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THE RICE INSTITUTE

PERMANENT MAGNETIC MOMENTS OF A SUPERCONDUCTING SPHERE

by

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Most respectfully submitted

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I. INTRODUCTION

A. Basic magnetic properties of the normal-superconducting transition

Superconductivity was discovered by H. Kamerling Onnes in 1911,²⁶ as he investigated the behavior of the electrical resistance of mercury at very low temperatures. The effect appeared as a complete loss of electrical resistance at 4.15 degrees Kelvin. Since that time, many metals and alloys, each with its own characteristic transition temperature, have been shown to exhibit the phenomenon of superconductivity. The enormous research effort on this fundamental problem has been reviewed in such excellent monographs as "Superconductivity" by D. Schoenberg,³² and "Superfluids" by F. London.¹⁸

The existence of a critical magnetic field for the destruction of superconductivity was discovered in 1914.²⁷ At a given temperature, the superconducting state of a metal may be destroyed by the application of a sufficiently large magnetic field. The magnetic field required to restore the normal state properties of a superconductor is a function of the temperature, and the temperature dependence of this field is a characteristic of a given material.

From the fact that the electric field in a superconductor is zero, it follows from Maxwell's equation $\text{curl } \vec{E} = -\frac{\partial \vec{B}}{\partial t}$, that the distribution of the magnetic induction in a superconductor cannot depend on the time. The statement that the electric field is zero limits the value of the induction only in so far as the magnetic induction at a point in the body must remain constant in time. It was not until 1933 that it was shown that the transition into the superconducting state is accompanied by the expulsion of magnetic flux from the material, the Meissner Effect.²¹

At the entrance into the superconducting state, currents are generated as the surface of the superconducting material. These "supercurrents," which screen the interior, are not dissipated; they change, with changes in the external field, only such that the magnetic induction in the superconducting material remains zero. The penetration depth, or the thickness of the current layer, and the layer in which magnetic fields penetrate into the superconducting material, is of the order of 10^{-8} - 10^{-7} meters. A reasonable sized specimen may be characterized for many purposes by both zero electric field and zero magnetic induction.

The transition between the two phases of a particular superconductive specimen is not always a sharp transition. There usually exist values of the parameters, specifying the state of the specimen, for which the specimen is neither in the normal state nor in the superconducting state. Not only do the particular properties of a given specimen depend on the previous history of the sample, but the state assigned to the specimen will depend on how the state is defined (i.e., whether the superconducting state is defined as the state of zero magnetic permeability, or zero electrical resistance). Moreover, the general features exhibited in the transition of superconductive materials differ radically for the various kinds of superconductors, the elements, compounds, and mixtures.

The reasons for the ambiguities which arise when trying to characterize the normal-superconducting transition may be divided into two main classes. The first of these, geometrical effects, are conceptually simple but, in reality, lead to complicated results. Because of the screening currents which shield a superconducting specimen from external

fields, the magnetic field will not usually be homogeneous over the surface of the specimen. The parameters affecting the state of the material will not, except under special conditions, be constant throughout the specimen, and, the specimen as a whole cannot be characterized as being in either state. For special geometrical systems, the fields are mathematically tractable, but rather complicated models are required to account even qualitatively for the observed behavior of superconductors.

Even for the case of a cylindrical specimen in a uniform longitudinal field (where there is no demagnetization in the idealized case) the transition does not exhibit "ideal" features. Imperities (either chemical or physical, such as lattice defects) tend, in general, to broaden or smear out the transition, and alloys usually have much broader transitions than the elements. But even the most pure single crystal specimens show supercooling and superheating, where one phase may persist for values of the parameters such that only the other phase should be present. These matters will be considered further after the experiments of this dissertation have been described.

B. The state of the theory

The first successful theoretical approach to an understanding of superconductivity was made when F. and H. London²⁰ proposed that the field distributions in the presence of superconductors obey a system of differential equations in which Ohm's law, for the electrical currents, is replaced by the London equations: (1a) $\text{curl } (\lambda \vec{J}_s) = -\vec{B}$, (1b) $\frac{\partial}{\partial t} (\lambda \vec{J}_s) = \vec{E}$. $\vec{J}_s$, the "supercurrent" is that part of the total current which is coupled with the field according to the London equations;

currents which obey the usual relationship, $\vec{j} = \sigma \vec{E}$, are by definition normal currents, and Maxwell's equations are assumed to be valid everywhere. In the present formulation of this theory,¹⁶ the magnetic permeability and the electrical conductivity are regarded as not being essentially affected by the transition. Rather, the transition arises from the creation of (or the condensation of electrons into) new energy states, such that the effect of the fields on the electronic motions in these states gives the currents of the London equations.

No rigorous complete test of the London equations has yet been made. The solutions of equations I show, in general, an exponential decay of the field towards the interior of the superconductor.¹⁷ However, the actual penetration is so small that the solutions of equations I cannot be differentiated experimentally from other types of penetration (for example, a constant field to a small depth δ , and zero field beneath the shell of thickness δ). Moreover, it has not been shown that there exist in metals electron states which would lead to currents obeying the London equations. At present, the London theory presents a mathematically convenient representation for the fields in the presence of superconductors, but an understanding of superconductivity in terms of a fundamental theory of metals has not yet been obtained.

Several theories based on the fundamental properties of solids have been proposed, but the one which has attracted the greatest attention recently is the Fröhlich-Bardeen^{3,6} lattice vibration theory. In this theory, direct interelectron interactions are ignored, but it is assumed that the interaction of the conduction electrons in a metal with the lattice vibrations leads to the creation of energy states which, presumably, give the superconductive properties. The most striking

success of the theory is the inclusion of an isotope effect, a change of critical temperature with isotopic mass. However, it is still to be shown that the model gives rise to nondissipative currents, and to the Meissner effect.

Considerable progress has been made in understanding the normal-supereconducting transition from the thermodynamic viewpoint. In the Gorter-Casimir⁹ formulation of the theory, the work which is done against any external fields, in executing the Meissner effect, is associated with the change in the Helmholtz thermodynamic potential of the electron structure. From this potential, the differences in entropy, specific heat, etc., for the two phases of the material, may be calculated in terms of the magnetic quantities associated with the transition.

Although it is not necessary to assign the free energy change to a particular mechanism for a calculation of the thermodynamic functions, the two-fluid theory of Gorter and Casimir¹⁰ does assume that the energy difference between the two phases is caused by some part of the conduction electrons passing into new states during the normal-supereconducting transition. By a proper choice of the free energy associated with the condensed "superconducting" and with the uncondensed "normal electrons," it is possible to derive thermodynamic functions which agree closely with the experimental quantities. The choice is dictated solely by the requirement that the calorimetric quantities should agree with the magnetic quantities, and is not based on a consideration of fundamental properties. There are, however, discrepancies which are difficult to interpret in terms of simple two-fluid behavior, and research work on the validity of this model is being continued.^{8,32}

The thermodynamic relations between the magnetic and the differences (for the two phases) of the calorimetric quantities are satisfied closely, but the theory meets with less success when applied to the intermediate state of superconductors. The intermediate state refers to the state of the specimen when, because of the effect of the superconducting specimen upon the external field, the specimen can be neither wholly in the superconductive state, nor wholly in the normal state. The failings of the theory are particularly obvious for small specimens, specimens with at least one diameter 10^{-4} meters or less.

Landau¹⁶ first put forward a model for the intermediate state. He proposed that, in the intermediate state, the specimen breaks up into a kind of domain structure, each domain of which is either normally conducting or superconducting. By including in the calculation of the thermodynamic potential (in addition to the volume free energy of the Gorter-Casimir theory) a free energy term proportional to the interface surface area between the normal and superconducting regions, many of the deviations of small specimens from the macroscopic theory could be accounted for. However, the agreement is still only semi-quantitative.^{2,15}

C. The investigations of this dissertation

An exclusion from a superconductor of all magnetic induction (except in the penetration layer) is usually achieved in practice only when the specimen is first placed in the superconducting state in zero magnetic field. When the transition is allowed to occur with a magnetic field in the interior of the specimen, the sample in the superconducting state may retain some of the magnetic flux. Indeed, for some 20 years the existence of "frozen in flux" prevented the recognition of the idea

that, in the "pure" superconducting state, both the electric field and the magnetic induction are zero.

Figure 1.1 depicts schematically the behavior, for two different conditions existing at the time of the transition, that a sphere would be expected to exhibit if the transition into the superconducting state were a transition to a state where only the electric field is zero. If the transition is effected with a magnetic field in the body (1.1a) then currents in the superconducting sphere will keep the internal induction constant when the external field is removed (1.1b). If the transition is made in zero field (1.1c), then external fields are prevented from later penetrating the sphere by surface currents, and the state shown in (1.1d) is reached. Figure 1.1d is now generally regarded as the state characteristic of a "pure" superconductor in a magnetic field, regardless of the history of the specimen. Actually, physical specimens exhibit to some extent the conditions portrayed by 1.1b, but no adequate understanding of the factors affecting this phenomenon is available.

The experimental study of the residual flux retained by a pure, polycrystalline tin sphere is reported here. The apparatus and techniques used in the first measurements (the torsion measurements) are the outgrowth of a number of experiments performed at The Rice Institute by W. V. Houston and his students.^{11,31} Their studies were on the pure superconducting state with a perfect Meissner effect, and great care was taken to avoid the existence of "frozen flux."

The first important questions that needed to be answered by the experiment were: Does the residual flux behave as a reproducible function of the parameters of the transition? If so, what is the dependence on temperature, magnetic induction, speed of the transition, specimen

orientation, etc.? Is the frozen flux to be found at the center along the polar axis, or otherwise? Is the frozen flux really constant in time, and, is it affected by perturbations (say, thermal or mechanical) of the superconductor? How is the frozen flux related to the mechanical forces which may be exerted, through the field, upon the superconducting sphere? How does this behavior fit in with the present concepts of the superconducting state, and of the phase transition? A modest beginning to answering some of these questions has been made.

II. EXPERIMENTAL ARRANGEMENT

Two cryogenic arrangements have been used to obtain the results to be described in part III. The first of these, a torsion pendulum, detected the magnetic moment of a superconducting sphere by means of the torque exerted on the sphere in the presence of an external magnetic field. The second set of equipment measured the frozen flux by measuring the charge transferred through a set of "flip" coils. The two different arrangements will now be described.

A. Cryogenic apparatus

A cross section of the torsion pendulum is shown in figure 2.I. The torsion system was suspended in a gas tight volume bounded by the helium devar, H, the main plate, F, and the brass tube, D. There were openings in plate F for the connection of pumps, manometers, and filling tubes.

A brass rod, A, passed through the stuffing box, B, and served as the torsion head. The height of the pendulum could be adjusted by the moving rod, A, and the stuffing box and torsion head, as a unit, could be rotated to any desired angular position.

