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THE RICE INSTITUTE

THE SCATTERING OF ALPHA PARTICLES FROM HELIUM

by

John Lynn Russell, Jr.

A THESIS

SUBMITTED TO THE FACULTY
IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY

GC Phillips

Houston, Texas

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TABLE OF CONTENTS

I.	Introduction	1
II.	Scope of the Thesis.	4
III.	Experiment	5
	A. Description of the Equipment	6
	B. Details and Procedure.	13
	1. The May Experiment	13
	2. The July Experiment.	18
	3. The December Experiment.	20
	C. Presentation of the Data	22
	D. Phase Shift Analysis	23
IV.	Analysis of the Data	28
	A. One Level Dispersion Theory.	30
	B. Alpha-Alpha Interaction Potentials	34
	C. Shape Independent Formalism.	42
V.	Conclusions.	51
VI.	Definition of Symbols.	53
	Appendix	56
	References	60
	Acknowledgements	62

I. INTRODUCTION

The alpha particle model was developed into a theory of the structure of light nuclei in the late nineteen-thirties by Wefelmeir¹, Wheeler², Weizsacker³ and Fano⁴. These authors were aware of the shortcomings of the alpha particle model and suggested that this model should consider the alpha particle to have only a short-lived identity rather than as a stable structure inside the nucleus. They proposed that the constituent protons and neutrons of an alpha particle in a nucleus would dissolve and recombine with a longer period than the periods of vibration and rotation of such a temporary structure. This proposal was made in an effort to make the alpha particle model consistent with the belief, at that time, that nuclear matter was comparable to a highly condensed "liquid" of protons and neutrons with little or no structure. When more of the properties of nuclei had been measured, and a high degree of order had been ascribed to the nucleus, the shell model of the nucleus was developed in the late nineteen-forties. The success of the shell model has been remarkable, but certainly not complete. Its success implies a highly ordered nucleus; hence, there has been an increased interest in the alpha particle model. Fairly reliable data are now available on the level structure of the light nuclei. The ones to which the alpha particle model would be expected to apply directly, Be^8 , C^{12} , O^{16} and Ne^{20} , show a tempting similarity in level structure.

It has been shown that the various level assignments of C^{12} and O^{16} are roughly consistent with the alpha particle model.^{5,6}

If one is to take the alpha particle model seriously, a systematic study of the parameters of the model is in order. Be^8 is crucial to the model. If the alpha particle is to be treated as a fundamental structure, then the Be^8 nucleus can be described by a two body wave function. This simplicity is the factor that makes a careful investigation of Be^8 significant. It has been shown by a number of investigators that scattering data can uniquely define a potential of interaction, provided there are no bound states and provided the potential meets certain reasonable mathematical requirements (see Section IV). The ground state of Be^8 is unbound.⁷ If the interaction between two alpha particles is expressible as a scalar potential, the scattering of alpha particles from helium should measure that interaction uniquely. From this interaction the properties of C^{12} , O^{16} and Ne^{20} can be computed to provide a rigorous check of the model.

The possibility of a two body analysis applying to alpha-alpha scattering and the lack of good data above three MEV bombarding energy prompted the work presented in this thesis. Before this work began, Heydenburg and Temmer,^{8,9} at the Carnegie Institution of Washington, measured alpha-alpha angular distributions at a number of energies from 0.18 to 3.0 MEV. Nilson at the University of Illinois¹⁰ completed similar experiments covering the energy range of 12 to 23 MEV last year. The present work covers the region from 3 to 6 MEV.

The three sets of data are apparently consistent with a two body description of Be^8 . The next step would be to compute the interaction potential from the scattering data. This has not yet been done.

II. SCOPE OF THE THESIS

The cross section for alpha particles scattered from helium was measured as a function of angle and energy from three to six million electron volts laboratory energy. Nuclear phase shifts were extracted from the data yielding phase shift versus energy for the region investigated. These data, combined with the results of other investigators, were compared with the predictions of the dispersion theory and of various simple potential shapes. The so called "shape independent" formalism,¹¹ originally developed for proton-proton scattering, was applied to the low energy data and the width of the ground state of Be^8 was computed. The qualitative shape of the alpha-alpha potential of interaction is discussed.

III. EXPERIMENT

The scattering of alpha particles from helium was first reported in 1927 by Rutherford and Chadwick,¹² and the experiment was repeated and extended in the nineteen-thirties by several investigators. This early work was summarized by Wheeler¹³ in 1938. The only sources of energetic alpha particles available at that time were natural alpha emitters. This plagued the early experimenters with all the problems associated with low intensity and poor resolution. The data were not sufficiently accurate to permit explicit extraction of phase shifts although Wheeler made the attempt.

It was apparent that the alpha-alpha experiment would not be practical until particle accelerators could be developed that would produce an intense monoenergetic beam of alpha particles. At the time this experiment was conceived the only reliable alpha-alpha data were from the Carnegie Institution of Washington, with three MEV bombarding energy the highest energy used up to that time. The Van de Graaff generator, at the Rice Institute, was modified to permit acceleration of singly ionized alpha particles in 1954, making possible the extension of reliable data to 6 MEV.

The construction of a large volume scattering chamber at the Rice Institute had just been completed in 1954 and had been designed to be adaptable to this experiment. There were several problems to be solved before the alpha-alpha experiment could be concluded--some peculiar to the alpha-alpha experiment--most characteristic of scattering problems

in general.

For the alpha-alpha experiment to be meaningful, the parameters of the experiment must be well defined. The beam of alpha particles emerging from the accelerator should be of known energy and it should be well collimated before reaching the target. The number of beam particles per data point must be accurately determined, and the target thickness should be known. The problem of detecting the scattered particles in the presence of an intense gamma ray flux is severe, since the energy of the detected alpha particles varies by a factor of twenty over the course of the experiment. Finally there should be an independent check of the entire apparatus under conditions as similar to the actual experiment as possible but where the results to be expected are accurately predictable.

Description of the Equipment

The design and construction of the large volume scattering chamber and the differential pumping system used for this experiment have been described elsewhere ^{14, 15, 16, 17}. Simply, the equipment consisted of a large cylindrical tank containing two particle detectors supported by coaxial shafts that could be rotated from outside the vacuum tight cylinder. A differentially pumped tube, re-entrant in one wall of the cylinder, permitted the accelerator beam to enter

TOP VIEW
LARGE VOLUME
SCATTERING CHAMBER
FIG. 1

the chamber without passing through a foil. The beam traversed the target volume at the center of the chamber and, to be integrated, passed through a foil out of the chamber volume into a Faraday cup assembly. The target was the gas in the chamber and was defined by slit systems on the detectors. Fig. 1 is a drawing of the top view of the chamber, showing only one detector and its slit system CD, Faraday cup B, differential pumping tube A. Fig. 2 is a photograph of the inside of the chamber, showing two detectors, the differential pumping tube and a re-entrant Faraday cup. Fig. 3 is a photograph of the chamber in operation.

The formula for computing the differential cross section from the experimental data may be derived by computing the number of particles scattered into the detector as a function of the experimental observables. Fig. 4 is a schematic diagram of the experimental arrangement. Particles are scattered from the beam along its entire path through the gas in the chamber. However, if the gas pressure is sufficiently low to make multiple scattering negligible, the detector is exposed to only those particles scattered from the beam in a short section of the beam of length l . If the diameter of the beam, the width of the front slit, w , and the area of the rear slit, A , are all small compared with the radius, R , and the slit separation, s , then:

$$l = w \cdot \frac{R}{s} \cdot \frac{1}{\sin \Theta_c}$$

The effective target thickness is, then, ld particles/cm²,

SCHEMATIC DIAGRAM OF THE EXPERIMENTAL ARRANGEMENT

FIG. 4

where d is the number density of the gas. The detector solid angle is A/R^2 . The number of particles, N , scattered into the detector is then:

$$N = I \cdot (\ell d) \cdot \frac{A}{R^2} \cdot \left(\frac{d\sigma}{d\Omega} \right)_{LAB}$$

where I is the number of beam particles and $\left(\frac{d\sigma}{d\Omega} \right)_{LAB}$ is the differential scattering cross section in the laboratory system. Letting $G = A\ell/Rs$ and rewriting:

$$\left(\frac{d\sigma}{d\Omega} \right)_{LAB} = \frac{N \sin \theta}{d I G}$$

This formula is an approximation, and is in error by about 0.2% at 15° for the equipment here described.

