Design of Coded Modulation Schemes for Orthogonal Transmit Diversity

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Abstract

In this paper, we propose a technique to decouple the problems of maximizing spatial diversity gain, temporal diversity gain, and coding gain, involved in the design of space-time codes in Rayleigh fading channels. For this purpose, we will use a set of concatenated codes with orthogonal transmit diversity system as their inner code. In the case of slowly fading channels, no temporal diversity is available, and the proposed technique decouples the problems of maximizing spatial diversity gain and coding gain. The design criterion for the outer code will be shown to be based on the maximization of the free Euclidean distance.

For fast fading channels, by using the new idea of constellation expansion (in dimension or size), we decouple the code design problem into two simpler problems, namely maximizing the spatial diversity gain, and maximizing the temporal diversity and coding gain. In this case, the code design criteria for the outer encoder will be shown to reduce to the maximization of Hamming and product distances in the expanded constellation.

The proposed techniques are illustrated by designing multilevel and multiple trellis coded modulation schemes for orthogonal transmit diversity systems. These codes are shown to have better performance compared to some existing space-time trellis codes with the same complexities.

Keywords

Coded Modulation, Orthogonal Transmit Diversity, Multilevel Coding, Multiple Trellis Coded Modulation, Multidimensional Trellis Coded Modulation

I. INTRODUCTION

The performance and code design criteria of single transmit and receive antenna systems in fading environments were initially studied by Divsalar and Simon [1]. It was shown in [1] that, unlike the additive white Gaussian noise (AWGN) channels, in fast fading environments, optimal performance cannot be guaranteed by just maximizing the free Euclidean distance of the code. But instead, the performance of the code is guided by two other factors: the *length of the shortest error event path*, and the *product of the squared Euclidean distances of the code symbols along the shortest error event path*. The first factor, which is essentially the minimum symbol Hamming distance of the code, determines the exponent of the signal to noise ratio in the error probability of the code, and is referred to as the *diversity gain*. The second factor, determines the multiplicative term in the error probability, and is referred to as the *coding gain*. Divsalar, *et al* [2], also proposed a new coded modulation scheme, called Multiple Trellis Coded Modulation (MTCM), which provides larger symbol Hamming distance, and thus a better performance in fast fading environments.

The diversity gain, achieved by increasing the minimum symbol Hamming distance of the code in fast fading environments, is due to the time variations of the channel and the assumption that the fading coefficients are independent from one symbol to the other. This requires very high velocity mobile terminals (e.g., 30 Km/s for a data rate of 100 Kbps at 1GHz), and is not achievable without interleaving/de-interleaving. As data rate increases, even with interleavers/de-interleavers of practical lengths, the conditions of a fast fading channel become less likely achievable, and channel starts looking more and more like slowly fading channel. The diversity gain due to the time variations of the channel (usually referred to as *temporal diversity*) is no longer available in these channels, and other sources of diversity have to be exploited to achieve performance improvement.

In a rich scattering environment, the fact that a signal is faded almost independently when traveling through two different paths (provided that the spacing between the starting or ending points of the two paths is large enough), provides another source of diversity, which is usually referred to as *spatial diversity*. In order to take advantage of this source of diversity, multiple transmit and/or receive antennas should be used. Information theoretic capacity analysis of the multiple antenna systems in [3], [4] shows that the capacity of the Gaussian channel with Rayleigh fading increases almost linearly with the minimum of the number of the transmit and receive antennas, and capacities as high as 19 b/s/Hz are achievable with just 4 transmit and receive antennas.

The code design problem for multiple transmit and receive antenna systems has recently received considerable attention, e.g., [5], [6], [7], [8], [9], [10]. In [5], the design criteria for

the codes to be used with multiple transmit and receive antenna systems (referred to as *space-time codes*) in different fading environments are derived. It was shown that the performance of the codes in slowly and fast fading environments is guided by different factors, and therefore, the code design criteria are different for those channels. In both cases, the overall performance improvement achieved by the space-time code, consists of two components: diversity gain, and coding gain. Most of the space-time code design techniques proposed so far (e.g. [5], [10]), attempt to construct codes which maximize both diversity and coding gains, simultaneously. As a result of the complexity involved in this problem, most of the proposed codes are handcrafted and the result of exhaustive searches over all possible codes.

The diversity gain of a space-time code in a slowly fading environment is due to the use of multiple transmit and receive antennas and cannot be increased to more than the product of the number of transmit and receive antennas, independent of the code used. On the other hand, the coding gain is essentially determined by the distance properties of the code, similar to the single transmit and receive antenna case. Decoupling the problems of maximizing the diversity gain and coding gain, if possible at all, could significantly simplify the code design procedure. In this paper, we will show that this decoupling is actually possible, and concatenated space-time codes with 1-2dB better performance can easily be designed using simple orthogonal space-time block codes and the well-known coded modulation techniques for single transmit and receive antenna systems.

In a fast fading environment, the diversity gain of a space-time code is achieved in two ways. A part of it results from the use of multiple antennas (spatial diversity gain), and the other part is provided by the redundancy added to the data through the coding scheme (temporal diversity gain). Decoupling the problems of maximizing these two diversity gains, if possible, could significantly simplify the code design procedure. In that case, we could use the existing coding schemes designed for fast fading channels and single transmit and receive antennas, since these schemes are already designed to maximize the temporal diversity gain. In this paper, we will show that such a decoupling is actually possible.

Considering a single antenna at the receiver, the maximum achievable spatial diversity gain is equal to the number of transmit antennas. Assuming a coherence time greater than or equal to the number of transmit antennas for the channel, one of the transmission schemes that can provide full spatial diversity gain and has a relatively simple structure is the *Orthogonal Transmit Diversity (OTD)* system proposed by Alamouti [7] and shown in Figure 1.

This system is a full-rate system (one symbol per transmission) and can easily be shown to provide full spatial diversity. Moreover, it has been shown in [11] that this system preserves

the capacity of two transmit and single receive antenna systems. These are motivations to consider the OTD system as a means of providing the spatial diversity gain and obtain the temporal diversity gain through an outer bandwidth efficient coded modulation scheme. In this paper, we will initially consider the case of two transmit and one receive antennas. The generalization of the discussion to any number of transmit antennas for which an orthogonal transmission scheme exists, is considered later.

In Section II, we will discuss the performance criteria of space-time codes in block fading environments, and will review the code design criteria of [5] for slowly and fast fading channels. In Section III, we will derive the design criteria for concatenated space-time codes with the OTD system as their inner code. It will be shown that depending on the type of the fading channel (slowly or fast), this will result in decoupling the problems of maximizing spatial diversity and coding gains or spatial and temporal diversity gains. For the case of fast fading channels, we will introduce the idea of *constellation expansion (in dimension or size)* to design coded modulation schemes for the orthogonal transmit diversity systems. It will be shown that the code design criteria reduce to the maximization of Hamming and product distances in the expanded constellation. Simulation results for two bandwidth efficient coded modulation schemes will be demonstrated in Section IV. In Section V, generalization of the proposed scheme to the case of space-time codes with more than two transmit and one receive antennas will be discussed. The case of frequency-selective multipath block fading channel will be addressed in Section VI, and Section VII presents the conclusions.

II. PERFORMANCE CRITERIA OF SPACE-TIME CODES

The criteria for designing optimum codes for data transmission over communication channels are usually derived from the error probability expressions (or appropriate upper bounds for the error probability) of the maximum likelihood decoder. For the case of multiple transmit and receive antenna systems with slowly or fast fading AWGN channels, Chernoff upper bounds for the pairwise error probabilities are derived in [5]. Based on these upper bounds, the design criteria for space-time codes in slowly and fast fading environments are introduced. Here, following the approach taken in [5], we derive the expression for the pairwise error probability of the space-time codes in a block fading environment, and use that to derive the design criteria.

