Multiuser Receivers for Code-Division Multiple-Access Systems with Trellis-Based Modulation

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Abstract—A code-division multiple-access (CDMA) communication system is studied where a trellis-based scheme is used for data encoding and modulation. The signature sequences (spreading codes), which are assigned to the direct-sequence spread-spectrum (DS/SS) modulator according to the encoding rule, are taken from a biorthogonal set. We derive the optimum detector by maximizing the likelihood ratio. In addition to the optimum multiuser detector with very high computational complexity, we present a multistage detector and a scheme based on a reduced tree search algorithm. The error probability is evaluated by deriving upper and lower bounds as well as by Monte Carlo simulations. We show that the optimum receiver is near-far resistant. The results from the numerical examples indicate that the suboptimum detectors are also capable of alleviating the near-far problem.

Index Terms— Multiuser detection, code-division multiple-access (CDMA), coding, multistage detector, M -algorithm.

I. INTRODUCTION

CODE-division multiple-access (CDMA) has been proposed for a variety of commercial systems and has already found application in wireless local-area networks (LAN's), cellular mobile radio systems, and satellite networks. Comprehensive studies on CDMA have been published addressing performance and capacity issues, code design, as well as optimum and various suboptimum receiver structures for multiuser reception. In a CDMA system, a number of users share a common transmission media at the same time instant and in the same frequency band. Different signature waveforms are assigned to different active users such that the receiver can demodulate the desired information. Due to the effective cross-correlations among the signature waveforms, the so-called near-far effect drastically reduces the performance of conventional matched filter receivers. The near-far problem refers to the case when the power levels of the received signal components are not well balanced and weak signals are interfered by one or more strong signals. Multiuser detection provides one possible solution to alleviate the near-far problem. Verdú derived and analyzed the optimum multiuser receiver [1], [2] which led to the investigations

of less complex suboptimum multiuser detectors by various authors [3]–[12].

Channel (or transmission) encoders transform the information bits into an encoded sequence such that the number of error events after decoding of the received signal is reduced. There are numerous point-to-point communication links where convolutional or block encoding is successfully employed [13]. Trellis-coded modulation (TCM) [14] is another favored encoding technique which is designed for signaling over bandwidth-constrained channels. Transmission coding for CDMA systems with direct-sequence spread-spectrum (DS/SS) modulation has been investigated in [15]–[18] and some interesting results were reported for single-user receivers. Woerner and Stark [19] presented a trellis-based encoding scheme that utilizes an extended set of signature waveforms (spreading codes). Their method employs a signal constellation of more than two dimensions and is superior to the method of “conventional” coding or TCM prior to spreading. The aforementioned investigations are restricted to single-user receivers. Giallorenzi and Wilson [20], [21] analyzed a CDMA system where the information bits are convolutionally encoded. They presented the optimum multiuser as well as suboptimum decorrelation and multistage detectors with reduced computational complexity.

In this paper, we investigate multiuser detectors for a CDMA system with a trellis-based modulation and coding scheme as proposed by Woerner and Stark [19]. The signal components from the K users arrive asynchronously with possibly different signal strength. The noise on the channel is modeled as additive and white with Gaussian distribution. Coherent reception is considered. We derive the optimum and a multistage detector as well as a receiver based on a reduced tree search algorithm.

This paper is organized as follows: after the description of the transmitter and the characterization of the channel in Section II, the maximum likelihood receiver is derived in Section III; suboptimum schemes are introduced in Section IV; and analytical and numerical results are presented in Sections V and VI, followed by the conclusions in the last section.

II. SYSTEM DEFINITION

In this study, we investigate a CDMA system where the data from the K active users arrive asynchronously at the receiver over K distinct propagation paths with possibly different signal strength. In other words, the system is assumed to operate in the absence of a power control scheme and with-

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Fig. 1. Illustration of the CDMA system.

out centralized time synchronization. The examined CDMA system is depicted in Fig. 1 and described in detail hereafter.

