Broadcast Space-Time Coding

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ABSTRACT

In this paper, we study techniques for broadcast channels with a single transmitter and multiple receivers. The transmitter is assumed to be equipped with multiple antennas. Further, each receiver is interested only in part of the transmitted information. Using Gaussian code based information theoretic bounds, we analyze two transmit beamforming techniques. The first technique, zero-forcing beamformer, uses the channel information for all users to send spatially orthogonal signals to different users, *i.e.*, no user receives interference from the other user signals. The second method uses the single-user optimal beamformer with no effort to reduce interference at the receivers. It is shown that the transmit beamforming techniques are similar to multiuser receivers used in non-orthogonal CDMA systems with random codes. In particular, the zero-forcing beamformer is similar to a decorrelating detector and the single-user beamformer is identical to a matched filter. The comparison with CDMA multiuser receivers is strengthened by results on spectral efficiency of proposed beamformers, which follow the behavior of decorrelating and matched-filter receivers.

Keywords: Downlink transmission, multiple antennas, beamformers, CDMA receivers.

1. INTRODUCTION

Multiple spatially separated antennas at the transmitter and receiver lead to large increase in achievable data rates over fading wireless channels, when compared to their single antenna counterparts.¹ Space-time coding for single-user communication was proposed in^{2,3} to realize the gains due to spatial diversity via multiple transmit and receive antennas. To support multiple users, space-time codes can possibly be used with any multiple access scheme, like code-division multiple access (CDMA) or time-division multiple access (TDMA). In this paper, we study methods to support multiple simultaneously active users by only exploiting their spatial diversity, a method which we label as *broadcast space-time coding**. Our motivation to study broadcast space-time coding is to understand the limits of using multiple antennas to not only increase the data rate per user but also increase user capacity of the system (maximum number of simultaneously supportable users). We emphasize that though we do not employ additional multiple access interference separation using TDMA or CDMA, a possible concatenation of them with broadcast space-time coding is not excluded.

Spatial diversity has been proposed to increase user capacity in space-division multiple access (SDMA) via beamforming for uplink channels (see for example⁵). Similarly, the beamforming based methods have been used for enhancing downlink channel capacity.⁶⁻⁸ In⁶ total downlink transmission power was minimized subject to a minimum on signal to noise plus interference ratio (SINR) for each user. The downlink formulation was further extended in⁸. In^{6,8} it was implicitly assumed that the optimal power and beamforming vectors are known to both transmitter and receiver. In a system where the base-station relies on feedback from the receiver to learn the channel, the methods proposed in^{6,8} require two rounds of training from the base-station for coherent demodulation, first training cycle to learn the channel at the receivers, followed by feedback to the transmitter and then training from transmitter to share the beamforming coefficients. Further, the optimization in⁶⁻⁸ requires that all the constraints on user SINR should be simultaneously satisfied for a solution to exist. If any user SINR cannot be met, then transmission for all the users is turned off, leading to poor utilization of resources.⁹ In fact, if a user is undergoing deep fading, then its transmission should be turned off to not only save power but also reduce the interference for other users. Thus, a more meaningful formulation should allow independently controlling transmission power for each user; a case analyzed in¹⁰ for single-antenna systems.

*The term *broadcast space-time coding* is motivated by the nomenclature used in.⁴

In this paper, we analyze two simple transmit beamforming techniques. The first method implements a simple *zero-forcing beamformer* such that no receiver receives any interference from the signal of other users. Again, since the beamformer depends on channel conditions of other users, two rounds of training are needed in practice to allow coherent demodulation at the receivers. The second beamformer simply uses the *single-user* optimal beamforming¹ for each user. The single-user beamformer requires no knowledge of other user channels and can be implemented in completely distributed manner. Further, it eliminates the second round of training regarding the beamforming coefficients. Both beamformers are analyzed using Gaussian code based lower bounds on multiuser broadcast capacity region.

We note that transmit beamforming is analogous to multiuser receiver design in non-orthogonal CDMA systems; this fact was independently discovered in.⁷ If the channel vectors are analogous to spreading signature, then beamformer design is equivalent to design of multiuser receivers. With the above analogy between CDMA reception and transmit beamforming, the *zero-forcing beamformer* is equivalent to a decorrelating detector (see¹¹ for different multiuser CDMA receivers). A decorrelating detector completely suppresses the interference from other users, much like zero-forcing beamformer suppresses interference at the transmitter. Similarly, the single-user beamformer is same as the matched filter receiver. In,⁷ an MMSE beamformer was derived using the same optimization problem formulation as in.^{6,8} Thus, our results strengthen the similarities between transmit beamforming and multiuser CDMA receivers. Equivalent to the results in CDMA multiuser receivers, we show that zero-forcing beamformer (decorrelator) is worse than single-user beamformer if the difference in transmitted powers of users is high (equivalent to a case of high near-far ratio in CDMA), and vice versa. We also show that for low values of $\alpha = n_u/n_T$, the ratio of number of users (n_u) to number of transmit antennas (n_T), the zero-forcing beamformer is better than single-user beamformer and worse for larger α . The last result is directly analogous to the result for CDMA receivers in Gaussian noise for decorrelating detectors and matched filters.¹²

