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Evanescent Wave Coupling in Terahertz Waveguide Arrays

by

Kimberly S. Reichel

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Dr. Daniel Mittleman, Chair
Professor, Electrical and Computer
Engineering

Dr. Qianfan Xu
Assistant Professor, Electrical and
Computer Engineering

Dr. Douglas Natelson
Professor, Physics and Astronomy

HOUSTON, TEXAS

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ABSTRACT

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At optical frequencies, evanescent wave coupling in waveguides is an important concept underlying key technologies such as optical fiber splitters and combiners. At terahertz (THz) frequencies, there is a lack of such devices. In order to fill this gap, we investigate evanescent wave coupling at THz frequencies in an array of narrow-width parallel-plate waveguides (PPWGs).

Although researchers have studied THz wave coupling between two adjacent wire waveguides, evanescent coupling in an array of PPWGs has not previously been considered. Metal PPWGs are ideal THz waveguide platforms since they offer low losses and negligible dispersion in the TEM waveguide mode. Additionally, PPWGs can exhibit energy leakage when the plates are narrow and the plate separation is large, indicating that an array of narrow-width PPWGs is a convenient platform for studying THz energy coupling between waveguides.

By using the presented design of an array of identical narrow-width PPWGs in close proximity with their unconfined sides facing each other, we have demonstrated evidence of evanescent wave coupling in THz PPWG arrays. Thereby, we observed stronger coupling with larger waveguide plate separations and longer propagation paths. We confirmed these results through THz time-domain

spectroscopy (THz-TDS) experiments and finite-element method (FEM) simulations. Based on evanescent wave coupling, this work establishes a platform to investigate new opportunities for THz waveguide devices and components such as splitters and power combiners.

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Contents

Acknowledgments.....	iii
Contents	1
List of Figures	2
Introduction	3
1.1. Terahertz Band	3
1.2. Evanescent Wave Coupling	4
Coupling Between Parallel-Plate Waveguides	6
2.1. PPWG Array Design	6
2.1.1. Experimental Design	7
2.1.2. Analysis Technique	8
2.1.3. Computational Design	10
2.2. Results	10
2.2.1. Plate Separation.....	12
2.2.2. Propagation Length	14
2.3. Discussion and Future Work	16
Notes on RF Modeling and Design in Comsol Multiphysics	18
3.1. General Comsol Design	18
3.1.1. Physics Design.....	19
3.1.2. Model Wizard	20
3.1.3. Definitions.....	20
3.1.4. Geometry	21
3.1.5. Materials	21
3.1.6. Physics Conditions	22
3.1.7. Mesh	22
3.1.8. Study	23
3.1.9. Analysis and Post Processing	23
References	25

List of Figures

List of Figures

Figure 2.1 PWWG array Design of PPWG array.....	7
Figure 2.2 Varying Plate Separation.....	9
Figure 2.3 Coupling ratios.....	11
Figure 2.4 FWHM	13
Figure 2.5 Varying Waveguide Length.....	14
Figure 2.6 Broadband Frequency Independence	16
Figure 3.1 Comsol Result	24

Chapter 1

Introduction

In this brief introduction, background information will be provided to give context to the supporting foundation of this experiment.

1.1. Terahertz Band

Until the late 1990's, the spectrum of frequencies from about 10GHz to 10THz had been known as the "Terahertz Gap." Since this time, the gap has rapidly been filling in with new terahertz generation and detection techniques, alongside new devices that enable better control of terahertz wave from waveguide to metamaterials.

One phenomenon that had been investigated was coupling to a metal wire waveguide. This successful demonstration of terahertz wave coupling forms the basis of a terahertz endoscope, a probe that can image even at an angle of 90° [1]. In

this paper, we were curious to further investigate wave coupling in the terahertz regime.

Terahertz has such a rich potential because it pulls from the neighboring frequency worlds of microwaves and infrared optics. In this work, we look to optical evanescent wave coupling as a launching pad to understand THz coupling.

