RICE UNIVERSITY

Optimal Reorientation of Spacecraft Using Only Control Moment Gyroscopes

by

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Abstract

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Spacecraft reorientation can require propellant even when using gyroscopes since these have momentum saturation limits at which control is lost until thrusters are fired to desaturate them. To eliminate this need and thereby reduce cost, this work seeks trajectories that avoid saturation altogether by taking advantage of known disturbance torques. This concept is formulated as an optimal control problem and a direct transcription method is applied to obtain numerical solutions. Unlike recent related work on attitude maneuvers and momentum desaturation using only gyroscopes, this thesis allows the full rotational state (attitude, rate, and momentum) at the start and end of the maneuver to be specified. This thesis establishes the viability of this technique, which can potentially extend the operational lifetime of any gyroscope-equipped spacecraft, by successfully demonstrating it in a flight test in which the International Space Station was rotated 90 deg without propellant.
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Chapter 1

Introduction

1.1 Motivation

The problem of spacecraft attitude control provides many applications for optimal control. Examples include fuel-optimal and time-optimal maneuvers [28], [19], [20]. In much of this literature, reaction control systems are typically used to perform the maneuver. Reaction control systems fire jet thrusters to torque the spacecraft. A disadvantage of this method of attitude control is that thrusters consume fuel, which is limited and expensive to replenish.

An alternative to reaction control systems are momentum exchange devices such as Control Moment Gyroscopes (CMGs). CMGs are spinning rotors that store angular momentum and induce a torque on the spacecraft when their axes of rotation are changed. CMGs have lower torque capacity than reaction control systems. Moreover, the total angular momentum the CMGs can store is limited, and when this threshold is reached, the CMGs are termed saturated. Saturation occurs when external disturbance torques acting on the spacecraft are larger than can be compensated for by the CMGs. Once the CMGs saturate, they cannot generate torque in some
direction. Thus attitude control with CMGs is lost, and some other method must be used to counterbalance bringing the CMG momentum back down (known as momentum dumping or desaturation). Usually, this means firing thrusters and burning costly propellant. It is desired to be able to perform maneuvers while maintaining the CMGs within their limits in order to avoid using propellant.

Some work has been done on optimal attitude maneuvers using momentum devices [35], [5], but in order to simplify the dynamics, external disturbance torques such as the gravity gradient and aerodynamic torques are not considered. Because of external disturbance torques on the spacecraft, its angular momentum is not conserved. Therefore the CMG momentum is a function of the attitude trajectory throughout a maneuver, not just of initial and final attitude. The key idea is to make use of this path dependence due to disturbance torques to avoid saturating the CMGs.

1.2 Concept Description

The concept is based on the observation that information about the spacecraft dynamics can be used to plan trajectories that reduce the overall cost (e.g. propellant, time, momentum, etc.) of executing the maneuver. The trajectory choice directly influences the cost of the maneuver. For example, an eigenaxis maneuver is kinematically the shortest path between two orientations but, in general, is not the lowest cost trajectory as it ignores the environmental dynamics [3]. By considering a “longer” kinematic path, the time to perform the maneuver is increased. However, the cost
of the maneuver can be reduced by taking advantage of the environmental dynamics instead of trying to overcome them as in the case of the eigenaxis maneuver.

The problem considered in this thesis is to perform a transition from a given initial set of rotational states (attitude, rate, CMG momentum) to given final states via an optimal trajectory that exploits the disturbance torques to ensure CMG thresholds are not exceeded. An optimal control problem is solved to find this trajectory satisfying the equations of motion as well as constraints on attitude, rate, and CMG momentum at the beginning and end of the maneuver, and peak CMG momentum and torque magnitudes. This optimal maneuver is termed Zero Prop Maneuver (ZPM), as it does not require the use of propellant. This approach can be used for any spacecraft with gyroscopes, but this thesis concentrates on the International Space Station (ISS). It is worth noting that such an optimal trajectory for the ISS is a function of the particular ISS configuration mass properties as well as the specific motion of articulating bodies (e.g. solar arrays and thermal radiators).

In general, most attitude maneuvers can be performed non-propulsively as long as sufficient momentum and time is available. The need for sufficient momentum is to allow for startup of the maneuver. In most instances, when the maneuver starts the momentum state is not zero due to previous operational mode requirements. Thus the peak momentum reached during the maneuver is a function of the initial momentum and the momentum necessary to build-up the rate. This total momentum and some margin to allow for system uncertainties must be less than some predetermined
fraction of the CMG capacity. The momentum needed to establish the rate is itself a function of the maneuver time. Generally, the longer the maneuver time, the lower the rate and also the peak momentum.

Recent related work using this concept includes CMG-only attitude maneuvers during ISS robotic payload operations [22], [7] and the ISS momentum dumping problem [26], [16], in which the CMG momentum was reduced by an optimal maneuver without firing thrusters. Momentum dumping with CMGs was also performed on Skylab during the dark side of the orbit to remove accumulated CMG momentum using the gravity gradient torque [11], [36]. Unlike those problems, in this thesis the full rotational state is constrained at the start and end of the maneuver rather than just the attitude or momentum.

The purpose of this thesis is not only to demonstrate conceptual viability, but to actually implement this technique for successful use in flight. Towards this goal, this thesis developed an optimal trajectory for a flight test demonstration of a Zero Prop Maneuver for the ISS. By executing a sequence of attitude and rate commands corresponding to the optimal trajectory, the ISS was rotated 90 deg without propellant, meeting all operational constraints, and requiring no changes to flight software.

This thesis is organized as follows. To begin, the attitude dynamics of a gyroscope-equipped rigid body in circular orbit are summarized in Chapter 2. The details of CMG attitude control for the ISS are also presented. In Chapter 3, the ZPM problem is constructed and stated formally along with the method used to solve it numerically.
Next, Chapter 4 describes the specifics of the ZPM flight test demonstration, presents performance of different solutions in high-fidelity simulation, and discusses robustness to parameter uncertainty. Finally, results of the flight test demonstration are given in Chapter 5. Chapter 6 contains concluding remarks and suggestions for future work.
Chapter 2

Spacecraft Attitude Dynamics

This chapter reviews the attitude dynamics of a rigid spacecraft equipped with gyroscopes in a circular low Earth orbit. After explaining the equations of motion and external disturbance torques, details of Control Moment Gyroscope (CMG) control of the International Space Station (ISS) are given. Finally, the last section describes a high-fidelity tool used in this thesis to simulate the dynamics.

2.1 Reference Frames

The following sections and the rest of this work employ the Local Vertical Local Horizontal (LVLH) and Body reference frames, both of which have the spacecraft center of mass as their origin. The Body frame (Figure 2.1) is fixed with respect to the ISS with the positive x-axis directed toward the nose and the positive y-axis along the main truss pointing starboard. As for the LVLH frame (Figure 2.2), the positive x-axis points in the direction of the velocity vector, the positive z-axis towards Earth, and the y-axis perpendicular to the orbit plane. Thus the LVLH frame makes one rotation about the Earth in one orbit. And since the LVLH frame depends solely on
the spacecraft’s position in orbit with the y-axis fixed throughout the orbit, its rate of rotation is just the negative orbital rate, $-n$, in the y-axis. For a circular orbit $n$ is constant.

Figure 2.1: ISS Body reference frame (Adapted from [24])

Figure 2.2: Local Vertical Local Horizontal (LVLH) reference frame (Adapted from [23])
2.2 Attitude Parameterization and Kinematics

This section contains the equations to parameterize attitude found in Hughes [15] and Wie [39], both of which provide a much more detailed exposition of the subject. Only the specific parameterization used in this thesis will be reviewed.

2.2.1 Quaternions

Euler observed that any transformation from one orientation to another can be characterized as a rotation of angle $\phi$ about an axis defined by a unit vector $a$. We refer to the $\phi$ as the eigenangle and the rotation axis as the eigenaxis. The difficulty with using $a$ and $\phi$ to parameterize attitude is that they are sometimes ambiguous [15]. For example, the identity rotation does not correspond to a unique $a$.

An alternative approach based on the concept of an eigenaxis and eigenangle are quaternions, $q \in \mathbb{R}^4$, defined by [39]:

\[
q = \begin{pmatrix} \cos \frac{\phi}{2} \\ a \sin \frac{\phi}{2} \end{pmatrix}.
\] (2.1)

Note that since $a$ is a unit vector, $\|q\|_2 = 1$. Quaternions are often preferred over other methods of attitude parameterization (e.g. direction cosine matrix, Euler angles, Rodriguez parameters) because of their computational efficiency [15]. Another advantage of quaternions over Euler angles and other three-parameter sets is that they avoid singularities via the extra parameter and unit norm constraint [15].
We wish to describe the spacecraft attitude relative to the rotating LVLH reference frame. First, a few definitions are necessary. The rotation matrix (also called direction cosine matrix) \( C: \mathbb{R}^4 \mapsto \mathbb{R}^{3 \times 3} \) corresponding to a rotation represented by \( q \) is [39]

\[
C(q) = \begin{pmatrix}
1 - 2(q_3^2 + q_4^2) & 2(q_2q_3 + q_1q_4) & 2(q_2q_4 - q_1q_3) \\
2(q_2q_3 - q_1q_4) & 1 - 2(q_2^2 + q_4^2) & 2(q_3q_4 + q_1q_2) \\
2(q_2q_4 + q_1q_3) & 2(q_3q_4 - q_1q_2) & 1 - 2(q_2^2 + q_3^2)
\end{pmatrix}.
\tag{2.2}
\]

Let \( C_i \) be the \( i \)th column of \( C \). Also define \( T: \mathbb{R}^4 \mapsto \mathbb{R}^{4 \times 3} \) by [39]

\[
T(q) = \begin{pmatrix}
-q_2 & -q_3 & -q_4 \\
q_1 & -q_4 & q_3 \\
q_4 & q_1 & -q_2 \\
-q_3 & q_2 & q_1
\end{pmatrix},
\tag{2.3}
\]

Next, let the spacecraft’s instantaneous angular rate relative to an inertial reference frame and expressed in the Body frame be given by \( \omega: \mathbb{R} \mapsto \mathbb{R}^3 \). To describe the attitude relative to LVLH, it is necessary to find the angular rate relative to LVLH. Using the angular rate inherent in the rotating LVLH reference frame expressed in the Body frame

\[
\omega_o(q) = -nC_2(q),
\tag{2.4}
\]

the quaternion attitude kinematics equation of the spacecraft orientation with respect
to LVLH is \[39\]

\[
\dot{q}(t) = \frac{1}{2} T(q)(\omega(t) - \omega_o(q)),
\]

(2.5)

where the term \(\omega(t) - \omega_o(q)\) denotes the angular rate relative to LVLH. Note that this thesis adopts the convention \(\dot{x}(t) = dx/dt\). It is worth emphasizing that to be physically meaningful, quaternions must satisfy the unit norm constraint.

