THE CROSS SECTION DEPENDENCE ON ENERGY NEAR A
THRESHOLD OF THE REACTION Be²(d,n)B¹⁰

A Thesis presented to the Faculty of
The Rice Institute in partial fulfillment
of the requirements for the degree of
Master of Arts

May 12, 1949
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INTRODUCTION

A new group of neutrons with negative Q has been reported by Evans, Malich, and Risser\textsuperscript{1} for the reaction
\[ \text{Be}^9 + \text{H}_2 \rightarrow \ast\text{B}^\text{11} \rightarrow \ast\text{B}^\text{10} + \text{n}^1 + Q \]
In these experiments the neutrons were detected with gas recoil proportional counters. One counter contained hydrogen and 2 per cent methane at one atmosphere, and the other was filled with commercial argon at one atmosphere.

Both counters showed a rise in the yield curve indicating a new group of neutrons with a Q value of - 0.74 Mev.

A check on the threshold for the new group was obtained by surrounding the argon counter from all directions except the forward with a slab of paraffin. The yield curve exhibited an upward break at 900 kev confirming the thresholds calculated from the first two methods. However, in none of these curves was the upward break sharply defined.

The purpose of this investigation was to confirm this threshold with a different counting technique and to develop an optimum counter geometry for detecting new slow neutron thresholds in other reactions. It is hoped that this method of counting might be able to resolve closely residing thresholds in nuclear reactions and thus yield important information on nuclear structure.

ACCELERATING APPARATUS

The Rice Institute Van de Graaff generator was used to obtain homogeneous beams of deuterons of energies (for this investigation) between 800 kev and 1100 kev. The energy of the deuterons was measured with a magnetic analyzer which deflected the beam through an angle of 90°. The energy defining slit was opened to a width of 3 kev for a bombarding energy of 1 Mev.
TARGETS

The targets used in these experiments were prepared by evaporating beryllium in a high vacuum onto clean polished silver disks about 3.2 cm in diameter. The beryllium was heated by small helical filaments of tungsten. A vacuum of \(5 \times 10^{-5}\) mm Hg or better was obtained before heating the beryllium. The targets ranged in thickness from about 3 kev to 35 kev. A microbalance was used to determine target thickness.

COUNTING TECHNIQUE

The neutrons were detected with a proportional counter containing BF\(_3\), and patterned after the one described by Hanson and McKibben\(^2\). The body of the counter was a 1 inch by 6 inch brass tube with a 1/32 inch wall, and was soldered to Kovar glass seals. The counter was filled with enriched BF\(_3\) (about 95 percent B\(^{10}\)) to a pressure of 30 cm Hg. The counter was supported as shown in Fig. 1 by a thin layer of ceresin wax inside an aluminum tube with a 1/16 inch wall. The effective length of the counter was about 4\(\frac{1}{2}\) inches.

![Diagram of counter](attachment:image.png)
Various sizes, shapes, and orientations of paraffin blocks around the counter were tried to find the optimum geometry for observing the slow neutron threshold. The objective was to obtain a sharp upward break at the threshold and to obtain a maximum ratio of slow neutron yield to fast neutron yield, the fast neutrons being those with Q values of 4.3, 3.7, 2.1, and 0.8 Mev. The paraffin arrangement shown in Fig. 1 gave satisfactory results.

Except on the back end the entire counter arrangement was covered with 0.3 mm sheet of cadmium to protect the counter from stray thermal neutrons.

Because of the background rate, due primarily to carbon deposited onto the energy defining jaws of the magnetic analyzer from the vacuum system, it was found advantageous to place the counter as close as possible to the target. The target was supported on a movable platform so that the target or a blank silver disk could be placed in the deuteron beam at will. The bar between the target disk and the blank was of soft iron and could be moved by a strong permanent magnet outside the target chamber. Stops on the central porcelain support prevented the disks from swinging past the point in line with the deuteron beam. The blank silver disk was placed in the beam to obtain the background rate. The target-counter orientation as shown in Fig. 2 shows a typical arrangement.

![Diagram](Fig. 2)
The equipment used with the counter consisted of an Atomic Instrument Company linear amplifier, Model 204-B, discriminator and scale-of-64 circuit, Model 101-A, and a regulated high voltage supply. The voltage used on the counter was between 1700 and 1750 volts, with a discriminator bias of 40 volts, and a rise time of 0.8 microsecond.

