A NOVEL ADAPTIVE PRE-DISTORTER USING LS ESTIMATION OF SSPA NON-LINEARITY IN MOBILE OFDM SYSTEMS

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ABSTRACT

Several LUT-based pre-distortion schemes using *Saleh's* model for compensation of RF non-linearity have been studied recently. In this paper we propose a novel parameter-based pre-distorter to enhance the power efficiency of mobile OFDM transmitters using solid-state power amplifiers (SSPA). Both the non-linearity and inverse of the non-linearity are estimated as memory-less polynomials using Least Squares (LS) criterion. A fully digital pre-distorter is then constructed with a closed-form inverse non-linearity to reduce the in-band non-linear distortion and out-of-band spectrum re-growth in the transmitter. Simulation results in 16-QAM OFDM show promising performance and simple implementation compared with the LUT method.

1. INTRODUCTION

Also known as orthogonal frequency-division multiplexing (OFDM), multi-carrier modulation (MCM) has been studied extensively because of its robustness in multi-path fading and simplification of the equalizer with the application of cyclic prefix (CP) as well as the application of FFT/IFFT for modulation. OFDM has been adopted in several wireless standards such as IEEE 802.11a WLAN and ETSI terrestrial broadcasting.

However, one major drawback in a practical system design with RF is the notoriously high Peak-to-Average Power Ratio (PAPR). This introduces in-band non-linear distortion and interference to adjacent channels due to out-band spectrum re-growth. Backoff schemes as well as PAPR reduction schemes were studied recently [1,2,3] to maintain linear transmission. However, backoff schemes reduce the power efficiency of RF transmitters dramatically and have deleterious effects on battery lifetime in mobile applications. In order to maximize the power efficiency of the transmitter, the power amplifier is typically forced to work near its saturation range, which will lead to inevitably high non-linearity in many cases.

A complicated pre-distortion technique was proposed for systems using high power amplifiers (HPA) to compensate for the non-linearity caused by the RF transmitter. The concept is to distort the input signals to the HPA with an inverse function of the HPA non-linearity and generate linearly amplified signals at the HPA output [4]-[7]. Almost all schemes applied *Saleh's* memory-less model for TWT type HPA [8]. However, SSPAs are generally used in mobile systems instead. Not much work has been reported yet for SSPA in the context of OFDM. As for

the implementation, most of them used the Look-up-table (LUT)[4,5,6] to provide numerical solution for the pre-distortion because it is difficult to get a closed-form inverse function of *Saleh's* model. However there are several disadvantages in LUT method: inherent quantization noise caused by the limited size of the LUT; large size of LUT increasing with the increase in the number of bits representing the signal; and a long time involved in the update of the LUT after estimating the property of HPA.

In [9], a gradient algorithm for pre-distorter using polynomial model is analyzed. But it didn't reach the initial goal to justify the normalized step better for pre-distortion, as claimed by the authors. In this paper, we show a LS this model accurately captures the AM/AM characteristics and provides several efficiencies for pre-distortion. We then derive the closed-form inverse of the non-linearity and construct a parameter-based pre-distorter in the base-band with a fully digital signal processing (DSP) implementation. The cost is two LS estimations of a linear equation with *Vandermonde* matrices. The simulation in 16-QAM OFDM shows very promising performance.

2. SYSTEM MODEL

In the OFDM transmitter, a set of information bits $[b_1 b_2 b_3 \dots b_M]$ are first mapped into the I/Q channel baseband symbols $\{S_n^{(i,r)}\}$ using a modulation schemes such as phase-shift-keying (PSK) or quadrature-amplitude-modulation (QAM). Each N symbols are made parallel into a block $[S_1^{r,i} S_2^{r,i} \dots S_N^{r,i}]^T$ at the input of IFFT. OFDM symbols in time domain over time interval $t \in [0, T_s]$ are generated by IFFT as,

$$s(k) = s_r(k) + js_i(k) = \frac{1}{\sqrt{N}} \sum_{n=1}^N S_n e^{j2\pi(k-1)(n-1)/N} \quad (1)$$

for $k=[1,2,\dots,N]$. Then the first G coefficients are repeated after N coefficients the original as Cyclic Prefix (CP) and made serial for quadrature modulation. We get the analog signal after DAC as,

$$\tilde{s}(t) = \Re\{x(t)e^{j(\omega_c t + \phi(t))}\} \quad (2)$$

with the amplitude and phase of the input signal to the RF transmitter,

$$x(t) = \sum_{i=0}^{+\infty} \sum_{k=0}^{N_s-1} \sqrt{s_i^2(k) + s_q^2(k)} p(t - k\frac{T_s}{N_s} - iT_s)$$

$$\phi(t) = \tan^{-1} \left(\frac{\tilde{s}_Q(t)}{\tilde{s}_I(t)} \right) \quad (3)$$

with $p(t)$ the pulse function, T_s the period for one symbol and N_s the number of samples after CP.