The torsion fiber, C, was soft soldered into the lower end of rod A, and the fiber extended to the lower end of tube D. The torsion fiber was a tungsten wire .003 inches in diameter, and 0.9 meters in length. The lower end of C was soft soldered to a brass screw set into the lucite cylinder, E. The solder did not bond directly to the surface of the tungsten, so that the fiber was not connected rigidly to the supports. However, the fiber did not slip in its supports sufficiently to influence these measurements.³⁰

The lucite cylinder had an octagonal cross section, and a mirror was waxed to each of the eight faces of E. The motion of the pendulum was observed through windows in tube D opposite the mirrors on E. The distance from the mirrors to the scale used to observe a hairline was 3.9 meters.

A glass tube, G, was attached rigidly to the lucite cylinder, and the tube extended to the lower part of the helium dewar. At the bottom of the glass tube, there was a lucite support, L, for the sphere M.

Surrounding the sphere, and inside the helium dewar, was a hollow glass cylinder, I, which was open at the top. The cylinder I separated the sphere from the helium bath, J, so that bubbling in the bath would not disturb the motion of the pendulum. Lucite washers were placed inside of cylinder I to prevent violent circulation of gas in the tube.

A second dewar, K, for liquid nitrogen, was set around the double walled vacuum jacket of the helium dewar. A pair of Helmholtz coils was placed outside the nitrogen dewar, and the axis of the coils was aligned with the horizontal component of the earth's field. In the measurements with the torsion pendulum, the vertical component of the earth's field was not compensated.

A second cryostat for studies on the residual flux is shown in figure 2.2. This gear was used to measure the frozen in flux directly, and to obtain its distribution. The gas tight helium volume was bounded by the main plate, F, and the helium dewar, J. There were openings in plate F for connection of pumps and manometers, for the insertion of liquid helium, and for the emergence of lead wires from the detection coils.

Mounted on the main plate F was a stuffing box, E, through which

passed a steel rod, G. The rod G could be rotated from outside the dewar, and a "stop" mounted on the plate controlled the angle of rotation. The lower end of rod G was fastened to a textolite rod I, which extended down into the helium bath and was clamped to the coil supports, N. Rotating the rods spun the detecting coils, K, about the sphere, C. The sphere and sphere support, D, were suspended from the plate by means of steel tubes, H, and the textolite tube, B, which surrounded rod I.

A cross section (actual size) of the coil and sphere arrangement is shown in figure 2.3. The sphere was supported by the aluminum stand, D, which rested on the textolite plate P. The aluminum coil forms, K, were mounted on an aluminum strip which fitted around the stand D, so that the coils rotated about a central axis which passed through the center of the sphere.

There were two sets of three detection coils each. The coils marked L, M, and S, in figure 2.3 had mean diameters of 1.022, .820, and .590 inches, respectively. The coils were wound with No. 48 formex insulated copper wire, and each of coils L, M, S, had 225, 254, 375 turns, respectively.

As with the torsion pendulum, a liquid nitrogen dewar and a set of Helmholtz coils, for producing horizontal fields, were placed outside the helium dewar. In some of the work, where it appeared that the vertical component of the earth's field was introducing complications, a large solenoid surrounding the nitrogen dewar was used to compensate the earth's vertical field.

B. Auxiliary equipment and operational procedure

The two cryostats were initially prepared in the same way. The inner helium dewar was flushed with pure helium, and the gas cylinder

was left connected to the dewar to maintain atmospheric pressure during the initial cooling. The double walled vacuum jacket was then flushed and left filled with several centimeters of air.

The precooling operation was accomplished by filling the outer dewar with liquid nitrogen. Thermal contact between the inner dewar and the nitrogen bath was maintained by the exchange gas in the vacuum jacket, and, in less than two hours, the experimental equipment had cooled to the temperature of the nitrogen bath (77°K.). The exchange gas was then pumped from the vacuum jacket, and liquid helium was introduced into the inner dewar.

For both cryostats, the temperature was controlled by regulating the vapor pressure above the helium bath. Since the specimen was tin, and the zero field transition temperature of tin is 3.73°K. , the required vapor pressures were below .47 meters of mercury. To attain these pressures, a vacuum pump was, in the first measurements, connected to the experimental dewar through a Wallace and Tiernan, type PA-149001 manostat.

Although this manostat is supposed to maintain a constant pressure independent of variations in either the pumping rate or the gas source, drifts of the pressure of the order of several millimeters of mercury were observed over the course of a few hours. The results for the lower temperatures (below 3.2°K.) appeared quite insensitive to these small temperature variations ($.008-.010^{\circ}\text{K.}$).

For the measurements between 3.2° and the transition temperature, however, it was found necessary to control the temperature more closely. To achieve greater control of the pressure, an oil differential manometer was connected to the mercury manometer used to measure the vapor pressure. Using the differential manometer continuously to correct the setting of

the manostat, the temperature of the bath should have been constant to within two millidegrees.

Despite these precautions taken to control the vapor pressure, it was found in measuring the critical field for the sphere, at temperatures above 3.70° , that the magnetization curves were not smooth. To correct this, the manostat was replaced by a sensitive automatic pressure control, the design and construction of which was carried out by Mr. Frank Jones, as a senior thesis, at The Rice Institute. The regulator consisted of two pumping lines placed in parallel, with the constrictions in the two lines adjusted so that only a small portion of the gas flowed through the smaller line. An electric, open-closed valve was placed in the smaller line, and the switch used to actuate the valve was a mercury differential manometer. As the pressure inside the cryostat decreased, the valve closed off the smaller line, and as the pressure increased the valve opened the small line. Under the conditions of these experiments, the vapor pressure over the bath oscillated by less than a millimeter of mercury, with a period of 4-5 seconds. In appendix A, it is shown that this oscillation should not have affected the temperature of the sphere. It is believed that the temperatures for runs 29-35 were constant to within one millidegree.

The Helmholtz coils used to generate the magnetic induction for these experiments produced 11.1×10^{-4} webers-meters⁻² per ampere of current. The induction was uniform over the volume of the sphere to within .1%. Currents of less than six amperes were supplied by a set of Edison cells, and currents of six to twenty amperes were supplied by a D.C. generator.

Inductions up to 15×10^{-4} w-m⁻² could be measured and controlled

to within $.01 \times 10^{-4}$ w-m⁻². Inductions between 15 and 65×10^{-4} w-m⁻² could be held constant to $.5 \times 10^{-4}$ w-m⁻², and inductions from 65 to 220×10^{-4} w-m⁻² could be controlled to within 1×10^{-4} w-m⁻². Because of heating in the coils, inductions greater than 150×10^{-4} w-m⁻² could not be maintained longer than 5-10 minutes.

The specimen used in this work was cast in a vacuum from spectroscopically pure tin. The rough sphere was then machined to have a diameter of $1.0000 \pm .0004$ inches. Despite the care taken to obtain a perfect sphere, the sphere showed magnetic effects characteristic of departure from spherical symmetry. If the sphere is approximated by a prolate ellipsoid of revolution, then the ellipsoid will experience a torque (caused by the asymmetry of the eddy current distribution) when a uniform magnetic field, which is neither parallel nor perpendicular to the axis of revolution, is changed. At 4.2° , the conductivity of the tin was so large that these torques were easily observable when an induction of 15×10^{-4} w-m⁻² was suddenly switched on. The orientation of the sphere was always adjusted, before the temperature was lowered below T_c , so that these torques showed a stable minimum, corresponding to the long axis being parallel to the external field. This orientation of the sphere and external magnetic induction is referred to as the position of stable equilibrium.

The results for runs 1-14 (except for runs 11 and 12 with another sphere) are for the "machined sphere." Tin is, of course, quite soft, and the effect of the machining was to produce an amorphous, or noncrystalline, layer on the surface of the sphere. Under a low-power microscope the surface appeared quite smooth, except for several tiny scratches. In order to remove the amorphous layer left by the shaping process, the

sphere was electropolished using the procedure developed by Jaquet.¹² After polishing, the surface of the sphere could be seen to be composed of small crystallites several tenths of a millimeter in diameter. Runs 15-35 were with the "polished sphere".

The measurement of the magnetic moment with the torsion pendulum was accomplished by measuring the torque exerted on the sphere per unit of applied magnetic induction. The torque on the sphere was measured either by observing the period of oscillation of the pendulum, or by observing the center of oscillation of the system. The equation of motion of the pendulum is

$$I \frac{d^2\theta}{dt^2} = -R \frac{d\theta}{dt} - K\theta + \tau_m$$

where I is the moment of inertia of the system, θ is the angular coordinate, t is the time, R is a resistive term, K is the torsion constant of the fiber, and τ_m is the torque due to magnetic forces. To a first approximation, the sphere behaved like a magnetic dipole in a uniform external field, so that the torques on the sphere caused by the permanent magnetic moments were given by

$$\tau_m = -MB \sin\theta$$

where B is the uniform, applied external magnetic induction, and M is an effective dipole moment.

Since all angular displacements used in the period measurements were less than .035 radians, the sine of θ may be replaced by θ with an error of less than one part in 4000, and the equation of motion may be rewritten as

$$\frac{d^2\theta}{dt^2} + 2\delta \frac{d\theta}{dt} + (\delta^2 + \omega^2)\theta = 0$$

where δ and ω are defined by

$$2\delta = \frac{R}{I}$$

$$\omega^2 = \frac{\kappa + MB}{I} - \frac{R^2}{4I^2}$$

A solution of the equation appropriate for this problem is

$$\theta = \theta_0 e^{-\delta t} \sin \omega t$$

Since the damping constant, δ , was very small ($\sim 2 \times 10^{-4}$ seconds $^{-1}$) for the sphere in the superconducting state, it was possible to measure the "period", T , of the pendulum, and to calculate the moment M from

$$\omega^2 = \frac{4\pi^2}{T^2} = \frac{\kappa + MB}{I} - \frac{R^2}{4I^2}$$

$$M = 4\pi^2 I \frac{d(T^{-2})}{dB}$$

The measured periods were of the order of 20 seconds, with a probable error of less than .02 seconds (usually ten oscillations were timed). Thus, the values of T^{-2} should be accurate to within .2%. The moment of inertia was determined to within .3% by using the sphere as a standard body of known moment of inertia. The ratio of the squares of the periods with and without the sphere in place is the ratio of the moments of inertia for the two cases, and since the moment of inertia of the sphere is known, the moment of inertia of the whole system may be calculated.

A typical set of measurements of T^{-2} plotted as a function of B is shown in figure 2.4a. (The values of all the measured quantities are given here in rationalized M.K.S. units.) The values of B were calculated

from the current in the Helmholtz coils, and they represent the homogeneous induction which would be present if the sphere (which is in the superconducting state) were nonmagnetic. The solid curve in the figure is a quadratic function of B which has been fitted to the experimental points by the method of least squares. The coefficient of the linear term in B is used as $\frac{d}{dB}(\tau^2)$ in the calculation of the effective dipole moment M .

The torque on the sphere was also measured by observing the change in the equilibrium position of the sphere as a function of the applied induction. For these measurements, the sphere was turned so that the axis of the moment was perpendicular to the axis of the external field. It is believed that the setting could not have been off by more than .1 radians, an error which would yield a moment .4% smaller than the correct value.

