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**Low Viscosity Channels and the Stability of Long Wavelength
Convection**

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Abstract

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Mantle convection simulations with a low viscosity channel, akin to the Earth's asthenosphere, are characterized by long wavelength flow structure. Boundary layer theory predicts that as the viscosity of the channel decreases, the wavelength that maximizes heat transfer increases. As a pattern selection criterion, this analysis is not complete. It provides no mechanism to relate the optimal heat transfer wavelength to the wavelength that is realized or preferred in nature. We present numerical simulation suites, for bottom and internally heated end-members, to demonstrate that the cell wavelengths that maximize heat transfer are also the most stable. This does not rule out the possibility of multiple wavelengths being realizable but it does imply that wavelengths near the stability peak will be preferred and, for the configurations we explore, the stability peak corresponds to the energetically most efficient flow configuration.

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Introduction

Seismic tomography indicates that convection in the Earth's mantle occurs at long wavelengths [e.g. Su and Dziewonski, 1992]. This length scale is particularly pronounced in the lower mantle. Isolating the factors that allow for long wavelength flow has been an active subject of research. For example, an insulating continent may be able to inhibit heat loss through the top surface where downwelling plumes form, lengthening the scale of convection [Gurnis and Zhong, 1991]. Another hypothesis focuses on depth dependence of the thermal expansivity [Hansen et al., 1992]. We will focus on the depth dependence of viscosity.

Viscosity is controlled largely by temperature and pressure, both of which increase with depth. The interaction between temperature and pressure dependence on viscosity can be represented by proximity to the melting temperature of the mantle, or solidus.

Information about mantle viscosity is primarily due to post-glacial rebound data.

Loading of the lithosphere occurs during glacial periods; during interglacials, the load is removed. This process can be modeled as a viscous fluid overlain by a rigid lithosphere.

Assuming that the depression has a constant lateral length scale, we can derive an ordinary differential equation in the vertical dependence of the stream function. The imposed boundary conditions are that the horizontal velocity is zero at the surface, and that the hydrostatic pressure head associated with the topography is balanced by the dynamic pressure plus the normal stress at the surface of the viscous half-space. The dependence that the relaxation rate of the depression has on mantle viscosity follows

from these assumptions. The relaxation period can be determined from dating fossil beach terraces. Data from rebound following the melting of the Fennoscandia ice sheet indicates the average mantle viscosity to be roughly $10^{21} \text{ Pa} \cdot \text{s}$ [Haskell, 1935]. The largely vertical flows below the depression in the Haskell model would attenuate small scale lateral variations relatively slowly. Numerical models predict that isoviscous assumptions yield deep troughs at the periphery [Cathles, 1975].

For smaller loads such as the antecedent glacier to Lake Bonneville, this pattern isn't apparent. Instead, if we assume a relatively thin low viscosity channel below the lithosphere, largely horizontal flow could quickly eliminate small wavelength anomalies. Following the melting of a glacier, a lateral pressure gradient would develop towards the center of the depressed region. The relaxation rate can be related to the channel viscosity, leading to estimates of the asthenosphere to be one to two orders of magnitude less viscous than the lower mantle [Bills et al., 1994]. However, significant uncertainty exists because the relaxation rate is also constrained by the depth of the channel. A number of numerical simulation studies have shown that a relatively low viscosity region in the upper mantle can promote long wavelength convection [e.g., Bunge et al., 1996; 1997; Tackley, 1996; Zhong and Zuber 2001].

Theoretical analysis has been used to argue that what is driving these numerical observations is confinement of lateral flow within low viscosity channels [Busse et al., 2006; Lenardic et al., 2006]. Boundary layer theory allows us to characterize the

temperature distribution in a convective system as an isothermal core, with thin boundaries on surfaces with appropriate boundary conditions across which conduction is the dominant form of heat transfer [Turcotte and Schubert, 1982]. The temperature distribution in the boundary layers is the canonical expression for half space cooling, and therefore the buoyancy force can be derived. Imposing energy balance between buoyancy and shear forces, we reach the conclusion that flow channelization lowers lateral viscous dissipation. This allows longer wavelength cells to be energetically more efficient, i.e., the wavelength that maximizes heat transfer increases as the channel viscosity decreases.

