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PROPAGATION OF 9.57μm RADIATION IN EPITAXIAL SILICON WAVEGUIDES

by

JOHN HAROLD JENNINGS II

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ABSTRACT

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Waveguides for CO$_2$ laser wavelengths consisting of high resistivity Si epilayers on low resistivity Si substrates were investigated. The 9.57 µm line of a CO$_2$ laser was used since Si has a lower absorption at this wavelength than at other CO$_2$ wavelengths.

Absorption measurements were made by focusing onto a cleaved edge and detecting the end-fire radiation from the other end. By measuring the transmitted power for different guide lengths, the absorption coefficient could be calculated assuming equal input and output coupling efficiencies. The measured absorption coefficient was compared to the theoretical value derived by assuming only losses due to free carrier absorption and lattice absorption are important.

The mode structure of the guide was determined by scanning the far-field radiation pattern.

Phase gratings were produced by coating wafers with photo resist and then exposing to two interfering Ar laser beams. Coupling to the guides with photo resist gratings and gratings that were sputter (RF) etched into the waveguide surface were investigated.
CONTENTS

Chapter

I. INTRODUCTION ....................................................... 1

1.1 Waveguides at CO\textsubscript{2} Wavelengths ......................... 1

1.2 Motivation for Current Research .................................. 2

II. THEORETICAL CONSIDERATIONS .................................. 4

2.1 Propagation of waves in a lossy medium ......................... 4

2.1.1 Electromagnetic field in a solid .............................. 4

2.1.2 Propagation of waves .......................................... 5

2.1.3 Classical dispersion theory ................................... 8

2.1.4 Free carriers in a crystal ..................................... 9

2.2 The dielectric slab waveguide .................................. 14

2.2.1 The lossless dielectric slab waveguide ....................... 14

2.2.2 The lossy slab waveguide .................................... 31

2.2.3 The epitaxial silicon waveguide .............................. 37

2.3 Input couplers ....................................................... 49

2.3.1 The grating coupler ........................................... 49

2.3.2 The prism coupler ............................................. 58

III. EXPERIMENTAL TECHNIQUES ..................................... 63

3.1 Grating fabrication .................................................. 63

3.2 Waveguide evaluation .............................................. 64

3.2.1 Focusing onto cleaved edge .................................. 64

3.2.2 Coupling via a phase grating ................................ 72

IV. RESULTS AND DISCUSSION ......................................... 78

4.1 Grating fabrication .................................................. 78

4.2 Waveguide evaluation .............................................. 78
4.2.1 Phase grating coupler................................. 78
4.2.2 Coupling by focusing on cleaved edge............. 81

V. CONCLUSIONS...................................................... 90

REFERENCES.......................................................... 92
I. INTRODUCTION

1.1 WAVEGUIDES AT CO\textsubscript{2} WAVELENGTHS

Waveguides for CO\textsubscript{2} laser wavelengths have received a great deal of attention recently. The need for efficient modulators at the longer wavelengths has, in large part, motivated this interest. In addition, the CO\textsubscript{2} laser is the main contender in the area of atmospheric and satellite optical communications. Consequently, there will be a need in the near future for complete integrated optical circuits capable of processing optical communication signals. The development of electronic circuitry to interface with the integrated optical circuits will require equal attention.

To date a number of materials have been tried as thin-film waveguides for 10.6 \textmu m radiation. Chang and Loh (1) have investigated a sputtered Ge film on an Intran II substrate and obtained losses < 20 \text{db/cm}. Spears et al. (2) formed waveguides by proton bombardment of Br-doped CdTe with a loss of 4.3 \text{db/cm}. Two dimensional waveguides of Pb\textsubscript{1-x}Sn\textsubscript{x}Te with PbTe cladding layers have been reported by Ralston et al. (3). They obtained losses of \leq 6.5 \text{db/cm} at 77 K and 33.9 \text{db/cm} at 300 K. GaAs waveguides have been studied by a number of investigators. Lotspsich (4) reported waveguides of GaAs with cladding layers of CdTe and As\textsubscript{2}S\textsubscript{3} which had losses as low as .7 \text{db/cm}. High resistivity GaAs epilayers on low resistivity substrates have been reported by Cheo (5), Black (6), and Sopori (7). For thicknesses greater than 20 \textmu m, Cheo obtained losses < 1 \text{db/cm} for a film carrier concentration, \(N_f \leq 10^{13} \text{cm}^{-3}\) and a loss of 1.7 \text{db/cm} for \(N_f = 4 \times 10^{15} \text{cm}^{-3}\). Sopori investigated two dimensional GaAs/n\textsuperscript{+}As waveguides with losses
low as 3.8 db/cm. McFee et al. (8) have reported epitaxial GaAs waveguides on Al$_x$Ga$_{1-x}$As substrates with losses of 2 db/cm. And finally, Chang et al. (9) obtained losses of 1 db/cm with GaAs epi-layers on GaAsP substrates.

1.2 MOTIVATION FOR CURRENT RESEARCH

In this work, n-type Si epilayers on n$^+$ Si substrates have been investigated for use as thin film waveguides at CO$_2$ laser wavelengths. Waveguides in Si at 3.39 µm wavelengths have been reported (10), but no work has been done at 10 µm wavelengths.

There are two main advantages of Si over previously reported materials. First, the technology for growing large area single crystal wafers is well established. Wafers suitable as waveguides can be obtained commercially and used as is, with no special preparation techniques required to make them suitable as waveguides. The well established technology for growth of Si wafers has also made them less expensive than any other material available. Secondly, as was mentioned in the previous section, electronic circuits will need to be interfaced with optical circuits. Since Si is the predominant material used for integrated circuits, it should be possible to integrate optical circuits and electronic circuits onto the same chip with very little difficulty.

The main disadvantage of Si is that it does not exhibit any linear electro-optic effect (Pockels effect). As a result, it will probably not make as efficient a modulator as a material such as GaAs (11-17) which does exhibit this effect. However, amplitude modulation and beam diffraction can be accomplished in Si by either optical or electrical injection of free carriers (18).
Absorption measurements were made by focusing an input laser beam on a cleaved edge of the guiding layer and measuring the end-fire radiation emerging from another cleaved surface. The 9.57\textmu m line of a CO\textsubscript{2} laser was used because silicon has a lower absorption coefficient at this wavelength than at other CO\textsubscript{2} wavelengths. Measurements of the power in the output beam for different lengths of guides allowed calculation of the absorption coefficients assuming equal input and output coupling efficiencies. Mode structure of the guide was determined from observation of the far-field radiation pattern.

Grating input and output couplers were produced by interfering two Ar laser beams (4880\textdegrees A) on wafers coated with photo resist. Input couplers of photo resist gratings and of gratings sputter (RF) etched into the waveguide surface were investigated.
II. THEORETICAL CONSIDERATIONS

2.1 PROPAGATION OF WAVES IN A LOSSY MEDIUM

2.1.1 Electromagnetic Field in a Solid

Consider the problem of optical radiation of frequency $\omega$ propagating in a solid. With sinusoidal time varying fields, Maxwell's equations become

\begin{align*}
\nabla \cdot \mathbf{D} &= 0 \quad (2.1-1a) \\
\nabla \cdot \mathbf{B} &= 0 \quad (2.1-1b) \\
\n\nabla \times \mathbf{E} &= \frac{\partial \mathbf{B}}{\partial t} = -j\omega \mu_0 \mathbf{H} \quad (2.1-1c) \\
\n\nabla \times \mathbf{H} &= \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} = j\omega \varepsilon_0 \mathbf{E} + \mathbf{J} \quad (2.1-1d)
\end{align*}

Also the electric and magnetic polarization vectors are defined by

\begin{align*}
\mathbf{P} &= \mathbf{D} - \varepsilon_0 \mathbf{E} = (\varepsilon - 1)\varepsilon_0 \mathbf{E} \quad (2.1-2a) \\
\mathbf{M} &= \frac{1}{\mu_0} \mathbf{B} - \mathbf{H} \quad (2.1-2b)
\end{align*}

respectively.

Equations (2.1-2a) and (2.1-2b) show that when time varying fields are applied to a material body the polarization vectors $\mathbf{P}$ and $\mathbf{M}$ also vary in time at the same frequency as the fields. For non-magnetic materials, $\mathbf{M}$ can be neglected. Also due to frictional damping forces $\mathbf{P}$ lags behind $\mathbf{E}$ somewhat. This means that in general $\varepsilon$ must be complex and this complex nature of $\varepsilon$ is a manifestation of the work that must be done to overcome the frictional damping forces. If we write the complex dielectric constant as

\[ \varepsilon = \varepsilon_1 + j\varepsilon_2 \tag{2.1-3} \]

then (2.1-1d) can be written as

\[ \nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} = j\omega \varepsilon_0 \mathbf{E} + \mathbf{J} \]
\[ \nabla \times \mathbf{H} = j\omega \varepsilon_1 \varepsilon_0 \mathbf{E} + (-\omega \varepsilon_2 \varepsilon_0 + \sigma) \mathbf{E} \]  
(2.1-4)

where we have used \( \mathbf{J} = \sigma \mathbf{E} \), \( \mathbf{J} \) being a conduction current. Hence (2.1-4) shows that having a complex dielectric constant is equivalent to changing the conductivity of the medium. When dealing with a dielectric material of finite conductivity \( \sigma \) it is convenient to define a new complex dielectric constant by

\[ \varepsilon = (\varepsilon_1 + j\varepsilon_2 - j \frac{\sigma}{\omega}) \]  
(2.1-5)

Or by lumping all the losses into \( \varepsilon_2 \) we can write

\[ \varepsilon = \varepsilon_1 + j\varepsilon_2 \]  
(2.1-6)

### 2.1.2 Propagation of Waves

Maxwell's equations for the average fields in a solid can now be written as

\[ \nabla \cdot \mathbf{D} = 0 \]  
(2.1-7a)

\[ \nabla \cdot \mathbf{B} = 0 \]  
(2.1-7b)

\[ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{H}}{\partial t} = -\mu_0 \frac{\partial \mathbf{H}}{\partial t} \]  
(2.1-7c)

\[ \nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} = \varepsilon_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \]  
(2.1-7d)

We seek plane wave solutions to these equations and so look for solutions of the form

\[ \mathbf{E} = E_0 e^{j(k \cdot r - \omega t)} \]  
(2.1-8)

If this wave is attenuated while propagating through a solid, then the wave vector \( k \) will be complex. If we take

\[ k = k_1 + jk_2, \]

then

\[ e^{j(k \cdot r)} = e^{j(k_1 \cdot r)} e^{-(k_2 \cdot r)} \]  
(2.1-10)
With an assumed solution of the form of (2.1-8), Maxwell's equations (2.1-7c) and (2.1-7d) become

\[ jk \times E = -\mu_0 \frac{\partial H}{\partial t} \]
\[ jk \times H = \varepsilon \varepsilon_0 \frac{\partial E}{\partial t} \]

Then

\[ -k \times k \times E = + \mu_0 \varepsilon \varepsilon_0 \omega \frac{E}{c} \]

or

\[ - (k \cdot E)k + (k \cdot k)E = \mu_0 \varepsilon \varepsilon_0 \omega \frac{E}{c} \]

For a plane wave, \( k \) and \( E \) are perpendicular and the first term on the left vanishes. So

\[ c^2 (k \cdot k) = \varepsilon \omega^2 \]  \hspace{1cm} (2.1-11)

If \( \varepsilon \) is real, then (2.1-11) can be written

\[ c^2 (k_1^2 - k_2^2 + 2j k_1 \cdot k_2) = \varepsilon \omega^2 \]  \hspace{1cm} (2.1-12)

and there are two solutions. The first solution is

\[ k_2 = 0 \]
\[ k_1 = \frac{\omega}{c} \sqrt{\varepsilon} \] \hspace{1cm} (2.1-13)

and this is the normal solution. There is also a solution for \( k_2 \neq 0 \). For this case we must have

\[ k_1 \cdot k_2 = 0 \] \hspace{1cm} (2.1-14)

This implies that the surfaces of constant phase are perpendicular to the surfaces of constant amplitude. This is the so-called evanescent wave. This wave is damped but there is no dissipation of energy.