For this arrangement the torques caused by the magnetic forces were

$$\tau_m = -MB \sin\left(-\frac{\pi}{2}\right) = MB$$

and the equation of motion may be written as

$$\frac{d^2\theta}{dt^2} + 2\delta \frac{d\theta}{dt} + (\delta^2 + \omega_0^2)\theta = \frac{MB}{I}$$

where δ and ω_0 are

$$2\delta = \frac{R}{I}$$

$$\omega_0^2 = \frac{\kappa}{I} - \frac{R^2}{4I^2}$$

The solution appropriate for this case may be expressed as

$$\theta = \frac{MB}{K} + \theta_0 e^{-\delta t} \sin \omega_0 t$$

so that the moment is given by

$$M = K \frac{d\theta}{dt}$$

The equilibrium position of the pendulum was determined from the end points of three oscillations. A typical plot of the center of oscillation against the magnetic induction is shown in figure 2.4b. The centers were reproducible to within .2 millimeters, and the points usually lay so close to a straight line that the slope was determined by a direct measurement from the graph. The calculation of the moment required a knowledge of the torsion constant for the system, but this was obtained from the same measurements which yielded the moment of inertia.

The circuit diagram for the detection coils of the second cryostat is shown in figure 2.5. The six individual coils were connected to the switch S, such that any of the coils could be put in series with the galvanometer. Since the external critical damping resistance of the galvanometer was much greater than the resistance of the coils at 4° , the resistances R_1 and R_2 were chosen such that the galvanometer was critically damped for any desired sensitivity.

The coils were always calibrated at 4.2° before each run. Figure 2.6 shows a typical calibration curve of the galvanometer deflection as a function of the average induction over the area of coils. The galvanometer deflection is the ballistic throw observed when the coils were rotated rapidly through π radians; the average induction was calculated from the current in the Helmholtz coils. Curves labeled 7,8,9, refer,

respectively, to the two L, M, S, coils connected in series. The ballistic throws were reproducible to within $.2 \times 10^{-3}$ meters for deflections less than .1 meters, and within $.5 \times 10^{-3}$ meters for deflections greater than .1 meters. An average induction greater than $.236 \times 10^{-4}$ w-m⁻² (the earth's horizontal induction) could be measured to better than 1%, and inductions greater than $.020 \times 10^{-4}$ w-m⁻² to better than 3%.

Each of the two cryostats had its inherent advantages and disadvantages for the measurement of the flux in the sphere, and the advantages of one were very nearly the disadvantages of the other. The torsion pendulum had the advantages of very high precision and accuracy, and that there was no magnetic material in the vicinity of the specimen. However, the measurements were lengthy and tedious, the sphere was not in direct thermal contact with the constant temperature bath, finite (although small) fields were required to make the measurements, measurements could not be made except in the superconducting state (because of eddy current damping in the normal state), and the measurements did not yield the magnetic flux directly.

The flux coil cryostat avoided the drawbacks of the torsion pendulum, but here the accuracy was lowered. Moreover, the sphere was very close to the aluminum stand and coil forms which, even if they were non-magnetic, could distort the field during field changes. Even though the flux threading the coils could be measured for any value of the applied induction, it was difficult to estimate the part of this which also threaded the sphere. However, there is no indication in the results that any serious difficulty arose from these sources.

III. EXPERIMENTAL RESULTS

In order to understand the experimental results better, it may be recalled that the magnetic field-temperature phase diagram for a superconductive sphere of tin is as is shown in figure 3.1. At the critical temperature, T_c , the sphere becomes superconducting in zero magnetic field. For temperatures below T_c , there is a critical induction, $B_c(T)$, such that the tin is stable in the superconducting state for inductions less than B_c .

The demagnetizing effect of the diamagnetic superconducting sphere introduces into the phase diagram a third region, called the intermediate state. This is the region $\frac{2}{3}B_c < B < B_c$ in figure 3.1; for this range of B the sphere can be neither wholly normal conducting, nor wholly superconducting. In the intermediate region, the sphere breaks up into a fine grained mixture of normal and superconducting domains (part 4).

Any point of the phase diagram may be reached by many paths, but only two methods of traversing the intermediate state will be discussed here. The first type of transition, called the "constant temperature transition," is represented by path 1 in figure 3.1; the second type, the "constant field transition," is represented by path 2 in the figure.

A. Torsion measurements

The constant temperature transitions were effected by setting the temperature at some value below T_c , applying an induction greater than $B_c(T)$, and then removing the external field. Some of the magnetic flux remained in the sphere, and so gave rise to a magnetic moment. The magnetic moment was measured by observing the torque on the sphere, for various small, external measuring induction fields. Figure 3.2 shows the

magnetic moments of the sphere for constant temperature transitions, and the data for each of the points are given in Table 3.1. Each plotted point represents several hours of precision measurement. The abscissa in figure 3.2 is the temperature at which the moment was frozen in. The dotted curve in the figure gives the moments for the machined sphere, and the solid curve through the crosses shows the moments obtained with the polished sphere. The two curves are similar at the higher temperatures, and much of the work was concentrated there.

The magnetic moments frozen in the sphere were permanent moments, with the magnetic flux tied to the crystal lattice. They were maintained by supercurrents which did not decay with time. That the induction was fixed in the sphere is shown by the magnetic torques exerted on the sphere, since these torques would exist, and would change when the sphere was rotated, only if the flux rotated with the sphere. This is in contrast to the superconducting screening currents which give rise to the Meissner effect, since this current distribution (and the field distribution) must be stationary with respect to the external fields as long as the axis of rotation is an axis of rotational symmetry for the specimen.

The magnetic forces exerted on the sphere increased the restoring torque on the pendulum when the external magnetic induction was in the same direction as the frozen field. This is such that the permanent internal induction in the sphere had the same direction as the field used to put the sphere into the normal state. As closely as could be determined from the measurements, there was no more than .07 radians difference in orientation between the horizontal component of the moment (which is all that could be seen with the pendulum) and the original external magnetic field used to produce the frozen flux.

When the temperature of the sphere was held constant, the moments were constant in time. Occasionally there appeared to be small moment changes during the first twenty minutes after the transition was completed. At least thirty minutes were required to make a complete determination of the moment and, in a few cases, the first readings would indicate moments larger (never smaller) than the equilibrium values. However, after twenty minutes, the torques remained constant and reproducible for as long as four hours (the longest they were observed).

In connection with the preceding paragraph it may be remarked that, when the torsion pendulum was moved just prior to making an experiment, the period of the pendulum in the absence of magnetic torques was observed to change for two to three hours. This change probably was caused either by the torsion fiber shifting in its supports, or by a relaxation of some strain in the fiber; since the change with time was regular, it was possible to correct the observed periods for this effect. However, the corrected periods also showed, in the first few minutes, deviations from the equilibrium values, so that it is believed that the observed changes were caused by rearrangements of the flux in the sphere, rather than by mechanical disturbances of the apparatus.

During the transition, two methods were used to clamp the sphere in a definite position. The first of these methods was simply to allow the torsion system to sit upon a "rest," and the second was to use the eddy currents of the normal state as an electromagnetic damper. The conductivity of the normal state tin was so large at these temperatures that the pendulum was critically damped in a horizontal induction of 5×10^{-4} w-m⁻².

Raising and lowering the pendulum for mechanical clamping of the sphere introduced vibrations which caused considerable delay in the experi-

ments. Since it was found that omitting this procedure did not affect the moments, the process was not used when the external field was removed rapidly.

The speed with which the external field was removed was varied over a wide range. The fastest removal of the external field sources (corresponding to opening a switch in the Helmholtz coil circuit) was of the order of 5×10^{-3} seconds. Moments also were frozen in where the external field was slowly lowered over a time greater than 9000 seconds. When the field was not removed by cutting the coil current, the sphere was clamped mechanically.

Figure 3.3 shows the high temperature region of figure 3.2 (for the polished sphere) with an expanded abscissa scale. The circled points are moments which were frozen in by lowering the field coil current slowly, and the crosses are moments frozen in by cutting the current. There is no apparent distinction between the moments for the two cases.

The moments through which the curves of figures 3.2 and 3.3 are drawn were all frozen in along the axis of stable equilibrium for the sphere. Table 3.2 gives a comparison of moments frozen in both parallel and inclined to the axis of stable equilibrium, for various temperatures. The differences of the moments, for different orientations in the sphere, at temperatures near T_0 (3.68 and 3.69°K.) were greater than the scatter of moments for a given temperature and a single orientation; but at 3.6, 3.2 and 2.4°K., the differences of the moments for different orientations were no greater than the differences that were obtained for successive moments with the same orientation in the sphere.

As remarked above, the magnetic moments frozen in the sphere were constant as long as the temperature was held constant. In fact, this is

one of the strongest arguments for the nondissipation of supercurrents. However, the frozen flux did change when the temperature of the sphere was changed.

Table 3.3 gives the moment changes for various temperature changes, and these results are shown graphically in figures 3.4. The ordinates in the figures are the moments, and the abscissas are the temperatures at which the moments were measured. The procedure in these measurements was to freeze in a moment at a given temperature, measure the moment, and then to change the temperature and remeasure the moment. The lines connecting the points in figures 3.4 represent only the sequence in which the measurements were taken, and not intermediate values.

Figures 3.4A and 3.4B show the values of the moments obtained, for the machined and the polished sphere respectively, when the moment was frozen in at constant temperature, and then the temperature of the sphere lowered. Since the critical field increases with decreasing temperature, and since all external fields were small, the sphere must have remained in the superconducting state during these temperature changes.

Figure 3.4C shows the preliminary studies on the moments obtained when the temperature of the sphere was raised. The designations "plus" and "minus" are arbitrary classifications of the direction of the moment. All horizontal fields were aligned with the earth's field; positive quantities were parallel to the earth's field, whereas negative quantities were antiparallel.

Although the decreasing temperature moments exhibited a regularity of behavior, the moment changes for increasing temperatures seemed to be unpredictable. It is believed that these apparent inconsistencies were caused by the sphere not remaining completely in the superconducting state

during the temperature changes, and that more careful work is required.

The evidence to support this conclusion is, first, the difference in the state of the constant temperature bath during the processes of raising and lowering the temperature. When the temperature is being lowered, bubbling and convection currents will keep the bath well stirred. In the process of raising the temperature however, large temperature gradients may exist for long periods of time. Moreover, the strong possibility exists that, because of the construction of the Wallace and Tiernan manostat, some air may have entered the system when this device was used to increase the pressure. The sphere was exposed to the helium atmosphere, and since the heat capacity of the sphere was small, there is the possibility of warm gas heating the sphere.

The sequence of events leading to the moment reversals also is suggestive that the reversed moments were really newly frozen in moments. The reversed moment of run 3, M3.7, occurred in the following manner: after the vapor pressure was changed, a period was measured with an induction of $-2 \times 10^{-4} \text{ v-m}^{-2}$, and this period indicated a zero, or small positive, moment; only after the application of the negative induction did the sphere exhibit the negative moment M3.7. The reversed moment M4.5b occurred as follows: after the vapor pressure was increased, the first ten periods gave M4.5a, which is close to M4.4; following a period measurement with $B = -1.8 \times 10^{-4} \text{ v-m}^{-2}$, the measurements suddenly gave M4.5b. It was not recorded whether the temperature change between M6.5 and M6.6 was made in a negative field, or in the earth's field.

