RICE UNIVERSITY

Δ⁺ PRODUCTION IN THE PP→PP⁺π⁰ REACTION AT 800 MeV

by

GREGORY PAUL PEPIN

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE MASTER OF ARTS

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HOUSTON, TEXAS

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ABSTRACT

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Fifth order differential cross sections, \( d^5\sigma/d\Omega_1d\Omega_2dp_1 \), have been measured for the reaction, pp→ppπ⁺, during two experiments using the 800 MeV External Proton Beam at the Los Alamos Meson Physics Facility (LAMPF). The kinematically complete data were obtained at seven angular settings covering a range of four-momentum transfers, \(-u_p\), from 0.20 to 0.49 (GeV/c)².

The protons were detected by MWPC counters on two arms which measured the reaction angles \( (\theta_1,\phi_1,\theta_2,\phi_2) \) for both charged particles. A magnetic spectrometer measured the momentum of one of the protons.

The measured cross sections were compared to calculations using a peripheral model of pion production. Results indicate that the data can be fit with a single value of the pion form factor parameter, \( \Lambda_\pi \), whose value is much larger than that predicted using optical model calculations.
ACKNOWLEDGMENTS

In any major undertaking, it is impossible for one person to do all of the work without the advice and assistance of others. I would like to express my appreciation to several members of the Bonner Labs for their help, especially Gordon Mutchler for his help in interpreting the data and for keeping me working when I felt lazy. Also, I would like to thank David Judd for his help with the efficiency calculations, John Clement for his assistance with the data analysis and for his Monte Carlo program, Ian Duck and Miro Furić for their work on the theory, and Helen Viereck for her work in typing the rough draft of the thesis.

Finally, I would like to express my appreciation to my parents for their financial and emotional support throughout the years of my education.
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CHAPTER 1

INTRODUCTION

The pion is one of the most important of the elementary hadrons because of its role as the mediator of the long range portion of the nuclear force in the meson theory of strong interactions (Yul). Any hope of fully understanding nuclear forces requires complete knowledge of the pion and its interactions with matter. Consequently, the pion has been one of the most extensively studied of the elementary particles since its discovery in 1947. Pion production mechanisms have been studied in nucleon-nucleon and nucleon-nucleus interactions at a wide range of energies above the pion production threshold. In addition, the advent of pion factories has enabled the use of the pion as a probe of nuclear structure in pion-nucleon and pion-nucleus interactions. This thesis will focus on pion production in nucleon-nucleon interactions at intermediate energies.

In the intermediate energy range from about 150 MeV to 1 GeV incident energy, the primary inelastic channel for nucleon-nucleon interactions is single pion production. In this energy range, pion production is dominated by the \( \Delta(1232) \) resonance, a broad pion-nucleon resonance with total spin and isospin quantum numbers of

\[ J = 3/2, \quad T = 3/2. \]
The various single pion production channels in nucleon-nucleon collisions are

\begin{align*}
pp&\rightarrow d\pi^+ \quad (1-1) \\
&\rightarrow pp\pi^0 \quad (1-2) \\
&\rightarrow pn\pi^+ \quad (1-3) \\
np&\rightarrow d\pi^0 \quad (1-4) \\
&\rightarrow pp\pi^- \quad (1-5) \\
&\rightarrow pn\pi^0 \quad (1-6) \\
nn&\rightarrow d\pi^- \quad (1-8) \\
&\rightarrow pn\pi^- \quad (1-9) \\
&\rightarrow nn\pi^0 \quad (1-10)
\end{align*}

Studies of the reactions 1-5 to 1-10 have been discouraged by the problems inherent in producing neutron targets and in detecting uncharged particles. The most extensive experimentation has been undertaken with reactions 1-1 to 1-5 (Bal, Col). Most of the experiments that have been performed have been inclusive counter experiments (Gli, Bil) or bubble and spark chamber experiments (Bul, Gul, Mol, Rol). Good data concerning the \( pp\rightarrow d\pi^+ \) (and the inverse reaction \( \pi^+d+pp \)) (Fel, Ril, Hel, Lo1) have been obtained at large momentum transfers using the inclusive counter techniques. However, for the three body final states, these experiments lack sufficient information to fully determine the reaction kinematics. Therefore, background due to competing reactions cannot be
removed, resulting in insufficient information to decipher details of
the reaction mechanisms. The bubble chamber and spark chamber experi-
ments do allow complete determination of the reaction kinematics, but
they typically suffer from poor statistics.

Recently, kinematically complete studies of pp+prn (Ril, Hul) have been performed with high statistical accuracy using two arm coinci-
dence techniques. In these experiments, the two charged final state
particles were measured in coincidence by detectors arranged on two
arms. The laboratory angles, θ and φ, for both particles and the
momentum of the proton were measured. Using time of flight measurements
and reaction kinematics, the competing reactions were removed. The
experiments, therefore, determined the contribution to the cross section
due to only one pion production channel with greater accuracy than was
previously available. The data taken in these experiments were used as
a test of peripheral model calculations similar to those of Brack, Riska,
and Weise (Hul, Brl). While the theory agreed well with the data, one
experiment is an insufficient test of the validity and limitations of
the theory.

The purpose of studying the pp+ppπ° reaction described in this
thesis was to provide another test of pion production models, especially
the peripheral model, in regions of phase space not probed in the
previous works. The data described here were obtained during two experi-
ments, E-81 and E-33/336, performed on the 800 MeV External Proton Beam
at the Los Alamos Meson Physics Facility (LAMPF). The fifth-order dif-
ferential cross section, d^5σ/dΩ_1 dΩ_2 dP_1, was extracted and compared with
peripheral model calculations of reference (Hul), modified for the
The pp→pp\pi^0 reaction. It was found that only a constrained set of parameters would give good agreement between the theory and the data. The best fit to the data occurred when \( A_\pi \), the pion form factor parameter, was 875 MeV/c. This value conflicts with the lower value, about 350 MeV/c (Gol), calculated using optical models, but agrees with the larger values of \( A_\pi \), 800 MeV/c-1000 MeV/c (Hul, Brl), calculated using peripheral models.

The thesis is divided into four chapters. In the following chapter the techniques used in both of the experiments are discussed and the differences between the two are outlined. In chapter 3, the analysis techniques are described in detail, followed by a discussion of the cross section and error calculations in chapter 4. The thesis concludes with the presentation of the data and discussion of the theory in chapter 5.
CHAPTER 2

EXPERIMENTAL TECHNIQUE

The data were obtained during two similar experiments performed at LAMPF: E-81 and E-33/336. Figures 2-1 and 2-2 show the arrangement of the experimental apparatus for each experiment.

Both experiments were performed on LAMPF's 800 MeV External Proton Beam. The beam (B) scattered off a liquid hydrogen target contained in a thin-walled kapton flask. The scattered charged particles were detected in coincidence by detectors arranged in two arms: a spectrometer arm (Arm 1) and a time of flight arm (Arm 2). The arms held six position-sensitive multiwire proportional counters (P1-P6) and four scintillation counters (S1-S4). In addition the spectrometer arm contained a picture frame bending magnet which, along with the MWPCs, measured the momentum of the particle detected in that arm. The beam was monitored by an ionization chamber (RION), a Profile Monitor (PM), and a monitor telescope (M1-M4).

The data was collected using a DEC PDP 11/45 computer which wrote all data on magnetic tape while performing on-line analysis of a portion of the data as a check of the data quality.

Further details about the equipment are discussed in the next three sections and in reference (Fel).
Figure 2-1. Experimental Configuration for E-81.

P1-P6 are the multiwire proportional counters.
S1-S4 are the scintillation counters.
M1-M4 are the components of the monitor telescope.
RION is the ionization chamber used to monitor the beam intensity.
PM is the profile monitor.
$^{18}$D $^{40}$ is the spectrometer magnet.
The arm containing the magnet is the spectrometer arm (arm 1).
The straight arm is the time of flight arm (arm 2).
LH$_2$ is the liquid hydrogen target.
Figure 2-2. Experimental Configuration for E-33/336.

See Figure 2-1 for the explanation of terms. Components of the two experiments are similar.
2.1 The Detectors

The six MWPCs were large solid angle, position sensitive multiwire counters. Each consisted of two planes of thin wires spaced 2 mm apart. The planes were oriented perpendicular to one another to form an x-y grid of wires which could measure position to within 1 mm (Bu2). These detectors determined the particle trajectories through each arm, thus measuring the reaction angles ($\theta_1$, $\theta_2$, $\phi_1$, and $\phi_2$) and the bend angle of the particle through the magnet.

The scintillation counters consisted of plastic scintillators optically coupled to RCA 8575 photomultiplier tubes using adiabatic plexiglas light guides. The scintillators had two major functions: to provide a fast trigger, thus improving data quality by requiring stricter time coincidences than were possible using MWPC electronics alone; and to measure the time of flight of the particle through each arm. In addition, several scalar quantities associated with the detectors were used to measure the relative beam intensity and to check detector operation.

2.2 The Spectrometer Magnet

The spectrometer magnet was a picture frame bending magnet in a "C" configuration with a pole face gap six inches high and eighteen inches wide. The depth of the magnet was thirty-six inches. The momentum resolution of the spectrometer was approximately 2%. Maps of the magnetic field indicated that the field was sufficiently uniform to allow the use of the uniform field model in calculating the momentum (Sel).
The momentum acceptance of the magnet was calculated using a Monte Carlo technique, discussed in the next chapter. The full width at half-maximum acceptance was found to be from 75% to 145% of the central momentum.

2.3 Beam Monitors

Three detectors provided the principal information about the beam intensity and position: the profile monitor, the monitor telescope, and the ionization chamber.

RION was the primary source of information about the beam intensity. It consisted of two thin, aluminized mylar foils separated by approximately 2 cm of Argon gas at atmospheric pressure. The gain of RION was calculated from the equation (Fe1)

\[
GAIN = \left( CF \cdot \frac{tA}{2.24 \times 10^4} \cdot \frac{P}{760} \cdot \frac{273.3}{T} \cdot \frac{dE}{dX}_{AR} \right) / W_{AR},
\]

where \( t \) was the distance between the foils,

\( A \) was the atomic weight of Argon (39.948),

\( P \) and \( T \) were the operating temperature and pressure respectively in mm Hg and °K,

\( \frac{dE}{dX}_{AR} \) was the energy loss per path length of 800 MeV protons in Argon (1.697 × 10^6 eV/gm/cm^2),

\( W_{AR} \) was the average energy per ion pair (26.4 eV),

\( CF \) was a calibration factor.

C.F. was found to be 1.00±.05 for E-81 using comparisons to another
calibrated ionization chamber (LION) (Bu2). It was found to be $0.95 \pm 0.05$ for E-33/336 using comparisons of elastic cross section measurements taken during this experiment with those taken by Willard et al. (Wil). The gain of RION was 160±8 for E-81 and 150±7 for E-33/336. This difference in gain was due to the differences in distances between the foils (Table 2.1), and in the operating pressure ($P_o = \text{ATM} + 2 \text{ lb.}$ for E-81, $P_o = \text{ATM}$ for E-33/336). The gain was fairly insensitive to changes in temperature and pressure throughout the experiments.