The efficiency of the counter was not accurately determined, but is believed to be on the order of a few per cent. Hanson and McKibben have shown that the efficiency of such an arrangement is fairly constant over a range of energy from 0 to about 100 keV.

EXPERIMENTAL RESULTS

Circumstances prevented an accurate determination of the threshold of the reaction at the time of this writing. The intentions were to calibrate the magnetic analyzer just below and above the approximate threshold by making use of the known gamma-ray resonances of the reaction $^{19}$F($p$, $\gamma$)$^{20}$Ne. The best estimate from the curves obtained is that the threshold occurs at about 920 keV. This value was obtained by using the magnetic analyzer calibration curve obtained in October, 1948, using the resonances from $^{19}$F($p$, $\gamma$)$^{20}$Ne. A typical experimental curve is shown in Fig. 3.

The counter geometry used to obtain this curve was the same as shown in Fig. 2. The target thickness was about 16 keV. The upward break in the curve is quite sharp, the indetermination being about the same as the width of the energy defining slit.

The presence of the threshold is very pronounced, the ratio of total neutron counts above threshold to the neutron counts below threshold being about 2.4. For about 100 keV below threshold, the curve is essentially flat, the height of the curve below threshold being due to the fast neutrons. Just above threshold,
most of the experimental curves exhibited a very slight drop, but the amount of the drop was not sufficient to be considered as evidence of a real decrease in counting rate. Most of the curves showed a gradual increase above threshold until a bombarding energy of 1 Mev was reached. Here another sudden rise in the yield curve occurred but was much less pronounced. The jump in the curve at this point was about 10 per cent. If this is another threshold, it indicates a Q value of about -0.82 Mev.

Because of the geometrical cone effect accompanying a threshold, one might expect the yield curve to show a decided drop above threshold as, for instance, did the yield curve obtained by Freier, Lampi, and Williams for the reaction Li⁷(p,n) Be⁷. But none of the different geometrical arrangements showed an appreciable drop above threshold. In order to explain this failure of the experimental curves to show a drop above threshold, the mechanics of the reaction have been worked out in great detail.

CALCULATIONS

\[ \text{H}^2 + \text{Be}^9 \rightarrow \text{B}^{11} \rightarrow \text{B}^{10} + n^1 - 0.74 \text{ Mev} \]

1 2 3 4

Letting the subscripts (1,2,3,4) refer to the particle in the equation above it, and applying the laws of conservation of energy and momentum, we obtain a number of relations between the various dynamical quantities of the particles. In these calculations we assume that the velocity of the bombarded nucleus is zero, \( V_2 = 0 \), before the collision. In the notation, a "superscript cm" indicates that the quantity is with respect to the center of mass coordinates. \( E_t \) is the threshold energy of the deuterons. The value of Q is given by Equa. (1).

\[ Q = - \frac{M_2}{M_1 + M_2} E_t = - \frac{2}{11} (90^4) = - 740 \text{ kev} \]
As already indicated, the threshold for this reaction might be about 920 kev. However, these calculations are based on a threshold value of 904 kev chiefly because they were made on the basis of the results obtained by Evans, Malich, and Rissel, and a 904 kev threshold gave a Q value of -740 kev. Since the energy enters most of the calculations in the form of the square root, these calculations involving \( E_t \) will be in error by less than 1 per cent if the value of \( E_t = 920 \) kev should prove to be correct. All of the calculations have been made within an accuracy better than 1 per cent.

The velocity of the center of mass in the laboratory system is given by Equa. (2).

\[
(2) \quad V_{cm} = \frac{M_1}{M_1 + M_2} V_1 = \frac{2}{11} V_1
\]

The velocity of the deuterons is about \( 1 \times 10^9 \) cm per sec in the neighborhood of the threshold.

