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Seismic Damage Accumulation of Highway Bridges in Earthquake Prone Regions

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Civil infrastructures, such as highway bridges, located in seismically active regions are often subjected to multiple earthquakes, such as multiple main shocks along their service life or main shock-aftershock sequences. Repeated seismic events result in reduced structural capacity and may lead to bridge collapse causing disruption in normal functioning of transportation networks. This study proposes a framework to predict damage accumulation in structures under multiple shock scenarios after developing damage index prediction models and accounting for the probabilistic nature of the hazard. The versatility of the proposed framework is demonstrated on a case study highway bridge located in California for two distinct hazard scenarios: a) multiple main shocks along the service life, and b) multiple aftershock earthquake occurrences following a single main shock. Results reveal that in both cases there is a significant increase in damage index exceedance probabilities due to repeated shocks within the time window of interest.

INTRODUCTION

Across the globe, many key critical infrastructure elements, such as buildings and highway bridges, are located in regions prone to earthquake excitations. In such seismically active regions, these structures are likely to experience continued exposure to earthquakes in the form of main shocks throughout their service life, as well as aftershocks immediately following a main shock event. Exposure to repeated earthquake pulses may lead to damage accumulation eventually causing exceedance of limiting threshold capacity and imminent structural collapse. Several field investigations have highlighted cases of structural failure as a result of earthquake damage accumulation. For instance, during the Umbria-Marche earthquake sequence in Central Italy (Amato et al. 1998) on September 26, 1997, several structures withstood a main shock of

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magnitude 5.9. However, the inherent weakening of structural capacities following the main shock event led to eventual collapse during the relatively weaker aftershock of magnitude 5.5 on October 14, 1997. With respect to this earthquake sequence, Abdelnaby (2012) reports about the Foligno Tower which collapsed as a consequence of repeated shaking. While this tower withstood the first two main shocks on September 26, the top of the tower collapsed during the aftershock on October 14. Additionally, Dolce and Larotonda (2001) mentions cases where buildings whose condition was still “satisfactory” after the first two main shocks, suffered “serious to very serious” (partial or total collapse) during the October 14 earthquake. Similarly, in Christchurch, New Zealand, several structures weakened from the magnitude 7.1 Darfield earthquake in September, 2010 suffered partial or complete collapse during a magnitude 6.2 earthquake the following year in February, 2011 (Bradley and Cubrinovski 2011). Consideration of the history of past earthquake events have been traditionally ignored when predicting the structural capacity or seismic reliability of existing structures. Moreover, existing studies on seismic life-cycle analysis of pristine or deteriorating structures (due to corrosion or fatigue or both) primarily assume that the structural damages following an earthquake event are always repaired completely to ensure pristine good-as-new conditions before the occurrence of subsequent earthquakes (Wen and Kang 2001; Ghosh and Padgett 2011; Sanchez-Silva et al. 2011). This assumption can often be impractical under conditions such as: a) when the level of damage resulting in structural weakening is visually insignificant to prompt retrofit actions; b) when economic constraints exist which render retrofit or structural upgrades infeasible after every earthquake; and c) when the duration between consecutive earthquakes is too short to initiate retrofit implementations, for instance, during aftershocks. In this regard, a recent study on main shock-aftershock sequence by Yin and Li (2011) revealed that aftershocks and associated downtime costs are critical contributors to the total seismic losses. However, their study did not account for the accumulation of structural damage along the lifetime of the structure while considering the temporal nature of main shock occurrences. In view of these existing drawbacks, it is of critical importance to develop a framework to predict the probability of structural damage as a result of repeated main shock events along the service life of the structure, or during repeated aftershock occurrences following a main shock.

While relatively little work exists on the damage accumulation in structures due to repeated earthquakes, several researchers have investigated accumulated damage from a single seismic

shock characterized by repeated cyclic loadings. For instance, Jeong and Iwan (1988) studied the effects of durations and loading-unloading cycles due to a single earthquake on the accumulation of strains in structural members. Ballio and Castiglioni (1994) conducted a series of linear and nonlinear analyses to ascertain the dependence of damage accumulation on absorbed energy and earthquake loading history. Some preliminary work on deterministic seismic damage accumulation of structures due to repeated earthquakes can be found in Elnashai et al. (1998) who showed that the ductility demand imposed on a structure following multiple earthquake ground motions is often several times higher than the ductility demand required by a single earthquake occurrence. Studies by Murià-Vila and Jaramillo (1998) revealed a significant reduction in lateral stiffness of a building founded in soft soil under repeated low magnitude earthquake excitations. Recently, Amadio et al. (2003) focused primarily on the behavior of inelastic single degree of freedom system under repeated earthquake ground motion and identified the effects of factors such as structural period, type of earthquake pulse and level of available ductility on damage accretion. However, all of the above mentioned studies are deterministic, without accounting for the inherent probabilistic nature of the hazard or uncertainty in the response of structures under repeated loading conditions. Additionally, these researchers considered a very short duration between earthquake occurrences which is incapable of capturing the aspect of damage accumulation along the service life of the structure for main shock events.

