Progressive Source-Channel Coding of Images Over Bursty Error Channels

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Abstract

Transmission of compressed image data over noisy channels is an important problem and has been investigated in a variety of scenarios. In this paper, we propose a progressive time-varying source-channel coding system for transmitting images over wireless channels. The core result of this paper is a systematic method of instantaneous rate allocation between the progressive source coder and channel coder. We develop closed form expressions for end-to-end distortion, as well as rate allocation, in memoryless channels. We extend the memoryless results to an algorithm for fading channels. Experimental results demonstrate the performance of this method.

1 Introduction

Progressive transmission is very useful in image communication, in particular for fast browsing and applications in non-homogeneous networks. Many of the powerful wavelet compression algorithms have this property, e.g. [1, 2]. However, progressive transmission is problematic in the presence of noisy channels. Progressive coders use adaptive forms of variable length coding, where the meaning of subsequent bits depends on the decoding of previous bits. This information is preserved in various “states” of the encoder/decoder. Even a single bit error will change the value of the states with high probability, and the decoded image will diverge sharply from the original. Thus high quality channels are needed to transmit such compressed data.

Various methods have been suggested in the literature to combat channel noise via error control coding for image transmission systems [3, 4, 5]. In this paper, we develop time-varying progressive source-channel coding strategies. We derive general results of instantaneous code rate assignment for memoryless

channels through an analysis of the local importance of the source code bits, as well as error propagation effects. This analysis is general and can be applied to any error correction codes and recursive source coders. Based on this analysis, we report a combined source-channel coding scheme using rate compatible punctured convolutional (RCPC) codes.

Our results for the memoryless channel are a generalization of the work of Sherwood and Zeger [5]. We add to the ideas in [5] by offering a systematic solution for the optimal operating point of the system (in terms of rate allocation). Furthermore, [5] focuses on a time-invariant scheme, while our method is generalized to time-varying as well as time-invariant strategies.

We then extended the memoryless results to fading channels. We use the two-state Gilbert-Elliott model for the fading channel (Figure 2). Our rate allocation scheme for fading channels assumes no channel state information is available at the encoder or decoder. Our results show that for most fading channels, the optimal strategy is to target the design at the “bad” state of the channel.

2 Problem formulation

We use the output of a progressive source coder as the input data stream into our algorithm. This stream is divided into blocks. Each block is supplemented with a Cyclic Redundancy Check (CRC) error detection code, and then goes through error control coding. The error detection code assures that the decoder will know if an unrecoverable error occurs, and will in that event stop decoding.

Assume a memoryless binary symmetric channel (BSC). We denote by e the cross-over probability or bit-error rate (BER) of the BSC, and P_n is the probability of at least one bit error in block n after decoding, i.e., P_n is the probability that block n is not recovered correctly at the decoder. The partial distortion due to successful decoding of all blocks prior to (and including) n is denoted by d_n . The total end-to-end distortion

*This work was supported in part by NSF under grant CDA-9617383.

tion of the system is D . ℓ_n is the length of packet n subsequent to channel coding.

The total distortion can be written in terms of probabilities of error-free decoding up to block n , which means there are no errors in blocks 1 through n , and there is an error on block $n + 1$. This probability is equal to $P_{n+1} \prod_{i=1}^n (1 - P_i)$. Keeping in mind that the first and last block need special treatment, the total distortion for a memoryless channel is

$$D = d_0 P_1 + \sum_{n=1}^{N-1} d_n P_{n+1} \prod_{i=1}^n (1 - P_i) + d_N \prod_{i=1}^N (1 - P_i) \quad (1)$$

where N is the total number of channel blocks. We need to minimize the distortion, subject to a constraint on the total channel rate.

$$\begin{aligned} \min_{\{\ell_i\}, N} \quad & d_0 P_1 + \sum_{n=1}^{N-1} d_n P_{n+1} \prod_{i=1}^n (1 - P_i) + d_N \prod_{i=1}^N (1 - P_i) \\ \text{s.t.} \quad & \sum_{n=1}^N \ell_n = L, \end{aligned} \quad (2)$$

where $\{d_n\}$ are given through the rate-distortion curves of the source coder. L is the total number of bits available, to be divided between the source coder and the channel coder. The optimization problem is over N and $\{\ell_i\}$. N determines the overall tradeoff between source coding bits and channel protection bits, and $\{\ell_i\}$ determines how the total available channel protection is spread over different blocks. To solve (2), we need to write ℓ_n in terms of P_n , the probability of block error. This characteristic can be approximated by a log-affine function, as shown by Figure 1:

$$\log P_n = A \ell_n + B. \quad (3)$$

Note that the channel BER e is now hidden inside model parameters A and B , and will not appear further. Using (3), we rewrite the constraint of the optimization:

$$\prod_{n=1}^N P_n = P. \quad (4)$$

For fixed N , the solution lies at the equilibrium of the Lagrangian:

$$\begin{aligned} J = \quad & d_0 P_1 + \sum_{n=1}^{N-1} d_n P_{n+1} \prod_{i=1}^n (1 - P_i) \\ & + d_N \prod_{i=1}^N (1 - P_i) + \lambda \left(\prod_{n=1}^N P_n - P \right) \end{aligned}$$