Throughout our analysis, we assume that all channels are known perfectly and the transmitter and receivers are aware of their own respective channels. The case of perfect channel knowledge forms a precursor to the more practical case of finite rate feedback.^{13–15} Note that by using beamforming, we are implicitly limited to rank one space-time coding. With asymmetric side information about the channel at the transmitter and receiver, it is possible that a combination of space-time coding (with rank more than one) and beamforming performs better than the proposed solutions in this paper. Further, in terms of its analogy with CDMA receivers, beamforming with imperfect channel estimation is similar to CDMA receivers with imperfect code knowledge, a relatively less studied problem. Finally, the transmitter and receiver are assumed to have no knowledge regarding the distribution of the channel coefficients. The assumption on Rayleigh fading of the channel is used only to compare the ergodic capacity performance using a standard channel model, but not to design the system. For the same reason, no power control over time is performed. In fact, the power for each user in each fading state is kept constant, in order to isolate and evaluate the beamforming gain.

In Section 2, we will review the channel model used in the paper. We analyze the decorrelating beamformer in Section 3 and the single-user optimal beamformer in 4. In Section 5, we provide several numerical results to compare the two beamforming approaches. We conclude in Section 6.

2. SYSTEM MODEL

The system model of a broadcast beamforming scheme is shown in Figure 1. We assume multiple-element antennas at the transmitter and the receiver of each user. The number of transmit and receive antenna elements are denoted by n_T and n_R , respectively and n_u is the total number of simultaneous users.

The system works as follows: at a certain symbol interval, the code symbol of the i^{th} user, c_i , chosen from a unit energy constellation distributed as $\mathcal{CN}(0, 1)$, is multiplied by its corresponding $n_T \times 1$ normalized beamforming vector, $\mathbf{W}_i$, summed over all users and then transmitted over the n_T antennas. The power assigned to user i is denoted by P_i , and the average power constraint of the system is expressed as

$$\text{trace}(\mathbb{E}[\mathbf{X}\mathbf{X}^H]) = \sum_{i=1}^{n_u} P_i \leq P. \quad (1)$$

The channel is modeled as Rayleigh fading AWGN, with independent and identically distributed (i.i.d.) fading coefficients between each pair of transmit and receive antennas of each user. The fading is also considered to be i.i.d.

Figure 1. Broadcast Beamforming Scheme

between different users. This means that the users are assumed to be located far enough from each other to guarantee independent fading channels. The above distributional assumptions are used only for evaluation of different methods but not for designing these methods. The baseband representation for the received signal of the i^{th} user is

$$\mathbf{Y}_i = \mathbf{H}_i^T \mathbf{X} + \mathbf{N}_i, \quad i = 1, 2, \dots, n_u, \quad (2)$$

where $\mathbf{X} = \sum_{i=1}^{n_u} \mathbf{X}_i$ is the $n_T \times 1$ vector of the sum of the simultaneous transmitted information data of all users as in Figure 1, $\mathbf{H}_i$ is the $n_T \times n_R$ channel matrix of user i , $\mathbf{N}_i$ is the complex circularly symmetric additive white Gaussian noise vector, and $\mathbf{Y}_i$ is the $n_R \times 1$ received signal. Both the channel matrix $\mathbf{H}_i$ and the noise vector $\mathbf{N}_i$ are distributed as $\mathcal{CN}(0, I)$.

For the beamforming schemes discussed in this paper, it is assumed that the transmitter has perfect channel state information of all users. Moreover, because of the necessity of performing training at the receiver for these schemes, the proposed schemes are better suited for slow fading channels. For simplicity, we also consider the case of single-element receive antenna at the receiver of all users. The generalization to more than one receive antenna will be presented elsewhere.

