Joint Signaling Techniques and Spectral Optimization for Symmetric Bit-Rate Communication Over Self-NEXT-Dominated Channels

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Abstract—We present a framework for maximizing the capacity of symmetric bit-rate communication services dominated by Gaussian crosstalk, in particular, *digital subscriber line* (DSL) services. We solve for optimal transmit power spectral densities (PSDs) that maximize the joint capacity of same-service users and yield significant gains in bit rates (or performance margins) over current schemes. Our results differ from previous work in that we develop transmit spectra in the presence of self-far-end crosstalk in addition to self-near-end crosstalk, present optimal contiguous spectra for practical modulation schemes, and derive optimal spectra under an additional frequency-domain peak-power constraint. Furthermore, by design, the optimal transmit PSDs are spectrally compatible with existing services on neighboring lines.

Index Terms—Crosstalk, digital subscriber line (DSL), information rates, multiuser communication, spectral compatibility.

I. INTRODUCTION

IN THIS LETTER, we derive optimal transmit power spectral densities (PSDs) that maximize the bit rates of symmetric bit-rate communication services dominated by Gaussian crosstalk. While our results are general, we will phrase them in the context of digital subscriber line (DSL) communication to be concrete. In a DSL scenario, a service $\mathcal{S}$ faces *self-far-end crosstalk* (self-FEXT), *self-near-end crosstalk* (self-NEXT), and additive Gaussian noise (AGN). We also derive practical contiguous PSDs and transmit PSDs in the presence of an additional frequency-domain peak-power constraint.

We assume the usual DSL environment setup [1] with standard crosstalk, channel, and noise assumptions [3] (see [2], [4], and [5] for detailed introductions to DSL and crosstalk). We divide the *transmission bandwidth* B of the channel into K narrow frequency bins, each of width W Hertz and center frequencies f_k , $k \in \{1, \dots, K\}$ (see Fig. 1). Let H_k , X_k , and F_k denote the magnitude squared of the channel, self-NEXT, and self-FEXT transfer functions in bin k . Further, let $N_k = N(f_k)$ denote the combined AGN from different service interference and channel noise [6], and P_k the power in bin k .

Let $S^u(f)$ and $S^d(f)$ denote the upstream and downstream transmit PSDs, and let $s^u(f)$ and $s^d(f)$ denote the same in a

Fig. 1. Magnitude-squared transfer function of the channel, 39 self-NEXT interferers, and 39 self-FEXT interferers for carrier serving area (CSA) loop 6. The figure also shows a narrow frequency bin centered around frequency f_k with the channel (H_k), self-NEXT (X_k), and self-FEXT (F_k) transfer functions approximated by constants over the bin.

single frequency bin. A *no-division signaling* (NDS)¹ scheme in frequency bin k is one for which $s^u(f) = s^d(f) \neq 0$ for all f in the bin; that is, both upstream and downstream transmissions occupy the band $|f - f_k| \leq (W/2)$. A *frequency-division signaling* (FDS) scheme in bin k is one for which $s^u(f) = 0$ when $s^d(f) \neq 0$ for all f in the bin and vice versa; that is, $s^u(f)$ and $s^d(f)$ are orthogonal within $|f - f_k| \leq (W/2)$ (see Fig. 2).

II. OPTIMIZED SIGNALING TECHNIQUES

Given the channel, noise, and crosstalk characteristics of the DSL cable, we maximize the overall capacity of all $\mathcal{S}$ -service lines by choosing between NDS and FDS in each frequency bin.

A. Optimal Discrete Transmit Spectra

In the presence of self-interference, our approach is similar to that in [2]. We can easily derive test conditions [3] to determine the signaling nature (FDS or NDS) in bin k

$$\text{If } X_k^2 - F_k^2 - H_k F_k < 0, \text{ then} \\ G_k = \frac{2P_k}{N_k W} \underset{\text{FDS}}{\overset{\text{NDS}}{\geq}} \frac{H_k - 2(X_k - F_k)}{X_k^2 - F_k^2 - H_k F_k} \quad (1)$$

$$\text{else if } X_k^2 - F_k^2 - H_k F_k > 0, \text{ then} \\ G_k = \frac{2P_k}{N_k W} \underset{\text{FDS}}{\overset{\text{NDS}}{\geq}} \frac{H_k - 2(X_k - F_k)}{X_k^2 - F_k^2 - H_k F_k}. \quad (2)$$

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¹NDS was referred to as EQPSD signaling in [2] and [7]–[10].

Fig. 2. Transmit spectra for NDS and FDS schemes in a single frequency bin k (see Fig. 1). In the FDS scheme, the spectra $s^u(f)$ and $s^d(f)$ share disjoint frequency bands and are orthogonal to each other. This eliminates self-NEXT, but reduces bandwidth (and capacity) by half.

The test conditions are physically reasonable: use FDS in the bin if the signal-to-noise ratio (SNR) (G_k) is low, compared with the self-interference transfer functions (X_k, F_k); otherwise, use NDS.