Moment M8.2 was obtained from the zero moment M8.1 by applying an induction of $1.8 \times 10^{-4} \text{ v-m}^{-2}$, and then raising the vapor pressure in the usual manner. The value of M8.2 is very nearly the moment obtained when

the sphere was cooled in an applied induction of 1.8×10^{-4} v-m⁻² (M7.1, table 3.4). This, together with the evidence of the preceding paragraphs, makes it seem certain that the moment reversals obtained when raising the temperature were caused by the temperature of the sphere rising above T_c .

The changes of the moment for small temperature variations are shown in figure 3.4D. Here it is seen that the precise moment changes were somewhat random, and were not reproducible to better than a few percent.

The determination of the moments frozen in by the constant field method was complicated by the moment changing when the temperature was changed. The constant field transitions were effected by applying a given magnetic field (with the sphere above the transition temperature), lowering the temperature until the sphere was superconducting, and then removing the external field. Unless the critical field for the temperature at which the moment was measured was just $\frac{3}{2}$ the applied magnetic field, then there should have been a temperature change inside the superconducting state.

The moments frozen in by the constant field method are given in table 3.4. The temperatures given in the table are the temperatures at which the moments were measured, and the transition temperatures quoted for the applied field were taken from the flux coil measurements (figure 3.6). A comparison with figure 3.3 shows that all the moments, except M6.1, were roughly equal to moments frozen in at the temperature corresponding to the critical temperature for the applied magnetic field; M6.1 was transported across the region where the moment changes rapidly with temperature changes.

B. Flux measurements

Except for the method of making the measurements, the handling of liquid helium and the magnetic fields was essentially the same (for the

flux coil cryostat) as for the operation of the torsion pendulum apparatus.

The flux coils were used to measure the magnetization curve for the specimen. These measurements were made primarily to determine the critical field of the specimen for temperatures near T_c . Measurement of the magnetization curves consisted in removing all fields from the interior of the specimen when the temperature was above T_c ; the temperature was then lowered to the value at which the measurement was to be made (so that the sphere had zero frozen flux). Then, the flux threading the sphere was measured as a function of the applied magnetic induction.

Figure 3.5 shows a double cycle magnetization curve for the specimen at 3.711°K . The ordinate, B_L , is the average induction over the two L coils, and the abscissa, B , is the magnetic induction calculated from the current in the Helmholtz coils; the data for these points are given in table 3.5. Since the transition was made in zero field, the sphere had zero frozen moment, and the curve started from zero applied induction with zero average induction in the coils. When a small induction was applied, the sphere was completely in the superconducting state and, because of its large size, distorted the external field as would a perfect diamagnetic sphere. The measured flux is just the leakage flux between the sphere and the coils. From the geometry of the sphere and the coils, the initial slope of the curve was calculated to be .28 (appendix B), and the measured value is .31.

When the applied induction reached two-thirds of the critical induction for the sphere, the induction at the magnetic equator of the sphere was equal to the critical induction. For further increases of the induction, the sphere could not remain wholly superconducting. The sphere entered the intermediate state for fields of induction greater than $\frac{2}{3} B_c$ and

less than B_c ; according to the zero order theory (which neglects both penetration and surface energy effects) the slope of the curve for the intermediate state should be constant.¹⁸ For the particular sphere and coils used here, the slope in the intermediate state should be 2.44; the measured value from the curve is 2.22.

Actually the curve does not show the sharp changes in slope to be expected at the entrance into, and exit from, the intermediate state, nor is the curve linear in this region. The critical fields were determined by extrapolating the linear portions of the curves, and the intersection point (A in figure 3.5) was taken as two-thirds the critical induction for the given temperature. The values of the critical induction measured this way are given in table 3.6, and the critical induction is plotted as a function of temperature in figure 3.6. From the curve

$$T_c = 3.733 \text{ } ^\circ \text{K}$$

$$\left(\frac{dB_c}{dT}\right)_{T_c} = -147 \times 10^{-4} \text{ webers-meter}^2\text{-degrees}^{-1}$$

B. Serin, who investigated the critical field in connection with the isotope effect, gives (for "natural" tin) the following values*

$$T_c = 3.740 \pm .009 \text{ } ^\circ \text{K}$$

$$\left(\frac{dB_c}{dT}\right)_{T_c} = -147 \pm 2 \times 10^{-4} \text{ webers-meter}^2\text{-degrees}^{-1}$$

The constant temperature transitions were effected in these experiments just as they were in the torsion pendulum experiment. Tables 3.7 and 3.8 give the flux frozen in at constant temperature, and figures 3.7 and 3.8 show these measurements of the flux plotted as a function of the temperature of the transition. The quantities B_0 , B_1 , B_2 , B_3 were

*Private communication with Dr. C. F. Squire

calculated from the observed values of the average induction over the L, M, S coils, using the method described in Appendix B. B_0 is the average induction over the whole cross section of the sphere. If r is the radius in the equatorial cross section, then B_1 is the average induction for the circle $r \leq .622 \times 10^{-2}$ m., B_2 is the average induction in the annular ring $.622 \times 10^{-2} < r \leq 1.04 \times 10^{-2}$ m., and B_3 is the average induction in the ring $1.04 \times 10^{-2} < r \leq 1.27 \times 10^{-2}$ m.

In constructing figure 3.8, only the measurements from experiments 30-35 were used at temperatures above 3.66°K . This was done because, in experiments prior to number 30, the vertical component of the earth's field was not compensated.

The values of the flux frozen in at constant field are given in table 3.9, and the measurements are shown plotted as triangles in figure 3.8. The temperature at which the points are placed in figure 3.8 are the critical temperatures for the applied inductions. Although these measurements of flux fit the constant temperature curve well, the effect of the flux changes with changing temperature, and the rather large scatter in the values of the flux makes it impossible to tell whether the constant field flux corresponded to $T_c(B_0)$ or to $T_c(\frac{3}{2} B_0)$ (i.e., to the points A or B in figure 3.1).

The flux change for changes of the temperature are shown in table 3.10, and figures 3.9. The average induction over the whole sphere is given as a function of the temperature in figure 3.9A. Figures 3.9B and C give the average induction in each of the three rings, as a function of the temperature at which the flux was measured, for two different experiments. The residual frozen flux at the lower temperatures, which did not change with temperature, was contained mostly in the interior of the

equatorial cross section of the sphere. There was very little flux near the equatorial surface.

The usual method of specifying the completeness of the Meissner effect is to give the "percent" of residual flux. This percent is defined at 100, times the ratio of the flux frozen in, to the minimum flux for which the specimen can remain completely in the normal state (i.e., to the flux threading the specimen when the induction is uniform and equal to the critical induction). Since the frozen flux is expressed here as the average induction over the cross section of the sphere, the percent of frozen flux is 100, times the ratio of the average induction to the critical induction. The critical inductions quoted in Tables 3.8 and 3.9 are, for temperatures above 3.68°K. , taken from figure 3.6. The critical induction for temperatures below 3.68°K. , where the value is rather insensitive to the particular value of T_c , was calculated using an empirical formulae³² (figure 3.1).

The temperature dependence of the percent of frozen flux, for $3.60 \leq T < T_c$, is shown in figure 3.10, where the percent is plotted as a function of the temperature at which the flux was frozen in. It should be emphasized that the sphere showed a very good Meissner effect; in the neighborhood of 3.3°K. , others workers commonly report frozen fluxes of 2-10%, whereas the sphere had only .1%.

The crosses shown in figure 3.10 are the percents of flux measured in experiments prior to number 30, whereas the circles are the measurements from runs 30-35. The triangles are percents of flux frozen in at a constant field B_0 , and the temperature at which they are placed is $T_c(B_0)$.

The magnetic properties of the sphere may be summarized as follows:

When the temperature of the sphere is held constant, the internal

induction of the sphere is a smoothly varying function of applied induction. The curve of internal induction as a function of external induction has three linear regions, each with its characteristic slope. The transition between the three regions is not sharp at high temperatures; the linear regions are joined by curved portions which extend over large ranges of the induction. The slopes for increasing fields and the slopes for decreasing fields are each reproducible; however, the slopes for decreasing fields are not the same as for increasing fields.

When the transition is made with a magnetic induction in the specimen, the superconducting sphere retains a part of the magnetic flux. The residual flux in the superconductor is fixed in the spherical specimen, and the flux is constant in time as long as the temperature of the sphere is held constant. The frozen magnetic flux can interact with external magnetic fields to produce a torque on the sphere.

The amount of flux frozen in the sphere shows a characteristic temperature dependence. The values of the flux are reproducible to within about 15% at any one temperature, and the residual flux seems not to be influenced by the speed with which the transition is made (for times of removing the external fields down to 5 milliseconds). Below 3.6 degrees, the frozen flux is independent of the orientation in the sphere to within the reproducibility of the measurements; at 3.68 degrees, the value of the frozen flux may change by as much as 30% with the orientation of the sphere and moment. An amorphous layer on the surface caused the sphere to freeze in much larger moments below 3.4 degrees.

The amount of residual flux shows a rapid increase (with decreasing temperature) near the critical temperature. The frozen in flux has a sharp peaked maximum .03-.04 degrees below T_c , and a broad minimum extending from 3.1 to 3.3 degrees. The frozen flux continues increasing,

with decreasing T , from 3.1 down to 2.3 degrees. An amorphous layer on the surface of the sphere sharpened the minimum and shifted it to a slightly higher temperature (although the minimum moments were equal).

The sphere shows a very good Meissner effect, only .1% of the flux being frozen in below 3.5 degrees. With increasing temperature the Meissner effect becomes progressively poorer until, for a critical induction of $.2 \times 10^{-4} \text{ w-m}^{-2}$, some 60% of the flux is frozen in the sphere.

The distribution of flux over the cross section of the sphere is not homogeneous, nor is either the flux or its distribution constant when the temperature is changed. Below 3.1 degrees the average induction is greatest at the edge of the equatorial cross section; with increasing T , the average induction first becomes concentrated near the center of the cross section, and, at about 3.71 degrees, becomes homogeneous. When the temperature of the sphere is lowered inside the superconducting region, the frozen flux decreases to a constant value, with most of the flux at the center of the cross sectional area.

IV. DISCUSSION

In section A, the relationship of the measured torques and fluxes will be discussed, and the overall consistency of the torque and flux measurements will be examined. In section B, these results, and the results for the amount of flux frozen in, will be examined in terms of the present knowledge and concepts of the structure of superconductors which have an internal induction.

A. The spherical specimen is assumed to occupy a volume over which a field that is uniform, and equal to B_0 when the sphere is in the normal state, may be established.

The presence of a superconducting sphere distorts the field very nearly as would the presence of a perfect diamagnetic sphere. The solution of the field for this case is well known, and may be obtained by placing a current sheet, of surface density $(I_1 \sin \theta)$, on the sphere $r = R$. R is the radius of the specimen; I_1 is equal to $-\frac{3B_0}{2\mu_0}$.