A RF transmitter using super-heterodyne architecture will include several analog components such as passive band-pass filter, passive mixer and power amplifier. Each of these RF components has certain levels of non-linearity. The power amplifier is of particular interest because its specification determines the maximum transmitted power. Equation (2) is considered as the input to a cascaded non-linear system of multi-stage analog circuits.

3. PROPOSED PRE-DISTORTER

3.1. Peak-to-Average Power Ratio (PAPR)

The distribution of the I-channel signal in (2) with 256 quantization level is depicted in Fig. 1. Because of the summation feature of OFDM signal, it can be approximated as a *Gaussian* distribution according to the central-limited theorem. Thus the signal of (2) has a very high PAPR, which is defined as maximum power over the average power of the signal [1].

$$\Gamma_{old} \approx \frac{1}{P_{av}} \max_{0 \leq t \leq T_s} |\tilde{s}(t)|^2 \quad (4)$$

The high PAPR of OFDM signal requires a very good linear transmission of the signal.

Fig. 1, Distribution of the amplitude of OFDM signal.

3.2. Non-linearity models

Power amplifiers are typically the most power-hungry components of RF transceivers. The design of PAs, especially for linear, low-voltage operations, remains a difficult problem defying an elegant solution. Two types of amplifiers are mostly used in communication: TWT and SSPA. TWT is mostly used for high power satellite transmitters while SSPA is used in many other applications including mobile transmitters because of its small size. Volterra series can be used to model a nonlinear system with memory. However this model involves the solution of high order filtering coefficients fitted by adaptive learning with extremely high complexity. Several previous papers used *Saleh's* model to analyze the HPA [4,5,6]. The complex output of RF with non-linear distortion can be expressed as

$$\tilde{x}(t) = A[\rho(t)]e^{j(\varphi(t) + \Phi(\rho(t)))} \quad (5)$$

where $\rho(t)$ and $\varphi(t)$ are the amplitude and phase of the input signal. The measured AM/AM and AM/PM for TWT is well presented by *Saleh's* model [3] as

$$\begin{cases} A(x) = \alpha_A x / (1 + \beta_A x^2) \\ \Phi(x) = \alpha_\Phi x^2 / (1 + \beta_\Phi x^2) \end{cases} \quad (6)$$

while SSPA's AM/AM and AM/PM can be captured by [6],

$$\begin{cases} A[\rho(t)] = \rho(t) / [1 + (\rho(t) / A_{max})^{2p}]^{1/2p} \\ \Phi[\rho(t)] = 0 \end{cases} \quad (7)$$

Here A_{max} is the maximum output amplitude and the parameter p controls the smoothness of the transition from the linear region to the limiting region.

However, these traditional models are based on single-tone amplitude sweeps and are not capable of truly representing the device behavior in the context of modern wideband digital modulations such as OFDM, WCDMA with non-constant envelope [4]. In this paper we propose to use a memory-less polynomial model in the transmitter, regardless of the SSPA type. The polynomial is written as,

$$y(t) = \sum_{i=0}^P \alpha_i x(t)^i = \alpha_0 + \alpha_1 x(t) + \sum_{k=2}^P \alpha_k x(t)^k \quad (8)$$

where $x(t)$ and $y(t)$ are the input and output of the nonlinear RF stage respectively. α_0 represents the DC offset, α_1 is the linear scalar, $\{\alpha_2, \dots, \alpha_P\}$ contribute to non-linearity in the system and P is the highest order. We will show that this model has several conveniences in compensation of the non-linearity.

In derivation of the proposed scheme, we will use the discrete formulation by assuming that we can get the samples of both $x(t)$ and $y(t)$ in (8). We also assume that the combinational effect of the thermal noise from the RF circuits is considered as white *Gaussian* noise. We denote the overall non-linearity for cascade non-linear systems $f_N(\dots f_2(f_1(x))\dots)$ as $f(x)$. The proposed pre-distorter includes two stages: the estimation stage with training data and the compensation stage for the actual signal.