The parameters of G , the geometry factor, were measured before and after each experiment. The area of the rear slit and the width of the front slit were measured with a cathetometer. ℓ and s were measured with a steel rule. G , as determined from these measurements, was 1.886×10^4 cm for the original detector and 2.138×10^4 cm for the second detector.

d , the number density of the target gas, was computed assuming validity of the ideal gas law. The pressure was determined with an oil manometer backed with a diffusion pump. The specific gravity of butyl sebacate, the manometer oil, was determined by a pycnometer technique to be 0.932. The temperature was measured with a mercury thermometer in thermal contact with the lid of the chamber.

The total number of particles, I , was determined by measuring their total charge by collecting the beam in a Faraday cup. Knowing the effective charge per particle, then,

determines the number of particles. The effective charge per particle of an energetic alpha particle beam after passing through matter is not $2e$ as might be expected. Because of continual pick-up and loss of electrons in passage through matter, the alpha particles at the Faraday cup consist of various percentages of doubly ionized, singly ionized, and uncharged helium atoms. Data on this effect are available¹⁸ so that the effective charge per particle can be determined.

Several modifications of the chamber were made after the forementioned descriptions : 14, 15, 16, 17 were written. All of the pumping lines and valves were increased in size to reduce the evacuation time, which was significant for such a large volume. The differential pumping tube support was reinforced to assure no deflection on evacuation. A remote manometer was constructed so that the pressure in the chamber could be measured at the accelerator control panel. A rotating foil holder was placed on the original detector to permit placing any of twelve graduated, absorbing foils between the exit of the detector slit system and the detector itself. The foil holder could be positioned without opening the chamber. A second detector, described by Henry,¹⁹ was added to the chamber. It, like the first, was a scintillation detector utilizing a Du Mont photo-tube and cathode follower housed in a brass can inside the chamber. The scintillator, in the chamber volume, rested on a lucite light pipe which extended through a vacuum seal to the photomultiplier. The interior of the brass housing was open to atmospheric pressure via a hollow supporting shaft. The second detector had

no absorber foil assembly. The angular resolution for the slit system for this detector was adjustable to two values, $\pm \frac{1}{2}$ degree (like the first detector) and $\pm 1/12$ degree. The scintillator used for the alpha-alpha experiment was thallium activated cesium iodide. First, a thin (1 mm. thick) polished crystal was tried, but the background was excessive. It was found that evaporating a thin layer of CsI(Tl) on glass produced a scintillator insensitive to gamma rays, but quite suitable for alpha detection. This device made possible the detection of alpha particles in a relatively intense flux of gamma radiation over the wide energy range required in this experiment.

The Faraday cup assembly was changed twice, owing to difficulties in integrating the alpha particle beam. Apparently two separate phenomena were causing difficulty. First, because of the high ionization per unit path length of an energetic alpha particle (about an order of magnitude greater than a proton of the same energy), the Faraday cup should be in a vacuum of about 5×10^{-5} mm. Hg to eliminate the possibility of ions, produced in the residual gas, conducting a significant amount of charge from the cup. This was insured by placing a liquid air trap in the Faraday cup vacuum chamber. Second, when the alpha particle beam passed through the foil, isolating the cup from the target gas, it picked up electrons of several hundred volts energy, apparently by head on collisions with electrons. A number of electrons, of the order of 30% of the number of beam particles, were apparently knocked over the potential barrier of the

electrostatic suppressor (at about -200 volts) into the Faraday cup. This was corrected by placing a small permanent magnet across the space between the isolating foil and the cup. These two provisions were apparently successful. Fig. 5 is a cross section of the final Faraday cup. The beam, in passing through the gas in the chamber, was spread by small angle, multiple scattering. The Faraday cup was made re-entrant, as shown, to reduce the path length of the beam in the gas. Current integration was accomplished by charging a standard condenser to a known voltage and then discharging it with the beam current from the Faraday cup. The zero of voltage was measured with a quartz fiber electrometer kindly made available by Dr. N. P. Heydenburg of the Carnegie Institution of Washington. A more complete description of the integrator is given by C. W. Reich in his dissertation.²⁰

The differential pumping tube, as originally designed, defined the beam at the first slit A (see Fig. 6) and at the last slit B with circular 10 mil tantalum slits of 1.5 mm aperture. The rest of the pumping constrictions were circular tantalum slits of 2 mm aperture. Scattering of beam particles from the slit edge at B gave rise to detected particles at small scattering angles when the chamber was evacuated. Placing a 1.5 mm slit at C and enlarging the slit at B to 2.3 mm, essentially eliminated the effect when the chamber was evacuated. However, under operating conditions with a gas target, there was gas in the differential pumping tube. This gas spread the edges of the beam in the tube so that it was still

FARADAY CUP ASSEMBLY

FIG. 5

DIFFERENTIAL PUMPING
TUBE

FIG. 6

possible for the edges of the slit at B to be illuminated. This was not corrected and was apparently the cause of a lack of reproducibility of the low energy, 15° , data of the alpha-alpha experiment.

To provide an energy calibration of the accelerator analyzing magnet after its shims and slits had been adjusted to bring the beam into the chamber in its operating position, a thin tantalum backed C^{13} target was mounted on a movable arm that could be put in the beam path. A $C^{13}(\alpha, n)^{16}$ excitation curve was then taken, and the energy calibration was determined by the level values of Schiffer, Bonner, Kraus and Marion.²¹ This provided an energy calibration of ± 20 KEV with a relative consistency between various sets of data of about ± 5 KEV.

Neglecting counting statistics and the uncertainty of forward angle data, the root mean square error estimated for the equipment as described above was about $\pm 3\%$. The estimated errors were:

Detection Efficiency		2.0%
Geometry	Radius	.2
	Slit separation	.2
	Front slit width	.2
	Rear slit area	.2
	Calculational Approximation	.2
	Angular uncertainty	.1 deg. <hr/>
Target	Purity	1.0
	Pressure	.3
	Temperature	.3
Current Integration	Faraday cup	.3
	Switching uncertainty	.5
	Condenser calibration	1.0
	Leakage	.1
	Charge state uncertainty	.2
Energy (assuming $2dE = d\sigma$)		<hr/> 1.0

Details and Procedure

The alpha-alpha experiment was first done at the Rice Institute in May, 1955. It was repeated in July and again in December, 1955. This section will be devoted to the chronologically ordered details of the manner in which the experiments were conducted and equipment changes made.

The May Experiment. The May experiment was exploratory. Conditions were far from ideal, since the equipment was still in the development stage, but it was hoped that enough could be learned to design a good experiment. Certain questions needed answers before development of the equipment could be profitably continued. The feasibility of using a scintillation detector for detecting low energy alpha particles in the presence of the expected high gamma ray flux was questionable. The application of the data on effective charge per particle¹⁸ to the cross section calculation was not yet a proven technique. But most important, were there any design blunders in the equipment or any unexpected problems that would show up in the course of an alpha-alpha scattering experiment? The conditions of the May experiment were as follows.

The lower limit of the beam energy was determined by the thickness of the foil in front of the Faraday cup. It was excessively thick aluminum, of the order of two centimeters air equivalence, and it placed the lower energy limit at about four MEV. The accelerator analyzing magnet had just been altered to permit bending energetic alpha particles and had not yet been recalibrated. The Van de Graaff metering

equipment included an uncalibrated electrostatic voltmeter which was used during the experiment. Later, an accurate calibration of the magnet current versus energy was obtained to 4.6 MEV. From this current calibration an energy scale for the voltmeter was established to 4.6 MEV and was extrapolated above that energy. The energy calibration derived on this basis is shown in Fig. 7. It was checked to within 10 KEV at 3.89 MEV via the scattering of alpha particles from oxygen which has a resonance at that energy.²²

To compute the energy of the beam at the target, the energy loss in the differential pumping tube must be known. The pressure of the first stage of the differential pumping tube was continuously monitored by an oil manometer. The maximum loss during the experiment, at the lowest energy, was 45 ± 9 KEV.

