RICE UNIVERSITY

ARTIFICIAL HEATING OF THE LOWER IONOSPHERE

BY

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ABSTRACT

Artificial Heating of the Lower Ionosphere

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The lower ionosphere has been heated in a controlled experiment at the Arecibo Ionospheric Observatory. Deposition of energy from a 40 Mhz transmitter has approximately doubled the ambient electron temperature at an altitude of 80 km. Energy from the radio wave is deposited in the ionosphere as a result of collisions of the electrons with the neutral molecules. Cross-modulation experiments (performed elsewhere) have gained information on electron number density and collision frequency between 60 and 90 km. The experiment described here will possibly allow determination of the electron collision frequency and thermal relaxation times between 70 and 120 km, and may increase the understanding of the interaction of plasmas and radio waves. The heating is measured with the 430 Mhz incoherent backscatter radar system. The temperature dependence of the electron backscatter cross-section enables the change in electron temperature to be inferred from backscatter power measurements. Heating with megawatt pulses of 1, 2 and 5 ms duration confirmed predictions of the magnitude of heating.

ANTENNA PATTERN
Ionospheric Heating Experiment

FRONTPIECE

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Artificial Heating of the Lower Ionosphere

I. INTRODUCTION

1. Introduction

This thesis describes an experiment designed to modify the ionosphere by deposition of energy from a powerful transmitter. The response of the ionosphere may allow determination of several ionospheric parameters, and increase the understanding of the interaction of plasmas and radio waves.

The 40 Mhz transmitter at the Arecibo Ionospheric Observatory is used to deposit energy in the lower ionosphere. The direct effect is primarily to increase the electron temperature. The 430 Mhz radar backscatter system is used to measure the change in temperature. The front-piece shows the antenna pattern for the heating experiment. Long pulses (1 to 5 ms) of the 40 Mhz transmitter significantly increase the electron temperature in the D and E regions. Since the effective radar backscatter cross-section is a function of electron temperature, it is possible to detect a change in the returned power in the 430 Mhz system due to the 40 Mhz heating pulse.

The experiment divides into two theoretical questions: predicting the heating, and deducing the heating from the backscatter data.

2. The Ionosphere

The ionosphere is the weakly ionized plasma surrounding the earth. It has been studied since the introduction of radio by propagation measurements, by ionospheric sounders (ionosondes), and more recently by rockets, satellites and incoherent backscatter.

The ionosphere is produced primarily by the ultra-violet and X-radiation of the sun. This radiation both dissociates and ionizes the molecular species in the atmosphere. The production of electrons and ions is balanced by recombination, attachment and diffusion processes, and rough equilibrium conditions exist. The regular diurnal, seasonal, and solar cycle variations and the occasional occurrences of solar eclipses and magnetic storms allow the ionosphere to be studied as it changes from equilibrium. Artificial disturbances, such as the Luxembourg effect, the explosion of atomic devices, and the heating experiment described here also permit investigation of the response of the ionosphere to non-equilibrium conditions.

Two sources explaining processes and observations in ionospheric physics are a book titled Physics of the Lower Ionosphere by Whitten and Poppoff (1965), and a review article by Donahue (1968) in Science titled "Ionospheric Composition and Reactions." The book gives a good theoretical discussion, and the article gives recent reaction rates and their implications. Both suffer, however, by neglecting to mention the technique of incoherent backscatter, which is discussed in section III.

Figure I-1 presents three parameters of importance to this experiment: the electron number density, the electron collision frequency, and the atmospheric temperature.

AVERAGE IONOSPHERIC CONDITIONS

FIGURE 1-1

Table I shows in addition the collision relaxation time and the Debye length. The collision relaxation time is the e-folding time required for heated electrons to return to thermal equilibrium with the neutral atmosphere. The Debye shielding length is the shortest scale size over which gross charge neutrality is to be expected. The values given in Figure II-1 and Table I are representative of average global ionospheric conditions and are given as a guide rather than as a definitive model. The electron density profile is a composite of several sources including results from the Arecibo Ionospheric Observatory; the collision frequencies above 100 km are taken from the Satellite Environmental Handbook (Johnson et al., 1964) and below 100 km from Thrane and Piggott (1966); the neutral atmospheric temperature is from CIRA curves for summer mean solar conditions.

An example of electron density profiles taken at Arecibo is given in Figure I-2, which shows profiles taken during sunrise (LaLonde, 1966). The heating experiment requires significant ionization at low altitudes, and so it is necessary to use only the daylight hours. Figure I-3 shows ion-temperatures and ion-collision frequencies as measured at Arecibo by Wand (1968). Wand has deduced from his data that the ion, electron, and neutral temperatures are all identical below 130 km, an important result for the heating experiment. The relevance of ion-collision frequency to this experiment lies in the complication it introduces into incoherent back-scatter theory, and is discussed in section III-4.

Cross-modulation studies (the Luxembourg effect) have given reasonable values of electron collision frequency in the region of 60 to 90 km. Predictions of collision-frequencies based upon laboratory measured cross-sections and