### 2.2.2 Euler Angles

To easily visualize spacecraft rotations, this thesis uses Euler angles \[15\] in plots to indicate the attitude as a succession of simple rotations each about a single axis beginning in the LVLH frame. Specifically, the Yaw, Pitch, Roll (YPR) sequence is used. Thus to find the attitude given by \([\text{Roll Pitch Yaw}] = [2 - 9 40]\) deg, start in LVLH, perform an initial rotation of 40 deg about the positive z-axis (yaw), then rotate about the new y-axis (pitch) by \(-9\) deg, and end with a 2 deg rotation about the resulting x-axis (roll). In this thesis, the YPR Euler angles are always listed in Roll, Pitch, Yaw (RPY) order.

### 2.3 Rotational Dynamics

For a rigid body in circular orbit, the rotational dynamics expressed in the Body frame are given by Euler’s equation \[39\]:

\[
J\ddot{\omega}(t) = \tau_d(t) - \omega(t) \times (J\omega(t) + h_{cmg}(t)) - \dot{h}_{cmg}(t),
\]

(2.6)
where \( J \in \mathbb{R}^{3\times3} \) is the inertia matrix of the body in slug-ft\(^2\) and \( \omega : \mathbb{R} \mapsto \mathbb{R}^3 \) is the body’s angular rate relative to an inertial reference frame in rad/sec. The other terms are \( h_{cmg} : \mathbb{R} \mapsto \mathbb{R}^3 \), the total angular momentum of the CMGs in units of ft-lbf-sec and \( \tau_d : \mathbb{R} \mapsto \mathbb{R}^3 \), the total external disturbance torque on the body in ft-lbf, discussed in the next section. All quantities are expressed in the Body frame.

### 2.4 External Disturbance Torques

Many external disturbance torques, generated by the environment, interact with a spacecraft in orbit. Examples of these types of torques include the gravity gradient, aerodynamic (i.e. drag), magnetic, solar pressure, and cosmic dust [15].

#### 2.4.1 Gravity Gradient Torque

Generally the largest external disturbance torque on a spacecraft in low Earth orbit, the gravity gradient torque, \( \tau_{gg} \), results when the spacecraft has a non-uniform distribution of mass, i.e. one part of the spacecraft experiences a weaker gravitational pull by the Earth than another part. The approximate formula for the gravity gradient torque is [15]

\[
\tau_{gg}(q) = 3n^2 C_3(q) \times (JC_2(q)).
\] (2.7)

It is evident that the gravity gradient torque is a function of a spacecraft’s orientation relative to LVLH \( q \), its orbit (which determines the orbital rate \( n \)), and its inertia, \( J \).
2.4.2 Aerodynamic Torque

Low Earth orbit spacecraft also experience atmospheric drag which produces an aerodynamic torque, $\tau_{aero}$ on the spacecraft. Computation of this torque requires an atmosphere and aerodynamics model as well as accurate knowledge of atmospheric parameters and spacecraft-specific properties such as drag coefficients, surface areas, centers of pressure, etc.

As a first example, consider an aerodynamics model that treats the ISS as a single body represented by three flat plates [38]:

\[
\tau_{aero}(q) = \sum_{p=1}^{3} \left( (c_p - cm) \times f_p(q) \right), \\
\]

\[
f_p(q) = -\frac{1}{2}\rho(t)V^2C_d|A_pC_{1,p}(q)|C_1(q) \tag{2.9}.
\]

For plate $p$, $f_p \in \mathbb{R}^3$ is the atmospheric drag force on the plate, $A_p \in \mathbb{R}$ is the projected area, and $c_p \in \mathbb{R}^3$ is the center of pressure. The remaining terms are center of mass $cm \in \mathbb{R}^3$, atmospheric density $\rho(t) \in \mathbb{R}$, magnitude of the spacecraft translational velocity relative to the atmosphere $V \in \mathbb{R}$, and drag coefficient $C_d \in \mathbb{R}$ (assumed to be the same for each plate). Here the first column $C_1 \in \mathbb{R}^3$ of the rotation matrix represents the unit vector parallel to the velocity vector expressed in the Body frame.

More complicated models involve several bodies and/or account for the interaction of atmospheric particles with the surface after a collision [15]. The ISS consists of many components, some of which (i.e. solar arrays) are in motion to face the sun while
others (i.e. thermal radiators) avoid the sun by rotating to keep an edge towards it. Like before, three flat plates can be used to represent each of these component bodies.

After colliding with a surface, atmospheric particles can either bounce off or “stick”. In a specular drag model, atmospheric particles are reflected from the plates after a collision, resulting in a force that is normal to the area of impact, and proportional to the product of the plate area and the square of the relative velocity normal to the plate area [31]. In a diffuse drag model, atmospheric particles stick to the plates after a collision, and temperature effects cause some reflection of particles normal to the surface of the plate [31]. Thus, two forces are created. The force from the particles sticking to the plate after a collision is parallel to the relative velocity vector, and proportional to the product of the plate area projected in a direction normal to the relative velocity, and the square of the relative velocity. The force from the reflection is normal to the plate and is proportional to the component of velocity normal to the plate. Figure 2.3 shows the vectors used to compute specular and diffuse drag forces. A combination model takes the percentage of diffuse collisions as an input.

Figure 2.3: Relevant vectors for area, $A$, and velocity, $V$ to compute specular and diffuse drag forces $F$ ([21])
and sums the forces due to both diffuse and specular collisions. The drag coefficient $C_d$ is computed as a function of this percentage.

A three-plate, multi-body model with $N_b$ total bodies and both diffuse and specular collisions is described next. For plate $p$ of body $b$, let $f_{(b,p)_i} \in \mathbb{R}$ be the $i$-th element of $f_{(b,p)} \in \mathbb{R}^3$, the drag force, $A_{b_p} \in \mathbb{R}$ be projected area, and $V_{b_p} \in \mathbb{R}$ be the relative velocity. Then the aerodynamic torque is [15], [31]

$$
\tau_{aero}(q) = \sum_{b=1}^{N_b} \sum_{p=1}^{3} (cp_{b,p} - cm_{comp}) \times f_{b,p},
$$

(2.10)

where $cm_{comp} \in \mathbb{R}^3$ is the ISS composite center of mass and for $i = 1, 2, 3$, 

$$
f_{(b,p)_i} = \begin{cases} 
-\rho(t)\sigma|A_{b_p}V_{b_p}|V_{b_i}, & \text{if } i = p \\
-\rho(t)\sigma|A_{b_p}V_{b_p}|V_{b_i} - \cdots \\
\rho(t)\sigma\alpha_n|A_{b_p}|V_{b_i} - \cdots \\
2\rho(t)(1 - \sigma)|A_{b_p}V_{b_p}|V_{b_i}. & \text{otherwise}
\end{cases}
$$

(2.11)

The constants $\sigma$ and $\alpha_n$ respectively are the fraction of diffuse collisions and the mean velocity of particles bouncing off in a direction normal to the surface.

The time-varying nature of atmospheric density necessitates an atmosphere model to characterize its dependence on position, seasonal variations, solar and geomagnetic activity, etc. For example, a spacecraft encounters more drag (higher density) during the sunlit portion of its orbit (due to outward expansion of the warmer atmosphere) than during eclipse. Also, solar activity (sunspots, solar flares, etc.) determines
the electromagnetic radiation which affects the Earth’s atmosphere. The 10.7 cm
wavelength solar radio noise flux, F10.7, is one widely-used measure of solar activity.
In addition, the interaction of the solar wind with the Earth’s magnetosphere is
gauged by Ap, the geomagnetic activity index. These two parameters vary throughout
the 11-year solar cycle and are inputs to atmosphere models.

An early atmosphere model is Jacchia 1970 [18], in which atmospheric density
is based on exospheric temperature and altitude. The Marshall Engineering Ther-
mosphere (MET) model, based on the Jacchia 1970 model, is the standard neutral
atmosphere density model used for control and lifetime studies involving all orbiting
spacecraft projects [25]. The MET model is an empirical model whose coefficients
were obtained from satellite data. Inputs to the model are time, position, solar flux,
and geomagnetic index. The exospheric temperature calculated from these inputs is
then used to determine the temperature for any altitude between the lower bound-
ary (90 km) and the upper boundary (2500 km) of the model from an empirically
determined temperature profile [25]. From the temperature, the total mass density
is calculated by summing the individual specie mass densities. The total density is
then further modified to include the effects of orbital and seasonal variations.

2.5 CMG Attitude Control

To counter the disturbance torques acting on the ISS, Control Moment Gyroscopes
(CMGs) are used. These electrically-powered spinning disks serve as angular momen-
tum storage devices. The ISS CMGs are mounted on a double-gimbal system, making it possible to point the momentum vector of each CMG in any direction by choosing appropriate inner and outer gimbal angles. By conservation of angular momentum, gimbaling the CMGs induces a torque on the ISS equal to the resulting change in the total momentum stored by the CMGs. To produce a desired torque on the ISS, inner and outer gimbal rates to command for each CMG are computed using a steering law [12]. Spinning at 6600 rev/min, each ISS CMG has a momentum magnitude of 3600 ft-lbf-sec and can produce a maximum torque magnitude of 50 ft-lbf.