The pressure of the gas in the chamber was determined before and after each data point with an oil manometer backed by a diffusion pump. Column height was about 10 cm (2/3 cm Hg) and was measured with a meter stick. Temperature of the gas was assumed to be the temperature of the chamber and was measured by a mercury thermometer in thermal contact with the lid of the chamber. Simultaneous temperature measurements of various parts of the chamber always agreed to one-half degree centigrade. The helium gas used for the experiment was obtained from the U. S. Navy, and the purity was somewhat better than 99.9%. The flow of helium into the chamber was satisfactorily adjusted with a mechanical leak, after passing through a standard reducing valve from a high pressure cylinder.

FIG. 7

The beam current was integrated with the standard neon bulb type integrator associated with the Rice Institute accelerator.²³ A cross section of the Faraday cup assembly is shown in Fig. 8. This assembly had been used successfully to integrate proton beams. The air cooled cup was supported on Kovar seals. Suppressor voltage was applied to a ring in front of the cup.

The geometry of the detector slit system and the zero scattering angle of the system were measured before and after the experiment. No discrepancy was apparent. A thallium activated CsI crystal 3 x 1 x 0.1 cm was used as a scintillator for the experiment. One angular distribution was taken with a scintillator made by evaporating 3 mg/cm² CsI(Tl) on glass to test the feasibility of such a detector. A five point integral bias curve was taken for every data point with a multichannel pulse height analyzer. The total number of detected alpha particles was chosen to be the point of inflection of the integral bias curve marking the onset of low level detector noise. Generally, this number was within two percent of the value obtained by straight line extrapolation of the flat portion of the bias curve to zero pulse height.

The first check of the equipment was to measure the alpha-alpha cross section as a function of gas pressure at 4.3 MEV. The cross section was independent of pressure up to a maximum used of 1.1 cm Hg; so any pressure effect was assumed negligible. For the rest of the experiment a pressure of about .7 cm Hg was maintained. Next studied was detector background. Counting rates at various angles and energies

FARADAY CUP ASSEMBLY

FIG. 8

were taken with no absorber in front of the detector, with just sufficient absorber to stop the alpha particles and with a thick absorber. The counting rate was independent of absorber thickness after the alpha particles were stopped completely, and the difference between total counting rate and background was fairly reproducible. The background amounted to only one percent in some forward directions where the counting rates were high and the alpha particles energetic, but at some backward angles where the recoiling alpha energy was low and the cross section small the background amounted to over 50% of the total counting rate. Obviously a better detector was in order. But the experiment was completed by taking a background count after every data point. Angular distributions were taken at energies of 5.63 MEV and 4.31 MEV. Points were taken every 2.5° (laboratory angle) from 15° to 70° for the 5.63 MEV data and every 5° for the 4.31 MEV experiment. The 4.31 MEV angular distribution was taken with an evaporated CsI(Tl) scintillator. Background for this detector was found to be negligible forward of 60° where noise from the photomultiplier began to contribute to the counting rate. For identical particle scattering, the cross section is symmetrical about 45° ; so this provides an accurate check on the symmetry of the equipment. Data points at about 4 MEV were taken at 30° and 60° on both sides of the beam as a further check of symmetry. Nothing in the data pointed to a lack symmetry in the equipment, and a consistent one to two percent deviation would have been observable. Excitation curves from 3.8 to 5.9 MEV were taken for angles of 15° to 17° ,

20° , $27^{\circ}22'$, and $35^{\circ}4'$. The angles were chosen to simplify phase shift analysis. The highest data points were taken at 5.9 MEV. This was the only Rice Institute Van de Graaff data obtained above 5.5 MEV.

To check the equipment, argon, at about 2.3 cm oil pressure, was put in the chamber, and an angular distribution was taken at 4.31 MEV to confirm that alpha-argon scattering is purely Rutherford at this energy. An excitation curve at 30° was obtained from 3.8 to 5.9 MEV. The alpha-argon angular distribution obeyed the $\csc^4 \theta/2$ law expected. However, all the experimental cross sections were somewhat higher than the theoretical Rutherford cross section. Fig. 9 shows the Ratio to Rutherford of the alpha-argon excitation curve assuming, for computing the experimental cross section, that the charge state of the integrated alpha particles was two. There was obviously a current integration problem. The thickness of the Faraday cup isolating foil was measured to compute the energy of the alpha particles striking the cup. The actual charge state of the alpha particles could then be determined from the results of experiments summarized by Allison and Warshaw in 1952.¹⁸ This predicted a Ratio to Rutherford of near unity at the higher energies and about 1.3 at the lowest energy. A discrepancy remained.

The alpha-alpha data had to be normalized to be useful at all. If the alpha-argon discrepancy was due to a fault of the Faraday cup assembly, and if that fault was not time dependent, then the proper normalization for the alpha-alpha

data would be obtained by dividing the alpha-alpha cross section at each energy by the Ratio to Rutherford of Fig. 9 at that energy. This was done, and the modified data proved to be consistent with the more dependable data taken later.

The July Experiment. The inadequacy of the Faraday cup for alpha particle integration altered the whole attitude toward the July experiment. It was hoped that the fault could be corrected. But if not, then the accuracy of the modified data of the previous run had to be determined. It was thought that the best way to check the modified data was to extend the Rice Institute data down to 3 MEV where it could be compared with the Carnegie Institution data.⁹ The conditions of the July experiment were as follows.

The Faraday cup isolating foil was an aluminum foil of about 0.7 cm air equivalence. This placed the practical lower energy limit at about 2.5 MEV. The accelerator analyzing magnet had not been altered since the previous alpha-alpha experiment so the same energy calibration applied. This calibration was checked with the chamber in operating position by obtaining an excitation curve for $C^{13}(\alpha, n)O^{16}$ and comparing the resonance energies with those of Schiffer, Bonner, Kraus and Marion.²¹ The energy scale was still somewhat uncertain above 4.6 MEV. The only change in target gas measurement was that the oil manometer measuring target gas pressure was read with a cathetometer calibrated to .05 mm.

The neon bulb type current integrator is difficult to calibrate more accurately than about three percent. For this reason a more accurate, electrometer type current integrator²⁰

was constructed. The limiting error for this instrument was the one percent condenser calibration. The Faraday cup assembly was replaced by the re-entrant assembly shown in Fig. 11. By placing the cup closer to the target volume the diameter, and hence the thickness, of the isolating foil could be reduced.

The geometry was measured before and after the experiment and no discrepancy found. The scintillator for the experiment was a CsI(Tl) screen made by evaporating 3 mg/cm^2 CsI on a glass microscope slide.

Three point angular distributions for alpha particles on argon at bombarding energies of 3, 3.5, 4 and 4.5 MEV were taken to determine the effective charge state of the integrated alpha-particles. It was found that the ratio to Rutherford of the alpha-argon cross section, shown in Fig. 10, was still not unity. The expected ratio to Rutherford, computed from the isolating foil thickness and the data given in reference 18, was about 1.04 at 3 MEV compared with the measured ratio of 1.3.

The alpha-alpha experiment was then done. Excitation curves at $27^{\circ}22'$ and 20° were taken from 3 to 5.5 MEV. Angular distributions were taken at 2.96, 3.96 and 4.98 MEV with points taken every 2.5° . The data was normalized, as before, with the alpha-argon ratio to Rutherford and found to be about 4% to 6% above the Carnegie data at 3 MEV and to be consistent at all angles and energies with the corrected May data. It was concluded that both sets of corrected alpha-alpha data were in agreement to about five percent below 4.6

FIG. 11

MEV. It was still desirable to obtain current integration in agreement with the known behavior of the charge state of the alpha particle, and to establish a reliable energy scale to 6 MEV.