This study focuses on the assessment of damage accumulation under repeated shocks while accounting for the probabilistic nature of the hazard. Within the scope of repeated shock events, this study will focus on two distinct scenarios: a) multiple earthquakes in the form of repeated main shocks along the service life of the structure, and b) main shock-aftershock sequences. Multiple earthquakes generating from proximate faults is however not considered within the scope of the present study. Additionally, while most of previous research has focused exclusively on buildings, this study will concentrate on highway bridges which constitute key elements of the transportation network. A critical step towards formulating the proposed framework is choosing an indicator which reflects the actual cumulative nature of damage under multiple earthquake pulses. Earthquake damage of structures is usually a combination of two limit states of failure (Kunnath and Jenne 1994): a) monotonic structural deformation or ductility, and b) dissipated hysteretic energy. While several damage indices existing in literature focus on these

limit states separately (Khashae 2005), the Park and Ang damage index (Park and Ang 1985) offers a combination of both limit states and has consistently resulted in good agreement with experimental test data for buildings as well as bridges (Chai et al. 1994; Kunnath and Jenne 1994; Williams and Sexsmith 1997). The Park and Ang damage index is used in this study to develop regression models to statistically predict damage accumulation based on the earthquake intensity and past damage history. These regression models are further used to predict the probability of damage index exceedance conditioned on the number of earthquake pulses incurred by the structure. Finally, time-dependent damage index exceedance probabilities are computed using site specific hazard curves for main shocks and aftershocks characterized by homogeneous and non-homogeneous Poisson process rates respectively. The regression model development and damage index exceedance probability computations are discussed in the mathematical formulation section of the paper. The proposed framework is applied on a representative case study single column box girder bridge located near the San Andreas Fault, California. While the readily available main shock hazard curve data for this site is obtained from USGS (2012), the time-dependent aftershock hazard rates are derived from the work by Yeo and Cornell (2009). Conclusions and opportunities for future work are presented in the end of the manuscript.

FORMULATION OF DAMAGE ACCUMULATION FRAMEWORK

PREDICTIVE DAMAGE ACCUMULATION REGRESSION MODELS

The Park and Ang index for damage measurement results from a combination of ductility demand induced by the earthquake and the dissipated hysteretic energy, as shown in Equation 1 (Park and Ang 1985; De Guzman and Ishiyama 2004):

$$D = \frac{\mu_m}{\mu_u} + \beta \frac{E_h}{M_y \theta_y \mu_u} \quad (1)$$

where D is the Park and Ang damage index, μ_m is the maximum ductility caused by the earthquake, μ_u is the ultimate ductility capacity under monotonic loading, E_h is the total hysteretic energy dissipated, M_y is the yield moment capacity, θ_y is the yield rotation angle and β is a dimensionless constant usually assumed to be 0.05 for reinforced concrete (RC) structures.

Additionally, Table 1 lists the classification of damage levels suggested by Park et al. (1985) used to relate empirical observed damages to calculated damage indices.

Table 1. Damage level classification and correlation with calculated damage indices and damage measures as proposed by Park et al. (1985)

Damage Level	Damage Index (D)	Damage Measure
I	$D < 0.1$	No damage; localized minor cracking
II	$0.1 < D < 0.25$	Minor damage; light cracking throughout
III	$0.25 < D < 0.4$	Moderate damage; severe cracking; localized spalling
IV	$0.4 < D < 1.0$	Severe damage; crushing of concrete; reinforcement exposed
V	$D > 1.0$	Loss of element load resistance

It is clear from Equation 1 that the engineering demand parameters, μ_m and E_h , are directly correlated with the characteristic of the structure and the ground motion. Previous research by Cornell et al. (2002) has revealed that for single shock events, engineering demand parameters (EDPs) can be related to the intensity of ground motion using the following functional form:

$$EDP = a(IM)^b \quad (2)$$

where a and b are regression coefficients and IM is the peak intensity of seismic shaking. Although the above relationship (Equation 2) was initially developed and validated by Cornell et al. (2002) for steel moment frames, it has also been widely adopted and implemented for reinforced concrete buildings or highway bridge structures (Nielson and DesRoches 2007; Dolšek 2012; Rajeev and Tesfamariam 2012). These past studies provide confidence in adopting a similar model form for demand modeling of RC bridges like our case study, and motivate the need to test its validity for predictive modeling of the damage index. The model form of Equation 2 can readily be transformed into the linear space to represent the linear regression model for seismic demand as:

$$\ln(EDP) = \ln(a) + b \ln(IM) \quad (3)$$

Hence, for single shock events, since both μ_m and E_h in the transformed space are linear functions of IM , the damage index D is also expected to be a linearly dependent on IM as shown in Equation 4:

$$\ln(D_1) = \alpha_1 + \beta_1 \ln(IM_1) \quad (4)$$

where D_1 is the damage index after the first earthquake pulse, α_1 and β_1 are regression coefficients and IM_1 is the ground motion intensity of the first earthquake shock. The goodness of fit measures obtained confirm that this model can be adopted to predict the damage index for single shock scenarios as will be elaborated later in the case study section.