The two ISS attitude control modes are momentum management and attitude hold [13]. In momentum management, the controller finds and follows Torque Equilibrium Attitudes (TEAs), a special combination of states at which the disturbance torques are balanced. Momentum management allows low CMG torque and momentum use [13], and is the primary mode of control for the ISS. On the other hand, the attitude hold controller maintains the commanded attitude, regardless of whether it is a TEA or not, at the cost of higher CMG momentum and torque. As described in the next chapter, the Zero Prop Maneuver (ZPM) transitions the ISS from one particular TEA held by a corresponding momentum manager controller to another. The ZPM itself is executed with the attitude hold controller.

2.5.1 Attitude Hold Controller

The attitude hold controller calculates the torque needed to achieve the commanded attitude at a commanded rate. To do this, the errors between the actual
and commanded attitude and rate are computed. Let the functions \( q_c : \mathbb{R} \mapsto \mathbb{R}^4 \) and \( \omega_c : \mathbb{R} \mapsto \mathbb{R}^3 \) give commanded attitude in quaternions and commanded rate respectively for a particular time. Then the attitude error \( \tilde{\epsilon} \in \mathbb{R}^3 \) and rate error \( \tilde{\omega} \in \mathbb{R}^3 \) are given by

\[
\tilde{\epsilon}(q, q_c) = 2T(q_c)^T q 
\]  \hspace{1cm} (2.12)

\[
\tilde{\omega}(\omega, \omega_c) = \omega - \omega_c,
\]  \hspace{1cm} (2.13)

where \( T \) is defined by equation (2.3). By restricting the commanded rate to be the LVLH rate for the commanded attitude (i.e. the rate necessary to maintain an attitude of \( q_c \) relative to LVLH), the rate command can be written in terms of the attitude command: \( \omega_c = \omega_o(q_c) \). This approach has been used in related work to control the ISS attitude [22], [26].

From these errors, the commanded torque \( u : \mathbb{R} \mapsto \mathbb{R}^3 \) is generated by a Proportional Derivative (PD) controller [39]:

\[
u(t) = J \left( K_P \tilde{\epsilon}(q, q_c) + K_D \tilde{\omega}(\omega, \omega_c) \right),
\]  \hspace{1cm} (2.14)

where \( K_P \) and \( K_D \) are scalar proportional and derivative gains. Then the time derivative of the total CMG momentum with respect to the Body frame is given by

\[
\dot{h}_{cmg}(t) = u(t) - \omega(t) \times h_{cmg}(t).
\]  \hspace{1cm} (2.15)
2.6 Space Station Multi-Rigid Body Simulation

The dynamics and attitude control described in this chapter are incorporated in the Space Station Multi-Rigid Body Simulation (SSMRBS) [31], a high-fidelity tool used at NASA to model ISS flight and operations. This tool represents the ISS as a multiple rigid body system and allows controlled individual motion of solar photovoltaic arrays and thermal control radiators based on the position of the sun. It also utilizes the same flight software as onboard the ISS for attitude control using CMGs and/or thrusters. Many options are available to specify the environment. For example, inputs to models of the gravitational and geomagnetic fields model allow the user to include higher order coefficients (zonal and tesseral harmonics) as well as choose between spherical and oblate Earth models. Also, one can choose from three different atmosphere models, including MET99, a 1999 update to the MET model [31].
Chapter 3

Formulating the Zero Prop Maneuver Problem

This chapter states the Zero Prop Maneuver problem (hereafter, ZPM) for the International Space Station (ISS), formulates it mathematically, and presents the solution method. The first few sections comment on the dynamics, constraints, and cost function. Then the optimal control problem is stated formally and subsequently transcribed to a discrete nonlinear program using the Legendre pseudospectral method. After describing the algorithm to solve the problem, some practical issues are addressed.

3.1 Problem Description

The goal here is to transition the ISS with the Control Moment Gyrosopes (CMGs) from a given set of initial rotational states (attitude, $q$; angular rate, $\omega$; and CMG momentum, $h_{\text{cmg}}$) to some given final rotational states without CMG saturation. Because of the disturbance torques acting on the ISS, two different trajectories starting and ending at the same states with the same maneuver time can have a different CMG momentum profile. Thus an optimal trajectory can exploit this
path dependence to avoid saturating the CMGs. Since for maneuvers with CMGs propellant would only be needed to desaturate the CMGs, a maneuver that avoids saturation would consume no propellant. Hence the name “Zero Prop Maneuver” (hereafter ZPM) is given to this problem.

3.2 Differential Algebraic Equations

Chapter 2 gave the dynamics equations (2.5), (2.6), (2.15), and (2.14) for a rigid, CMG-equipped spacecraft in circular orbit that must be satisfied during the maneuver. These equations of motion become the differential algebraic equations for the ZPM optimal control problem:

\begin{align}
\dot{q}(t) &= \frac{1}{2} T(q)(\omega(t) - \omega_0(q)) \\
\dot{\omega}(t) &= J^{-1}\left( \tau_d(q) - \omega(t) \times (J\omega(t) + h(t)) - \dot{h}(t) \right) \\
\dot{h}(t) &= u(t) - \omega(t) \times h(t) \\
u(t) &= J\left( K_P \tilde{\varepsilon}(q, q_c) + K_D \tilde{\omega}(\omega, q_c) \right),
\end{align}

where \(\|q(t)\|_2 = 1\) and \(\|q_c(t)\|_2 = 1\) must hold for all \(t\) (see Section 2.2.1). Note that the subscript in \(h_{cmg}\) has been dropped here and for the rest of the thesis. Also, substituting equation (3.1c) into (3.1b), the \(-\omega(t) \times h(t)\) term cancels out. In this thesis, \(K_P = 0.000128\) and \(K_D = 0.015846\).
3.3 Boundary Conditions

The boundary conditions constrain the initial and final attitude, angular rate, and CMG momentum. For the problems considered in this thesis, the ZPMs occur between two periods of momentum management. In other words, the ZPM transitions the ISS from one particular Torque Equilibrium Attitude (TEA) maintained by a Momentum Manager (MM) controller, to a different TEA to be held by another MM. Thus the attitude and CMG momentum targets are chosen according to the MM controller used. Since the maneuver is from TEA to TEA, the angular rates at the beginning and end of the maneuver are the LVLH rates $\omega_o$ for the initial and final attitudes. Thus the boundary conditions are

$$
q(t_0) = \bar{q}_0 \\
\omega(t_0) = \omega_o(\bar{q}_0) \\
h(t_0) = \bar{h}_0 \\
q(t_f) = \bar{q}_f \\
\omega(t_f) = \omega_o(\bar{q}_f) \\
h(t_f) = \bar{h}_f
$$

where the bars indicate given constants to be specified, and the subscripts “0” and “f” refer to initial and final respectively. The total time to complete the maneuver must also be pre-selected for the ZPM. Generally, shorter maneuver times require
greater momentum use. A maneuver time of about one orbital period is typically sufficient. For longer maneuver times, analysis of the ISS attitude during the optimal maneuver must be performed to ensure ISS power and thermal restrictions are met (see Section 4.5).

### 3.4 Path Constraints

Four path constraints are necessary. The first two, as discussed in Section 3.2, ensure that the actual and commanded quaternions have unit norm:

\[ \|q(t)\|_2 = 1, \quad \forall t \in [\bar{t}_0, \bar{t}_f] \]  
(3.2)

\[ \|q_c(t)\|_2 = 1, \quad \forall t \in [\bar{t}_0, \bar{t}_f] \]  
(3.3)

The next two come from the physical limitations of the CMGs. Namely, with four active CMGs, the maximum total CMG momentum and torque magnitudes are \( h_{\text{max}} = 14400 \text{ ft-lbf-sec} \) and \( \dot{h}_{\text{max}} = 200 \text{ ft-lbf} \) respectively. Therefore, two inequality path constraints guarantee that the CMG momentum and torque at each point during the maneuver stay at or below their saturation thresholds:

\[ \|h(t)\|_2 \leq h_{\text{max}}, \quad \forall t \in [\bar{t}_0, \bar{t}_f] \]  
(3.4)

\[ \|\dot{h}(t)\|_2 \leq \dot{h}_{\text{max}}, \quad \forall t \in [\bar{t}_0, \bar{t}_f] \]  
(3.5)
Since all of these constraints involve the 2-norm, they are not differentiable at zero. Thus, all path constraints are squared to remedy this.

A related restriction concerns the health of the CMGs. As described in Chapter 2, changing the inner and outer gimbal angles alters the axis of rotation of the CMGs, thereby producing a torque on the ISS. Due to CMG health concerns, low gimbal rates are preferred, with a suggested range of ±1 deg/sec for each CMG gimbal rate. Decreasing the CMG torque magnitude upper bound \( \dot{h}_{\text{max}} \) results in reduced gimbal rates.