1.2. Evanescent Wave Coupling

Evanescent wave coupling in waveguides and waveguide arrays has been well studied in the visible and infrared and is an important concept underlying key technologies such as optical fiber splitters and combiners [2, 3]. For example, evanescent wave coupling enables a directional coupler to function based on the evanescent field of one fiber spatially overlapping into the mode profile of another fiber in close proximity [4]. Although evanescent coupling is very well developed in optics, it has only barely been studied in the terahertz (THz) range between two adjacent wire waveguides [1]. In the terahertz range, a common waveguide platform is the metal parallel-plate waveguide (PPWG). This geometry is often employed because of the relatively low loss and the possibility of broadband dispersionless operation in the TEM waveguide mode. However, evanescent coupling between adjacent PPWGs, or among an array of narrow PPWGs, has not previously been considered.

To explore waveguide coupling in the THz regime, we consider an array of finite-width PPWGs running parallel to each other. Finite-width PPWGs can exhibit

energy leakage when the plates are narrow and the plate separation is large [5-8], suggesting that an array of narrow PPWGs is a convenient platform for studying THz energy coupling between waveguides. With broadband single-cycle pulses and coherent detection of the THz field, extending evanescent wave coupling to the terahertz regime may enable interesting new applications in THz waveguide devices and components such as splitters and power combiners.

Chapter 2

Coupling Between Parallel-Plate Waveguides

2.1. PPWG Array Design

To observe THz evanescent wave coupling, we constructed finite-width PPWGs in the form of an array (Figure 2.1a). Each array consisted of seven PPWGs. We chose a plate width (w) on the same order of wavelength of 2 mm to ensure a strong evanescent wave transverse to the waveguide axis. We fixed the gap (g) between plates of 2 mm to allow spatial overlap of the modes between waveguides. For ease of assembly, we fabricated the arrays by machining two solid aluminum pieces to have a series of corresponding “fins”, one set comprising the top plate of each PPWG and the other set comprising the bottom plate (Figure 2.1b). By varying the distance between the top and bottom sets of the PPWG array, we controlled the plate separation (b) using precision micrometer mounts. We fabricated arrays of

various propagation lengths (L) ranging from 5 to 50cm. We choose the depth (d) from waveguide edge to aluminum back plate of 18 mm so as to be large enough that no reflections occur from the aluminum; therefore, this back plate serves as a structural support only, and can be ignored from the point of view of the propagation of THz radiation in the waveguides.

Figure 2.1 PPWG array Design of PPWG array.

Design of finite-width PPWG array with the orange circle illustrating the THz beam spot on the input facet of the center waveguide. (a) Transverse cross section of array with dimensions: width $w=2$ mm, gap $g=2$ mm, and variable plate separation b . Output is measured along horizontal dashed line in steps of 0.5mm. (b) Experimental design of array from two machined Al plates showing length L in the propagation direction.

2.1.1. Experimental Design

To measure coupling across the array, the field emerging from the output of the waveguide array was measured using a THz time-domain spectroscopy (THz-TDS) system with fiber-coupled photoconductive antennas. The beam was vertically

polarized to dominantly excite the fundamental transverse-electromagnetic (TEM) mode [9]. From the transmitter antenna, the light was focused using a pair of lenses in confocal configuration and an aperture was used at the input face to ensure the incident THz beam excited only the central waveguide of the array. At the output, a 0.8 mm diameter aperture was placed in front of the receiver antenna to improve the spatial resolution. A total of 57 THz waveforms at 0.5 mm step intervals were measured along the x-direction of the array, shown by the dashed line in Figure 2.1a. This was repeated for each plate separation and propagation length that we investigated.

2.1.2. Analysis Technique

We obtained the electric field distribution across the array as a function of b , L , and frequency using the method described above. Typical results are shown in Figure 2.2a depicting electric field map of frequency versus horizontal position along the waveguide array at a length L of 30 cm, for four different plate separations. Assuming each waveguide supports a mode of approximately Gaussian shape along the x-direction, for a selected frequency we fit Gaussian curves centered at each waveguide (Figure 2.2b) using the power normalized to the maximum value at the center of the array. We constrained the width of the Gaussian curves to be the same for a particular plate separation and waveguide width while allowing the fitting algorithm to optimize the amplitude, so that the number of free parameters in the fitting is only one per waveguide. This analysis yields the total power contained in each waveguide at the output aperture of the array.

Figure 2.2 Varying Plate Separation

(a) Electric field maps of frequency versus x-position across array. **(b)** Frequency slice at $f=125\text{GHz}$ of $L=30\text{cm}$, $b=1\text{mm}$ device showing experimental data (black line) and Gaussian fits (red lines). The vertical stripes (gray) indicate the presence of a PPWG.