TABLE I

Values of Selected Ionospheric Parameters

Altitude z, km	Electron No. Density		Electron Temperature		Electron Coll. Freq.		Coll. Relax. Time		Debye Length	
	Day n, cm^{-3}	Night n, cm^{-3}	Day $T, ^\circ\text{K}$	Night $T, ^\circ\text{K}$	Day ν_M, s^{-1}	Night ν_M, s^{-1}	Day τ_O, ms	Night τ_O, ms	Day D, cm	Night D, cm
50	<10		250	250	6×10^7	6×10^7	.003	.003	>35	
60	50	<10	230	230	2×10^7	2×10^7	.01	.01	15	>35
70	80	<20	210	210	5×10^6	5×10^6	.04	.04	11	>22
80	400	<100	200	200	10^6	10^6	0.2	0.2	5	>10
90	5×10^3	700	200	200	2×10^5	2×10^5	1	1	1.4	3.7
100	5×10^4	10^3	230	230	2×10^4	2×10^4	10	10	0.47	3.3
110	10^5	2×10^3	300	300	10^4	10^4	20	20	.38	2.7
120	3×10^4	10^3	350	350	6×10^3	5×10^3	40	40	.46	4.4
130	10^5	500	400	400	4×10^3	3×10^3	50	66	.43	6.2
150	1.5×10^5	300	500	500	2×10^3	10^3	100	200	.40	8.9
200	3×10^5	10^3	1100	900	500	100	400	2000	.42	6.6
250	3×10^5	3×10^4	1200	950	700	140	280	1400	0.27	1.2
300	10^6	2.5×10^5	1300	1000	900	400	220	500	0.25	0.44
350	8×10^5	2.2×10^5	1350	1000	800	350	250	400	0.28	0.46
400	6×10^5	2×10^5	1400	1000	600	300	330	670	0.33	0.48
500	3×10^5	9×10^4	1400	1000	300	180	670	1100	0.47	0.73

TABLE 1

Selected electron density
profiles during sunrise

FIGURE 1-2

Measured Ion Temperatures and Momentum-Transfer Collision Frequencies of Ions and Neutral Particles on 5 September 1967.

FIGURE 13

model atmospheres agree with the data in this altitude range. Above 90 km cross-modulation experiments won't work (since the collision frequency is too small) and the best measurements so far are wave propagation experiments carried on rockets. These measurements, as shown in Figure I-4, are not in good agreement with each other or with the predictions (Thrane and Piggott, 1966).

Comparison of measured values of ν_M with calculated values.

FIGURE 14

3. The Luxembourg Effect

"In 1933 it was observed that reception at Eindhoven of the wireless programme from Beromünster was frequently marred by a weak background of the programme from Luxembourg . . . and the suggestion was made that it was due to interaction between the two waves as they passed through the ionosphere. The phenomenon became known as the 'Luxembourg effect' or 'wave interaction'" (Huxley and Ratcliffe, 1949).

Since the accidental discovery described above, several experiments designed to learn more about the lower ionosphere have used powerful transmitters to disturb this region.

The Luxembourg effect forced a change in outlook. Until then the only question asked was how did the Electric layer, now the E-layer, affect electromagnetic radiation. After Luxembourg it became necessary to also ask how an EM wave changed the E-region. Bailey and Martin (1934) gave the correct explanation for the Luxembourg effect. They suggested that the impressed EM wave changed the instantaneous electron collision frequency and hence the amount of attenuation suffered by the wanted signal as it passed near the powerful disturbing transmitter. Figure I-5 shows the geometry of this phenomenon as it was originally reported in Europe.

Fejer (1955) described a method for obtaining electron density and collision frequency profiles using the cross-modulation technique. Ferraro and Lee (1967) have presented data on collision frequency between 50 and 90 km using a high power interaction facility. The method is shown in Figure I-6. The amplitude of a disturbed and undisturbed train of wanted waves is compared while changing the interaction altitude. By assuming a scale height for the electron collision frequency, it is possible to estimate separately the electron density and collision frequency.

THE LUXEMBOURG EFFECT

FIGURE 1-5

FIGURE 1-6

4. Riometer Technique

The relative ionosphere opacity instrument, the riometer, has given a large amount of data about the D-region. The riometer is a receiver tuned to a frequency above the penetration frequency which observes cosmic sky background noise after being attenuated by the lower ionosphere. In section II it will be shown that attenuation of radio waves is proportional to nv/ω^2 . If a model for the electron collision frequency ν is assumed, then an estimate of the electron number density n at low altitudes can be made. The wave frequency ω must be sufficiently small (usually below 30 Mhz) to allow significant attenuation. This technique is used for polar storm studies when large numbers of electrons are suddenly introduced into the D-region.

In an interesting experiment at the Jicamarca Observatory, Klemperer (1963) modulated the attenuation of the cosmic noise background at 50 Mhz. His experiment used a 3 msec 4MW disturbing pulse which heated the D and E region. This decreased the cosmic noise received as long as the ionosphere remained heated. Figure I-7 shows the decrease in received signal power following the heating pulse. The delay between the end of the pulse and the minimum noise is merely due to the time required for the signal to reach an altitude where significant absorption occurs, and then for that reduced noise level to reach the ground. The effect seems to decay rapidly at first, and then to have a much longer recovery constant.

Recovery of 50 Mc/s sky brightness temperature after 4 megawatt, 3 millisecond disturbing pulse.

FIGURE 1-7

5. Suggestions for Heating the Ionosphere

Several authors have discussed altering the ionosphere with powerful radio waves. V. A. Bailey (1959) has suggested an experiment where a "powerful extraordinary circular gyro-wave in the nocturnal lower E-region and in the day-time D-region . . . could produce a glow discharge which is about fifty times as bright as the night sky on a moonless night." He suggests using 500 kw of radiated power with a 400 dipole antenna. In an article titled "Alteration of the Electron Density of the Lower Ionosphere with Ground Based Transmitters," P. P. Lombardini (1964) suggests that using the Arecibo dish, breakdown (further ionization) of the atmosphere at 80 km could be achieved with a 220 Mw 40 Mhz transmitter!

On a less spectacular scale, D. T. Farley (1963) describes an experiment where a powerful radio wave just above the penetration frequency heats the F-region. That experiment requires a tuneable transmitter to adjust to the local penetration frequency, and then as the heated plasma expands, to follow the penetration frequency while the number density decreases. In the F-region heat is lost by the electrons not only to collisions but also by conduction along the magnetic field lines.