### 3.5 Cost Function

Solutions that are as far as possible from CMG saturation are preferred since they provide the most margin. That is, minimizing the peak momentum magnitude leaves the most margin for the CMG controller to expend extra momentum to account for uncertainty in the initial conditions and other parameters as well as inaccuracy in the model. Hence, a good cost function for this problem is the maximum CMG momentum magnitude during the maneuver. Instead of the max function, minimizing a parameter \( \gamma \) which serves as the upper bound to the momentum constraint keeps the cost function differentiable.
3.6 Optimal Control Problem Statement

The preceding sections define the Optimal Control Problem (OCP) as follows:

\[
\begin{align*}
\min & \quad \gamma \\
\text{s.t.} & \\
\dot{q}(t) &= \frac{1}{2} T(q)(\omega(t) - \omega_o(q)), \quad \forall t \in [\bar{t}_0, \bar{t}_f] \\
\dot{\omega}(t) &= J^{-1}(\tau_d(q) - \omega(t) \times (J\omega(t)) - u(t)), \quad \forall t \in [\bar{t}_0, \bar{t}_f] \\
\dot{h}(t) &= u(t) - \omega(t) \times h(t), \quad \forall t \in [\bar{t}_0, \bar{t}_f] \\
u(t) &= J(K_P\tilde{\epsilon}(q, q_c) + K_D\tilde{\omega}(\omega, q_c)), \quad \forall t \in [\bar{t}_0, \bar{t}_f] \\
\|q(t)\|_2^2 &= 1, \quad \forall t \in [\bar{t}_0, \bar{t}_f] \\
\|q_c(t)\|_2^2 &= 1, \quad \forall t \in [\bar{t}_0, \bar{t}_f] \\
\|h(t)\|_2^2 &\leq \gamma, \quad \forall t \in [\bar{t}_0, \bar{t}_f] \\
\|\dot{h}(t)\|_2^2 &\leq \dot{h}_{\text{max}}^2, \quad \forall t \in [\bar{t}_0, \bar{t}_f] \\
q(\bar{t}_0) &= \bar{q}_0 \\
\omega(\bar{t}_0) &= \omega_o(\bar{q}_0) \\
h(\bar{t}_0) &= \bar{h}_0 \\
q(\bar{t}_f) &= \bar{q}_f \\
\omega(\bar{t}_f) &= \omega_o(\bar{q}_f) \\
h(\bar{t}_f) &= \bar{h}_f.
\end{align*}
\]
The variables are the states $q : \mathbb{R} \mapsto \mathbb{R}^4$, $\omega : \mathbb{R} \mapsto \mathbb{R}^3$, and $h : \mathbb{R} \mapsto \mathbb{R}^3$; the controls $q_c : \mathbb{R} \mapsto \mathbb{R}^4$, and the parameter $\gamma$.

Solution methods for optimal control problems fall into two broad classes: indirect and direct. Indirect methods explicitly derive and solve the optimal control necessary conditions using the Pontryagin Maximum Principle [4], [27]. While this approach can lead to very accurate solutions, there are several disadvantages. Among these are the need to find a good initial guess (including one for the costate variables), numerical sensitivity, and difficulty of incorporating path inequalities [2]. In contrast, direct methods transform the optimal control problem into a NonLinear Program (NLP) by discretization and are generally easier to use. There are several types of discretization schemes that can be used [1], [8], [9]. This thesis uses the Legendre pseudospectral collocation method to numerically solve the OCP.

### 3.7 Legendre Pseudospectral Collocation Method

In this section, the Legendre pseudospectral collocation method is applied to the OCP (3.6). This approach approximates the states and controls with Legendre polynomials $P_i$, orthogonal on the interval $[-1, 1]$ with weighting function $w(t) = 1$. The $N^{th}$ degree Legendre polynomial is [6]

$$P_N(t) = \frac{1}{2^N N!} \frac{d^N}{dt^N} (t^2 - 1)^N. \quad (3.7)$$
The states and controls are interpolated at pre-selected node points, and the values at these points become the optimization variables. This method uses the Legendre-Gauss-Lobatto (LGL) points for the nodes, which are the extrema of the Legendre polynomials. The differential algebraic equations (3.1) and path constraints are enforced at these points in what is known as collocation. It is assumed that satisfying the constraints at the node points will be sufficient to satisfy them for the entire interval of interest.

Define $x : \mathbb{R} \rightarrow \mathbb{R}^{10}$ to be the vector of all states $x(t) = [q(t) \ \omega(t) \ h(t)]^T$ for this problem and let $f(x(t), q_c(t))$ represent the corresponding right-hand sides of the differential algebraic equations (3.1) so that $\dot{x}(t) = f(x(t), q_c(t))$. Let $N$ be the degree of the polynomials $x^N$ and $q_c^N$ that interpolate the states $x$ and controls $q_c$ at the LGL points $t_i$. The nodes $t_i$ are in the interval $[-1, 1] = [t_0, t_N]$, so the transformation $\hat{t} : [-1, 1] \mapsto [\bar{t}_0, \bar{t}_f]$ given by $\hat{t}(t) = \frac{(\bar{t}_f - \bar{t}_0)t + (\bar{t}_f + \bar{t}_0)}{2}$ is applied. Thus

$$\frac{dx(\hat{t}(t))}{dt} = \frac{dx(\hat{t}(t))}{d\hat{t}(t)} \frac{d\hat{t}(t)}{dt} = \frac{\bar{t}_f - \bar{t}_0}{2} f(x(\hat{t}(t)), q_c(\hat{t}(t))).$$

(3.8)

In the following, $x, q_c, x^N$, and $q_c^N$ are written simply as functions of $t$ instead of $\hat{t}(t)$.

Now for $t \in [-1, 1]$, the Lagrange basis [6]

$$\phi_i(t) = \frac{1}{N(N + 1)P_N(t_i)} \frac{(t^2 - 1)}{(t - t_i)} \hat{P}_N(t), \quad i = 0, 1, \ldots, N$$
satisfies

\[ \phi_i(t_k) = \delta_{ik} = \begin{cases} 
0 & \text{if } i \neq k \\
1 & \text{if } i = k. 
\end{cases} \]

Thus interpolating

\[ x(t) \approx x_N(t) = \sum_{i=0}^{N} x_i \phi_i(t) \]

and

\[ q_c(t) \approx q_{cN}(t) = \sum_{i=0}^{N} q_{ci} \phi_i(t) \]

with \( x_i = x(t_i) \) and \( q_{ci} = q_c(t_i) \) gives \( x(t_i) = x_N(t_i) \) and \( q_c(t_i) = q_{cN}(t_i) \) as desired.

Differentiation yields \( \dot{x}(t) \approx \dot{x}_N(t) = \sum_{i=0}^{N} x_i \dot{\phi}_i(t) \). This can be written as \( \dot{x}_N(t_k) = \sum_{i=0}^{N} D_{ki} x_i \) for each \( t_k \) by defining the Legendre differentiation matrix

\[ D_{ki} = \begin{cases} 
\frac{P_N(t_k)}{(t_k-t_i)P_N(t_i)} & \text{if } k \neq i \\
-\frac{N(N+1)}{4} & \text{if } k = i = 0 \\
\frac{N(N+1)}{4} & \text{if } k = i = N \\
0 & \text{otherwise.} 
\end{cases} \]

Recalling equation (3.8), the collocation conditions

\[ \dot{x}_N(t_k) = \frac{\bar{t}_f - \bar{t}_0}{2} f(x_k, q_{ck}), \quad k = 0, 1, \ldots, N \quad (3.9) \]

force the approximations to satisfy the differential algebraic equations at each node point.
3.8 Nonlinear Program Statement

Collecting the above transforms the OCP (3.6) into a nonlinear program (NLP):

\[
\begin{align*}
\min_{\gamma} \quad & \gamma \\
\text{s.t.} \quad & \frac{2}{t_f - t_0} \sum_{i=0}^{N} D_{ki} q_i = \frac{1}{2} T(q_k)(\omega_k - \omega_o(q_k)), \quad k = 0, 1, \ldots, N \\
& \frac{2}{t_f - t_0} \sum_{i=0}^{N} D_{ki} \omega_i = J^{-1}\left( \tau_d(q_k) - \omega_k \times (J\omega_k) - u_k \right), \quad k = 0, 1, \ldots, N \\
& \frac{2}{t_f - t_0} \sum_{i=0}^{N} D_{ki} h_i = u_k - \omega_k \times h_k, \quad k = 0, 1, \ldots, N \\
& u_k = J\left( K_P \tilde{\varepsilon}(q_k, q_{c_k}) + K_D \tilde{\omega}(\omega_k, q_{c_k}) \right), \quad k = 0, 1, \ldots, N \\
& \|q_k\|_2^2 = 1, \quad k = 0, 1, \ldots, N \\
& \|q_{c_k}\|_2^2 = 1, \quad k = 0, 1, \ldots, N \\
& \|h_k\|_2^2 \leq \gamma, \quad k = 0, 1, \ldots, N \\
& \frac{2}{t_f - t_0} \sum_{i=0}^{N} D_{ki} h_i \bigg\|^2_2 \leq \dot{h}_{\text{max}}^2, \quad k = 0, 1, \ldots, N \\
& q_0 = \bar{q}_0 \\
& \omega_0 = \omega_o(\bar{q}_0) \\
& h_0 = \bar{h}_0 \\
& q_N = \bar{q}_f \\
& \omega_N = \omega_o(\bar{q}_f) \\
& h_N = \bar{h}_f.
\end{align*}
\]
Here, the optimization variables are the discrete parameters given by the vector $X = [x_0 \ x_1 \ \cdots \ x_N \ q_{c0} \ q_{c1} \ \cdots \ q_{cN} \ \gamma]^T$.

3.9 DIDO, An Optimal Control Solver using the Legendre Pseudospectral Method

The Legendre pseudospectral collocation method discussed above is implemented by the software package DIDO from the Naval Postgraduate School [30]. DIDO solves the NLP resulting from direct transcription of the OCP by calling the Sparse Nonlinear OPTimizer (SNOPT) [10], which employs sequential quadratic programming. To call SNOPT, DIDO uses TOMLAB [14], an optimization environment for MATLAB. The DIDO user interface itself is in MATLAB, and as mentioned elsewhere [26], is easy to use, though the inability to provide analytical derivatives and control convergence criteria for the NLP are disadvantages.