The profile monitor was an 8.1 cm by 8.1 cm multiwire counter that was used to monitor the beam position and shape. The x- and y-outputs of the profile monitor were integrated and displayed on an oscilloscope with a .25 cm spatial resolution.

A monitor telescope consisting of the four scintillation counters, M1-M4, was used as a relative beam intensity monitor. In E-81, the four counters were arranged in two horizontal arms at 40° with respect to the beam, viewing a 7.62 cm x 7.65 cm x .5 cm CH$_2$ target housed in a helium-filled chamber. In E-33/336, the arms were oriented vertically at 40° above and below the beam line, viewing the LH$_2$ target. The fourfold coincidence signals were scaled and used to monitor changes in the beam intensity. In addition, these signals were gated by an electronic busy signal and used to determine the live time of the system.

While the experiments were similar, physical and electronic differences between the two did exist. Physical differences consisted of target, MWPC, and scintillation counter dimensions; parameters describing the position and orientation of the spectrometer magnet; and distances between the detectors on each arm. These differences can be most easily
seen using the comparisons in Tables 2.1 to 2.3. The differences in electronic logic, due mainly to improvements in technology during the three years between the experiments, are discussed in the following section.

2.4 Electronic Logic

Figures 2-3 and 2-4 show the electronic logic used in E-81 and E-33/336 respectively. The description of each figure is given on the caption page accompanying each diagram. The major differences between experiments occurred in the logic that generated the strobe signal to the MWPCs and in the logic that determined the time of flight.

The differences in strobe logic were due to improvements in the MWPC electronics. For E-81, the electronics that amplified the signal from each wire produced a pulse 500 ns wide. The strobe logic produced a pulse 200 ns wide. In order for this strobe pulse to be in time with the amplified pulse from the MWPC, it was necessary for the strobe electronics to be located in the experimental area as close to the MWPCs as possible to avoid delays in long cable runs. The combination of the 200 ns strobe pulse and the 500 ns MWPC pulse resulted in a total time resolution of 700 ns (Bu2). If more than one event was recorded by a MWPC during the resolving time, a multiple readout occurred, lowering the detector efficiency. For E-33/336, the MWPC amplifier electronics produced pulses 20 ns wide, which were delayed by 600 ns to allow for delay of the strobe signal in the cables. Because of the delay of the MWPC signal, the strobe logic could be located in the experimental
### Table 2.1. Target and Detector Sizes

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<tr>
<td><strong>Target (cylinder)</strong></td>
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<tr>
<td>Length</td>
<td>6.4 cm</td>
<td>6.9 cm</td>
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<tr>
<td>Radius</td>
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<tr>
<td><strong>MWPCs</strong></td>
<td></td>
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<tr>
<td>P1</td>
<td>12.2</td>
<td>12.2</td>
</tr>
<tr>
<td>P2</td>
<td>28.4</td>
<td>28.4</td>
</tr>
<tr>
<td>P3</td>
<td>12.2</td>
<td>7.6</td>
</tr>
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<td>P4</td>
<td>20.3</td>
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<td>P5</td>
<td>77.2</td>
<td>28.4</td>
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<td>P6</td>
<td>77.2</td>
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<td><strong>Scintillators</strong></td>
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<tr>
<td>S1</td>
<td>10.2</td>
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<td>S2</td>
<td>35.6</td>
<td>35.6</td>
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<tr>
<td>S3</td>
<td>10.2</td>
<td>12.7</td>
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<tr>
<td>S4</td>
<td>76.2</td>
<td>35.6</td>
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<tr>
<td><strong>RION</strong></td>
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<tr>
<td>t (distance between foils)</td>
<td>1.905 cm</td>
<td>1.938 cm</td>
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<td><strong>Gas Mixtures</strong></td>
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<td>MWPCs</td>
<td>Argon-30%</td>
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<td>Dimethylpropane-70%</td>
<td>Freon-.5%</td>
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Table 2.2. Magnet Parameters

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</tr>
<tr>
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<tr>
<td>Height</td>
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</tr>
<tr>
<td>$\theta_{in}$</td>
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</tr>
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<td>Effective Length</td>
<td>113.3 cm</td>
<td>113.3 cm</td>
</tr>
<tr>
<td>Effective Gap</td>
<td>10.2 cm</td>
<td>10.2 cm</td>
</tr>
<tr>
<td>Magctr—Beam</td>
<td>0.0 cm</td>
<td>11.7 cm</td>
</tr>
<tr>
<td>Component Pair</td>
<td>E-81 cm</td>
<td>E-33/336 cm</td>
</tr>
<tr>
<td>------------------</td>
<td>---------</td>
<td>-------------</td>
</tr>
<tr>
<td>TGT-S1</td>
<td>72.7</td>
<td>125.8</td>
</tr>
<tr>
<td>S1-P1</td>
<td>29.5</td>
<td>16.8</td>
</tr>
<tr>
<td>P1-P2</td>
<td>177.2</td>
<td>234.0</td>
</tr>
<tr>
<td>P2-S2</td>
<td>13.0</td>
<td>18.6</td>
</tr>
<tr>
<td>TGT-S3</td>
<td>64.1</td>
<td>98.8</td>
</tr>
<tr>
<td>S3-P3</td>
<td>25.4</td>
<td>16.2</td>
</tr>
<tr>
<td>P3-P4</td>
<td>103.9</td>
<td>81.1</td>
</tr>
<tr>
<td>P4-MAGCTR</td>
<td>112.5</td>
<td>115.7</td>
</tr>
<tr>
<td>MAGCTR-P5</td>
<td>114.6</td>
<td>106.0</td>
</tr>
<tr>
<td>P5-P6</td>
<td>101.6</td>
<td>87.3</td>
</tr>
<tr>
<td>P6-S4</td>
<td>13.6</td>
<td>20.5</td>
</tr>
</tbody>
</table>
trailer, eliminating the need for electronics in two places, and the resultant problems with system timing. The strobe logic produced a gate signal no more than 100 ns wide that encompassed the amplifier signal. The time resolution was improved to at least 120 ns (Bu3) decreasing the probability of multiple readouts and increasing the allowable event rates.

The differences in time of flight logic pertained to which information was retained for later analysis. For E-81, a coincidence between S3 and Strobe provided the start to a time to digital converter (TDC). A signal from either of the two phototubes from S2 provided one stop signal while a coincidence between two of the four phototubes in S4 provided the other stop signal. The final times of flight were thus determined in the electronics. TOFM (time of flight-magnet arm) was the time between the S4 stop signal and the start signal. TOFT (time of flight-TOF arm) was the time between the S2 stop signal and the start signal.

For E-33/336, signals from all of the phototubes in S1, S2, and S4 provided separate TDC stop signals while a coincidence between S3 and STROBE provided the TDC start signal. Each raw time of flight, the time between the TDC start and each stop signal, was saved on magnetic tape. The final times of flight were calculated by the analysis program discussed in chapter 3.
Figure 2-3. Electronic Logic Diagrams for E-81.

The following is the list of abbreviations and terminology used for all of the logic diagrams of figure 2-3:

FI(FO)—LRS 428 linear fan in/fan out
OR—LRS 429 logic fan in/fan out
AND—LRS 365 coincidence
DISC—LRS 621 discriminator
20F4—EGG C144 majority coincidence
GATE GEN—LRS 222 gate and delay generator
LEV CONV—LRS 688 level adapter
TDC—LRS 2226 time to digital converter
MWPC CONTROL—Rice MWPC control module
CURR. INT.—Brookhaven Instr. Corp. Model 1000 current integrator

All quantities in parentheses are scalars.
Figure 2-3a. Logic Used to Strobe the MWPCs. (E-81)

A 3-fold coincidence among scintillators $S_2$, $S_3$, and $S_4$ produced a signal that enabled the MWPCs to detect an event. An electronic busy signal inhibited the production of a strobe signal during the electronic dead time of the system. These electronics were located in the experimental area.
TAG- The TAG signal was a stronger coincidence requirement than a Strobe since S1 was required in coincidence with S2, S3, and S4. Only if a TAG occurred could an event be recorded.

Time of Flight- The TDC start signal came from a coincidence between S3 and Strobe, while S2 and S4 provided separate stop signals.

These electronics were located in the data acquisition-trailer.
M1-M4—The four-fold coincidence, M1-M4, was scaled and used as a relative beam intensity monitor. The monitor signal was also gated by a system busy signal to determine the dead time. The free and gated signals from M1-M2 were delayed by 30 ns. and used in coincidence with the M3-M4 signals to determine monitor accidental.

RION—The current output from RION was integrated and scaled to measure the absolute beam intensity.
Figure 2-4. Electronic Logic Diagrams for E-33/336.

The following is the list of abbreviations and terminology used for all of the logic diagrams of figure 2-4:

FO—LRS 428A linear fan in/fan out
OR/AND—LRS 622 Quad coincidence
DISC—LRS 521BL Quad discriminator
GATE GEN—LRS 222 dual gate generator
TDC—LRS 2248 Time to digital converter
LEV CONV—LRS 688 level adapter
MWPC CONTROL—Rice MWPC control module
CI—Ortec Model 439 current integrator

All of the electronics for E-33/336 were located in the data acquisition trailer.
Figure 2-4a. Logic Used to Strobe the MWPCs. (E-33/336)

This figure shows the strobe logic for E-33/336.
It is similar to Figure 2-3a.
Figure 2-4b. Logic Used to Produce the TAG Signal and Determine the Time of Flight. (E-33/336)

TAG–The TAG logic for E-33/336 is similar to that for E-81.

Time of Flight–The TDC start signal came from a coincidence of S3 and Strobe. Each phototube in S1, S2, and S4 provided separate stop signals.
DS2A
FO
TDC STOP 1

DS2B
FO
TDC STOP 2

DS3
FO
TDC START

DS4A
FO
TDC STOP 3

DS4B
FO
TDC STOP 4

DS4C
FO
TDC STOP 5

DS4D
FO
TDC STOP 6

MWPC CONTROL
TAG D IN
STROBE IN
STROBE
STROBE
BUSY
BUSY OUT
BUSY
BUSY
CLEAR OUT
CLEAR OUT
CLEAR OUT
CLEAR OUT

MWPC CONTROL
TAG D IN
STROBE IN
STROBE
STROBE
BUSY
BUSY OUT
BUSY
BUSY
CLEAR OUT
CLEAR OUT
CLEAR OUT
CLEAR OUT

TDC CLEAR
LEV
LEV
CONV
CONV
CONV
CONV

TDC CLEAR
LEV
LEV
CONV
CONV
CONV
CONV

TDC CLEAR
LEV
LEV
CONV
CONV
CONV
CONV

TDC CLEAR
LEV
LEV
CONV
CONV
CONV
CONV
The beam monitor logic for E-33/336 was the same as for E-31.
3.1 Introduction

Data from the two experiments were analyzed for three reactions at nine angular settings:

\[
\begin{aligned}
pp\rightarrow pp & \quad (E-33/336) \\
pp\rightarrow d\pi^+ & \quad (E-81) \\
pp\rightarrow pp\pi^0 & \quad (E-81, E-33/336)
\end{aligned}
\]

The two body reactions were analyzed to test the beam normalization and momentum calibration. Table 3.1 lists the data by experiment, reaction, and angular setting. In addition, two final columns contain the magnetic field and central momentum of the spectrometer magnet. Most of the three body data from the two experiments required two settings of the magnetic field to completely cover the kinematic range of the \(pp\rightarrow pp\pi^0\) reaction.