Assume that

$$\mathbf{c} = \begin{pmatrix} c_1^1 & c_2^1 & \cdots & c_L^1 \\ c_1^2 & c_2^2 & \cdots & c_L^2 \\ \vdots & \vdots & \ddots & \vdots \\ c_1^{n_T} & c_2^{n_T} & \cdots & c_L^{n_T} \end{pmatrix} \quad \text{and} \quad \mathbf{e} = \begin{pmatrix} e_1^1 & e_2^1 & \cdots & e_L^1 \\ e_1^2 & e_2^2 & \cdots & e_L^2 \\ \vdots & \vdots & \ddots & \vdots \\ e_1^{n_T} & e_2^{n_T} & \cdots & e_L^{n_T} \end{pmatrix}$$

represent the matrices of transmitted and erroneously decoded symbols, respectively, where n_T is the number of transmit antennas and L is the frame length. We assume that the channel is block fading with block length M , i.e., fading coefficients are constant across blocks of length M and are independent from one block to the other block. We also assume that there are K such blocks in a frame, i.e., $L = K \times M$. If complete channel state information is available at the receiver, the Chernoff upper bound for the conditional pairwise error probability given fading coefficients, can be expressed as

$$P(\mathbf{c} \rightarrow \mathbf{e} \mid \alpha_k^{i,j}, i = 1, 2, \dots, n_T, j = 1, 2, \dots, n_R, k = 1, 2, \dots, K) \leq \exp(-d^2(\mathbf{c}, \mathbf{e})E_s/4\mathcal{N}_0), \quad (1)$$

where n_R is the number of receive antennas, $\alpha_k^{i,j}$ represents the complex Gaussian fading coefficient with zero mean and variance 0.5 per dimension, corresponding to the channel between the i th transmitter and j th receiver in the k th block of the frame, $\mathcal{N}_0/2$ is the variance per dimension of the complex additive white Gaussian noise, $\sqrt{E_s}$ is a scaling factor to make the average energy of the constellation equal to 1, and

$$d^2(\mathbf{c}, \mathbf{e}) = \sum_{j=1}^{n_R} \sum_{k=1}^K \sum_{m=1}^M \left| \sum_{i=1}^{n_T} \alpha_k^{i,j} (c_{(k-1)M+m}^i - e_{(k-1)M+m}^i) \right|^2. \quad (2)$$

Taking the expected value of (1) with respect to the fading coefficients $\{\alpha_k^{i,j}\}$, we will have

$$P(\mathbf{c} \rightarrow \mathbf{e}) \leq \prod_{j=1}^{n_R} \prod_{k=1}^K \mathbb{E}_{\Omega_k^j} \left\{ \exp \left(- \sum_{m=1}^M \left| \sum_{i=1}^{n_T} \alpha_k^{i,j} (c_{(k-1)M+m}^i - e_{(k-1)M+m}^i) \right|^2 E_s / 4\mathcal{N}_0 \right) \right\}, \quad (3)$$

where $\Omega_k^j = (\alpha_k^{1,j}, \dots, \alpha_k^{n_T,j})$.

It can be easily seen that

$$\sum_{m=1}^M \left| \sum_{i=1}^{n_T} \alpha_k^{i,j} (c_{(k-1)M+m}^i - e_{(k-1)M+m}^i) \right|^2 = \Omega_k^j \mathbf{D}_k(\mathbf{c}, \mathbf{e}) \mathbf{D}_k^H(\mathbf{c}, \mathbf{e}) \Omega_k^{jH}, \quad (4)$$

where

$$\mathbf{D}_k(\mathbf{c}, \mathbf{e}) = \begin{pmatrix} c_{(k-1)M+1}^1 - e_{(k-1)M+1}^1 & \cdots & c_{kM}^1 - e_{kM}^1 \\ c_{(k-1)M+1}^2 - e_{(k-1)M+1}^2 & \cdots & c_{kM}^2 - e_{kM}^2 \\ \vdots & \ddots & \vdots \\ c_{(k-1)M+1}^{n_T} - e_{(k-1)M+1}^{n_T} & \cdots & c_{kM}^{n_T} - e_{kM}^{n_T} \end{pmatrix}$$

Substituting (4) in (3), we will have

$$P(\mathbf{c} \rightarrow \mathbf{e}) \leq \prod_{j=1}^{n_R} \prod_{k=1}^K E_{\Omega_k^j} \left\{ \exp \left(-\Omega_k^j \mathbf{D}_k(\mathbf{c}, \mathbf{e}) \mathbf{D}_k^H(\mathbf{c}, \mathbf{e}) \Omega_k^j E_s / 4\mathcal{N}_0 \right) \right\}. \quad (5)$$

Since Ω_k^j is assumed to be a complex Gaussian random vector with zero mean and covariance matrix I_{n_T} , it is easy to see that the above expression can be simplified to

$$P(\mathbf{c} \rightarrow \mathbf{e}) \leq \prod_{j=1}^{n_R} \prod_{k=1}^K \frac{1}{\det(I_{n_T} + \mathbf{D}_k(\mathbf{c}, \mathbf{e}) \mathbf{D}_k^H(\mathbf{c}, \mathbf{e}) E_s / 4\mathcal{N}_0)} = \prod_{k=1}^K \frac{1}{\det(I_{n_T} + \mathbf{D}_k(\mathbf{c}, \mathbf{e}) \mathbf{D}_k^H(\mathbf{c}, \mathbf{e}) E_s / 4\mathcal{N}_0)^{n_R}}. \quad (6)$$

The above upper bound can further be simplified and approximated as

$$P(\mathbf{c} \rightarrow \mathbf{e}) \leq \prod_{k=1}^K \frac{1}{\prod_{i=1}^{n_T} (1 + \lambda_k^i(\mathbf{c}, \mathbf{e}) E_s / 4\mathcal{N}_0)^{n_R}} < \prod_{k=1}^K \prod_{i; \lambda_k^i(\mathbf{c}, \mathbf{e}) \neq 0} (\lambda_k^i(\mathbf{c}, \mathbf{e}) E_s / 4\mathcal{N}_0)^{-n_R}, \quad (7)$$

where $\lambda_k^i(\mathbf{c}, \mathbf{e})$'s are the eigenvalues of the matrix $\mathbf{D}_k(\mathbf{c}, \mathbf{e}) \mathbf{D}_k^H(\mathbf{c}, \mathbf{e})$.

Based on the above upper bound, in the next two subsections, we will review the criteria of [5] for designing space-time codes for two special cases of slowly and fast Rayleigh fading channels.

A. Slowly Fading Channels

For slowly fading channels, the fading coefficients are assumed to be constant across the whole frame, so we have only one block of size L (i.e., $M = L$ and $K = 1$). Therefore

$$P(\mathbf{c} \rightarrow \mathbf{e}) < \prod_{i; \lambda_i(\mathbf{c}, \mathbf{e}) \neq 0} (\lambda_i(\mathbf{c}, \mathbf{e}) E_s / 4\mathcal{N}_0)^{-n_R}, \quad (8)$$

where $\lambda_i(\mathbf{c}, \mathbf{e})$'s are the eigenvalues of the matrix $\mathbf{D}(\mathbf{c}, \mathbf{e}) = \mathbf{c} - \mathbf{e}$. This bound results in the *Rank* and *Determinant* criteria of [5] for slowly fading channels:

- *The Rank Criterion:* In order to achieve the maximum diversity gain of $n_T n_R$, the matrices $\mathbf{D}(\mathbf{c}, \mathbf{e})$ have to be full rank for all pairs of codewords $(\mathbf{c}, \mathbf{e})$.
- *The Determinant Criterion:* Assuming that the above rank criterion is satisfied, in order to achieve maximum coding gain, the minimum of the determinants of the matrices $\mathbf{D}(\mathbf{c}, \mathbf{e}) \mathbf{D}^H(\mathbf{c}, \mathbf{e})$, taken over all pairs of codewords $(\mathbf{c}, \mathbf{e})$, has to be maximized.