If \( \varepsilon \) is complex, then the only solutions of (2.1-11) are for \( k \) complex. Consider the case where \( k_1 \) and \( k_2 \) are in the same direction.
Then (2.1-11) becomes

\[ c^2(k_1^2 - k_2^2 + 2jk_1k_2) = \omega^2(\varepsilon_1 + j\varepsilon_2) \]  

(2.1-15)

So

\[ (k_1^2 - k_2^2) = \frac{\omega^2}{c^2} \varepsilon_1 \]  

(2.1-16a)

and

\[ 2k_1k_2 = \frac{\omega^2}{c^2} \varepsilon_2 \]  

(2.1-16b)

Define a complex index of refraction, \( n \), by

\[ k = \frac{\omega}{c} n = \frac{\omega}{c} (n + j\kappa) \]  

(2.1-17)

or

\[ k_1 = \frac{\omega}{c} n, \quad k_2 = \frac{\omega}{c} \kappa \]  

(2.1-18)

where \( n \) is the refractive index and \( \kappa \) is the absorption index. Then

(2.1-16a) and (2.1-16b) are

\[ n^2 - \kappa^2 = \varepsilon_1 \]  

(2.1-19a)

\[ 2n\kappa = \varepsilon_2 \]  

(2.1-19b)

and combining gives

\[ \varepsilon = (n + j\kappa)^2 \]  

(2.1-20)

After substituting (2.1-9), equation (2.1-8) becomes, in one-dimension

\[ E_x = E_0 e^{i(k_1x - \omega t)} e^{-k_2x} \]  

(2.1-21)

The power absorption coefficient \( \alpha_p \) is defined by the condition that the energy falls to 1/e of its original value in a distance \( 1/\alpha_p \). Then

\[ \alpha_p = 2k_2 = \frac{2\omega\kappa}{c} = \frac{4\pi\kappa}{\lambda_0} (m^{-1}) \]  

(2.1-22)
2.1.3 **Classical Dispersion Theory**

In this section we use classical dispersion theory to obtain expressions for $n$ and $k$. Both this section and the next section follow the derivation given by Moss (19).

The classical theory of optical dispersion in a solid considers the solid as made up of electrons bound to their equilibrium positions by elastic forces. Under the influence of an electromagnetic wave the electrons are set into forced oscillations. In one dimension the equation of motion of the electrons is

$$\frac{m}{2}\frac{d^2x}{dt^2} + m\gamma \frac{dx}{dt} + mw_0^2x = -eE_x e^{-j\omega t}$$  \hspace{1cm} (2.1-23)

where $x$ is the displacement from the equilibrium position and $E_x e^{-j\omega t}$ is the assumed form of the perturbing field. Try a solution of the form

$$x = x_0 e^{-j\omega t}$$  \hspace{1cm} (2.1-24)

Then

$$-mw_0^2x_0 - j\omega \gamma x_0 + mw_0^2x_0 = -eE_x$$

or

$$x_0 = \frac{-eE_x/m}{(w_0^2 - \omega^2 - j\omega \gamma)}$$  \hspace{1cm} (2.1-25)

Now the polarization vector $\mathbf{P}$ (electric dipole moment per unit volume) for charges separated by a distance $d$ is given by

$$\mathbf{P} = -eN\mathbf{d}$$  \hspace{1cm} (2.1-26)

where $N$ is the number of electrons per $m^3$. In one dimension (2.1-26) can be written

$$P_x = -eNx$$  \hspace{1cm} (2.1-27)

or from (2.1-24)
\[ P_x = -eN_x e^{-j\omega t} \]  \hspace{1cm} (2.1-28)

Using (2.1-2a) we find
\[ \varepsilon = 1 + \frac{P}{\varepsilon_o E} \]  \hspace{1cm} (2.1-29)
\[ \varepsilon = 1 + \frac{P_x}{\varepsilon_o E e^{-j\omega t}} \]  \hspace{1cm} (2.1-30)
\[ \varepsilon = 1 - \frac{eN_x}{\varepsilon_o E \varepsilon X} \]

Using (2.1-25)
\[ \varepsilon = 1 + \frac{e^2}{\varepsilon_o N/m\varepsilon_0} \frac{e N/m\varepsilon_0}{(\omega_0^2 - \omega^2 + j\omega \gamma)} \]

So
\[ (\omega_1 - 1) = \frac{e^2}{\varepsilon_o m} \frac{\omega_0^2}{(\omega_0 - \omega)^2 + \omega \gamma} \]  \hspace{1cm} (2.1-31a)

and
\[ \varepsilon_2 = \frac{e^2}{\varepsilon_o m} \frac{\omega \gamma}{(\omega_0^2 - \omega^2)^2 + \omega \gamma} \]  \hspace{1cm} (2.1-31b)

Using (2.1-19a) and (2.1-19b)
\[ n^2 - \kappa^2 - 1 = \frac{e^2}{\varepsilon_o m} \frac{\omega_0^2 - \omega^2}{(\omega_0^2 - \omega^2)^2 + \omega^2 \gamma^2} \]  \hspace{1cm} (2.1-32a)
\[ 2nk = \frac{e^2}{\varepsilon_o m} \frac{\omega \gamma}{(\omega_0^2 - \omega^2)^2 + \omega^2 \gamma^2} \]  \hspace{1cm} (2.1-32b)

\subsection*{2.1.4 Free Carriers in a Crystal}

For the case of "free" carriers in a semiconductor the equation of motion of the electrons is the same as (2.1-23) except there is no
restoring force \((\omega_0 = 0)\) and the effect of the periodic potential due to the lattice is taken into account by replacing the electron mass by an effective mass \(m^*\). Then (2.1-32a) and (2.1-32b) become

\[
\begin{align*}
\frac{n^2 + \kappa^2 - 1}{\omega^2 + \gamma^2} &= \frac{-Ne^2/m^* \varepsilon_0}{\omega^2 + \gamma^2} \\
2n\kappa &= \frac{Ne^2\gamma/m^* \varepsilon_0}{\omega(\omega^2 + \gamma^2)}
\end{align*}
\]  

(2.1-33a) \hfill (2.1-33b)

The damping constant \(\gamma\) can be determined by considering the motion of electrons in a constant electric field \(E\). The equation of motion is

\[
\frac{m}{dt^2} + \frac{m\gamma}{dt} dx = -eE
\]  

(2.1-34)

Under steady-state conditions, the average velocity of an electron is constant. If (2.1-34) is integrated over a time \(T\) in which the electron has many collisions then the first term can be dropped and we have

\[
\begin{align*}
\left[ m^* \gamma \frac{dx}{dt} \right]_0^T &= -eET \\
\left[ m \gamma \frac{dx}{dt} \right]_0^T &= -eET
\end{align*}
\]  

(2.1-35)

If we recall the definition of mobility, \(\mu_n\), as the average electron drift velocity per unit electric field, i.e.,

\[
\mu_n = \frac{\langle \dot{V}_x \rangle}{E} = \frac{\left[ \frac{dx}{dt} \right]_0^T / T}{E},
\]  

(2.1-36)

then (2.1-35) becomes

\[
-m^* \gamma \mu_n ET = -eET
\]
\( \gamma = \frac{e}{m^* \mu_n} = \frac{1}{\tau} \) \hspace{1cm} (2.1-37)

Then (2.1-33a) and (2.1-33b) can be rewritten as

\[ n^2 - \kappa^2 - 1 = \frac{-Ne^2/m^* \varepsilon_o}{(\omega^2 + \frac{\varepsilon^2}{m^*^2 \mu_n^2})} \] \hspace{1cm} (2.1-38a)

\[ 2n\kappa = \frac{Ne^3/m^*^2 \mu_n \varepsilon_o}{\omega(\omega^2 + \frac{\varepsilon^2}{m^*^2 \mu_n^2})} \] \hspace{1cm} (2.1-38b)

Recalling that the power absorption coefficient is given by

\[ \alpha_p = \frac{2\omega \kappa}{c} \quad \text{or} \quad 2n\kappa = \frac{c\alpha_p}{\omega} \]

Equation (2.1-38b) gives

\[ \alpha_p = \frac{\mu_n Ne/\varepsilon_o \, c \, n}{1 + (m^* \omega/e)^2} \] \hspace{1cm} (2.1-39)

For \( m^* \mu_n \omega/e \gg 1 \) (usually satisfied for \( \lambda \leq 100 \mu m \)),

\[ \alpha_p \approx \frac{\lambda^2 e^3}{4\pi^2 c^3 n \varepsilon_o} \frac{N}{m^*^2 \mu_n} \] \hspace{1cm} (m\(^{-1}\)) \hspace{1cm} (2.1-40)

We now want to find the change in the index of refraction due to the presence of free carriers. To do this we break the polarization into two parts:

\[ P = P_{BE} + P_{FC} \] \hspace{1cm} (2.1-41)

where \( P_{BE} \) is the polarization due to bound electrons while \( P_{FC} \) is the polarization due to free carriers.

From (2.1-2a)
\[ \varepsilon = 1 + \frac{P}{\varepsilon_0 E} = 1 + \frac{P_{BE}}{\varepsilon_0 E} + \frac{P_{FC}}{\varepsilon_0 E} \] (2.1-42)

Also,
\[ \varepsilon_{BE} = 1 + \frac{P_{BE}}{\varepsilon_0 E} \] (2.1-43)
\[ \varepsilon_{FC} = 1 + \frac{P_{FC}}{\varepsilon_0 E} \] (2.1-44)

Hence
\[ \varepsilon = \varepsilon_{FC} + \varepsilon_{BE} - 1 \] (2.1-45)

Now it is clear that
\[ \varepsilon_{BE} = n_o^2 \] (2.1-46)

where \( n_o \) is the index of refraction of the solid with no free carriers present. We are assuming that the only absorption in the material is due to free carriers.

Hence \( \varepsilon_{BE} = n_o^2 \) is real and
\[ \varepsilon_{FC} = (n + j\kappa)^2 \] (2.1-47)

Then (2.1-38a) becomes
\[
\begin{align*}
n^2 - \kappa^2 - n_o^2 &= -\frac{Ne^2}{m*\varepsilon_0} \\
&= \frac{(\omega^2 + \frac{e^2}{m*\mu_n^2})}{(\omega^2 + \frac{e^2}{m*\mu_n^2})}
\end{align*}
\] (2.1-48)

Assuming \( \kappa \) small,
\[
\begin{align*}
n^2 - n_o^2 &= -\frac{Ne^2}{m*\varepsilon_0} \omega^2 \\
&= \frac{(1 + \frac{e^2}{m*\mu_n^2})}{(1 + \frac{e^2}{m*\mu_n^2})}
\end{align*}
\] (2.1-49)
Assuming

\[ \frac{e^*}{m \mu_n} \ll 1, \]

\[ n_0^2 - n^2 = \frac{Ne^2}{m \epsilon_0 \omega^2} \quad (2.1-50) \]

or

\[ n^2 = n_0^2 - \frac{Ne^2}{m \epsilon_0 \omega^2} \quad (2.1-51) \]

This shows that the refractive index of a region with excess carriers is suppressed below that of a region with no excess carriers.

