x_2^1 \). One solution of this problem is to tell the raters the sizes, \( N_a^1 \), and ask them assess the term \( N_a^1 B_{IA,i}^j \) as a whole instead of each individual \( B_{IA,i}^j \).
III.C. A Transformation of the Evaluation Vector into a Single Value**

Thus far we have obtained, using the model in III.B., a 7-vector $x_j$ measure for each of the $N$ project concepts $PC_j$, $j=1,...,n$. To compare these vectors it is necessary to relate the seven independently scaled benefit factors. One way of doing this involves identification and evaluation of verbal anchor points for each component. However, an acceptable verbal anchoring will not be available until several as yet unresolved difficulties have been handled.

Another approach is to find a common scale and a mapping into it for each of the seven benefit scales. In other words, we need a special measurement scale $T$ and seven multipliers, $m_1, m_2, ..., m_7$, so that the products, $m_1x_j^1, ... , m_7x_j^7$, are evaluations of Project $j$ on scale $T$ with respect to the seven benefits. The number $m_i$ is clearly the ratio of a unit on $T$ to a unit on the $i$-th scale. We generate $m_1, ..., m_7$ by a procedure based on the relative importance of the seven factors. Numerically, the relative importance can be presented by assigning a set of weights:

$$\lambda = \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \\ \lambda_5 \\ \lambda_6 \\ \lambda_7 \end{bmatrix} \quad \sum_{i=1}^{7} \lambda_i = 1$$

** Part of the AARPS report April 27, 1979.
The numbers \( \lambda_i \) may be assigned by the decision maker alone or he may use a summary of judgements of auxiliary evaluators. Either way the responsible decision maker plays a key role. For example, if each of five auxiliary evaluators who represent different points of view (e.g. a potential user of rehabilitation service, a RSA manager, a state administrator, a counselor and a research) is asked to select a weighting vector, then the decision maker assigns weights

\[
U = \begin{bmatrix}
  u_1 \\
  u_2 \\
  u_3 \\
  u_4 \\
  u_5
\end{bmatrix} \quad \sum_{\nu=1}^{5} u_{\nu} = 1
\]

to these five vectors, \( \lambda^1, \lambda^2, \lambda^3, \lambda^4, \lambda^5 \), and selects for his \( \lambda \) the weighted average

\[
\lambda = u_1 \lambda^1 + u_2 \lambda^2 + u_3 \lambda^3 + u_4 \lambda^4 + u_5 \lambda^5
\]

The vectors \( \lambda^j \) and \( U \) are input to the averaging process as independent data. The components of \( U \) can be varied as the decision maker senses shifts in the relevance of each evaluator's opinion. Also one of the evaluators may be the decision maker himself.

For example let the columns of the following matrix represent the five weighting vector \( \Lambda \)
Furthermore, since the project evaluations are separated by time intervals, the decision maker may wish to update the vector $\lambda^0$ that was used in the previous period by adding new data from the above five evaluators. In that case $\lambda^0$ can enter the averaging process as an additional term of which the significance is presented by its weight, $u^0$. That is

$$\lambda = u^0 \lambda^0 + u^1 \lambda^1 + u^2 \lambda^2 + u^3 \lambda^3 + u^4 \lambda^4 + u^5 \lambda^5$$

In our example, if it is decided that the old and the new information should be equally-proportioned in the calculation of $\lambda$, then $u^0 = .5$ and thus $u^1 = .1$, $u^2 = .15$, $u^3 = .1$, $u^4 = .1$. Moreover, assume

$$\lambda^0 = \begin{bmatrix} .65 \\ .10 \\ .10 \\ .05 \\ .06 \\ .02 \\ .02 \end{bmatrix} \quad \text{then} \quad \lambda = \begin{bmatrix} .575 \\ .125 \\ .15 \\ .05 \\ .0505 \\ .025 \\ .02 \end{bmatrix}$$
Next, we need to set up a representative 7-vector

\[
Y = \begin{bmatrix}
y_1 \\
\vdots \\
y_7
\end{bmatrix}
\]

for which

\[
\begin{bmatrix}
y_1 m_1 \\
\vdots \\
y_7 m_7
\end{bmatrix}
\sim
\begin{bmatrix}
\lambda_1 \\
\vdots \\
\lambda_7
\end{bmatrix}
\]

where "\(\sim\)" is the symbol of proportionality and means that one vector equals a constant times the other. For example, if a set of verbal anchor points were available, their values \(y_i\) on their respective scales would be obvious choices for the components of \(Y\). In the current situation we may choose \(y_i\) as a representative of the \(n\) individual \(i\)-th components, \(x_1, \ldots, x_n\).