B. Fast Fading Channels

For fast fading channels, the fading coefficients are assumed to be independent from one symbol to the other, so we have L blocks of length one (i.e., $M = 1$ and $K = L$). In this case,

$\mathbf{D}_k(\mathbf{c}, \mathbf{e})$ is just a column vector, and $\mathbf{D}_k(\mathbf{c}, \mathbf{e})\mathbf{D}_k^H(\mathbf{c}, \mathbf{e})$ has at most one non-zero eigenvalue, namely $\sum_{i=1}^{n_T} |c_k^i - e_k^i|^2$ (since $\sum_{i=1}^{n_T} \lambda_k^i(\mathbf{c}, \mathbf{e}) = \text{trace}(\mathbf{D}_k(\mathbf{c}, \mathbf{e})\mathbf{D}_k^H(\mathbf{c}, \mathbf{e})) = \sum_{i=1}^{n_T} |c_k^i - e_k^i|^2$), and the upper bound can be written as

$$P(\mathbf{c} \rightarrow \mathbf{e}) < \prod_{k; \sum_{i=1}^{n_T} |c_k^i - e_k^i|^2 \neq 0} \left(\sum_{i=1}^{n_T} |c_k^i - e_k^i|^2 E_s / 4\mathcal{N}_0 \right)^{-n_R}, \quad (9)$$

which results in the *Distance* and *Product* criteria of [5] for fast fading channels:

- *The Distance Criterion:* In order to achieve the diversity gain of νn_R in a fast fading environment, any two codeword matrices $\mathbf{c}$ and $\mathbf{e}$ must be different in at least ν columns.
- *The Product Criterion:* In order to achieve the maximum coding gain for a given diversity gain, the minimum of the products of the squared Euclidean distances of those columns of $\mathbf{c}$ and $\mathbf{e}$ in which the two matrices are different, taken over all pairs of codeword matrices $(\mathbf{c}, \mathbf{e})$, has to be maximized.

In both cases, the given criteria do not provide a systematic way to design good space-time codes for slowly or fast fading channels. In the next section, we propose a concatenated space-time code structure, which simplifies the code design procedure by decoupling the problems of maximizing spatial and temporal diversity gains.

III. CODE DESIGN CRITERIA FOR ORTHOGONAL TRANSMIT DIVERSITY SYSTEM

In this section, we consider the design of space-time codes in Rayleigh fading channels, using the orthogonal transmit diversity system of Figure 1. We will also develop guidelines for the design of bandwidth efficient coded modulation schemes for this system in slowly and fast fading channel conditions.

A. The System Model

The model of our proposed system is shown in Figure 2. The input information bits are first encoded using a coded modulation block. These encoded symbols are later passed through the OTD transmitter, acting as an inner encoder in this scenario. The two symbol streams resulting from the Alamouti's orthogonal transformation (Figure 1) are then transmitted through the two transmit antennas.

The channel is modeled to be Rayleigh fading AWGN, with statistically independent fading coefficients between each pair of transmit and receive antennas. The additive noise terms at different symbol intervals are assumed to be independent samples of a zero-mean complex Gaussian random variable with variance $\mathcal{N}_0/2$ per dimension.

The received symbol at the receiver during each symbol interval is the sum of the two faded symbols from the two transmitters affected by the additive white Gaussian noise. Later in

this section, we will show that the optimum decoder for this scheme is the concatenation of Alamouti's linear combiner (standard OTD receiver) and a maximum likelihood decoder for the outer coded modulation scheme. The OTD receiver performs a combining operation on the received symbols and finally, the combined symbols are sent to the outer coded modulation decoder to recover the data bits.

B. Design Criteria for Slowly Fading Channels

As explained in Section II, in a slowly fading environment, we have $K = 1$ and $M = L$. Thus, for the system of Figure 2, (6) reduces to

$$P(\mathbf{c} \rightarrow \mathbf{e}) \leq \frac{1}{\det(I_2 + \mathbf{D}(\mathbf{c}, \mathbf{e})\mathbf{D}^H(\mathbf{c}, \mathbf{e})\frac{E_s}{4N_0})} , \quad (10)$$

Because of the orthogonal transmission structure of the OTD system shown in Figure 1 and assuming that L is even, we have

$$\begin{aligned} c_{l-1}^1 &= c_{l-1} & c_l^1 &= -c_l^* \\ c_{l-1}^2 &= c_l & c_l^2 &= c_{l-1}^* \end{aligned} , \text{ for } l = 2, 4, \dots, L.$$

Thus, the difference matrix $\mathbf{D}(\mathbf{c}, \mathbf{e})$ can be expressed as

$$\mathbf{D}(\mathbf{c}, \mathbf{e}) = \begin{pmatrix} c_1 - e_1 & -(c_2 - e_2)^* & \cdots & c_{L-1} - e_{L-1} & -(c_L - e_L)^* \\ c_2 - e_2 & (c_1 - e_1)^* & \cdots & c_L - e_L & (c_{L-1} - e_{L-1})^* \end{pmatrix}.$$

Substituting the above matrix in (10), it follows that

$$P(\mathbf{c} \rightarrow \mathbf{e}) \leq \frac{1}{\left[1 + \sum_{l=1}^L |c_l - e_l|^2 \left(\frac{E_s}{4N_0}\right)\right]^2} . \quad (11)$$

An upper bound for the pairwise error probability follows from the Chernoff bound of (11) as

$$P(\mathbf{c} \rightarrow \mathbf{e}) < \left[\left(\sum_{l=1}^L |c_l - e_l|^2 \right) \left(\frac{E_s}{4N_0} \right) \right]^{-2} . \quad (12)$$

The upper bound of (12), shows a full spatial diversity of 2 resulting from the OTD system. It should also be noticed that because of the slowly fading nature of the channel during the transmission of the whole block of symbols, no temporal diversity can be provided by the coded modulation scheme. Instead, the code design criterion in this case would involve the maximization of the coding gain, which is expressed as

$$d_E(\mathbf{c}, \mathbf{e}) = \sum_{l=1}^L |c_l - e_l|^2.$$

The above expression is the definition of the free Euclidean distance of the code. Thus, the criterion for the design of optimum coded modulation schemes for the system of Figure 2 is exactly equivalent to the code design rule for single transmit and receive antenna systems in AWGN channels. The important conclusion drawn from this argument is that any coded modulation scheme, already designed for optimum performance in an AWGN channel with single transmit and receive antennas, would also be optimum for the system of Figure 2.

Moreover, it is evident that designing a code based on just maximization of the free Euclidean distance and benefiting the inherent spatial diversity gain of the OTD system, is much simpler than designing space-time codes according to the rank and determinant criteria of [5]. Interestingly, the error performance simulation results reported in Section IV, show that the system of Figure 2 provides a higher coding gain as compared to some existing space-time trellis codes with the same complexities, both designed for optimum performance in a slowly fading environment.