3.10 Implementation Issues

To solve the problem numerically with DIDO, a few items must be chosen carefully. First, the order of the method, $N$ (i.e., the degree of the approximating polynomials or the number of nodes minus one) must be high enough to produce solutions of sufficient accuracy. The simplest method for discovering the proper number of nodes is to start small, check for accuracy by simulating the solution, and then increase the nodes until the simulation agrees satisfactorily with the solution. In this process, one
can start the higher node optimization runs by using the coarse grid solutions as the initial guess. While a solution generated with this grid refinement procedure (with intermediate coarse-grid solutions) could possibly be less optimal than a solution found using only a fine grid, the procedure improves convergence and computation time. In fact, the ZPM problem is very nonlinear, and starting at a fine grid typically results in the optimization not converging to a solution. The initial guess used for the coarsest grid was a linear interpolation between the boundary conditions for the states and zero for the controls. Therefore the guess for the attitude resembles an eigenaxis maneuver, the shortest kinematic path between the initial and final attitude. Besides selecting the number of nodes and initial guess, it is critical to appropriately scale the problem. The variables, constraints, and cost function were scaled to be of roughly unit magnitude by dividing by expected maximum values over the course of the maneuver.
Chapter 4

Solving the Zero Prop Maneuver Problem for the Flight Test

This chapter describes the process leading to the International Space Station (ISS) flight test demonstration of the Zero Prop Maneuver (ZPM) and presents different numerical solutions verified in simulation. The relevant parameters, operational procedures and constraints for the flight test are discussed throughout the chapter. The final section considers the effect of uncertainty in the inertia, initial conditions, and other parameters on the ZPM trajectory.

4.1 Operational Concept

To implement the ZPM, a sequence of attitude and rate pair commands would be determined for an optimal trajectory to transition the ISS between given attitude, rate, and CMG momentum states while limiting peak CMG momentum, torque, and gimbal rates. The Attitude Determination and Control Officer (ADCO) would receive these commands to use as inputs to a software tool which builds a time-tagged command pair sequence for uplink to the Command and Control Multi-
plexer/DeMultiplexer (C&C MDM) computer prior to the maneuver start time. As the C&C MDM command buffer is limited to 200 slots, the ZPM was allocated 160 slots composed of 80 command pairs (attitude and rate). The attitude and rate commands cannot be issued simultaneously with the current flight software and must be separated by at least 1 sec.

The controller sequence for the demonstration is as follows. From an ISS Momentum Manager (MM) controller which maintains a particular Torque Equilibrium Attitude (TEA), at a pre-designated time the ISS attitude hold controller would be activated to perform the ZPM with CMG desaturation inhibited. After the ZPM is complete, another MM with a different target TEA would take over. For the test, it was decided that the ZPM phase must operate under 90% of CMG momentum capacity. For four active CMGs (the nominal case), this gives $0.9 \times 4 \times 3600 = 12960$ ft-lbf-sec. If this limit was reached, the test would be terminated, and thrusters would be fired to complete the maneuver and bring the momentum down.

### 4.2 Demonstration Maneuver Details

An ISS Assembly Stage 12A reorientation on November 5, 2006 was chosen for the ZPM flight test demonstration. Specifically, the ZPM should transition the ISS from an initial TEA with attitude in YPR Euler angles (listed in RPY order) of $[2 \quad -9.75 \quad 13]$ deg and CMG momentum of $[-496 \quad -175 \quad -3892]$ ft-lbf-sec to a final TEA with attitude $[-2.19 \quad -7.88 \quad -89.85]$ deg and CMG momentum.
[\[-9 \quad -3557 \quad -135\]] \text{ ft-lbf-sec.} The initial and final angular rates are the LVLH rates at the initial and final attitudes. At the particular initial attitude listed above, the positive x-axis of the ISS is approximately in the direction of the Velocity Vector (+XVV). Analogously, the final attitude has the ISS positive y-axis roughly in the direction of the Velocity Vector (+YVV). Thus a maneuver from a +XVV TEA to +YVV TEA should essentially amount to a $-90$ deg rotation about the positive z-axis. This observation is verified by the large difference in the initial and final yaw angles of the specified attitudes, with only small differences in the roll and pitch axes.

The ZPM was designed to start at orbit noon, the midpoint in the sunlit phase of the ISS orbit. Since some or all of the solar arrays rotate to track the sun, the atmospheric drag is affected by their movement. Thus starting at different points in the orbit will alter the aerodynamic torque time history. A ZPM trajectory clearly depends on the particular orbit (and position in that orbit) as well as the precise position throughout the maneuver of each of the solar arrays and thermal radiators, described next.

4.2.1 Motion of Articulating Bodies

Figure 2.1 depicts the particular ISS Stage 12A configuration, including docked visiting vehicles, as planned for the flight test demonstration. The motion of the articulating bodies during the maneuver was specified as follows. Refer to Figure 4.1, a photograph of ISS Stage 12A taken two months before the test. Solar arrays and radiators are labeled along with the positive x and y axes. The port and starboard
thermal control radiators were to be fixed at 0 deg (in the x-y plane as shown). For both P4 arrays, let an orientation of 0 deg indicate that the active solar surface (which contains the solar cells) is pointing inboard. Then the aft P4 solar array would be at an angle of 76 deg about the -x axis while the foreword P4 array would be at 281 deg about the +x axis. All other solar arrays would be rotating to track the sun.
Figure 4.1: ISS Assembly Stage 12A (Adapted from [37])
4.3 Trajectory for 6000 sec ZPM with 4 CMGs

The first optimal trajectory presented here is for four active CMGs and takes 6000 sec (just over an orbit) to complete the maneuver. The values of system parameters used to obtain the trajectory are as follows. The orbital rate is \( n = 1.1461 \times 10^{-3} \) rad/sec (corresponding to an altitude of 185 nautical miles), and the inertia matrix \( J \) in slug-ft\(^2\) is given by [24]

\[
J = \begin{pmatrix}
17834580 & 2787992 & 2873636 \\
2787992 & 27738150 & -863810 \\
2873636 & -863810 & 38030467
\end{pmatrix}.
\]

(4.1)

Also, the atmosphere model inputs for the solar radio noise flux (F10.7) and geomagnetic activity index (Ap) are F10.7=71.3 \( (10^4 \text{ Jansky}) \) and Ap=10.1 [29].

The target endpoint values of the states discussed earlier (the +XVV and +YVV TEAs) were used and are given in Tables 4.1 and 4.2. In the tables, the subscripts “0” and “f” refer to, respectively, the initial and final states. Also, the initial and final angular rates are forced to be the LVLH rates \( \omega_o \) at the initial and final attitudes. The initial and final attitudes are given in terms of both YPR Euler angles \( (\vec{e}_0 \text{ and } \vec{e}_f) \) and quaternions.

<table>
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<th>( \vec{e}_0 )</th>
<th>2</th>
<th>-9.75</th>
<th>13</th>
<th>deg</th>
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<tbody>
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<td>( \vec{q}_0 )</td>
<td>0.98966</td>
<td>0.02690</td>
<td>-0.08246</td>
<td>0.11425</td>
</tr>
<tr>
<td>( \omega_o(\vec{q}_0) )</td>
<td>(-2.5410 \times 10^{-4})</td>
<td>(-1.1145 \times 10^{-3})</td>
<td>8.2609 \times 10^{-5}</td>
<td>rad/sec</td>
</tr>
<tr>
<td>( h_0 )</td>
<td>-496</td>
<td>-175</td>
<td>-3892</td>
<td>ft-lbf-sec</td>
</tr>
</tbody>
</table>

Table 4.1: ZPM initial conditions
\[
\begin{array}{|c|c|c|c|}
\hline
\bar{e}_f & -2.19 & -7.88 & -89.85 \text{ deg} \\
\bar{q}_f & 0.70531 & -0.06201 & -0.03518 & -0.70531 \\
\omega_{o}(\bar{q}_f) & 1.1353 \times 10^{-3} & 3.0062 \times 10^{-6} & -1.5713 \times 10^{-4} \text{ rad/sec} \\
h_f & -9 & -3557 & -135 \text{ ft-lbf-sec} \\
\hline
\end{array}
\]

**Table 4.2:** ZPM final conditions

With the above values, a solution was generated with DIDO satisfying the boundary conditions exactly (as well as meeting the other constraints of the problem). However, the dynamics in the optimization are simplified, and the optimal trajectory should be verified in the high-fidelity Space Station Multi-Rigid Body Simulation (SSMRBS). The attitude from the optimization versus SSMRBS is shown in YPR Euler angles in Figure 4.2. The dotted vertical lines separate the maneuver from the

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure4.2.png}
\caption{Optimal and Simulated Attitude for 6000 sec ZPM}
\end{figure}

+XVV MM and +YVV MM phases. Note that the simulation is run long enough

(seven orbits) for the +XVV MM to settle to a steady state (as would normally be the case in flight) before entering the ZPM phase. The MM follows a dynamic TEA, and so the initial states at the start of the ZPM may not exactly match the values used for the boundary conditions in the optimization. These initial errors between optimization and simulation are given in Table 4.3. The attitude error is less than half a degree, the angular rate error is less than half a millidegree per second (mdeg/sec), and the error in initial CMG momentum is less than 300 ft-lbf-sec.

Despite these errors, the ZPM is completed with close agreement between the computed and simulated attitude trajectories. The same can be said of the angular rates, $\omega(t)$ (Figure 4.3). To execute the ZPM in the simulation, 80 pairs of commands consisting of attitude and rate commands were used. These command pairs were issued with equal spacing, resulting in a command update interval of 75 sec for the 6000 sec maneuver. To fully comply with operational restrictions, the attitude command was separated from the rate command by 1 sec, with the rate command coming first. Since the attitude hold controller is designed to follow the commanded attitude and rate closely, the close agreement between simulated attitude and rate is not surprising.