Unlike single shock scenarios, damage index evaluation under multiple shocks is further involved owing to its dependence on the history of shock occurrences. Under multiple earthquake events, the only parameter which can be considered strictly cumulative is the total energy dissipated (E_h), whereas, the maximum ductility μ_m could have been achieved during the most immediate seismic pulse or in any of the previous pulses, depending on the nature of the earthquake shocks. For clarity, let us suppose that the bridge structure is subjected to two earthquake pulses along its service life and let the corresponding maximum curvature ductilities be μ_{m1} and μ_{m2} and hysteretic energies dissipated be E_{h1} and E_{h2} respectively. In order to calculate the damage index, the total energy dissipated is given by:

$$E_h = E_{h1} + E_{h2} \quad (5),$$

and the maximum curvature ductility is given by:

$$\mu_m = \max(\mu_{m1}, \mu_{m2}) \quad (6)$$

Hence, from the above equations it is clear that while total dissipated energy increases with the number of shocks, the maximum curvature ductility is solely dependent on the strongest pulse in the history of shocks the bridge is subjected to along its service life. It is however noted that the curvature ductility also depends also on the level of degradation since it reflects, for instance, a reduction in the stiffness. Thus it is expected that μ_{m2} should always be greater than μ_{m1} ; however, the influence of other aspects, such as geometry or direction of the movement,

may in a few specific cases lead to a case where $\mu_{m1} > \mu_{m2}$ (Abdelnaby 2012). Regardless of these instances, the damage index of a structure is a quantity that is strictly increasing with the number of earthquake shocks as shown in Equation 7:

$$D_n > D_{n-1} > \dots > D_2 > D_1 \quad (7)$$

where, D_n is the damage index after the structure has been subjected to n shocks. Consequently, the damage index after n earthquake shocks can be described as a multilinear regression model as follows:

$$\ln(D_n) = \alpha_n + \beta_n \ln(IM_n) + \gamma_n \ln(D_{n-1}) + \delta_n \ln(IM_n) \ln(D_{n-1}) \quad (8)$$

where D_n is the damage index after the n^{th} earthquake shock with ground motion intensity IM_n ; α_n , β_n , γ_n and δ_n are regression coefficients, and D_{n-1} is the damage index after $n-1$ earthquake shocks. This multilinear regression model with interaction can be seen as an extension of the model previously presented in Equation 4 since the damage index of the structure after the n^{th} shock naturally depends on how ‘weak’ the structure has become after being exposed to the previous ($n-1$) shocks (quantified by D_{n-1}). While the Park and Ang damage index has consistently emerged as a good indicator of damage for reinforced concrete columns following single shock earthquake events (Kappos 1997; Kunnath et al. 1997), test data validating the performance of this damage measure for multiple shock scenarios is lacking in literature. The methodology presented in this study for probabilistic prediction of the damage index from multiple shocks offers opportunities for possible validation of the presented analytical models as test data for structures subjected to repeated shocks become prevalent in the future.

In order to arrive at either of the regression equations 4 or 8, the structure needs to be subjected to a series of ground motions either individually (for one shock) or in combinations as earthquake trains (for multiple shocks). It is noted that the predictive regression equations for the damage index based on the earthquake intensity are approximate statistical relationships and the error in model prediction is propagated throughout the results developed in this study. Additionally, in this study, the authors have chosen peak ground acceleration (PGA) as the earthquake intensity (IM) measure owing to its excellent predictive capability of the damage index as indicated by the goodness-of-fit estimates (shown in the case study section).

Furthermore, PGA is adopted for the case study given the tradition of adopting PGA as the predictor for demand modeling and fragility analysis of bridges, owing to past studies that have proven its superior performance on the basis of such measures as practicality, efficiency, sufficiency, proficiency, and hazard computability (Padgett et al. 2008). The framework presented in this manuscript is however not limited only to PGA as the intensity measure and is flexible to incorporate any other intensity parameter the user might choose to use; however, the predictive model error propagated in the framework may differ as a function of this selection.

The representative case study example of a single column bridge pier presented in a later section will demonstrate the predictive capabilities and viability of Equation 8 which conditions D_n only on IM_n and D_{n-1} instead of the entire damage history ($D_1, \dots, D_{n-1}$). It is however noted that for multiple column piers, Equation 8 should be derived for the damage index data corresponding to each column of the bridge following the non-linear time history analysis. In addition to the adopted model form (Equation 8), regression models with linear, interaction and quadratic terms were also tested. The improvement in model goodness-of-fit estimates was however found to be negligible and hence the present model form is adopted in this study for simplicity.

DAMAGE INDEX EXCEEDANCE PROBABILITIES: LIFETIME MAIN SHOCK HAZARD AND AFTERSHOCK HAZARD ANALYSIS

After predictive regression equations are formulated, the probability of exceeding different levels of damage indices are computed, given that the structure is subjected to a certain number of shocks. This probability, represented as $P[D > d \mid n \text{ shocks}]$, can be evaluated using Monte Carlo simulation. In this approach, first a large number of earthquake intensity measures are sampled based on earthquake occurrence probabilities corresponding to site specific seismic hazard curves. Second, the total energy dissipated and subsequent damage indices are computed using Equations 4 – 8, while accounting for the uncertainty about the predictive regression models. Finally, the probability of exceeding a certain level of energy dissipated is computed as:

$$P[D > d \mid n \text{ shocks}] = \frac{1}{N_{MC}} \sum_i I[D_{n_i} > d] \quad (9)$$

where, N_{MC} is the total number of Monte Carlo trials, D_{n_i} is a realization of the damage index after n shocks for the i^{th} Monte Carlo trial, $I[\cdot]$ is the indicator function which equals 1 when $[\cdot]$ is true or equals 0 if $[\cdot]$ is false. This study employs 50,000 Monte Carlo trials (N_{MC}) to arrive at accurate estimates of damage index exceedance probabilities as per Equation 9 although a preliminary investigation revealed that the results stabilize with fewer trials.