We consider a CDMA system where each transmitter comprises a binary memoryless source, a linear encoder and a memoryless modulator. At time index i , the binary source of the k th transmitter generates β information bits. These bits form the information symbol $b_k^{(i)}$ which takes on letters from a 2^β -ary alphabet. The information bits are mutually independent and identically distributed. To provide a more compact notation, we introduce the vector $\mathbf{b}_k = (b_k^{(-I)}, \dots, b_k^{(I)})^\top$ which forms a packet of $(2I + 1)$ transmitted symbols $b_k^{(i)}$.¹ For the K users, the packets of uncoded symbols are contained in the matrix $\mathbf{B} = (\mathbf{b}_1, \dots, \mathbf{b}_K)^\top$ which belongs to the set of possible symbol patterns $\mathcal{B} = \{\mathbf{B} \in \{0, 1, \dots, 2^\beta - 1\}^{K \times (2I+1)}\}$. In order to describe the transmitted signals, we introduce the mapping of the 2^β -ary symbol $b_k^{(i)}$ to a Q -ary symbol which, subsequently, determines the transmitted waveform. This mapping is implemented by an encoder and can be described in the setup of a finite state machine where the i th encoded symbol

$$c_k^{(i)} = f(b_k^{(i)}, \sigma_k^{(i)}) \in \mathcal{Q} \quad (1)$$

is taken from the Q -ary alphabet $\mathcal{Q} = \{0, 1, \dots, Q - 1\}$ and depends on the current symbol $b_k^{(i)}$ and on the state $\sigma_k^{(i)}$. A transition of the finite state machine from state $\sigma_k^{(i)}$ to $\sigma_k^{(i+1)}$ is written as $\sigma_k^{(i)}: b_k^{(i)} \rightarrow \sigma_k^{(i+1)}$. The state $\sigma_k^{(i)}$ is determined by L previously generated information bits (not information symbols). Hence, we have 2^L different states. The parameter L is often referred to as the total encoder memory since it describes the number of information bits currently memorized by the encoder. At the beginning of the packet, the encoder's state has to be set to a predetermined value. The encoding rule $f(\cdot, \cdot)$ in (1) can be implemented by a convolutional encoder [13] with rate β/n if $Q = 2^n$ where n is a positive

¹The transpose operator is denoted by $(\cdot)^\top$.

integer.² The modulator of the k th user generates a continuous-time signal according to its unique set of signature sequences $\{\mathbf{a}_{k,0}, \mathbf{a}_{k,1}, \dots, \mathbf{a}_{k,Q-1}\}$ and its inputs, the encoded symbols $c_k^{(i)} \in \mathcal{Q}$. The generated signature waveforms, which are defined for all possible input values $l \in \mathcal{Q}$, can be described as

$$\phi_k(t; l) = \frac{1}{\sqrt{T_s}} \left(\sum_{j=0}^{N_c-1} a_{k,l}^{(j)} \Pi_{T_c}(t - jT_c) \right). \quad (2)$$

The generated signals are of unit energy and take on nonzero values over one symbol period $T_s = N_c T_c$. In (2), $\Pi_{T_c}(t)$ defines a unit rectangular pulse of duration of one chip T_c and $a_{k,l}^{(j)} \in \{-1, +1\}$ denotes the j th spreading code symbol (chip) of the signature sequence $\mathbf{a}_{k,l} = (a_{k,l}^{(0)}, \dots, a_{k,l}^{(N_c-1)})$ of length N_c . The signature sequences are chosen such that we have "good" cross-correlation properties between all sequences of different users. A set of bi-orthogonal sequences for the k th user can be obtained by extending a set of $Q/2$ orthogonal sequences³ $(\mathbf{a}_{k,l} \perp \mathbf{a}_{k,l'}, \forall l \neq l', l < Q/2 \text{ and } l' < Q/2)$ by its antipodal counterparts $\mathbf{a}_{k,l} = -\mathbf{a}_{k,l+Q/2}, \forall l < Q/2$. The design of the encoder and the assignment of the signature sequences to its trellis needs to be carried out such that the Euclidean distance is maximized between any merging paths that originate from the same node [19].

The signals generated by the K users reach the receiver over K distinct propagation paths. The amplitude of the signal received from the k th user is modeled through the complex-valued⁴ coefficient α_k which accounts for the power of the emitted signal and the channel attenuation. The experienced delay on this propagation path is denoted by τ_k (cf. Fig. 1). The received signal can be described as

$$r(t) = s(t; \mathbf{B}) + n(t) \quad (3)$$

where $s(t; \mathbf{B})$ is the information bearing part and $n(t)$ represents the base-band noise which is modeled by a zero mean complex Gaussian process with a flat, two-sided power spectral density with height $N_0/2$ for both the real and imaginary parts. The noise-free part of the received signal results as

$$s(t; \mathbf{B}) = \sum_{k=1}^K \sum_{i=-I}^I s_k(t - \tau_k - iT_s; c_k^{(i)}) \quad (4)$$

where $s_k(t; c_k^{(i)}) = \alpha_k \phi_k(t; c_k^{(i)})$ represents the received signature waveform with regard to the k th user and the i th time interval. To simplify the problem and without loss of generality, we shall suppose that the delays are taken modulo T_s and that the users are numbered such that $0 \leq \tau_1 \leq \dots \leq$

²The convolutional encoder is frequently characterized by the constraint length which is $L + 1$ for $\beta = 1$.