The analysis proceeded in four steps. First, the events were separated into two classes: good and bad events, depending on whether or not trajectories could be unambiguously determined. The detector system efficiency was calculated as a ratio of good events to total events after which bad events were withheld from further analysis. Second, several geometric quantities were calculated for the good events and used to remove accidental events and events in which multiple
Table 3.1. Analyzed Data

<table>
<thead>
<tr>
<th>Reaction</th>
<th>Experiment</th>
<th>$\theta_1$</th>
<th>$\theta_2$</th>
<th>$+u_p (\text{GeV}/c)^2$</th>
<th>B (KG)</th>
<th>$*P_c$ (MeV/c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>pp$^+d\pi^+$</td>
<td>81</td>
<td>14°</td>
<td>25°</td>
<td>----</td>
<td>15.6</td>
<td>1020.</td>
</tr>
<tr>
<td>pp$^+pp$</td>
<td>33/336</td>
<td>45°</td>
<td>35°</td>
<td>----</td>
<td>7.9</td>
<td>650.</td>
</tr>
<tr>
<td>pp$^+pp\pi^+$</td>
<td>81</td>
<td>14°</td>
<td>25°</td>
<td>.21, .49</td>
<td>7.6</td>
<td>500.</td>
</tr>
<tr>
<td>pp$^+pp\pi^+$</td>
<td>81</td>
<td>14°</td>
<td>25°</td>
<td>15.3</td>
<td>1000.</td>
<td></td>
</tr>
<tr>
<td>pp$^+pp\pi^+$</td>
<td>14°</td>
<td>30°</td>
<td>7.9</td>
<td>520.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>pp$^+pp\pi^+$</td>
<td>14°</td>
<td>30°</td>
<td>15.3</td>
<td>1000.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>pp$^+pp\pi^+$</td>
<td>15°</td>
<td>41°</td>
<td>8.6</td>
<td>560.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>pp$^+pp\pi^+$</td>
<td>15°</td>
<td>41°</td>
<td>15.5</td>
<td>1020.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>pp$^+pp\pi^+$</td>
<td>15°</td>
<td>21°</td>
<td>.23, .34</td>
<td>9.2</td>
<td>600.</td>
<td></td>
</tr>
<tr>
<td>pp$^+pp\pi^+$</td>
<td>20°</td>
<td>22°</td>
<td>.33, .37</td>
<td>9.2</td>
<td>600.</td>
<td></td>
</tr>
<tr>
<td>pp$^+pp\pi^+$</td>
<td>33/336</td>
<td>13.05°</td>
<td>20.8°</td>
<td>.20, .36</td>
<td>5.0</td>
<td>410.</td>
</tr>
<tr>
<td>pp$^+pp\pi^+$</td>
<td>13.05°</td>
<td>20.8°</td>
<td>9.0</td>
<td>740.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>pp$^+pp\pi^+$</td>
<td>19.05°</td>
<td>22°</td>
<td>5.0</td>
<td>410.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>pp$^+pp\pi^+$</td>
<td>19.05°</td>
<td>22°</td>
<td>.31, .40</td>
<td>9.0</td>
<td>740.</td>
<td></td>
</tr>
</tbody>
</table>

$+u_p$ is the square of the momentum transfer to the proton in the intermediate state: pp$^+p\Delta^+$. The lower values of $u_p$ indicate a $\Delta^+$ produced at more forward angles in the center of mass. Two values of $u_p$ are indicated for those settings in which the kinematics allow a $\Delta^+$ between either of the final state protons and the $\pi^0$. The first value listed corresponds to the $\Delta^+$ with the larger phase space.

*$P_c$ is the central momentum of the magnet, given by the equation

$$P_c = \frac{.3001 \times B \times \text{EFFL}}{\sin \theta_{in} + \sin \theta_{out}}$$

where EFFL is the effective length of the magnet in cm.,
B is the central magnetic field in KG,
and $\theta_{in}$ and $\theta_{out}$ are the angles with respect to the magnet pole faces with which the particle enters and exits the spectrometer magnet.
scattering occurred. Third, time of flight differences were used to separate out events from competing reactions. Fourth, the system solid angle was calculated using the Monte Carlo techniques.

The following sections describe in detail the analysis techniques. In general, data manipulation was identical for all data. Differences between experiments are pointed out as they occur.

3.2 Detector Efficiency

Not all events that were recorded could be analyzed due to detector malfunctions that resulted in insufficient or ambiguous information about the event. Since previous measurements (Fel, Hol, Wi2) have determined that the scintillation detectors were 100% efficient, all data that was lost by the detectors was assumed to be due to MWPC failures. The MWPC efficiency was a function of gas mixture, pressure, and operating voltage, all of which varied; therefore, the MWPC efficiency was calculated on a run by run basis.

Two types of MWPC inefficiency caused loss of data: failure of a coordinate to readout, called a zero readout (ZRO) resulted in insufficient position information for analysis of an event, while more than one readout in a coordinate, called a multiple readout (MRO) resulted in ambiguous position information.

Since the MWPC efficiency was dependent on the type of particle being detected, it was desirable to remove background events from the efficiency calculation. In theory, the time of flight and pulse height information could be used to remove background events. In practice, however, the time of flight resolution in the TOF arm was insufficient
to separate proton events from pion events. Previous analysis (Jul) determined that a particle from one reaction could mask a particle from another reaction if they had similar times of flight and pulse heights. In order to calculate the efficiency correctly, it was important to use all $ppp^0$ events in the efficiency calculation. Therefore, cuts could only be placed at smaller pulse heights and larger times of flight than those for the desired reaction in order to avoid biasing the efficiency calculation.

Only events satisfying the time of flight and pulse height restrictions were used in the efficiency calculation. Because zero and multiple readout efficiencies were independent of one another, the total data efficiency was the product of the two.

The zero efficiency (ZEF) was a measure of the probability that each MWPC coordinate had at least one readout. Zero events were due to insufficient gain, electronic failures, and the nonuniformity of the electric field near the edges of the MWPC (Fel). Since the inefficiency near the edges would give a deceptively low estimate of the zero efficiency, the events used in the calculations were required to pass through the center of the MWPC. This was accomplished by placing cuts around the center of the event distribution in each MWPC coordinate and requiring that the zero efficiency for the coordinate be calculated only if an event occurred between the cuts in the adjacent MWPC. MWPCs 1 and 2, 3 and 4, 5 and 6 were the pairs of adjacent detectors (Jul).

The ZEF for each coordinate was then calculated:
\[(ZEF)_i = \frac{N_i}{(N_i + N^Z_i)}, \quad (3-1)\]

where \(N_i\) was the number of events in the \(i^{th}\) coordinate which had at least one readout,

\(N^Z_i\) was the number of events in the \(i^{th}\) coordinate which failed to have at least one readout.

The zero efficiency of the system was the product of the individual zero efficiencies:

\[ZEF = \prod_{i=1}^{12} (ZEF)_i. \quad (3-2)\]

Multiple readouts were events in which at least one coordinate detected more than one readout within the resolving time of the system. Electronic malfunctions, accidental events, and true two-particle events caused the MROs. Due to the multiparticle events, the multiple readout efficiency (MREF) was calculated for the system as a whole and not coordinate by coordinate. Each coordinate was tested for more than one readout. If a MRO and no zero event occurred in any coordinate, the event was called a multiple event. If each coordinate had one and only one readout, the event was called "good," \(G\), and passed on for further analysis. The MREF was calculated as the ratio of good events to total events:

\[MREF = \frac{N^G}{(N^G + N^M)}, \quad (3-3)\]

where \(N^G\) was the number of good events and \(N^M\) was the number of multiple events.
3.2.1 Analysis of 13 Readout Events

Due to poor statistics and low MREF in some of the data, it was desirable to attempt to retrieve some of the multiple events. This could be accomplished if the particle trajectories for the multiple events could be uniquely determined.

If an event had one extra readout in only one coordinate, the trajectory calculation, discussed in the next section, could remove the spurious readout. The trajectory of the 13 readout event was calculated twice. First, one readout from the multiple event coordinate was combined with the eleven other coordinates to calculate a trajectory. Then the second of the two readouts was used to calculate the trajectory. If one of the calculated trajectories satisfied the trajectory requirements, the event was called "good" and the extra readout was ignored.

This technique was used only if there were 13 readouts and no zero events because, in all other cases, the possibility of unambiguously calculating the trajectory of the event was low. References (Hol) and (Jul) contain further discussion about the retrieval of 13 readout events.

This technique of recovering 13 readout events was used only in the analysis of the E-33/336 data. The E-81 data did not have the problems with low statistics that were evident in the E-33/336 data. Analyzing the multiple events improved the statistics by almost a factor of two. The data efficiency also increased by the same amount, leaving the cross section unchanged, within statistics.
3.3 Geometric Quantities

The particle trajectories measured using the MWPCs were used to calculate several geometric quantities which proved to be useful in removing the background due to accidental events.

The position of closest approach of the particle trajectories was calculated and histogrammed as X-TGT, Y-TGT, and Z-TGT corresponding to the x-, y-, and z-coordinates with respect to the origin of a right-hand coordinate system centered at the middle of the target. Any events not originating within the target volume were discarded using appropriate cuts. Depending on the data being analyzed, either DVTGT, the distance of closest approach (E-81), or DYTGT, the y-projection of the distance of closest approach (E-33/336), was calculated and histogrammed. DV-TGT, calculated from

\[ DV-TGT = \sqrt{(X_1 - X_2)^2 + (Y_1 - Y_2)^2 + (Z_1 - Z_2)^2} \]

was replaced by DY-TGT, calculated from DY-TGT = \( Y_1 - Y_2 \) in order to remove the geometric effects inherent in the calculations of X and Z, which reduced the accuracy of DV-TGT at small angles. Typical distributions of the target geometry are shown in Figures 3-1 and 3-2.