Molmud (1964) has postulated that the energy dependence of the electron attachment coefficient might change the electron number density in the D-region when disturbed by a radio wave. At altitudes below 65 km a majority of negative charges are electrons attached to atoms (negative ions). The ratio of free to attached electrons is a function of the attachment coefficient, and as the free electrons are heated, they can be driven onto molecular oxygen, thus reducing the

free electron number density. This experiment requires a much greater electron number density at D-region altitudes than is normally present. Molmud suggests that under the disturbed condition of an atmospheric atomic blast, which would allow his experiment to work (as well as blocking normal communications), his process might "provide a channel of decreased electron density in the D-region through which a wanted wave may propagate with greatly reduced attenuation."

II. THEORY OF HEATING

1. Introduction

The principle effect of deposition of radio-wave energy in the ionosphere is an increase in the electron temperature. The electrons are heated as a result of suffering collisions while immersed in the radio wave. The ordered velocity induced by the wave in the electrons is transformed into a random super-thermal velocity after a collision. An electron which shared energy with the wave in half of a wave cycle cannot return the energy the next half if the excess velocity, as a result of a collision, is no longer in the plane of the E-vector. The collision can be either elastic or non-elastic; what is important is that the momentum rather than the energy of the electron be changed.

2. Deposition of Energy

The rate at which energy is taken out of a radio-wave and absorbed by a plasma can be calculated most directly by the volumetric ohms law equation

$$Q_{\Omega} = \frac{1}{2} \operatorname{Re}\{\sigma\} E^2 . \quad \text{II-1}$$

Q_{Ω} is the power absorbed per unit volume in watts/ M^3 , σ is the conductivity in mhos/ M , and E is the electric field strength in volts/ M . The factor of one-half arises by considering E to be the maximum value of a sinusoidally varying field, and then averaging over one period. Only the real part of the conductivity contributes to the losses; the imaginary part propagates the wave.

Due to the earth's magnetic field, the ionospheric

plasma is nonisotropic, and hence σ in general is a tensor. The significant parameters to compare to determine whether a tensor approach is necessary are the electron gyro frequency and the frequency of the heating wave. If the wave frequency is much greater than the gyro frequency, the disturbed electron will only complete a small portion of a gyration before the driving E-field reverses polarity, so the magnetic field effects are negligible. In this experiment, the heating wave is 40 Mhz, while the electron gyro frequency is near 1 Mhz, so σ is considered to be a scalar. With a lower frequency heating wave it might be possible to notice the birefringent properties of σ by transmitting alternately right or left circular polarizations.

3. Conductivity

To find the form of the conductivity, we use the classical Appleton-Hartree magnetoionic theory. This theory includes the effects of collisions between the electrons and neutrals by introducing a friction term in the equation of motion of the electrons (the Langevin equation). Neglecting the magnetic field, the conductivity becomes the simple Drude or zero-field conductivity

$$\sigma = \frac{ne^2}{m(\nu_{AH} - i\omega)} \quad \text{II-2}$$

Here n is the electron number density, e is the electron charge, m the electron mass, ν_{AH} the Appleton-Hartree collision frequency, ω the angular frequency of the radio-wave. The real part becomes

$$\text{Re}\{\sigma\} = \frac{ne^2\nu_{AH}}{m(\nu_{AH}^2 + \omega^2)} \approx \frac{ne^2\nu_{AH}}{m\omega^2} \quad \text{II-3}$$

and the approximation holds for altitudes greater than 70 km when ω is 40 Mhz.

The Appleton-Hartree theory assumes that the electron collision frequency is independent of electron energy, an assumption that the Luxembourg effect has shown to be false. The Sen-Wyller (1960) modification of the Appleton-Hartree theory assumes that the collision frequency is proportional to energy (in agreement with recent laboratory experiments), and that the electron velocities are given by the Maxwell-Boltzmann distribution.

The Sen-Wyller approach shows that in the limiting case of $\nu_{AH} \ll \omega$, the form of the conductivity remains the same, but the Appleton-Hartree formula overestimates by a factor of $5/2$ the collision frequency. Hence the absorption is underestimated by the same factor for a given collision frequency. The physical meaning of this discrepancy is that a small number of electrons in the high energy tail of the velocity distribution are more influential than the bulk of electrons near the peak of the distribution.

Thrane and Piggott (1966) report their collision frequencies in terms of ν_M , the collision frequency for a mono-energetic electron gas at energy kT . ($\nu_{AH} = 5/2\nu_M$). The current literature is reporting ν_M for uniformity.

The mono-energetic collision frequency ν_M is proportional to the electron collision cross-section, the electron velocity, and the neutral gas density. For extremely low temperatures, below $12^\circ K$, the cross-section appears to be constant, but for reasonable ionospheric temperature the cross-section experimentally seems to be $\propto T^{\frac{1}{2}}$, hence $\nu_M \propto T$ (Phelps, 1960).

4. The Magnitude of the Heating Wave

Evaluation of the magnitude of the E-field in the heating wave requires knowledge of the gain of the antenna and the power of the transmitter.

The peak gain of the antenna at 40 Mhz was measured with a drift scan through the radio source Taurus at a zenith angle of 3.7° . The effective aperture was calculated by Watkins (1967) to be $A_e = 2.2 \times 10^4 \text{ m}^2$, which corresponds to an aperture efficiency of 30%. The half-power beam width is 2.1° , and the dish is fed with a set of four 3 element Yagis. The error of the peak gain measurement is guessed by the author to be no greater than 10%.

The 40 Mhz transmitter was usually operated at 1 Mw peak power, which is half the rated (but unobtainable) power. The transmitted power delivered to the feed was calibrated by calorimetry techniques using a power splitter on the feed platform, and the 40 Mhz power, P_{40} , should be accurate to 5%.