Consider now two regions of a solid; one having carrier concentration \( N_2 \) and the other having concentration \( N_3 \). Then, from (2.1-51), the two indices of refraction are

\[ n_2^2 = n_0^2 - \frac{N_2 e^2}{m \epsilon_0 \omega^2} \quad (2.1-52a) \]

\[ n_3^2 = n_0^2 - \frac{N_3 e^2}{m \epsilon_0 \omega^2} \quad (2.1-52b) \]

Then

\[ n_3^2 - n_2^2 = -\frac{N_3 e^2}{m \epsilon_0 \omega^2} + \frac{N_2 e^2}{m \epsilon_0 \omega^2} \quad (2.1-53) \]

Defining

\[ \Delta N = N_3 - N_2 \quad (2.1-54) \]

\[ n_3^2 - n_2^2 = -\frac{\Delta N e^2}{m \epsilon_0 \omega^2} \]
= (n_3 + n_2)(n_3 - n_2)

Define

$$\Delta n = n_2 - n_3$$  \hspace{1cm} (2.1-55)

Then

$$\Delta n = \frac{\Delta N e^2}{(n_3 + n_2)m \varepsilon_o \omega^2}$$  \hspace{1cm} (2.1-56)

For not too high carrier concentrations we can approximate (2.1-56) using the approximation

$$(n_3 + n_2) \approx 2n_o$$  \hspace{1cm} (2.1-57)

Then

$$\Delta n = \frac{\Delta N e^2}{2n_o m \varepsilon_o \omega^2}$$  \hspace{1cm} (2.1-58)

or

$$\Delta n = \frac{\Delta N e^2 \lambda^2}{8 \pi n_o m \varepsilon_o c^2}$$  \hspace{1cm} (2.1-59)

Note that if N_3 > N_2, then $\Delta n$ is positive; that is $n_3 < n_2$ and the region with the larger number of free carriers has a lower index of refraction.

2.2 THE DIELECTRIC SLAB WAVEGUIDE

2.2.1 The Lossless Dielectric Slab Waveguide

A schematic of the dielectric slab waveguide is shown in Figure 2.2-1. It consists of a core region of index $n_2$ surrounded by a substrate of index $n_3$ and a superstrate (usually air) of index $n_1$. These surrounding regions are sometimes referred to as the cladding layers.

The guide is said to be symmetrical if $n_1 = n_3$, otherwise it is called
Fig. 2.2-1. The dielectric slab waveguide.

Fig. 2.2-2. Crosssection of the slab waveguide.
asymmetrical. The extent in the y-direction is taken to be infinite and the positive z-direction is taken to be the direction of propagation of the guided waves.

As with all guiding structures the slab waveguide supports a finite number of guided modes and an infinite continuum of unguided radiation modes. Guiding occurs because of total internal reflection at the core-substrate interface and at the core-superstrate interface.

If we assume

\[ n_3 \geq n_1 \]  \hspace{1cm} (2.2-1)

then total internal reflection can occur only if

\[ n_2 > n_3 \geq n_1. \]  \hspace{1cm} (2.2-2)

The modes of the slab waveguide can be classified as either TE or TM modes. TE or transverse electric modes have no longitudinal components of the electric field while TM or transverse magnetic modes have no longitudinal magnetic field component. These modes have been studied extensively by Marcuse (20) from whom I quote only the results.

**TE Modes**

For TE modes the only nonzero field components are \( E_y, H_x, \) and \( H_z. \) The geometry of the problem is shown in Figure 2.2-2. The time dependence is assumed to be

\[ e^{j\omega t} \]  \hspace{1cm} (2.2-3)

while the z-dependence is assumed to be

\[ e^{-j\beta z}. \]  \hspace{1cm} (2.2-4)

\( \beta \) is called the propagation constant. Since the waveguide is assumed to be of infinite extent in the y-direction we assume there is no
y-dependence; that is,

\[
\frac{\partial}{\partial y} = 0 \tag{2.2-5}
\]

The field equations satisfied by the TE modes (which follow directly from Maxwell's equations) are

\[
\frac{\partial^2 E_y}{\partial x^2} + (n^2 k^2 - \beta^2) E_y = 0, \tag{2.2-6}
\]

\[
H_z = \frac{i}{\omega \mu_0} \frac{\partial E_y}{\partial x} \tag{2.2-7}
\]

and

\[
H_x = -\frac{\beta}{\omega \mu_0} E_y \tag{2.2-8}
\]

where \( n \) is the refractive index and \( k \) is the free space wave number.

Equation (2.2-6) is the well known Helmholtz or reduced wave equation, and its solutions can be divided into three types of modes depending on the range of \( \beta \). The first and most important group is the guided modes. These modes consist of a finite number of discrete modes which are evanescent in the superstrate and substrate and are propagating in the core. The other two groups of modes are continuous radiation modes. For \( n_1 k \leq |\beta| < n_3 k \), the field is evanescent in the superstrate and propagating in the core and substrate. These modes are called substrate modes. The so-called air modes occur when the propagation constant is in the range \( 0 \leq \beta < n_1 k \). The fields for these modes are everywhere propagating. The three groups of modes — guided, air, and substrate modes — form a complete orthogonal set of modes, and any field distribution can be written as a series expansion of these modes.
Guided Modes

Guided modes occur for $\beta$ in the range $n_3 k < \beta_m < n_2 k$. Since they are discrete modes, $\beta$ has been subscripted by the mode number, $m$. The fields for these modes are obtained by seeking solutions of (2.2-6) for which the fields vanish at $x = \pm \infty$. If we write

$$E_{ym}(x, z, t) = E_{ym}(x)e^{j(\omega t - \beta_m z)}, \quad (2.2-9)$$

then the solution of (2.2-6) that satisfies the boundary conditions (tangential $E$ continuous at $x = 0, -d$) is

$$E_{ym}(x) = \begin{cases} 
A e^{-p_1 x}, & x \geq 0 \\
A(\cos p_2 x - \frac{p_1}{p_2} \sin p_2 x), & 0 \geq x \geq -d \\
A(\cos p_2 d + \frac{p_1}{p_2} \sin p_2 d)e^{p_3(x+d)}, & x \leq -d 
\end{cases} \quad (2.2-10)$$

where

$$p_1 = (\beta_m^2 - n_1^2 k_0^2)^{1/2}$$

$$p_2 = (n_2^2 k_0^2 - \beta_m^2)^{1/2} \quad (2.2-11)$$

$$p_3 = (\beta_m^2 - n_3^2 k_0^2)^{1/2}$$

A plot of the electric field distribution for the first three guided modes in an epitaxial silicon waveguide is shown in Figure 2.2-3. $H_{zm}(x)$ is obtained from (2.2-7) and (2.2-10):
FIG. 2.2-3. Field distribution for first three guided TE modes.
20

\[
H_{zm}(x) = \begin{cases} 
-j \frac{p_1}{\omega_{\mu_0}} A e^{-p_1 x} & \text{, for } x \geq 0 \\
-j \frac{p_2}{\omega_{\mu_0}} A \sin p_2 x + \frac{p_1}{p_2} \cos p_2 x , & \text{for } -d \leq x \leq 0 \\
-j \frac{p_3}{\omega_{\mu_0}} A \cos p_2 x + \frac{p_1}{p_2} \sin p_2 x e^{p_3 (x+d)} , & \text{for } x \leq -d
\end{cases}
\]

\(H_{zm}(x)\) can be obtained from (2.2-8). Boundary conditions on \(H_{zm}(x)\) (normal \(H\) continuous at \(x = 0, -d\)) give an eigen-value equation for the propagation constant, \(\beta_m\):

\[
d^{(m)} = \frac{1}{p_2} \left[ \tan^{-1} \frac{p_1}{p_2} + \tan^{-1} \frac{p_3}{p_2} + m\pi \right]
\]

where \(m\) is again the mode number and \(d^{(m)}\) is the thickness of the guide.

A plot of \(d^{(m)}\) versus the normalized propagation constant, \(\beta_m / k\), is given in Figure 2.2-4 for an n-type Si epitaxial layer (\(N \approx 6 \times 10^{14} \text{cm}^{-3}\)) on an n+ Si substrate (\(N \approx 3 \times 10^{18} \text{cm}^{-3}\)). Note that \(\beta_m / k\) increases with increasing waveguide thickness, \(d\), and with increasing mode number, \(m\).

When \(\beta_m = n_3 k\) the waves are no longer confined to the core but begin radiating into the substrate, and the wave is said to be cutoff. The cutoff thickness of the \(m^{th}\) mode is then given by

\[
d_{c}^{(m)} = \frac{1}{k(n_2^2 - n_3^2)^{1/2}} \left[ \tan^{-1} \frac{n_3^2 - n_1^2}{n_2^2 - n_3^2} + m\pi \right]
\]
Fig. 2.2-4. Waveguide thickness vs. $\beta_m/k$ for T E modes with $N_3 = 3 \times 10^{18}$ cm$^{-3}$. 
This is the thickness of a waveguide that will have its $m^{th}$ mode just cutoff. For a guide with $d_{c}^{(m)} > d_{c}^{(m)}$ the $m^{th}$ mode will be a propagating mode. Note that for a symmetric waveguide ($n_{1} = n_{3}$) there is always at least one propagating mode. However, this is not the case for the asymmetric waveguide. It is possible to have an asymmetrical guide that supports no modes.

To obtain the power carried by a mode we consider the complex Poynting vector:

$$S_{c} = \frac{1}{2}(E \times H^{*})$$

(2.2-15)

where the star denotes complex conjugation. The time averaged complex power flow across a closed surface $\Sigma$ is

$$P = \oint_{\Sigma} S_{c} \cdot n \, da$$

(2.2-16)

For the TE modes of the slab waveguide, the power flow in the $z$-direction is

$$P_{z} = -i \int_{-\infty}^{\infty} E_{ym} H_{xm}^{*} \, dx = \frac{\beta_{m}}{2\omega_{0}} \int_{-\infty}^{\infty} |E_{ym}|^{2} \, dx$$

(2.2-17)

$P_{z}$ is the power flow per unit distance in the $y$-direction. The modes are usually normalized to one watt and all modes are orthogonal. Thus normalization and orthogonality give

$$\frac{\beta_{m}}{2\omega_{0}} \int_{-\infty}^{\infty} E_{yn} E_{ym}^{*} \, dx = \delta_{nm}$$

(2.2-18)

where $\delta_{nm}$ is the usual Kronecker delta function defined by

$$\delta_{nm} = \begin{cases} 1 & n=m \\ 0 & n \neq m \end{cases}$$

(2.2-19)
Then from the normalization condition (2.2-18), the constant $A$ can be written as

$$A^2 = \frac{4p_2^2 \omega_0}{\beta_m (d + 1/p_3 + 1/p_1)(p_2^2 + p_1^2)}$$

(2.2-20)

**Radiation Modes**

The radiation modes are obtained by relaxing the restriction that the fields must vanish at $x = \pm \infty$. It is easy to see that two types of radiation modes result for the asymmetric waveguide. If a plane wave is incident from below then it is possible to have the wave incident at such an angle that the wave is refracted at the core-substrate interface and totally reflected at the core-air interface. This corresponds to the **substrate modes**. The second type of modes corresponds to incidence at an angle such that refraction occurs at both interfaces, and these modes are the **air modes**. Both types of modes occur for a continuous range of the propagation constant, $\beta$.