Taking the derivative of J with respect to P_j and setting it to zero yields a system of equations. Inverting

Figure 1: Block error rate under different channel code rate while channel BER is $e = 0.1$

and subtracting two subsequent elements of this system of equations gives:

$$\left(\frac{1}{P_{j+1}} - \frac{1}{P_j} \right) (-\lambda P) = (d_j - d_{j-1}) \prod_{i=1}^j (1 - P_i), \quad (5)$$

which is a recursive solution for P_j . Solving for the special case of $j = N$ gives $\lambda < 0$. Substituting back in the solution gives

$$P_{j+1} \geq P_j \quad \forall j,$$

i.e., the optimal end-to-end block errors are non-decreasing, as one would expect.

This analysis is completely general for memoryless channels, and can be applied with any progressive source code and any family of channel codes. The results in this section build on [5], and offer a systematic way of finding optimal time-invariant or time-varying rate allocation between a progressive source coder and channel coder.

Rate Overhead:

In this scheme, some overhead is necessary in the transmission of data: since the channel protection is time-varying, the channel decoder needs to know how to decode the stream at each point in time. To limit the rate impact of the overhead, we take into account the following facts: (a) there are only a finite number of channel codes available to the system (b) The channel code is constant over each block and (c) the channel error protection is uniformly non-increasing.

The header string starts with the number of blocks, followed by the index of the first code rate, then the

Figure 2: The Gilbert-Elliott model

location of transitions. Transition T_i 's are presented by their distance from the end of the stream, motivated by the fact that transitions are more dense at the end of the stream. Since $T_i \geq T_{i-1}$, the number of bits needed to specify T_i is no more than $\lceil \log_2(T_{end} - T_i) \rceil$. This overhead is protected strongly, so it is received at the decoder with error probability better than 10^{-6} . Despite the strong protection, the header is relatively small and its impact on bit rate is negligible, as demonstrated in Section 4.

3 Fading channels

The wireless fading channel can be modeled as a finite state Markov process [6]. A popular model is the two-state Gilbert-Elliott model, as shown in Figure 2. In each state, the channel is described by a BSC with a given BER (denoted e_b for bad state and e_g for good state). The Markov model is specified by transition probabilities P_{gb} and P_{bg} . We will assign the code rate for the Gilbert channel assuming that no realization-specific channel state information is available to either transmitter or receiver.

Because we cannot know the instantaneous state of the channel and its varying BER, we design for a single BER, which we call “design BER” and denote by ϵ_d . This design process is guided by the results obtained in the last section. However, we still need to determine the operating point for our “design BER” ϵ_d . Intuitively, one would expect the best design to be somewhere between the BER of the good state and BER of the bad state of the channel.

Recall that a high setting for ϵ_d means more channel error protection, with fewer bits available for source coding, and vice versa. To gain insight into the trade-offs involved in this case, we go back to the memoryless channel. Assuming a fixed total rate, we designed systems with varying rate allocation (hence different ϵ_d). The results are shown in Figure 3. The drop to the left shows what happens when ϵ_d is too high: fewer bits are available for source coding. But the sharper drop to the right shows that the system collapses if channel error protection is inadequate. This is the case where the decoder cannot recover from errors. The behavior shown by this curve motivates a conservative approach

Figure 3: Effect of rate allocation on end-to-end PSNR. Experiment on 512×512 Lena.

to error protection in time-varying channels. This conclusion is supported by the experiments in Section 4.2.

4 Numerical results

In the following simulations, a 512×512 Lena image, compressed with the progressive wavelet coder of Said and Pearlman [2] is transmitted through a noisy binary symmetric channel (BSC). A rate compatible punctured convolutional (RCPC) code is used for error protection, with list Viterbi decoding. The RCPC code is punctured from a $1/4$ parent code with 6 memory cells. The puncturing period is 8 and the puncturing matrices are from [7], allowing code rates between $8/32 \sim 8/9$. Number of candidate paths in list decoding is 70. The source block length is 202, to which is added 16 bits of CRC.

4.1 Code rate assignment for memoryless channels

In this experiment, the Lena image is transmitted through a memoryless BSC with BER 0.1. The block error rates of the system under different code rate are obtained through simulations. Figure 1 illustrates the log-affine characteristic of the error correction performance, which justifies (3).