Since the first component already has a well-defined unit of measurement namely, the dollar, it is taken as basic. Thus \(m_1 = 1\). And for \(i = 2, \ldots, 7\), we have

\[
\frac{y_i m_i}{y_1 m_1} = \frac{y_i m_i}{y_1} = \frac{\lambda_i}{\lambda_1}
\]

or

\[
m_i = \frac{(y_1 / \lambda_1)}{(y_1 / \lambda_i)}
\]

The value of \(m_i\) then has the interpretation "the number of dollars that a unit of the \(i\)-th benefit factor is worth".

Now each vector \(X^j\) can be transformed into a vector \(X^j(M)\) defined by
The product \( m_i x_i^j \) is the dollar value of \( x_i^j \) unit of the \( i \)-th benefit factor. Thus the sum

\[
y_j = \sum_{i=1}^{7} m_i x_i^j
\]

of the components of \( X_j^j(M) \) is the computed dollar value of Project \( j \).

One obvious choice for \( y_i \) is the average of the \( i \)-th row, i.e.

\[
y_i = \frac{\sum_{j=1}^{n} x_i^j}{n}
\]

Two other possibilities are the median value and the maximum value of the \( i \)-th row.

Suppose that we have five project concepts and suppose that the five vectors are

<table>
<thead>
<tr>
<th></th>
<th>( x^1 )</th>
<th>( x^2 )</th>
<th>( x^3 )</th>
<th>( x^4 )</th>
<th>( x^5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_1 )</td>
<td>4,400,000</td>
<td>1,000,000,000</td>
<td>26,550,000</td>
<td>850,000</td>
<td>1,100,000</td>
</tr>
<tr>
<td>( x_2 )</td>
<td>8,000</td>
<td>2,400</td>
<td>65,000</td>
<td>12,800</td>
<td>3,500</td>
</tr>
<tr>
<td>( x_3 )</td>
<td>53,200</td>
<td>25,000</td>
<td>3,333</td>
<td>41,350</td>
<td>9,900</td>
</tr>
<tr>
<td>( x_4 )</td>
<td>14,000</td>
<td>10,880</td>
<td>78,000</td>
<td>35,300</td>
<td>106,000</td>
</tr>
<tr>
<td>( x_5 )</td>
<td>828</td>
<td>1,100</td>
<td>3,000</td>
<td>9,500</td>
<td>1,500</td>
</tr>
<tr>
<td>( x_6 )</td>
<td>8</td>
<td>1</td>
<td>32</td>
<td>.90</td>
<td>.45</td>
</tr>
<tr>
<td>( x_7 )</td>
<td>11</td>
<td>0</td>
<td>18</td>
<td>55</td>
<td>88</td>
</tr>
</tbody>
</table>
Then the value of $y_1$ resulting from the above three criteria would be:

1. **Mean criterion**

   $y_1 = \frac{4,400,000 + 1,000,000 + 26,550,000 + 850,000 + 1,100,000}{5} = \$206,580,000$

2. **Median criterion**

   $y_1 = \$4,400,000$

3. **Max criterion**

   $y_1 = \$1,000,000,000$

   If we use the max criterion in this example, then

   \[
   Y_{\text{max}} = Y = \begin{bmatrix}
   1,000,000,000 \\
   65,000 \\
   53,200 \\
   106,000 \\
   9,500 \\
   90 \\
   88 
   \end{bmatrix}
   \]

   Moreover, if we use the weights

   \[
   \lambda = \begin{bmatrix}
   .50 \\
   .15 \\
   .20 \\
   .05 \\
   .05 \\
   .03 \\
   .02 
   \end{bmatrix}
   \]

   then

   \[
   M_{\text{max}} = M = \begin{bmatrix}
   1 \\
   4,615.4 \\
   7,518.4 \\
   943.4 \\
   10,526 \\
   666,666 \\
   454,545 
   \end{bmatrix}
   \]
\[ x^1(M) = \begin{bmatrix}
4,400,000 \\
36,923,200 \\
400,000,000 \\
13,207,600 \\
8,715,528 \\
5,333,330 \\
5,000,000 
\end{bmatrix} \quad \text{and} \quad V^1 = 4.7358 \times 10^8 \]

There are arguments for and against each of the three criteria for choosing \( Y \). Suppose, for instance, that we add another project concept, \( PC^{n+1} \) and consider its vector \( x^{n+1} \). The selection of \( Y \) can be very sensitive to the components of \( x^{n+1} \) if we use the max criterion. It can also be sensitive if we use the median criterion in case \( n \) is small. In our example, if an additional project \( PC^6 \) has economic benefit \( x_6 = 5,000,000,000 \), then the max criterion implies \( y_1 = 5,000,000,000 \), which is five times of the old figure. On the other hand, the median criterion will have to decide \( y_1 \) by the two median values 26,550,000 and 4,400,000 and will also make much difference in later computation.