**Substrate Modes.** These occur for

$$n_1k < |\beta| < n_3k$$

(2.2-21)

The solution to (2.2-6) which satisfies the boundary conditions is

$$E_y(x) = \begin{cases} 
A^{(s)} e^{jh_1 x}, 	ext{ for } x \geq 0 \\
A^{(s)} \left[ \cos h_2 x + j \frac{h_1}{h_3} \sin h_2 x \right], 	ext{ for } -d \leq x \leq 0
\end{cases}$$

(2.2-22)
\[
E_y(x) = C^{(s)} \cos h_3(x+d) + D^{(s)} \sin h_3(x+d),
\]

for \( x \leq -d \)

where

\[
h_1 = (n_1^2 k^2 - \beta^2)^{1/2}
\]

\[
h_2 = (n_2^2 k^2 - \beta^2)^{1/2}
\]

\[
h_3 = (n_3^2 k^2 - \beta^2)^{1/2}
\]

Note that in the range (2.2-21), \( h_1 \) is positive imaginary. \( H_z \) and \( H_x \) are again obtained from (2.2-7) and (2.2-8). Boundary conditions on \( H_z \) give \( C^{(s)} \) and \( D^{(s)} \) in terms of \( A^{(s)} \):

\[
C^{(s)} = A^{(s)}(\cos h_2 d - j \frac{h_1}{h_3} \sin h_2 d)
\]

\[
D^{(s)} = A^{(s)}(\frac{h_2}{h_3} \sin h_2 d + j \frac{h_1}{h_3} \cos h_2 d)
\]

In general, the normalization condition for radiation modes (either TE or TM) is

\[
\frac{1}{2} \int_{-\infty}^{\infty} E(x, h_3) \times H^*(x, h_3') \cdot \hat{e}_z \, dx = s \delta(h_3 - h_3')
\]

where \( \delta(h_3 - h_3') \) is the Dirac delta function and \( h_3 \) and \( h_3' \) label any two radiation modes. For \( h_3 \neq h_3' \), the integral vanishes and hence the modes are orthogonal. The factor \( s \) is chosen either \( +1 \) in order to keep the right hand side of (2.2-25) positive. For TE modes, \( s = +1 \). Then the normalization condition becomes

\[
\frac{\beta^*}{2 \omega \mu_0} \int_{-\infty}^{\infty} E_y(x, h_3) E_y^*(x, h_3') \, dx = \delta(h_3 - h_3')
\]
This gives
\[
A(s) = \left\{ 4\omega_0 h_3^2 h_2^2 / \pi |\beta| \left[ h_3^2 (h_2 \cos h_2 d) - j h_1 \sin h_2 d \right]^2 + h_2^2 (h_2 \sin h_2 d) + j h_1 \cos h_2 d \right\}^{1/2}
\]

**Air Modes.** These modes have propagation constants in the range
\[
0 \leq \beta < n_1 k
\]

There are two air modes for each value of \( \beta \). For the symmetrical waveguide these two modes are the even and odd modes corresponding to cosine and sine variations in \( x \) (21). For asymmetrical waveguides the \( x \) variation is more complex. However, the two modes still correspond to \( x \)-variations which differ in phase by \( \pi/2 \) radians (22).

The electric field can be written as
\[
E_y(x) = \begin{cases} 
A^{(a)} (\cos h_1 x + \frac{h_2}{h_1} F_1,2 \sin h_1 x), & \text{for } x \geq 0 \\
A^{(a)} (\cos h_2 x + F_1,2 \sin h_2 x), & \text{for } 0 \geq x \geq -d \\
A^{(a)} \left[ (\cos h_2 d - F_1,2 \sin h_2 d) \cos h_3 (x+d) + \frac{h_2}{h_3} (\sin h_2 d + F_1,2 \cos h_2 d) \right. & \\
& \left. \sin h_3 (x+d) \right], & \text{for } x \leq -d
\end{cases}
\]

where \( h_1, h_2, \) and \( h_3 \) are given by (2.2-23). Note that \( h_1 \) is now positive and real.
F_1 and F_2 are chosen such that the two types of modes are mutually orthogonal and such that even and odd radiation modes result in the limit of a symmetric guide. This gives

\[
F_{1,2} = \frac{1}{(h_2^2 - h_3^2)\sin 2h_2d} \left\{ (h_2^2 - h_3^2)\cos 2h_2d \\
+ \frac{h_3}{h_1} (h_2^2 - h_1^2) + \left[(h_2^2 - h_3^2)^2 \\
+ 2\frac{h_3}{h_1} (h_2^2 - h_3^2)(h_2^2 - h_1^2)\cos 2h_2d \\
+ \frac{h_3^2}{h_1} (h_2^2 - h_1^2)^2 \right]^{\frac{1}{2}} \right\}
\]

(2.2-30)

The plus (minus) sign corresponds to the odd (even) modes.

The normalization condition (2.2-26) again determines the constant A^{(a)}:

\[
A^{(a)} = \frac{(4\omega_{10})}{\left\{ \pi |\beta| \left[(\cos h_2d - F_{1,2}\sin h_2d)^2 \\
+ \frac{h_2^2}{h_3^2} (\sin h_2d + F_{1,2}\cos h_2d)^2 \\
+ (1 + \frac{h_2^2}{h_1^2} F_{1,2}^2) \frac{h_1}{h_3} \right] \right\}}
\]

(2.2-31)

As \(\beta\) covers the range (2.2-28), \(h_3\) covers the range

\[
(n_3^2 - n_1^2)^{\frac{1}{2}} k < h_3 < n_3 k.
\]
h_3 is also allowed to be in the range
\[ n_3 k < h_3 < \infty \quad (2.2-32) \]
For this range of h_3, \( \beta \) is imaginary. These modes are also described by (2.2-29).

**TM Modes**

For TM modes the only nonzero field components are \( H_y \), \( E_x \), and \( E_z \). The field equations obeyed by these modes are:

\[ \frac{\partial^2 H_y}{\partial x^2} + (n^2 k^2 - \beta^2) H_y = 0 \quad (2.2-33) \]
\[ E_x = \frac{\beta}{n^2 \omega \varepsilon_0} H_y, \quad (2.2-34) \]

and

\[ E_z = -\frac{1}{n^2 \omega \varepsilon_0} \frac{\partial H_y}{\partial x}. \quad (2.2-35) \]

There are again three types of modes depending on the range of \( \beta \), just as for TE modes. Only the guided modes will be considered here.

**Guided Modes**

These modes occur for values of \( \beta \) in the range
\[ n_3 k < \beta < n_2 k \quad (2.2-36) \]
The solution to (2.2-33) which satisfies the boundary conditions at \( x = 0, -d \) is:

\[ H_{ym}(x) = \begin{cases} 
  Be^{-p_1x}, & \text{for } x \geq 0 \\
\end{cases} \]
\[
H_{ym}(x) = \begin{cases} 
B(\cos p_2 x - \frac{n_2^2}{n_1^2} \frac{p_1}{p_2} \sin p_2 x), \\
\text{for } 0 \geq x \geq -d \\
B(\cos p_2 d + \frac{n_2^2}{n_1^2} \frac{p_1}{p_2} \sin p_2 d) e^{p_3(x+d)}, \\
\text{for } x \leq -d 
\end{cases} 
\] (2.2-37)

where \( p_1, p_2, \) and \( p_3 \) are given in (2.2-11). A plot of the field distribution for the first three guided TM modes is shown in Figure 2.2-5 for \( n_1 = 1, n_2 = 3.42, \) and \( n_3 = 3.26. \)

The eigenvalue equation for the TM modes is

\[
d^{(m)} = \frac{1}{p_2} \left[ \tan^{-1} \frac{n_1^2 p_3}{n_3^2 p_2} + \tan^{-1} \frac{n_2^2 p_1}{n_1^2 p_2} + \frac{1}{i} \right] 
\] (2.2-38)

In Figure 2.2-6, \( d^{(m)} \) is plotted versus the normalized propagation constant, \( \beta_m/k. \) The cutoff thickness is again obtained by letting

\[
\beta_m \rightarrow n_3 k: 
\]

\[
d_c^{(m)} = \frac{1}{k(n_2^2 - n_3^2)^{1/2}} \left[ \tan^{-1} \frac{n_2^2 (n_3^2 - n_1^2)^{1/2}}{n_1^2 (n_2^2 - n_3^2)^{1/2}} + \frac{1}{i} \right] 
\] (2.2-39)

The amplitude coefficient \( B \) obeys the normalization condition

\[
\frac{\beta_m}{2\omega \mu_o} \int_{-\infty}^{\infty} \frac{1}{n^2} H_{yn} H_{ym} \ dx = \delta_{nm} \] (2.2-40)

which demands that
Fig. 2.2-5. Field distribution for first three guided TM modes.
Fig. 2.2-6. Waveguide thickness vs. $\beta_m/k$ for TM modes and $N_3 = 3 \times 10^{18}$ cm$^{-3}$. 
Absorption losses due to free carrier absorption and lattice absorption can be calculated by using a complex index of refraction

\[ n = n + j \kappa \]  

(2.2-42)

where \( n \) is the normal refractive index and \( \kappa \) is the absorption index. The time dependence is still given by (2.2-3) but now the \( z \) dependence is

\[ e^{-j\beta z - \alpha z} = e^{-\gamma z} \]  

(2.2-43)

so

\[ \gamma = \alpha + j\beta. \]  

(2.2-44)

**TE Guided Modes**

Helmholtz equation for the lossy waveguide becomes

\[
\frac{\partial^2 E_{ym}}{\partial x^2} + (n^2 k^2 + \gamma_m^2) E_{ym} = 0
\]  

(2.2-45)

If we write

\[ E_{ym} = E_{ym}(x) e^{(j\omega t - \gamma_m z)} \]  

(2.2-46)

then \( E_{ym}(x) \) is still given by (2.2-10). However, \( p_1, p_2, \) and \( p_3 \) are now
\[ p_1 = j(\gamma_m^2 + \eta_1^2k^2)^{1/2} \]
\[ p_2 = (\eta_2^2k^2 + \gamma_m^2)^{1/2} \quad (2.2-47) \]
\[ p_3 = j(\gamma_m^2 + \eta_3^2k^2)^{1/2} \]

\[ H_{zm}(x) \] can be written

\[
H_{zm}(x) = \begin{cases} 
- \frac{j p_1}{\omega \mu_0} A e^{-p_1 x}, & \text{for } x > 0 \\
- \frac{j p_2}{\omega \mu_0} (A \sin p_2 x - B \cos p_2 x), & \text{for } -d < x < 0 \\
- \frac{j p_3}{\omega \mu_0} (A \cos p_2 d - B \sin p_2 d) e^{p_3(x+d)}, & \text{for } x \leq -d
\end{cases}
\]