The antenna power equation in Mks units is

$$\frac{1}{2} \left(\frac{\epsilon_0}{u_0} \right)^{\frac{1}{2}} E^2(z) = \frac{P_{40} G}{4\pi z^2} \frac{\text{Watts}}{M^2} \quad \text{II-4}$$

Using the gain relationship $G = 4\pi A_e / \lambda^2$ the above becomes, after inserting the value of 377 ohms for $(u_0 / \epsilon_0)^{\frac{1}{2}}$, the impedance of free space

$$E^2(z) = \frac{2 \cdot 377 P_{40} A_e}{z^2 \lambda^2} \frac{V^2 \text{olts}}{M^2} \quad \text{II-5}$$

In Equation II-4 it is assumed that the local plasma frequency never approaches the wave frequency, and that absorption

does not materially affect the strength of the wave.

5. Magnitude of Deposition

From Equations II-1, II-3, and II-5, we can evaluate the ohmic deposition rate, Q_{Ω} , remembering that $v_{AH} = 5/2 v_m$ and $2\pi c = \omega\lambda$:

$$Q_{\Omega} = \frac{5 \cdot 377 e^2 n v_m A P_{40}}{8\pi^2 c^2 m z^2} \frac{\text{Watts}}{M^3} \quad \text{II-6}$$

6. Equilibrium Equation

The amount of increase of the electron temperature is determined by the rate of ohmic power deposition, and by the rate of energy loss. At low altitudes collisional losses dominate, and the heat conductivity of the electron gas may be neglected.

In the F-region, the heat conductivity is significant, because the collisional losses are small. Farley (1963) has calculated the heating possible in the F-region with a transmitter tuned to the penetration frequency, and he includes the effects of heat conductivity.

An estimate of the relative importance of heat conduction and collisional losses can be obtained by comparing the time required for a sound wave to traverse the beam of the 430 Mhz transmitter with the thermal relaxation time due to collisions. If a heat pulse would take many relaxation times to escape the beam, then heat conductivity is unimportant.

The rate of collisional loss is assumed to be $(U - U_0)G_M v_M$, where U is the instantaneous energy of the electron gas per unit volume, U_0 is the undisturbed energy, and G_M is the fraction of excess energy lost per collision. This is

merely Newton's law of cooling, which states that the rate of cooling of a body is proportional to the temperature difference between the body and the surroundings. G_M is a small number because the electrons are colliding with heavy molecules, and it is difficult for them to transfer momentum. A good discussion of both measured and theoretical values for G_M is given by Thrane (1966). The reason heating is possible is because it takes only one collision for the electron to steal energy from the wave, but hundreds of collisions are necessary to reach thermal equilibrium. The thermal relaxation time constant is $\tau = (G_M v_M)^{-1}$.

The thermal equilibrium equation is

$$\frac{dU}{dt} = Q_\Omega(t) - (U - U_0)G_M v_M \quad \text{II-7}$$

$Q_\Omega(t)$ is a square pulse, and equals zero for $0 > t > t_{40}$, where t_{40} is the pulse length of the 40 Mhz transmitter. Equation II-7 becomes, after inserting $U = \frac{3}{2} nkT$, for $0 < t < t_{40}$:

$$\frac{3}{2} k \left(n \frac{dT}{dt} + T \frac{dn}{dt} \right) = Q_\Omega - \frac{3}{2} nk(T - T_0)G_M v_M \quad \text{II-8}$$

The second term on the LHS is nearly zero, as the number density remains practically constant over the short time scales involved here. Multiplying and introducing the constant ΔT_{ss} , the steady state change in the electron temperature, we obtain

$$\frac{dT}{dt} = \Delta T_{ss} G_M v_M - (T - T_0)G_M v_M \quad \text{II-9}$$

where

$$\Delta T_{ss} = \frac{Q_{\Omega} \tau T_0}{U_0} = \frac{5 \cdot 377 e^2 A_e P_{40}}{12 \pi^2 c^2 \text{kmz}^2 G_M} \quad \text{II-10}$$

ΔT_{ss} is the asymptotic limit that the electron temperature approaches as the heating pulse length t_{40} becomes much greater than the thermal relaxation time τ . Notice that ΔT_{ss} is independent of the electron number density, as well as the collision frequency ν_M . The only unknown in Equation II-10 is the collisional loss coefficient G_M . Thus we have the possibility of measuring G_M if we can keep the heating pulse length significantly larger than the thermal relaxation time.

In the opposite limit of a short pulse ($t_{40} \ll \tau$) the solution of Equation II-9 is

$$\Delta T(t_{40}) = \Delta T_{ss} \frac{t_{40}}{\tau} = \frac{Q_{\Omega} t_{40} T_0}{U_0} \quad \text{II-11}$$

Inserting the definitions of ΔT_{ss} and τ into the above gives

$$\Delta T(t_{40}) = \frac{5 \cdot 377 e^2 A_e P_{40} \nu_M t_{40}}{12 \pi^2 c^2 \text{kmz}^2} \quad \text{for } t_{40} \ll \tau. \quad \text{II-12}$$

The only unknown in Equation II-12 is ν_M , so we have the possibility of determining ν_M when $t_{40} \ll \tau$. Another restriction on Equation II-12 is that $\Delta T(t_{40})/T_0$ should be small or the estimate of ν_M will be in error.

It remains to find the solution of Equation II-9 for the general case of any t_{40} . The general solution is not a simple exponential because ν_M is dependent on temperature.