2.1.3. Computational Design

Our experimental results were confirmed through finite element method (FEM) simulations [10]. We exploit the symmetry of the problem to model only one quarter of the waveguide array, using perfect electric conductor (PEC) boundary conditions horizontally (along the x-direction) and perfect magnetic conductor (PMC) boundary conditions vertically (along the y-direction) on the inside boundaries [11]. Perfectly matched layers (PML) surrounded the model to absorb radiation at external boundaries without reflections. More details on modeling can be found in Chapter 3. At a particular frequency, plate separation, and length, the resultant electric field was extracted along the output of the array for comparison to experiment. FEM simulation is beneficial since there is no closed-form analytical solution to describe the modes of a finite-width PPWG.

2.2. Results

In order to characterize mutual coupling between THz finite-width PPWGs, we studied two variables: plate separation (b) and propagation length (L). The plate separation directly affects the energy confinement factor and the propagation length influences the cumulative degree of interaction between waveguides. To quantify the coupling ratio, using the Gaussian curve fitting described above, we determined the area under these curves which describes the power contained in a particular waveguide. Assuming the amount of power that couples from the central waveguide to the first most adjacent waveguides is the same as the amount of power that

couples from the first most adjacent waveguides to the second most adjacent, we took the ratio of power between waveguides in the array. This involved taking the ratio of: 1) the left most adjacent of the central waveguide to the center, 2) the right most adjacent of the central waveguide to the center, 3) the second left most adjacent waveguide to the first left most adjacent, and 4) the second right most adjacent waveguide to the first right most adjacent. Then we averaged these ratios for all plate separations and propagation lengths. Figure 2.3 shows the values of the coupling ratio as a function of b , L , and frequency.

Figure 2.3 Coupling ratios.
Coupling ratios of experimental data (solid lines) and FEM simulations (dashed lines). Stronger coupling is more evident for larger plate separations and longer propagation path lengths.

Using finite-element-method (FEM) simulations, we also numerically computed the solution for a given geometry and applied the same analysis of Gaussian fitting to this computational result. Within the range of lengths for which the simulations are feasible (given finite computing resources, $L \leq 20$ cm), we see good agreement of the coupling ratios between the experimental results (solid lines) and the numerical simulations (dashed lines), which indicates the robustness and reliability of the experimental results.

2.2.1. Plate Separation

As mentioned, the plate separation (b) has the most influence over the energy confinement factor. As seen in Figure 2.2a, greater plate separations yield more energy distribution across the array, which is evidence of greater coupling from the central waveguide to adjacent waveguides. In finite-width PPWGs, energy confinement decreases as plate separation increases [8]. Thus, when the energy is less confined, the mode profile is broader and there is more evanescent leakage from the open sides of the waveguide. The evanescent energy then overlaps into adjacent waveguides and the effect continues spreading energy across the array. This confinement can be attributed to plasmonic edge effects. Plasmonic excitations have been seen at optical frequencies in slot waveguides [12] and recently, evidence of plasmonic edge modes has been demonstrated on THz PPWG plate edges [8, 11, 13]. When the plates are closer together, these plasmonic edge modes couple more strongly to each other yielding higher energy confinement within the waveguide [14]. Higher energy confinement implies that the evanescent tails of the propagating

mode profile must be less pronounced, and thus the mode width should be smaller. Since the width of each Gaussian fit to the mode profile shape is fixed for each plate separation and propagation length, the full-width half maximum (FWHM) can be extracted as a function of these two variables, as shown in Figure 2.4. Indeed, we see smaller widths for plates that are closer together, evidence of the plasmonic energy confinement.

Figure 2.4 FWHM

Full-width half-maximum (FWHM) of Gaussian fits to experimental data. Clearly from lowest to highest plate separation, the FWHM increases as plate separation increases, indicating a change of the spatial mode profile from narrow to broad.

2.2.2. Propagation Length

Figure 2.5 Varying Waveguide Length

Electric field maps of frequency for various propagation lengths (L) at a fixed plate separation of $b=1$ mm. Each frame shows frequencies for 10 – 300 GHz and is normalized to the maximum value.

The propagation length (L) also influences the amount of coupling.