What may not match is the CMG torque and momentum, since the control torque $u(t)$ required to stay on the commanded trajectory could differ from that computed

<table>
<thead>
<tr>
<th>Variable</th>
<th>Roll</th>
<th>Pitch</th>
<th>Yaw</th>
<th>Magnitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>YPR Euler angle error [deg]</td>
<td>0.2</td>
<td>-0.2</td>
<td>-0.0</td>
<td>0.3</td>
</tr>
<tr>
<td>Angular rate error [mdeg/sec]</td>
<td>-0.3</td>
<td>0.2</td>
<td>0.3</td>
<td>0.4</td>
</tr>
</tbody>
</table>

Table 4.3: Initial condition errors between optimization and simulation for 6000 sec ZPM
by the optimization because of the finite number of commands, errors in the initial states at the start of the ZPM (see Table 4.3), and differences in modeling complexity, for example. However, the difference in the control torques, $u(t)$ (Figure 4.4) between DIDO and SSMRBS is small enough in this case. Thus the CMG momentum $(h(t))$ comparison is reasonably close as seen in Figure 4.5. Consequently the CMG momentum magnitude from SSMRBS (9371 ft-lbf-sec) does not go much higher than the peak computed by DIDO (Figure 4.6) and is well below the four CMG saturation threshold of 14400 ft-lbf-sec. As for the CMG torque magnitude path constraint, it is also higher than predicted by optimization (Figure 4.7). But since the CMG torque magnitude is well below its actual capacity of 200 ft-lbf, this is not a problem. The reason for maintaining low CMG torque magnitude was to have low outer gimbal rates. As Figure 4.8 illustrates, the peak outer gimbal rate is 0.49 deg/sec which is less than the limit of 1 deg/sec.

Table 4.4 lists errors between optimization and simulation at the end of the maneuver. Notice this trajectory does not drastically deviate from an eigenaxis maneuver, the initial guess used to obtain this solution. That is, the oscillations about the straight line connecting the initial and final Euler angle for each axis are not very large.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Roll</th>
<th>Pitch</th>
<th>Yaw</th>
<th>Magnitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>YPR Euler angle error [deg]</td>
<td>-0.1</td>
<td>-0.1</td>
<td>-0.0</td>
<td>0.1</td>
</tr>
<tr>
<td>Angular rate error [mdeg/sec]</td>
<td>0.1</td>
<td>-0.0</td>
<td>-0.0</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Table 4.4: Final condition errors between optimization and simulation for 6000 sec ZPM
Figure 4.3: Optimal and Simulated Angular Rates for 6000 sec ZPM

Figure 4.4: Optimal and Simulated Control Torque for 6000 sec ZPM
Figure 4.5: Optimal and Simulated CMG Momentum for 6000 sec ZPM

Figure 4.6: Optimal and Simulated CMG Momentum Magnitude for 6000 sec ZPM
Figure 4.7: Optimal and Simulated CMG Torque Magnitude for 6000 sec ZPM

Figure 4.8: Simulated CMG Outer Gimbal Rates for 6000 sec ZPM
4.4 Trajectories for 7200 sec ZPM with 3 CMGs

About one month before the flight test, problems with one of the CMGs forced NASA to make it inactive and spin it down. This meant the ZPM demonstration would have to be done using only three active CMGs. Consequently, the cutoff value of CMG momentum magnitude for the test became 9720 ft-lbf-sec, leaving almost no margin for the solution presented in Section 4.3 since the peak in that case was 9371 ft-lbf-sec. Obviously, that solution could not handle any uncertainty or modeling error. A new trajectory had to be sought with a peak momentum closer to 6000 ft-lbf-sec if possible to maintain 1 CMG momentum margin (3600 ft-lbf-sec).

To find trajectories with lower peak momentum, two different approaches were taken. First, the solution obtained is by no means globally optimal, and so a different initial guess might result in convergence to a better solution. The other option was to extend the maneuver time. This was done in order to slow the rate of rotation and hence require less momentum use to build up the rate. The resulting solutions discussed below take 7200 sec (compared to 6000 sec for the original solution). The momentum manager phases surrounding the ZPM were simulated in SSMRBS just as before. Also, all simulations in the rest of the thesis have an inactive CMG that is spun down.
4.4.1 Large Roll Rotation Trajectory for 7200 sec ZPM with 3 CMGs

This section shows an optimal trajectory for a 7200 sec maneuver using all the same system parameters and boundary conditions as before (see Section 4.3). This solution, depicted in Figures 4.9-4.15, was obtained by using a different initial guess, producing a trajectory with a large excursion (120 deg) in the roll axis. Again, the dotted vertical lines in the figures separate the ZPM from the +XVV MM and +YVV MM phases. As before, 80 pairs of attitude and rate commands were used. With the longer maneuver time of 7200 sec, the command pair update interval is 90 sec, 15 sec more than the 6000 sec maneuver case.

The peak momentum of 6670 ft-lbf-sec provides 3050 ft-lbf-sec margin with respect to 90% capacity. While the peak outer gimbal rate increased to 0.71 deg/sec, it is still within the acceptable range of ±1 deg/sec. The initial errors between simulation and optimization are the same as before (Table 4.3), while the final errors are listed in Table 4.5. Notice the large oscillations in the simulated control torque (and hence CMG torque magnitude) not present in the optimal control torque. These oscillations are due to the long interval between command updates as well as the large angle excursion. See Section 4.5 for more on this issue.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Roll</th>
<th>Pitch</th>
<th>Yaw</th>
<th>Magnitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>YPR Euler angle error [deg]</td>
<td>−0.1</td>
<td>0.0</td>
<td>−0.0</td>
<td>0.1</td>
</tr>
<tr>
<td>Angular rate error [mdeg/sec]</td>
<td>−0.2</td>
<td>−0.2</td>
<td>−0.1</td>
<td>0.3</td>
</tr>
</tbody>
</table>

**Table 4.5:** Final condition errors between optimization and simulation for 7200 sec ZPM with Large Roll Rotation
Figure 4.9: Optimal and Simulated Attitude for 7200 sec ZPM with Large Roll Rotation

Figure 4.10: Optimal and Simulated Angular Rates for 7200 sec ZPM with Large Roll Rotation
Figure 4.11: Optimal and Simulated CMG Momentum for 7200 sec ZPM with Large Roll Rotation

Figure 4.12: Optimal and Simulated Control Torque for 7200 sec ZPM with Large Roll Rotation
Figure 4.13: Optimal and Simulated CMG Momentum Magnitude for 7200 sec ZPM with Large Roll Rotation
**Figure 4.14:** Optimal and Simulated CMG Torque Magnitude for 7200 sec ZPM with Large Roll Rotation

**Figure 4.15:** Simulated CMG Outer Gimbal Rates for 7200 sec ZPM with Large Roll Rotation
4.4.2 Large Yaw Rotation Trajectory for 7200 sec ZPM with 3 CMGs

Next, Figures 4.16-4.22 show the result of using a different initial guess. Again, all parameters are the same as before. The figures show the ZPM (delineated by dotted vertical lines) with part of the surrounding MM phases. With a peak momentum of 5710 ft-lbf-sec, this is the best solution in terms of momentum margin. The final errors are given in Table 4.6.

![Graph](image)

**Figure 4.16:** Optimal and Simulated Attitude for 7200 sec ZPM with Large Yaw Rotation

<table>
<thead>
<tr>
<th>Variable</th>
<th>Roll</th>
<th>Pitch</th>
<th>Yaw</th>
<th>Magnitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>YPR Euler angle error [deg]</td>
<td>−0.1</td>
<td>0.1</td>
<td>0.0</td>
<td>0.2</td>
</tr>
<tr>
<td>Angular rate error [mdeg/sec]</td>
<td>0.3</td>
<td>−0.6</td>
<td>−0.2</td>
<td>0.7</td>
</tr>
</tbody>
</table>

**Table 4.6:** Final condition errors between optimization and simulation for 7200 sec ZPM with Large Yaw Rotation
Figure 4.17: Optimal and Simulated Angular Rates for 7200 sec ZPM with Large Yaw Rotation

Figure 4.18: Optimal and Simulated CMG Momentum for 7200 sec ZPM with Large Yaw Rotation
Figure 4.19: Optimal and Simulated Control Torque for 7200 sec ZPM with Large Yaw Rotation

Figure 4.20: Optimal and Simulated CMG Momentum Magnitude for 7200 sec ZPM with Large Yaw Rotation
**Figure 4.21:** Optimal and Simulated CMG Torque Magnitude for 7200 sec ZPM with Large Yaw Rotation

**Figure 4.22:** Simulated CMG Outer Gimbal Rates for 7200 sec ZPM with Large Yaw Rotation
4.5 Flight Test Demonstration Final Trajectory for 7200 sec ZPM with 3 CMGs

In addition to the constraints and issues focused on thus far, a few other items require scrutiny before a trajectory attains approval for flight. These include thermal, power, and operational requirements for the ISS. For example, to keep various components of the ISS within their appropriate temperature ranges, any proposed optimal trajectory must be analyzed to ensure the attitude is within a certain envelope. It is also important to avoid staying in orientations that hinder communication efforts or do not allow the solar arrays to get the most sunlight for long periods of time. For the flight test, these constraints eliminated the possibility of using the trajectories in the previous section due to their large angle excursions which would require further analysis in order to be certified for use. It was decided that an optimal trajectory closer to the one in Section 4.3 resembling an eigenaxis maneuver would be allowable for a 7200 sec maneuver time.