If the permanent magnetic induction is assumed to be uniform inside the sphere, then the permanent frozen induction may also be represented as produced by a spherical current shell of radius R . The current density here is $(I_p \sin \theta')$, with I_p equal to $\frac{3B_p}{2\mu_0}$. B_p is the uniform frozen magnetic induction inside the sphere, and (r', θ', φ') are spherical polar coordinates with the polar axis z' lying in the $y - z$ plane and inclined at an angle α with the polar axis of (r, θ, φ) (figure 4.1).

The external field sources define a unique direction (the direction of B_0) which is fixed, so that all quantities may be specified by their

values in the (r, θ, ϕ) coordinate system. The sphere and torsion pendulum also define a unique direction, and it is assumed that the permanent flux is embedded rigidly in the (r', θ', ϕ') system. The current distribution I_1 , although embedded in (r', θ', ϕ') system, is specified with respect to the (r, θ, ϕ) system. The experiments on the damping,¹¹ and the gyromagnetic ratio,^{14,31} for a superconductor, indicate that the current shell I_1 may, in the absence of frozen flux at least, slip freely over the surface of the sphere.

Assuming that far from the sphere the actual field of the permanent frozen flux has the same form as the dipole field given by B_p ,

$$B_{pr} = B_p \frac{R^3}{r^3} \cos \theta'$$

$$B_{p\theta} = B_p \frac{R^3}{2r^3} \sin \theta'$$

the torque on the sphere may be calculated by considering the work done in a virtual rotation of the external field sources, while holding constant the field produced by the sphere. The currents generating the external field B_0 may be represented as two current loops (one of which is shown in figure 4.1) of radii a , lying in the planes $z = \pm \frac{a}{2}$ with centers on the z -axis. I_0 is determined by the condition that I_0 must produce $\frac{B_0}{2}$ (since there are two loops) at $r = 0$. Hence

$$I_0 = \frac{(a^2 + \frac{a^2}{4})^{3/2}}{\mu_0 a^2} B_0 = \frac{R_0^3}{\mu_0 a^2} B_0$$

From the axial symmetry of the system, only the field of the frozen induction can exert a net torque on the currents, I_0 , in the loops. The torque exerted by the field B_p on a current element of I_0 is given by

$$\begin{aligned} d\vec{\tau} &= d\vec{r} \times \vec{F} = (d\vec{I}_0 \times \vec{B}_p) \times \vec{r} = (d\vec{I}_0 \cdot \vec{r}) \vec{B}_p - (\vec{B}_p \cdot \vec{r}) d\vec{I}_0 \\ &= -B_p r d\vec{I}_0 \end{aligned}$$

since $(d\vec{I}_0) \cdot \vec{r} = 0$. For the component of torque about the X-axis,

$$d\tau_x = B_p r_0 I_0 a \sin \theta d\theta$$

Transforming B_p to the (r, θ, ϕ) system, and summing over all the current elements gives, for the total torque on the external currents,

$$\tau_x = \frac{2\pi R^3}{\mu_0} B_p B_0 \sin \alpha$$

The work done during a virtual rotation (about the X-axis) of the currents I_0 through the angle $-\delta\alpha$, is given by

$$\delta W = -\tau_x \delta\alpha = -\frac{2\pi R^3}{\mu_0} B_p B_0 \sin \alpha \delta\alpha$$

If all the currents are maintained constant during the virtual displacement $-\delta\alpha$, then δW is equal to the decrease of the potential function, $-\delta U$. Since the final configuration is identical with what would be obtained if the sphere were given a virtual displacement $\delta\alpha$, it follows that

$$-\delta U = \tau^* \delta\alpha = -\frac{2\pi R^3}{\mu_0} B_p B_0 \sin \alpha \delta\alpha$$

where τ^* is the torque on the sphere. Hence, the torque on the sphere is

$$\tau^* = -M^T B_0 \sin \alpha ; \quad M^T = \frac{2\pi R^3}{\mu_0} B_p$$

The ratio of the effective dipole moment $M^{\dagger}$ to the average frozen induction, B_p , depends only on the radius of the sphere. For the sphere used here, $R = 1.27 \times 10^{-2}$ m., so that $\frac{M^{\dagger}}{B_p} = 10.24 \pi - \pi^5 - v^{-2}$.

A quantitative comparison of the observed moments and the average induction is not possible because of the scatter in the measured quantities, and because of the dependence of the moments and flux on the orientation of the sphere and the external field. Since the torsion measurements were all made prior to the flux measurements, it was not possible to reproduce these orientations of the sphere and external field in the flux measurements. Moreover, the vertical component of the earth's field was not compensated for the moment measurements. Perhaps the best temperatures at which to make a comparison are the low temperatures, where the moments do not depend on orientation, and where the effect of the earth's vertical field should be small.

When the sphere has zero frozen flux, the magnetic induction should be represented accurately by the currents I_0 and I_1 . In appendix B, the leakage flux, measured by the L coils, is calculated assuming the room temperature measurements for the sphere and coil geometries. The agreement between the calculated and measured leakage fluxes is strong evidence that the geometrical specifications of the coils and sphere are correct at 4.2 ° K. If this is so, and, if the field of the permanent flux is a dipole field outside the sphere, then a large correction must be used to obtain the average induction in the sphere from the observed average induction in the L coils (appendix B). A comparison of figures 3.2 and 3.7 shows that the moment is roughly proportional to the average flux for the whole temperature range. When the dipole correction is made to the observed average induction, the ratio (at 2.4°) of the magnetic moment to the average induction in the sphere is $7.5 \pi - \pi^5 - v^{-2}$.

The difference between the measured and predicted constants of proportionality (7.5 and 10.24, respectively) is greater than may reasonably be attributed to experimental errors.

That the currents shielding the sphere, and the currents producing the permanent flux, are superposed independently on the sphere is indicated by the magnetization measurements. From figure 3.5, it is seen that the flux for increasing inductions (with a residual flux) is just the sum of the leakage flux (with zero residual flux) and the value of the frozen flux.

If the shielding current distribution is independent of the history of the specimen (independent of the frozen flux), then the difference between the average induction for increasing and decreasing fields (figure 3.5) can only be caused by flux emerging from the sphere during the decrease from $B = \frac{2}{3} B_c$ to $B = 0$. Only a part of the flux threading the sphere when it should first be completely superconducting is retained as a permanent frozen flux. For the case of figure 3.5, the ratio of the permanent flux to the flux threading the sphere at $B = \frac{2}{3} B_c$ is only .55. This effect is analogous to the moment changes for changing temperature, and it can account for the torque changes which were occasionally observed in the first few minutes following the transition. The usual procedure in making the torsion pendulum measurements was to remove the field of the Helmholtz coils, and then to make successive period determinations in the earth's field, a positive field, and a negative field. Almost invariably, it was the first two measurements (in positive fields) which yielded the torques larger than the equilibrium values; from figure 3.5 it is seen that, even in the earth's field, there is still additional "permanent" flux which disappears when the external field is reduced to zero. Although

no determinations were made of the stability (in time, with the external field constant) of this additional "permanent" flux, it may be stated that the total elapsed time required to determine the decreasing field curve was greater than the elapsed time between the transition and the time the torques assumed their equilibrium values.

Overall, the measurements seem internally consistent. The greatest objections are the difference between the observed and predicted constants of proportionality between the moment and average induction, and that the sphere froze in flux above the apparent critical temperature. However, neither of these effects appears too serious in light of a more realistic model of the internal induction.

B. From figure 3.10, it is seen that the sphere begins to show a "poor" Meissner effect for temperatures near to T_c . As the temperature is increased, the average frozen induction becomes larger compared to B_c , and there is even some induction frozen in above the apparent critical temperature. At the higher temperatures, the magnetization curves are rounded and the transition is spread over a large range of induction, so that the definition of the critical field becomes somewhat vague. The critical fields used to calculate Φ for figure 3.10 were determined by the first penetration of the magnetic induction into the sphere, when there was zero permanent induction in the specimen. It is seen that the percents of flux frozen in for small constant fields (where the flux threading the normal state sphere is known) agree well with the percents of flux obtained using the measured values of the critical induction.

The freezing in of 100% of the flux is the behavior that would be expected if the transition into the superconducting state were simply a

transition into a state where the conductivity is infinite (figure 1.1b). Since the discovery of the Meissner effect, the idea has gained acceptance that the "pure" superconducting state is specified not only by $B =$ constant, but that the constant is zero (except in the penetration layer).¹⁸ It is assumed that average fields, of the form assumed for B_p in the preceding section, cannot represent the true internal state of a superconductor.

The Meissner effect is itself, however, an idealization which is not achieved in practice.* The experimental data for the completeness of the Meissner effect in various superconductive materials does tend to show some correlation with the particular chemical or physical state of the individual samples; however, there are deviations from these trends which appear inconsistent with even the simplest hypotheses concerning the effect of chemical and physical properties upon the magnetic behavior of superconducting specimens. In general, the elemental superconductors exhibit the most "ideal" properties, good Meissner effects and sharp transitions. Superconductive chemical compounds usually show much the same features as the elements, whereas superconductive mixtures have large hysteresis effects and very broad, rounded transitions (although this seems to be not always true³⁵). The departures from the idealized properties seem to be associated with the presence of impurities and imperfections in the specimens, but the kinds of imperfections involved, and their precise roles, are not understood.

In the present work, the sphere shows a gradual transition, with increasing temperatures, from the properties exhibited by the pure superconducting elements, to the behavior which seems characteristic of superconductive mixtures. Pippard has recently given preliminary results for

*Schoenberg³² reviews the experimental work on this subject in sections 2.1 to 2.8.

the residual flux trapped in tin-indium cylinders in transverse magnetic fields.²⁹ He finds that pure tin cylinders trap a small percent of the flux, and that this percent decreases with increasing temperature. However, the percent of flux frozen in cylinders containing 2% indium impurity increases with increasing temperature. But this does not necessarily mean that the large percent fluxes observed here are caused by impurities. The symmetry properties of the particular arrangement seems to affect the amount of frozen flux,³³ and Pippard's experiment has a much lower degree of symmetry than is obtained with the sphere.

The existence of frozen in magnetic flux is "explained" by postulating the creation of superconducting rings which can conserve the magnetic flux which they embrace. Although the formation of superconducting rings was introduced ad hoc to account for the residual magnetic moments,³² there is recent experimental evidence that indicates that enclosed normal regions really do exist in superconductors.^{23,24,25,36} However, the factors controlling the formation and stability of the rings are not understood.

Landau¹⁶ first formulated quantitatively a model for a superconductor, containing an internal induction, which took account of the difference between the average induction in the specimen and the induction at a given point. He assumed that, in the body of the superconductor, the magnetic induction at a point must be either the critical induction or zero. The interior of the superconductor is divided into domains; normal conductive properties are assigned to the regions containing the critical induction, and regions with zero induction have superconductive properties. In order to obtain agreement between the predicted properties, and the properties observed with small specimens, thermodynamic contributions from the interface between the N-S domains must be included with the usual

volume contributions.^{1,2,15}

The domain structure used by Landau has been observed experimentally,^{23,24,25,36} but the actual domains are not either regularly shaped or very reproducible. The magnetic induction distribution in the channel between the two plane surfaces of closely spaced hemispheres has been measured with very small bismuth probes. Meshkovsky²⁵ has given maps of the field distribution over the cross section of a sphere in the intermediate state, and a distribution for a tin sphere at 3°K., $B = 70 \times 10^{-4}$ w-m⁻² is shown in figure 4.2. The shaded regions in the figure have the critical induction (normal domains) and the light areas have zero induction.