7. Solution in the General Case

Solution of the equilibrium Equation II-9 in the general

case requires knowledge of the energy dependence of G_M and ν_M , which we now take to be

$$\nu_M = \nu_{Mo} \left(\frac{T}{T_o} \right), \quad G_M = G_{Mo} \quad \text{II-13}$$

in agreement with many laboratory and cross-modulation experiments. The nought subscript implies undisturbed or initial conditions. Equations II-13 lead to the following convenient definitions

$$\tau_o = \frac{1}{G_{Mo} \nu_{Mo}}, \quad \tau_\Omega = \tau_o \frac{T_o}{T_{ss}} \quad \text{II-14}$$

τ_o is the thermal relaxation time and τ_Ω will appear shortly. Equation II-9 can now be written

$$\frac{dT}{dt} = \frac{T_{ss}}{\tau_o T_o} T - \frac{T^2}{\tau_o T_o} \quad \text{II-15}$$

which is a separable differential equation and has the solution

$$T(t) = T_{ss} [1 + x e^{-t/\tau_\Omega}]^{-1} \quad \text{II-16}$$

The constant x depends on initial conditions and equals $T_{ss}/T_o - 1$ when $T(o) = T_o$. The change in electron temperature with time during the heating pulse is

$$\Delta T(t) = T_o \left[\frac{1 + \Delta T_{ss}/T_o}{1 + \Delta T_{ss}/T_o e^{-(t/\tau_o)(1 + \Delta T_{ss}/T_o)}} - 1 \right]$$

for $0 < t < t_{40}$. II-17

This expression is plotted in Figure II-1 (after normalization) and compared with an exponential function which would be the solution if the collision frequency were independent of energy.

The solution of II-15 following the end of the heating pulse is now given. The constant x becomes $T_o/T(t_{40}) - 1$ when $T(t_{40})$ is the temperature at the end of the heating pulse, and τ_Ω becomes τ_o since $T_{ss} = T_o$ when the heating transmitter is off.

$$\Delta T(t) = T_o \left\{ 1 + \left[\frac{1 + \Delta T_{ss}/T_o e^{-(t_{40}/\tau_o)(1 + \Delta T_{ss}/T_o)}}{(1 + \Delta T_{ss}/T_o)} - 1 \right] e^{-(t-t_{40})/\tau_o} \right\} -$$

for $t > t_{40}$.

II-18

This is plotted in Figure II-2.

8. Numerical Calculations

The rate of deposition of energy Q_Ω is given analytically by Equation II-6, and its numerical value in mks units is

$$Q_\Omega = 1.64 \times 10^{-23} n v_M \left(\frac{100}{z} \right)^2 \left(\frac{P_{40}}{1 \text{ MW}} \right) \frac{\text{Watts}}{M^3} \quad \text{II-19}$$

where n in M^{-3} , v_M in sec^{-1} , z in km, and P_{40} in MW.

Q_Ω is plotted as a function of altitude in Figure II-3 for typical day and night-time values of the product nv_M .

The maximum increase in electron temperature that can be achieved, ΔT_{ss} is given in Equation II-10, and has the value

$$\Delta T_{ss} = 158 \left(\frac{P_{40}}{1 \text{ MW}} \right) \left(\frac{100}{z} \right)^2 \left(\frac{5 \times 10^{-3}}{G_m} \right) \text{ } ^\circ\text{K} . \quad \text{II-20}$$

FIGURE II-1

FIGURE II-2

FIGURE 11-3

ΔT_{ss} is shown in Figure II-4 for two values of G_M . These values are the extremes that are likely from cross-modulation experiments.

For short pulse heating ($t \ll \tau_o$), the value $\Delta T(t_{40})$ from Equation II-12 has numerically

$$\Delta T(t_{40}) = 7.9 \times 10^{-4} \left(\frac{P_{40}}{1 \text{ MW}} \right) \left(\frac{100}{z} \right)^2 \left(\frac{t_{40}}{1 \text{ MS}} \right) v_M \text{ } ^\circ\text{K} \quad \text{II-21}$$

where t_{40} is in msec. These are plotted along with ΔT_{ss} in Figure II-4 and are labelled for three pulse lengths of 0.2, 2, and 5 msec. The achievable change in electron temperature falls off rapidly with altitude. To obtain heating at high altitudes requires long pulse lengths.

FIGURE II-4

III. DETECTION OF HEATING

1. The 430 Mhz Radar System

Regular observations of the ionosphere are made with the 430 Mhz radar system of Arecibo. Gordon and LaLonde (1961) and Carlson (1965) have described the structure, capabilities, and operation of the Arecibo Ionospheric Observatory. The antenna is a 1000 ft. diameter spherical dish and the transmitter can routinely give 2.4 Mw peak power with a 6% duty cycle. The transmitter and receiver are controlled by a digital computer which partially processes each echo in real time and provides a density profile and a spectra as soon as a run is completed. This experiment uses a 40 μ s pulse length which gives an altitude resolution of 6 km.

The transmitted pulse can be cycled through nine discrete frequencies with a rapid pulse repetition rate in order to achieve better time resolution than a single frequency could achieve. The shift in frequencies avoids range ambiguities since the 200 kc increment shifts are greater than the returning spectra.

2. Incoherent Backscatter

The 430 Mhz interrogating wave is weakly scattered in the ionosphere by a process that has only recently been understood, called incoherent backscatter. Papers by Dougerty and Farley (1960), Fejer (1960), Salpeter (1960) and others have developed the basic theory of incoherent scatter.

When the wavelength of the radar wave is much greater than the Debye shielding length, the EM wave "views" the plasma as a continuum rather than as discrete particles. In this case, the scattering can be thought of as caused

by variations in the dielectric constant of the plasma, i.e., fluctuations in the electron number density. These fluctuations are associated with the thermal motion of the ions (because the ions and electrons are coupled by the coulomb forces), and are inherent in the plasma because it is hot. The amplitude of these acoustic fluctuations is a function of the plasma properties, and determines the magnitude of the backscatter. The scattered spectra has a width corresponding to the doppler shift expected from the thermal velocity of the ions.