Using energetic arguments to predict convection patterns in natural systems has a long history extending from theories of optimal convection [Malkus, 1954], to dissipative systems theory [Glansdorff and Prigogine, 1971], to maximum entropy production ideas [Paltridge, 1979; Ozawa et al., 2003]. It has been argued that all of these approaches reflect the unifying principal that natural, far from equilibrium systems will configure themselves, subject to constraints, so as to minimize flow resistance [Bejan, 2000]. For convection, the minimization is for the effective thermal resistance of the system. A criticism of these ideas is that they lack a mechanistic explanation, i.e., why the patterns observed in nature should be associated with a minimum (or maximum) of some flow property is not directly addressed. For the particular case we are considering, a pressure gradient argument has been offered to compliment the energetic argument [Lenardic et al., 2006]. Beyond the flow wavelength that maximizes heat flux, the lateral pressure

gradients needed to drive long wavelength flow start to inhibit vertical flow into low viscosity channels. Thus, cells longer than energetically optimal tend to become unstable. This connects the heat transfer characteristics of flow wavelengths at or greater than energetically optimal to their associated stability. The argument does not address the stability of wavelengths shorter than optimal.

In Lenardic et al. [2006] it was shown that for any specific viscosity profile, a range of wavelengths were stable to small amplitude perturbations, i.e., order of one percent. The range moved to longer wavelengths as the viscosity of the asthenosphere decreased but all wavelengths shorter than optimal were stable regardless of the viscosity structure. The stability of flow configurations to relatively large, finite amplitude perturbations was not tested. Such finite amplitude perturbations can be associated with physical factors not included in the idealized models. For example, plate re-organizations associated with more complex yielding rheologies and/or the presence of continents [Zhong et al., 2007] could introduce temperature and/or velocity changes that can be effectively viewed as perturbations to the idealized model. Mapping the finite amplitude stability of flow states within model parameter space can isolate a band of wavelengths that will be realizable for the largest range of potential perturbations. If the band of most stable wavelengths coincides with the wavelength region associated with maximum heat transfer then this can provide mechanistic insight as to why optimal wavelengths are favored: Significantly longer or shorter ones are less stable to finite amplitude perturbation introduced to the natural mantle system by, for example, the presence of continents. With this in mind, we use numerical simulations to examine the stability properties of convective flows with the

configurations used by Lenardic et al. [2006]. Specifically, we seek to determine if the most stable wavelength cells are associated with a minimum in the total system thermal resistance.

Model Equations

We begin with the idea of conservation in a fluid system. The time rate of change of a quantity over some volume must equal rate of its production within the volume combined with the net flux into the surface bounding the volume. We apply this principle to arrive at equations for the conservation of mass, momentum, and energy of the fluid, expressed in terms of density (ρ), velocity (u), and temperature (T), respectively. Fluxes and production may be related to the conserved quantity through a constitutive law, informed by empirical observation.

Changes in the fluid's density can be neglected except for its contribution to the buoyancy force, known as the Boussinesq approximation. The momentum flux, or stress, is linearly dependent on strain rate with a constant of proportionality referred to as the kinematic viscosity (μ). This rate is represented as its ratio to the density, the dynamic viscosity (ν). The isotropic component of stress is the pressure (ω). The source and sink of momentum is the buoyancy force, driven by acceleration due to gravity (g) and temperature-driven changes in density ($\alpha\Delta T$). The heat flux is linearly dependent on the temperature gradient with a constant of proportionality referred to as the thermal

conductivity (k). The rate of heat production is constant (H). The conservation equations are non-dimensionalized by setting the length scale by the depth of the system (d), and the time scale by the rate of thermal diffusion (d^2 / κ).

Thermal convection in the mantle can be considered in terms of two end members: an internally heated system and a bottom heated system. In internally heated cases, the temperature is normalized by the rate of heat production and other parameters of the system ($\Delta T = d^2 H \rho / k$). In bottom heated cases, the temperature is normalized by its change across the height of the system (ΔT). The model then becomes characterized by the non-dimensional parameters, the Rayleigh number $\left(Ra = \frac{g \alpha d^3 \Delta T}{\nu \kappa} \right)$ and the Prandtl number ($P = \nu / \kappa$). We take the infinite Prandtl number limit, leaving inertial terms in the momentum balance negligibly small, and the following equations are remaining [e.g. Turcotte and Schubert, 1982].

$$\begin{aligned} \partial_i u_i &= 0 \\ 0 &= -\partial_i \omega + \partial_j^2 u_i + Ra T \delta_{iz} \\ \partial_i T + u_i \partial_i T &= \partial_i^2 T + \begin{cases} 1 & \text{Internal heating} \\ 0 & \text{Bottom heating} \end{cases} \end{aligned}$$

In general, these equations cannot be solved analytically. The boundary layer theoretical arguments discussed in the introduction have limited applicability to asymmetric systems, so we rely primarily on a numerical analysis.