³It may be convenient to choose a set which is only approximately orthogonal.

⁴The coefficient α_k is complex-valued since we formulate the transmitted signals using the base-band-form. The bandpass signal $x(t)$ centered at the carrier frequency f_c can be expressed by means of its equivalent base-band signal (i.e., the complex envelope) $y(t)$ as follows: $x(t) = \sqrt{2} \operatorname{Re}\{y(t) \exp(j2\pi f_c t)\}$ where $\operatorname{Re}\{\cdot\}$ represents the real part of its argument.

$\tau_K < T_s$. It is convenient to introduce the received bit energy of the k th user

$$E_k = \frac{1}{\beta} \int |s_k(t; l)|^2 dt = \frac{|\alpha_k|^2}{\beta} \quad (5)$$

as well as the near-far factor $\gamma = E_k/E_1$ for all $k > 1$ and all $l \in \mathcal{Q}$. The near-far effect is characterized with respect to user one.

III. MAXIMUM LIKELIHOOD DETECTOR

In this study it is assumed that the values of the channel parameters are constant over the observation period (i.e., over the transmission of one data packet) and to be known at the receiver or "perfectly estimated" through a synchronization algorithm. The overall performance of a system with additional parameter estimation (cf. [10], [22]–[24] for uncoded systems) is often very close to a system with known channel parameters. In this paper, we concentrate on an algorithm for demodulating the information symbols. Let us, therefore, formulate the log-likelihood function of the received signal $r(t)$, that is the logarithm of the probability density function of the received signal conditioned on the transmitted information symbols $\mathbf{B}$ up to an additive constant [25]

$$\Lambda(\mathbf{B}) = \frac{2}{N_0} \int \text{Re}\{r(t)s^*(t; \mathbf{B})\} dt - \frac{1}{N_0} \int |s(t; \mathbf{B})|^2 dt \quad (6)$$

where $(\cdot)^*$ yields the complex conjugate value. Note that the likelihood function depends indirectly on the bits, i.e., through the encoding scheme (1). With (3) and (4), the log-likelihood function can be evaluated as

$$\Lambda(\mathbf{B}) = \sum_{(k,i) \in \mathcal{J}} \frac{1}{N_0} \text{Re} \left\{ 2z_k^{(i)}(c_k^{(i)}) - \sum_{(k',i') \in \mathcal{J}} R_{k,k'}(c_k^{(i)}, c_{k'}^{(i')}, i-i') \right\} \quad (7)$$

where $\mathcal{J} = \{(k,i): k \in \{1, 2, \dots, K\}, i \in Z \text{ with } |i| \leq I\}$ defines the set of relevant index pairs and

$$z_k^{(i)}(l) = \int r(t)s_k^*(t - iT_s - \tau_k; l) dt \quad (8)$$

is the i th output of a filter matched to the k th user's signature waveform with index l . In (7),

$$R_{k,k'}(l, l', i) = \int s_k^*(t - \tau_k; l) s_{k'}(t + iT_s - \tau_{k'}; l') dt \quad (9)$$

expresses the cross-correlations of the l th and (l') th received signature waveform, and of the k th and (k') th user, respectively. The parameter i accounts for the additional offset in terms of symbol periods T_s between the two signals. We have zero cross-correlation if $|i| > 1$. Since the users are ordered according to their delays τ_k , we can observe for all $l \in \mathcal{Q}$ and $l' \in \mathcal{Q}$ that $R_{k,k'}(l, l', 1) = 0$ if $k \geq k'$ and $R_{k,k'}(l, l', 1) = 0$ if $k \leq k'$.