Recall that \( m_i = (y_1/\lambda_1)/(y_1/\lambda_i) \) where \( \lambda_1 \) and \( \lambda_i \) are fixed numbers. Now if we consider the situation that some of the projects promise fairly good economic benefits but are rated extremely low on the \( i \)-th factor. Then the mean and median criterion may make \( y_1 \) unreasonably small and hence make \( m_i \) unreasonably large. For example, if we have five 2-dimensional vectors:

<table>
<thead>
<tr>
<th>( x^1 )</th>
<th>( x^2 )</th>
<th>( x^3 )</th>
<th>( x^4 )</th>
<th>( x^5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_1 )</td>
<td>( 4.7 \times 10^8 )</td>
<td>( 2 \times 10^8 )</td>
<td>( 1.5 \times 10^8 )</td>
<td>( 8.5 \times 10^8 )</td>
</tr>
<tr>
<td>( x_2 )</td>
<td>( 40 )</td>
<td>( 4000 )</td>
<td>( 850 )</td>
<td>( 100 )</td>
</tr>
</tbody>
</table>
Also if \( \lambda = \begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix} = \begin{bmatrix} .6 \\ .4 \end{bmatrix} \) then the values of \( Y, M \), \( X^j(M) \), and \( v^j \) by the three criteria are respectively:

1. **Mean criterion**

\[
Y = \begin{bmatrix} 5 \times 10^8 \\ 1005 \end{bmatrix}, \quad M = \begin{bmatrix} 1. \\ 331,674 \end{bmatrix}
\]

<table>
<thead>
<tr>
<th>( X^j(M) )</th>
<th>( x^1 )</th>
<th>( x^2 )</th>
<th>( x^3 )</th>
<th>( x^4 )</th>
<th>( x^5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_1^j(M) )</td>
<td>( 4 \times 10^8 )</td>
<td>( 2 \times 10^8 )</td>
<td>( 1.5 \times 10^8 )</td>
<td>( 8.5 \times 10^8 )</td>
<td>( 9 \times 10^8 )</td>
</tr>
<tr>
<td>( x_2^j(M) )</td>
<td>( 1.3 \times 10^7 )</td>
<td>( 1.3 \times 10^9 )</td>
<td>( 2.8 \times 10^8 )</td>
<td>( 3.3 \times 10^7 )</td>
<td>( 1.16 \times 10^7 )</td>
</tr>
<tr>
<td>( v^j )</td>
<td>( 4.1 \times 10^8 )</td>
<td>( 1.5 \times 10^9 )</td>
<td>( 4.3 \times 10^8 )</td>
<td>( 8.8 \times 10^8 )</td>
<td>( 9.1 \times 10^7 )</td>
</tr>
</tbody>
</table>

2. **Median criterion**

\[
Y = \begin{bmatrix} 4 \times 10^8 \\ 100 \end{bmatrix}, \quad M = \begin{bmatrix} 1 \\ 2,666,667 \end{bmatrix}
\]

<table>
<thead>
<tr>
<th>( X^j(M) )</th>
<th>( x^1 )</th>
<th>( x^2 )</th>
<th>( x^3 )</th>
<th>( x^4 )</th>
<th>( x^5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_1^j(M) )</td>
<td>( 4 \times 10^8 )</td>
<td>( 2 \times 10^8 )</td>
<td>( 1.5 \times 10^8 )</td>
<td>( 8.5 \times 10^8 )</td>
<td>( 9 \times 10^8 )</td>
</tr>
<tr>
<td>( x_2^j(M) )</td>
<td>( 1.07 \times 10^8 )</td>
<td>( 1.07 \times 10^10 )</td>
<td>( 2.3 \times 10^9 )</td>
<td>( 2.67 \times 10^8 )</td>
<td>( 9.3 \times 10^7 )</td>
</tr>
<tr>
<td>( v^j )</td>
<td>( 5.07 \times 10^8 )</td>
<td>( 1.09 \times 10^10 )</td>
<td>( 2.45 \times 10^9 )</td>
<td>( 1.12 \times 10^9 )</td>
<td>( 9.9 \times 10^8 )</td>
</tr>
</tbody>
</table>