Three geometric quantities associated with the spectrometer magnet were also histogrammed. Typical spectra for the three quantities are shown in Figure 3-3. DX-MAG and DY-MAG were the differences between the x- and y-position of the particle at P5 and the position at P5 calculated using the uniform field model (Sel). DSY was the difference between the measured and calculated vertical slope of the particle from P5 to P6.
Figure 3-1. x-, y-, x- and z-position of the target. The vertical scale is the number of counts. The vertical dashed lines are the position cuts.
Figure 3-2. DV-TGT and DY-TGT. The vertical scale is number of counts. The vertical dashed lines are the cuts.
Figure 3-3. Spectrometer Geometry Cuts.

DX-, DY-, and DSY-MAG are the differences between the measured and calculated position and slope of the particle through the magnet. The vertical scale is the number of counts.
These three quantities were used to cut events in which scattering from the magnet pole faces occurred.

3.4 Removal of Background Reactions

Geometry cuts could only remove events with anomalous trajectories. Events from other kinematically-allowed reactions passed these cuts. The events due to the background reactions were rejected using the particle times of flight.

The only possible background reactions with appreciable cross sections at 800 MeV were the \( pp \rightarrow d \pi^+ \) and \( pp \rightarrow p \pi^+ n \) reactions. Due to the mass difference between the different particles, the velocity of the deuterons was much less than that of the protons while the velocity of the pions was greater. Therefore, over comparable flight paths, the time of flight for the deuteron was much larger than for the proton and the time of flight of the pion was much smaller than for the proton. Cuts on the time of flight measured by the scintillators could thus remove the events due to these competing reactions.

Since the electronic logic which determined the time of flight was different for the two experiments (Section 2.4), the analysis techniques were different. Because of the differences, the removal of the background reactions will be discussed separately for the two sets of data.

3.4.1 Time of Flight Analysis for E-81

Figure 3-4 contains histograms of the times of flight determined by the electronic logic. The time of flight spectra for particles in the spectrometer arm (TOFM) had two well separated peaks corresponding to protons and deuterons. The time of flight resolution in the
Figure 3-4. Time of Flight Information. E-81.

Time of flight for the magnet arm (TOFM) and TOF arm (TOFT) in nanoseconds. The vertical dashed line in TOFM is a cut used to remove deuteron events from pp→dπ⁺. The counts to the right of the proton peak in TOFT are due to the second kinematic locus of pp→ppπ°.
spectrometer arm was sufficient to easily separate the protons from the deuterons. The time of flight spectra for particles in the TOF arm (TOFT) had several badly resolved structures. The counts at low time of flight were accidental events, due to the five nanosecond microstructure of the incident beam, which were present across the spectrum. The first peak was due to pions from \( pp \rightarrow p\pi^+n \) while the second was due to protons from \( pp \rightarrow pp\pi^0 \). The tail to the right of the proton peak was due to slow protons from the low phase space kinematic locus for \( pp \rightarrow pp\pi^0 \). Often the separation of the peaks in TOFT was worse than that shown in the figure, with the proton peak completely hidden by the pion peak.

However, the kinematics offered a way to improve the separation between the time of flight peaks. The times of flight for the particles in each arm were calculated using the measured kinematic quantities and subtracted from the measured times of flight. These calculations included energy loss in the measured momentum, \( P_1 \). Since pions and protons in the TOF arm with the same momentum had different times of flight, the new quantity, called DTOFT, had more easily separated peaks (Figure 3-5).

After the time of flight cuts, the unmeasured kinematic quantities were calculated and histogrammed. These calculations also included an energy loss correction to the proton momentum measured by the spectrometer. Figure 3-6 shows the size of the energy loss correction, which was small at low momenta and negligible at momenta above about 500 MeV/c.

Even with the improvements in resolution using DTOFT, it was not always possible to remove all of the background using one set of cuts around the proton peak. In all runs, the background due to accidental
Figure 3-5. Results of DTOF Calculation for TOF Arm.

The counts to the left of the $\pi^+$ peak are the accidental events and those to the right of the proton peak are events due to the second kinematic locus. The vertical dashed lines represent the data cuts.
Figure 3-6, Energy Loss Calculation.

Calculation of the energy lost by the momentum analyzed proton in travelling from the target to the center of the spectrometer magnet.
events was present under the proton peak. In addition for some runs, DTOFT had insufficient resolution to remove all of the pions from the \( pp \rightarrow pp^+n \) reaction or all of the protons from the second kinematic locus.

The accidental events were corrected for by placing a second set of cuts, equal in width to the primary cuts, around the flat background at negative DTOFT values. These events were then analyzed as normal events and subtracted from the kinematic spectra.

A grid search curve fitting method (Bel), that assumed the DTOFT spectrum to be made up of two Gaussian curves superimposed on a flat background, was used to estimate the fraction of counts between the primary cuts that were due to pion events.

To remove this background, a set of tight cuts was placed around the pion peak in DTOFT. These events were analyzed as normal events and the kinematic quantities were histogrammed. The good event spectra were then normalized to the appropriate proton spectra using

\[
N^N_\pi = N\pi \cdot \text{PCT} \cdot \text{TEV/GEV},
\]

where \( N^N_\pi \) was the normalized background spectrum,

\( N\pi \) was the non-normalized spectrum,

PCT was the percentage of the proton spectrum due to the pion background,

TEV was the total number of events in the DTOFT spectrum,

GEV was the number of events between the primary DTOFT cuts.

The normalized pion spectrum was subtracted bin by bin from the good event spectrum.
In addition to estimating the fraction of pions in the good event spectra, the curve fitting routine also estimated the percentage of protons rejected by the primary DTOFT cuts. The data were increased by this percentage to give the final spectra. Table 3.2 lists the size of the corrections calculated by the curve-fitting procedure. Finally, the number of events for the \( i^{th} \) bin was

\[
N_i = N_{p1} - N_{A1} - N_{\pi1} + N_{i}^{\text{cut}},
\]

(3-5)

where \( N_{p1} \) was the number of good events,

\( N_{A1} \) was the number of accidental events,

\( N_{\pi1}^N \) was the normalized number of pion events,

\( N_{i}^{\text{cut}} \) was the estimated number of protons cut by DTOFT.

The final source of contamination of the data was the second kinematic locus. As can be seen from Figure 3-7, the three body kinematics allowed two values of \( P_2 \) for each value of \( P_1 \). The phase space for the two loci was very different (Fel, Nil), with the small \( P_2 \) locus having, in general, a phase space about 10% as large as the locus with larger \( P_2 \). Usually only the large \( P_2 \) locus contained a \( \Delta^+ \) in the final state with sufficient phase space to affect the differential cross section. The removal of the events due to the locus with the smaller phase space would thus decrease the background contaminating the cross section for \( \Delta^+ \) production.

Since \( P_2 \) was not measured in the experiment, the events from the lower locus could not be removed directly with momentum cuts. Since
Table 3.2. Curve Fit Correction.

<table>
<thead>
<tr>
<th>( \theta_1 )</th>
<th>( \theta_2 )</th>
<th>B (KG)</th>
<th>Percent Pions</th>
<th>Amt. Data Cut(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>14°</td>
<td>25°</td>
<td>7.6</td>
<td>0.1%</td>
<td>2.1%</td>
</tr>
<tr>
<td>14°</td>
<td>25°</td>
<td>15.3</td>
<td>4.0%</td>
<td>6.3%</td>
</tr>
<tr>
<td>14°</td>
<td>30°</td>
<td>7.9</td>
<td>0.0%</td>
<td>7.6%</td>
</tr>
<tr>
<td>14°</td>
<td>30°</td>
<td>15.3</td>
<td>4.1%</td>
<td>5.6%</td>
</tr>
<tr>
<td>15°</td>
<td>21°</td>
<td>9.2</td>
<td>0.5%</td>
<td>2.0%</td>
</tr>
<tr>
<td>15°</td>
<td>41°</td>
<td>8.6</td>
<td>0.0%</td>
<td>0.6%</td>
</tr>
<tr>
<td>15°</td>
<td>41°</td>
<td>15.5</td>
<td>0.1%</td>
<td>0.4%</td>
</tr>
<tr>
<td>20°</td>
<td>22°</td>
<td>9.2</td>
<td>0.6%</td>
<td>2.5%</td>
</tr>
</tbody>
</table>

This table indicates the size of the correction to the data introduced by the curve fitting procedure. The Percent Pions is the percentage of pions estimated by the curve fit to have passed the proton cuts in DTOFT. Amt. Data Cut is the estimate of the percentage of the data that was rejected by the primary cuts in DTOFT.
The plot of $P_2$ vs. $P_1$ is shown for an arbitrary angle setting to show the double valued nature of the kinematic locus. The analysis was unable to remove events from the lower $P_2$ locus from the data. This low $P_2$ background accounted for about 15% to 20% of the cross section in the region between $P_1=200$ MeV/c and $P_1=400$ MeV/c.
Time of flight measured for scintillators in the TOF arm. The final time of flight for a particle in the TOF arm is the difference between TOFS1 and TOFS2. Note the lack of the peak for protons from the second kinematic locus.
The first peak is due to $\pi^+$ while the second is due to protons. The $\pi^+$ peak is present only at small magnetic fields of the spectrometer.
events with different values of $P_2$ had different times of flight, TOFT and DTOFT could remove some of the unwanted events. However, near the kinematic limits, $P_2$ changed slowly enough that the resolution of TOFT and DTOFT was insufficient to separate the events from the two loci. Fortunately, this background was significant only at small values of $P_1$, far away from the peak in the cross section due to the $\Delta^+$ resonance. At small $P_1$, it was estimated that this background was between 10% and 25% of the cross section. In the region near the $\Delta^+$ peak, the background due to the second kinematic locus was found to be negligible.

3.4.2 Time of Flight Analysis for E-33/336

While the E-81 electronics determined the final times of flight, the E-33/336 electronics determined only raw times of flight for each phototube. For scintillators with more than one phototube, the time of flight was determined from the phototube which detected the event first. The times of flight measured for each scintillator using this method are shown in Figures 3-8 and 3-9.

TOFS1 was the time difference between an event at S1 and the TDC start signal from S3. The peak in the spectrum was due to both protons and pions. The counts on either side of the peak were accidentals, present across the spectrum, due to the five nanosecond microstructure present in the proton beam.

Because of a longer flight path, the pion and proton peaks in TOFS2 were distinguishable from one another. The peak at lower time of flight was due to pions while the peak at higher times of flight was due to the protons. The counts away from the two peaks were due to accidental
events. The events from the second kinematic locus, discussed previously, were not apparent in this data. It is possible that better timing eliminated these events or that they were hidden by the accidentals at higher time of flight.

TOFS4, shown in Figure 3-9, had two peaks in the data with the low magnetic field. The peak at very low time of flight was due to pions from the $pp \rightarrow \pi^+pn$ reaction. The broad peak contained protons from both the $pp \rightarrow p\pi^0$ and $pp \rightarrow p\pi^+n$ reactions. The TOFS4 spectrum at the high magnetic field did not contain the pion peak but was otherwise similar to the spectrum at the low magnetic field.

Since the final times of flight were calculated in the analysis program, corrections to offset timing, pulse height, and position differences could be made to improve separation of the peaks.