The probability of damage index exceedance calculated using Equation 9 is dependent on the number of shock occurrences. However, it is often of practical importance to compute the chance of exceeding limiting values of damage index given a time period of interest. Such time durations may include the service life of a structure for life-cycle analysis or the time interval immediately following a main shock when aftershock occurrences are highly probable. Using the total probability theorem, the time-dependent exceedance probabilities may be computed as:

$$P[D > d|T] = \sum_{n=1}^{\infty} P[D > d|n \text{ shocks}] P[n, T] \quad (10)$$

where, T is the time period of interest, $P[n, T]$ is the probability of experiencing n shocks in time T , and $P[D > d | n \text{ shocks}]$ is computed using Equation 9.

This paper computes the probability $P[n, T]$ for two distinct circumstances:

- 1) *Main shocks*: using a constant main shock hazard occurrence rate λ_m for the service lifetime of the structure (e.g., $T = 50$ years), and
- 2) *Aftershocks*: using time-dependent aftershock hazard occurrence rate $\lambda_a(t)$ for a time interval of one year (i.e., $T = 365$ days) following a main shock event, after which the threat of aftershock occurrence usually decays to an insignificant level (FEMA 2000; Luco et al. 2002).

These aforementioned constant (for main shocks) or time varying (for aftershocks) hazard occurrence rates can be obtained from region specific hazard curves, as will be demonstrated in the case study section of this paper. Using the constant or time varying hazard rates and characteristics of a homogeneous or non-homogeneous Poisson process for main shocks or aftershocks respectively, the probability $P[n, T]$ can be computed as:

$$P[n, T] = \begin{cases} \frac{(\lambda_m T)^n}{n!} e^{-\lambda T} & \text{for main shock scenario} \\ \frac{\left[\int_0^T \lambda_a(t) dt \right]^n}{n!} e^{-\int_0^T \lambda_a(t) dt} & \text{for aftershock scenario} \end{cases} \quad (11)$$

While the present study investigates the evolution of the damage index for these two scenarios separately, future studies will consider both main shocks and aftershocks jointly within the damage accumulation framework.

CASE STUDY EXAMPLE

CASE STUDY BRIDGE AND FINITE ELEMENT MODEL

The formulations and the framework developed in the preceding section will be demonstrated using a typical representative case study single column integral concrete box girder bridge located in California (Figure 1). Based on the dimensions and material properties of the deck superstructure the weight of the superstructure is 2850kN. While the superstructure elements such as the bridge deck and the abutments are not explicitly modeled in this study, the superstructure mass is assumed to be represented as a lumped mass on top of the bridge column and the superstructure weight propagated as axial load in addition to the self-weight of the column. The axial load ratio of the column is assumed to be 0.06, typical of single column bridges in California (Brandenberg et al. 2011). The diameter of the bridge column is assumed to be 1.28m and the longitudinal steel ratio in the column is 2.5% of the gross cross sectional area distributed as 22 #14 rebars, each with a nominal diameter of 43mm. The nonlinear finite element model of the bridge column, idealized as a beam-column element fixed at the base, is analyzed for seismic excitation using the finite element software package OpenSees (Mazzoni et al. 2009). The column section is modeled using a nonlinear fiber section with distributed plasticity in which the column concrete is modeled using the Concrete04 material and the steel is modeled using the *uniaxialMaterial Hysteretic* capable of capturing strength degradation from repeated loading cycles. While the simplistic modeling assumptions are adopted to demonstrate the damage accumulation framework, future studies will consider explicit finite element modeling of the overall bridge system in addition to investigating the sensitivity of ground

motion direction on bridge damage. With the present case study bridge, the following sections will demonstrate:

- a) Computation of the damage index under a single shock or train of earthquake shocks,
- b) Formulation of the regression equations to predict the damage index from future shocks, and
- c) Evaluation of the damage index exceedance probability under two distinct cases of main shock and aftershock hazards.

[insert Figure 1 here]

Figure 1. Representative case study single column box girder bridge

DAMAGE INDEX MEASUREMENT FOR SINGLE AND MULTIPLE EARTHQUAKE OCCURRENCES

Damage index measurement following the nonlinear time history analysis of the bridge structure under seismic excitation requires estimation of maximum curvature ductility demand and the total hysteretic energy dissipated. In this study earthquake pulses from a suite of 100 ground motions for California developed by Gupta and Krawinkler (2000) and Krawinkler et al. (2003) are adopted for the finite element simulations. The selected ground motion records are characterized by PGAs between 0.03g to 1.3g, and durations between 18.7 seconds to 99.96 seconds. Additionally magnitudes ranged between 4.7 to 6.5, and distances between 3.6 km to 60 km. Pertinent structural characteristics required for the damage index estimation, such as, yield moment capacity, ultimate curvature ductility, and yield rotation angle are presented in Table 2.