The December Experiment. The December experiment completed the series of scattering experiments. The equipment changes made for this experiment were apparently successful in that the experimental alpha-argon cross section accurately obeyed a Rutherford law when corrected for the known alpha particle state of ionization. Further, the December experiment determined the accuracy and reliability of the results of the previous cross section measurements.

The practical low energy limit for this experiment, as obtained from the Faraday cup isolating foil thickness, was about 2.5 MEV. The pole pieces of the accelerator analyzing magnet had been altered again to make possible measurement of field strength with a proton moment device. The magnet had been calibrated to 5.6 MEV--a calibration being necessary since the effective radius of curvature was strongly energy dependent above 4 MEV. The calibration was checked with the chamber in place by the $C^{13}(\alpha, n)O^{16}$ technique. The energy loss of the alpha particles before arriving at the target was a maximum of 35 ± 6 KEV.

The target gas was maintained at a pressure of about 4 mm Hg.

The electrometer type current integrator previously described was used. The Faraday cup assembly is shown in Fig. 5. This assembly was designed after a two day alpha-argon scat-

tering experiment devoted entirely to determining the faults of the previous Faraday cup assemblies.

A second scintillation detector, described previously, was installed in the chamber before the December experiment. This addition facilitated obtaining relative angular distributions, and allowed an independent check on the other detector. CsI(Tl) evaporated on glass was used as the scintillator for both detectors.

Counting statistics for this experiment varied from point to point, as in the other two experiments, since a fixed amount of charge was taken for each data point. However, the statistical uncertainties were generally better than one percent. Exceptions occur at the minima of the cross section.

A number of alpha-argon cross sections were measured at various angles between 2.5 and 4 MEV. Fig. 12 shows the data as ratio to Rutherford, assuming the charge state of the integrated alpha particles to be two. The solid curve is the expected ratio to Rutherford,¹⁸ assuming the Faraday cup isolating foil to be of 0.78 cm air equivalence (as obtained from its weight of 1.17 mg/cm²). The agreement was most gratifying with maximum deviations of three percent.

Alpha-alpha excitation curves at angles of 27°22' and 20° were obtained from 3 to 5.5 MEV, and angular distributions were measured at energies of 2.97, 4.01 and 5.01 MEV. The results agree quite well with the previous modified data.

FIG. 12

Presentation of the Data

The formula for computing cross section from the experimental data, as derived previously, is:

$$(1) \quad \left(\frac{d\sigma}{d\Omega} \right)_{LAB} = \frac{N \sin \theta_L}{dIG}$$

Laboratory cross section is converted to center of mass cross section by the formula:²⁴

$$(2) \quad \left(\frac{d\sigma}{d\Omega} \right)_{CM} = \frac{1}{2\sqrt{1+\cos \theta_L}}$$

For elastically scattered, identical particles the laboratory scattering angle is one-half the center of mass angle, and the laboratory energy is twice the center of mass energy.

From equations (1) and (2) is obtained the center of mass differential cross section,

$$\left(\frac{d\sigma}{d\Omega} \right)_{CM} = \frac{N}{2dIG} \tan \theta_L$$

and since the center of mass cross section is the function of immediate theoretical usefulness, the data was tabulated in this form.

Figs. 13 and 14 are angular distributions taken during the May experiment. Figs. 15, 16 and 17 are angular distributions taken during the July experiment. And Figs. 18, 19 and 20 are angular distributions taken during the December experiment.

Figs. 21 and 22 are composite excitation functions on which all the alpha-alpha excitation data taken have been plotted. The 27° and 22° curves include overlapping data

FIG. 13

FIG. 14

FIG. 15

FIG. 16

$\alpha\cdot\alpha$ ANG. DISTRIBUTION

$E_L = 4.98$ MEV

• DATA

— THEORY $\delta_0 = 98.8^\circ$

$\delta_2 = 29.3^\circ$

.4

.3

.2

.1

BARNS/STERADIAN(CM)

FIG. 17

θ (CM)

10°

20°

30°

40°

50°

60°

FIG. 18

FIG. 19

FIG. 20

FIG.21

$\alpha\cdot\alpha$ EXCITATION FUNCTIONS

+ DATA $\theta(\text{CM}) = 70^\circ 7'$

• DATA $\theta(\text{CM}) = 54^\circ 44'$

— THEORY (PHASES FROM FIG

EXCITATION FUNCTIONS

+ DATA $\theta(\text{CM}) = 70^\circ 7'$

• DATA $\theta(\text{CM}) = 54^\circ 44'$

— THEORY (PHASES FROM FIG.23)

$\alpha\cdot\alpha$ EXCITATION FUNCTIONS

- DATA $\theta(\text{CM})=40^\circ$
- + DATA $\theta(\text{CM})=30^\circ 33'$
- THEORY (PHASES FROM FIG

EXCITATION FUNCTIONS

• DATA $\theta(\text{CM}) = 40^\circ$

+ DATA $\theta(\text{CM}) = 30^\circ 33'$

— THEORY (PHASES FROM FIG.23)

from all three experiments. The $15^{\circ}17'$ and $35^{\circ}3'$ excitation curves were taken in the first (May) experiment only. It is noted that there is apparently an excellent agreement of the energy scale of the three sets of data up to 5.5 MEV. This agreement of energy scale was used to justify probable energy error on the 5.9 MEV data of about ± 50 KEV. The solid curves shown with the excitation functions are computed from the phase shifts given in Fig. 23. Apparently the modified alpha-alpha data obtained in May and July are of an accuracy comparable to the December data.

Phase Shift Analysis

The quantum mechanical treatment of an energy characteristic state describing the collision process is the Schroedinger equation with the time removed:²⁴

$$\left[-\frac{\hbar^2}{M} \nabla^2 + V(r) \right] \psi = E \psi$$

In this expression, $V(r)$ is the interaction potential, and M the alpha particle mass.

Solutions to the equation must match certain boundary conditions in order to describe the scattering experiment. Before collision the particles are described by a plane wave, and after collision by a spherical wave emerging from a scattering center. The plane wave corresponds to the incident beam and the outgoing spherical wave to the scattered particles. The boundary condition at infinity is then: (For coulomb scattering, the boundary condition at infinity is actually a distorted plane wave. This complication does not change the

argument presented here.)

$$\lim_{r \rightarrow \infty} \psi = e^{ikz} + f(\theta) \frac{e^{ikr}}{kr}$$

The coefficient of the scattered wave determines the number of particles scattered at angle θ , so that by proper normalization:

$$\left(\frac{d\sigma}{d\Omega} \right)_{CM} = |f(\theta)|^2$$

If the interaction between the two particles can be described by a coulomb potential modified at small interparticle distances by a short range potential, then the solution for $f(\theta)$ may be written,

$$f(\theta) = -\frac{\eta}{2k} \csc^2 \frac{\theta}{2} e^{i\eta \ln \csc^2 \frac{\theta}{2}} + \frac{1}{k} \sum_{L=0}^{\infty} (2L+1) e^{i(2\alpha_L + \delta_L)} \sin \delta_L P_L(\cos \theta)$$

$$\alpha_L = \sum_{S=1}^L \arctan \eta_S$$

$$\eta = \frac{ZZ'e^2}{\hbar v}$$

δ_L = Phase shift of partial wave of order L .