In the alternate case, when the Debye length is much longer than the probing wave, the wave scatters as if from the individual charged particles, and since the ions are relatively massive, only the electrons contribute to the returned power. The spectra of the scattered power corresponds to the thermal velocities of the electrons.

Consider again the case when the Debye length is much less than the incident wavelength, and the scattering is due to fluctuations in the medium. The power returned near the transmitted frequency with a doppler shift corresponding to ionic thermal velocities is called the ion line. The spectra of the ion line is saddle shaped, and is presented in Figure III-1. The peak frequency on each side corresponds to the velocity of an ion-acoustic wave traveling either toward or away from the observing receiver.

Irregularities in the medium (due to the random thermal motions) may be thought of as deviations in the dielectric constant. The dielectric constant can then be fourier analyzed with respect to the probing direction into a spectrum of spatial harmonics at a given instant. This fourier spectrum is taken for convenience in units of the probing

FIGURE III-1

wavelength λ . Then the spectra has spatial components of frequency, λ , $\lambda/2$, $\lambda/3$, and these are considered to represent corrugations in the dielectric constant which have the velocity of the ion-acoustic waves they represent. Only those corrugations which act as dielectric mirrors are responsible for backscatter, namely only $\lambda/2$. Thus from our simple picture we would expect to have as backscatter two delta functions at plus and minus the corrugation (acoustic) velocity. But these corrugations are damped by the Landau process, and so the delta functions are smeared in frequency due to their short time durations.

3. The Temperature Dependent Cross-section

An approximation for the backscatter cross-section of the plasma is given by Moorcroft (1963)

$$\frac{\sigma}{n \sigma_e} = \frac{(kD)^2}{1 + (kD)^2} + \frac{1}{[1 + (kD)^2][1 + (kD)^2 + T_e/T_i]} \quad \text{III-1}$$

σ_e is the Thompson cross-section for a single electron, D is the Debye length, and k is $4\pi/\lambda$ (twice the wave number of the transmitted wave). The first term on the RHS is called the electronic component, because as $kD \rightarrow \infty$ (i.e., very small λ) the second term vanishes and $\sigma \rightarrow n \sigma_e$. The second term is the ionic component, and as $kD \rightarrow 0$, Equation III-1 becomes

$$P \approx \frac{n \sigma_e}{1 + (T_e/T_i)} \quad (\text{as } kD \rightarrow 0). \quad \text{III-2}$$

Here P is the unnormalized backscatter power that is returned. The power is a function of the electron density and

the temperature ratio of the electron and ions. If we assume that both n and T_i remain constant during and after the heating pulse, then a measurement of P before and after a heating pulse gives an indication of the change in T_e .

Letting P_o be the power returned when the electrons are at ambient temperature (which is assumed to be the same as the ion and neutral temperatures), and letting P_1 be the power returned immediately after a heating pulse,

$$\frac{P_1 - P_o}{P_o} = \frac{\Delta P}{P} = \frac{-\Delta T/T_o}{2 + \Delta T/T_o} \quad \text{III-3}$$

An increase in T_e of 10% reduces the power by about 5%. We form the fraction $\Delta P/P$ in order to eliminate all variables except $\Delta T/T_o$. P_1 and P_o are the experimental observables, and the fractional change in P , $\Delta P/P$, is the parameter used in calculating $\Delta T/T_o$ (the fractional change in electron temperature).

Equation III-1 was derived assuming steady state temperatures, a condition which is not met in this experiment. It might be expected that the equation would not describe the instantaneous changes in T_e , since it takes a finite time for the ion-acoustic wave responsible for the scattered power to relax to the new conditions. This problem in incoherent backscatter theory has not yet been solved. Perhaps this experiment will provide the impetus for such work.

4. Collision Dominated Spectra

Another problem arises at low altitude when the backscattered spectra is said to be collision dominated (below 100 km). Because of the high ion-neutral collision frequency the spectra is significantly narrowed, as displayed in

Figure III-2. The collision dominated case is treated for thermal equilibrium (i.e., $T_e = T_i$) by Dougherty and Farley (1963). They show that the scattered cross-section is independent of the ion-neutral collision frequency. But our heating problem requires an analysis allowing $T_e \neq T_i$.

An alternate solution of the scattering problem is possible using continuum equations instead of the kinetic theory (Tanenbaum, 1968). This approach is possible only when the ion-neutral mean free path is less than the wavelength of the probing wave (the collision dominated condition). Seasholtz and Tanenbaum (1968) have calculated the effect of unequal electron and ion temperatures for the collision dominated case. They show in Figure III-3 that only for large T_e/T_i is there a significant difference from the $(1 + T_e/T_i)^{-1}$ dependence of Equation III-2. Hence it seems that for $\Delta T/T_0 < 3$ we can neglect ion-neutral collisions.

5. Debye Length Considerations

The relationship in Equation III-3 was derived assuming that the Debye length was negligibly small. At low altitudes, however, the Debye length correction is important.