Assuming that the distribution of neutrons from the excited compound nucleus is spherically symmetrical in the center of mass coordinates, the neutrons will be distributed over a cone at some energy above threshold in the laboratory coordinates. For \( j = 0 \), the neutrons will leave the *B^11 nucleus with zero velocity in the center of mass system just at the threshold. The energy of the emitted neutrons in the center of mass coordinates is given by Equa. (3), where \( \Delta E_1 \) is the energy of the deuteron above threshold.

\[
(3) \quad E_1^{cm} = \frac{M_2 M_3}{(M_1 + M_2)^2} \Delta E_1 = \frac{90}{121} \Delta E_1
\]

So approximately three-fourths of the increase in bombarding energy go to the neutrons with respect to the center of mass coordinates.

The velocity of the center of mass in the laboratory system and the velocity of the neutrons in the center of mass system are given by Equa. (4). The energies
are in units of Mev. A plot of these velocities vs. $\Delta E_1$ is given in Fig. 4.

\[ V_{cm} = 1.79 \times 10^{8} \sqrt{E_1} \text{ cm/sec} \]
\[ V_{4cm} = 12.0 \times 10^{6} \sqrt{\Delta E_1} \text{ cm/sec} \]

The half-angle of the cone of neutrons in the laboratory system is given by Eq. (5).

\[ \sin \theta_m = \frac{V_{4cm}}{V_{cm}} = \left( \frac{M_2 M_3}{M_1 M_4} \frac{\Delta E_1}{E_1} \right)^{\frac{1}{2}} = \left( \frac{45 \Delta E_1}{E_1} \right)^{\frac{1}{2}} \]

The variation of $\theta_m$ with $\Delta E_1$ is given by the curve in Fig. 5. At a bombarding energy of 20.6 kev above threshold, $V_{4cm} = V_{cm}$, and the cone has opened to a half-angle of 90° or a solid angle of 2$\pi$ steradians, half the total solid angle. Any further increase in bombarding energy will yield neutrons throughout the entire solid angle.

To determine the geometrical effect of the cone on the counting rate, we determine the fraction $F$ of the neutrons that lie within the constant cone subtended by the counter as the total cone of neutrons opens up. Let $b = V_{cm}$, $a = V_{4cm}$, $\theta$ is the half-angle of the cone subtended by the counter, $\phi_1$ and $\phi_2$ are the angles of neutron emission in the center of mass system corresponding to an angle of emission of $\theta$ in the laboratory system. The following analysis is for an infinitely thin target.

\[ \text{Fig. 6.} \]
From the figure

\[
\phi_1 = \Theta + A
\]

(6)

\[
\phi_2 = \Pi - \Theta - B
\]

\[
B = \Pi - A
\]

therefore,

\[
\phi_1 = \Theta + \sin^{-1} \left( \frac{b}{a} \sin \Theta \right)
\]

(7)

\[
\phi_2 = -\Theta + \sin^{-1} \left( \frac{b}{a} \sin \Theta \right)
\]

now the fraction \( F \) of neutrons inside the cone of half-angle \( \Theta \) is

\[
F = \frac{1}{4\Pi} \left[ 2\Pi \left( 1 - \cos \phi_1 \right) + 2\Pi \left( 1 - \cos \phi_2 \right) \right]
\]

(8)

\[
= 1 - \frac{1}{2} \left( \cos \phi_1 + \cos \phi_2 \right)
\]

\[
= 1 - \frac{1}{2} \left( 2 \cos \frac{1}{2} \left( \phi_1 + \phi_2 \right) \cos \frac{1}{2} \left( \phi_1 - \phi_2 \right) \right)
\]

(9)

\[
\frac{1}{2} \left( \phi_1 + \phi_2 \right) = \sin^{-1} \left( \frac{b}{a} \sin \Theta \right)
\]

and

\[
\cos \frac{1}{2} \left( \phi_1 + \phi_2 \right) = \sqrt{1 - \frac{b^2}{a^2} \sin^2 \Theta}
\]

(10)

also from Equa. (7)

\[
\frac{1}{2} \left( \phi_1 - \phi_2 \right) = \Theta
\]

(11)
then

\[(12) \quad \cos \frac{1}{2}(\phi_1 - \phi_2) = \cos \theta\]

therefore

\[(13) \quad F = 1 - \cos \theta \sqrt{1 - \frac{b^2}{a^2} \sin^2 \theta}\]

The foregoing derivation is for the case when \(b > a\). When \(b < a\),

![Diagram](image)