It follows from the likelihood function (7) that a bank of $K \times Q$ matched filters⁵ provides the receiver with the sufficient statistics. The receiver decides on the transmitted information symbols by maximizing the likelihood function

$$\hat{\mathbf{B}} = \arg \left\{ \max_{\mathbf{B} \in \mathcal{B}} [\Lambda(\mathbf{B})] \right\}. \quad (10)$$

It is convenient at this point to define the one-to-one mapping $\theta(k, i) = k + iK$ with $k \in \{1, 2, \dots, K\}$ and $i \in Z$. The function $\theta(k, i)$ provides an increasing index for all symbols $b_k^{(i)}$ according to the position $iT_s + \tau_k$ of the associated received signal $s_k(t - iT_s - \tau_k; l)$ along the time axis. Let us introduce the metric

$$\mathcal{M}_{k,i} = \mathcal{M}_{k'',i''} + \lambda_{k,i}(b_k^{(i)}, \mu_k^{(i)}) \quad (11)$$

where the index pair (k'', i'') is selected such that $\theta(k'', i'') = \theta(k, i) - 1$. In (11), the cumulative part follows as

$$\lambda_{k,i}(b_k^{(i)}, \mu_k^{(i)}) = \text{Re} \left\{ z_k^{(i)}(c_k^{(i)}) - \frac{\beta E_k}{2} - \sum_{(k',i') \in \mathcal{J}_{k,i}^{(K)}} R_{k,k'}(c_k^{(i)}, c_{k'}^{(i')}, i-i') \right\} \quad (12)$$

where $\mathcal{J}_{k,i}^{(K)} = \{(k', i'): 0 > (k' - k, i' - i) > -K\}$ describes the index pairs of $K - 1$ symbols that precede the current one $b_k^{(i)}$. Equations (11) and (12) are defined for $|i| \leq I$ by further defining $R_{k,k'}(c_k^{(i)}, c_{k'}^{(i-1)}, 1) = 0$ and $\mathcal{M}_{k,i} = 0$ for $i \leq -I$. Due to the ordering of the users according to their delays, the samples $z_k^{(i)}(l)$ and the cross-correlations in (12) only depend on the information symbol $b_k^{(i)}$ and on the state variable $\mu_k^{(i)}$ which contains $K - 1$ preceding symbols and K encoder states

$$(b_k^{(i)}, \mu_k^{(i)}) = (b_k^{(i)}, \dots, b_{k''}^{(i'')}, \sigma_k^{(i)}, \dots, \sigma_{k''}^{(i'')}). \quad (13)$$

Here, the index pair $\theta(k'', i'')$ is such that $\theta(k'', i'') = \theta(k, i) - K$. The entries in (13) are ordered with respect to the decreasing indexing function $\theta(\cdot, \cdot)$. The received signal of the multiuser system can therefore be modeled by a finite state machine. The state transition is written as $\mu_k^{(i)}: b_k^{(i)} \rightarrow \mu_{k''}^{(i')}$ with (k'', i'') such that $\theta(k'', i'') = \theta(k, i) + 1$. Note that the state variable $\mu_k^{(i)}$ contains $(KL + (K - 1)\beta)$ previously transmitted uncoded bits. It is easy to show that the metric (11) yields a decomposed version of the log-likelihood function with $\mathcal{M}_{K,I} = (N_0/2)\Lambda(\mathbf{B})$. Therefore, the maximum in (10) can be obtained through a sequential maximum likelihood estimator and can be realized by means of the Viterbi algorithm [13], [26]. The latter computes the metric (11) for all possible input patterns and further exploits remerging branches by transforming the search tree into a trellis. Here, the trellis has K levels for every time interval T_s and $2^{KL + (K-1)\beta}$ nodes (states) with each node having 2^β emanating and remerging

⁵Since the set of signature sequences is biorthogonal, only half the matched filters need actually to be implemented.

branches. If no encoding scheme is applied ($L = 0$), the above scheme is identical to the one presented by Verdú [1].

The complexity C of the detection scheme with respect to one symbol interval T_s is expressed by the number of visited branches of the trellis, i.e., by the number of evaluated metric expressions. The complexity of the optimum detector is $C_O = K2^{KL+(K-1)\beta}$ and grows exponentially with the number of active users K . Hence, it seems to be very unlikely that such a scheme is implemented for a reasonably large number of users.

IV. SUBOPTIMUM DETECTORS

In the previous section, we developed a sequential decomposition of the log-likelihood function that exploits the cumulative characteristics of the metric. Since the implementation cost of the optimal detector is in most cases extremely high, we investigate in this section two suboptimal algorithms with the aim to reduce the size of the search tree.