3.3. Least Square (LS) Estimation of Non-linearity

In the estimation stage, we will send a training sequence with sufficient dynamic range to probe the non-linearity. Then the RF output with this sequence is fed back to the base-band of the transmitter and sampled as,

$$y_n = f(x_n) + w_n = \sum_{i=0}^P \alpha_i x_n^i + w_n \quad (9)$$

By defining $N \times 1$ signal vectors $Y_t = [y_1 \ y_2 \ \dots \ y_N]^T$, $X_t = [x_1 \ x_2 \ \dots \ x_N]^T$, noise vector $\underline{w} = [w_1 \ w_2 \ \dots \ w_N]^T$ and non-linearity coefficients vector $\underline{a} = [\alpha_0 \ \alpha_1 \ \alpha_2 \ \dots \ \alpha_P]^T$, we form a $N \times P$ Vandermonde matrix $\tilde{X}_t = [1_{N \times 1} \ X_t \ X_t^2 \ \dots \ X_t^P]$. We get a matrix formulation of (9) as,

$$Y_i = \bar{X}_i^H * \underline{a} + \underline{w} \quad (10)$$

with $\underline{w} \sim N(0, \sigma^2 \mathbf{I})$ a Gaussian noise vector. The LS estimation of the coefficients is,

$$\hat{\underline{a}} = [\bar{X}_i^H \bar{X}_i]^{-1} \bar{X}_i^H * Y_i \quad (11)$$

It is known that when the noise is *Gaussian*, this estimation becomes a minimum variance unbiased estimation (MVUB).

3.4. Pre-distorter

Several LUT-based pre-distorter can be found in the literature such as Mapping pre-distorter, and Gain-based pre-distorter [4]. Although they also estimated the non-linearity characteristics (most of these works used TWT though), the final solution is not parameter-based and has inherent disadvantages as analyzed before. Here we derive a closed-form pre-distorter using a polynomial model.

As in Fig. 2, the purpose of pre-distortion is to find another pre-distortion function $g(x)$, so that the overall effect of the RF output will be linear as,

$$\tilde{y}(n) = f(\tilde{x}(n)) = f(g(x(n))) \approx \alpha_1 x(n) \quad (12)$$

By denoting the pre-distortion function $g(x)$ as another Q-order polynomial $\tilde{x}(n) = g(x(n)) = \sum_{i=0}^Q \beta_i x^i(n)$, we get

$$\tilde{y}(n) = \sum_{i=0}^P \alpha_i \left(\sum_{j=0}^Q \beta_j x^j(n) \right)^i = \sum_{k=0}^{PQ} \gamma_k x^k(n) \approx \alpha_1 x(n) \quad (13)$$

with $\gamma_k = \zeta(\alpha_i, \beta_j)$ a function of both α_i and β_j . The direct solution of this is extremely difficult. We noticed that

$$g(x(n)) = f^{-1}(\tilde{y}(n)) = f^{-1}(\hat{\alpha}_1 x(n)) \quad (14)$$

With training data, $f^{-1}(y)$ is fitted by another polynomial,

$$x_i(n) = f^{-1}(y_i(n)) = \ell(y) = \sum_{i=0}^Q \lambda_i y_i(n), \quad (15)$$

Similar to the solution of forward non-linearity, we get

$$\hat{\Lambda} = (\bar{Y}_i^H \bar{Y}_i)^{-1} \bar{Y}_i^H X_i \quad (16)$$

with $\bar{Y}_i = [Y_i^0 \ Y_i^1 \ \dots \ Y_i^Q]$ the *Vandermonde* matrix of the output signal vector and $\hat{\Lambda} = [\hat{\lambda}_0 \ \hat{\lambda}_1 \ \dots \ \hat{\lambda}_Q]$ the estimated inverse non-linearity coefficients. Then the solution of $g(x)$ is

$$g(x(n)) = \ell(\hat{\alpha}_1 x(n)) = \sum_{i=0}^Q \lambda_i \hat{\alpha}_1^i x^i(n) = \sum_{i=0}^Q \beta_i x^i(n) \quad (17)$$

where $\beta_i = \lambda_i \hat{\alpha}_1^i$ can be written in a matrix form as

$$\hat{\mathbf{B}} = [\hat{\beta}_0 \ \hat{\beta}_1 \ \dots \ \hat{\beta}_Q]^T = \hat{\Lambda} \cdot \text{diag}(1, \hat{\alpha}_1, \hat{\alpha}_1^2, \dots, \hat{\alpha}_1^Q) \quad (18)$$

The architecture of the pre-distorter is depicted in Fig. 3. A linear demodulator will sample the RF output of training data. Then the non-linearity and inverse non-linearity coefficients are estimated in the DSP base-band as in (11) and (16). The pre-distortion $g(x)$ is then constructed as (17).