3. **Max criterion**

\[
Y = \begin{bmatrix} 9 \times 10^8 \\ 4000 \end{bmatrix}, \quad M = \begin{bmatrix} 1 \\ 150,000 \end{bmatrix}
\]
The big value for \( m_2 \) in (1) and in (2) has the effect that \( PC^2 \) rules out all other projects despite the fact that the first component has been assigned heavier weight. The max criterion is thus more stable in the sense that the relative importance of the i-th benefit factor would be less affected by the presence of research projects which have very low ratings on this factor.

The problem of choosing projects among \( PC^1, \ldots, PC^n \) for grant awards is obviously under the constraint of limited funds. Suppose the research cost for \( PC^j \) is \( C^j \) and the total amount of funds available is \( S \). With the calculated data, \( V^j \), and thereof the benefit/cost ratio \( R^j = V^j / C^j \), we can formulate the process of selection as an integer programming problem.

\[
\begin{align*}
\text{max} & \quad R^1Z^1 & & \ldots & & R^nZ^n \\
\text{subject to} & \quad C^1Z^1 & & \ldots & & C^nZ^n \\
\text{where} & \quad Z^j = 1 & & \text{if } PC^j \text{ is to be funded} \\
& \quad Z^j = 0 & & \text{if } PC^j \text{ is rejected}
\end{align*}
\]

One intuitive solution is to arrange the \( PC^j \) in order according to their benefit/cost ratios beginning with the largest \( R^j \) and then implement projects in order until the total resource \( S \) has been allocated.
For example, if $S = \$1,750,000$, and the figures $V^j$, $C^j$, and $R^j$ of the previous five project concepts are listed in the following table

<table>
<thead>
<tr>
<th>Project</th>
<th>$PC^1$</th>
<th>$PC^2$</th>
<th>$PC^3$</th>
<th>$PC^4$</th>
<th>$PC^5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V^j$</td>
<td>$1.736 \times 10^8$</td>
<td>$1.22 \times 10^9$</td>
<td>$4.863 \times 10^8$</td>
<td>$5.89 \times 10^8$</td>
<td>$2.755 \times 10^8$</td>
</tr>
<tr>
<td>$C^j$</td>
<td>$3 \times 10^5$</td>
<td>$1.05 \times 10^6$</td>
<td>$8 \times 10^5$</td>
<td>$1.5 \times 10^5$</td>
<td>$6.5 \times 10^5$</td>
</tr>
<tr>
<td>$R^j$</td>
<td>157.9</td>
<td>97.7</td>
<td>60.8</td>
<td>393</td>
<td>42.7</td>
</tr>
</tbody>
</table>

Thus $R^4 > R^1 > R^2 > R^3 > R^5$ and

$C^4 + C^1 + C^2 = 1,500,000 < S$

$C^4 + C^1 + C^2 + C^3 = 2,300,000 > S$.

Therefore the final decision is to accept project 1, 2, and 4.

There is an apparent defect in this "order and choose" rule. Suppose for instance, $S = \$1,300,000$, then $C^4 + C^1 + C^2 > S$ and hence only two projects $PC^4$ and $PC^1$ should be funded. However, the implementation of $PC^4$, $PC^1$ and $PC^3$ is within the range of total funds and implies more benefit than the previous selection. This problem may be solved in either of two ways: (a) apply a computer algorithm to solve the Knapsack programming problem [9], (b) allow the funds $S$ be a little flexible so that the project listed on the margin of total budget is also included in the funding set.
IV Summary

This thesis is concerned with an evaluation process for selection of rehabilitation research projects. It begins with a review of general cost-benefit analysis. The strength and the weakness of various cost-benefit models are considered. Then an existing several-variable model is studied and extended. The main chapters of this thesis deal with the parts of this model that relate to construction of a utility function. This function assigns to each project proposal a value that summarizes its expected benefit in all respects. Finally, the projects to be funded are selected according to their benefit/cost ratios.