**Fig. 7**

From Fig. 7, we see that

\[(14) \quad \phi = \theta + A = \theta + \sin^{-1} \left( \frac{b}{a} \sin \theta \right)\]

then proceeding as before,
\[ F = \frac{1}{4\pi} \left[ 2\pi \left( 1 - \cos \phi \right) \right] \]

\[ = \frac{1}{2} \left( 1 - \cos \phi \right) = \frac{1}{2} \left\{ 1 - \cos \left[ \theta + \sin^{-1} \left( \frac{b}{a} \sin \theta \right) \right] \right\} \]

\[ = \frac{1}{2} \left\{ 1 - \cos \theta \cos \left[ \sin^{-1} \left( \frac{b}{a} \sin \theta \right) \right] + \sin \theta \sin \left[ \sin^{-1} \left( \frac{b}{a} \sin \theta \right) \right] \right\} \]

\[ = \frac{1}{2} \left\{ 1 + \frac{b}{a} \sin^2 \theta - \cos \theta \sqrt{1 - \frac{b^2}{a^2} \sin^2 \theta} \right\} \]

when \( b = a \), both forms (13) and (15) yield

\[ F = \sin^2 \theta \]

The geometrical cone effect for counter half-angles of \( 15^\circ, 30^\circ, 45^\circ, \) and \( 60^\circ \) is given in Fig. 8.

The half-angle of the counter arrangement shown in Fig. 2 was taken arbitrarily to be the angle at the target between the axis of the counter and a line drawn so as to penetrate one mean free path of the paraffin before emerging from the paraffin block. At the energies of the slow neutrons (15-35 kev) the mean free path in paraffin is about 4 mm. This angle was found to be about \( 63^\circ \).

With the counter so close to the target, there are a number of apparent corrections. For instance, the neutrons inside a cone of \( 12^\circ \) were not counted at all since they went through the counter before being slowed down by the paraffin, and the cross-section for the reaction \( \text{B}^{10}(n,\alpha)\text{Li}^7 \) which occurs in
the counter is very small for these neutrons. However, for half-angles considerably greater than 12°, the solid angle contained within the 12° cone is small compared to the solid angle of the entire cone. So this correction is unimportant. The threshold would not be detected until the cone had opened to more than 12°. This would occur at 0.9 kev above the true threshold, but again this is not appreciable.

The spot of impinging deuterons on the target was about 1 cm in diameter. The uncertainty in counter half-angle caused by the size of this spot is only about 1°.

Another correction could be applied. The probability of a neutron's being counted is a function of its position in the cone; that is, a neutron entering the paraffin fairly close to the counter has a better chance of entering the counter as a thermal neutron than one far out on the edge of the paraffin cylinder. The correction is very complicated and subject to a number of statistical uncertainties. So no analytical treatment is possible. However, some graphical corrections of this nature were applied, but the change in shape of the geometrical curve was not great enough to warrant the crudeness and arbitrariness of the corrections. So the geometrical curve obtained with Equas. (13) and (15) and shown in Fig. 9 is taken to be accurate enough for our purposes.

DISCUSSION

A comparison of the experimental yield curve in (Fig. 3) and the calculated geometrical cone effect curve shows that some factor so far not considered must enter into the reaction. It is known that the probability that a compound nucleus will disintegrate in a particular way is related to the cross section for the corresponding inverse capture process. The ratio of the probabilities for the direct and inverse processes is equal to the ratio of the densities of
the final and initial states of the system at corresponding energies. The disintegration probability is given by Eq. (17), where $\Gamma_4$ is the partial level width due to the neutron. 

$$W = \frac{\Gamma_4}{\hbar}$$

For low energies, $\Gamma_4$ is proportional to the neutron velocity in the center of mass system. The cross section for the reaction is equal to the product of the cross section for the formation of the compound nucleus (which is practically constant in the neighborhood of the threshold) and the disintegration probability. Therefore, we would expect the cross section for the reaction to be proportional to the square root of the neutron energy in the center of mass system. And since the neutron energy is proportional to the difference between bombarding energy and threshold energy, $(\Delta E_1)$, according to Eq. (3), the cross section for the reaction will be proportional to $(\Delta E_1)^{1/2}$. Then in order to get a theoretical yield curve, we would multiply the geometrical curve by $(\Delta E_1)^{1/2}$. This has been done, and is the curve marked $P_3$ in Fig. (9).