A. Multistage Detector

Multistage detection schemes have been successfully applied to various CDMA scenarios with severe near-far problems [4], [10], [20]. The achieved performance was for most cases very close to the optimum. The multistage approach to the maximization problem (10) requires for the k th user to compute

$$\hat{\mathbf{b}}_k(m+1) = \arg \left\{ \max_{\substack{\mathbf{b}_k \in \{0, \dots, 2^\beta - 1\}^{(2I+1)} \\ \mathbf{b}_{k'} = \hat{\mathbf{b}}_{k'}(m), \forall k' \neq k}} [\Lambda(\mathbf{B})] \right\} \quad (14)$$

where $\hat{\mathbf{b}}_k(m+1)$ describes all estimated symbols for that particular user at the $(m+1)^{\text{st}}$ stage. We provide the estimator with estimates for the symbols $\hat{\mathbf{b}}_{k'}(m)$ of all the other users $k' \neq k$ that are obtained at a previous stage. Introducing these symbol estimates in (7) yields the cumulative metric for the k th user

$$\mathcal{M}_{k,i}(m+1) = \mathcal{M}_{k,i-1}(m+1) + \hat{\lambda}_{k,i}^{(m)}(b_k^{(i)}, \sigma_k^{(i)}) \quad (15)$$

where

$$\hat{\lambda}_{k,i}^{(m)}(b_k^{(i)}, \sigma_k^{(i)}) = \text{Re} \left\{ z_k^{(i)}(c_k^{(i)}) - \sum_{\substack{k'=1 \\ k' \neq k}}^K \sum_{i'=i-1}^{i+1} R_{k,k'} \times (c_k^{(i)} \hat{c}_{k'}^{(i')}(m), i - i') \right\}. \quad (16)$$

Here, the metric of the k th user depends only on the symbols $\mathbf{b}_k$. For all users $k' \neq k$, the estimates of the encoded symbols $\hat{c}_{k'}^{(i')}(m)$ are obtained through the encoding rule (1) from the estimated symbols of the m th stage $\hat{\mathbf{b}}_{k'}(m)$. At each stage and for the k th user, the sequential maximum likelihood estimator operates every T_s seconds on a trellis with only 2^L states and can again be implemented by the Viterbi algorithm. For the first stage, one possibility is to obtain the $\hat{\mathbf{b}}_k(1)$'s through a conventional detector which does not exploit any knowledge of the interfering signals of the other users. This can be achieved

by setting the summation term in (16) to zero. For a realization, the matched filter samples need to be appropriately delayed before passing to the next higher stage and the values for the cross-correlations (9) have to be set to zero in (16) if $|i'| > I$.

By introducing a multistage detection scheme, the original problem is separated into different stages which allows to split the original trellis into K trellis of reduced complexity. The total number of states to be considered equals $C_{MD} = SK2^L$ for a K -user S -stage detector. For all cases of interest, the implementation costs is clearly much smaller when comparing to the optimum demodulator.

B. Reduced Tree Search Algorithm

There exist several tree search algorithms [27], [28] which do not visit all branches on the tree (or trellis). In this paper, we apply the M -algorithm [27] which is a breadth-first search procedure and belongs to the class of sorting algorithms. The M -algorithm provides the advantage that it is a synchronous algorithm in the sense that all paths under consideration have the same length and that one output symbol is released at each level of the algorithm. This algorithm is considered in [27] to be a good candidate for practical realizations and has also been applied to a joint detection and channel estimation problem in a CDMA system [6].

The M -algorithm accesses a memory with M retained paths of a certain length. At every time instant, characterized by $\theta(k, i)$, the algorithm moves forward by hypothesizing these M paths which then yields $2^\beta M$ new paths to examine. Only the M paths with highest metric are going to be held in memory. The following steps are performed at each tree level:

- 1) extend the M paths in memory by updating the metric (11) according to all possible input bit combinations;
- 2) perform an ambiguity check, i.e., search for merging paths, and keep only the survivor;
- 3) retain only the M paths with highest metric—this step involves a sorting procedure;
- 4) release the oldest symbol of the path with highest accumulated metric and go to step 1 unless all bits of the packet have already been decoded.

The M -algorithm only takes into account a certain subset of the tree nodes that appear to belong with high probability to the correct path; that is, only $C_{MA} = KM$ nodes are visited for every symbol period of length T_s . If $C_{MA} = C_O$ then the M -algorithm becomes equivalent to the optimum detector; however, M is chosen much smaller in practice. The M -algorithm has to be appropriately initialized since it may otherwise not be able to track the correct path. The initialization can easily be performed by setting all state variables to a predefined value at the encoder and decoder or to employ a conventional detection scheme at the beginning of the transmission. A drawback for the M -algorithm is the need for a sorting procedure (step 3) which consumes a nonnegligible part of the total computational time.