For identical (Bose) particles, as obtains in alpha-alpha scattering, the wave function must be symmetrical under interchange of the two particles, i.e., rotation of the system by π . This alters the boundary condition at infinity to:

$$\lim_{r \rightarrow \infty} \psi = e^{ikz} + [f(\theta) + f(\pi - \theta)] \frac{e^{ikr}}{kr}$$

So that:

$$(3) \quad \left(\frac{d\sigma}{d\Omega} \right)_{CM} = |f(\theta) + f(\pi - \theta)|^2$$

And:

$$\begin{aligned}
[f(\theta) + f(\pi - \theta)] = & -\frac{\eta}{2k} \csc^2 \frac{\theta}{2} e^{i\eta \ln \csc^2 \frac{\theta}{2}} \\
& - \frac{\eta}{2k} \csc^2 \left(\frac{\pi - \theta}{2}\right) e^{i\eta \ln \csc^2 \left(\frac{\pi - \theta}{2}\right)} \\
& + 2 \sum_{L=0, 2, 4, \dots}^{\infty} (2L+1) e^{i(2\alpha_L + \delta_L)} \sin \delta_L P_L(\cos \theta)
\end{aligned}$$

Odd values of the sum vanish since $P_L(\cos \theta) = P_L(\cos(\pi - \theta))(-1)^L$. More fundamentally, states of odd parity cannot be used to describe a symmetrical system.

The utility of equation (3) for alpha-alpha scattering is that up to 23 MEV only three terms of the sum on L are needed to fit the experimental data. In the region of 3 to 6 MEV it was found that only two terms were needed, $L = 0$ and $L = 2$. From just the practical point of view this means the experimentally determined alpha-alpha cross section surface versus energy and angle can be reduced to three energy dependent parameters; and any angular distributions at energies E , $0 < E < 23$ MEV, can be described by three numbers.

The technique used to solve for the phase shifts for the Rice Institute data from 3 to 6 MEV was as follows. First it was assumed that only S and D-wave phase shifts would be necessary to fit the data. One notes that at a zero of the second order Legendre polynomial, $P_2(\cos \theta)$, equation (3) can be explicitly solved for the S-wave shift, δ_0 . Then by using this value of δ_0 , data at some other angle determines the D-wave phase shift for that energy.

Let:

$$x+iy = -\frac{1}{2} \csc^2 \frac{\theta}{2} e^{i \ln \csc^2 \frac{\theta}{2}} - \frac{1}{2} \csc^2 \left(\frac{\pi-\theta}{2} \right) e^{i \ln \csc^2 \frac{\pi-\theta}{2}} + i P_2(\cos \theta) e^{2i\alpha_2}$$

at $54^\circ 44'$,

$$\sin(2\delta_0 - \xi) = \frac{1}{A} \left[k^2 \left(\frac{d\sigma}{d\Omega} \right)_{cm} - A^2 - 1 \right]$$

where: $A^2 = x^2 + (y+1)^2$

$$\tan \xi = \frac{y+1}{x}$$

at $\theta \neq 54^\circ 44'$

$$\sin(2\delta_2 + 2\alpha_2 - \xi) = \frac{1}{10 A P_2(\cos \theta)} \left[k^2 \left(\frac{d\sigma}{d\Omega} \right)_{cm} - A^2 - 25 P_2(\cos \theta) \right]$$

Where:

$$A^2 = (x + \sin 2\delta_0)^2 + (y + 2 \sin^2 \delta_0)^2$$

$$\tan \xi = \frac{y + 2 \sin^2 \delta_0}{x + \sin 2\delta_0}$$

The next step in the analysis was to take these trial phase shifts and compute the entire angular distribution, and then to compute angular distributions for $\delta_0 + 1^\circ$ and $\delta_2 + 1^\circ$. This procedure gave a family of curves which when compared with the data made possible an accurate estimate of the best phase shifts to try. These new, estimated phase shifts were then converted to an angular distribution, and it was generally found to be an acceptable fit to the data. The ability to fit the data with only two phase shifts, δ_0 and δ_2 , justified the original assumption. All of the data were not of the same dependability, and it was decided that this fact justified not making a least squares fit. The various final fits of the angular distributions are shown

with the data in Fig. 13 to 20.

The phase shifts, obtained as described above, were plotted versus energy, and are shown in Fig. 23. These phase shift curves were converted to excitation curves at center of mass angles of $30^{\circ}33'$, 40° , $54^{\circ}44'$ and $70^{\circ}7'$, and are shown with the data at those angles in Fig. 21 and 22.

While fitting the angular distributions it was found that the two data points forward of 20° in the laboratory were inconsistent with the rest of the angular distribution; i.e., a theoretical curve passing through about twenty data points back of 20° would not pass through the two most forward data points. The deviation for these two angles was in the direction to be explained by scattering into the detector of additional particles whose origin was other than the target volume. The discrepancy was about ten percent in the worst cases. For these reasons, the phase shifts were chosen to fit the data backwards of 20° . Above 5 MEV the effect disappeared, and a satisfactory fit to the data at all angles was obtained. The estimated error for S and D-wave phase shifts is $\pm 1.5^{\circ}$ from 3 to 5.5 MEV and $\pm 2^{\circ}$ from 5.5 to 5.9 MEV.

The three reliable sets of data on alpha-alpha scattering, obtained at the Carnegie Institution of Washington, the Rice Institute, and the University of Illinois, are apparently consistent. Fig. 24 shows all three sets of data.

EXPERIMENTAL S AND D-WAVE PHASE SHIFTS FOR $\alpha\cdot\alpha$ SCATTERING

FIG. 23

AL S AND D-WAVE PHASE R $\alpha\cdot\alpha$ SCATTERING

FIG. 23

TOTAL S-D-G AND I-WAVE PHASE SHIFTS FOR α - α SCATTERING

IV. ANALYSIS OF THE DATA

Alpha-alpha scattering is but one of many techniques of studying the unstable nucleus of Be^8 . Fig. 25 is a reproduction of a diagram from Ajzenberg and Lauritsen⁷ showing what is known of the level structure and the various nuclear reactions that give information about Be^8 . Several things are apparent from this diagram. First, Be^8 is unstable to alpha decay by 96 KEV.^{7,25} The next mode of decay that is energetically possible, a break-up into $\text{Li}^7 + \text{H}^1$, occurs at 17.242 MEV^{7,26} excitation. This decay could be attained by an alpha-alpha scattering experiment of alpha energy equal to 34.676 MEV. Below the proton emission threshold eight levels are proposed by Ajzenberg and Lauritsen.

The existence of the 2.9 state is unquestioned, and even before the experiments reported in this thesis were completed there was good evidence that this level was a 2^+ state.²⁷ The existence of the 4.2 and 5.4 states²⁸ is questionable, in that they have not been observed in most experiments. None of the experimental evidence for these states is unambiguous. The 7.55 and the 10.8 states are based on the alpha-alpha data of Steigert at Indiana.²⁹ These results are in rough agreement with the University of Illinois data¹⁰ above about 9 MEV excitation. However, the two experiments differ radically at the lower energies. The University of Illinois data have more internal consistency, and from the necessary continuity of phase shifts appear to be more reasonable when compared with the Rice Institute data. It will be

assumed that the University of Illinois data supersedes the data obtained by Steigert, et. al. On this basis the 7.55 state is eliminated and the 10.8 state confirmed (though moved up to 11.6 MEV). The existence of the 14.7 state is doubtful because of the ambiguity of the interpretation of the Li(d,n) experiment³⁰ in which the inelastic neutrons are detected with photographic plates. The 16.06 and 16.72 MEV states are accessible to neutron threshold techniques via ${}^7\text{Li(d,n)}{}^8\text{Be}$ and have been confirmed by Bonner and Cook.³¹

Fig. 26 shows a diagram of ${}^8\text{Be}$ which includes only the confirmed states and the low lying nuclear parents. This level scheme is consistent with that proposed by Moak and Wisseman³² as a result of a very careful ${}^6\text{Li}({}^3\text{He},\text{p}){}^8\text{Be}$ experiment. The most striking feature of this diagram is its simplicity below 15 MEV. A quantized rigid rotator has energy states in the ratio $L(L+1)$ where L is the angular momentum quantum number of the rotator. The ratio of energy states for $L = 0, 2, 4$ is then 0, 3, 10. The comparison of this to the ${}^8\text{Be}$ spectrum has been subject to much speculation. Any nuclear model which permits low lying states to be explained by rotation of the entire nucleus can fit these three levels. It is apparent that above ~ 15 MEV this simplicity vanishes.