C. Design Criteria for Fast Fading Channels

In this section, it is assumed that the block length of the fast fading channel of interest for the system of Figure 2 is equal to two symbol intervals. This means that the fading coefficients of the channel remain constant during the transmission of a block of two symbols and are statistically independent between different blocks. To derive the Chernoff upper bound as in (6), the whole block of length L can be partitioned into $K = L/2$ blocks of length 2 each (assuming that L is an even number). Thus, (6) can be expressed as

$$P(\mathbf{c} \rightarrow \mathbf{e}) \leq \prod_{k=1}^{L/2} \frac{1}{\det(I_2 + \mathbf{D}_k(\mathbf{c}, \mathbf{e}) \mathbf{D}_k^H(\mathbf{c}, \mathbf{e}) \frac{E_s}{4N_0})} . \quad (13)$$

Because of the orthogonal transmission structure of the OTD system, we have

$$\begin{aligned} c_{2k-1}^1 &= c_{2k-1} & c_{2k}^1 &= -c_{2k}^* \\ c_{2k-1}^2 &= c_{2k} & c_{2k}^2 &= c_{2k-1}^* \end{aligned} , \text{ for } k = 1, 2, \dots, L/2.$$

So, the matrix $\mathbf{D}_k(\mathbf{c}, \mathbf{e})$ can be written as

$$\mathbf{D}_k(\mathbf{c}, \mathbf{e}) = \begin{pmatrix} c_{2k-1} - e_{2k-1} & -(c_{2k} - e_{2k})^* \\ c_{2k} - e_{2k} & (c_{2k-1} - e_{2k-1})^* \end{pmatrix}.$$

Substituting the matrix $\mathbf{D}_k(\mathbf{c}, \mathbf{e})$ in (13) results in

$$P(\mathbf{c} \rightarrow \mathbf{e}) \leq \prod_{k=1}^{L/2} \frac{1}{[(1 + (|c_{2k-1} - e_{2k-1}|^2 + |c_{2k} - e_{2k}|^2) \frac{E_s}{4N_0})]^2} . \quad (14)$$

An upper bound for the pairwise error probability of the system of Figure 2 in a fast fading channel with block length of two, follows from (14) as

$$P(\mathbf{c} \rightarrow \mathbf{e}) < \prod_{\substack{(c_{2k-1}, c_{2k}) \neq (e_{2k-1}, e_{2k}) \\ k=1, \dots, L/2}} \left[(|c_{2k-1} - e_{2k-1}|^2 + |c_{2k} - e_{2k}|^2) \left(\frac{E_s}{4\mathcal{N}_0} \right) \right]^{-2}. \quad (15)$$

As can be seen from (15), a full spatial diversity gain of 2 results from the orthogonal transmission system. Hence, applying the OTD system, we have maximized the spatial diversity gain achievable by two transmit antennas. It should also be noticed that, unlike the slowly fading scenario, in a fast fading channel, a considerable temporal diversity gain can be provided by the coding scheme. Hence, the optimum coded modulation scheme in this case is the one which maximizes the temporal diversity gain as well as the coding gain.

In [1], it has been shown that the performance of codes in fast fading channels with single transmit and receive antennas is controlled by two factors: the minimum symbol Hamming distance (length of the shortest error event path), and the product of squared symbol distances along the shortest error event path. These two factors determine the temporal diversity gain and the coding gain, respectively. From (15) however, it follows that the optimum code for transmission over the OTD system is not simply obtained by the criteria in [1], but instead it involves maximization of new distance quantities defined in terms of pairs of consecutive symbols. These new *pairwise Hamming* and *pairwise product* distances, denoted by $d_{PW,H}$ and $d_{PW,P}$ respectively, are defined as:

$$d_{PW,H}(\mathbf{c}, \mathbf{e}) = \sum_{\substack{(c_{2k-1}, c_{2k}) \neq (e_{2k-1}, e_{2k}) \\ k=1, \dots, L/2}} 1, \quad (16)$$

and

$$d_{PW,P}(\mathbf{c}, \mathbf{e}) = \prod_{\substack{(c_{2k-1}, c_{2k}) \neq (e_{2k-1}, e_{2k}) \\ k=1, \dots, L/2}} (|c_{2k-1} - e_{2k-1}|^2 + |c_{2k} - e_{2k}|^2). \quad (17)$$

These pairwise distances motivate us to introduce a new code design technique for the OTD system in a fast fading channel, based on expanding the signal constellation. The expansion can be performed in either dimension or size of the constellation (going to higher orders of modulation). Each point in the new constellation can be considered as the concatenation of two consecutive signal points from the original signal set. Denoting the sequences of transmitted and erroneously decoded symbols in the new constellation by $\mathbf{C} = \{C_1, C_2, \dots, C_{L/2}\}$ and $\mathbf{E} = \{E_1, E_2, \dots, E_{L/2}\}$, respectively, we have:

$$\begin{aligned} C_k &= (c_{2k-1}, c_{2k}) \quad k = 1, 2, \dots, L/2, \\ E_k &= (e_{2k-1}, e_{2k}) \quad k = 1, 2, \dots, L/2. \end{aligned}$$

Thus, (16) and (17) can be rewritten as

$$d_{PW,H}(\mathbf{c}, \mathbf{e}) = \sum_{\substack{C_k \neq E_k \\ k=1, \dots, L/2}} 1 = d_H(\mathbf{C}, \mathbf{E}), \quad (18)$$

and

$$d_{PW,P}(\mathbf{c}, \mathbf{e}) = \prod_{\substack{C_k \neq E_k \\ k=1, \dots, L/2}} |C_k - E_k|^2 = d_P(\mathbf{C}, \mathbf{E}). \quad (19)$$

Hence, the code design criteria will be based on maximizing the minimum symbol Hamming and product distances in the expanded constellation.

Thus, to achieve a diversity gain of L equal to the length of the block, only a minimum symbol Hamming distance of $L/2$ in the expanded constellation is needed. This does not necessarily require a minimum Hamming distance of L in the original signal set. In fact, it is clear from the upper bound of (15) that adopting the orthogonal transmit diversity, the Hamming distance requirement of the codewords halves. This is the consequence of the inherent spatial diversity gain of two resulting from the orthogonal transmission system. The reduction in the Hamming distance requirement allows us to have larger subsets in the signal set partitioning, which in turn results in higher achievable code rates with less code complexities. This will be explained with more detail in Section IV.

D. Optimal Decoding Algorithm

In this subsection, we will prove that the optimum decoder for the system of Figure 2 can be obtained by concatenating Alamouti's linear combiner (standard OTD receiver [7]) and a maximum likelihood decoder for the outer coded modulation scheme.

Assuming perfect channel state information at the receiver, the decision metric of the optimum (maximum likelihood) decoder for a space-time coding scheme with block length of L , can be expressed as

$$\sum_{j=1}^{n_R} \sum_{l=1}^L \left| r_l^j - \sum_{i=1}^{n_T} \alpha_l^{i,j} c_l^i \right|^2, \quad (20)$$

where r_l^j denotes the received signal at the j^{th} receiver during the l^{th} symbol interval, and $\alpha_l^{i,j}$ is the complex Gaussian fading coefficient of the channel between the i^{th} transmitter and the j^{th} receiver in the l^{th} symbol interval. So, the maximum likelihood decoder decides in favor of the codeword which maximizes the decision metric of (20).

For the orthogonal transmission system of Figure 2 with two transmit and single receive

antennas, (20) reduces to

$$\sum_{p=1}^{L/2} \left(|r_{2p-1} - \alpha_{2p}^1 c_{2p-1} - \alpha_{2p}^2 c_{2p}|^2 + |r_{2p} + \alpha_{2p}^1 c_{2p}^* - \alpha_{2p}^2 c_{2p-1}^*|^2 \right), \quad (21)$$

where the index j in the expression $\alpha_{2p}^{i,j}$ is dropped as the number of receive antennas is assumed to be one. The received signal at each symbol interval is the sum of the two faded transmitted symbols affected by the additive white complex Gaussian noise with variance $\mathcal{N}_0/2$ per dimension

$$\begin{aligned} r_{2p-1} &= \alpha_{2p}^1 c_{2p-1} + \alpha_{2p}^2 c_{2p} + n_{2p-1}, \\ r_{2p} &= \alpha_{2p}^2 c_{2p-1}^* - \alpha_{2p}^1 c_{2p}^* + n_{2p} \end{aligned} \quad \text{for } p = 1, 2, \dots, L/2. \quad (22)$$

Expanding (21), the maximum likelihood decision rule can be written as

$$\sum_{p=1}^{L/2} \left\{ (|r_{2p-1}|^2 + |r_{2p}|^2) + (|\alpha_{2p}^1|^2 + |\alpha_{2p}^2|^2) (|c_{2p-1}|^2 + |c_{2p}|^2) - 2\Re \left[(r_{2p-1} \alpha_{2p}^{1*} + r_{2p}^* \alpha_{2p}^2) c_{2p-1}^* \right] + 2\Re \left[(r_{2p-1} \alpha_{2p}^{2*} - r_{2p}^* \alpha_{2p}^1) c_{2p}^* \right] \right\}, \quad (23)$$

where $\Re[x]$ denotes the real part of x .