V. ERROR PROBABILITY

In order to derive expressions for upper and lower bounds on the error probability, we first introduce some notation

needed for the formulation. The symbol error probability of the k th user over finite horizon (i.e., assuming that we have an input sequence of finite length $2I + 1$) is defined as $P_k^{(i)} = \Pr\{b_k^{(i)} \neq \hat{b}_k^{(i)}\}$ where $\hat{b}_k^{(i)}$ is the symbol estimate provided by the detector. It is often possible to extend such a result to the case of infinite horizon with $P_k = \lim_{I \rightarrow +\infty} P_k^{(i)}$. The symbol error matrix⁶ is defined as $E = [e_k^{(i)}]_{K \times (2I+1)}$ with $e_k^{(i)} = b_k^{(i)} - \hat{b}_k^{(i)}$. Detection errors are indicated by nonzero values for $e_k^{(i)}$. The matrix E belongs to the set

$$\mathcal{E} = \{E \in \{-2^\beta + 1, \dots, 2^\beta - 1\}^{K \times (2I+1)}, e_k^{(i)} \neq 0 \text{ for some } (k, i)\}. \quad (17)$$

Restricting the detection errors to the k th user yields the set of nonzero error matrices $\mathcal{E}^{(k)} = \{E \in \mathcal{E}, e_{k'}^{(i)} = 0, \forall k' \neq k, \text{ and } \forall i\}$. We further introduce the set of admissible errors conditioned on the event that B was transmitted

$$\mathcal{A}(B) = \{E \in \mathcal{E}, \text{ such that } (B + E) \in \mathcal{B}\} \quad (18)$$

and particularly the admissible errors with respect to the i th symbol of the k th user $\mathcal{A}_k^{(i)}(B) = \{E \in \mathcal{A}(B), e_k^{(i)} \neq 0\}$.

Let us first define the error probability of pairwise comparison for maximum likelihood detection $P_2(B, \tilde{B}) = \Pr[\Lambda(\tilde{B}) - \Lambda(B) > 0 \mid B \text{ sent}]$ where it is assumed that $B \in \mathcal{B}$ was transmitted and $\tilde{B} \in \mathcal{B} \setminus \{B\}$ is the *only* alternative⁷. For the considered channel model with the additive white Gaussian noise process, we have after some manipulations with (3), (7), and (8)

$$P_2(B, \tilde{B}) = Q\left(\frac{\|s(t; B) - s(t; \tilde{B})\|}{\sqrt{2N_0}}\right) \quad (19)$$

where $\|s(t)\|^2 = \int |s(t)|^2 dt$ and $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{+\infty} e^{-t^2/2} dt$. As expected, the pairwise error probability decreases with increasing distance between the two signals $s(t; B)$ and $s(t; \tilde{B})$.

The symbol error probability is evaluated by averaging over all possible symbol patterns and by considering all admissible error events. Hence, we have with regard to the i th symbol of the k th user

$$P_k^{(i)} = \sum_{B \in \mathcal{B}} \Pr \left[\bigcup_{E \in \mathcal{A}_k^{(i)}(B)} [\Lambda(B + E) > \Lambda(B)], B \in \mathcal{B} \right]. \quad (20)$$

This expression can be upper bounded by means of the Union Bound [26] and (19) as

$$P_k^{(i)} \leq \sum_{B \in \mathcal{B}} \sum_{E \in \mathcal{A}_k^{(i)}(B)} Q\left(\frac{\|s(t; B) - s(t; B + E)\|}{\sqrt{2N_0}}\right) \cdot \Pr[B \in \mathcal{B}]. \quad (21)$$

⁶For optimal detection, it is essential that the branches of the encoder's trellis merge at one single terminal node. It is therefore necessary to add some few predefined tail symbols to the information bearing symbol packet B . Obviously, the error matrix need also to be extended by the same number of zero-valued symbols. These two matrices are therefore of dimension $K \times (2I + 1 + I_{\text{tail}})$. This detail is neglected in the formal description but it is taken care of in the numerical experiments.

⁷The operator $\setminus$ denotes set exclusion.