Continuity of \[ H_{zm}(x) \] at \( x = 0 \) and at \( x = -d \) give

\[ p_1 A + p_2 B = 0 \quad (2.2-49) \]
\[ A(p_2 \sin p_2 d - p_3 \cos p_2 d) + B(p_2 \cos p_2 d + p_3 \sin p_2 d) = 0 \quad (2.2-50) \]

We wish to solve these two equations to get expressions for the propagation constant, \( \beta_m \), and the absorption constant, \( \alpha_m \). To do this we assume the absorption is small and obtain an approximate solution by considering only first order terms, neglecting all second and higher order terms in \( \alpha \) and \( \kappa \). Then \( p_1, p_2, \) and \( p_3 \) are, to first order:
\[ p_1 = \left( \beta_m^2 - n_1^2 k_1^2 \right)^{1/2} - j \frac{\alpha_m \beta_m + n_1 \kappa_1 k_1^2}{\left( \beta_m^2 - n_1^2 k_1^2 \right)^{1/2}} \]  
\[ = p_1' + j p_1'' \]

\[ p_2 = \left( n_2^2 k_2^2 - \beta_m^2 \right)^{1/2} + j \frac{\alpha_m \beta_m + n_2 \kappa_2 k_2^2}{\left( n_2^2 k_2^2 - \beta_m^2 \right)^{1/2}} \]  
\[ = p_2' + j p_2'' \]

and

\[ p_3 = \left( \beta_m^2 - n_3^2 k_3^2 \right)^{1/2} - j \frac{\alpha_m \beta_m + n_3 \kappa_3 k_3^2}{\left( \beta_m^2 - n_3^2 k_3^2 \right)^{1/2}} \]  
\[ = p_3' + j p_3'' \]

and the \( p \)'s have been separated into real (\( p' \)) and imaginary (\( p'' \)) parts. These expressions hold as long as the following inequalities are satisfied

\[ \frac{\alpha_m \beta_m + n_1 \kappa_1 k_1^2}{\left( \beta_m^2 - n_1^2 k_1^2 \right)^{1/2}} \ll 1, \]  
\[ \alpha_m \beta_m + n_2 \kappa_2 k_2^2 \ll 1, \]  
\[ \frac{\alpha_m \beta_m + n_3 \kappa_3 k_3^2}{\left( \beta_m^2 - n_3^2 k_3^2 \right)^{1/2}} \ll 1. \]  

Equations (2.2-49) and (2.2-50) have nontrivial solutions only if the determinant of the coefficients of \( A \) and \( B \) is zero:
\[(p_1 p_2 \cos p_2 d + p_1 p_3 \sin p_2 d)
- (p_2^2 \sin p_2 d - p_1 p_3 \cos p_2 d) = 0 \quad (2.2-53)\]

Substituting (2.2-51a, b, c) into the above equation yields two equations — one for the real part and one for the imaginary part. All second and higher order terms in the \(p_1''\)'s are dropped and the approximations \(\sinh p_2''d \approx p_2''d\) and \(\cosh p_2''d \approx 1\) are used.

The real part of (2.2-53) then yields after manipulation:
\[
d^{(m)} = \frac{1}{p_2} \left[ \frac{\tan^{-1} \frac{p_1'}{p_2} + \tan^{-1} \frac{p_3'}{p_2}}{\pi} + m\pi \right] \quad (2.2-54)
\]

If we recall the form of \(p_1', p_2',\) and \(p_3'\) from (2.2-51a, b, c) and compare (2.2-54) with (2.2-11) we note that they are identical. That is, to first order, the propagation constant, \(\beta_m\), in a lossy waveguide is the same as for the lossless case.

The imaginary part of (2.2-53) gives the absorption coefficient, \(\alpha\), of the waveguide modes:
\[
\alpha = \frac{b}{a} (m^{-1}) \quad (2.2-55)
\]

where
\[
a = (\tan p_x d) \left[ -p_1'^2 p_2' p_3'^2 - p_2' p_3'^2 + p_1'^2 p_2' \right]
- 2p_1' p_2' p_3' - p_1' p_2' p_3'^2 + \beta \left[ -p_2'^2 p_3' \right]
+ p_1'^2 p_3' + p_1'^2 p_3'^2 - p_1' p_2'^2 p_3' d
- p_1' p_2'^2 + p_1' p_3'^2 \quad (2.2-56)
\]
The power absorption coefficient is twice the value obtained from (2.2-55). So

\[ \alpha_p = 2\alpha = \text{power absorption coefficient} \quad (2.2-58) \]

**TM Guided Modes**

The derivation of the propagation constant, \( \beta_m \), and the absorption coefficient, \( \alpha_m \), for the TM guided modes is identical to that for the TE guided modes. To first order, assuming small absorption, \( p_1, p_2, \) and \( p_3 \) are given by (2.2-51a-c). Then continuity of \( E_{zm} \) gives the equation

\[
\begin{bmatrix}
\frac{p_1 p_2}{n_2^2 n_3^2} \\
\cos p_2 d + \frac{p_1 p_3 \sin p_2 d}{n_3^4}
\end{bmatrix}
- \begin{bmatrix}
\frac{p_2^2}{n_2^4} \\
\sin p_2 d
\end{bmatrix}
- \begin{bmatrix}
-\frac{p_2 p_3}{n_2^2 n_3^2} \\
\cos p_2 d
\end{bmatrix} = 0
\quad (2.2-59)
\]
Substituting (2.2-5la-c) into (2.2-58a) yields two equations - one involving the real terms and one involving the imaginary terms.

Again with the approximations sinh $p_2''d \approx p_2''d$ and cosh $p_2''d \approx 1$ and neglecting all second and higher order terms, the real part of (2.2-58a) yields the same eigenvalue equation as for the lossless case (2.2-38):

$$d^{(m)} = \frac{1}{p_2'} \left[ \tan^{-1} \frac{n_1^2 p_3'}{n_3^2 p_2'} + \tan^{-1} \frac{n_2^2 p_1'}{n_1^2 p_2'} + m\pi \right]$$

(2.2-60)

The imaginary part of (2.2-58a) gives the field absorption coefficient, $\alpha$:

$$\alpha = \frac{b}{a} (m^{-1})$$

(2.2-61)

where

$$a = \beta\tan p_2' d \left[ -\frac{p_1'^2 p_2' p_3' d}{n_2^4 n_3^2} - \frac{p_2'^2 p_3'^2}{n_3^4} - \frac{p_1'^2 p_2'}{n_3^4} \right]$$

$$+ \beta \left[ -\frac{p_2'^2 p_3'}{n_2^2 n_3^2} + \frac{p_1'^2 p_3'}{n_2^2 n_3^2} + \frac{p_2'^2 p_3'^2}{n_3^4} - \frac{p_1'^2 p_2'^2 p_3'^2}{n_2^4} \right.$$

$$- \frac{p_1'^2 p_2'^2}{n_2^2 n_3^2} + \frac{p_1'^2 p_3'}{n_2^2 n_3^2} \right]$$

(2.2-62)

and
The power absorption coefficient is again equal to

\[ \alpha_p = 2\alpha = 2 \frac{b}{a} \text{ (m}^{-1} ) \]  \hspace{1cm} (2.2-64)

### 2.2.3 The Epitaxial Silicon Waveguide

This structure consists of a high resistivity silicon epitaxial layer on a low resistivity silicon substrate (Figure 2.2-7). The index of refraction of the substrate is suppressed below that of the core because of the large number of free carriers. Equation (2.1-59) gives the difference in index of refraction between the epi layer and substrate:

\[ \Delta n = \frac{\Delta n \epsilon_0^2}{8\pi n_m \epsilon_0 c^2} = n_2 - n_3 \]  \hspace{1cm} (2.2-65)

or

\[ n_3 = n_2 - \Delta n \]  \hspace{1cm} (2.2-66)

For \( N_3 \gg N_2 \),
Fig. 2.2-7. The epitaxial silicon waveguide.
\[ n_3 = n_2 - \frac{N_3 e^2 \lambda^2}{8\pi^2 \hbar \epsilon_0 c \mu_n c^2} \]  \hspace{1cm} (2.2-67)

where \( N_3 \) is the carrier concentration of the substrate. For \( \lambda = 9.57 \mu m \), this reduces to

\[ n_3 = (3.42 - 4.43 \times 10^{-20} N_3) \]  \hspace{1cm} (2.2-68)

where the assumption \( n_2 = n_0 = 3.42 \) has been used and \( N_3 \) is the concentration per cm\(^3\).

Plots of normalized propagation constant, \( \beta_m/k \), versus \( d \) for epitaxial Si waveguides with varying substrate concentrations are shown in Figures 2.2-8 - 2.2-10 for TE modes and in Figures 2.2-11 - 2.2-13 for TM modes.

Absorption in Si at 9.57 \( \mu m \) is due to both lattice absorption and free carrier absorption (23). The free carrier absorption coefficient was derived in section 2.1.4 and is given by

\[ \alpha_{FC} = \frac{\lambda^2 e^3}{4\pi^2 c^3 n_0 \epsilon_0 \mu_n} \frac{N(m^{-3})}{m^2 \mu_n} (m^{-1}) \]  \hspace{1cm} (2.2-69)

or

\[ \alpha_{FC} = (1.92 \times 10^{-18}) \frac{N(cm^{-3})}{\mu_n} (cm^{-1}) \]  \hspace{1cm} (2.2-70)

where we have used \( m^* = .27 m_0 \) (19) and the electron mobility \( \mu_n \) has dimensions of \( m^2/v\cdot sec \). For \( N > 10^{17} \) cm\(^{-3}\), the lattice absorption is negligible compared to the free carrier absorption. Therefore, the absorption coefficient in the substrate is given by

\[ \alpha_3 = (1.92 \times 10^{-18}) \frac{N_3}{\mu_n} \]  \hspace{1cm} (2.2-71)

Then the absorption index of the substrate is obtained from (2.1-22)
Fig. 2.2-8. Waveguide thickness vs. $\beta_m/k$ for TE modes and $N_3 = 3 \times 10^{18}$ cm$^{-3}$. 
Fig. 2.2-9. Waveguide thickness vs. $\beta_m/k$ for TE modes and $N_3 = 7.5 \times 10^{17}$ cm$^{-3}$. 
Fig. 2.2-10. Waveguide thickness vs. $\beta_m/k$ for TE modes and $N_3 = 2.5 \times 10^{17}$ cm$^{-3}$.
Fig. 2.2-11. Waveguide thickness vs. $\beta_m/k$ for TM modes and $N_3 = 3 \times 10^{18} \text{cm}^{-3}$. 
Fig. 2.2-12. Waveguide thickness vs. $\beta_m/k$ for TM modes and $N_3 = 7.5 \times 10^{17}$ cm$^{-3}$.
Fig. 2.2-13. Waveguide thickness vs. $\beta_m/k$ for TM modes and
$N_3 = 2.5 \times 10^{17} \text{ cm}^{-3}$. 
\[ \kappa_3 = (1.5 \times 10^{-22}) \frac{N_3}{\mu_n} \]  \hspace{1cm} (2.2-72)