Table 2. Structural characteristics of the example bridge column required for damage index measurement

Structural Characteristic	Symbol	Unit	Value
Yield curvature	ϕ_y	1/m	0.0052
Ultimate ductility capacity under monotonic loading	μ_u	--	17.024
Yield moment	M_y	kN-m	8751.35
Yield rotation angle	θ_y	rad	0.0042

To demonstrate the procedure for damage index quantification, consider the case study bridge column subjected to two consecutive earthquake shocks with peak ground acceleration (PGA) intensities of 0.21g and 0.35g (Figure 2a). The second pulse is appended to the first pulse following a gap representing the no-loading condition such that the vibration response from the first pulse dampens out prior to re-loading (taken as a period of 100 sec in this study). Additionally, in a generic sense, if the first pulse represents a main shock, the second pulse can either represent another main shock pulse occurring at a later point in time along the service life of the bridge, or an aftershock immediately following the main shock. Figure 2b shows the force-displacement curve of the bridge structure which can be used to compute the individually dissipated hysteretic energy for the two earthquake pulses: $E_{h,1} = 119.07$ kN-m and $E_{h,2} = 173.36$ kN-m for earthquake shocks 1 and 2 respectively. Additionally, Figure 2c shows the moment-curvature relation at the column plastic hinge location corresponding to the two-pulse earthquake train with the lower intensity earthquake (pulse 1) leading to a lower value of maximum curvature ductility $\mu_{m,1} = 4.93$ as compared to the relatively stronger earthquake (pulse 2) which results in a maximum curvature ductility $\mu_{m,2} = 6.98$. It is noted that maximum ductility for the n^{th} ($\mu_{m,n}$) shock is obtained by normalizing the maximum observed curvature by the yield curvature as shown in the following equation:

$$\mu_{m,n} = \frac{\phi_{m,n}}{\phi_y}$$

where, $\phi_{m,n}$ is the maximum curvature observed during the n^{th} earthquake pulse, and ϕ_y is the yield curvature.

[insert Figure 2a here]

(a)

[insert Figure 2b here]

(b)

[insert Figure 2c here]

(c)

Figure 2. (a) Train of two earthquake pulses used for deterministic illustration of damage index computation; (b) force –displacement plot of bridge column response depicting the total hysteretic energy dissipation; and (c) moment-curvature plot depicting the maximum curvature ductilities incurred during the two-pulse shock scenario

To demonstrate the concept of damage accumulation with number of shocks, the damage index will be evaluated for two cases. The first case involves damage index measurement for the bridge structure subjected only to the first shock. Using Equation 1 discussed earlier, this damage index D_1 can be computed using as:

$$D_1 = \frac{4.93}{17.02} + 0.05 * \frac{119.07}{8751.35 * 0.0042 * 17.02} = 0.30 \quad (12)$$

In the second case, when both earthquake pulses are considered, the dissipated hysteretic energy used to predict the damage index after two shocks is now cumulative and equals $E_h = E_{h1} + E_{h2} = 119.07 + 173.36 = 292.43$ kN-m, while the maximum curvature ductility is $\mu_m = \max(\mu_{m1}, \mu_{m2}) = 6.98$. The damage index is therefore computed as shown in Equation 13:

$$D_2 = \frac{6.98}{17.02} + 0.05 * \frac{292.43}{8751.35 * 0.0042 * 17.02} = 0.44 \quad (13)$$

The above example demonstrates the phenomena of damage accumulation with increasing number of shocks. While the presented results correspond to the case where the second shock has a stronger intensity than the first shock (as might occur when the structure is subjected to two independent main shocks along the service life), the proposed framework is capable of capturing damage accumulation when the first shock is predominantly stronger than the second shock (for example, during main shock-aftershock scenarios). For instance, when the structure is subjected to the above earthquakes, but in the reverse order, maximum curvature ductility for the second shock is 2.9 as opposed to 3.3 for the first shock. However, even though there is a reduction in the curvature ductility, the computed Park and Ang damage index increases from 0.19 to 0.21. The following section will formulate the regression equations to predict the damage index and quantify the associated uncertainty following single or multiple shock scenarios based on the earthquake intensity and past damage history.

FORMULATING REGRESSION EQUATIONS TO PREDICT DAMAGE INDEX

Regression equations represent statistical relationships between the predictors and predicted variable and can be employed to approximate the damage index for future shocks as a function of ground motion intensity and previous earthquake history (in case of multiple shock scenarios).