As the flight test drew closer, more accurate values of the parameters for the ZPM became available. Thus the altitude was updated to 182 nautical miles, making the orbital rate $n = 1.1475 \times 10^{-3}$ rad/sec. New predicted values for solar radio noise flux, $F_{10.7}=85$ ($10^4$ Jansky), and geomagnetic activity index, $A_p=5$ were taken from the NOAA 27-day forecast [33].
Also, the inertia matrix in slug-ft$^2$ was updated to

$$J = \begin{pmatrix}
18836544 & 3666370 & 2965301 \\
3666370 & 27984088 & -1129004 \\
2965301 & -1129004 & 39442649
\end{pmatrix}.$$

The TEAs tracked by the $+XVV$ MM would reflect the change in mass properties, and so the initial conditions for the ZPM were updated. The new initial attitude and momentum are listed in Table 4.7 along with the angular rate (the LVLH rate for the new attitude). The attitude only changed 0.41 deg in pitch, but the momentum increased by a substantial 1500 ft-lbf-sec in roll. The final conditions were left unchanged.

| $\vec{e}_0$ | 2 | -9.34 | 13 | deg |
| $\vec{q}_0$ | 0.98996 | 0.02650 | -0.07891 | 0.11422 |
| $\omega_o(\vec{q}_0)$ | $-2.5470 \times 10^{-4}$ | $-1.1159 \times 10^{-3}$ | $8.0882 \times 10^{-5}$ | rad/sec |
| $\vec{h}_0$ | 1000 | -500 | -4200 | ft-lbf-sec |

Table 4.7: Updated ZPM initial conditions

Incorporating these adjustments, a new trajectory is plotted in Figures 4.23-4.29. Again, part of the simulated periods of momentum manager control before and after the ZPM (between dotted vertical lines) are shown. Note the similarity to the solution in Section 4.3. The peak momentum is 7832 ft-lbf-sec and the peak outer gimbal rate 0.74 deg/sec. The initial and final condition errors between optimization and simulation are shown in Tables 4.8 and 4.9. As in previous solutions, there are oscillations in the simulated control torques not seen in the optimal control torques.
This behavior stems from the large interval between command pair updates (90 sec) chosen to restrict the total number of (attitude and rate) commands to 160. To verify this, the optimal solution was executed using a command pair update every 5 sec. As seen in Figures 4.33 and 4.35, there is a much better match between simulated and optimal torques (compare with Figures 4.26 and 4.28).

<table>
<thead>
<tr>
<th>Variable</th>
<th>Roll</th>
<th>Pitch</th>
<th>Yaw</th>
<th>Magnitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>YPR Euler angle error [deg]</td>
<td>0.2</td>
<td>0.5</td>
<td>-0.0</td>
<td>0.5</td>
</tr>
<tr>
<td>Angular rate error [mdeg/sec]</td>
<td>0.1</td>
<td>-0.2</td>
<td>0.1</td>
<td>0.2</td>
</tr>
<tr>
<td>CMG momentum error [ft-lbf-sec]</td>
<td>536.</td>
<td>71.</td>
<td>118.</td>
<td>554.</td>
</tr>
</tbody>
</table>

**Table 4.8:** Initial condition errors between optimization and simulation for 7200 sec ZPM Final Trajectory

<table>
<thead>
<tr>
<th>Variable</th>
<th>Roll</th>
<th>Pitch</th>
<th>Yaw</th>
<th>Magnitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>YPR Euler angle error [deg]</td>
<td>-0.0</td>
<td>-0.1</td>
<td>-0.0</td>
<td>0.1</td>
</tr>
<tr>
<td>Angular rate error [mdeg/sec]</td>
<td>0.0</td>
<td>0.4</td>
<td>0.2</td>
<td>0.4</td>
</tr>
</tbody>
</table>

**Table 4.9:** Final condition errors between optimization and simulation for 7200 sec ZPM Final Trajectory
**Figure 4.23:** Optimal and Simulated Attitude for 7200 sec ZPM Final Trajectory

**Figure 4.24:** Optimal and Simulated Angular Rates for 7200 sec ZPM Final Trajectory
Figure 4.25: Optimal and Simulated CMG Momentum for 7200 sec ZPM Final Trajectory

Figure 4.26: Optimal and Simulated Control Torque for 7200 sec ZPM Final Trajectory
Figure 4.27: Optimal and Simulated CMG Momentum Magnitude for 7200 sec ZPM Final Trajectory

Figure 4.28: Optimal and Simulated CMG Torque Magnitude for 7200 sec ZPM Final Trajectory
Figure 4.29: Simulated CMG Outer Gimbal Rates for 7200 sec ZPM Final Trajectory

Figure 4.30: Optimal and Simulated Attitude for 7200 sec ZPM Final Trajectory with command pairs updated every 5 sec
Figure 4.31: Optimal and Simulated Angular Rates for 7200 sec ZPM Final Trajectory with command pairs updated every 5 sec

Figure 4.32: Optimal and Simulated CMG Momentum for 7200 sec ZPM Final Trajectory with command pairs updated every 5 sec
Figure 4.33: Optimal and Simulated Control Torque for 7200 sec ZPM Final Trajectory with command pairs updated every 5 sec

Figure 4.34: Optimal and Simulated CMG Momentum Magnitude for 7200 sec ZPM Final Trajectory with command pairs updated every 5 sec
Figure 4.35: Optimal and Simulated CMG Torque Magnitude for 7200 sec ZPM Final Trajectory with command pairs updated every 5 sec
4.6 Robustness of Final Trajectory to Parameter Uncertainties

There is of course no guarantee that all of the assumptions for the ZPM demonstration will hold when the optimal trajectory is actually flown. To evaluate robustness of the optimal trajectory to uncertainties in various parameters, the final solution (from the previous section) was simulated using perturbed parameters selected from a uniform distribution. These uncertain parameters included altitude, position in orbit at ZPM startup (initial angle since orbit noon), solar radio noise flux, geomagnetic activity index, initial conditions, and inertia. For each of these, the optimal trajectory was run in SSMRBS with 1000 perturbations of only that particular uncertain parameter to isolate its impact. In all cases, only the ZPM phase was simulated with SSMRBS initialized to the ZPM initial conditions (except of course, when the uncertainties being considered were in the initial conditions themselves). Thus the errors at the transition from +XVV MM to ZPM seen before were not present in the following.

The first case considered is altitude. Figures 4.36, 4.37, and 4.38 plot the peak CMG momentum magnitude, CMG final momentum error magnitude, and peak CMG outer gimbal rate against the altitude for each of the 1000 samples. The altitude was chosen from a uniform distribution with range 177 to 187 nautical miles (nmi). Recall the nominal value (used to design the trajectory) was 182 nmi, the solid vertical line in the figures. The peak momentum and gimbal rate do not vary greatly with changes
in altitude, but the final momentum error is rather large for lower altitudes. The final attitude and rate errors are insignificant as usual and not shown. The peak gimbal rate exhibits a jump discontinuity at about 181 nmi. This behavior appears to be due to discontinuities in the steering law [12] used to generate the gimbal rates. Certain criteria trigger a gimbal redistribution event resulting in a jump discontinuity in the gimbal rates therefore changing the peak gimbal rate.

![Figure 4.36](image)

**Figure 4.36:** Peak CMG Momentum Magnitude versus Altitude for ZPM Final Trajectory, 1000 samples

Next, the robustness to starting the ZPM at a different point in the orbit can be seen in Figures 4.39-4.41. Here, the initial angle since orbit noon was varied from −45 to 45 deg. Since the trajectory was designed to start at orbit noon, the nominal value is 0 deg. Starting earlier than orbit noon produced slightly lower momentum peaks, but higher final momentum error. The opposite can be said for starting later.
Figure 4.37: Final CMG Momentum Error Magnitude versus Altitude for ZPM Final Trajectory, 1000 samples

Figure 4.38: Peak CMG Outer Gimbal Rate versus Altitude for ZPM Final Trajectory, 1000 samples
The best results were around 0 to 10 degrees after orbit noon.

![Figure 4.39: Peak CMG Momentum Magnitude versus Initial Angle Since Orbit Noon for ZPM Final Trajectory, 1000 samples](image)

Next, the effects of the atmospheric model input parameters are displayed. The results of varying solar radio noise flux, F10.7, (measured in units of $10^4$ Jansky) between 65 to 95 (nominally 85) are shown in Figures 4.42-4.44. For the geomagnetic activity index, Ap, a range of 0 to 40 (nominally 5) was used to produce Figures 4.45-4.47. The peak momentum ranges for both parameters are bigger than the previous cases but still only about a 1000 ft-lbf-sec. The final momentum error increases rapidly with increasing Ap. The F10.7 and Ap ranges were chosen according to the predicted variance in solar activity.

Discrepancies in the predicted altitude, mass properties, environmental conditions (e.g. F10.7, Ap) could all contribute to errors in the initial states at the start of the
Figure 4.40: Final CMG Momentum Error Magnitude versus Initial Angle Since Orbit Noon for ZPM Final Trajectory, 1000 samples

Figure 4.41: Peak CMG Outer Gimbal Rate versus Initial Angle Since Orbit Noon for ZPM Final Trajectory, 1000 samples
Figure 4.42: Peak CMG Momentum Magnitude versus Solar Radio Noise Flux for ZPM Final Trajectory, 1000 samples

Figure 4.43: Final CMG Momentum Error Magnitude versus Solar Radio Noise Flux for ZPM Final Trajectory, 1000 samples
Figure 4.44: Peak CMG Outer Gimbal Rate versus Solar Radio Noise Flux for ZPM Final Trajectory, 1000 samples

Figure 4.45: Peak CMG Momentum Magnitude versus Geomagnetic Activity Index for ZPM Final Trajectory, 1000 samples
Figure 4.46: Final CMG Momentum Error Magnitude versus Geomagnetic Activity Index for ZPM Final Trajectory, 1000 samples

Figure 4.47: Peak CMG Outer Gimbal Rate versus Geomagnetic Activity Index for ZPM Final Trajectory, 1000 samples
ZPM since +XVV MM searches for TEAs based on the environment. To test robustness to initial state errors, the initial attitude, angular rate, and CMG momentum errors were each treated separately. Note that each of these three cases perturbs a vector rather than a scalar quantity, but the results are plotted against a single variable, the magnitude of the initial error.