In the core region, absorption is due mainly to the lattice absorption since the free carrier concentration is low. For wavelengths in the range 6-20 \( \mu \text{m} \), the absorption is due to two phonon absorption (24) -- a photon is absorbed by the lattice with the creation of two phonons. Collins (25) has obtained experimentally the absorption coefficient due to this absorption over the range 7-22 \( \mu \text{m} \). He found a low absorption around 9.5 \( \mu \text{m} \). For \( \lambda = 9.57 \mu \text{m} \) the absorption coefficient is

\[ \alpha_2 = \alpha_L \approx 0.5 \text{ cm}^{-1} \]  \hspace{1cm} (2.2-73)

This compares to an absorption coefficient of approximately 2 \( \text{cm}^{-1} \) at 10.6 \( \mu \text{m} \). From (2.1-22), the absorption index at 9.57 \( \mu \text{m} \) is

\[ \kappa_2 \approx 3.8 \times 10^{-5} \]  \hspace{1cm} (2.2-74)

Using (2.2-72) and (2.2-74), (2.2-58) gives the absorption coefficient for the guided modes of the waveguide. At the wavelengths considered, the losses due to scattering from surface imperfections should be very small (21, 26, 27). Hence, the absorption coefficient from (2.2-55) - (2.2-57) should give the actual losses of the guiding structure. This coefficient has been plotted versus waveguide thickness, \( d \), in Figure (2.2-14) for TE modes and in Figure (2.2-15) for TM modes. The important trends to notice from these plots are:

1) The absorption is very large for small thicknesses. As the thickness increases, the absorption first drops rapidly and then levels off to an almost constant value. 2) The lower the mode the lower are the absorption losses. This is especially true for small waveguide widths. Cheo (5) and Chang (28) have shown experimentally for GaAs/n+GaAs
Fig. 2.2-14. Power absorption coefficient vs. waveguide thickness for TE modes of an epitaxial Si waveguide with substrate doping concentration: $N_3 = 3 \times 10^{18} \text{cm}^{-3}$. 
Fig. 2.2-15. Power absorption coefficient vs. waveguide thickness for TM modes of an epitaxial Si waveguide with substrate doping of $3 \times 10^{18}$ cm$^{-3}$. 
waveguides that this is the case. Figure 2.2-16 shows a plot of $\alpha$ versus $d$ for the $TE_0$ mode for different substrate concentrations.

For $N_3 = 3 \times 10^{18} \text{ cm}^{-3}$, $\mu_n = 0.03 \text{ m}^2/\text{v sec}$, and $d = 20 \mu\text{m}$, (2.2-72) gives

$$\kappa_3 \approx 1 \times 10^{-2}$$

(2.2-75)

and (2.2-55) - (2.2-58) give an absorption coefficient for the $TE_0$ mode of

$$\alpha_p \approx 0.82 \text{ cm}^{-1}$$

(2.2-76)

The corresponding value for the $TM_0$ mode is

$$\alpha_p \approx 1.9 \text{ cm}^{-1}$$

(2.2-77)

2.3 INPUT COUPLERS

Coupling of radiation into thin film dielectric waveguides is accomplished mainly by the use of two couplers: the phase grating coupler or the prism-film coupler. The prism-film coupler has the advantage of versatility, in that one prism may be used as a coupler for many different waveguides. The grating coupler has the advantage of being very rugged, and it is compatible with integrated optical circuits since the grating is fabricated right on the waveguide structure. Although the theoretical coupling efficiencies are the same, experimentally it has been found that the prism coupler is a more efficient coupler than the grating coupler. Both couple radiation to only one guided mode at a time.

2.3.1 The Grating Coupler

The grating coupler consists of a phase grating which is either deposited on top of, or etched into, the guiding structure (Figure
Fig. 2.2-16. Absorption coefficient for the TE_{03} mode of an epitaxial Si waveguide for (A) $N_3 = 1 \times 10^{18}$ cm$^{-3}$, (B) $N_3 = 7.5 \times 10^{17}$ cm$^{-3}$, (C) $N_3 = 4 \times 10^{17}$ cm$^{-3}$, and (D) $N_3 = 1 \times 10^{17}$ cm$^{-3}$. 
2.3-la and 2.3-lb). The theory of these structures has been extensively studied in the literature (22), (29-31). For coupling to occur two conditions must be met:

1. Polarization matching

For coupling to TE (TM) modes the incident beam must have a component of the electric (magnetic) field parallel to the lines of the grating. If the incident beam is linearly polarized parallel to the lines of the grating only TE modes can be excited, while if the polarization is rotated 90° from this then only TM modes can be excited. By rotating the polarization 45°, both TE and TM modes can be excited.

2. Phase matching

For coupling to take place, the z-component of the phase velocity of a diffracted beam must be equal to the z-component of the phase velocity of the guided mode into which the beam is to be coupled.

The phase matching condition can be understood by considering the diffraction grating. Although this is different from the thick phase grating of the coupler, both gratings have the same conditions on the direction of the diffracted beams (32).

The case for a plane wave incident from the air is shown in Figure 2.3-2. A plane wave is incident at an angle of θ degrees from the normal to the surface. To have a diffracted beam at an angle of θ₂ from the normal to the surface the two rays shown must be in phase at points B and D. This requires

\[ O.P.L. (AB) - O.P.L. (CD) = p\lambda \]  \hspace{1cm} (2.3-1)

where O.P.L. stands for optical path length and p is the diffraction
Fig. 2.3-1. The phase grating coupler: (a) deposited grating, (b) etched grating.
Fig. 2.3-2. Incidence on diffraction grating from air region.
order. This gives
\[ L \sin \theta - n_2 L \sin \theta_2 = p\lambda \]  
(2.3-2)
where \( L \) is the periodicity of the grating. Phase matching of the
diffracted beam at \( \theta_2 \) to the \( m \)th guided mode requires
\[ \beta_m = n_2 k \sin \theta_2 \]  
(2.3-3)
where \( k \) is the free space wave number. Then the phase matching condition
for incidence from the air is
\[ \sin \theta_{SYNC} = \frac{\beta_m}{k} + \frac{p\lambda}{L} \quad (p=0, \pm 1, \pm 2, \ldots) \]  
(2.3-4)
The incident angle \( \theta_{SYNC} \) required for phase matching is called the
**synchronous angle**. This angle is plotted versus \( \beta_m/k \) in Figure 2.3-3
with the periodicity of the grating as a parameter. Note that the
\( \theta_{SYNC} \) is much more sensitive to the periodicity than to \( \beta_m/k \).

For incidence from the substrate (Figure 2.3-4), a diffracted
beam at \( \theta_2 \) from the normal to the surface occurs if
\[ \text{O.P.L. (AB)} + \text{O.P.L. (BC)} = p\lambda \]  
(2.3-5)
or
\[ n_2 L \sin \theta + n_2 L \sin \theta_2 = p\lambda \]
and using (2.3-3) the phase matching condition for incidence from the
substrate is
\[ n_2 \sin \theta_{SYNC} = \frac{p\lambda}{L} - \frac{\beta_m}{k} \quad (p=0, \pm 1, \pm 2, \ldots) \]  
(2.3-6)

Conditions for maximum input coupling efficiency have been
studied using a perturbation analysis of the grating coupler by Ogawa
et al. (22). These conditions are listed below:

1. The grating length \( d \) should be chosen to be
Fig. 2.3-3. Synchronous coupling angle vs. $\beta_m / k$ for different grating periodicities.
Fig. 2.3-4. Incidence on diffraction grating from substrate region.
\[ d = \frac{1.25}{m} \]  
\text{(2.3-7)}

where \( a_m \) is the attenuation constant of the guided field in the grating region with no incident beam.

2. The incident beam diameter should be the same as the grating length.

3. The grating period, \( L \), should be chosen so as to minimize the number of diffracted orders (equations (2.3-4) and (2.3-6)). For backward excitation from the substrate (Figure 2.3-4), \( L \) can be chosen so that there is only one diffracted order. This case gives the maximum coupling efficiency of 81%. However, excitation from the substrate requires the use of a prism on the substrate in order to obtain the required coupling angles. For this reason, coupling is usually done from the air side.

The angular range of the incident excitation angle is called the beamwidth of the excitation. This beamwidth is directly proportional to the total number of lines in the phase grating. The more lines there are the more critical the excitation angle is (22).

The output coupling efficiency of the grating coupler is 100% as long as the grating length is long enough ( \( d \gg 1/a_m \) ) (29).

Experimentally, Dakss et al. (33) have used a photo-resist grating on a glass waveguide with a coupling efficiency of 40%. Dalgoutte (34) used a deposited grating with backward excitation from the substrate and obtained an efficiency of 70%. Kogelnik and Sosnowski (35) have used a holographic coupler with an efficiency of 70%. Cheo (5) used an etched grating coupler for a GaAs/\( n^+ \)GaAs waveguide and obtained an efficiency of 10% at \( \text{CO}_2 \) laser wavelengths. Tracy et al. (36) used
electron resist gratings to couple into vinyl trimethyl silane (VTMS) waveguides for different wavelengths in the range 0.4765 – 1.06 μm. They found that the maximum coupling efficiency decreases for increasing wavelength. At 1.06 μm they obtained a maximum coupling efficiency of 10%.

2.3.2 The Prism Coupler

The prism coupler has also been extensively studied (5), (37-43). It is shown in Figure 2.3-5. If a laser beam is totally internally reflected from the bottom surface of the prism then coupling of energy can occur via the evanescent wave in the gap. In order for appreciable transfer of energy to take place, four conditions must be met (40):

1. Phase matching

   The z-component of the phase velocity in the prism must be matched to the phase velocity of the guided mode to be excited.

2. Polarization matching

   The polarization of the incoming beam must be chosen to correspond to the polarization of the guided mode; that is, to excite only TE modes the incident beam must be linearly polarized in the y-direction, while to excite only TM modes the incident beam must be linearly polarized in the x-direction.

3. Small gap

   For efficient coupling the air gap, s, must be small (typically of the order of λ/2). Experimentally, one uses a wedge on the back of the waveguide (Figure 2.3-6) and pressure tunes for maximum coupling.

4. Sharp termination of prism
Fig. 2.3-5. The prism-film coupler.
Fig. 2.3-6. Pressure tuning of prism coupler to obtain efficient coupling.
To achieve high coupling efficiency the prism should be sharply terminated and the incident laser beam should fill the region between the pressure point and the rectangular corner of the prism.

Phase matching is achieved by varying the angle of incidence. The angle for which coupling occurs is called the synchronous angle. The phase matching condition for a 45-45-90 coupling prism can be derived by again referring to Figure 2.3-5. The laser beam is incident at an angle of $\theta$ from the normal to the prism. The wave in the prism is phase matched to the $m^{th}$ guided mode if

$$n_o k = \beta_m$$ \hspace{1cm} (2.3-8)

or

$$n_o k \sin(45^\circ - \phi) = \beta_m$$

so

$$\phi = 45^\circ - \sin^{-1} \frac{\beta_m}{n_o k}$$

Then from Snell's law

$$\sin \theta_{SYNC} = n_o \sin \left[ 45^\circ - \sin^{-1} \left( \frac{\beta_m}{k n_o} \right) \right]$$ \hspace{1cm} (2.3-9)

Knowing the propagation constant for a given mode one can calculate the synchronous coupling angle from this equation.