It was shown in [1] that, after some modifications, the corresponding bound converges for sufficiently low noise even for the infinite horizon case. One key step was to consider only "indecomposable" error sequences. Unfortunately, a straightforward application of this method to the problem on hand does not seem to be possible due to the introduced memory at the encoder. We will show in some numerical examples that the presented bound seems to be insensitive to variations of the packet length for a sufficiently high signal-to-noise ratio (SNR). We observe for a near-far factor $\gamma \rightarrow +\infty$ at constant E_1 that the distance $\|s(t; B) - s(t; B + E)\|$ approaches $+\infty$ in (21) if $E \in \mathcal{E} \setminus \mathcal{E}^{(1)}$. In this case, the evaluated Q -function in (21) goes to zero. This means that, for a large near-far factor and well-designed codes, the sum over all possible errors in (21) is dominated by errors with regard to user 1 and that the upper bound of the symbol error probability approaches the one of a one user system: $P_{UB,1}^{(i)} = \lim_{\gamma \rightarrow +\infty} P_{UB,K}^{(i)}$ where $P_{UB,K}^{(i)}$ is the right hand side of (21) considering user $k = 1$ in a system with K active users. Hence, the numerical evaluation of the upper bound in (21) is much less complex since only the symbols of user 1 play a role.

The next step is to find a lower bound on the error probability. We apply the common technique of providing the receiver with some side information. This bound can be expressed through the free Euclidean distance (with respect to the i th symbol of the k th user) which is defined as

$$D_{k,\text{free}}^{(i)} = \inf_{B \in \mathcal{B}} \left[\inf_{E \in \mathcal{A}_k^{(i)}(B) \cap \mathcal{E}^{(k)}} \|s(t; B) - s(t; B + E)\| \right]. \quad (22)$$

For sufficiently long packets, the distance $D_{k,\text{free}}^{(i)}$ is independent of the index i which is therefore dropped. Note that the normalized free distance $D_{k,\text{free}}/\sqrt{\beta E_k}$ is the same for all of the K users. Let us suppose that the transmitted sequence B is chosen such that $D_{k,\text{free}}$ can be achieved through an arbitrary hypothetical error pattern $E \in \mathcal{A}_k^{(i)}(B)$. If we assume that the receiver obtains the side information that either B or $B + E$ was transmitted then the detector needs only to decide on $\Lambda(B) \geq \Lambda(B + E)$. The probability of an erroneous decision results as $P_2(B, B + E)$. Therefore, the error event probability can be lower bounded with (19) by $P_{k,\text{event}} \geq Q(D_{k,\text{free}}/\sqrt{2N_0})$. Considering all error events that affect a particular symbol leads to a lower bound on the symbol error probability. If we neglect higher order power terms of the Q -function, we obtain as a good approximation of the bound

$$P_k \geq \bar{N} \cdot Q\left(\frac{D_{k,\text{free}}}{\sqrt{2N_0}}\right) \quad (23)$$

where $\bar{N}$ denotes the average number of symbol errors with respect to one error event. It is sometimes convenient to express the lower bound by means of the asymptotic coding gain which is defined as $G = (D_{k,\text{free}}/D_{k,\text{min}})^2$. Here, $D_{k,\text{min}}$ denotes the minimum free Euclidean distance of the k th user with regard to the signaling waveforms of the baseline system (e.g., $D_{k,\text{min}} = 2\sqrt{E_k}$ for noncoded antipodal modulation).

The definition of the asymptotic efficiency [1] has to be modified for the trellis-based modulation scheme and is de-

finied with respect to the symbol with index pair (k, i) as

$$\eta_k^{(i)} = \sup_{\rho} \left[0 \leq \rho \leq 1; \lim_{\sigma \rightarrow 0} \frac{P_k^{(i)}(\sigma)}{Q\left(D_{k,\text{free}} \sqrt{\rho/(2\sigma)}\right)} < +\infty \right]. \quad (24)$$

The notation $P_k^{(i)}(\sigma)$ emphasizes the dependence of the symbol error probability (20) upon the noise level σ (i.e., $\mathcal{N}_0$ in the considered case). Let us further define the near-far resistance as $\bar{\eta}_k = \inf[\eta_k^{(i)}]$ where the infimum is taken with respect to $E_{k'} > 0$ over all index pairs $(i, k') \neq (0, k)$. If we substitute $P_k^{(i)}(\sigma)$ in (24) by the expression for the upper bound (21), we observe that the optimum detector is near-far resistant over finite horizon since the expression for the upper bound cannot approach infinity. This is due to the fact that the argument of the Q -function in the summation term is always greater than zero.