Now, let's define z_{2p-1} and z_{2p} as

$$\begin{aligned} z_{2p-1} &= r_{2p-1} \alpha_{2p}^{1*} + r_{2p}^* \alpha_{2p}^2, \\ z_{2p} &= r_{2p-1} \alpha_{2p}^{2*} - r_{2p}^* \alpha_{2p}^1. \end{aligned} \quad \text{for } p = 1, 2, \dots, L/2. \quad (24)$$

These quantities are exactly the outputs of the standard OTD receiver/combiner proposed by Alamouti in [7]. The first important point of the discussion so far is that based on the Neyman-Fisher factorization theorem [12], z_{2p-1} and z_{2p} defined as above are sufficient statistics for the maximum likelihood decoder. This means that using the standard OTD combiner at the front end of the receiver causes no information loss in estimating the symbol sequence $c_1, c_2, \dots, c_L$.

Considering that the first term of the summation in (23) is independent of the symbol sequence to be detected, it can be replaced by any other expression which is also independent of $c_1, c_2, \dots, c_L$. Thus, replacing $|r_l|^2$ with $|z_l|^2/\beta_l$ for $l = 1, 2, \dots, L$, where

$$\beta_l = |\alpha_l^1|^2 + |\alpha_l^2|^2, \quad \text{for } l = 1, 2, \dots, L, \quad (25)$$

it can be easily seen that minimizing the decision metric of (21) is equivalent to minimizing the new maximum likelihood metric

$$\sum_{p=1}^{L/2} \left\{ \left(\frac{|z_{2p-1}|^2}{\beta_{2p}} + \frac{|z_{2p}|^2}{\beta_{2p}} \right) + \beta_{2p} (|c_{2p-1}|^2 + |c_{2p}|^2) + 2\Re (z_{2p-1} c_{2p-1}^* + z_{2p} c_{2p}^*) \right\}. \quad (26)$$

Equation (26) can also be expressed as

$$\sum_{l=1}^L \left(\frac{|z_l|^2}{\beta_l} + \beta_l |c_l|^2 - 2\Re(z_l c_l^*) \right) = \sum_{l=1}^L \frac{|z_l - \beta_l c_l|^2}{\beta_l}. \quad (27)$$

On the other hand, substituting (22) in (24), the linear combiner outputs at two consecutive symbol intervals would be

$$\begin{aligned} z_{2p-1} &= (|\alpha_{2p}^1|^2 + |\alpha_{2p}^2|^2) c_{2p-1} + \alpha_{2p}^{1*} n_{2p-1} + \alpha_{2p}^2 n_{2p}^*, \text{ for } p = 1, 2, \dots, L/2. \\ z_{2p} &= (|\alpha_{2p}^1|^2 + |\alpha_{2p}^2|^2) c_{2p} + \alpha_{2p}^{2*} n_{2p-1} - \alpha_{2p}^1 n_{2p}^* \end{aligned} \quad (28)$$

Defining N_{2p-1} and N_{2p} as the noise terms at the output of the OTD receiver during two consecutive symbol intervals, we have

$$\begin{aligned} N_{2p-1} &= \alpha_{2p}^{1*} n_{2p-1} + \alpha_{2p}^2 n_{2p}^*, \text{ for } p = 1, 2, \dots, L/2, \\ N_{2p} &= \alpha_{2p}^{2*} n_{2p-1} - \alpha_{2p}^1 n_{2p}^* \end{aligned} \quad (29)$$

It can be easily shown that N_l 's for $l = 1, \dots, L$, are zero mean, iid complex Gaussian random variables with variance $\beta_l \mathcal{N}_0/2$ per dimension.

Therefore, the decision metric of (27), is in fact the metric for the maximum likelihood decoding algorithm performed on the output symbols of the linear combiner (OTD receiver). Thus, it is concluded that the optimum decoder for the orthogonal system of Figure 2 is achieved by concatenating the linear combining scheme of (24) with a standard maximum likelihood decoder operating on the combined symbols.

IV. DESIGN OF TRELLIS CODED MODULATION SCHEMES FOR THE OTD SYSTEM

In Section III, it was explained that the system in Figure 2 can considerably simplify the design procedure of maximum diversity gain coding schemes for multiple antenna communication systems in fading channels. We later derived upper bound expressions for the pairwise error probability of this system in slow and fast fading channel conditions. Based on these bounds, guidelines for the design of concatenated space-time coded modulation schemes were proposed and discussed in detail. In this section, we will put these criteria into practice by designing bandwidth efficient trellis coded modulation schemes for the system in Figure 2. The simulation results reported in the next two subsections show that codes designed for the system of Figure 2 have better performance compared to the space-time trellis codes of [5] with similar complexities.

A. Code Design for Slowly Fading Channels

It was shown in Section III-B that the criterion for designing optimum codes for transmission over the orthogonal system of Figure 2 in slowly fading channels, is based only on the

maximization of the free Euclidean distance. To analyze the error performance of the codes designed as such, consider designing rate 2 b/s/Hz codes for the system in Figure 2. We will use Ungerboeck's 4 and 8-state TCM codes with 8PSK modulation, designed based on the maximum free Euclidean distance criterion, for optimum performance in AWGN channels [13]. In Figures 3 (a) and (b), the simulation results for the frame error performance of these codes for different levels of SNR are shown (each frame consists of 130 code symbols).

For comparison purpose, we have also plotted the error performance of two space-time trellis codes proposed in [5] with two transmit and single receive antennas and the same trellis complexities and code rates, designed based on the rank and determinant rules of [5] for slowly fading channels. While both codes demonstrate a diversity gain of two (the curves are parallel with slope of -2), it can be seen that the system of Figure 2 shows more than 1 and 1.6 dB gain over the space-time trellis codes of [5] for the two cases of 4 and 8-state trellises, respectively. Besides, it is observed that the concatenated orthogonal space-time system performs closer to the outage probability.

B. Code Design for Fast Fading Channels

In Section III-C, it was described that the code design criteria for the OTD system in fast fading channels, is based on the maximization of Hamming and product distances in the expanded constellation, where each point is the concatenation of two points from the original signal set. Assuming that the original signal set is two dimensional (2D), one way of constructing the new constellation is to consider the four dimensional (4D) Cartesian product of the original 2D signal set by itself. Another way is to construct a new 2D constellation of size M^2 , where M is the size of the original signal set. These two approaches are not conceptually different. The only difference is in the simplicity of the implementation depending on the modulation type. For MPSK constellations, expansion in dimension is not desirable, because the expanded constellation will not have the same circular structure and the existing set partitioning techniques cannot be exploited. Hence, in this case, we use expansion in size. However, for rectangular constellations (QAM), expansion in dimension results in a constellation with the same rectangular structure, and the existing set partitioning and code design techniques can be easily extended to this case.

The code design will be performed for the new constellation trying to maximize the Hamming and product distances. At the output of the coded modulation block, each encoded symbol from the new constellation will be considered as the concatenation of two signal points from the original constellation, and will be transmitted in two consecutive symbol intervals through the OTD system as in Figure 1. Multiple Trellis Coded Modulation (MTCM) and

MultiLevel Coding (MLC) design techniques satisfying the Hamming and product criteria of Section III-C, have already been proposed in the literature ([2], [14]). These are appropriate coded modulation schemes for fast fading channels, as they can be designed to achieve good distance properties required by the criteria derived in [1].

In the following examples, we will use the set partitioning scheme of [15] to design MLC codes for QAM constellations, and that of [2] to design MTCM codes for MPSK signal sets.