It should be noted that $\beta_m/k$ is in the range

$$n_3 < \frac{\beta_m}{k} < n_2$$

and if $n_o < n_2$, then it is possible that $\beta_m/k n_o$ is greater than one. In this case (2.3-9) cannot be solved and coupling to this mode is impossible. For $n_o > n_2$, this is not a problem. Hence, to excite all possible modes of the guide, the refractive index of the prism must be
greater than the refractive index of the guide.

The maximum theoretical input coupling efficiency is 81% (37), (43). In practice, coupling efficiencies of 60% or more have been obtained in the visible (43) and near infra-red. At CO₂ laser wavelengths, Chang (1) obtained a coupling efficiency of 10% while Cheo (5) obtained a coupling efficiency of 40%. Both used Ge prisms as couplers.
III. EXPERIMENTAL TECHNIQUES

3.1 GRATING FABRICATION

Grating couplers have been fabricated in three different ways:

1. exposing photo resist to two interfering laser beams (45-51);
2. writing a grating in electron resist using a scanning electron microscope (SEM) (36), (52); and
3. by using conventional photolithography (5). This last technique can only be used for infra-red waveguides because of the small periodicity required in the visible and near infra-red. The method chosen for this work was the first -- interfering two Ar laser beams (4880°A) to form a phase grating in a photo resist film.

Silicon epitaxial wafers were first cleaned using the "RCA Cleaning Procedure" (53) and then were baked at 170° C for eight hours to remove water from the surface of the Si. If the water is not removed, adhesion problems result and crazing of the photo resist during developing can occur.

The wafers were then spin coated with Shipley AZ1350B photo resist. The spinning time was 30 seconds at a speed of 4500 rpm resulting in a thickness of approximately 1.8 μm.

Next the wafers were baked at 70° C for ½ hour to drive off all the solvent and insure equal sensitivity of the photo resist.

Using a UV lamp the photo resist was exposed to a mask which, after development, left two square pads of photo resist each 3 mm on a side and separated by 3 mm. After development the wafers were baked at 70° C for ½ hour.

The two photo resist pads were then exposed to two interfering
laser beams. The exposure set-up is shown in Figure 3.1-1. The 4880°A line from an Ar laser (single mode TEM\(_{00}\)) was first split into two beams. Each was individually spatial filtered and beam expanded to approximately 5 cm in diameter. The two beams were then made to intersect on the photo resist pads at an angle \(\theta_0\). This gave a grating periodicity of

\[
L = \frac{\lambda}{2 \sin(\theta_0/2)}
\]

(3.1-1)

\(\theta_0\) was chosen to be 7.5° so that

\[
L = 3.75 \mu m
\]

(3.1-2)

This periodicity ensures that there are only two diffracted orders (see equations 2.3-4 and 2.3-6).

The output of the laser was 1 watt. Typical exposure times were 11 minutes with a corresponding development time of 10-20 seconds (1:1 Az Developer/H\(_2\)O). After development, the wafers were post baked for 15 minutes at 110° C to lock the photo resist.

Some gratings were sputter etched (RF) into the Si wafer. Sputtering times of 4-5 minutes at a power of \(2\frac{1}{2}\) watts/cm\(^2\). After sputtering the photo resist was removed with acetone.

3.2 WAVEGUIDE EVALUATION

3.2.1 Focusing Onto Cleaved Edge

The silicon epitaxial wafers used in this experiment were obtained from Texas Instrument, Inc., and are described in detail in Table 3.2-1. In order to determine the absorption coefficient of the wafers a CO\(_2\) laser beam was focused onto a cleaved end of the guide. Transmission measurements were then obtained by measuring the power in the beam emitted
Fig. 3.1-1. Experimental arrangement for producing photo resist gratings by the interference of two laser beams.
from the other end (3), (54).

TABLE 3.2-1
WAFFER SPECIFICATIONS

<table>
<thead>
<tr>
<th>Substrate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Growth Method: Czochralski</td>
</tr>
<tr>
<td>Resistivity: (0.013,\Omega\cdot\text{cm} (N_3 \approx 3.0 \times 10^{18} , \text{cm}^{-3}))</td>
</tr>
<tr>
<td>n-type Antimony doped</td>
</tr>
<tr>
<td>Etch Pit Count: &lt; 3000/cm²</td>
</tr>
<tr>
<td>Thickness: 16.6 mils (420μm)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Epitaxial Layer</th>
</tr>
</thead>
<tbody>
<tr>
<td>n-type Phosphorus doped</td>
</tr>
<tr>
<td>Resistivity: (8-10,\Omega\cdot\text{cm} (N_2 \approx 6 \times 10^{14} , \text{cm}^{-3}))</td>
</tr>
<tr>
<td>Thickness: 16-20μm</td>
</tr>
</tbody>
</table>

The wafers were scribed with a diamond stylus and broken to a length of approximately 15 mm. These portions were then scribed again to permit breakage after mounting. These second scribes were designed to give a waveguide of approximately 6 mm in length after breaking. In this manner guides of two different lengths were obtained having the same geometry and the same input coupling. The only possible difference between the guides would be in output coupling.

The samples were mounted as shown in Figure 3.2-1. The waveguide was sandwiched between two low resistivity (~0.009 -cm) Si strips. The sandwich was then mounted with epoxy on a glass microscope slide. Since
Fig. 3.2-1. Waveguide mounting arrangement.
the epoxy was not completely opaque at CO$_2$ laser wavelengths, the sample was epoxied only at the edges and clamped in the middle.

The absorption coefficient of the substrate is, from (2.2-72) for $N_3 = 3 \times 10^{18}$ cm$^{-3}$ and $\mu_n = 0.03$ (m$^2$/v$^{-3}$)

$$\alpha = 192 \text{ (cm}^{-1})$$

(3.2-1)

So for a length of 6 mm, the power is down by a factor of $10^{-52}$

For the bounding silicon strips, the absorption is even greater. Hence as a beam is scanned across the sample, (Figure 3.2-2) any transmitted beam will be due to propagation through the waveguide. This assumes, of course, that there are no air spaces between the Si slices. To be certain that the CO$_2$ laser radiation propagated in the waveguide and not through air spaces three checks were made on each waveguide:

1. Before scanning with a CO$_2$ beam, the sample was scanned using a He-Ne laser ($\lambda = 6328^{\circ}$A). Detection of an output would have meant that propagation was through an air gap, since Si is opaque at $\lambda = 6328^{\circ}$A. However, in all cases no radiation was detected even on the lowest scale (corresponding to $10^{-8}$W) of the lock-in amplifier.

2. While detecting a transmitted CO$_2$ laser beam, the pressure of the clamp was adjusted. In all cases there was very little change in the detected power, indicating that propagation was taking place through the waveguide and not through an air gap.

3. The sample was viewed end on under a microscope (400x) with illumination from below. If any light was visible through cracks between the silicon slices the sample was rejected.

The measurement set-up is shown in Figure 3.2-3.
Fig. 3.2-2. Diagram of mounted sample showing the scan path of the laser beam.
Fig. 3.2-3. Experimental arrangement for detecting power transmitted through guide.
The 9.57 micron line of a GTE Sylvania 941S laser was chopped at a frequency of 100 Hz. The 9.57 \( \mu \text{m} \) line was chosen because Si has lowest losses at this wavelength than at other CO\(_2\) lines (25). The laser was linearly polarized in the \( x \)-direction (vertically). With this polarization only TM modes were excited. After chopping, the beam was focused to a spot on the sample by a Ge lens (F.L. = 3.81 cm). The diffraction limited spot radius is given by

\[
r = 1.22 \frac{f \lambda}{a}
\]

(3.2-2)

where \( a \) is the diameter of the laser beam (55). For a beam diameter of 4.5 mm (1/\( e^2 \) power points), the diameter of the spot is

\[
d \approx 220 \mu \text{m}
\]

(3.2-3)

The sample was attached to the detector, and the detector and sample were raised and lowered using a micrometer stage. The detector was about 5 cm from the output edge of the sample. The detector was a Laser Precision pyroelectric detector model KT-1110S with a 1 mm diameter element. The sensitivity (using \( 10^{10} \) load resistance) is

\[
r_v = 49.0 \text{ volts/watt}
\]

(3.2-4)

The detector also had a special metal coating to allow for detection in the visible.

The output of the detector was fed into a Princeton Applied Research (PAR) Lock-in Amplifier (Model 124) along with a reference signal from the chopper which allowed for a synchronous detection. Voltage signals down to 100 nanovolts (corresponding to \( 10^{-8} \) watts) could be detected in this manner.

A high resistivity Si wafer was used as a beam splitter so that the power in the incident beam could be monitored. After a warm-up
time of one hour or more the laser was very stable — both with respect to power and frequency. Input powers were approximately 700 mw for the CO$_2$ laser and 3 mw for the He-Ne laser.

To determine the order of the guided modes a scan of the far field radiation pattern was made (3). The experimental arrangement is shown in Figure 3.2-4. The laser remains focused onto a cleaved edge of the guide while the detector is scanned across the path of the output beam in the x-direction (vertically). The distance of the detector from the output face was 10 cm.

3.2.2 Coupling Via A Phase Grating

Coupling into the waveguide using a phase grating was investigated also. The gratings were prepared by the procedure outlined in section 3.1. The important properties of the silicon wafers are (from Table 3.2-1):

\[
\begin{align*}
N_2 &= 6 \times 10^{14} \text{ cm}^{-3} \\
N_3 &= 3 \times 10^{18} \text{ cm}^{-3} \\
d &\approx 18.8 \mu \text{m }
\end{align*}
\]

The geometry is shown in Figure 2.2-7. Both photo resist gratings and gratings that had been sputter etched into the waveguide were investigated. With the carrier concentrations given above, the indices of refraction are (from 2.2-68):

\[
\begin{align*}
n_1 &= 1 \\
n_2 &= 3.42 \\
n_3 &= 3.26
\end{align*}
\]  

With these parameters, then, this waveguide can propagate 4 TE
Fig. 3.2-4. Experimental arrangement for scanning the far-field radiation pattern.
and 4 TM modes. The normalized propagation constants, $\beta_m/k$, are obtained from equations (2.2-13) and (2.2-38), while the synchronous coupling angles $\theta_{SYNC}$ for each are obtained from equation (2.3-4). These are listed for each of the modes in Table 3.2-2.

<table>
<thead>
<tr>
<th>MODE</th>
<th>$\beta_m/k$</th>
<th>$\theta_{SYNC}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>TE 0</td>
<td>3.412</td>
<td>38.6°</td>
</tr>
<tr>
<td>1</td>
<td>3.389</td>
<td>36.8°</td>
</tr>
<tr>
<td>2</td>
<td>3.350</td>
<td>34.1°</td>
</tr>
<tr>
<td>3</td>
<td>3.297</td>
<td>30.5°</td>
</tr>
<tr>
<td>TM 0</td>
<td>3.412</td>
<td>38.6°</td>
</tr>
<tr>
<td>1</td>
<td>3.387</td>
<td>36.7°</td>
</tr>
<tr>
<td>2</td>
<td>3.347</td>
<td>33.9°</td>
</tr>
<tr>
<td>3</td>
<td>3.292</td>
<td>30.2°</td>
</tr>
</tbody>
</table>

The experimental arrangement for coupling into the waveguide via the grating is shown in Figure 3.2-5. Synchronous detection methods were again used as in section 3.2.1. The sample was mounted so that it could be rotated about the vertical axis. Relative rotation angles could be measured to within 1 minute of arc. Absolute values of $\theta_o$ could be measured to $\pm$ 1 degree. The laser beam was polarized in the vertical direction and the diffraction grating was oriented so that the lines of the grating were parallel to the electric field vector of the
Fig. 3.2-5. Experimental arrangement for detection of coupling to guide using a grating coupler.
incident radiation. This permitted coupling to only TE modes. Three experiments were run to try to determine whether coupling of radiation into and out of the waveguide was taking place.