It was originally believed that a domain structure of the kind shown in figure 4.2 could not exist at a free surface of a superconductor, because of the difficulty of satisfying the magnetic boundary conditions.¹⁶ There is experimental evidence, however, that indicates that the normal regions do come directly out to the surface of the sphere.^{24,36} If the flux threads, or tubes, of which the permanent flux consists, approach the surface of the superconductor uniformly, then it might be expected that the tubes of magnetic induction would remain cylindrical for a small distance beyond the surface of the specimen. This effect could, qualitatively at least, account for the difference between the measured and predicted values of the proportionality constant for the magnetic moment and the average induction. The large dipole correction to the observed values of the average induction arises from the discontinuity of the field of a uniformly magnetized sphere at the surface, and from the large curvature of the stream lines outside the sphere. If the constant induction tubes remained cylindrical beyond the plane of the detection coils, and

the field lines did not curve too sharply back, then the correction to be applied to the observed values of the flux would be different from the dipole correction. Without making the dipole correction to the observed values of the flux, the ratio of measured moment to flux agrees with the predicted value. This argument is, of course, speculative, since it is not known how far from the superconducting surface the flux threads retain their individuality. The distance between the sphere and coils is about 1.5×10^{-3} meters, which is of the order of magnitude of the diameter of the flux tubes. But the actual field distribution at the surface of the sphere appears too complicated to give a quantitative treatment.

It is necessary to postulate a positive surface free energy in order to obtain the domain structure of the Landau theory and, indeed, to obtain the Meissner effect at all.¹⁹ An interpretation is that, in the pure elements, which show good Meissner effects, the positive surface energy prevents the superconducting rings (some of which are already formed in figure 4.3) from being stable when the external magnetic field is lowered. In the superconductive mixtures, which show poor Meissner effects, the surface energy is lowered (presumably by impurities) so that superconducting threads may persist for fields greater than the critical field, and superconducting rings may persist in zero external induction. This leads to the rounding of the magnetization curve, and to the residual moments, respectively.²⁶

The change of the magnetic properties of the sphere from those of the pure elements to those of the superconductive mixtures may be ascribed to a decrease of the surface energy with increasing temperature. This is not surprising (or very enlightening), since the surface energy is a

construct which was invented primarily to account for the differences shown by various superconductors. Although some qualitative suggestions for the origin and meaning of the surface energy term have been proposed,^{7,28} these ideas have not yet been developed to the point where the influence of specific properties of the superconductor upon the surface energy may be estimated.

The conservation of the magnetic flux "trapped" in a superconductor follows from Maxwell's equation, when combined with the experimental observation that the electric field is zero in a superconductor. When a flux tube is completely enclosed by a superconducting ring, the total magnetic flux threading the ring should remain constant, even though the distribution of induction inside the normal part of the material may change (specific heat measurements show that a superconductor with a large frozen flux has mostly properties characteristic of the normal state²²). According to the London theory, the conservation of magnetic flux should be valid only for a superconducting ring which is thick compared with the penetration layer. However, this restriction should be satisfied by the flux threads in a large specimen (figure 4.2). In view of this, it is plausible that the moment and flux changes (which occur when the temperature of the sphere is changed) are caused by the individual flux tubes moving to the outer surface of the specimen, where they are no longer completely enclosed by superconducting rings.

The foregoing would be straightforward if the moment and flux measurements agreed better. The 30% difference between the two methods may, however, cause a reasonable doubt about what has actually been measured in the experiments. But it is significant that the method proposed for removing the discrepancy (although the method is not rigorously justified)

is in such a direction as to lend greater support for the interpretation that the observed values of the flux are, at least, proportional to the values of the flux in the sphere. Moreover, the systematic 30% difference may be considered small, compared with the total changes in moment and flux which were obtained.

The discussion presented in this section has been qualitative and speculative, but this should not be allowed to obscure the principal experimental features. Even allowing very wide (and, perhaps, generous) limits for the interpretation of the experimental data, two important questions concerning the superconducting state emerge from this work. The first of these is concerned with the normal-superconducting transition, and the behavior which an "ideal" superconductor would exhibit at the transition into the superconducting state. The second question is concerned with the internal state of a superconducting sample, and the effect of temperature changes on the internal structure of the specimen.

The temperature dependence of the surface energy which appears to be necessary to account for the present results (i.e., positive below 3.2°K. , but decreasing and becoming negative with increasing temperature), is different from the dependence used to account for other experiments.^{2,4,15} In particular, the supercooling experiments,⁴ and the measurements on the magnetization of small specimens,^{2,15} indicate a surface energy which is positive at all temperatures.

A correlation of the dependence of the surface energy with specific parameters of superconductors seems not to be possible with the experimental data available at present. The experiments with superconductive mixtures indicate that the presence of impurities should lead to negative surface energies,^{29,34} whereas some experiments indicate that

carefully prepared mixtures may have a positive surface energy.³⁵ Much more research work on the properties of, and the factors influencing, the phase transition in superconductive metals is required before adequate and convincing interpretations of experiments such as those described here may be given.

V. CONCLUSIONS

The residual flux trapped in a chemically pure tin sphere has been studied in detail by two methods. It has been shown that the amount of magnetic flux trapped in the superconducting sphere exhibits a pronounced temperature dependence. The characteristic features of the functional dependence of the residual flux on the temperature (the maximum and minimum) are only slightly affected by the secondary properties of the transition, within the limits which these properties were varied in these experiments. The magnitude of the frozen flux does, in some temperature ranges at least, depend on the surface condition of the sphere, and on the orientation of the flux in the sphere. Comparison of the torque and flux measurements shows that the observed torques are proportional to the average induction over the detection coils, as would be expected if the permanent induction were uniform inside the sphere. The constant of proportionality is of the right order of magnitude to satisfy the uniform internal induction model, but the difference between the calculated and observed values is greater than may be accredited to experimental errors. It is proposed that the field of the frozen induction in the sphere may not be represented, near the sphere, as the field produced by a uniform internal induction, but the distribution appears to be too complicated for a quantitative treatment. The term "frozen flux" seems appropriate in the sense that the sphere "freezes in" the very complex intermediate state structure, which exists in a sphere during the transition between the two phases.

These results are discussed in terms of the phenomenological surface energy parameter. Although the temperature dependence of this para-

meter, which seems to be required for the present work, is different from that used in other experiments, no conclusions can be drawn from this until a quantitative method of treating the frozen flux has been developed. If the behavior of this specimen is caused by physical inhomogeneities, then the effects should change when the physical state is altered, or should be different in other specimens. If the effects are, however, real properties of the superconducting state (which have been masked previously by other features of the transition), then with suitable care it should be possible to observe these effects in other specimens. The apparent migration of the normal conducting domains, when there are temperature perturbations, poses a fundamental problem for understanding the factors which influence the mobility of the normal-superconducting interface.

Appendix A:

A simple calculation suffices to show that vapor pressure oscillations should not seriously affect the temperature of the helium bath. Consider a semiinfinite right circular cylinder filled with a liquid. Let the surface of the bath be at $z = 0$, and positive z measured down into the bath. Then the differential equation satisfied by the temperature, in the absence of convection currents, is the following

$$\nabla^2 T = \frac{c}{k} \frac{\partial T}{\partial t}$$

where c is the heat capacity per unit volume, and k is the thermal conductivity of the liquid. This equation is, of course, separable into space and time equations.

Since the axis of the cylinder is an axis of symmetry, the angular variable does not appear in the equation, and, if the walls transmit no heat, then the equation reduces to

$$\frac{\partial^2 T}{\partial z^2} = \frac{c}{k} \frac{\partial T}{\partial t}$$

The boundary conditions to be imposed on the solution are

$$T(z=0, t) = T_0 (1 + \delta \sin \omega t)$$

$$T(z, t) \rightarrow T_0 \text{ as } z \rightarrow \infty$$

which represents a bath with a uniform temperature, T_0 , on which is superposed a sinusoidal variation at the surface.

The solution which satisfies the boundary conditions may be written as

$$T = T_0 \left[1 + \delta e^{-\lambda z} \sin(\omega t - \lambda z) \right]$$

$$\lambda = \sqrt{\frac{\omega c}{2k}}$$

For liquid helium at the temperatures of interest here, the heat capacity is 9×10^4 calories - meters⁻³ - degrees⁻¹, and the thermal conductivity is 6×10^{-3} calories - meters⁻¹ - seconds⁻¹ - degrees⁻¹.¹³ Hence, for a period $\tau = 5$ seconds,

$$\lambda = \sqrt{\frac{\pi c}{\tau k}} = 3.1 \times 10^3 \text{ meter}^{-1}$$

This means that if the temperature at the surface of the bath oscillated with an amplitude $T_0 \delta$, then the amplitude one centimeter below the surface is $T_0 \delta e^{-31}$. Since the sphere was always well below the surface of the bath, the oscillations should not have affected the temperature of the sphere.

Appendix B

The relationship between the measured galvanometer deflections and the flux distribution is discussed here.

The coils were calibrated before each experiment by measuring the galvanometer deflection as a function of the uniform magnetic induction applied over the area of the coils (figure 2.6). The deflections are proportional to the total flux threading the coils, so that, if the field is not uniform, then the indicated values of the induction are averages over the plane of the coils.

1. The quantities B_s , B_1 , B_2 , and B_3 were derived from the experimental data by assuming that the detecting coil loops (for the S and M coils) lay in the surface of the sphere. This is equivalent to the assumption that the field distribution produced by the permanent currents in the sphere has stream lines which are uniform within infinite circular cylinders. The three cylinders (one solid, two annular) are bounded by the S coil loop, the S and M coil loops, and the M coil loop and the equatorial perimeter of the sphere.

A given galvanometer deflection for the L coils corresponds to a total flux, $\Phi_{LC} = A_{LC} B_{LC}^{(c)}$ threading the coil, where A_{LC} and $B_{LC}^{(c)}$ are the area of the L coils, and the uniform, calibration induction, respectively. If A_s and B_s are the area of the sphere and the average induction in the sphere, respectively, then assuming that the stream lines are straight, B_s is given by

$$B_s = \frac{A_{LC}}{A_s} B_{LC}^{(c)} = 1.04 B_{LC}^{(c)}$$

If subscripts "SC" and "MC" mean quantities obtained for the S and M coils, respectively, then B_1 , B_2 , and B_3 are defined by

$$B_1 = B_{sc}^{(c)}$$

$$B_2 = \frac{A_{MC}}{A_{MC} - A_{SC}} B_{MC}^{(c)} - \frac{A_{SC}}{A_{MC} - A_{SC}} B_{SC}^{(c)} = 2.07 B_{MC}^{(c)} - 1.07 B_{SC}^{(c)}$$

$$B_3 = \frac{A_S}{A_S - A_{MC}} B_S - \frac{A_{MC}}{A_S - A_{MC}} B_{MC}^{(c)} = 3.05 B_S - 2.05 B_{MC}^{(c)}$$

B_1 is the average, uniform induction in the tube defined by the S coil loops. B_2 and B_3 are the average, uniform induction in the two annular tubes between the S and M coils, and the M coils and the edge of the sphere, respectively.