Minor timing differences between the signals from the separate phototubes in S2 or S4 resulted in smearing of the time of flight spectra. This was reduced by adding constant timing offsets to the raw times of flight until the times determined for separate phototubes in the same scintillators were about equal.

The pulse height correction offset the time of flight differences due to varying pulse heights. The long distance (approximately 125 ft.) that the analog signals were required to travel before reaching the discriminators degraded the rise time for the pulse height resulting in substantial walk. This caused signals of different height to cross the discriminator threshold at different times. Figure 3-10 outlines the nature of the problem. The correction was made using a linear approximation of the time of flight due to the pulse height.
Figure 3-10. Pulse Height Correction.

This figure shows the problem introduced by pulse height differences among separate phototubes. The signal with the lower pulse height crosses a threshold voltage at a later time than the signal with the larger pulse height. Two phototubes with different pulse height that detected a particle at the same time would thus indicate different times of flight for the event. This is corrected for using the equation

\[ \text{TOF} = \text{TOF} + 0.04 \times \text{pulse height}. \]
TCOR = .04 • Pulse Height (3-6)

where .04 was an adjustable parameter describing the slope of the pulse height. This time was added to the time measured for each phototube.

The position of an event in each scintillator affected the time of flight due to the varying time needed for the light to travel through the scintillator. To offset this effect, the x-position of the event in each scintillator was determined from the adjacent MWPC. The time for light to travel from the event to the center of the scintillator was calculated. The measured raw times of flight were then corrected to give the time of flight for an event at the center of the scintillator.

After the time of flight corrections were made the kinematics were used to calculate the DTOF spectra, further improving the separation of the peaks. Figure 3-11 shows the DTOFS1 spectrum, which displays the effect of the time of flight corrections on the measured time of flight. Figure 3-12 shows the DTOFT spectrum. A loose cut was placed around the proton peak in the DTOFS1 spectrum to remove the majority of the pions. The remaining pion events were rejected using tighter cuts in DTOFT. The additional information from TOFS1 thus permitted removal of almost all the pion events, eliminating the need for the curve fitting procedures required in the E-81 data. The accidental background was, however, still present across the time of flight spectrum. These events were removed using a second set of cuts in DTOFS1 around the accidental peaks at high time of flight. These events were then analyzed as normal events and subtracted from the good event spectra.
Figure 3-11. DTOFS1 Spectrum. (E-33/336)

The figure shows clearly the effect of the position and pulse height corrections on the time of flight. The solid curve includes corrections while the dashed curve does not. The small peaks at high and low DTOFS1 are accidentals. The cuts are omitted for clarity.
Figure 3-12. DTOFT Spectrum.

DTOFT is the difference between the measured and calculated values of the quantity

\[ \text{TOFT} = \text{TOFS2} - \text{TOFS1}. \]

The solid curve includes cuts on DTOFS1 while the dashed curve represents open cuts on DTOFS1. Cuts are omitted for clarity.
After all cuts were made, the kinematic quantities were calculated and histogrammed. The calculations included the energy loss correction discussed earlier. Finally, the number of events in the $i^{th}$ bin was

$$N_i = N_{pi} - N_{Ai},$$  \hspace{1cm} (3-7)

where $N_{pi}$ was the number of good events from the primary analysis, $N_{Ai}$ was the number of events due to the accidental background.

3.5 Determination of the Solid Angle

The experimental solid angle for a two-arm experiment can theoretically be found using

$$\Delta \Omega = \int \frac{d\hat{A}_1 \cdot \hat{r}_1}{r_1^2} \cdot \int \frac{d\hat{A}_2 \cdot \hat{r}_2}{r_2^2}.$$

Unfortunately, such a straightforward calculation is possible only for a simple system in which all events originate from a point and the areas and distances for detectors are well known. This experiment was complicated by several factors: energy loss and multiple scattering, the extended target, the presence of more than one detector in each arm, the spectrometer magnet, and kinematically-forbidden regions of phase space (Jul).

Energy loss was taken into account earlier in the analysis so its effect was ignored in the solid angle calculation. In addition, multiple scattering was ignored due to the small multiple scattering angles and large angular acceptance of the detectors.
While corrections for the extended target and large number of detectors could be handled using ray-tracing techniques, the spectrometer magnet and kinematic limits introduced momentum-angle correlations that could not be as easily handled. Due to the spectrometer magnet, particles with certain momenta were bent out of the system while other particles with different momenta but the same initial trajectory were accepted. The kinematic limits placed different upper and lower limits on the allowed momenta for particles with different trajectories, which needed to be taken into account in the solid angle calculation. Because these factors made an analytic calculation of the solid angle impossible, the Monte Carlo technique was used to approximate the system.

The Monte Carlo technique used for this problem is a numerical integration technique in which a physical situation is modeled by choosing random numbers in such a way that they simulate the system under study. The desired solution is then determined from the behavior of the random numbers (Hal, Nil).

For this system, the desired final solution was the curve of the system solid angle as a function of momentum. This was obtained using a computer simulation of the experiment in which all equipment was set up exactly as in the true experiment. Inputs were the beam, target, and detector dimensions, distances between equipment, and magnet pole face sizes. Any offsets due to equipment misalignment were also included. The calculation used the uniform field model to approximate bend angles for the simulated events (Jul).

The specific steps that were used in the calculation follow:
1) Initial solid angle and momentum limits were chosen within which all events occurred. These limits were large enough to encompass the physical limits of the experiment to avoid any bias in the result.

2) An 800 MeV incident proton beam was chosen. The beam was taken to be a cylinder of radius 0.1 inches.

3) A random interaction point was chosen within the beam-target overlap region. From this point, random interaction angles $\theta_1$, $\phi_1$, $\theta_2$, $\phi_2$ were chosen along with a random momentum $P_1$. The random trajectories and momentum were within the limits of step 1.

4) If the event was kinematically-allowed, the number of events tried ($N_T$) was incremented. If it was not allowed, a new event was chosen (step 2).

5) The spectrometer analyzed proton was tested to see if it missed any of the magnet arm detectors. If it did, a new event was chosen (step 2).

6) If the event passed the magnet arm limitations, the other proton was sent through the time of flight arm. If the proton missed any of the TOF arm detectors, the event failed and a new event was chosen (step 2).

7) If the event was "seen" by all detectors, it was a successful event and the number of events passed ($N_P$) was incremented. The momentum of each successful event was histogrammed in bins of the same size as the data bins. The final momentum distributions represented the experimental solid angle as a function of momentum. A representative graph of the solid angle as a function of momentum
is shown in Figure 3-13.

8) The process was repeated until enough successful events were accumulated that the statistical error of the Monte Carlo calculation was much smaller than the statistical errors in the data.

The solid angle as a function of momentum was proportional to the number of events tried per bin:

\[
\Delta \Omega_j(P_i) = \Delta \Omega_1 \Delta \Omega_2 \cdot \frac{N_i^P}{N_B^T},
\]

where \(\Delta \Omega_j(P_i)\) was the solid angle for the \(j^{th}\) momentum bin,

\(\Delta \Omega_1\) and \(\Delta \Omega_2\) were the initial solid angle ranges for the spectrometer and time of flight arms, respectively,

\(N_i^P\) was the number of successful events in the \(j^{th}\) bin,

\(N_B^T\) was the number of events tried per bin. Since the events were tried uniformly over the allowed momentum range, this was just the number of events tried divided by the total number of momentum bins.

For elastic scattering, all of the data occurred in very few momentum bins. Since the solid angle was limited by the spectrometer arm, the experimental solid angle was given by

\[
\Delta \Omega = \Delta \Omega_1 \cdot \frac{N^P}{N_B^T},
\]

where \(N^P\) was the total number of events passed,

\(N_B^T\) was the number of events tried per bin,

\(\Delta \Omega_1\) was the initial spectrometer arm solid angle range.
Result of a Monte Carlo calculation of the solid angle for a typical angle setting. The number of counts was inserted into equation 3-8, as $N_0$, to get the actual solid angle as a function of $P_1$. 

Figure 3-13. Momentum Dependence of the Solid Angle.
CHAPTER 4

CROSS SECTION CALCULATIONS

4.1 Two Body Cross Sections

The differential cross section \( \frac{d\sigma}{d\Omega} \) was calculated as a function of \( \theta_{\text{CM}} \), the center of mass spectrometer angle, for all data with two body final states. In the center of mass, the cross section was (Jul):

\[
\left( \frac{d\sigma}{d\Omega} \right)_{\text{CM}} = \frac{N^A}{N^T \cdot N^B \cdot \Delta\Omega^L} \cdot J(\Omega^L, \Omega^\text{CM}), \tag{4-1}
\]

where \( N^A \) was the adjusted number of events, the number of events passing all analysis cuts corrected for system efficiency and live time,

\( N^T \) was the number of target protons,

\( N^B \) was the number of beam protons incident on the target,

\( \Delta\Omega^L \) was the spectrometer arm solid angle in the lab frame, and

\( J(\Omega^L, \Omega^\text{CM}) \) was the Jacobian that transformed the solid angle from the lab to the CM frame.

\( N^A \) was calculated from the number of events passing cuts, the total data efficiency, and the live time.

\[
N^A = \sum_{i=1}^{R} \frac{N_i}{\text{DEF}_i \cdot \text{LT}_i}. \tag{4-2}
\]
DEF is discussed in section 3.2. The live time was determined by comparing the gated and ungated number of two arm coincidences in the monitor telescope. In addition, the live time for E-81 included a correction for strobe pileups:

\[
[E-81]LT = \frac{\text{MONG-MACG}}{\text{MONF-MACF}} \frac{\text{STROBE}}{\text{STROBE + PU}}, \quad (4-3a)
\]

\[
[E-33/336]LT = \frac{\text{MONG-MACG}}{\text{MONF-MACF}}. \quad (4-3b)
\]

MONG and MONF were the gated and ungated number of true two-arm coincidences while MACG and MACF were corrections for accidental coincidences.

The number of target protons per cm\(^2\) was calculated using

\[
N_T = \frac{1 \cdot \rho \cdot N_A}{A}, \quad (4-4)
\]

where \(l\) was the target length (Table 2.1),
\(\rho\) was the density of LH\(_2\) (.0708 gm/cm\(^3\)),
\(N_A\) was Avogadro's number (6.022 \times 10\(^{23}\)),
\(A\) was the atomic weight of hydrogen (1.008 gm/mole).