From the nuclear parentage point of view, one would expect that ${}^8\text{Be}$ could be described by a linear combination of wave functions of the form ${}^4\text{He} + {}^4\text{He}$, ${}^7\text{Li} + \text{H}^1$, ${}^7\text{Be} + \text{n}^1$, ${}^6\text{Li} + \text{H}^2$, etc. On the basis of this model, one would suspect that the ground state would be largely of alpha-alpha parentage because the other parents are so far removed in energy. At the

PROPOSED LEVEL STRUCTURE OF Be^8

FIG. 26

higher energies of excitation the ${}^8\text{Be}$ nucleus should consist of a mixture of the various parents. The model is completely general, since all permutations of the eight fundamental particles may be classed as parents. This feature makes a quantitative calculation virtually impossible unless all but one or two possible parents are of negligible importance. The rest of this discussion will assume that below 15 MEV the only nuclear parent of ${}^8\text{Be}$ is the two body alpha-alpha configuration. Experimental justification for this assumption will never be complete because of the nature of a negative experiment. However, the known stability of ${}^4\text{He}$ and the width of the 2^+ and 4^+ states lend credence to the supposition.

One Level Dispersion Theory

The extraction of phase shifts from alpha-alpha scattering data does not in itself assume that only a two body mechanism is operative in the scattering process. According to the Wigner-Eisenbud formalism³³ the experimental phase shifts can be expanded in terms of resonance and potential scattering regardless of the mechanism. The usual practice when applying this formalism to an isolated resonance is to assume the potential scattering is due to a hard sphere with a radius independent of energy. The single level form is:³³

$$\delta_L = \delta_{\lambda,R} - \phi_L$$

$$\phi_L = \arctan \left[\frac{F_L}{G_L} \right]_{\rho=kR}$$

$$\delta_{\lambda,R} = \arctan \frac{\frac{1}{2} \bar{\Gamma}_{\lambda,L}}{E_{\lambda,L} - \Delta_{\lambda,L} - E_{cm}}$$

$$\frac{1}{2} \bar{\Gamma}_{\lambda,L} = \left[\frac{\rho \gamma_{\lambda}^2}{A_L^2} \right]_{\rho = kR}$$

$$A_L^2 = F_L^2 + G_L^2$$

$$\Delta_{\lambda,L} = \gamma_{\lambda}^2 [g_L + L]_{\rho = kR}$$

$$g_L = \rho \left[\frac{1}{F_L} \frac{\partial F_L}{\partial \rho} - \frac{1}{A_L^2} \frac{G_L}{F_L} \right]_{\rho = kR}$$

With symbols defined:

E_{cm} = Energy of center of mass system, MEV.

$E_{\lambda,L}$ = Constant expansion parameter, MEV.

F_L = Regular coulomb wave function.

G_L = Irregular coulomb wave function.

$$\rho = kr$$

$$k = \sqrt{ME_{cm}}/\hbar^2$$

R = Nuclear radius, cm.

γ_{λ}^2 = Reduced width (constant), MEV.

$\bar{\Gamma}_{\lambda,L}$ = Laboratory width, MEV.

Φ_L = Potential, or hard sphere, phase shift.

$\delta_{\lambda,R}$ = Resonance phase shift.

The resonance energy is defined to be that energy at which $\delta_{\lambda,R} = 90^\circ$.

An infinite number of resonance terms is a complete set of functions so that there is no question of being able to fit

the data with an expansion of this sort. The form is especially useful for analyzing scattering anomalies for which only one resonance term is needed in a large energy region and for which off resonance scattering is readily described by some slowly varying function of the energy such as hard sphere scattering. This approach has been notably successful in describing the scattering of protons from carbon.²⁰ Of course the potential phase shift cannot be completely arbitrary or no information is gained. The single level formula can be useful in determining approximate resonance parameters even if the description is not accurate off resonance. Fitting the data in the region of the resonance yields an approximate resonance energy and reduced width. This fitting of the resonance is done for alpha-alpha scattering for the ground, 2.9, and 11.6 MEV states and shown in Fig. 27. A G-wave analysis was included in the data kindly sent by Nilson,¹⁰ and also shown is a G-wave analysis, done by the author, based on the parameters shown in Fig. 27. The parameters for the D-wave analysis were obtained by assuming that the resonant energy is 6 MEV bombarding energy corresponding to an excitation of 2.9 MEV in ${}^8\text{Be}$, and that the hard sphere size necessary to fit the data at the resonance is an adequate description of the non-resonant phase shift near the resonance. The experimental D-wave phase shift at 6 MEV is 70° . The resonant phase shift at resonance is 90° . Therefore the hard sphere phase shift must be -20° at 6 MEV. This gives a hard sphere radius of 5.0×10^{-13} cm. Since the parameter $E_{\lambda,l}$ is adjusted to force the resonance to occur at 6 MEV, the only free para-

COMPARISON OF THE PREDICTIONS OF ONE-LEVEL DISPERSION THEORY WITH THE DATA

(LAB)

M

(SPHERE SCATTERING)

$$\left(\frac{\gamma^2}{\lambda^4}\right) = 0.27 \text{ MEV(LAB)}$$

$$\left(\frac{\gamma^2}{\lambda^4}\right) = 1.08 \text{ MEV(LAB)}$$

♦ S-WAVE DATA
+ D-WAVE DATA
* G-WAVE DATA

COMPARISON OF THE PREDICTIONS OF ONE-LEVEL DISPERSION THEORY WITH THE DATA

- ◆ S-WAVE DATA
- + D-WAVE DATA
- * G-WAVE DATA

FIG. 27

meter is the resonance width which is adjusted to fit the low energy data. The most important feature of this analysis is the wide deviation of theory from data at the higher energies. Now, hard sphere phase shifts decrease monotonically to minus infinity as the energy goes to infinity. However, the phase shift due to a finite interaction potential goes to zero as the potential energy becomes negligible compared with the kinetic energy of the particles. The lack of fit to the high energy data indicates that there is an extremely broad D-state just wide enough to counteract the effect of hard sphere scattering in this energy region, or that the kinetic energy of the alpha particles is comparable to the potential energy of the interaction in this region. The latter is more likely since a state as broad as would be required in the first implication would be many times the width of the first D-state and that state is some 70% of the Wigner limit.³⁴ The Wigner limit is defined as $3\hbar^2/2\mu R^2$ and is the theoretical upper limit of a reduced width. The theory is not exact, of course, and depends directly on the not too well defined nuclear radius. An experimental reduced width approaching the Wigner limit implies that the state is a single particle configuration.³⁴

It is of interest to compare the parameters required by the one level formula to fit the D and G states. The G-state parameters in the table are based on the re-analysis of the high energy data shown in Fig. 27.

	D-state	G-state
Excitation	2.9 MEV	11.6 MEV
Reduced Width	0.9 MEV (CM)	1.5 MEV (CM)
Wigner Limit	1.3 MEV	1.6 MEV
Hard Sphere Radius	5.0×10^{-13} cm	4.5×10^{-13} cm
$\delta_\lambda^2 / \frac{3\hbar^2}{2\mu R^2}$	0.72	0.95

Since the nucleus apparently has a different effective size for the two energies the maximum resonance width as determined by the Wigner formula,³⁴ $3\hbar^2/2\mu R^2$ has changed. The extreme width of the states implies that they are single particle states. The fact that the ratio of the reduced width to the Wigner limit is near unity for both resonances makes possible the speculation that this ratio should be near unity for the S-state. No scattering data is available for the ground state, the nearest point being at 400 KEV alpha particle energy.⁸ The S-wave phase shift was computed assuming the ground state widths equal to 1/2 and 1/8 of the Wigner limit and assuming a hard sphere radius of 5.7×10^{-13} cm, which was chosen to fit the low energy scattering data. These curves are also shown on Fig. 27. The above reduced widths corresponds to laboratory widths of 20 and 5 electron volts (CM), respectively.