In the absence of noise ($N_k = 0$), we can derive the signaling test condition

$$X_k^2 - F_k^2 - H_k F_k \underset{\text{FDS}}{\overset{\text{NDS}}{\geq}} 0. \quad (3)$$

To obtain the complete discrete optimal spectra, we apply the test conditions (1) and (2) to each frequency bin in $[0, B]$. To distribute the total power over the NDS and FDS regions, we employ a variation of waterfilling [1]. A simple iterative algorithm yields the complete discrete optimal transmit spectra [2] (see Fig. 3).

B. Optimal Grouping of Bins

Discrete transmit spectra must use discrete multitone (DMT) for modulation. A scheme like contiguous spectra with a reduced number of transitions are desirable for implementation using broadband modulation schemes such as carrierless amplitude/phase modulation (CAP) or quadrature amplitude modulation (QAM) [5]. We now present a scheme to group FDS bins that yields contiguous spectra that achieve *equal performance margins* and *equal upstream and downstream average powers* (see Fig. 4). Let E_{FDS} denote the FDS spectral region and E_{NDS} the NDS spectral region. A simple iterative algorithm yields the optimal grouping of bins.

- 1) Solve for the discrete optimal spectrum $S^u(f)$ according to the algorithm in Section II-A.
- 2) Obtain $S^d(f)$ from $S^u(f)$ by symmetry, i.e., $S^d(f) = S^u(f) \forall f \in E_{\text{NDS}}$ and $S^d(f)$ orthogonal to $S^u(f) \forall f \in E_{\text{FDS}}$. Merge $S^d(f)$ and $S^u(f)$ to form $S(f)$ as

$$S(f) = \begin{cases} S^u(f) = S^d(f) & \forall f \in E_{\text{NDS}} \\ S^u(f) + S^d(f) & \forall f \in E_{\text{FDS}}. \end{cases} \quad (4)$$

Fig. 3. Upstream and downstream transmit spectra showing regions employing NDS and FDS. The bins $[1, M_N]$ employ NDS and the bins $[M_F, K]$ employ FDS, as determined by evaluating test conditions (1) and (2) in each bin. The bin M_{N2F} is the optimal NDS-to-FDS switchover bin. Bins $[1, M_{N2F}]$ employ NDS, and bins $[M_{N2F} + 1, K]$ employ FDS.

- 3) Choose a bin $M_C \in (M_{N2F}, K]$, where M_{N2F} is the optimal NDS-to-FDS switchover bin. Estimate bin $M_G \in (M_C, K]$ such that we have equal average powers in both directions. Group the bins of $S(f)$ to obtain upstream and downstream transmit spectra

$$S_{\text{opt}}^u(f) = \begin{cases} S(f) & \forall f \in E_{\text{NDS}} \\ & \forall f \in \text{bins } (M_C, M_G] \\ 0, & \text{otherwise} \end{cases} \quad (5)$$

$$S_{\text{opt}}^d(f) = \begin{cases} S(f) & \forall f \in E_{\text{NDS}} \\ & \forall f \in \text{bins } (M_{N2F}, M_C] \\ & \forall f \in \text{bins } (M_G, K] \\ 0, & \text{otherwise.} \end{cases} \quad (6)$$

- 4) Iterate Step 3 for different choices of M_C and M_G . Choose the final values for bins M_C and M_G to achieve equal performance margins and equal average powers in both directions of transmission.

Since we have two degrees of freedom (bin choices M_C and M_G) to obtain equal margins and equal average powers, the algorithm is guaranteed to converge [3].

Fig. 4. Optimal upstream ($S^u(f)$) and downstream ($S^d(f)$) transmit spectra for HDSL2. (a) On CSA loop 6. (b) On CSA loop 4 (having bridged taps) with 39 self-NEXT and 39 self-FEXT interferers and AGN of -140 dBm/Hz. NDS takes place to the left of switchover frequency M_{N2F} , and FDS to the right.

C. Optimization Under a Frequency-Domain Peak-Power Constraint

The need to limit the transmit spectrum could arise due to a standardized fixed PSD mask, or from spectral compatibility considerations. Here, we impose a frequency-domain peak power constraint, in addition to the average power constraint. We obtain optimal transmit spectra for an S service under these constraints in the absence of self-interference (refer to [3] for the solution in the presence of self-interference).

The problem statement is similar to that in [11], but without the equal-capacity constraint and with the additional frequency-domain peak-power constraint. Consider a line carrying a service S in the presence of different service interference and channel noise. The twisted-pair channel can be treated as a Gaussian channel with colored Gaussian noise [6]. The channel capacities (in bits per second) along with the average-power constraints are given by [2], [3], and [11]. In addition, we have new peak-power constraints

$$S^u(f) \leq Q^u(f) \forall f, \quad S^d(f) \leq Q^d(f) \forall f. \quad (7)$$

The average-power and peak-power constraints are linear and differentiable. Further, the capacity objective function is concave [12]. Any solution to this nonlinear optimization problem must therefore satisfy the Karush–Kuhn–Tucker (KKT) necessary conditions for optimality [13]. For a concave objective function and linear, differentiable constraints, any solution that satisfies the KKT necessary conditions is a unique globally optimal solution [13]. Thus, we seek any solution that satisfies these conditions.