For the single shock scenario, regression models similar to Equation 4 are constructed after subjecting the case study bridge structure to the adopted 100 ground motion pulses. The data cloud and the fitted regression line in logarithmic space are depicted in Figure 3 and the regression equation is shown in Equation 14.

$$\ln(D_1) = 1.91 + 2.51 \ln(PGA_1) \quad (14)$$

[insert Figure 3a here]

(a)

[insert Figure 3b here]

(b)

Figure 3. (a) Linear regression model for predicting the damage index following single shock occurrences, and (b) Multilinear regression model for predicting the damage index after three shocks as a function of the PGA of the third shock and damage index incurred up to the second shock

A high value of the coefficient of determination, $R^2 = 0.87$, and a relatively low estimate mean square error $\varepsilon_1 = 0.70$ indicates an adequate model fit to the generated damage index data. To construct similar polynomial regression models to predict the damage index for two or more consecutive shocks, the bridge structure is subjected to train of appended earthquake records (similar to Figure 2a), randomly selected and paired from the same suite of 100 ground motions. The fitted multilinear regression models will now follow the form shown earlier in Equation 8, conditioned on the PGA intensity of the latest pulse and the damage index incurred until the previous shock. For instance, Equation 15 shows the fitted regression model for 2 shocks with an R^2 value of 0.86 and a mean squared error $\varepsilon_2 = 0.68$.

$$\ln(D_2) = 1.82 + 0.77 \ln(PGA_2) + 0.12 \ln(D_1) - 0.33 \ln(PGA_2) \cdot \ln(D_1) \quad (15)$$

These multilinear regression models consistently perform well for a higher number of shocks as tabulated in Table 3 and shown in Figure 3b for three consecutive shock scenario. Additionally, it is observed that the coefficients of the regression models for 2 shocks and higher are marginally different from one another suggesting that earthquake damage has the Markovian property. Furthermore, this similarity can be used with advantage to develop an “average” model to predict the damage index efficiently. The regression coefficients for the average model in addition to those for original models for the multiple repeated shocks are also presented in Table

3. Consistently high R^2 values confirm that the average model performs adequately as well as the original models with negligible loss of accuracy and imparts confidence to adopt this model to capture damage accumulation and predict damage index exceedance probabilities as shown in the next section.

Table 3. Comparison of regression model coefficients and goodness of fit estimates for the original and average models for more than one shock scenario

Model	Number of shocks (n)	α_n	β_n	γ_n	δ_n	Original Model R^2	Average model R^2
Original	2	1.82	0.77	0.12	-0.33	0.86	0.85
Original	3	1.48	0.64	0.19	-0.33	0.90	0.89
Original	4	1.64	0.74	0.27	-0.32	0.91	0.90
Average	--	1.65	0.71	0.19	-0.33	--	--

COMPUTATION OF DAMAGE INDEX EXCEEDANCE PROBABILITIES: LIFETIME MAIN SHOCK AND AFTERSHOCK OCCURRENCES

In addition to the regression models developed in the preceding section, estimation of damage index exceedance probabilities requires information on the hazard potential where the bridge is located. In this study it is assumed that the case study bridge is located near the Stanford University campus site, 10 km away from the San Andreas Fault in California. This site is specifically chosen in this study to aid in exceedance probability computations (demonstrated in a later section), after adopting the available aftershock probabilistic seismic hazard data available in Yeo and Cornell (2009). It is noted that development of aftershock probabilistic seismic hazard curves is not an easy task in itself because of the time-dependent nature of the problem and is an area of ongoing research. Consequently, the amount of literature on this topic is limited. Unlike aftershocks, probabilistic seismic hazard curves for main shocks can be easily obtained from the USGS (2012). The following sections of the paper will exemplify damage index exceedance probability calculation for two distinct scenarios: 1) lifetime main shock hazard, and 2) aftershock hazard.

Scenario I: Main shock hazard

Figure 4 shows the main shock hazard curve for the chosen site near the San Andreas Fault, representing the annual probability of exceeding different PGA intensities. Main shock hazard occurrence is typically considered to be a Poisson process such that the annual earthquake

exceedance probabilities are constant over the service life of the structure. The earthquake *occurrence* probabilities for different PGA intensities ranges can be obtained by calculating successive differences of the PGA *exceedance* probabilities from the hazard curve. A critical step in the lifetime damage index exceedance probability computation involves evaluation of $P[D > d \mid n \text{ shocks}]$. This solution was obtained by using Monte Carlo simulation (outlined in Equation 9) with the following characteristics: a) sampling 50,000 earthquake intensity measures based on earthquake occurrence probabilities; and b) using the regression equations with the associated uncertainties developed in the previous section to estimate damage indices depending on the number of shocks. In this study the authors have considered earthquake intensities with PGA levels of 0.1g and higher because earthquake intensities below this level are found to cause insignificant bridge damage (Nielson and DesRoches 2007).

[insert Figure 4 here]

Figure 4. Main shock hazard near Stanford University campus site, 10 km away from the San Andreas Fault

Figure 5a depicts the probability of exceeding different levels of damage index as the structure is subjected to repeated earthquake shocks. This increasing probability of failures clearly indicates the need to consider multiple shocks within the damage accumulation framework. Additionally, the number of shocks is restricted to 8 because the probability of having more than these many number of shocks in the lifetime of the bridge structure (assumed as $T = 50$ years) is found to be negligible. The probability of number of shock occurrences during the lifetime of the structure is shown in Figure 5b after computing earthquake occurrence probabilities using the Poisson assumption (Equation 11). The constant Poisson hazard occurrence rate λ_m is calculated from the hazard curve after adding the individual annual occurrence rates of different PGA intensity ranges, which are individually Poissonian.