\(N^B\), the number of beam protons, was determined using the ionization chamber RION

\[
N^B = \frac{\text{RION}}{\text{GAIN} \cdot 1.602 \times 10^{-19} \text{coul}}, \quad (4-5)
\]

where RION was the digitized output of the Rice ion chamber in units of 2 \times 10^{-11} coul/pulse (E-81) or 1 \times 10^{-10} coul/pulse (E-33/336). GAIN was the gain of RION, discussed in Chapter 2.
The spectrometer arm solid angle was calculated using the Monte Carlo calculation discussed in section 3.5:

\[ \Delta \Omega_1^L = \frac{N^P}{N_B^T} \cdot \Delta \Omega_1 \cdot \]  

(4-6)

The Jacobian was given by (Jul)

\[ J(\Omega_1^L, \Omega_1^{CM}) = \frac{\gamma^{CM} \cdot p^{CM} \cdot p_{1L}^L \cdot \beta^{CM} \cdot E_{1TOT} \cdot \cos \theta_{1L}^L}{(p_{1L}^L)^2}, \]  

(4-7)

where \( p^{CM} \) was the momentum of the center of mass of the two body system in the lab frame,

\( \gamma^{CM} \) was the gamma of the center of mass,

\( \beta^{CM} \) was the beta of the CM \( [\beta = (1 - \gamma^2)^{-\frac{1}{2}}] \),

\( p_{1L}^L \) was the laboratory momentum of the particle measured by the magnet,

\( E_{1TOT} \) was the total energy of the spectrometer analyzed particle, and

\( \theta_{1L}^L \) was the lab scattering angle of the spectrometer analyzed particle.

The results of the two body cross section measurements are discussed in section 5.1.
4.2 Three Body Cross Sections

For the reaction with three bodies in the final state, the differential cross section per momentum bin was calculated and graphed as a function of $P_1$. The cross section for the $j^{th}$/momentum bin was

$$\left(\frac{d^5 \sigma}{d\Omega_1 \cdot d\Omega_2 \cdot dP_1}\right)_j = \frac{N_j^A}{N_B \cdot N_T \cdot \Delta\Omega_j(P_1) \cdot \Delta P_1}, \quad (4-8)$$

where $N_j^A$ was the adjusted number of events in the $j^{th}$ momentum bin, defined earlier:

$$N_j^A = \sum_{i=1}^{R} \left( \frac{N_{ji}^P}{LT_i \cdot DEF_i} \right)$$

$N_{ji}$ is defined in section 3.4:

$$N_{ji} = N_{ji}^P - N_{ji}^A - N_{ji}^N + N_{ji}^{cut}, \quad (E-81)$$

$$N_{ji} = N_{ji}^P - N_{ji}^A, \quad (E-33/336)$$

$DEF_i$, $LT_i$, and $N_B$, and $N_T$ are defined in the previous section.

$\Delta\Omega_j(P_1)$ was the solid angle for the $j^{th}$ bin, calculated in section 3.5 as

$$\Delta\Omega_j(P_1) = \frac{N_{ji}^P}{N_B} \cdot \Delta\Omega_1 \cdot \Delta\Omega_2. \quad (4-9)$$

$\Delta P_1$ was the histogram bin size for the proton measured by the spectromter arm.
Section 5.2 discusses the results of the three body measurements.

4.3 Error Analysis

As with any statistical measurement, this experiment had associated with it uncertainties which could be divided into two classes: systematic errors and statistical errors.

4.3.1 Systematic Errors

Systematic errors were those due to limitations in the experimental technique and equipment. These included finite spatial and time of flight resolution, uncertainty in data efficiency and beam normalization, and uncertainty in the measured momentum.

Previous measurements (Fel, Hol, Jul, Ho2) have determined that the systematic error in the beam measurement using RION was about 5%. Additionally, pp elastic cross section measurements in E-33/336 indicated about a 5% drift in the gain of RION.

The efficiency measurement constituted the other major contribution to the systematic error in the absolute normalization. Varying the position and width of the efficiency cuts on the separate time of flight and pulse height histograms changed the data efficiency by as much as 15%. Using the criteria discussed in section 3.2 to limit the position of the cuts reduced this uncertainty to about 5%. This value was used for the systematic error in the data efficiency.

As discussed in section 2.2, the uncertainty in the measured momentum was 2%, approximately 15 MeV/c in the momentum range of this experiment.
Because measurements of the pp→pp and pp→dπ⁺ reactions aided in correcting errors in angle measurements and magnetic fields, the errors in the quantities were assumed to be small.

Thus, by quadratic addition, as in equation 4-10, the systematic error was about 7% in the absolute normalization and 2% in the momentum calibration.

\[
\frac{(\Delta \sigma)}{\sigma}_{\text{syst}} = \sqrt{\left(\frac{(\Delta \sigma)}{\sigma}_{\text{RION}}\right)^2 + \left(\frac{(\Delta \sigma)}{\sigma}_{\text{EFF}}\right)^2} \tag{4-10}
\]

4.3.2 Statistical Errors

The terms that contributed to the statistical errors were \(N_j\), \(N^B\), \(N^T\), DEF, LT, and \(\Delta \Omega(P)\). \(\Delta \Omega(P)\) contributed through the statistical error introduced by the Monte Carlo calculation. Because of the large (≈10%) statistical errors in some of these terms, quantities whose statistical errors were less than 0.5% were ignored in the error calculation. Terms whose errors were ignored were \(N^B\), \(N^T\), and LT.

Of the terms that had appreciable statistical errors, DEF obeyed Binomial statistics while the other terms obeyed Poisson statistics.

The statistical error was calculated using the equation (Bel)

\[
\Delta f = \left\{ \sum_i \left[ \left( \frac{3 f}{\partial x_i} \right)^2 \Delta x_i^2 \right] \right\}^{\frac{1}{2}}, \tag{4-11}
\]

where \(\Delta f\) and \(\Delta x_i\) were the errors in \(f(x_i)\) and \(x_i\) respectively.

\(\Delta x_i\) was given by the relations...
\[ \Delta x_i = (x_i \cdot P_i)^{1/2} \quad (4-12a) \]

for the Poisson distribution and

\[ \Delta x_i = (x_i \cdot P_i(1-P_i))^{1/2} \quad (4-12b) \]

for the Binomial distribution.

\( x_i \) was the total number of possible occurrences of an event and \( P_i \) was the probability of the occurrence of \( x \). The term \( x_i \cdot P_i \) could be replaced by \( N_i \), the actual number of occurrences of \( x_i \).

Using these equations, the statistical errors in the three body cross sections could be calculated. From section 3.4, the number of events for the \( j^{th} \) momentum bin was

\[ N_j = N_j^P - N_j^A - N_j^\pi + N_j^{\text{cut}} \quad [E-81] \]

\[ N_j = N_j^P - N_j^A \quad [E-33/336] \]

with the associated error

\[ \Delta N_j = (N_j^P + N_j^\pi + N_j^A)^{1/2} \quad [E-81] \]

\[ \Delta N_j = (N_j^P + N_j^A)^{1/2} \quad [E-33/336] \]

From section 3.5, the statistical error in \( \Delta \tilde{N}_j(P_i) \) was due to \( N_j^P \); therefore,
where

\[ \Delta N_j^p = \left( N_j^p \right)^{\frac{1}{2}}. \]  

(4-14)

The data efficiency was made up of two terms, the zero and multiple readout efficiencies:

\[ \text{DEF} = \text{ZEF} \times \text{MREF} \]

where, from section 3.2,

\[ ZEF_i = \frac{12}{\pi} \frac{\text{ZEF}}{i=1} \]

\[ ZEF_i = \frac{N_i}{N_i + N^z_i}, \]

and \( \text{MREF} = \frac{N^G}{N^G + N^M}. \)

The error in \( \text{MREF} \) was

\[ \Delta \text{MREF}_i = \left\{ \frac{(N_i^G)^2 (N_i^G)^2 + (N_i^M)^2 (N_i^M)^2}{(N_i^G + N_i^M)^4} \right\}^{\frac{1}{2}} \]  

(4-15)

where \( N^G = [N^G(1-P^G)]^{\frac{1}{2}} \),

\( N^M = [N^G(1-P^M)]^{\frac{1}{2}} \),

\( P^G = \text{probability of the occurrence of a good event} \),

\( P^M = \text{probability of the occurrence of a multiple event} \).
Similarly, the error in the zero efficiency for each coordinate was

\[ \Delta ZEF_{ki} = \left\{ \frac{(N^Z_{ki})^2(\Delta N^1_{ki})^2 + (N^Z_{ki})^2(\Delta N^2_{ki})^2}{(N^Z_{ki})^2(N^1_{ki} + N^2_{ki})^2} \right\}^{1/2}, \]

and for all coordinates,

\[ \Delta ZEF_i = ZEF_i \left\{ \sum_{k=1}^{12} \frac{\Delta ZEF_{ki}}{ZEF_{ki}} \right\}^{1/2}. \quad (4-16) \]

Therefore, the error in the adjusted number of events for the \( i^{th} \) run and the \( j^{th} \) momentum bin was

\[ \Delta N^A_{ji} = N^A_{ji} \left\{ \left( \frac{\Delta N^A_{ji}}{N^A_{ji}} \right)^2 + \left( \frac{\Delta ZEF_i}{ZEF_i} \right)^2 + \left( \frac{\Delta MREF_i}{MREF_i} \right)^2 \right\}^{1/2}, \]

and the error for all runs was

\[ \Delta N^A_j = \left\{ \sum_{i=1}^{R} (\Delta N^A_{ji})^2 \right\}^{1/2}. \quad (4-17) \]

The total error in the cross section was

\[ \Delta \left( \frac{d^5\sigma}{d\Omega_1 d\Omega_2 dP_1} \right)_j = \left( \frac{d^5\sigma}{d\Omega_1 d\Omega_2 dP_1} \right)_j \cdot \left\{ \left( \frac{\Delta N^A_j}{N^A_j} \right)^2 + \left( \frac{\Delta N^P_j}{N^P_j} \right)^2 \right\}^{1/2}. \quad (4-18) \]

The same equation was used for the statistical error in the two body cross section with the following changes:

- \( j=1 \) because all data came in one momentum bin
- \( N^A \) was defined in section 4.1 meaning that \( \Delta N^A_j = (N^A_j)^{1/2} \) for each run.
All other facets of the calculation were unchanged.

The magnitude of the statistical error was approximately 7% per bin for the three body cross sections for all angular settings.
CHAPTER 5

RESULTS AND DISCUSSION

5.1 Two Body Cross Sections

5.1.1 pp→pp

The pp elastic cross section measured in E-33/336 tested beam normalization, angle calibrations, and momentum calibrations.

The elastic cross sections were measured at two angles and compared to the cross sections of Willard et al. (Wil). The cross sections measured here are listed in Table 5.1, along with the results of the previous measurement. The data are also graphed as a function of $\theta_{\text{cm}}$ in Figure 5-1. Comparison of the two sets of data indicated that the absolute normalization required a value of $150 \pm 7$ for the gain of RION.