Equation III-1 can be rewritten as

$$P = \frac{(B_i/B_e)\alpha^2}{1 + \alpha^{-2}} + \frac{1}{[1 + \alpha^{-2}][1 + \alpha^{-2} + \Delta T/T]} \quad \text{III-4}$$

where P is the backscattered power received with a bandwidth of B_i , the width of the filter normally used for ion-line density measurements. B_i is about 25 kc, and B_e , the width of the electron component, is one-half a megahertz. The parameter $\alpha^{-1} = kD = 4\pi D/\lambda$. Assuming that we can drop the

FIGURE III-2

FIGURE III-3

first term because B_i/B_e is a small number, $\Delta P/P$ becomes, when α^{-1} is not negligible,

$$\frac{\Delta P}{P} = \frac{(1 + \alpha_o^{-2})(2 + \alpha_o^{-2})}{[1 + (1 + \Delta T/T)\alpha_o^{-2}][2 + (1 + \Delta T/T)\alpha_o^{-2} + \Delta T/T]} - 1 \quad \text{III-5}$$

where $\alpha_o^{-1} = \frac{4\pi}{\lambda}(6.9)\sqrt{\frac{T_{eo}}{n}}$ after inserting the definition of Debye length, D . We now have $\Delta P/P$ as a function of α_o^{-2} , which is plotted in Figure III-4. The altitude dependence of α_o^{-2} is also indicated for daytime values of T_{eo} and n . For α_o^{-2} less than 1/10 it seems that the simple form for $\Delta P/P$ of Equation III-3 can be used. When $\alpha_o^{-2} = 1/10$, the error in using Equation III-3 to estimate $\Delta T/T$ from $\Delta P/P$ is 20%. When $\alpha_o^{-2} = 1$, the error is 50%. Since it is difficult to obtain n and T at low altitudes where α_o^{-2} becomes significant, the Debye length correction is a serious handicap to estimating $\Delta T/T$ and hence v_M below 80 km.

FIGURE III-4

IV. THE HEATING DATA

1. Experimental Procedure

Two similar sets of experiments are presented here: one from the summer of 1967, and another from January, 1969. The 1967 heating experiment attempted to measure the amount of heating, and the 1969 experiment attempted to measure in addition the thermal relaxation time constant.

A typical pulse heating sequence is shown in Figure IV-1. The 40 Mhz heating pulse is immediately followed by a 430 Mhz interrogating pulse. The electron temperature is of course highest immediately after the end of the 40 Mhz pulse. The ionospheric echo from the 430 Mhz pulse is received and the data necessary to compute an electron density profile is stored in the computer. The inter-pulse period is constrained by the 4% maximum duty cycle limit of the 40 Mhz transmitter. This heating sequence is run for a few minutes, and then the 40 Mhz transmitter is turned off while a measurement of the backscattered 430 Mhz power is made when the electrons are at their ambient temperature. Then the 40 Mhz transmitter is turned on again for another heating sequence. Several heated and unheated sequences are separately summed until a sufficient number of pulses are recorded to provide adequate noise statistics. Then the quotient $\Delta P/P$ is formed as a function of altitude.

The 1969 experiment was similar, but the single 430 Mhz interrogating pulse between each 40 Mhz heating pulse was replaced by a series of nine stepped-frequency transmissions. A measurement could thus be made of the backscattered power not only immediately after the end of the heating pulse, but at eight times afterward. Eight separate $\Delta P/P$'s could in

Pulse Heating Sequence

FIGURE IV-1

theory be calculated, using the ninth as a reference, but in practice only the first 2 or 3 are calculated, and the remaining ones are averaged to give a better reference.

Determining the backscattered power requires careful handling of the receivers. It is difficult to detect a weak echo in a noisy background, especially when the noise is changing. At low altitudes where the signal to noise ratio is small due to a low electron density, the accuracy of the measurements is reduced.

The fractional standard deviation (standard deviation over signal) when the signal to noise ratio is large ($S/N \gg 1$) is approximately $\sigma_S/S = 1/\sqrt{n}$; when $S/N \ll 1$, $\sigma_S/S = \frac{1}{\sqrt{n}} \frac{1}{S/N}$. When two noisy numbers are subtracted, the statistics become worse by the factor $\sqrt{2}$. These standard deviations have been plotted on the $\Delta P/P$ data to give an idea of the statistical validity of the data.

The presence of the transmitted pulse, which requires that the receivers be gated off, is an additional complication. There is a residual effect from the transmitted power which increases the noise temperature of the receiver. A separate monitor - the recovery channel - is necessary to subtract any extra noise from the data. Accurate estimation of the noise is necessary to detect a weak echo, or small changes in a weak echo due to heating.

2. The Summer 1967 Data

Several runs with heating pulses of 1/2, 1, 2, and 5 msec duration were made in the summer of 1967. They all showed changes in the expected direction between the heated and unheated sequences. Two runs, however, showed changes at some altitudes in the wrong direction. They may have

been caused by gross changes in the ionosphere during the course of the run. Figures IV-2 and IV-3 show the data plotted as $\Delta P/P$ vs. altitude for all the summer 1967 runs.

The first of these has plotted the shorter heating pulse lengths of 0.1, 0.5, and 1 msec, and the second includes two 2 msec and one 5 msec heating run. The 5 msec and 0.5 msec data each has apparently significant deviations of $\Delta P/P$ in the positive direction in addition to the negative excursions which are predicted. All the runs used a 40 μ s interrogating pulse length except the 0.1 and 0.5 msec runs used a 100 μ s probing pulse. In Figure IV-2, the small deviation in $\Delta P/P$ for the 0.1 msec run is deemed of no statistical importance. The expected heating from this short pulse length would not be detectable with the size of the error bars which are given. Both the 0.5 and 1.0 msec runs show the predicted decrease in $\Delta P/P$ below 85 km, and each goes off scale. In Figure IV-3, the 2 msec runs differ by about a factor of two in the decrease in $\Delta P/P$, and the reason for this is not understood. The 5 msec run has a wild excursion in the positive direction around 80 km.