Alpha-alpha Interaction Potentials

No discussion of alpha-alpha scattering would be complete without a comparison of the data with the predictions of various alpha-alpha interaction potentials. The assump-

tion of the existence of a potential that will describe all the scattering data is the basis of the alpha particle model. It is obviously impossible to guess the shape of such a potential. But it is interesting to see the comparison of data with the predictions of simple potentials. The procedure for a calculation is first to assume a short range potential, and then to determine solutions of the separated Schroedinger equation in the region of the potential. At some radius, R , it is assumed that the interaction potential becomes coulomb. The logarithmic derivative of the interior solution is matched to the logarithmic derivative of a linear combination of the regular and irregular coulomb wave functions at R , and the relative mixture of coulomb solutions needed determines the phase shift. The first restriction of the data on the nuclear potential is that the ground state of ${}^8\text{Be}$ must occur at 192 KEV. A trial and error procedure, that is satisfactory, is to vary one parameter of the potential until an S-wave phase shift of 90° at 192 KEV is predicted. For any two parameter potential, such as a square well, there is only one parameter left which can be used to fit the rest of the data.

An example of this technique for the square well follows.
Let

$$\begin{aligned} V(r) &= -b & r < R \\ V(r) &= 4e^2/r & r > R \end{aligned}$$

The wave equation to be solved is:

$$\frac{d^2 u_L}{dr^2} + \frac{M}{\hbar^2} \left(E - V - \frac{L(L+1)}{r^2} \right) u_L = 0$$

For $L = 0$, the solution inside the nuclear radius is $\sin K_w r$, where $K_w = \sqrt{M(E - b)/\hbar^2}$. Outside R , the solution is a linear combination of the regular and irregular coulomb wave functions, F_L and G_L . The phase shift, δ_L , is defined by:

$$r > R \quad \left| \quad u_L = F_L \cos \delta_L + G_L \sin \delta_L \right.$$

The two solutions must match in slope and magnitude at the nuclear surface, so that equating their logarithmic derivatives at $r = R$ gives for $L = 0$,

$$K_w \cot K_w R = \left[\frac{F_0' + \tan \delta_0 G_0'}{F_0 + \tan \delta_0 G_0} \right]_{r=R}.$$

This expression can be solved for the phase shift, δ_0 .

$$\tan \delta_0 = - \left[\frac{F_0' - F_0 \frac{K_w}{k} \cot K_w R}{G_0' - G_0 \frac{K_w}{k} \cot K_w R} \right]_{r=R}$$

The ground state requirement is that the denominator be zero at 192 KEV (LAB).

$$K_w R \cot K_w R = \left[k R \frac{G_0'}{G_0} \right]_{k=R_{\text{res. } E_1}, r=R}$$

Or, for $R = 5.7 \times 10^{-13} \text{ cm}$;

$$K_w R \cot K_w R = -1.435$$

Solved, this gives: $b = 2.79 \text{ MEV (LAB)}$, or 1.395 MEV (CM) .

For comparison, $4e^2/r$ at $r = 5.7 \times 10^{-13} \text{ cm}$ is 1.01 MEV (CM) .

The results of a square well of radius $5.7 \times 10^{-13} \text{ cm}$ and depth 1.395 MEV (CM) are shown in Fig. 28. The low energy S-

COMPARISON OF PREDICTIONS OF A SQUARE WELL WITH THE DATA

$R = 5.7 \times 10^{-13}$ CM
 $b = 2.79$ MEV (LAB)

COMPARISON OF PREDICTIONS OF A SQUARE WELL WITH THE DATA

$R = 5.7 \times 10^{-13}$ CM
 $b = 2.79$ MEV (LAB)

◆ S-WAVE DATA
† D-WAVE DATA
* G-WAVE DATA

$\delta_0^{-\pi}$

FIG. 28

δ_2

δ_4

AB), MEV

15

20

wave data is in excellent agreement since the well was chosen to fit this region. Obviously, the true potential shape is not even similar to a square well. This calculation points out the great amount of shape information available in the data and is to be compared with the fact that for low energy proton-proton scattering virtually any two parameter potential will fit the data.¹¹

The true shape of the potential being unknown, there is considerable calculational advantage in choosing, for comparison, interaction potentials that permit solution of the wave equation in terms of tabulated functions. Haefner³⁵ proposed a potential of the form:

$$\begin{aligned} V &= 4e^2/r & r > R \\ V &= -b + 30 \hbar^2/Mr^2 & r < R \end{aligned}$$

which gives interior solutions as higher order Bessel functions. This potential is physically justified by the possibility that the alpha particles, although obeying Bose-Einstein statistics as a whole, are repulsed at close distances by the Pauli exclusion principle acting between individual nucleons. The predictions of this potential were published by Haefner in 1951,³⁵ and the Illinois group¹⁰ extended the calculations to cover their energy range. This potential is a two parameter well and its predictions for various values of range and depth are shown in Fig. 29. The predicted variation of phase shift with energy is in better agreement with the data than the square well.

Another potential, tried by Margenau in 1941,³⁶ is the

COMPARISON OF PREDICTIONS OF HAEFNER TYPE POTENTIALS WITH THE DATA

FIG. 29

♦ S - WAVE
† D - WAVE
* G - WAVE
() = R , 10⁻

COMPARISON OF PREDICTIONS OF HAEFNER TYPE POTENTIALS WITH THE DATA

FIG. 29

♦ S - WAVE DATA
+ D - WAVE DATA
* G - WAVE DATA
() = $R, 10^{-13}$ CM

hard core well defined by:

$$\begin{array}{ll} V(r) = 4e^2/r & r > R \\ V(r) = -b & R > r > \text{core radius} \\ V(r) = \text{infinity} & r < \text{core radius} \end{array}$$

Margenau's purpose was primarily to study the alpha particle model on the basis of Van der Waals forces. He pointed out that any reasonable hard core potential gave the level structure for Be^8 as 0-2-4, in agreement with all recent data, but abandoned his attempt because the data available at that time implied a 0-0-X level structure.¹³ The predictions of this well are quite similar to the Haefner potential discussed previously.

The calculations, described above, somewhat justify the supposition that a central scalar potential of interaction between two alpha particles is the proper approach to a description of Be^8 . However, they emphasize the hopelessness of a trial and error technique of finding that potential.

Certain generalities become apparent on closer study of the predicted phase shift variations with energy of the two types of wells considered above. The phase shifts, δ_L , are a measure of the deviation of the true potential from the coulomb potential. (In the corresponding neutron problem the phase shift is a measure of the deviation of the actual potential from zero potential). The phase shifts tend to zero at energies large compared with the deviation of the true potential from coulomb. It is clear that any potential that does not have a pole at the origin of the form $4e^2/r$ will differ

from the coulomb potential by an infinite amount at the origin. This is the reason for the large phase shift variation predicted by the square well at the higher energies. The potential can alter the wave function, and thus the phase shift, only in spatial regions where the wave function, u_L , is not zero. Since u_L is required to be zero at the origin, no information is obtainable about the potential at the origin. But the size of the wave function near the origin increases at the higher energies (by virtue of the smaller wave length) so that the high energy S-wave phase shift contains information about the core of the potential. The position of the ground state is a restriction on the potential volume in that it requires deep wells to be of short range and shallow wells to be long range. The higher order partial waves are repulsed by the centrifugal barrier so that their positions of maximum probability are somewhat removed from the origin. These waves, then, give virtually no information about the core of the potential. However, the energy at which the higher order phase shifts begin to deviate from zero, and their exact energy variation, are a very sensitive function of the long range nature of the potential. With these general statements it is possible to construct a rather specific picture of the alpha-alpha interaction potential.

First, the position of the ground state limits the net attractive strength of the potential. The square well of radius 5.7×10^{-13} cm of depth 1.4 MEV is to be compared with the Haefner potential of radius 3.5×10^{-13} cm which requires a maximum depth of 23 MEV to fit the ground state. Because

of the strong repulsive core, this Haefner well is negative only for a distance of 1×10^{-13} cm, i.e., from 2.5 to 3.5×10^{-13} cm. This gives a net attractive volume, defined as the square root of the average well depth times the distance over which the well is negative, of about $3.3 \times 10^{-13} \sqrt{\text{MEV}}$ -cm for both wells.