VI. NUMERICAL RESULTS

In this section we consider two particular encoding schemes which both are of relatively modest complexity. The trellis which defines the encoder and the assigned spreading sequences is depicted in Fig. 2 for both codes. The encoder can be realized by a half-rate convolutional encoder with memory $L = 2$ for code 1 and by a 1/3 rate encoder with $L = 3$ for code 2 resulting in a coding gain of $G = 3.98$ dB and 4.77 dB, respectively. The employed spreading sequences of length $N_c = 31$ are derived from the set of AO/LSE m -sequences [29]. The expansion to an (approximately) orthogonal set is performed by setting $\mathbf{a}_{k,l} = D^l \mathbf{a}_{k,0}$ for $\forall l < Q/2$ where D^j denotes the cyclic shift operator by j positions. The channel is distorted by additive white Gaussian noise and the delays τ_k 's and the angles of α_k 's are assumed to be mutually independent and uniformly distributed over the entire range $[0, T_s)$ and $[0, 2\pi)$, respectively. At the receiver, the information on evaluated paths is stored in memory by both the Viterbi algorithm and the M -algorithm. A common way to keep the size of this memory reasonably small is achieved by truncating the memorized paths and to release a decision before the demodulation of the entire packet. In this setup, the length of the path memory equals five times the total encoder's memory (cf. [13], [27]).

Fig. 3 depicts the lower bound (23) and upper bound $P_{\text{UB},1}^{(0)}$ (21) on the bit-error probability versus SNR E_b/N_0 for a two-user system using code 1 or 2. For comparison we also plot simulation results of the optimum receiver for a near-far factor of 10 dB and the bit-error probability of an uncoded single-user system with antipodal modulation. We can observe that the upper bound, which is evaluated over a finite horizon, is insensitive to the packet length $(2I+1)$ for sufficiently high SNR. The performance of the M -algorithm and of the optimum receiver is shown in Figs. 4 and 5 for code 1 at an SNR of 5 dB. These plots indicate that the M -algorithm is near-far resistant for sufficiently large M and near-far factor. In case of low near-far factor, where the M -algorithm does not perform well, we propose to demodulate the received signal by the conventional detector (i.e., one-stage detector). Fig. 5 contains results obtained through such

Fig. 2. Trellis of the encoder with assigned spreading sequences for codes 1 and 2 in (a) and (b), respectively (the first entries for the signature sequences are assigned to the upper outgoing branches referring to a zero input and the second entries to the lower ones).

a modified scheme. The decrease in computational complexity when using the M -algorithm instead of the Viterbi algorithm can be very high, e.g., the number of considered nodes for the six user system is only 36 compared to 2^{17} states for the optimum detector. It is, however, rather difficult to provide a fair comparison between these two algorithms since the M -algorithm includes a sorting procedure whose complexity is very implementation specific. In Fig. 6, the bit-error probability versus near-far factor of a two, four, and six-user system indicates the good performance of the suboptimal two-stage receiver. Note that the M -algorithm outperforms the two-stage demodulator in some situations and vice versa. The near-far problem, which affects the performance of the conventional receiver (cf. Fig. 5), is alleviated by all three presented demodulators.

VII. CONCLUSION

A CDMA system with a trellis-based coding and modulation scheme was investigated in the absence of a power control scheme. The optimum receiver and two suboptimum schemes were derived. These receivers exploit the cross-correlations among the transmitted signature waveforms as well as the data encoding rule in a combined manner. The optimal receiver was proven to be near-far resistant and, as indicated by the numerical experiments, the suboptimum multiuser receivers also alleviate the near-far problem. Compared to the uncoded

(a)

(b)

Fig. 3. Bit-error probability versus SNR of an optimum receiver for user 1 in a two-user system with a near-far factor of 10 dB compared to lower and upper bound (LB and UB) for code 1 in (a) and code 2 in (b). Upper bounds are given for various packet lengths.

Fig. 4. Performance of the detector with M -algorithm versus near-far factor in a two-user system for various M at an SNR of 5 dB compared to the optimum receiver (code 1).

system at the same data rate and signal bandwidth, a substantial gain in performance with regard to the SNR is achieved by the proposed receivers. There is a tradeoff between implementation costs and performance: the receivers based on

Fig. 5. Performance of the M -algorithm/conventional receiver in a system with two, four, and six users at an SNR of 5 dB (code 1).

Fig. 6. Performance of the multistage detectors in a system with two, four, and six users at an SNR of 5 dB compared to the optimum receiver (code 1).

the M -algorithm and on the multistage approach show only near optimal performance; on the other hand, their complexity grows roughly linearly with the number of users K whereas this relation is exponential for the optimum scheme.

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