“Peak-constrained waterfilling”² yields the optimal solution to [11] and (7) [3]. The optimal transmit spectrum is given by

$$S_{\text{opt}}^u(f) = \begin{cases} \lambda - \frac{N(f)}{[H_C(f)]^2}, & f \in E_{\text{pos}} \\ Q^u(f), & f \in E_{\text{max}} \\ 0, & \text{otherwise} \end{cases} \quad (8)$$

with λ a Lagrange multiplier. The spectral regions E_{pos} and E_{max} are specified by

$$\begin{aligned} E_{\text{pos}} &= \{f : 0 \leq S^u(f) \leq Q^u(f)\} \\ E_{\text{max}} &= \{f : S^u(f) > Q^u(f)\}. \end{aligned} \quad (9)$$

We vary the value of λ to achieve the optimal transmit spectrum $S_{\text{opt}}^u(f)$ that satisfies the average- and peak-power constraints. It is easily shown that this solution satisfies the KKT conditions [3].

III. SIMULATION RESULTS

We consider HDSL2, a 1.544 Mb/s symmetric bit-rate service, for our simulations [14]–[16]. Fig. 4 illustrates the optimal transmit spectra for the standard test case CSA loop 6 [5] for HDSL2 in the presence of 39 HDSL2 self-NEXT and self-FEXT interferers.³ High self-NEXT at high frequencies forces the optimal upstream and downstream spectra to separate in frequency, giving rise to an FDS region.

The performance margin of a channel measures the maximum SNR degradation that a channel can sustain before being unable to transmit at a particular bit rate and BER [20]. Table I lists the uncoded performance margins of the optimal transmit spectra versus those obtained using the OPTIS transmit spectra.⁴ For HDSL and T1 interferers, only the NEXT powers were considered; for HDSL2, “self” comprises both self-NEXT and self-FEXT. Equal performance margins were obtained for upstream and downstream optimal transmit spectra. In all cases, the optimal scheme significantly outperforms OPTIS (which uses fixed transmit spectra), with performance gains up to 16 dB.

²Peak-constrained waterfilling can be likened to filling water in a closed vessel with uneven top and bottom surfaces.

³Simulation details: Bit rate fixed at 1.552 Mb/s. Total average input power in each direction $P_{\text{max}} = 19.78$ dBm. Different service interference models obtained from [17, Annex B of T1.413-1995], the ADSL standard, with exceptions as in [15]. Self-NEXT interference modeled as a two-piece Unger model [18]. Margins calculated according to [19]. OPTIS (ANSI T1E1.4’s HDSL2 standard for transmit PSD masks) transmit spectra obtained by tracking 1 dBm/Hz below the OPTIS PSD masks [16]. OPTIS performance margin figures from [16]. AGN of -140 dBm/Hz added to the interference. DMT scheme: Sampling frequency $f_s = 1000$ kHz. Bin width $W = 2$ kHz. Start frequency = 1 kHz. Bit-error rate (BER) = 10^{-7} . SNR gap = 9.8 dB. No cyclic prefix. No limitation on maximum number of bits per tone. See [20] for more details.

⁴OPTIS numbers were obtained from [16]. Diff = Difference between worst-case optimal and worst-case OPTIS.

TABLE I
UNCODED (NO CODING GAIN) PERFORMANCE MARGINS (IN DECIBELS) FOR
HDSL2 ON CSA LOOP 6: OPTIS [16] VERSUS OPTIMAL

Crosstalk source	DSL service	OPTIS		Optimal		Diff
		Up	Dn	Up	Dn	
49 HDSL	HDSL2	2.7	12.2	18.5	18.5	15.8
25 T1	HDSL2	19.9	17.5	21.3	21.3	3.8
39 self	HDSL2	2.1	9.0	18.3	18.3	16.2
24 self+24 T1	HDSL2	4.3	1.7	5.4	5.4	3.7

IV. CONCLUSIONS

Optimal transmit spectra adapt to surrounding interference and yield large performance-margin gains (up to 16 dB), compared with current fixed-spectra schemes for DSL communication. We can trade these increased performance margins for increased bit rates or decreased average transmit power. Optimal spectra are not bound to any particular modulation scheme. Vendor advantage can be achieved by adapting spectra underneath fixed, standardized PSD masks. By design, optimal transmit spectra achieve good spectral compatibility with existing services [3]. Our results assume some knowledge of the characteristics of the neighboring interfering services [3], but these are often known or can be estimated in practice.

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