3. ZERO-FORCING BEAMFORMER

In this transmit beamforming technique, the beamforming vectors at the transmitter are computed such that the power of undesired signals at each receiver is completely canceled out. This can be achieved by choosing the beamforming vector of the i^{th} user, $\mathbf{W}_i$, as the projection of its own channel vector, $\mathbf{H}_i$, onto the intersection of the null spaces of the channel vectors of all other users. Thus,

$$\mathbf{W}_i = \Pi_i(\mathbf{H}_i) \quad (3)$$

where Π_i is the projection operator onto the null space of the span of $\{\mathbf{H}_1, \mathbf{H}_2, \dots, \mathbf{H}_{i-1}, \mathbf{H}_{i+1}, \dots, \mathbf{H}_{n_u}\}$. The optimal receiver for this beamforming scheme is a simple single-user matched filter. To perform coherent detection, it is assumed that the receiver of each user, has perfect knowledge of its own beamforming vector in addition to the channel state information. This requires the existence of two sets of training sequences from the transmitter to the receiver, one for the channel information and the other for the beamforming vectors.

The baseband representation of the received signal of the users in this scheme is obtained from (2) as

$$\mathbf{Y}_i = \mathbf{H}_i^T \mathbf{X}_i + \mathbf{N}_i, \quad i = 1, 2, \dots, n_u. \quad (4)$$

This means that each user will receive only that part of the data which is primarily intended for him. It can be further shown that the achievable rates region of the users, using the zero-forcing beamformer can be expressed as

$$R_i \leq \mathbb{E}_{\mathbf{H}_i} [\log (1 + P_i |\mathbf{H}_i^T \mathbf{W}_i|^2)], \quad i = 1, 2, \dots, n_u. \quad (5)$$

The fact that the beamformer is obtained by projecting onto the null space of other user channels is analogous to the action of decorrelating detector in multiuser CDMA systems. In a decorrelating detector, the signal of other users is completely suppressed by considering the signal components of a particular user which are orthogonal to interfering user signals. By using only the orthogonal components of user of interest, the code design for each user is identical to that of single-user Gaussian channels, since the detector removes all multiple access interference. The same applies to zero-forcing beamformer, though the suppression of interference is done at the transmitter instead of the receiver.

Another important issue in the implementation of this scheme is the assumption of perfect channel information at the transmitter. When only an imperfect version of the the channel is available at the transmitter, the performance of the zero-forcing beamformer is affected significantly. The significant loss in performance results from the beamformers ability of completely suppressing other user signals, leading to reduction in signal of interest and increase in interference from other user signals.

4. SINGLE-USER OPTIMAL BEAMFORMER

When perfect channel state information is available at the transmitter, the optimal single-user beamforming scheme of^{1,11} suggests choosing the beamforming vectors as

$$\mathbf{W}_i = \frac{\mathbf{H}_i^*}{\|\mathbf{H}_i\|}, \quad i = 1, 2, \dots, n_u. \quad (6)$$

The advantage of using (6) is that no extra training is required to learn the beamformer coefficients for coherent detection. Knowing its own channel information, each user can find its beamforming vector according to (6). This is in contrast to the zero-forcing beamformer where a second round of training sequence is also required for the beamforming vectors. This makes the single-user optimal beamforming technique attractive for ad hoc networks with distributed channel state information among the users.

The single-user optimal beamforming scheme, does not cancel out the power of undesired signals at the receivers. Thus, the baseband representation of the received signal of the users can be expressed as

$$\mathbf{Y}_i = \mathbf{H}_i^T \mathbf{X}_i + \sum_{\substack{j=1 \\ j \neq i}}^{n_u} (\mathbf{H}_i^T \mathbf{X}_j) + \mathbf{N}_i, \quad i = 1, 2, \dots, n_u. \quad (7)$$

The achievable rates region of the users is

$$R_i \leq \mathbb{E}_{\mathbf{H}_i} \left[\log \left(1 + \frac{P_i |\mathbf{H}_i^T \mathbf{W}_i|^2}{1 + \sum_{\substack{j=1 \\ j \neq i}}^N P_j |\mathbf{H}_i^T \mathbf{W}_j|^2} \right) \right], \quad i = 1, 2, \dots, n_u. \quad (8)$$

The single-user optimal beamformer is equivalent to the matched-filter receiver in CDMA systems, where each user signal is detected using its optimal receiver for a non-interference Gaussian channel. Since matched-filtering is merely an inner product operation (or equivalently linear filtering), it is robust [†] to errors in filter coefficients. This implies that if there is a mismatch regarding information about the channel at the transmitter and receiver, the performance loss using a single-user beamformer is not as significant as the zero-forcing beamformer.

[†]Inner product operator is a continuous and differentiable operator. Therefore small errors in matched filter receiver or single-user beamformer lead to small loss in overall performance.

5. NUMERICAL RESULTS

In this section, the numerical results for the achievable data rates of the two broadcast beamforming schemes, discussed in Sections 3 and 4, are reported. All results were obtained using Monte-Carlo integration. Since no power control is performed, the power is equally assigned between all the antennas and the total power constraint of the system is normalized to one. It is also assumed that user channels are statistically identical.