The construction of the utility function is composed of two parts (III.C. & III.D.). The second part has been better-defined and has a more complete structure. The first part, however, is more tentative and incomplete. The difficulty lies in the enormity of the rating tasks and in the lack of a concrete measurement scale. More research in this area seems desirable.
Appendix 1

The first-order personal benefits:

1. Personal functioning of the handicapped
2. Personal well-being and self identity
3. Individual vocational sufficiency
4. Personal economic improvement
5. Containment of personal illness and disability
6. Family functioning stability
7. Access to material components of the quality of life
8. Child welfare
9. Quality of service delivery
10. Containment of institutionalization
11. Coping behavior
12. Counselling, education and training
13. Personal-social fulfilment
14. Social involvement
15. Containment of personal cost
16. Better physical environment
17. Adaptive behavior
18. Client-initiative
19. Expanded potential to benefit from services
20. Client awareness and expectation of service
21. Consumer participation
22. Understanding rights and obligations
23. Service containment
Appendix 1 (continued)

The first-order non-personal benefits:

1. Legislative impact
2. Program improvement strategies
3. Program improvement tactics
4. Improved evaluation of program
5. Policy refinement
6. Service process refinement
7. Improved providers' efficiency and effectiveness
8. Amelioration of societal disturbances
9. Cost-effectiveness and benefit/cost
10. Improved information systems
11. Improved productivity of tax expenditures
12. Validation of benefits
13. More effective planning processes
14. Expanded R&D potential
15. Improved data quality
16. Improved and more flexible administration
17. Program containment
18. Administrative information dissemination
19. Coordination of federal linkages with other political entities
20. Efficiency and effectiveness of public assistance
21. Mediation of social change
22. Improved staff training
23. Generalizability of services
Appendix 2

Verbal designation of the 9 cousters (second-order benefit factors) yielded by analyzing the 23 personal benefit (numbered in parentheses)

1. Fostering Consumer Involvement
   (3) Client Expectation of Service
   (7) Understanding Rights and Obligations
   (19) Consumer Participation

2. Enhancing the Quality and Accessibility of Services
   (18) Counseling, Education, and Training
   (12) Quality of Service Delivery
   (20) Expanding Potential for Benefits of Services

3. Containing Institutionalization
   (10) Containment of Institutionalization

4. Containing Personal Costs and Need for Services
   (11) Service Containment
   (17) Containment of Personal Cost

5. Encouraging the Individual’s Social Participation
   (2) Family Functioning and Stability
   (15) Social Involvement
   (16) Personal-Social Fulfillment

6. Improving the Physical Environment
   (22) Better Physical Environment

7. Enhancing Individual Coping Skills
   (8) Personal Well-Being and Self-Identity
   (13) Client Initiative

I. Enhancing Service Quality

II. Containing Costs of Rehabilitation Services

III. Improve Individual Client Outcomes
(14) Adaptive Behavior
(21) Coping behavior

8. **Minimizing Functional Limitations and Personal Disability**
   (6) Personal Function of Handicapped
   (9) Containment of Personal Illness and Disability

9. **Improved Personal Vocational Status and Material Well-Being**
   (5) Individual Vocational Sufficiency
   (4) Personal Economic Improvement
   (23) Material Components of Quality of Life

Verbal designation of the 9 clusters (second-order benefit factors) yielded by analyzing the 23 non-personal benefit factors (numbered in parentheses)

10. **Enhancing the Effectiveness of Service Providers**
    (14) Improved Staff Training
    (19) Improving Providers' Efficiency & Effectiveness

11. **Expanding the Knowledge Base**
    (12) Expanded R & D Potential
    (15) Improved Information Systems
    (18) Improved Data Quality

12. **Improving Program Performance and Performance Measures on Benefit-Cost Criteria**
    (10) Validation of Benefits
    (9) Cost-Effectiveness and Benefit/Cost
    (21) Improved Productivity of Tax Expenditures

IV. Improving Administrative Bases for Service Provision
13. **Improving Program Development and Evaluation**
   (20) Service Process Refinement
   (5) Program Containment
   (16) Improved Evaluation of Program
   (8) Program Improvement Strategies
   (22) Program Improvement Tactics

14. **Facilitating Societal Change**
   (1) Mediating Social Change
   (2) Public Acceptance

15. **Promoting Generalizability of Services**
   (17) Generalizability of Services

16. **Developing and Communicating Policies, plans and Procedures**
   (3) Administrative Information Dissemination
   (4) Planning Processes
   (7) Policy Refinement

17. **Facilitating Administrative Flexibility and Improvement**
   (13) Administrative Flexibility & Improvement

18. **Improving Legislative Impact and the Coordination of Governmental Entities**
   (11) Legislative Impact
   (23) Federal Linkages With Other Political Entities
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