Fig. 2. Mathematical model for pre-distortion.

Since the power amplifier property varies with time due to aging, temperature changes, supply voltage variations and the power control, the predistortion requires adaptive updating. The computation cost is the estimation of both the $f(x)$ and $f^{-1}(x)$ in the form of $[(X^H X)^{-1} X^H]$ of *Vandermonde* matrix. The computation of it can be simplified by using the structure of the *Vandermonde* matrix. For example, by designing the training data $x_i, x_j \neq 1, \forall i, j \in \{1 \ 2 \ \dots \ P\}$, we have

$$X^H * X = (y_{ij})_{P \times P} = (1 - \{x_i x_j\}^N) / (1 - \{x_i x_j\})_{i,j} \quad (19)$$

The computation of (11) and (16) can be further simplified using advanced linear algebra methods exploring the structure of *Vandermonde* matrix.

4. SIMULATION RESULTS

The performance of the proposed pre-distorter is analyzed in a simulation with the specification of IEEE 802.11a OFDM WLAN standard. $N=64$ sub-carriers are used for orthogonal modulation and $G=16$ cyclic prefix are applied. We used 16-QAM as the modulation scheme. We also used the model in (7) with $p=1, A_{max}=2$ to generate the RF non-linearity. Notice that although this model is not exactly same as the real specifications of RF circuits such as P1dB, IP3 etc, it is appropriate to use it for analysis because it has a relation with those real specifications.

The responses of several functions of pre-distortion are shown in Fig.4. It can be seen that the selected parameters generate smooth non-linearity $f(x)$, which means that there is no sharp saturation point for the amplifier. The pre-distortion $g(x)$ is a rotation of the inverse function $l(x)$ with a new scaled slope. We finally get very linear overall response $f(g(x))$ with slope α_1 . However, we also noticed a slight divergence from the linearity for pre-distortion when the input is increased to a critically high level and the output can be considered deeply saturated (almost flat-top). This is because the corresponding range of the inverse non-linearity becomes very sharp when the non-linearity is almost flat. However α_1 is typically much larger than 1. $\alpha_1 x$ might exceed the operating range of x in the inverse function $l(x)$. Although the mathematical solution of polynomial pre-distortion can generate a deep sink or a sharp cliff because of approximation, in the actual simulation the response become flat because of the effect of compression by $f(x)$.

The BER curves of pre-distorted scheme with different "output backoff (OBO)" levels are depicted in Fig.5, together with the non-linearly distorted BER with different OBO levels.

OBO is defined as the maximum output power (saturation power) over the mean power of the transmitted signal as “ $OBO(dB)=10\log_{10}(P_{max}/P_{avo})$ ”. It determines the operating point of the amplifier. It is found that the pure non-linear distortion caused the larger BER compared with linear transmission at $OBO=5, 7$ dB. Even when OBO is as large as 10dB, there is a degradation of more than 2 dB in SNR to achieve BER 10^{-4} . But with pre-distortion, the BER curve is very close to linear case with $OBO=7$ dB. However, when OBO decreased to 5 dB, we found the BER increases. This means that pre-distortion works in a limited input range without very deep saturation. But this is still much better than pure non-linearity at the same OBO level. Actually this is the inherent limit of most pre-distortion schemes, including the LUT-based pre-distortion.

5. CONCLUSION

In this paper¹ we derived a closed-form estimation both of the non-linearity and inverse of non-linearity with memory-less polynomial model of SSPA used for mobile OFDM transmitter. Then a fully digital parameter-based pre-distorter is derived for OFDM. Simulation shows promising performance in terms of spectrum and BER.

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Fig. 3. Architecture of the proposed pre-distorter.

Fig. 4 Response curve of the functions in the pre-distorter.

Fig. 5. BER performance with different OBO level for 16-QAM with 64 sub-carriers OFDM.

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