2. Actually, the field produced by the sphere cannot be uniform within the infinite cylinders, as assumed above. Another simple case is to assume that the field produced by the specimen is the same field as that of a uniformly magnetized sphere.

The sphere and coils are oriented as shown in figure B.1.

FIGURE B.1

It is convenient to define the quantities I_{LC} , I_{MC} , and I_{SC} , given by

$$I_i = \int_{d_i}^{D_i} u(r, \theta) 2\pi r_1 dr_1 = 2\pi \int_{d_i}^{D_i} \frac{2\cos^2\theta - \sin^2\theta}{r^3} r_1 dr_1$$

$$I_i = 2\pi \left\{ \frac{D_i^2}{(a_i^2 + D_i^2)^{3/2}} - \frac{d_i^2}{(a_i^2 + d_i^2)^{3/2}} \right\}$$

The values of the constants for the three coils are

i	$d_i \times 10^2 - \text{m}$	$D_i \times 10^2 - \text{m}$	$a_i \times 10^2 - \text{m}$
LC	1.14	1.30	.565
MC	.82	1.04	.971
SC	.354	.75	1.22

The integrals are given by

$$\begin{aligned} I_{LC} &= - .040 \times 2 \pi \text{ cm}^{-1} \\ I_{MC} &= .049 \times 2 \pi \text{ cm}^{-1} \\ I_{SC} &= .133 \times 2 \pi \text{ cm}^{-1} \end{aligned}$$

If the field distribution is the same as that of a uniformly magnetized sphere, with a uniform internal induction B_p , then the z-component of the field is given by

$$B_z = B_p \quad r < R$$

$$B_z = \frac{B_p R^3}{2} u(r, \theta) \quad r \geq R$$

The total flux threading the coil is

$$\begin{aligned} \Phi_{pi} &= \int_0^{d_i} B_{pi} 2\pi r_i dr_i + \int_{d_i}^{D_i} \frac{B_{pi} R^3}{2} \mu(r, \theta) 2\pi r_i dr_i \\ &= \left[\pi d_i^2 + \frac{R^3 I_i}{2} \right] B_{pi} \end{aligned}$$

For a given galvanometer deflection, Φ_{pi} is equal to a particular calibration value, $\Phi_i^{(c)} = \pi D_i^2 B_i^{(c)}$. Hence, B_p and $B_i^{(c)}$ are related by

$$B_{pi} = C_i B_i^{(c)} ; \quad C_i = \left\{ \left(\frac{d_i}{D_i} \right)^2 + \left(\frac{R}{D_i} \right)^2 \frac{R I_i}{2\pi} \right\}^{-1}$$

For the L, M, and S coils, the constants C_i are

$$C_{Lc} = 1.40 ; \quad C_{Mc} = 1.40 ; \quad C_{Sc} = 1.42$$

The quantities B'_s , B'_1 , and B'_2 (which are the quantities B_s , B_1 , and B_2 corrected for the dipole curvature of the field) may be obtained from B_s , B_1 , and B_2 by changing the scale of the latter quantities. B'_1 and B'_2 are 1.4 times B_1 and B_2 , respectively, and B'_s is $\frac{1.40}{1.044} = 1.35$ times B_s . B'_3 will be slightly different from 1.4 times B_3 , since B_s (rather than $B_{LC}^{(c)}$) was used to obtain B_3 .

3. When the sphere is completely superconducting, the z component of a uniform, applied magnetic induction B is given by¹⁸

$$B_z = 0 \quad r < R$$

$$B_z = B \left[1 - \frac{R^3}{2r^3} \right] \quad r \geq R$$

For this case, the total flux threading a coil is given by

$$\Phi_i = \int_{A_i} B_z dA = B \left[\pi (D_i^2 - d_i^2) - \frac{R^3 I_i}{2} \right]$$

This corresponds to the measured, average calibration flux $\Phi_i^{(c)} = \pi D_i^2 B_i^{(c)}$, so that

$$B_i^{(c)} = B \left[1 - \left(\frac{d_i}{D_i} \right)^2 - \left(\frac{R}{D_i} \right)^2 \frac{R I_i}{2\pi} \right]$$

For figure 3.5, B_L is $B_{LC}^{(c)}$, so that the initial slope of the curve should be given by

$$\frac{dB_L}{dB} = \frac{dB_{LC}^{(c)}}{dB} = 1 - \left(\frac{d_{LC}}{D_{LC}} \right)^2 - \left(\frac{R}{D_{LC}} \right)^2 \frac{R I_{LC}}{2\pi}$$

$$\frac{dB_L}{dB} = .280$$

When the sphere is in the intermediate state ($\frac{2}{3} B_c < B < B_c$) the z-component of the field is given by¹⁸

$$B_z = 3B - 2B_c \quad r < R$$

$$B_z = B - (B_c - B) R^3 u \quad r \geq R$$

The flux threading a coil is given by

$$\Phi_i = \int_{A_i} B_z dA = \left[2\pi d_i^2 + \pi D_i^2 + R^3 I_i \right] B - \left[2\pi d_i^2 + R^3 I_i \right] B_c$$

Again, this corresponds to $\pi D_i^2 B_i^{(c)}$, so that the slope of figure 3.5 for $\frac{2}{3} B_c < B < B_c$ should be

$$\frac{dB_L}{dB} = 2 \left(\frac{d_{LC}}{D_{LC}} \right)^2 + 1 + \left(\frac{R}{D_{LC}} \right)^2 \frac{R \bar{I}_{LC}}{\pi}$$

$$= 2.44$$

FIGURE 11

FIGURE 2.1

FIGURE 2.2

FIGURE 2.3

FIGURE 2.4A

FIGURE 2.4B

FIGURE 2.5

FIGURE 31

FIG

FIGURE 3.2

ACHINED SPHERE

FIGURE 33

FIGURE
34A

FIGURE
3.4B

FIGURE
3.4C

FIGURE 3.4D

FIGURE 3

FIGURE 3.5

FIGURE 3.6

FIGURE 3.7

FIGURE 3.7

FIGURE 3.8

FIGURE 3.9A

FIGURE 3.9B

FIGURE 3.9C

FIGURE 3.10

FIGURE 4J

FIGURE 4.2

Table 3.1

The magnetic moment obtained by transitions at constant temperature.

Units: T - °K.

M - newton - meters³ - webers⁻¹

Number	T	$M \times 10^5$ $n \cdot \pi^3 \cdot v^{-1}$
A. Machined sphere		
M5.3	2.850	29.86
M5.1	2.984	-21.95
M4.1	3.050	18.56
M1.4	3.333	-7.91
M1.3	3.339	7.90
M1.2	3.341	7.41
M1.1	3.352	6.85
M3.6	3.399	8.03
M2.3	3.432	4.65
M2.2	3.437	5.41
M2.1	3.447	5.60
M1.6	3.547	6.32
M1.5	3.552	5.95
M1.9	3.637	20.88
M1.8	3.637	20.56
M1.7	3.639	20.67
M2.8	3.668	-49.68
M2.6	3.670	47.72
M2.7	3.670	47.57
M3.1	3.670	46.74
M2.5	3.676	55.29
M2.9	3.676	56.82

B.1 Polished sphere - cutting current in Helmholtz coils

M19.5	2.380	17.0
M19.4	2.581	12.9
M19.2	2.801	10.3
M17.9	2.850	8.5
M17.7	3.220	4.64
M17.6	3.424	5.76
M17.4	3.598	23.6
M17.1	3.658	42.7
M17.3	3.658	49.8
M18.1	3.682	67.1
M19.1	3.682	62.2
M20.3	3.692	79.9

Table 3.1 (continued)

N20.6	3.700	70.0
N21.2	3.702	65.9
N21.3	3.712	50.0

B.2 Polished sphere - lowering current slowly

N18.2	3.682	61.7
N22.1	3.702	72.5
N24.1	3.702	73.1
N21.1	3.704	65.8
N23.1	3.710	67.2
N21.4	3.712	57.6
N24.2	3.712	62.5
N24.3	3.718	34.3
N24.6	3.722	18.9
N24.4	3.729	.5

Table 3.2

The dependence of the magnetic moment on the orientation of the sphere and external field.

θ is the angle between the external field and the axis of stable equilibrium.

Units: T - °K.

M - newton - meters³ - webers⁻¹

Number	T	Mx10 ⁵	θ
M19.5	2.380	17.0	0
M19.6	2.364	18.0	$\pi/2$
M17.7	3.220	4.6	0
M17.8	3.217	3.5	$\pi/2$
M17.4	3.598	23.6	0
M17.5	3.598	21.6	$\pi/2$
M18.1	3.682	67.1	0
M18.3	3.682	85.3	$\pi/2$
M18.2	3.682	61.2	0
M18.4	3.682	87.5	$\pi/2$
M19.1	3.682	62.2	0
M19.2	3.682	86.0	$\pi/2$
M20.5	3.692	70.0	0
M20.6	3.692	79.2	$\pi/4$
M20.3	3.692	79.9	0
M20.2	3.694	86.1	$\pi/2$

Table 3.3

The moment changes for temperature changes inside the superconducting state.

Units: T - °K.

M - newton - meters³ - webers⁻¹

Number	T	Mx10 ⁵
A. Machined sphere		
M3.1	3.670	46.74
M3.2	3.611	7.43
M3.3	3.513	4.46
M3.4	3.409	3.66
M3.5	3.658	0
M6.1	3.602	10.5
M6.2	3.529	6.4
M6.3	3.437	4.5
M6.4	3.311	4.4
M6.5	3.487	5.3
M6.6	3.558	-13.4
M8.3	3.639	28.7
M8.4	3.558	13.4
M8.5	3.417	5.40
M8.6	2.868	4.16
M8.7	3.662	0
M7.1	3.649	31.9
M7.2	3.620	0
M7.3	3.605	0
M7.4	3.626	0
M7.7	3.626	26.4
M7.8	3.598	23.1
M7.9	3.540	18.1
M7.10	3.501	14.5
M7.11	3.607	14.2

Table 3.3 (continued)

N3.6	3.399	8.03
N3.7	3.545	-13.43
N3.8	3.522	-10.56
N3.9	3.402	-7.82
N4.1	3.050	18.56
N4.2	3.202	17.76
N4.3	3.355	15.37
N4.4	3.499	5.23
N4.5a	3.653	5.67
N4.5b	3.653	-37.57
N4.6	3.050	4.62
N8.1	3.018	0
N8.2	3.639	31.6
N9.1	3.602	12.6
N9.2	3.620	11.9
N9.3	3.602	11.8
N9.4	3.581	11.0
N9.5	3.602	10.9
N9.6	3.620	11.0
N9.7	3.581	10.4
N9.8	3.563	9.5
N9.9	3.602	14.8
N9.10	3.620	14.2
N9.11	3.602	13.9
N10.1	3.602	15.4
N10.2	3.620	13.6
N10.3	3.602	13.6
N10.4	3.602	15.3
N10.5	3.620	13.6
N10.6	3.602	13.2

Table 3.3 (continued)

B. Polished sphere

N22.1	3.702	72.5
N22.2	3.692	67.2
N22.3	3.672	54.8
N22.4	3.630	36.5
N22.5	3.520	14.6

N23.1	3.710	67.0
N23.2	3.700	61.1
N23.3	3.690	57.2
N23.4	3.670	49.6
N23.5	3.630	37.1
N23.6	3.547	20.4
N23.7	3.320	12.0

Table 3.4

The magnetic moment obtained by transitions at constant field.