Numerical Simulations

We consider infinite Prandtl number thermal convection in two-dimensional Cartesian domains with free slip upper and lower boundaries and reflecting side walls. Three end-member configurations are explored: 1) Bottom heating with symmetric low viscosity channels at the surface and base, 2) Internal heating with a single low viscosity channel at the surface, and 3) Internal heating with a single low viscosity channel submerged a specified distance below the top of the modeling domain, 4) Internal heating with both a submerged channel and a highly viscous lid at the surface, and 5) Internal heating with a submerged channel at a higher Rayleigh number. For bottom heated cases the upper and lower boundaries have fixed non-dimensional temperatures of zero and one, respectively. For internal heating the lower boundary is zero heat flux. Control parameters are the Rayleigh number based on the viscosity of the central region (Ra), the viscosity ratio between the channels and the central region (r), and the channel thickness (d). For bottom heating, Ra is set to $5e3$. For internal heating, it is set to $1e4$ and $1e5$. For all cases $d=0.1$. The viscosity ratio between the channel and interior is varied from 0.1 to 0.001. We solve the equations describing our system using the CITCOM finite element code [Moresi and Solomatov, 1995]. Two different density finite element meshes are used for the calculations. For one mesh density we apply 32×32 elements over any 1×1 patch of the modeling domain, with enhanced vertical resolution over the upper and lower regions. For the other mesh density we apply 64×64 finite elements over any 1×1 patch. By comparing solutions on different meshes we confirmed that results were adequately resolved for the Ra range covered.

For a simulation suite with fixed heating mode and viscosity ratio, a number of initial states with variable convective wavelengths are generated. The ratio of convective cell length to depth, the aspect ratio, is $\frac{1}{2}$ of the wavelength. The heat transfer properties of different aspect ratio cells are determined as in Lenardic et al. [2006]. The stability of any initial state is determined by applying variable wavelength and amplitude thermal perturbations. If an initial state is unstable to a particular perturbation, it will transition to a new wavelength (Figure 1). If it is stable, the perturbation decays and the initial wavelength is maintained. To determine a minimum instability amplitude, a range of perturbation wavelengths are applied to any initial state (Figure 2). The transitional amplitude, between stable and unstable, is determined for each wavelength. In comparing the stability of different aspect ratio cells, the minimum transitional amplitude, from all perturbation wavelengths, is considered. The only practical limitation on the number of wavelength perturbations explored was set by our choice to limit the size of our modeling domains to be at most 12×1 .

Figure 3a shows the non-dimensional heat flux, Nusselt number, as a function of cell aspect ratio for bottom heated simulations, along with a schematic of the viscosity configuration. The minimum instability amplitude is plotted in Figure 3b. The perturbation amplitude is normalized by the temperature variation across the system. Figure 3a confirms the theoretical prediction that the wavelength that maximizes surface heat flux should shift toward larger values as the viscosity ratio decreases [Busse et al.,

2006]. Figure 3b shows that the shift in the thermally optimal wavelength is associated with a shift in the most stable wavelength.

Figure 4a shows non-dimensional internal temperature as a function of cell aspect ratio for internally heated simulations, along with a schematic of the viscosity configuration. The associated minimum instability amplitude is plotted in Figure 4b. Perturbation amplitudes are normalized by the temperature variation from the surface to the mid-depth for each specific initial wavelength. Figure 4a confirms the theoretical prediction that the wavelength that maximizes internal cooling should shift towards larger values as the channel viscosity decreases [Lenardic et al., 2006]. As with bottom heating, the shift in the thermally optimal wavelength corresponds to a shift in the most stable wavelength (Figure 4b).

Figure 5 shows results from internally heat models in which the low viscosity channel is submerged a non-dimensional distance of 0.05 below the domain surface. Figure 5a shows the non-dimensional internal temperature for a given aspect ratio, and Figure 5b shows the associated minimum instability. Perturbation amplitudes are normalized by the temperature variation across the system. Although the scaling theory characterizing surface channels fails to extend to this parameter space [Lenardic et al., 2006], Figure 5a mirrors the prediction of longer wavelength cells maximizing internal cooling as the channel viscosity decreases. As with the other simulation suites, there is a correlation between the shift in the thermally optimal wavelength and the shift in the most stable

wavelength (Figure 5b).

Figure 6 measures the lateral pressure gradient across the convection cell averaged over the low viscosity channel for the submerged channel case, with a viscosity contrast of 0.001. The pressure gradient peak is roughly coincident with the stability peak. As the wavelength increases past the stability peak, the pressure gradient decreases. It becomes insufficiently large to accommodate the vertical flow into the channel needed to maintain the same length scale, and the pattern starts to break up.

Figure 7 results from imposing a highly viscous surface region with a contrast of 10 with the interior, corresponding to the lithosphere, on the original submerged channel case with a viscosity contrast between the channel and interior of 0.001 and Ra of $1e4$. The optimal wavelength decreases significantly. Continuing the pressure gradient argument that we have been making, the more viscous surface layer imposes a greater shear on the channel, which balances the pressure gradient corresponding to a smaller wavelength than was optimal in the case with a less viscous lid.