Equivalent to fiber optics, as the propagation length is increased, the effective interaction length between waveguides is increased [4]. In Figure 2.5, we see how energy spreads across the array for longer propagation lengths. As the THz signal propagates for longer distances in a PPWG, the energy loss out of the unconfined sides of the waveguide increases due to the evanescent leakage [6, 9]. In the design of

our array, the energy of the central excited waveguide spills out into the adjacent waveguides on either side; then, this energy in turn spills into the next most adjacent waveguides.

As in the case of evanescent wave coupling between two adjacent fibers [15,16], we anticipate that the degree of energy coupling between adjacent PPWGs should grow exponentially with propagation length. With this assumption, we extract a coupling coefficient at each wavelength by fitting the measured coupling ratios (e.g., Figure 2.3) to an exponential function the form $y = e^{Cx} - 1$ where C is the power coupling coefficient, and x represents propagation distance. From fits such as this, we extract the values of C as a function of wavelength and plate separation. Figure 2.6 shows these results for $C(\lambda)$, for several values of the plate separation. This result indicates that the coupling coefficient is approximately independent of wavelength, over a relatively broad range

This weak wavelength-dependence of $C(\lambda)$ suggests that the confinement mechanism for the guided mode is not strongly wavelength-dependent, and therefore, cannot depend critically on the plate width (e.g., the plate width w , which effectively ranges from $\sim 4\lambda/3$ to $\sim \lambda/3$ within the measurement bandwidth). Instead, this result is consistent with the aforementioned description in which the confinement mechanism is mediated by the excitation of edge plasmons at the waveguide's corners, as discussed previously [8]. We also observe that the coupling coefficient depends on the plate separation b in a non-linear fashion. This is not

surprising, since the energy confinement varies in a non-linear fashion with respect to b as observed previously [8].

Figure 2.6 Broadband Frequency Independence
Coupling coefficients for several plate separations at different frequencies, which shows evidence of frequency invariant coupling.

2.3. Discussion and Future Work

Through this design of a finite-width PPWG array, we have seen evidence of evanescent wave coupling in THz finite-width PPWG arrays. By varying plate separation and prorogation length, we have investigated factors that influence this

coupling. We have found the relationship that greater plate separation and longer propagation lengths lead to stronger coupling. These results were confirmed through experiment and FEM simulation. This work establishes a platform to investigate new opportunities for THz devices based on evanescent coupling.

Chapter 3

Notes on RF Modeling and Design in Comsol Multiphysics

This section will go over the details of the model used for the simulations presented in this paper to serve as a guide to future readers on setting up Comsol designs using the Radio Frequency (RF) Module.

3.1. General Comsol Design

Comsol Multiphysics is a powerful package, but it is important understand to the concepts underlying these calculations. This section covers general set up of a RF model and uses the computational model in this work as an design example, all using Comsol 4.2a. There are several steps: physics design, choosing an appropriate model, setting definitions, geometry, and materials, defining physics conditions, meshing, study conditions, and post-processing results.

3.1.1. Physics Design

The first step of any computational model is to consider the physics of the desired problem to model. In the case of coupling between an array of PPWGs, the model must account for the presence of the waveguides, and the air between the waveguides; it must also excite the central waveguide in the TEM mode, and be able to extrapolate the electric field distribution at various propagation lengths. Since electromagnetic (EM) waves do not propagate through the metal (although there is a finite skin-depth that the waves penetrate in a metal, which can be accounted for by setting proper material boundaries), there is only the need to build the regions of air where the EM waves will propagate. A port boundary can be used to excite the central waveguide, and after the calculation the resultant electric field resultant can be extracted.

Next, consider if there is any symmetry in the model. It is always beneficial to exploit symmetry in order to cut down on the model size which saves computational time and memory requirements. In this design, the array is symmetrical along both the x- and y-directions so only a quarter of the array needs to be modeled.

In order to properly meet the physical constraint that all fields diminish to zero at infinity, Perfectly Matched Layers (PML) can be employed. These are perfect absorbing boundaries with no reflections. In this model, PML is used at the output of the array and along the side on the outer perimeter of the array.

3.1.2. Model Wizard

Comsol has many options and packages, but for electromagnetic wave modeling use the RF Module. Using the “Model Wizard” will get the bare bones of your model set up. Simulations can be run in 3D or 2D but if not necessary to model volumes use 2D since it saves much computational time and memory usage. In the RF package there is an option to do steady-state or transient calculations, and several options of boundary mode-analysis, eigenfrequency, frequency domain. In this work, the model was set up with 3D, steady-state, and frequency domain conditions.