The product curve is much more like the experimental curves than the plain geometrical curve $F$. This fact tends to substantiate the relation between the cross section and the neutron velocity in the center of mass system.

If the neutrons were emitted with orbital angular momentum $\ell = 1$, the partial level width due to the neutron (and hence the cross section for the reaction) would be proportional to $(\Delta E_1)^{1/2}$. The product curve of the geometrical curve and $(\Delta E_1)^{1/2}$ is also shown in Fig. (9) and is marked $P_{3'}$. Clearly, this curve is not consistent with the experimental data, so the original assumption that the neutrons were emitted with zero orbital angular momentum is at least partially justified.
As already stated, the above calculations and curves are based on an infinitely thin target. Since the experimental curve shown in Fig. 3 was made with a 16 kev target, the product curve $P_2$ has been computed for a 16 kev target by adding the ordinates of 16 identical $P_2$ curves, each one displaced 1 kev from the one preceding. The ordinates were computed at intervals of 1 kev, and a plot of the integrated curve is shown in Fig. (10).

The similarity between the experimental curve and the integrated theoretical curve is indeed remarkable. The fact that both curves show the same small dip just above the threshold is believed to be due more to circumstances of chance than to the exactness of the experimental data or counting technique.

A close examination of the two curves shows that the rise of the experimental curve took place over a range of about 20 kev, whereas the computed curve required almost 30 kev to reach a local maximum. This is not significant because the target thickness was known only to within plus or minus a few kev, and the arbitrary method of choosing the counter half-angle could easily cause a variation of a few kev in the range of rise.

Further examination indicates that the experimental curve rises slightly less rapidly than the calculated curve. This could be caused by a slightly lower counter efficiency for the fast neutrons as the bombarding energy is increased, and by the fact that the cross section for the reaction is proportional to $(\Delta E_1)^{1/2}$ only for small neutron velocities in the center of mass system. For high neutron energies, the cross section for the reaction is equal to a constant for a small range of bombarding energies.

One might expect that the cross section dependence on energy near a threshold of the reaction $^7\text{Li} (p,n)^7\text{Be}$ would show the same relation as has been shown to
to be the case for the reaction $\text{Be}^9(d,n)\text{B}^{10}$. However, experimental curves obtained by Taschek and Hemmendinger\textsuperscript{5}, Freier, Lampi, and Williams\textsuperscript{3}, and others for the threshold of 1.83 Mev indicate that the cross section cannot be explained so simply. A geometrical curve for an infinitely thin target for the geometry used by Freier, Lampi, and Williams\textsuperscript{3} is given in Fig. 11. This curve closely resembles the experimental yield curve obtained by them with a 24 kev target. About 140 kev above threshold, the geometrical curve has dropped to about $1/16$ of the maximum. At the same energy, the experimental curve dropped to about $1/3$ of the maximum. But if the geometrical curve were integrated over a target thickness of 24 kev, it would drop only to about $1/4$ or $1/5$ of the maximum at the same energy.

The product curve of the geometrical curve and $(\Delta E_p)^{1/2}$ for an infinitely thin target is also shown in Fig. 11. When this curve is integrated over a 24 kev target, the percentage drop above threshold is very small compared to the drop observed experimentally. Clearly, the cross section cannot be represented as it was in the $\text{Be}^9(d,n)\text{B}^{10}$ reaction. The possibility of a resonance very near this threshold has been discussed by Taschek and Hemmendinger\textsuperscript{5}. They considered a resonance near threshold unlikely or weak because of the total yield. However, since there is no other apparent reason for this threshold to behave differently from the one discussed for the $\text{Be}^9(d,n)\text{B}^{10}$ reaction, this investigation indicates that a resonance is quite likely near the threshold.

If a number of other slow neutron thresholds were investigated in this manner and found to agree with the $\text{Be}^9(d,n)\text{B}^{10}$ reaction, there would be little doubt that some resonance phenomenon is taking place near the threshold of the $\text{Li}^7(p,n)\text{Be}^7$ reaction.
BIBLIOGRAPHY

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FIG. 4

VEL IN UNITS OF $10^7$ CM PER SEC