B.1 *Constellation Expansion in Dimension: Design of Multidimensional MLC for OTD*

Suppose that the goal is to design a coded modulation scheme with rate 3 b/s/Hz and total diversity gain of 4. We use a 4D 256-point lattice constellation with 2D constituents coming from a 16QAM constellation, along with a two-level MLC [16], [17], [14], [18], [19], which provides minimum Hamming distance of 2. Two convolutional encoders of rates 2/3 and 4/5 are used as the first and second level encoders. The 4D set partitioning chain $Z^4/D_4/RZ^4/RD_4$ is used to partition the 256-point constellation into 8 subsets of size 32, as explained in [20] and [15]. One of the 8 subsets is chosen by the outputs of the first level encoder. The outputs of the second level encoder are then used to choose one of the 32 points inside the chosen subset. Each 4D point is then mapped into its two 2D coordinates to produce a sequence of signal points from a 16QAM constellation. This sequence is later transmitted through the OTD system of Figure 1.

The error rate performance of the above code is shown in Figure 4(a) and is compared to the uncoded 8PSK OTD scheme. As can be seen from the error rate curves, the coded scheme shows more than 5 dB gain over the uncoded scheme at error rates of 2×10^{-4} and lower.

B.2 *Constellation Expansion in Size: Design of MTCM for OTD*

Consider designing a code with the overall diversity gain of 4 and rate 1.5 b/s/Hz using QPSK modulation. In order to achieve a minimum Hamming distance of 2 resulting in a total diversity gain of 4 using the OTD system, it suffices to consider an MTCM code with multiplicity of 4 and perform the set partitioning task for a 2-fold Cartesian product of a 16PSK signal set. Each point in the 16PSK signal set is considered as the concatenation of two consecutive QPSK symbols. If the set partitioning scheme of [2] is adopted for a 2-fold Cartesian product of 16PSK symbols, a maximum of 16 codewords can be assigned to each subset. This means that a maximum of 16 parallel paths can be considered for the trellis. So, if a 4 state fully connected trellis is considered, the encoder would be capable of encoding 6 input bits. Together with the 4 QPSK symbols assigned to each transition of the trellis, this results in the desired rate of $6/4 = 1.5$ b/s/Hz. Note that if we wanted to

use the same trellis to design a code with diversity gain of 4 for single transmit and receive antenna system, the maximum achievable rate would be 1 b/s/Hz. That is because the set partitioning of the 4-fold Cartesian product of QPSK symbols, would result in subsets with a maximum size of four [2]. Thus, for comparison purpose, an 8 state fully connected trellis with 8PSK modulation has been used to design an MTCM code with rate 1.5 b/s/Hz and diversity gain of 4 for single transmission scheme.

The error performance comparison of these two transmission schemes is shown in Figure 4(b). It can be seen that while both codes provide a diversity gain of 4 (the slope of the curves is -4), the MTCM code designed for the orthogonal system of Figure 2, outperforms the single transmit and receive antenna transmission scheme by 1 dB.

To demonstrate the robustness of the system of Figure 2, its performance has also been compared to the smart-greedy space-time code of [5] in Figures 5 (a) and (b). Smart-greedy codes are designed to provide a good performance in both slowly and fast fading channels. Thus, even if the transmitter doesn't know the channel, the code is constructed to take advantage of both the spatial diversity provided by the use of multiple antennas, and the possible temporal variations of the channel. As such, the smart-greedy codes guarantee a diversity gain of r_1 in slow and $r_2 \geq r_1$ in fast fading channel conditions [5].

For comparison purpose, we picked up the 2-state smart-greedy space-time code of [5] with QPSK modulation which is designed to achieve diversity gains of 2 and 3 in slowly and fast fading channels, respectively, using two transmit and one receive antennas. The error performance has been simulated in both cases and compared to the concatenated orthogonal space-time code of Figure 2, using an MTCM inner code of multiplicity 4. The complexities of the two trellises are similar (both have 2 states), and the code rate is 1 b/s/Hz in both cases.

As can be seen from the simulation results of Figures 5 (a) and (b), the concatenated space-time code shows a better performance in both slowly and fast fading channels. In slowly fading case, the concatenated system provides just a spatial diversity gain of 2, and no additional temporal diversity is gained from the inner MTCM scheme. That is why the two error curves are parallel with slope of almost -2 . However, the concatenated system presents a higher coding gain. In the case of fast fading channel, the concatenated space-time code demonstrates an asymptotic diversity gain of 4 (2 spatial, and 2 temporal from the inner code), as compared to the diversity 3 resulting from the smart-greedy code.

V. DESIGN OF CONCATENATED SPACE-TIME CODES WITH GENERALIZED ORTHOGONAL TRANSMIT DIVERSITY SYSTEMS

In this section, we will extend the design criteria of Section III to orthogonal systems with more than two transmit and one receive antennas. We refer to these as *Generalized Orthogonal Transmit Diversity (GOTD)* systems.

In [6], it was proven that for complex signal constellations, full rate, full diversity orthogonal systems of size n_T exist if and only if $n_T = 2$ (n_T is the number of transmit antennas). For more than two transmit antennas, full diversity generalized orthogonal designs with rates less than one, have been introduced in the literature [6], [21]. These designs fall into two main categories: rate halving codes, and square matrix embeddable codes. The rate halving codes [6] are built by concatenating real orthogonal matrices and their complex conjugates together, halving the overall rate of the resulting complex orthogonal design.

The square matrix embeddable codes are based on orthogonal square code matrices with dimension n_T (assuming to be a power of 2). For number of transmit antennas that are not a power of two, the code matrix dimension is assumed to be $n_T \times Q$ ($Q \geq n_T$ for the code to be linearly decodable), which is obtained by deleting some rows from an orthogonal design of higher dimension. For example, for the case of three antennas, the code is constructed by deleting one row from the 4×4 square code. Codes for 5, 6 and 7 antennas are built similarly from the 8×8 square orthogonal design and so on.

Some examples of the square matrix embeddable codes are the sporadic codes of [6] for $n_T = 3$ and 4 (with rate $3/4$), and the unitary designs of [21]. It has been proven in [21] that the maximum achievable rates of these codes are $\frac{\lceil \log_2 n_T \rceil + 1}{2^{\lceil \log_2 n_T \rceil}}$ ($\lceil x \rceil$ is the integer greater or equal to x). Unitary designs and their construction have been introduced and fully explained in [21]. The code matrix for a unitary design with four transmit antennas and rate $3/4$, constructed in [21] has the code matrix

$$\mathbf{c} = \begin{pmatrix} c_1 & -c_2^* & -c_3^* & 0 \\ c_2 & c_1^* & 0 & c_3^* \\ c_3 & 0 & c_1^* & -c_2^* \\ 0 & -c_3 & c_2 & c_1 \end{pmatrix}, \quad (30)$$

For a generalized orthogonal design of rate R and code matrix size of $n_T \times Q$, we assume that RQ symbols are transmitted during Q consecutive symbol intervals, through the GOTD transmitter. As an example, for the orthogonal design of (30) where $Q = 4$ and $R = 3/4$, 3 coded symbols are transmitted during the transmission period of the code block.

The important characteristic of all the above designs is the orthogonality of their code

matrix, $\mathbf{c}$

$$\mathbf{c}\mathbf{c}^H = \sum_{q=1}^{RQ} |c_q|^2 I_{n_T}. \quad (31)$$

This causes the orthogonal systems to be capable of providing a full spatial diversity gain of $n_T n_R$ (n_R is the number of receive antennas), as will later be shown in this section. Moreover, in a similar way to the discussion of Section III-D, it can be shown that the optimum decoder for the concatenated space-time codes with GOTD systems, is also obtained by concatenation of a linear combiner with the maximum likelihood decoder for the outer coded modulation scheme.

In the next two sections, we will study the criteria for designing concatenated space-time codes with the GOTD system in slow and fast fading channels.