The pp elastic data also indicated that the reaction angle measured in the initial survey of the system was too large by approximately $0.95^\circ$. This error was determined by comparing the measured reaction angle of the second final state proton with the value calculated using the kinematics for the reaction. The measured and calculated values were found to agree only if $\theta_{\text{LAB}}^1$ was decreased by $0.95^\circ$. This error was verified by performing a crude resurvey which showed that the reaction angle was too large by about $1^\circ$. The error was taken into account in the analysis of the data from E-33/336.
Table 5.1. pp+pp Elastic Cross Sections

<table>
<thead>
<tr>
<th>$\theta_L$</th>
<th>$\theta_{cm}$</th>
<th>$(d\sigma/d\Omega)_{cm}$ (mb/sr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^*9.8^\circ$</td>
<td>23.3°</td>
<td>8.44±0.40</td>
</tr>
<tr>
<td>$^*15.0^\circ$</td>
<td>35.5°</td>
<td>5.76±0.26</td>
</tr>
<tr>
<td>$^*20.0^\circ$</td>
<td>47.0°</td>
<td>4.02±0.18</td>
</tr>
<tr>
<td>$^*24.9^\circ$</td>
<td>58.0°</td>
<td>2.58±0.06</td>
</tr>
<tr>
<td>$^*29.9^\circ$</td>
<td>69.0°</td>
<td>1.55±0.05</td>
</tr>
<tr>
<td>$^{**}30^\circ$</td>
<td>70°</td>
<td>1.55±0.11</td>
</tr>
<tr>
<td>$^*34.9^\circ$</td>
<td>79.6°</td>
<td>1.14±0.04</td>
</tr>
<tr>
<td>$^{**}35^\circ$</td>
<td>82°</td>
<td>1.10±0.08</td>
</tr>
<tr>
<td>$^*39.9^\circ$</td>
<td>89.9°</td>
<td>0.99±0.03</td>
</tr>
</tbody>
</table>

*Taken from reference (Wil).
**Data from this experiment.
Figure 5-1. pp +pp (elastic) Results.

The solid circles in the figure are the results of Willard, et. al. while the triangles are the results from E-33/336. The agreement using a value of 150 ±7 for the gain of RION is good, verifying the absolute beam normalization.
5.1.2 pp+dπ⁺

The pp+dπ⁺ reaction measured previously by Felder (Fel) was reanalyzed to test the techniques employed to analyze the pp+ppπ⁰ data from E-81. A center of mass differential cross section of 121 μb ± 3 μb for θ₁ = 14°, θ₂ = 25° was obtained using present techniques. Comparison of Felder's cross section of 120 μb ± 4 μb (Fel) indicated that the present results were consistent with previous work.

5.2 Three Body Cross Sections

The fifth-order differential cross sections, \( \frac{d^5 \sigma}{d \Omega_1 d \Omega_2 dp_1} \), was calculated using equation 4-7. Table 5.2 lists the results of the measurements as a function of the measured momentum, \( P_1 \), for all of the data from Table 3.1 which contained three body final states. Figure 5-2 shows the cross sections.

For each angular setting the cross section peaked at a momentum between 800 MeV/c and 900 MeV/c, corresponding to the \( \Delta^+ \) resonance. The momentum at which the kinematics predict a \( \Delta^+ \) between the second proton and the pion is indicated by an arrow in each figure. The increase in cross section at the high and low ends of the spectra were phase space effects due to tangency at the ends of the kinematic locus.

Table 5.2 indicates that, in two cases, near repetitions of angle settings occurred between experiments 81 and 33/336. The first two columns of the table compare the cross sections from \( \theta_1, \theta_2 = 20°, 22° \) (E-81) with the data from \( \theta_1, \theta_2 = 19.05°, 22° \) (E-33/336). The second two columns compare the data from \( \theta_1, \theta_2 = 15°, 21° \) (E-81) with that from \( \theta_1, \theta_2 = 13.05°, 21° \). These two near repetitions of data proved
### Table 5.2. Differential Cross Sections

<table>
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<th>P₁</th>
<th>9₁₂⁹</th>
<th>9₁₃⁸</th>
<th>9₁₄⁸²</th>
<th>9₂₃⁸²</th>
<th>9₂₄⁸²</th>
<th>9₃₄⁸²</th>
<th>9₄₅⁸²</th>
<th>9₅₆⁸²</th>
<th>9₆₇⁸²</th>
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<td>MeV/c</td>
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<td>10²-3²</td>
<td>10²-3²</td>
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<td>1.1000</td>
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<td>315.8700</td>
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</tbody>
</table>

**Notes:**
- P₁ is the energy in MeV/c.
- The values represent the differential cross sections in barns (b) per steradian (sr) at the specified energy.
This figure shows the fifth-order differential cross section for the data listed in Table 5.2. The data are graphed as a function of the measured momentum, $P_\perp$. The peak corresponds to the $\Delta^+$ resonance. The error bars in the figure are statistical only.

The arrow in each figure indicates the position of the $\Delta^+$ between the second proton and the pion.
$\theta_1 = 20^\circ, \theta_2 = 22^\circ$

E-Si

$\frac{d^2\sigma}{d\Omega_1 d\Omega_2} (\mu b/sr^2 MeV/c)$

$P_1$ (MeV/c)
$\theta_1 = 9.05^\circ$, $\theta_2 = 22^\circ$

E-33/336

$d^6\sigma/d\Omega_1 d\Omega_2 dP_1$ (µb/sr² MeV/c)

$P_1$ (MeV/c)
$\theta_1 = 15^\circ, \theta_2 = 21^\circ$

$E - 81$

$\frac{d^3\sigma}{dQ_1 dQ_2 dp_1} \ (\mu b/s^2 MeV/c)$

$P_1 (MeV/c)$
\( \theta_1 = 30.5^\circ, \theta_2 = 20.8^\circ \)

E-33/336
\[ d^2 \sigma / d\Omega dP, \mu b / sr^2 MeV/c \]

\[ \theta_1 = 14^\circ, \theta_2 = 25^\circ \]

E-81

\[ P_1 \text{ (MeV/c)} \]
useful in checking the consistency of the two measurements. The comparison is more easily seen using Figure 5-3, in which the two sets of data are compared graphically. The figures show that the two sets of measurements are consistent with one another, with differences probably due to differences in angle settings between the two experiments.

The measured cross sections have been compared to the peripheral model calculations of reference (Hul), which are discussed in the following section.

5.3 Theory

The peripheral model calculations of reference (Hul), performed originally to describe \( \Delta^{++} \) production in the \( pp'p\pi^+n \) reaction, have been modified by Furic and Duck to calculate the \( pp'pp\pi^0 \) cross sections (Ful, Dul). The calculations included the exchange terms shown in Figure 5-4.

Figures 5-4 (a) and (b) contain \( \pi \) and \( p \) exchange terms acting through the \( \Delta^+ \) resonance as an intermediate state. Because of the two identical final state particles, terms were also included in which the positions of the \( p_1 \) and \( p_2 \) lines in the diagrams were exchanged. The presence of \( \Delta^+ \) resonances between either of the final state protons and the \( \pi^0 \) had a major effect on the cross sections at some angles, shifting both the position and magnitude of the peaks (Ful).

The pion exchange term was parameterized by form factors introduced at the \( NN\pi \) and \( \pi N\Delta \) vertices:

\[
f_\pi(t) = \frac{\Lambda^2_\pi - M^2_\pi}{\Lambda^2_\pi + t} - \frac{M^2_\pi}{\Lambda^2_\pi + t}
\]
Comparisons of the cross sections for $\theta_1, \theta_2 = 15^\circ, 21^\circ$ with $\theta_1, \theta_2 = 13.05^\circ, 20.8^\circ$ and for $\theta_1, \theta_2 = 20^\circ, 22^\circ$ with $\theta_1, \theta_2 = 19.05^\circ, 22^\circ$ show slight differences between the two experiments. The data from E-81 are denoted by dots while the data from E-33/336 are denoted by triangles. The differences in Figure (a) may be attributed to the difference in $\theta_1$. In Figure (b), there is some question about the normalization of the data from E-81 ($\theta_1, \theta_2 = 15^\circ, 21^\circ$).
\( \theta_1 = 20^\circ, \theta_2 = 22^\circ \)

\( \theta_1 = 13.05^\circ, \theta_2 = 22^\circ \)
where $f_\pi(t)$ was the pion form factor,

- $M_\pi$ was the mass of the pion,

- $\Lambda_\pi$ was an adjustable form factor parameter, and

- $-t$ was the square of the four-momentum transferred by the exchanged pion (Hul).

The experiment measured the pion-nucleon form factors, with the data serving primarily to fix a value of the form factor parameter, $\Lambda_\pi$.

The inclusion of the $\rho$ exchange in diagrams 5-4 (a) and (b) increased the value of $\Lambda_\pi$ required to fit the data, due to the destructive interference between the $\rho$ exchange and $\pi$ exchange terms. The $\rho$ exchange was also parameterized by form factors at the $NN\rho$ and $\Delta N\rho$ vertices. The cross section was fairly insensitive to the $\rho$ form factor parameter, $\Lambda_\rho$. This parameter was set equal to 1.8 GeV/c, the value used in reference (Hul).

The theory also included one pion exchange terms with an $N^*(1470)$ intermediate state, shown in Figure 5-4 (c) and (d); and the nonresonant one pion exchange terms of Figure 5-4 (e) and (f). These terms were included in an attempt to explain the large off-resonance cross sections; however, they were found to have a negligible effect on the cross section at these energies.

It was found that only a constrained set of parameters gave good agreement between the theory and the data. The parameters used, which are listed in Table 5.3, were similar to those used in reference (Hul) to give the best fit to the $pp\rightarrow p\pi^+ n$ data. The only differences occurred in the values of $\Lambda_\pi$ and $M_\Delta$. The agreement with the $pp\pi^-$ data was
Figure 5-4. Diagrams Used in the Peripheral Model Calculations.

The calculations include the $\pi$ and $\rho$ exchanges with a $\Delta^+$ intermediate state [fig. (a) and (b)], $\pi$ exchange through the $N^*$ intermediate state [fig. (c) and (d)], and nonresonant $\pi$ exchange [fig. (e) and (f)]. The terms in which $p_1$ and $p_2$ are interchanged are calculated in the theory, but they are not shown in the figure.
Table 5.3. Values of Parameters Used for the Best Fit.

<table>
<thead>
<tr>
<th>Type of Parameter</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Δ resonance parameters</td>
<td>$M_\Delta = 1232$ MeV $\Gamma_\Delta = 100$ MeV</td>
</tr>
<tr>
<td>N* parameters</td>
<td>$M_{N\ast} = 1470$ MeV $\Gamma_{N\ast} = 250$ MeV</td>
</tr>
<tr>
<td>π coupling constants</td>
<td>$f_{NN\pi} = 1.0$ $f_{\Delta N\pi} = 2.0$ $f_{N\ast N\pi} = 0.61$</td>
</tr>
<tr>
<td>ρ coupling constants</td>
<td>$\Lambda_{NN\rho} = 6.24$ $f_{\Delta N\rho} = 10.54$</td>
</tr>
<tr>
<td>Form factor parameters</td>
<td>$\Lambda_\pi = 875$ MeV/c $\Lambda_\rho = 1800$ MeV/c $\kappa = 160$ MeV/c</td>
</tr>
</tbody>
</table>

The values used for the best fit were the same as those used to fit the data of reference (Hu) except for the values of $\Lambda_\pi$ and $M_\Delta$ which were increased to 875 MeV/c and 1232 MeV respectively. The parameters are explained fully in the reference, along with the details of the calculation.
improved by increasing $A_\pi$ from 780 MeV/c to 875 MeV/c and increasing $M_\Delta$ from 1220 MeV to 1232 MeV. Figure 5-5 compares the data and the theory and demonstrates the effect of the changes in $A_\pi$ and $M_\Delta$.