In Figure 2, the ergodic capacity region of a two-user system for different number of transmit antennas is plotted using (5) and (8). It can be seen that for both of the transmit beamforming schemes, zero-forcing beamformer and single-user optimal beamformer, increasing the number of transmit antenna elements results in greater capacity region. The gain in achievable data rates resulting from the use of multiple transmit antennas is expected according to the results obtained in¹; for a fixed user population, more antennas imply more degrees of freedom per user in fading channels.

Further from Figure 2, it follows that the single-user optimal beamformer performs better than the zero-forcing beamformer scheme for small number of transmit antennas (4 and 8). As the number of transmit antennas increases, the achievable rates using the zero-forcing beamformer, approaches and exceeds from that of the single-user optimal beamformer. This fact is more evident in Figure 3, where the normalized rates (rate per user) of the same two-user system is plotted for different number of transmit antennas. This result is obtained for the case where power is equally assigned between the two users. The reason for poor performance of zero-forcing beamformer compared to single-user beamformer is due to appreciable loss in signal power due to nulling operation. The same phenomenon occurs when comparing decorrelator with matched filter.¹¹ As the number of antennas increases, this loss due to projection onto null space of other user channels reduces due to availability of additional degrees of freedom. For example, the cross-over point is $n_T = 12$ antennas in Figure 3, for the zero-forcing beamformer to outperform the single-user beamformer, when both users receive equal power.

To obtain different rates for the users, the power allocation among users is varied. That is, to reduce R_1 and increase R_2 , user 1 is given lower power than user 2. Note that this power allocation remains fixed for all fading states, *i.e.*, there is no power control over time. For high or low values of P_1/P_2 (low- R_2 or low- R_1), the single-user beamformer is better than zero-forcing method. The reasons for the relative performance of the two beamformers can be explained as follows. A zero-forcing beamformer nulls the other user channel irrespective of the amount of power transmitted to that user. So, for the cases where P_1 is low, the loss in signal power for user 2 is appreciable due to nulling. In fact, for these cases, the single-user beamformer is approximately the mutual information maximizing beamformer and hence it performs better in that regime (see plots for 16 and 32 antennas in Figure 2 for low values of R_1 or R_2). The above behaviour is identical to relative performance of decorrelator with matched filter for low or high values of near-far ratio.

The effect of the user load (number of simultaneous users) of the network on the achievable per user capacity is shown in Figures 4 and 5. In these plots, the normalized rate is plotted versus number of transmit antennas. Each line represents a fixed user load ($\frac{n_u}{n_T} = \alpha$). For example $\alpha = 1$ represents the case where the number of users is equal to the number of transmit antennas. Figures 4 and 5 show that for both of the two studied beamforming schemes, there is a decrease in achievable normalized rates as the system becomes more loaded (that is, as α increases). However, the drop in the user capacity is more severe for the case of zero-forcing beamformer scheme. Thus, the zero-forcing beamforming scheme is not well suited for highly loaded systems; a conclusion similar to CDMA systems.

The other interesting point from Figures 4 and 5 is the gain of multiple transmit antennas. For the zero-forcing beamformer scheme, no gain is provided by increasing the number of transmit antennas, as long as the number of users are also increasing proportionally (that is for fixed α). In contrast, the single-user optimal beamformer continues to improve with more antennas. This again emphasizes that among the two beamforming schemes, the single-user optimal beamformer is the preferred solution for highly loaded networks.

The performance of the normalized rates for different loads of the network is shown in Figure 6. This result shows a striking similarity of the the broadcast beamforming techniques to spectral efficiency analysis of multiuser CDMA receivers in.¹²

6. CONCLUSIONS

In this paper, we presented preliminary analysis of two simple transmit beamforming methods to support multiple simultaneously active users, which rely solely on spatial diversity. We expect that the connection between transmit

beamforming and multiuser receivers will allow us to borrow efficient and computationally efficient techniques from mature literature of CDMA receivers. Other issues of importance include the form and amount of feedback. Preliminary results in¹³ have shown that a small amount of feedback is enough to achieve most of the gains due to channel state information; similar conclusions were reached for single-user systems in.^{14–16}

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Figure 2. Two-user ergodic capacity regions for different number of transmit antennas (no SNR difference between users).

Figure 3. Single user capacity of a two user system vs number of transmit antennas

Figure 4. Normalized achievable rates (rate per user) of a zero-forcing beamformer scheme.

Figure 5. Normalized achievable rates of a single-user optimal beamformer scheme.

Figure 6. Normalized achievable rates versus user load of the network.