A. Design Criteria for Slowly Fading Channels

As in Section II-A, in a slowly fading environment, we have $K = 1$ and $M = L$. Thus, (6) results in

$$P(\mathbf{c} \rightarrow \mathbf{e}) \leq \frac{1}{\det(I_{n_T} + \mathbf{D}(\mathbf{c}, \mathbf{e})\mathbf{D}^H(\mathbf{c}, \mathbf{e})\frac{E_s}{4N_0})^{n_R}}. \quad (32)$$

On the other hand, because of the linearity of all the orthogonal designs discussed so far, the code difference matrix $\mathbf{D}(\mathbf{c}, \mathbf{e})$, will also inherit the orthogonality property (31) of the code block matrix [21]. Thus for the code difference matrix $\mathbf{D}(\mathbf{c}, \mathbf{e})$ expressed as

$$\mathbf{D}(\mathbf{c}, \mathbf{e}) = (\mathbf{D}_1(\mathbf{c}, \mathbf{e}), \mathbf{D}_2(\mathbf{c}, \mathbf{e}), \dots, \mathbf{D}_{L/Q}(\mathbf{c}, \mathbf{e})), \quad (33)$$

we will have

$$\mathbf{D}_p(\mathbf{c}, \mathbf{e})\mathbf{D}_p^H(\mathbf{c}, \mathbf{e}) = \left(\sum_{q=1}^{RQ} |c_{(p-1)RQ+q} - e_{(p-1)RQ+q}|^2 \right) I_{n_T}, \quad \text{for } p = 1, 2, \dots, L/Q. \quad (34)$$

So, (32) reduces to

$$P(\mathbf{c} \rightarrow \mathbf{e}) \leq \frac{1}{\det \left(I_{n_T} + \left(\sum_{p=1}^{L/Q} \sum_{q=1}^{RQ} |c_{(p-1)RQ+q} - e_{(p-1)RQ+q}|^2 \frac{E_s}{4N_0} \right) I_{n_T} \right)^{n_R}}, \quad (35)$$

or equivalently

$$P(\mathbf{c} \rightarrow \mathbf{e}) \leq \frac{1}{\left(1 + \sum_{l=1}^{RL} |c_l - e_l|^2 \frac{E_s}{4N_0} \right)^{n_T n_R}}. \quad (36)$$

An upper bound for the pairwise error probability of concatenated space-time codes employing GOTD systems in slowly fading channel conditions, follows from the Chernoff bound

of (36) as

$$P(\mathbf{c} \rightarrow \mathbf{e}) < \left[\left(\sum_{l=1}^{RL} |c_l - e_l|^2 \right) \left(\frac{E_s}{4\mathcal{N}_0} \right) \right]^{-n_T n_R}. \quad (37)$$

The above upper bound shows a full spatial diversity gain of $n_T n_R$ provided by the GOTD system. It can also be seen that due to the slowly fading characteristic of the channel, the coding scheme cannot provide any temporal diversity gain. The criterion for the design of optimum coded modulation schemes in this case, is based on the maximization of the code free Euclidean distance

$$d_E(\mathbf{c}, \mathbf{e}) = \sum_{l=1}^{RL} |c_l - e_l|^2. \quad (38)$$

Thus, it is concluded that for slowly fading channels, the design criterion of concatenated space-time codes with GOTD systems is the same as the code design rule for the system of Figure 2.

In Figure 6, the simulation results for the frame error probability of codes designed for the unitary design of (30), with $n_T = 3$ and 4 antennas are provided. As the outer code, we have used the 8-state Ungerboeck's TCM code [13] designed based on the maximization of the code free Euclidean distance for optimum performance in AWGN channels. The rate of the code is 2 b/s/Hz, which together with the rate 3/4 of the unitary design, results in an overall rate of 1.5 b/s/Hz.

It can be noticed that there is an increase in the slope of the frame error probability curves as the number of transmit and receive antennas increase. This increase in the diversity gain is expected according to (37). Moreover, the codes with 2 receive antennas provide a higher coding gain with respect to the single receive antenna codes. It is seen that at frame error rates of 10^{-3} and lower, the code with 4 transmit and 2 receive antennas gives more than 6 dB gain over the case of 4 transmit and single receive antennas.

The performance comparison with the outage probability is demonstrated in Figures 7 (a) and (b). It can be seen that at frame error rate of 0.1 (in these simulations, each frame consists of 176 coded symbols transmitted from each transmit antenna), the code for 4 transmit and single receive antennas performs within approximately 2.5 dB of the outage probability.

B. Design Criteria for Fast Fading Channels

In this section, we assume that the block length of the block fading channel equals to the length of the code block matrix ($M = Q$). Thus, for a total of $K = L/Q$ blocks, it follows

from (6) that

$$P(\mathbf{c} \rightarrow \mathbf{e}) \leq \prod_{k=1}^{L/Q} \frac{1}{\det(I_{n_T} + \mathbf{D}_k(\mathbf{c}, \mathbf{e}) \mathbf{D}_k^H(\mathbf{c}, \mathbf{e}) \frac{E_s}{4N_0})^{n_R}}. \quad (39)$$

Substituting (34) into (39), an upper bound for the pairwise error probability of concatenated space-time codes with the GOTD systems in fast fading channels, can be derived as

$$P(\mathbf{c} \rightarrow \mathbf{e}) < \prod_{\substack{(\mathbf{c}_{(k-1)RQ+1}, \dots, \mathbf{c}_{kRQ}) \neq (\mathbf{e}_{(k-1)RQ+1}, \dots, \mathbf{e}_{kRQ}) \\ k=1, \dots, L/Q}} \left[\left(\sum_{q=1}^{RQ} |c_{(k-1)RQ+q} - e_{(k-1)RQ+q}|^2 \right) \left(\frac{E_s}{4N_0} \right) \right]^{-n_T n_R}. \quad (40)$$

It should be noted from (40) that in the case of fast fading channels, in addition to the full spatial diversity gain of $n_T n_R$ resulting from the GOTD system, the coding scheme is also capable of providing temporal diversity gain. Similar to the discussion of Section III-C, the code design criteria in this case is based on the maximization of the minimum product and Hamming distances in the expanded signal set. This expansion can be performed in both dimension or size, however, for the generalized orthogonal designs, the constellation points in the new signal set are the concatenation of RQ signal points (2 for the system of Figure 2) from the original signal set.

To demonstrate the above design procedure, we have simulated the performance of codes designed for the unitary design of (30) with 3 and 4 transmit and single receive antennas in fast fading channels. The simulation results are reported in Figures 8 (a) and (b). In Figure 8(a), the outer encoder is a 4-state fully connected MTCM code with multiplicity 6, using QPSK modulation. The code design would be based on the maximization of Hamming and product distances in the expanded constellation, where each point is the concatenation of $RQ = 3$ signal points from the original QPSK signal set. Thus, the set partitioning of [2] has been performed for the 2-fold Cartesian product of 64PSK signal set. This results in a rate of 4/3 b/s/Hz. Since the rate of the unitary design of (30) is 3/4, the overall rate of the concatenated code will be 1 b/s/Hz.

In Figure 8(b), the outer code is a three level MLC, designed for a 6D QAM constellation. Each 2D constituent constellation is a 16QAM. The encoders at all levels are convolutional encoders, with rates 2/3, 3/4, and 4/5, respectively. The overall rate of the code is 2.25 b/s/Hz.

It can be seen from Figures 8 (a) and (b) that asymptotically, the codes are achieving their expected diversity gains of 6 and 8 for three and four transmit antennas, respectively. The code for 4 transmit antennas shows more than 1 dB gain over the 3 antenna code at symbol error rates of 10^{-4} and lower.

VI. PERFORMANCE CRITERIA OF SPACE-TIME CODES IN MULTIPATH CHANNELS

In this section, we will consider the space-time code design criteria in the case of frequency-selective multipath fading channels. We assume that the delay spread of the channel and the delays of different paths at each receive antenna are multiples of $1/W$ where W is the signal bandwidth. We also assume that the modulation signal (including the spreading code in the case of CDMA systems) has the property that its shifted (delayed) versions by different multiples of $1/W$, have zero correlation. This assumption, though seeming very unrealistic, is fairly achievable by using long pseudorandom spreading codes in wideband CDMA systems. We further assume that at each receive antenna, a RAKE receiver with S fingers is used, where S/W is equal to the delay spread of the channel, and that the additive white Gaussian noise terms at different fingers of RAKE receivers are independent.