An error range of 3 deg per axis was used for the results in Figures 4.48-4.50, which plot the values of interest against the magnitude (2-norm) of the initial attitude error. The graphs indicate the ZPM trajectory can tolerate an attitude error magnitude of 3 deg without saturation, but the gimbal rates will be greater than 1 deg/sec. From 1000 samples, 5% reached 90% capacity for three CMGs, and 11% had gimbal rates violating the limit. The inner gimbal rates, usually less than 0.5 deg/sec, became larger than 1 deg/sec as well. Also, 1% of samples actually passed 99% of capacity and thus saturated the CMGs in the simulation and did not finish the maneuver. With the exception of a handful of outliers (due to the instances of saturation), the final momentum error is not large.

Next, the uncertainty in the angular rate was set to 5 mdeg/sec per axis to produce Figures 4.51-4.53. Similar comments apply here as well, but this time 46% and 16% of samples hit 90% and 99% capacity respectively. It appears an initial rate error magnitude of 3 mdeg/sec or less is acceptable (with no other errors present).

The last initial condition error case is the initial CMG momentum, with samples of up to 2000 ft-lbf-sec in each axis (Figures 4.54-4.56). The relationship between
Figure 4.48: Peak CMG Momentum Magnitude versus Initial Attitude Error Magnitude for ZPM Final Trajectory, 1000 samples

Figure 4.49: Final CMG Momentum Error Magnitude versus Initial Attitude Error Magnitude for ZPM Final Trajectory, 1000 samples
Figure 4.50: Peak CMG Outer Gimbal Rate versus Initial Attitude Error Magnitude for ZPM Final Trajectory, 1000 samples

Figure 4.51: Peak CMG Momentum Magnitude versus Initial Angular Rate Error Magnitude for ZPM Final Trajectory, 1000 samples
Figure 4.52: Final CMG Momentum Error Magnitude versus Initial Angular Rate Error Magnitude for ZPM Final Trajectory, 1000 samples

Figure 4.53: Peak CMG Outer Gimbal Rate versus Initial Angular Rate Error Magnitude for ZPM Final Trajectory, 1000 samples
initial momentum error magnitude and peak momentum is close to linear, as the peak momentum seems to be just the initial error added to the nominal peak momentum. The gimbal rates all stay below 0.9 deg/sec, and the final momentum error is again well-correlated with the initial momentum error. The peak momentum reached 90% of capacity for 12% of samples.

![Figure 4.54: Peak CMG Momentum Magnitude versus Initial CMG Momentum Error Magnitude for ZPM Final Trajectory, 1000 samples](image)

Finally, uncertainty in inertia of up to 5% in each element was considered. That is, entries of the inertia matrix were individually scaled by factors ranging from 0.95 to 1.05. Figures 4.57-4.59 plot the quantities of interest versus the average percent error over all elements of the inertia matrix. The peak momentum reached 90% capacity for 20% of the samples. While the gimbal rates are acceptable, the final momentum errors are substantially large.
Figure 4.55: Final CMG Momentum Error Magnitude versus Initial CMG Momentum Error Magnitude for ZPM Final Trajectory, 1000 samples

Figure 4.56: Peak CMG Outer Gimbal Rate versus Initial CMG Momentum Error Magnitude for ZPM Final Trajectory, 1000 samples
Overall, the trajectory was least robust to changes in inertia and initial conditions, especially attitude and rate.

Figure 4.57: Peak CMG Momentum Magnitude versus Inertia Scaling for ZPM Final Trajectory, 1000 samples
Figure 4.58: Final CMG Momentum Error Magnitude versus Inertia Scaling for ZPM Final Trajectory, 1000 samples

Figure 4.59: Peak CMG Outer Gimbal Rate versus Inertia Scaling for ZPM Final Trajectory, 1000 samples
Chapter 5

Flight Test Demonstration

The Zero Prop Maneuver (ZPM) flight test demonstration performance is shown in this chapter. Predicted and flight results are compared.

On November 5, 2006, the Zero Prop Maneuver started at GMT 15:57 and ended at GMT 17:57. The demonstration was successful, as revealed by screenshots (Figures 5.2-5.3) of flight data taken in the Mission Evaluation Room at Johnson Space Center shortly after entering +YVV momentum management control with no problems. The optimal trajectory reoriented the ISS 90 deg in two hours without using any propellant.

The commanded and actual attitude of the ISS are shown in Figure 5.1. The gaps in the data are due to expected loss of signal periods. Figure 5.2 shows the CMG momentum (including magnitude and percentage of capacity). The CMG momentum magnitude only reached 7557 ft-lbf-sec, just under 70% capacity for three CMGs. The CMG inner and outer gimbal rates are depicted in Figure 5.3, with the third CMG (showing noisy data) inactive. The maximum outer gimbal rate was 0.55 deg/sec. These momentum and gimbal rate peaks were even lower than predicted in
simulation. Figures 5.4-5.10 compare the flight test performance against simulation and optimization results. The initial and final errors between flight and optimization are in Tables 5.1 and 5.2.

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<thead>
<tr>
<th>Variable</th>
<th>Roll</th>
<th>Pitch</th>
<th>Yaw</th>
<th>Magnitude</th>
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<tr>
<td>YPR Euler angle error [deg]</td>
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<td>0.5</td>
<td>0.6</td>
<td>0.6</td>
</tr>
<tr>
<td>Angular rate error [mdeg/sec]</td>
<td>−0.3</td>
<td>0.4</td>
<td>−0.1</td>
<td>0.5</td>
</tr>
<tr>
<td>CMG momentum error [ft-lbf-sec]</td>
<td>15.</td>
<td>−223.</td>
<td>−175.</td>
<td>284.</td>
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**Table 5.1:** Initial condition errors between flight and optimization for ZPM Flight Test

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<th>Magnitude</th>
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</thead>
<tbody>
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<td>−0.1</td>
<td>−0.0</td>
<td>0.1</td>
</tr>
<tr>
<td>Angular rate error [mdeg/sec]</td>
<td>0.1</td>
<td>0.2</td>
<td>0.0</td>
<td>0.2</td>
</tr>
</tbody>
</table>

**Table 5.2:** Final condition errors between flight and optimization for ZPM Flight Test
Figure 5.1: ZPM Flight Test Commanded and Actual Attitude
Figure 5.2: ZPN Flight Test CMG Momentum
Figure 5.3: ZPM Flight Test Outer Gimbal Rates

CMG Gimbal Rates (deg/s)

1 - CMG1 M031GRate 2 - CMG1_M030GRate

Z1GC02FC0186R
Z1GC02FC0176R
0.004
0.003

Z1GC04FC0186R
Z1GC04FC0176R
0.015

Z1GC06FC0186R
Z1GC06FC0176R
0.008

Z1GC08FC0186R
Z1GC08FC0176R
0.000
-0.021

309:16:09.38 309:16:08.38 309:15:56.00
-0.54600
-0.54600
-0.54600

-0.00871
-0.00871
-0.00871

309:17:31.05 309:17:31.05 309:17:31.05
0.03452
0.03452
0.03452

309:17:54.28 309:17:54.28 309:17:54.28
0.11379
0.11379
0.11379

309:18:40.00 309:18:40.00 309:18:40.00
309:18:40.00
309:18:40.00
309:18:40.00
309:18:40.00
309:18:40.00
309:18:40.00
309:18:40.00
309:18:40.00
309:18:40.00
Figure 5.4: Optimal, Simulated, and Flight Attitude for ZPM Flight Test

Figure 5.5: Optimal, Simulated, and Flight Angular Rates for ZPM Flight Test
Figure 5.6: Optimal, Simulated, and Flight CMG Momentum for ZPM Flight Test

Figure 5.7: Optimal, Simulated, and Flight Control Torque for ZPM Flight Test
Figure 5.8: Optimal, Simulated, and Flight CMG Momentum Magnitude for ZPM Flight Test

Figure 5.9: Optimal, Simulated, and Flight CMG Torque Magnitude for ZPM Flight Test
Figure 5.10: Simulated and Flight CMG Outer Gimbal Rates for ZPM Flight Test
Chapter 6

Conclusion

This thesis considered the problem of performing rotational maneuvers of spacecraft by only using gyroscopes. To keep the gyroscopes within operational limits and thus avoid the need to fire thrusters to maintain control, optimal trajectories were found which account for external disturbance torques during the maneuver. In particular, by executing the optimal commands developed in this thesis, a large-angle reorientation (about 90 deg) of the International Space Station (ISS) was demonstrated in flight with only three active Control Moment Gyroscopes (CMGs) and no propellant burned. This is of great practical importance as propellant is costly to get into orbit and replenish and takes up limited payload space. Not only that, in the case of the ISS, these non-propulsive maneuvers have the added benefit of not having any negative impact due to thruster firings. For example, thruster firings stress the ISS structure and gradually contaminate the solar arrays. Thus non-propulsive maneuvers reduce lifetime solar array erosion as well as constraints on solar array operations since the arrays do not have to be pre-positioned to avoid contamination. Also, this technique did not require any changes to the flight software. Ultimately,
these types of maneuvers can extend the operational lifetime of spacecraft equipped with gyroscopes.

Chapter 2 reviewed spacecraft attitude dynamics for a rigid body in circular orbit and discussed CMG attitude control for the ISS. The equations of motion presented were then used to formulate the Zero Prop Maneuver (ZPM) optimal control problem in Chapter 3. The ZPM problem is to use CMGs to transition from one given set of rotational states (attitude, angular rate, and CMG momentum) to another in a fixed amount of time while operating within the momentum and torque capacity of the CMGs. By doing so, CMG control is sufficient for the entire maneuver, and no propellant is necessary. The peak CMG momentum magnitude was minimized to allow margin for greater momentum use without saturation in the presence of parameter uncertainties and modeling errors. Robustness to these uncertainties for the optimal trajectory used for the flight test was studied in Chapter 4. Flight test demonstration results were presented in Chapter 5.

Future work will include streamlining the ZPM trajectory optimization process and putting these types of maneuvers into regular operational use. Another suggestion is to design trajectories that are robust to parameter uncertainty.
Bibliography