One of these runs (9 August, a 2 ms run) is shown in Figure IV-4 with a prediction for $\Delta P/P$ based on an assumed model for v_M . This run shows significant heating below 100 km, and follows the prediction rather closely between 105 km and 80 km. It is unclear why the data does not follow the prediction better below 80 km, but at lower altitudes the backscatter measurements are less precise, and the ionosphere is more apt to have changed during the course of the run.

By inverting the problem, and asking what collision frequency is necessary to give the observed data, we can obtain

FIGURE IV-2

FIGURE IV-3

FIGURE IV-4

collision frequency profiles as shown in Figure IV-5. Here we have taken only those altitudes where it is reasonable to assume that the heating is in the short pulse limit and hence $\Delta T/T_0 \propto \nu_M$ as in Equation II-12.

3. The January 1969 Data

Four heating runs of 1, 2 and 5 msec pulses were made and all showed heating. A 2 msec experiment shown in Figure IV-6 shows heating effects in both the first and second interrogating pulses. From the magnitude of $\Delta P(1)/P$, shown as solid dots in the figure, $\Delta T(1)/T_0$ can be inferred. Similarly, $\Delta P(2)/P$ gives $\Delta T(2)/T_0$. Then the ratio $\Delta T(2)/\Delta T(1)$ gives the thermal relaxation time since the period between measurements (5.6 msec) is known. The collision frequency ν_M is obtainable from $\Delta T(1)/T_0$, so we can also deduce G_M . Calculations for two altitudes are produced here.

<u>Altitude</u>	<u>$\Delta T(1)/T_0$</u>	<u>$\Delta T(2)/T_0$</u>	<u>τ</u>	<u>ν_M</u>	<u>G_M</u>
105 km	0.22	0.09	6.3ms	$3.15 \times 10^4 \text{ s}^{-1}$	5.0×10^{-3}
111 km	0.10	0.07	15.7ms	1.43×10^4	4.5×10^{-3}

The change in ν_M with altitude implies a lapse rate of the neutral atmosphere of 7.5 km, which is a reasonable value. The value of G_M is close to measurements made by cross-modulation experiments at 85 km, and agrees with theoretical predictions (Thrane, 1966).

FIGURE IV-5

FIGURE IV-5

Figure IV-7 shows deduced electron collision frequency profiles for the four runs obtained in January 1969. All four runs show collision frequencies higher than the model for electron collision frequency adopted in Table I. These values of collision frequency deduced from the Arecibo heating experiment are close to the values presented in Figure I-4. These four runs agree quite closely with each other, and have the same lapse rate as the model. It is felt that there is no reason to accept the model or previous measurements as more accurate than the electron collision profiles deduced from the January 1969 data. The January 1969 data generally gives a higher collision frequency than the summer 1967 data. The difference could either be due to a change in the receivers or a change in the atmosphere. The receivers are using a different method for subtracting noise from the echo, and the change in season and solar cycle could have changed the electron collision frequency.

FIGURE IV-7

V. CONCLUSIONS

1. Validity of Data

The principle assumptions will be restated here. These assumptions are either of a theoretical or practical nature.

The theoretical assumptions in predicting the magnitude of heating are straightforward. The rate of deposition of energy is a simple computation. It assumes that the electron collision frequency is directly proportional to energy and that the electron velocities are maxwellian. The magnetic effects on the conductivity are neglected and the only heat loss process considered is collisional losses to neutrals or ions.

The validity of the major assumption in measuring the heating with incoherent backscatter is more difficult to assess. That assumption is that Equation III-1 is valid over the short time scales used in this experiment. Equation III-1 was derived assuming steady state temperature, a condition which this experiment violates. After discussions with several of the originators of the incoherent backscatter theory it is felt that the necessary theoretical corrections to allow for a time dependent electron temperature will be slight, or possibly non-existent.

The most critical experimental problem is the accurate estimation of receiver noise, especially following the 40 Mhz heating pulse. The receiver noise is monitored, and while the measurement seems not to be exact, contamination of the data is thought to be slight.

2. Prospects for Future

The nine-step frequency interrogation experiment will be repeated with a longer integration time to reduce the

noise level in the $\Delta P/P$ plots. Perhaps both the second and third probing pulse after the heating pulse will be able to detect heating. Instead of dividing the heating interpulse period into nine equal parts as is presently done, the nine probes can be placed closer to the heating, and hence obtain more measurements of the cooling curve.

An experiment has been designed to attempt 430 Mhz heating; that is, the heating pulse would be at 430 Mhz, with the nine-stepped frequency pulses varying around 430 Mhz as they do now. The advantage of this is that only one transmitter is required, thus increasing the probability of a successful run (the 40 Mhz transmitter is an inherently unreliable instrument). The ohmic deposition rate, Q_{Ω} , is independent of frequency, and is a function of the power-aperture product only. The 430 Mhz transmitter can produce almost 2.5 times the peak power of the 40 Mhz transmitter, but the effective aperture at 430 Mhz is smaller because we must use the filled beam gain rather than the peak forward gain as was done with the 40 Mhz beam. Calculations show that heating at 430 Mhz should be about 90% as effective as at 40 Mhz. Even though the beam size is ten times smaller at 430 Mhz, problems of ionospheric winds or heat conduction should still be negligible.

A bistatic station for use with the ionospheric program at Arecibo is coming into operation near Fajardo on the eastern end of Puerto Rico. It will enable backscatter measurements in the E-region to be made with an altitude resolution of 1.5 km (LaLonde, 1968). It is desirable to have better altitude resolution than the present experiment allows (6 km) since the collision frequency we are trying to measure has a lapse rate on the order to 6-8 km.

Both pulse and continuous wave heating could be attempted.

If the equilibrium constants are changed in the D or E-region by continuous wave heating, then the electron number density could be altered. Molmud's arguments for the D-region (presented in section I) can be examined experimentally if the D-region can be observed from Fajardo.

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