[insert Figure 5a here]

(a)

[insert Figure 5b here]

(b)

Figure 5. (a) Probability of exceedance of different levels of damage index depending on the number of shocks ($P[D > d \mid n \text{ shocks}]$), and (b) Probability of incurring $n \text{ shocks}$ in lifetime $T = 50$ years

In order to make the probability of damage index exceedance independent of the number of shocks, the lifetime exceedance probabilities $P[D > d \mid T]$ are computed using Equation 10. These

probabilities are of particular interest to bridge owners and decision makers since they provide information on the chance of damage index exceedance for the structural service life. Such information may aid in devising potential retrofit strategies or structural upgrades to reduce lifetime risks associated with bridges located in seismic zones. Figure 6 shows the accumulation of lifetime damage index exceedance probabilities for the case study bridge structure. Each color band within Figure 6 represents the contribution from the exceedance probabilities given the number of shocks and the chance of incurring that many shocks within structural lifetime (i.e., $P[D > d \mid n \text{ shocks}] \times P[n, T]$). Also shown is the cumulative contribution of exceedance probabilities for all shocks experienced by the bridge structure, which is equivalent to the lifetime probability of exceedance as indicated in the figure.

[insert Figure 6 here]

Figure 6. Probability of exceeding different damage index levels along the lifetime of the structure. The different color bands in the figure correspond to the joint contributions of exceedance probabilities given the number of shocks and the chance of incurring that many shocks within structural lifetime (i.e., $P[D > d \mid n \text{ shocks}] \times P[n, T]$)

Scenario II: Aftershock hazard

Unlike main shock hazards, aftershock hazard rates are not constant over time and depend heavily on the number of days elapsed since the main shock event (Utsu and Ogata 1995; Yeo and Cornell 2009). While data on site specific aftershock exceedance rates is scarce, Yeo and Cornell (2009) recognize the non-homogeneous Poisson characteristics of this phenomenon and provide sufficient information from which time-dependent aftershock probabilistic hazard curves can be derived. It is however noted that the assumed non-homogeneous Poissonian nature of the aftershocks in Yeo and Cornell's (2009) model is yet to be validated using available techniques. Additionally, this model assumes that aftershocks are uniformly distributed along the fault rupture or concentrated at the ends, which according to Boyd (2012) is unrealistic. The purpose of this research, however, is to present a framework to compute damage index exceedance during main shock-aftershock sequences. While aftershock modeling is not the primary focus of this research, the proposed methodology is flexible to incorporate any emerging aftershock models and Yeo and Cornell's (2009) model is adopted herein for simplicity.

Since aftershock occurrence rates are significantly influenced by the magnitude of the main shock (Ōmori 1894; Utsu and Ogata 1995), this study will focus on aftershock occurrences

following a magnitude 7 main shock event. Such an event is simulated in this study by subjecting bridge structure to the Imperial valley earthquake record from the PEER ground motion database (PEER 2012). With respect to a magnitude (M_w) 7 earthquake, Yeo and Cornell (2009) provide: a) instantaneous daily aftershock rates as a function of time elapsed from the main shock, and b) the probability of hazard exceedance at the site given an aftershock of random magnitude in the aftershock zone. This data is reproduced in Figures 7a and 7b respectively. Additionally, Yeo and Cornell (2009) also indicate that the instantaneous daily aftershock rates multiplied to the probability of hazard exceedance will generate time-dependent aftershock probabilistic hazard curves, as derived in Figure 7c. The number of aftershock occurrences within a year ($T = 365$ days) following the main shock is shown in Figure 7d calculated using the non-homogeneous Poisson process rate $\lambda_a(t)$ from the time-dependent aftershock hazard curves.

[insert Figure 7a here]

(a)

[insert Figure 7c here]

(c)

[insert Figure 7b here]

(b)

[insert Figure 7d here]

(d)

Figure 7. (a) Instantaneous daily aftershock exceedance rate as a function of time following the main shock (adopted from Yeo and Cornell (2009)), (b) probability of hazard exceedance at site given an aftershock of random magnitude in the aftershock zone (adopted from Yeo and Cornell (2009)), (c) time-dependent aftershock probabilistic seismic hazard curve for the case study site, and (d) probability of incurring n shocks in 365 days following the main shock occurrence calculated using the non-homogeneous Poisson process rate from the time-dependent aftershock hazard curves

Due to the initial magnitude 7 Imperial valley earthquake, which already induces some level of structural damage, the shock dependent exceedance probabilities $P[D > d | n \text{ shocks}]$ for the aftershock scenario are higher compared to the main shock scenario presented earlier. This conditional exceedance probability is shown in Figure 8a for the first seven shocks, beyond which the probability of an aftershock occurrence is minimal (Figure 7d). Additionally, Figure 8b depicts the probability of exceeding different levels of damage index for 365 days following the main shock, after which the chance of aftershock occurrence is minimal. A closer observation of the color bands in Figure 8b reveals that contribution of the first three shocks to the cumulative probability of damage is most significant attributed to their high probability of occurrence as compared to other shocks.