Figure 8 shows the extension of our parameter space into a ten times larger Ra , $1e5$. We see that the stability peak once more coincides with the thermally optimal wavelength. However, the band of stability extends to much larger wavelengths, with the stability

peaking at aspect ratios 8 and 10, significantly larger than aspect ratio 5, associated with $Ra = 10^4$. A trend of increasing aspect ratios with increasing Ra is at variance with the pattern of mantle-like Ra leading to smaller length scales. However, secondary instabilities associated with the boundary layer are relatively insignificant at the vigor of convection represented by these figures. We presume that greater lateral dissipation associated with the boundary layer instabilities at high Ra dominate over the dynamics underlying the trend presented here.

Conclusion and Future Work

Our simulations show that the introduction of low viscosity channels to a thermally convecting layer allows longer aspect ratio convection cells to be energetically more efficient and also more stable to finite amplitude perturbations. More specifically, the peak in the convective efficiency versus cell wavelength curve is shown to coincide with the peak in stability versus wavelength space and both peaks shift toward longer wavelengths as the relative viscosity of the channels decreases.

Our main conclusion is limited to the model parameter space we have explored, in particular relatively low to intermediate values of convective vigor, i.e., Ra . Although our calculations are 2-D, the methodology of mapping stability space does require substantial computer and wall time given that a range of perturbation amplitudes and wavelengths need to be considered for a range of cell aspect ratios. Moving to high Ra values would

increase the time needs as denser meshes would be required. However, with academic institutions acquiring computer farms designed for multiple single processor jobs to more efficiently map parameter space, the extension of our results in parameter space is not an impractical line of future work. In terms of extensions to 3-D, recent work has shown that, in terms of heat transfer properties, the general results from 2-D simulations for the submerged channel case carry over to 3-D simulations [Hoink and Lenardic, 2008, 2009]. Determining if the cell stability properties carry over in the same way would also be a useful line of future work (particularly in spherical domains).

Our main conclusion is also limited to the idealized model configurations we explored. Our choice to use these configurations as a starting point is driven by the fact that theoretical analysis of the heat transfer properties for the first two configurations is in place [Busse et al., 2006; Lenardic et al., 2006]. For the third configuration, a reasonably extensive set of numerical simulations, in both two- and three-dimensional domains [Lenardic et al., 2006; Hoink and Lenardic, 2008; 2009], have been used to explore the systems heat transfer properties. A fruitful next step is to consider the stability of convection cells in models with temperature-, depth-, and yield stress-dependent rheologies [e.g., Tackley, 2000a; 2000b; Richards et al., 2001; Stein et al., 2004]. Such models can generate convection with plate-like surface velocities, which moves them closer to representing convection in the Earth's mantle. Our approach of mapping both the heat transfer and stability properties of different cell wavelengths for such models can be used to explore the idea that the Earth's asthenosphere stabilizes long wavelength mantle convection and enhances the rate of mantle cooling.

Finally, a theoretical basis for more general systems would be helpful. An application of Lyapunov stability [Perko, 2001] with a careful selection of the Lyapunov function [Glansdorff, 1972] has been applied to isoviscous systems to yield optimally stable wavelengths [Roberts, 1966]. Unfortunately we have not been able to duplicate this work. Weakly nonlinear stability analysis [Manneville, 1990] may be a fruitful avenue of research, but we would still be limited to a narrow parameter space with little application to convection in the Earth's mantle.

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Figure 1: Thermal fields from representative bottom heated simulations. The fields shown display a situation in which a perturbation amplitude was large enough to cause a wavelength transition.

Figure 2: Thermal fields from representative internally heated simulations. The fields show that the critical perturbation amplitude needed to initiate a wavelength transition depends on the perturbation wavelength.

Figure 3: Plots of Nusselt number (a) and minimum instability amplitude (b) versus cell aspect ratio for bottom heated simulation suites.

Figure 4: Plots of internal temperature (a) and minimum instability amplitude (b) versus cell aspect ratio for internally heated simulation suites with a near surface low viscosity channel.

Figure 5: Plots of internal temperature (a) and minimum instability amplitude (b) versus cell aspect ratio for internally heated simulation suites with a submerged low viscosity channel.

Figure 6: Plots of the normalized horizontal pressure gradient in the submerged channel, vertical velocity in the downward direction, and the minimum instability amplitude, versus the aspect ratio for the internally heated suites with a viscosity contrast of 0.001.

Figure 7: Plots of internal temperature (a) and minimum instability amplitude (b) versus cell aspect ratio for internally heated simulation suites with a submerged low viscosity channel and a viscosity contrast of 0.001. The viscous lid suites are compared against the non-viscous lid suites.

Figure 8: Plots of internal temperature (a) and minimum instability amplitude (b) versus cell aspect ratio for internally heated simulation suites with a submerged low viscosity channel and high Ra .