3.1.3. Definitions

Defining parameters will ease the modeling building process. Global definitions such as parameters, variables, and functions, can be set up that will govern the whole simulation. Local definitions can also be set. The ability to access parameters throughout the model allows for efficiency by avoiding the necessity of typing in all values every time in each place of the model. It also makes it quite convenient if there is a need to change some sizes or values. By adjusting the value of the definition, when rebuilding or remeshing the model the new parameters will be used. It is a good idea to include units when defining parameters. For RF models such as this, it is convenient to define the working frequency and any dimension for the geometry of the model.

3.1.4. Geometry

There is the ability to draw or import model geometry. On the geometry settings, length units can be defined to the desired scale. The geometry building interface functions much like any CAD program. There are many shapes built in, and there is also the possibility of drawing on a 2D work plane that can be extruded into 3-Dimension.

The geometry of this model is an array of five parallel-plate waveguides (PPWGs), but due to symmetry only half of the central waveguide is modeled and two full waveguides. The dimensions were defined in the global definitions to match the experimental model. Since the array is invariant in the propagation direction (z-direction), using a work plane in the x-y plane at $z=0$, a transverse cut of a quarter of the array is modeled, then extruded to the appropriate propagation length. At the end of the array a section of air is built. There is also need to include PML on the outer side and output of the model to absorb all fields.

3.1.5. Materials

Material properties need to be assigned to the geometry, whether air, dielectric, or metal. Comsol conveniently has a material library to choose from, or if needed custom designed materials can be constructed. This model uses air in all domains.

3.1.6. Physics Conditions

Setting up model equations and boundary conditions is probably the most important part of any model. In the RF Module, full field or scattered field calculations can be made. Some of the options of domains conditions are: electric wave equation, far-field, or PMLs. Some examples of the boundary conditions are: PEC, PMC, port, or periodic boundary conditions. The automatic default setting for all boundaries is PEC. In this model, the central waveguide is excited by a port at the input, PMC is used on any surface in y-z plane at $x=0$, and PML along the outer side and output. PEC is the default condition elsewhere.

3.1.7. Mesh

Meshing determines how to split up the model into finite elements. There are very elegant, established, and efficient algorithms used for creating the mesh. The most common element shape is triangular (2D and 3D boundaries) and tetrahedral (3D) since these algorithm are most efficient. For RF modles, an element size of $\lambda/10$ good rule of thumb but $\lambda/3$ is alright for coarse simulations.

In Comsol the options for meshing are physics-controlled where element size is detmined by the model setting, user-controlled where there is the ability to define all element sizes per geometry as desired. This model uses $\lambda/10$ at the edges every 5 cm in the z-direction, $\lambda/6$ along the bottom of the model which corresponds to the center between the waveguide plates, and $\lambda/4$ otherwise.

3.1.8. Study

The last step before computing is to set up the study and solver. The solvers can be broken into two categories: iterative and direct. Iterative solvers, as the name would imply, iterates a solution over the whole model until a relative tolerance is fulfilled which means a solution is reached. It is necessary to use preconditioner so that the model has a good starting guess to iterative from. The benefit of iterative methods is that it uses little memory, but the drawback is that it takes a long computational time. In other solver type, direct methods set up a sparse linear system of equations into a matrix A with the model variables. This matrix is set up using preordering algorithm to make the matrix as sparse as possible, such that the number of nonzero values is minimized. The solution is determined using LU factorization and inverting matrix A to compute the solution of unknown. The opposite of iterative solves, the benefit of direct methods is that computes very quickly, but the drawback is that it takes a very large amount of computational memory which scales exponentially with number of elements. Direct method was used in this work. Once the solvers are set, the model is ready to compute!

3.1.9. Analysis and Post Processing

When the computation is complete, Comsol offers very beautiful visualizations of the results. At the user's discretion, any variable on any edge, boundary, or domain can be plotted. Data can also be exported for further analysis.

The figure below shows an example of the output from the modeled quarter PPWG array.

Figure 3.1 Comsol Result
Slice of electric field result (Enorm) on the transverse x-y plane at z=50 cm.

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