1. To determine when and if coupling occurred, the transmitted beam T1 and the reflected beam R1 were monitored while continuously varying the incident angle $\theta_0$. As coupling of power into the guide is achieved the power in the transmitted beam should decrease and the power in the reflected beam should increase (22), (36). The distance, x, of lens 1 from the sample was varied but normally it was at a distance of $x = 30$ cm so that the beam was expanding when hitting the grating and so that the beam just covered the whole area of the grating (3 mm x 3 mm).

2. The pyroelectric detector was used to try to detect a beam coupled out of the guide by the second diffraction grating. The sample was rotated by 1 degree increments and at each step the detector was scanned over the area where the output beam was expected. The input angle varied over the range $0^\circ - 45^\circ$.

3. The wafer was cleaved at the edge of the grating and an attempt was made to detect any end-fire radiation (1) (Figure 3.2-6). The detector was positioned 10 cm from the cleaved end of the guide. The sample was rotated in $\frac{1}{2}$ degree increments and after each increment the detector was scanned in both the x and y directions. In this way the input angle was varied $0^\circ$ to $45^\circ$. 
Fig. 3.2-6. Experimental arrangement for detecting any end-fire radiation after coupling using a grating coupler.
IV. RESULTS AND DISCUSSION

4.1 GRATING FABRICATION

Optical photographs of typical photo resist gratings prepared as outlined in section 3.1 are shown in Figures 4.1-1 and 4.1-2. The first photograph (Figure 4.1-1) is of a good area of a grating at high magnification (400x). It shows that the grating lines are very uniform and that there is no resist left between the lines. However, the second photograph (Figure 4.1-2) is of the same grating but magnified only 200x. It clearly shows that the photo resist is not cut through to the silicon over the entire grating. The third photograph (Figure 4.1-3) shows a different grating at the same magnification. This grating does seem to have been developed through to the silicon everywhere. However, as can be seen, the lines did not adhere to the wafer, but instead lifted up in places.

These imperfections occurred not just for the samples shown but for many samples. Either the photo resist was not developed through to the silicon everywhere or else the grating lifted off. The adhesion problem is probably due to the presence of water. To correct this situation the wafers should be vacuum baked both before applying the photo resist and after the first development.

Grating periodicities were measured using the micrometer stage of the microscope. Measured periodicities were in the range 3.7 μm - 3.9 μm.

4.2 WAVEGUIDE EVALUATION

4.2.1 Phase Grating Coupler
Fig. 4.1-1. Photograph of photo resist grating #1 (x400).

Fig. 4.1-2. Photograph of photo resist grating #1 (x200).
Fig. 4.1-3. Photograph of photo resist grating #2 (x200).
Coupling into the waveguide by using a grating coupler (both photo resist and etched) was never accomplished. When monitoring the transmitted power, while varying the angle of incidence on the coupler, no dip in the transmitted power occurred. The same was true for the reflected beam—no irregular increase or decrease was observed. No output radiation was observed either coupled out of a second grating coupler or coupled out of a cleaved end.

The reasons for this are believed to be the following. The photo resist \( (n = 1.7) \) grating does not introduce much phase retardation at the wavelengths used \( (9.57 \ \mu m) \). This can be overcome by sputter etching the grating into the waveguide layer \( (n = 3.42) \). However, because of the problem with uniformity of the photo resist discussed in section 4.2, the actual depth of the sputtered grating is very small. Fradin and Cheo (56) have found that for efficient coupling into GaAs, using a sputter etched grating coupler, the depth of the grooves should be \( 1 \ \mu m \) or larger and that the greater the depth the more efficient the coupler will be. For the sputter etched gratings studied here, a depth groove of \( < .1 \ \mu m \) is estimated based on the intensities of reflected diffracted beams (for visible light) before and after sputtering.

4.2.2 Coupling by Focusing on Cleaved Edge

The results of scanning the \( \text{CO}_2 \) laser beam across two samples cut from the same wafer are shown in Figures 4.2-1 and 4.2-2. Parts (a) of the figures give the transmission through the guides before the second cleave. Parts (b) give the transmission after cleaving to a shorter length. In all cases, the peaks of the graphs correspond to the
Fig. 4.2-1a. Relative transmitted power vs. scanning position for waveguide TI#10-1A.
Fig. 4.2-1b. Relative transmitted power vs. scanning position for waveguide TI#10-1B.
Fig. 4.2-2a. Relative transmitted power vs. scanning position for waveguide TI#10-2A.
Fig. 4.2-2b. Relative transmitted power vs. scanning position for waveguide TI#10-2B.
position of the waveguide layers within the experimental accuracy of the measurement.

Since the spot diameter \( d = 220 \mu m \) of the incident laser beam is much larger than the thickness of the waveguides, the width of the peaks in Figures 4.2-1 and 4.2-2 should be approximately equal to the spot diameter. This is roughly the case.

Table 4.2-1 gives the relevant data for each waveguide. Knowing the power transmitted through two different lengths of guide, the power absorption coefficient, \( \alpha \), is calculated from

\[
P_0 = P_c e^{-\alpha p z}
\]

where \( P_0 \) is the peak transmitted power and \( P_c \) is the incident power coupled into the guide. The input and output coupling efficiencies are assumed to be equal for guides (a) and (b) of each sample. The calculated values of the absorption coefficient are also given in Table 4.2-1. Note that the absorption of two samples is very nearly the same even though the relative detected powers are widely different. Averaging the two values gives an absorption coefficient of \( \approx 1.2 \, cm^{-1} \) or 5.1 db/cm.

**TABLE 4.2-1**

GUIDE LENGTHS, TRANSMITTED POWERS AND ABSORPTION COEFFICIENTS FOR SILICON WAVEGUIDES

<table>
<thead>
<tr>
<th>SAMPLE</th>
<th>WAVEGUIDE LENGTH</th>
<th>PEAK RELATIVE TRANSMITTED POWER</th>
<th>( \alpha )</th>
</tr>
</thead>
<tbody>
<tr>
<td>TI#10-1A</td>
<td>14.7 mm</td>
<td>70</td>
<td>1.1cm(^{-1})(4.8 db/cm)</td>
</tr>
<tr>
<td>1B</td>
<td>6.4 mm</td>
<td>175</td>
<td>1.2cm(^{-1})(5.4 db/cm)</td>
</tr>
<tr>
<td>TI#10-2A</td>
<td>15.4 mm</td>
<td>21</td>
<td></td>
</tr>
<tr>
<td>2B</td>
<td>6.3 mm</td>
<td>65</td>
<td></td>
</tr>
</tbody>
</table>
To determine the modes that are propagating a scan of the far field radiation pattern was made. This is shown in Figure 4.2-3 for sample TI#10-1b. It has the single lobe that is characteristic of the fundamental TM$_0$ mode (Figure 2.2-5). This is to be expected since the higher modes have more losses than the fundamental mode. The peak power detected in the scan of the far field radiation pattern was 12.4 microwatts.

A (very) crude estimate of the coupling efficiency can be obtained by assuming the input and output coupling efficiencies are equal. Then using the values for the absorption coefficient obtained earlier, the coupling efficiency can be written as

$$ (\eta P_i)^2 = P_0 e^{1.2z} $$  \hspace{1cm} (4.2-2)

where $P_i$ is the incident laser power and $P_0$ is the detected output power. Using the values of $P_i = .6\mu W$, $z = .63$ cm, and $P_0 = 12.4 \mu W$, gives

$$ \eta = .85\% $$  \hspace{1cm} (4.2-3)

Theoretically, one would expect that the input coupling efficiency would depend on the ratio of the area of the beam incident on the waveguide to the total area of the beam and also on the reflection coefficient. Mathematically, this is

$$ \eta = \frac{A_w}{A_T} \rho = \frac{A_w}{A_T} \left( \frac{n - 1}{n + 1} \right)^2 $$  \hspace{1cm} (4.2-4)

For a spot diameter of 220\mu m and a guide thickness of 18.8 \mu m this gives

$$ \rho = .3 \frac{A_w}{A_T} $$
$\theta = \tan^{-1} \left( \frac{x}{z} \right)$

Fig. 4.2-3. Far-field radiation pattern.
\[ \eta = 3.3\% \quad (4.2-5) \]

which agrees reasonably well with the experimental coupling efficiency 
(4.2-3) considering the roughness of the calculations.
V. CONCLUSIONS

Epitaxial Si/n⁺Si structures have been investigated for use as waveguides at CO₂ laser wavelengths. For an epilayer concentration of \( N_2 = 6 \times 10^{14} \text{ cm}^{-3} \), substrate concentration of \( N_3 = 3 \times 10^{18} \text{ cm}^{-3} \), guide thickness of \( d = 18.8 \mu\text{m} \), and \( \lambda = 9.57 \mu\text{m} \), an absorption coefficient of \( 1.2 \text{ cm}^{-1} \) (5.1 db/cm) has been measured for the \( \text{TM}_0 \) mode. However, because of variations in the coupling efficiencies, the actual absorption coefficient could be as large as \( 3 \text{ cm}^{-1} \) (13 db/cm). The calculated value of the absorption coefficient (assuming no radiation losses due to surface imperfections) is \( 1.9 \text{ cm}^{-1} \) for the \( \text{TM}_0 \) mode which agrees, within the experimental accuracy of the experiment, to the measured value.

The absorption coefficient for the TE modes was not measured, but calculations show that it should be less than for the TM modes. The calculated absorption coefficient for the above guide for the \( \text{TE}_0 \) mode is \( 0.86 \text{ cm}^{-1} \) (3.7 db/cm).

The absorption can be lowered by using lower doped substrates and very high resistivity epilayers. However, the lattice absorption places a minimum value of \( 0.5 \text{ cm}^{-1} \) (2.2 db/cm) at 9.57 \( \mu\text{m} \). At 10.6 \( \mu\text{m} \), the lower limit on the absorption is \( 2 \text{ cm}^{-1} \) (8.7 db/cm). Although the measured and calculated absorption coefficients are not as low as for GaAs/n⁺GaAs waveguides (\( < 1 \text{ db/cm} \) at 10.6 \( \mu\text{m} \)), they are still low enough to be considered for use in integrated optical circuits with the advantages of lower cost and a more highly developed technology.

A scan of the far-field radiation pattern was made to determine the mode structure of the guide. This revealed a single lobe characteristic of the fundamental \( \text{TM}_0 \) mode. This is the expected result since the
higher order modes have much higher losses (Figure 2.2-15).

As noted in sections 4.1 and 4.2.1, no coupling was obtained using a grating coupler because of insufficient groove depth after etching. This could be improved by (1) vacuum baking of the Si wafers before coating with photo resist, and (2) thinning the photo resist with AZ thinner.

It should also be possible to achieve coupling by using a Ge prism coupler. This technique should also give higher coupling efficiencies than the grating coupler.
REFERENCES