While a comparison between Figure 8b and Figure 6 may potentially indicate that aftershock damage exceedance risks are higher than lifetime main shocks, it should be noted that the aftershock results presented in this are valid only following a strong initial main shock of magnitude 7. Hence it is intuitive to expect a potentially higher risk of cumulative damage following a given strong earthquake event than in the case of uncertain lifetime main shocks. Future studies on this topic will investigate aftershock damage index exceedance probabilities for different main shock magnitudes, in addition to building a methodology to study main shocks and aftershocks damages under the same framework.

[insert Figure 8a here]

(a)

[insert Figure 8b here]

(b)

Figure 8. a) Probability of exceedance of different levels of damage index depending on the number of aftershocks ($P[D > d | n \text{ shocks}]$), and b) Probability of exceeding different damage index levels for 365 days after main shock occurrence

CONCLUSIONS

Highway bridges located in seismically active regions are subjected to repeated main shocks as well as main shock-aftershock sequences along their service lives. Each earthquake event leads to weakening of the structure and the remaining structural capacity may be insufficient to resist future events. Existing literature on bridge reliability and life cycle analysis of structures tends to neglect damage accumulation from repeated events and hence potentially under predicts the probability of incurring different levels of bridge damage. In this study a framework is developed to estimate the probability of structural damage due to repeated earthquake occurrences while also accounting for the random nature of hazard occurrence. As an indicator of accumulated damage the Park and Ang damage index is chosen which helps to quantify damage using the maximum curvature ductility induced by the earthquake and the amount of energy dissipated by the structure. A preliminary step in the damage accumulation framework involves the development of linear (for single shocks) or multilinear (for multiple shocks) regression equations to statistically predict the damage index as a function of the earthquake intensity and past damage history. The developed regression equations are independent of the nature of the hazard and can be applied to the bridge structure without any prior knowledge on the site specific hazard. Since the probability of earthquake occurrences are different for different PGA intensities, the site specific hazard curves are used in conjunction with a Monte Carlo strategy to develop probabilistic estimates of damage index exceedance conditioned on the

number of shocks. Finally, time-dependent damage index exceedance probabilities are estimated after computing the likelihood of occurrences of different number of shocks using the constant homogeneous (for main shocks) or time-dependent non-homogeneous (for aftershocks) Poisson process rates.

The proposed damage accumulation framework is applied to a representative case study single column box girder bridge located near the San Andreas Fault, California. Regression equations for single and multiple shocks are developed using individual or trains of appended ground motion records. It is observed that the linear (for single shocks) and multilinear (for multiple shocks) regression regressions provide adequate fits to the simulated data. Additionally, for multiple shock scenarios an “average models” are developed to efficiently predict the damage index for future shock occurrences. While these average models are found to consistently perform well up to the number of shocks used to construct the regression models in this paper, future studies will further investigate the accuracy of prediction for higher number of earthquake sequences.

The developed regression models are applied to two distinct scenarios to demonstrate the versatility of proposed framework. The first scenario focuses only on main shock occurrences at different points in time along the service life of the structure, considered to be 50 years in this study. A region specific main shock hazard curve for the chosen bridge location site is adopted from USGS (2012) to calculate earthquake exceedance probabilities conditioned on the number of shocks as well as the lifetime of the structure. The latter is computed after estimating the probability of incurring different number of shocks using the constant earthquake occurrence rate and homogeneous Poisson process assumption. The second scenario focuses on aftershock occurrences following an initial main shock of magnitude 7. While the same regression equations can still be employed, computation of aftershock exceedance probabilities is slightly more involved than lifetime main shock occurrences. This is due to the time-dependent nature of aftershocks which are most likely to occur in the days immediately following the main shock and have limited likelihood of occurrence after a year. Additionally, unlike lifetime main shock hazard, probabilistic aftershock hazard curve data is scarce. This study derives time-dependent aftershock hazard curves using the data provided by Yeo and Cornell (2009). Consequently time-dependent damage index exceedance rates are computed after using the time-dependent aftershock hazard occurrence rate and non-homogeneous Poisson process characteristics. This

study revealed that for both main shock and aftershock scenarios, there is a significant increase in the probability of damage index exceedance under repeated shock scenarios within the chosen time windows. While the present study treats lifetime main shocks and aftershocks as two separate scenarios, future work will focus on joint consideration of these two hazard scenarios with the same framework. Additionally consideration of multiple earthquakes from proximate faults affecting the seismic performance of highway bridge structures provides opportunities for future research directions. This study highlights the importance of considering multiple shocks and the subsequent accumulation of damage in seismic design and risk assessment. It offers a framework to support retrofit decisions and design upgrades of existing highway bridge structures in earthquake prone regions. For instance the developed methodology can support targeted risk based design wherein the bridge structure can be structurally designed for a target level of damage index exceedance at any point of time along the service life of the structure. Additionally, the regression models for damage index prediction after single or multiple earthquake shocks can prompt decisions on viable repair strategies to be adopted immediately after a seismic event. Furthermore, the proposed damage accumulation models maybe used to update fragility curves, which are of great importance in risk analysis. The developed probabilistic estimates of damage index exceedance will inform bridge owners or stakeholders about the associated seismic risks and assist in devising potential retrofit strategies or structural upgrades to reduce lifetime risks associated with bridges located in seismic zones.

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Daily Hazard Exceedance Rate