B_0 is the magnetic induction in which the transition was made.

Units: T - $^{\circ}K$.

B - webers - meters⁻²

Number	$B_0 \times 10^4$	T	$T_c(B_0)$	$T_c(\frac{3}{2}B_0)$	$M \times 10^5$
Unpolished sphere					
M6.1	6.98	3.602	3.686	3.662	10.5
M7.1	2.01	3.649	3.720	3.713	31.9
Polished sphere					
M20.1	.23	3.692	3.732	3.730	12.7
M20.4	1.00	3.692	3.726	3.723	45.5

Table 3.5

The magnetization of the sphere at 3.711°K . B_L is the average induction in the L coils, and B is the applied (free stream) magnetic induction.

Units: B - webers - meters⁻²

$B \times 10^4$	$B_L \times 10^4$
.000	.00
.226	.08
.500	.15
.773	.23
1.047	.32
1.322	.41
1.593	.50
1.868	.61
2.144	.93
2.418	1.47
2.694	2.12
2.968	2.76
3.242	3.09
3.512	3.42
3.838	3.78
4.156	4.09
4.336	4.35
4.336	4.31
4.062	4.06
3.730	3.69
3.512	3.41
3.242	3.14
2.968	2.77
2.694	2.42
2.418	2.08
2.144	1.87
1.868	1.69
1.490	1.45
1.322	1.33
1.046	1.19
.758	1.03
.500	.89
.226	.75
.000	.63
.226	.69
.500	.79
.773	.85
1.050	.94
1.323	1.04

Table 3.5 (continued)

1.595	1.13
1.872	1.24
2.008	1.29
2.144	1.37
2.279	1.46
2.418	1.72
2.556	1.91
2.693	2.15
2.968	2.61
3.242	3.13
3.687	3.65
4.336	4.31
3.687	3.65
3.242	3.11
2.966	2.73
2.829	2.54
2.667	2.36
2.418	2.11
2.143	1.87
1.868	1.69
1.593	1.51
1.322	1.34
1.047	1.18
.773	1.04
.500	.86
.226	.74
.000	.61

Table 3.6

Measurements of the critical magnetic induction.

Units: T - °K.

B - webers - meters⁻²

Number	T	B_c × 10⁴
F35.2	3.661	10.7
F35.1	3.687	6.75
F29.1	3.699	4.63*
F32.1	3.708	3.85
F31.1	3.711	3.08
F27.2	3.717	2.76*
F32.3	3.720	2.17
F28.1	3.721	1.65*
F30.1	3.722	1.50

*The vertical component of the earth's field was not compensated, but the experimental values have been corrected for this effect.

Table 3.7

The magnetic induction obtained for transition at constant temperature - field coil current cut.

Units: T - ° K

 B_1 - webers - meters⁻²

Number	T	$B_1 \times 10^6$	$B_2 \times 10^6$	$B_3 \times 10^6$	B_3/B_1	$B_0(T) \times 10^4$	ϕ
729.12	2.313	17.7	16.3	21.7	1.45	179	.099
729.13	2.313	18.0	15.8	21.0	1.25	179	.100
729.10	2.703	12.3	10.4	14.8	1.29	136	.090
729.11	2.703	11.9	10.8	15.2	1.58	136	.088
729.8	2.900	8.6	8.2	10.1	1.37	112	.077
729.9	2.900	9.0	7.8	10.8	1.32	112	.080
729.5	3.019	5.4	4.0	7.1	1.69	97	.056
729.6	3.111	8.4	5.4	15.2	3.04	79	.106
729.7	3.111	7.4	5.6	7.4	.81	79	.094
729.3	3.344	4.0	2.7	4.5	.94	74	.074
729.4	3.344	6.1	3.3	4.8	.49	74	.113
729.5	3.344	4.7	3.3	4.3	.67	74	.087
728.10	3.531	14.1	11.1	8.8	.39	29	.486
728.9	3.601	28.1	23.7	19.1	.47	19	1.48
728.7	3.692	50.5	48.0	37.1	.55	12	4.21
728.6	3.698	62.6	61.7	52.0	.70	6.67	9.38
726.2	3.699	74.3	53.4	55.1	1.01	5.03	11.0
726.4	3.699	74.2	50.0	66.1	1.30	5.03	11.0
729.2	3.699	69.9	68.7	64.6	.85	5.03	13.9
727.5	3.700	55.9	53.2	45.8	.66	4.89	11.4
734.11	3.701	74.3	73.3	69.4	.89	4.74	15.7

Table 3.7 (continued)

733.3	3.703	75.6	77.7	75.5	73.9	.95	4.45	17.0
727.4	3.708	36.8	45.3	33.2	32.0	.71	3.68	10.0
728.4	3.713	47.4	50.3	46.5	45.5	.91	2.96	16.0
733.6	3.713	66.5	66.3	66.3	66.8	1.01	2.96	22.5
727.3	3.717	41.1	43.4	34.4	40.5	.93	2.38	17.3
733.9	3.719	59.6	59.0	59.1	60.9	1.03	2.08	28.7
728.2	3.721	29.9	29.4	28.1	31.9	1.08	1.80	16.6
730.2	3.722	49	49	47	51	1.02	1.65	29.7
733.11	3.723	51.5	52.2	49.8	57.4	1.10	1.48	34.8
733.13	3.726	45.2	45.5	44.5	45.7	1.00	1.06	42.7
733.14	3.728	36.3	36.8	35.1	34.0	.92	.77	45.8
734.9	3.728	27.4	29.0	27.4	26.1	.90	.77	35.6
734.7	3.731	16.4	18.0	15.1	16.1	.90	.32	51.3
734.5	3.733	7.5	9.0	8.1	5.5	.61	.02	
734.6	3.733	9.7	11.7	9.1	8.4	.72	.02	
734.2	3.737	4.0	5.0	4.5	2.4	.48	-	

Table 3.8

The magnetic induction obtained for transitions at constant temperature - field coil current lowered slowly.

Units: $T - ^\circ K$

B_1 - webers - meters⁻²

Number	T	$B_1 \times 10^6$	$B_2 \times 10^6$	$B_3 \times 10^6$	B_3/B_1	$B_0(T) \times 10^6$	φ
F28.8	3.652	52.8	50.0	39.4	.57	12×10^2	4.39
F28.1	3.680	52.6	50.5	50.8	.90	784	6.72
F28.5	3.688	62.1	59.7	52.7	.71	667	9.34
F26.1	3.699	55.6	53.8	57.6	1.04	503	11.1
F26.3	3.699	56.0	54.5	58.5	1.06	503	11.1
F29.1	3.699	57.2	57.8	49.4	.76	503	11.4
F33.2	3.703	74.3	75.0	70.7	.91	445	16.7
F31.1a	3.711	65.0	65.4	66.5	1.05	328	19.8
F31.1b	3.711	64.2	66.0	63.5	1.00	328	19.6
F31.1c	3.711	63.4	66.0	63.7	1.04	328	19.3
F28.3	3.713	48.6	47.1	46.4	.89	296	16.4
F33.5	3.713	66.1	65.9	66.8	1.01	296	22.3
F27.1	3.716	39.5	38.1	28.5	.69	253	15.6
F27.2	3.717	35.1	36.1	30.4	.76	238	14.7
F33.8	3.719	59.6	58.8	60.9	1.03	208	28.7
F28.1	3.721	30.1	29.4	30.7	1.02	180	16.7
F30.1	3.722	47	45	49	1.04	165	28.5
F33.10	3.723	51.9	50.5	53.1	1.02	148	35.1
F33.12	3.726	45.3	44.3	45.7	1.02	106	42.7
F34.10	3.728	26.3	27.3	21.5	.70	77	34.2
F34.8	3.731	16.3	16.4	14.1	.76	32	50.8
F34.4	3.733	9.7	9.3	7.3	.62	2	-
F34.5	3.737	4.6	4.3	2.7	.40	-	-

Table 3.9

The magnetic induction obtained for transitions at constant field.

B_0 is the magnetic induction in which the transition was made.

Units: T - °K.

B - webers - meters⁻²

Number	T	$B_0 \times 10^4$	$\tau_c(B_0)$	$\tau_c(\frac{3}{2}B_0)$	$B_0 \times 10^6$	$B_1 \times 10^6$	$B_2 \times 10^6$	$B_3 \times 10^6$	α
F33.1	3.713	.236	3.732	3.731	14.2	14.0	14.5	14.2	60.2
F33.4	3.713	.472	3.730	3.728	26.6	26.6	26.2	26.9	56.4
F33.7	3.713	.708	3.728	3.726	37.3	37.8	36.3	37.8	52.7
F30.3,7	3.708	1.033	3.726	3.723	47.0	48.3	45.2	47.5	45.5

Table 3.10

The flux changes for temperature changes inside the superconducting state.

Units: T - °K.

B_1 - webers - meters⁻²

Number	T	$B_0 \times 10^6$	$B_1 \times 10^6$	$B_2 \times 10^6$	$B_3 \times 10^6$
F25.1	3.680	51.1	56.4	51.6	46.5
F25.2	3.600	19.4	26.5	16.4	15.3
F25.3	3.452	6.4	11.0	5.4	2.7
F25.4	3.020	2.0	5.8	.6	-.5
F30.3,7	3.708	47.0	48.3	45.2	47.5
F30.3,8	3.701	44.3	46.0	44.6	43.6
F30.3,9	3.682	40.0	42.7	38.8	38.5
F30.3,10	3.631	29.7	25.3	39.7	25.0
F30.3,11	3.457	13.1	20.1	10.8	8.5
F30.3,12	3.257	9.3	14.7	7.8	5.4
F35.2	3.687	76.0	83.3	77.1	67.7
F35.3	3.670	68.1	77.7	70.3	57.1
F35.4	3.687	66.0	75.2	66.2	57.1
F35.5	3.710	49.8	52.0	53.6	44.6
F35.6	3.687	47.6	50.3	51.3	41.9
F35.7	3.670	45.1	48.8	46.9	40.1
F35.8	3.708	44.6	48.6	46.8	38.9
F35.9	3.670	43.6	47.6	46.8	36.9
F35.10	3.709	43.6	47.6	46.0	37.7

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