The code rate assignment of this system is given by a solution to (2). Figure 4 shows the optimal error protection profile at BER 0.1 and transmission rate 0.2 bpp. The two staircase characteristics represent rounding to nearest rate and nearest lower rate (because channel codes are not available for arbitrary rates). For comparison, we also present the fixed-rate

Figure 4: Optimal code rate profile at channel BER 0.1 and transmission rate 0.2

solution. The final end-to-end distortion of the system is tested through Monte Carlo simulations and presented in Table 1. The impact of overhead on bit rate is approximately 0.001 bpp.

This experiment indicates up to 0.3 dB gain for a variable-rate strategy, compared to fixed-rate. Most of this gain is at lower transmission rates, due to the flatness of rate-distortion curve at higher rates.

4.2 Code rate assignment for fading channels

The characteristics of various fading channels used for our experiments are given in Table 2. In our first experiment, we use Channels 1, 2, and 3. They all share an average BER of 0.1, but have different dynamics. We designed a system for a bit error rate of 0.1, and then simulated the system on the three channels mentioned above, as well as a memoryless BSC with BER 0.1. Table 3 gives the mean and variance of PSNR in each case.

All fading channels have lower average PSNR compared to memoryless BSC. Channel 3, which has the highest e_b , suffers the most. This is due to the fact

rate	0.1	0.2	0.4	0.6	0.8	1.0
fixed	25.33	27.64	30.23	32.00	33.20	34.24
variable	25.64	27.87	30.45	32.11	33.32	34.26

Table 1: Average PSNR for 512×512 Lena over memoryless BSC with BER 0.1 (0.001 bpp overhead not accounted for in variable-rate case)

	e_g	e_b	P_{gb}	P_{bg}
channel 1	0.001	0.111	9/100	1/100
channel 2	0.01	0.11	9/100	1/100
channel 3	0.068	0.12	8/100	5/100
channel 4	0.001	0.12	0.005	0.0031
channel 5	0.001	0.12	0.00125	0.005

Table 2: Channel cross-over and Markov parameters

rate	Average PSNR			Variance of PSNR		
	0.1	0.5	1.0	0.1	0.5	1.0
BSC	25.64	31.33	34.26	0.32	0.70	0.80
ch 1	25.52	31.06	33.90	0.59	1.08	1.42
ch 2	25.49	31.10	33.89	0.70	1.12	1.56
ch 3	25.14	30.13	32.51	1.19	1.99	2.75

Table 3: Mean and variance of PSNR for different channels

that decoding may collapse if the channel slips into a state with a BER higher than what the system is designed for. In comparison, the bad states of Channel 1 and 2 are almost the same but the good state of channel 1 is 10 times better than channel 2. Still, their performance is almost the same. The effect of bad state is dominant in these channels due to the sharp drop in PSNR as e_b increases from e_d .

This experiment suggests that the design BER e_d should be chosen to be at, or very close to, e_b . The tradeoff in the choice of e_d is to have less image data and more protection, or more data and less protection. But the sensitivity to protection overrides the marginal gain due to more data, and hence has to be the overriding concern.

The previous fading parameters were designed to illustrate the importance of e_b . But what about realistic fading channels? Channel 4 has fading parameters that correspond to the GSM system with carrier frequency 900 MHz, transmission rate 33.8 kbps/user, and a mobile at 35 mph. The average burst length for this set of parameters is around several hundred bits. Figures 5 and 6 show the mean and variance of PSNR for this system as the design BER is varied. It is easily observed that the best design BER again coincides with the BER of the “bad” state of the channel.

Finally, noting that in the previous experiment, the channel stays in the “good” state 38.5% of the time, we performed our last experiment on Channel 5, which stays in the “good” state 80% of the time. The results are shown in Figures 5 and 6. This time, while the

Figure 5: Average PSNR under different design BER: solid line for channel 4, dashed line for channel 5

optimal point is not exactly e_b , it is very close. Furthermore, by choosing $\epsilon_d = e_b$ the PSNR performance is also very close to the optimal point (not a sharp drop off). It seems that one can use, very generally, the design criterion of targeting the “bad” state, and always remain at or near optimality.

5 Conclusions

In this paper, we present a systematic way to find the optimal time-varying rate allocation for progressive joint source-channel coding. This scheme can be applied to any progressive source code and any family of channel codes. For memoryless channels, we analyze the effect of channel error propagation in the decoder, and formulate an optimal instantaneous trade-off between source and error protection bits. In the case of memoryless channel, we find closed form expressions for the end-to-end PSNR as well as instantaneous rate allocation. This result is generalized to an algorithm for the fading channel. Experiments show time-varying allocation provide up to 0.3 dB gain over the time-invariant case. Most of the gain is obtained at high BER and low transmission rate. We find that in fading channels, the optimal design BER is always on or very close to the BER of the bad state.

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Figure 6: PSNR variance under different design BER: solid line for channel 4, dashed line for channel 5

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