The solid curve in the figure is the prediction of the theory using the larger values of $A_\pi$ and $M_\Delta$. This calculation gives good agreement with the data at small values of $u_p$, the square of the momentum transfer to the proton in the $p\Delta^+$ state. The momentum transfers are defined in Figure 5-6. For data with large $u_p$ the theory peaks at the wrong value of $P_\perp$ and misestimates the magnitude of the cross section.

The dashed curve in the figure represents the prediction with the smaller values of $A_\pi$ and $M_\Delta$. The calculation with these parameters gives poor agreement with all of the data, with incorrect predictions of both the position and magnitude of the peak in the cross section.

In order to test the effect of the larger values of $A_\pi$ and $M_\Delta$ on the earlier calculations of the $pp\to p\pi^+n$ cross sections, the theory was recalculated and compared to the data from (Hul). The results are shown in Figure 5-7. The solid curve in the figure shows the results of the calculation with $A_\pi = 875$ MeV/c and $M_\Delta = 1232$ MeV. The present calculations exclude the n-p final state interactions terms which were included in reference (Hul). In regions where the momentum transfer to the $\Delta^{++}$ ($t_{\Delta^{++}}$) is small, corresponding to large values of $u_n$, the momentum transfer to the neutron in the $n\Delta^{++}$ state (Figure 5-6), the theory gives a poor prediction of the peak in the cross section. It also badly overestimates the size of the cross section for $\theta_1 = 20^\circ$, $\theta_2 = 22^\circ$. For large values of $t_{\Delta^{++}}$, the theory overestimates the magnitude of the
Figure 5-5. Comparison Between the Theory and the Data.

The solid curve is the fit to the data using the values of 875 MeV/c and 1232 MeV for \( \Lambda_\pi \) and \( M_\Delta \) respectively.

The dashed curve is the fit using the values of 780 MeV/c and 1220 MeV for \( \Lambda_\pi \) and \( M_\Delta \) respectively.

All other parameters are identical to those listed in Table 5.3.

The arrow in each figure indicates the position of the \( \Delta^+ \) between the second proton and the pion.
$pp \rightarrow pp \pi^0$

$\theta_1 = 15^\circ, \theta_2 = 21^\circ$

$\varepsilon = 81$

$\frac{d^3\sigma}{dQ_1 dQ_2 dP_1} (\mu b/sr^2 MeV/c)$

$P_1$ (MeV/c)
\[ d^2 \sigma / d\Omega_1 d\Omega_2 dP_1 \] (\(\mu b / \text{sr}^2 \text{MeV/c}\))

\(pp \rightarrow pp\pi^0\)

\(\theta_1 = 3.05^\circ, \theta_2 = 20.8^\circ\)

E-33/336
$d^6\sigma/d\Omega_1 d\Omega_2 dP_i$ (µb/sr² MeV/c)

$\theta_1 = 14^\circ$, $\theta_2 = 25^\circ$

$E_{\pi^0} = 81$
\[ \frac{d^3 \sigma}{d \Omega_1 d \Omega_2 d \mathbf{P}_t} \left( \mu b/\text{sr}^2 \text{MeV}/c \right) \]

**pp-ppπ⁺**

\[ \theta_1 = 14^\circ, \theta_2 = 30^\circ \]

**E-81**

---

\[ \frac{d^3 \sigma}{d \Omega_1 d \Omega_2 d \mathbf{P}_t} \left( \mu b/\text{sr}^2 \text{MeV}/c \right) \]

**pp-ppπ⁺**

\[ \theta_1 = 15^\circ, \theta_2 = 41^\circ \]

**E-81**

---
\[ P = (f_B + A^\pm A^+), A^{\pm 2} \]

\[ \Delta^+ (\Delta^{++}) \]

\[ t_\Delta = (P_B - P_\Delta)^2 \quad \Delta^- \Delta^+ \]

\[ u_N = (P_B - P_N)^2 \quad N \rightarrow p, n \]

\( P \) is the four-momentum

**Figure 5-6:** Momentum Transfers.

\( t_\Delta \) is the square of the four-momentum transfer from the beam proton to the final state \( \Delta \).

\( u_N \) is the square of the four-momentum transfer from the beam proton to the final state nucleon.

The terms in parentheses in the diagram pertain to the pp+\( \pi^+ \)n reaction.
cross section slightly but does well in predicting the position of the peaks.

The dashed lines in Figure 5-7 show the theory with the smaller values of \( \Lambda_\pi \) and \( M_\Delta \), used in reference (Hul). These calculations also exclude the n-p final state interaction terms. These curves have slightly better agreement with the data, although they underestimate the cross sections for most of the data.

The discrepancy in the value of \( \Lambda_\pi \) that fits the two sets of data is due to differences in the range of the momentum transfers, \( u_p \) and \( u_n \). The momentum transfers for the pp->p\pi^+n data were smaller than those for the pp->p\pi^0 data. The peripheral model calculations were expected to fit the data best in regions where the momentum transfers were small and the \( \Delta \) was completely dominant.

The discrepancy in \( M_\Delta \) may have been caused by the method of averaging the theory over the angular acceptance of the experimental system or by ambiguities in the beam energy (Mul). In the angle averaging, the theory was calculated at sixteen points within \( \pm 2^\circ \) of the central angles. The values of the prediction at all sixteen points were then averaged together with an equal weight. For the pp\pi^0 data, this was an acceptable method because the phase space was a fairly flat function of \( \Delta^+ \) production angles for these data. For the p\pi^+n data, phase space was rapidly increasing as the \( \Delta^{++} \) center of mass production angle decreased. The method of angle averaging used thus gave too much weight to the theory at large \( \Delta^{++} \) production angles and too little weight to the theory at smaller angles. The ambiguity in the beam energy arises
Figure 5-7. Comparison Between the Theory and the pp+π+πn Data.

The solid curves in the figure are the predictions using the values $A_\pi=875 \text{ MeV/c}$ and $M_\Delta=1232 \text{ MeV}$.

The dashed curves are the predictions using the values $A_\pi=780 \text{ MeV/c}$ and $M_\Delta=1220 \text{ MeV}$.

All other parameters are listed in Table 5.3.

The arrow in each figure indicates the position of the $\Delta^{++}$ between the proton and the pion.
from the tendency during E-81 of the beam energy to drift downward from the nominal energy of 800 MeV to 785 MeV. The results were to cause $M_{\Delta}$ to decrease in order to obtain a peak in the cross section at the correct values of $P_L$. This problem was circumvented for the $p\pi^+n$ theory by recalculating the theory as a function of the relative energy $T_{\pi^+p}$. Since this quantity was calculated using the kinematics, it took into account the phase space differences so that the errors in angle averaging were removed. The data and the theory, when plotted as a function of $T_{\pi^+p}$, agreed well using the 1232 MeV for $M_{\Delta}$. Figure 5-8 shows a plot of the cross section as a function of $T_{\pi^+p}$ for $\theta_1 = 15^\circ$, $\theta_2 = 21^\circ$. This calculation was not performed for the $pp\pi^o$ data due to the ambiguity in $T_{\pi^o_p}$ introduced by the presence of two final state protons.

5.4 Conclusions

The fifth-order differential cross section for the $pp\rightarrow pp\pi^o$ reaction has been measured in kinematically complete experiments. Data have been obtained at five angular settings covering a range of momentum transfer, $u_p$, from $-0.2$ to $-0.5$ (GeV/c)$^2$. The data are dominated by the presence of the $\Delta^+$ resonance in the final state.

Comparisons between the data and the results of the peripheral model calculations of reference (Hul) indicate that the pion form factor parameter, $\Lambda_\pi$, has a value of about 875 MeV/c. This value is 12% higher than the value used in reference (Hul) to fit the $pp\rightarrow p\pi^+n$ data taken earlier. Despite the discrepancy, both calculations indicate that the value of $\Lambda_\pi$ should be large. This disagrees with the value of $\Lambda_\pi = 350$ MeV/c found using optical model calculations (Gol).
Figure 5-8. $pp \rightarrow p\pi^+n$ Cross Section as a Function of $T_{\pi^+p}$

The invariant cross section (differential cross section divided by the phase space) is plotted as a function of the relative energy between the proton and the $\pi^+$. The angle plotted, $\theta_1=15^\circ$, $\theta_2=21^\circ$, was chosen arbitrarily. The curve on the graph is the result of the peripheral model calculation with $M_A=1232$ MeV. The improvement of the prediction of the position of the peak is apparent.
While the present model agrees reasonably well with the data, the calculated cross section peaks at too low a value of $P_\perp$ or overestimates the cross section at some angles. Also, $M_\Lambda$ is too low for the theory for $pp+\pi^+n$. Presently, work is under way to modify the theory by replacing the $\Lambda$ and $N^*$ resonances with $\pi N$ phase shifts for the one pion exchange contribution to the theory. This calculation should not be as sensitive to the value of $M_\Lambda$, and it should be valid over a wider momentum transfer range. It should also give better agreement with the data in the region away from the $\Lambda^+$ resonance peak. These improvements should decrease the existing discrepancies in $\Lambda_\pi$ and $M_\Lambda$ and will hopefully predict both the $pp+\pi^+n$ and $pp+pp\pi^0$ differential cross sections using the same value of $\Lambda_\pi$.

This study of $pp+pp\pi^0$ provides a new, stringent test of the models concerning the inelastic channels of the nucleon-nucleon interaction. Any model will have to be able to fit data covering all of the pion production channels in all regions of phase space if it strives to explain the nature of the force between two nucleons in the intermediate energy region.

5.4.1 Future Work

Experiments are presently planned or in progress which will provide more stringent tests of the present theory and help to correct its deficiencies.

Polarized beam and target experiments, such as $p\uparrow p\uparrow \pi^+ n$, $p\uparrow p\uparrow d\pi^+$, and $p\uparrow p\uparrow pp$ measurements of cross section asymmetries should provide stringent tests of the theory and aid in determining a more dependable
value of the $\rho$ form factor parameter, $\Lambda^\rho$.

Experiments which focus on nonresonant pion production, such as the study of the $np\rightarrow pp\pi^-$ reaction, should aid in correcting the inability of the present theory to accurately predict cross sections which are not dominated by the $\Delta$ isobar.
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Du1 I. Duck, private communication.


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Mul  G. S. Mutchler, private communication.