With the above assumptions, the Chernoff upper bound for the conditional pairwise error probability given fading coefficients, can be expressed as

$$P(\mathbf{c} \rightarrow \mathbf{e} \mid \alpha_{k,s}^{i,j}, i = 1, \dots, n_T, j = 1, \dots, n_R, k = 1, \dots, K, s = 1, \dots, S) \leq \exp(-d^2(\mathbf{c}, \mathbf{e})E_s/4N_0), \quad (41)$$

where $\alpha_{k,s}^{i,j}$ represents the fading coefficient for the s th path from the i th transmit antenna and the j th receive antenna, in the k th block of the block fading channel, and

$$d^2(\mathbf{c}, \mathbf{e}) = \sum_{j=1}^{n_R} \sum_{s=1}^S \sum_{k=1}^K \sum_{m=1}^M \left| \sum_{i=1}^{n_T} \alpha_{k,s}^{i,j} (c_{(k-1)M+m}^i - e_{(k-1)M+m}^i) \right|^2. \quad (42)$$

If we further assume that $\alpha_{k,s}^{i,j}$'s are independent complex Gaussian random variables with zero mean and variance 0.5 per dimension, and take the expected value of (41) with respect to the fading coefficients $\{\alpha_{k,s}^{i,j}\}$, we will have

$$P(\mathbf{c} \rightarrow \mathbf{e}) \leq \prod_{j=1}^{n_R} \prod_{s=1}^S \prod_{k=1}^K E_{\Omega_{k,s}^j} \left\{ \exp \left(- \sum_{m=1}^M \left| \sum_{i=1}^{n_T} \alpha_{k,s}^{i,j} (c_{(k-1)M+m}^i - e_{(k-1)M+m}^i) \right|^2 E_s/4N_0 \right) \right\}, \quad (43)$$

where $\Omega_{k,s}^j = (\alpha_{k,s}^{1,j}, \dots, \alpha_{k,s}^{n_T,j})$. It is obvious that the above expression is equivalent to the expression (3) with n_R replaced by $n_R S$, and therefore, the upper bound for pairwise error probability (equivalent of (6)) for this case can be written as

$$P(\mathbf{c} \rightarrow \mathbf{e}) \leq \prod_{k=1}^K \frac{1}{\det(I_{n_T} + \mathbf{D}_k(\mathbf{c}, \mathbf{e})\mathbf{D}_k^H(\mathbf{c}, \mathbf{e})E_s/4N_0)^{n_R S}}. \quad (44)$$

This implies that the design criteria for space-time codes, as well as the concatenated orthogonal space-time coding scheme proposed in this paper, in frequency-selective multipath

block fading channels, are the same as the ones for the case of no multipath. The only difference is the performance improvement due to the diversity gain of S in the multipath case, achieved by using RAKE receivers at each receive antenna.

The error rate performance of the concatenated orthogonal space-time code of Figure 2 in a multipath fading channel, is illustrated in Figure 9. It is assumed that there are 3 paths between each pair of transmit and receive antennas and the paths are modeled by Jakes' channel. The Doppler frequency of the Jakes' model is chosen to be 110 Hz, which corresponds to a mobile speed of 60 Km/h at carrier frequency of 2 GHz. With the symbol period chosen to be 0.1 ms, the coherence time of the channel will be around 90 symbol intervals, which makes the channel more like slowly fading, if no interleaving is used. A Gold sequence with spreading factor of 31 is used to enable the receiver to resolve the multipath components. The outer code is Ungerboeck's 8-state TCM designed for AWGN channel. As it can be seen from the figure, the expected diversity gain of 6 is almost achieved at SNR's as low as 4 dB (the spreading gain 31 of the CDMA system is also taken into account).

VII. CONCLUSIONS

In this paper, we introduced a method to decouple the problems of maximizing spatial and temporal diversity and/or coding gain, by using a concatenated code structure with the orthogonal transmit diversity system as its inner code. We also derived the criteria for the design of coded modulation schemes for the orthogonal systems in slowly and fast fading channels.

In the case of slowly fading channels, the proposed technique decouples the problems of maximizing spatial diversity gain and coding gain, and the design criterion for the outer code is based on the maximization of the code free Euclidean distance. Simulation results show that the proposed 4 and 8-state concatenated codes perform 1-2 dB better than the trellis codes of [5] having the same rates and number of states.

For the case of fast fading channel, the new idea of constellation expansion is proposed to decouple the code design problem into two simpler problems, namely maximizing the spatial diversity, and maximizing the temporal diversity and coding gain. It is shown that this expansion can be performed in dimension or size of the signal set. The design criteria in this case are shown to be based on the maximization of the Hamming and product distances in the expanded constellation. Simulation results show that concatenated codes, with multilevel and MTCM codes as their outer codes, can achieve the expected overall diversity gains with more than 5 and 1 dB performance improvement compared to the uncoded orthogonal transmission and single transmission MTCM schemes, respectively. Comparison of the codes

designed for the fast fading channel with the smart-greedy code of [5] having the same complexity, shows that the concatenated code can achieve better performance than the smart-greedy code in both fast and slowly fading environments.

The generalization of the proposed structure to the systems with more than two transmit and/or one receive antenna is also considered in this paper. Similar concatenated code structures are obtained for slowly and fast fading channels, by using generalized orthogonal transmission schemes as the inner code. Simulation results show that these codes can also achieve the expected overall diversity gains in both fast and slowly fading environments.

We also considered the case of frequency-selective multipath channels with resolvable paths at the receiver. The code design criteria are shown to be the same as the ones for the case of no multipath. It is also shown that the resolvable multipath components at the receiver result in performance improvement by increasing the overall diversity gain of the system. Simulation results show that the expected diversity gains can actually be achieved, even with non-ideal channel models such as Jakes' channel.

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Fig. 1. Orthogonal Transmit Diversity (OTD) System

Fig. 2. Concatenated Orthogonal Space-Time Code

(a) 4-state trellis

(b) 8-state trellis

Fig. 3. Performance comparison of rate 2 b/s/Hz space-time trellis codes, with two transmit and one receive antennas, designed for slowly fading channels.

(a) Multidimensional MLC scheme ($R = 3$ b/s/Hz)

(b) MTCM scheme ($R = 1.5$ b/s/Hz)

Fig. 4. Symbol error performance of concatenated orthogonal space-time code with (a) Multidimensional MLC and (b) MTCM schemes as its outer code, designed for fast fading channels.

(a) Slowly fading channel

(b) Fast fading channel

Fig. 5. Performance comparison of concatenated orthogonal space-time MTCM code with AT&T smart-greedy space-time trellis code, with two transmit and one receive antennas ($R = 1$ b/s/Hz).

Fig. 6. Error performance of rate 1.5 b/s/Hz codes designed for generalized orthogonal transmit diversity systems with more than two transmit or one receive antennas. Codes are designed for slowly fading channel.

(a) 1 receive antenna

(b) 2 receive antennas

Fig. 7. Performance comparison of rate 1.5 b/s/Hz codes designed for generalized orthogonal transmit diversity systems in a slowly fading channel , with the outage probability.

(a) MTCM scheme ($R = 1$ b/s/Hz) (b) MLC scheme ($R = 2.25$ b/s/Hz)

Fig. 8. Error performance of (a) MTCM and (b) MLC designed for generalized orthogonal transmit diversity system with three and four transmit and single receive antennas, in a fast fading channel.

Fig. 9. Error performance of concatenated orthogonal space-time code with two transmit and one receive antennas, designed for a slowly fading multipath channel, and simulated using Jakes' model.