The radial extension of the potential is best determined by the higher order partial waves. δ_2 and δ_4 both begin deviating from zero in the positive direction. This observation requires the long range features of the potential to be attractive. The two potentials tried indicate that the radius of this outer effect of the potential should be about 5×10^{-13} cm. Implied, then, is a potential trough, a few MEV deep, located at a radius of about 5×10^{-13} cm. The dimensions of the trough are limited by the S-wave data. It cannot be so deep as to produce bound states. And the potential cannot remain negative into the origin or another S-wave scattering state would be predicted as in the square well case (Fig. 28).

The S-wave phase shift actually becomes negative above 20 MEV(LAB) requiring the existence of a region, between the trough and the origin, in which the potential is more repulsive than the coulomb potential (see Appendix). For the Haefner potential, of radius 5.1×10^{-13} , δ_0 is negative at about 10 MEV(LAB). This implies that the actual core potential must lie somewhere between $4e^2/r$ and $30h^2/Mr$. Whether the potential is actually infinite at the origin is unobservable since the magnitude of the potential at $r = 0$ does not affect the scattering data. In fact, the potential shape at

radii less than 10^{-13} cm is probably meaningless since potentials at distances that small could be studied by scattering experiments only at energies in excess of 30 MEV. The two body model of alpha-alpha scattering would be expected to break down at such an energy.

These arguments yield a qualitative description for the alpha-alpha interaction potential of an attractive potential trough at about 5×10^{-13} cm and a moderately repulsive core. The width of the trough cannot be more than about 2×10^{-13} cm, because of the hard core, and the depth from the volume argument must be of the order of 3 MEV(CM). It is interesting to note that had the Indiana group²⁹ been correct in assigning an S-state at 7.5 MEV excitation, the above arguments would have indicated a "soft", or attractive, core alpha-alpha potential.

It has been shown by P. Juaho³⁷ that for the coulomb scattering problem a nuclear phase shift defined for all energy uniquely defines a potential provided (1) there are no bound states, (2) the nuclear potential falls off at large radius more rapidly than exponentially, and (3) the nuclear potential has no negative poles greater than first order. Jost and Cohn³⁸ developed three similar techniques for approaching the unknown potential problem for uncharged particles. It is not clear that any of the three methods could be easily modified to apply to the case with coulomb forces. A grossly approximate technique was used to determine the proton-proton potential from experimental phase shift data by Froberg.³⁹ However, the direct approach to the unknown

potential problem requires more investigation and it is hoped that the alpha-alpha data now available will provide the motivation for such an investigation.

The Shape Independent Formalism

There is another technique available for the study of alpha-alpha scattering data which was used by Jackson and Blatt in the analysis of low energy proton-proton scattering.¹¹ A development of the principle behind the "shape independent" formalism is presented here that is due to Bethe.⁴⁰

The separated wave equation for S-wave alpha particles acted upon by Coulomb forces and a short range nuclear potential $V(r)$ is:

$$\frac{d^2 u}{dr^2} + \frac{M}{\hbar^2} \left(E_{cm} - \frac{e^2}{r} - V(r) \right) u = 0$$

Writing this for two different energies E_a and E_b , multiplying the a equation by u_b and the b equation by u_a , and subtracting gives:

$$\begin{array}{l} u_b \left[-\frac{d^2}{dr^2} + \frac{1}{Dr} - V \right] u_a = k_a^2 u_a \\ u_a \left[-\frac{d^2}{dr^2} + \frac{1}{Dr} - V \right] u_b = k_b^2 u_b \\ \hline u_a'' u_b - u_b'' u_a = (k_b^2 - k_a^2) u_a u_b \end{array}$$

Integrating over r from ϵ to ξ gives:

$$(4) \quad u_b' u_a - u_a' u_b \Big|_{\epsilon}^{\xi} = (k_a^2 - k_b^2) \int_{\epsilon}^{\xi} u_a u_b dr$$

If this is repeated for the pure coulomb potential ($V(r) = 0$) and these solutions designated by U_a and U_b a similar result

is obtained.

$$(5) \quad u'_b u'_a - u'_a u'_b \Big|_{\epsilon}^{\frac{1}{2}} = (k_b^2 - k_a^2) \int_{\epsilon}^{\frac{1}{2}} u_a u_b dr$$

u can be normalized to unity at infinity. v can be similarly normalized and be defined as a linear combination of the regular and irregular coulomb wave functions such that outside the range of nuclear forces $u = v$, i.e.,

$$v = F_0 \cos \delta_0 + G_0 \sin \delta_0$$

However, for convenience in the calculations, v and u are normalized instead such that:

$$v = \frac{C_0}{\sin \delta_0} (F_0 \cos \delta_0 + G_0 \sin \delta_0) = C_0 (G_0 + F_0 \cot \delta_0)$$

If the limit $\frac{1}{2}$ is taken to infinity, v equals u .

Subtracting equations (4) and (5) gives:

$$(6) \quad (u'_b u'_a - u'_a u'_b) - (u'_b u'_a - u'_a u'_b) \Big|_{\epsilon}^{\infty} = (k_b^2 - k_a^2) \int_{\epsilon}^{\infty} (u_a u_b - u_a u_b) dr$$

As $r \rightarrow 0$, $u \rightarrow 0$ since this is the true solution to the potential problem and must match the boundary condition at the origin. Since v includes the irregular coulomb solution it does not go to zero as $r \rightarrow 0$. In terms of an expansion about the origin:

$$F_0 = C_0 k r + \mathcal{O}(r^2) \dots \quad F'_0 = C_0 k + \mathcal{O}(r) + \dots$$

$$G_0 = \frac{1}{C_0} \left[1 + \frac{F}{D} (\ln \frac{F}{D} + 2\gamma - 1) + h(\eta) \frac{F}{D} (1 + \frac{F}{D}) + \mathcal{O}(r) \dots \right]$$

$$G'_0 = \frac{1}{C_0} \left[\frac{1}{D} (\ln \frac{F}{D} + 2\gamma) + h(\eta) \frac{1}{D} + \mathcal{O}(r) \dots \right]$$

$\gamma = \text{Euler's Constant}$

$$h(\eta) = \operatorname{Re} \left\{ \Gamma'(\eta) / \Gamma(\eta) \right\} - \ln \eta$$

so that:

$$V = 1 + O(r) \dots \dots$$

$$V' = C_0^2 k \cot \delta_0 + \frac{1}{D} \left[\ln \frac{r}{D} + 2\gamma h(\eta) \right] + O(r) \dots \dots$$

Writing equation (6) with these expansions and letting ϵ go to zero, the diverging logarithmic terms cancel leaving:

$$C_{01} k \cot \delta_{01} + h(\eta_1) - C_{02} k \cot \delta_{02} + h(\eta_2) = D(k_1^2 - k_2^2)^2 \int_{\epsilon}^{\infty} (V_2 V_1 - U_2 U_1) dr$$

This is an expression that relates the phase shifts at two energies by an integral over the wave functions at those energies. If one energy, E_0 , is taken to be zero, then:

$$\frac{\pi \cot \delta_0}{e^{2\pi\eta_0 - 1}} + h(\eta) = D \left[-\frac{1}{2} + k^2 \int_0^{\infty} (V_0 V - U_0 U) dr \right]$$

since:

$$k D C_0^2 = \frac{\pi}{e^{2\pi\eta_0 - 1}}$$

$$h(\eta) \rightarrow 0 \text{ as } E \rightarrow 0$$

$$\lim_{E \rightarrow 0} \frac{\pi \cot \delta_0}{e^{2\pi\eta_0 - 1}} \equiv -\frac{1}{2}$$

If V (and U) can be expanded in terms of k^2 , then

$$V = V_0 + k^2 V(1) + k^4 V(2) + \dots$$

$$U = U_0 + k^2 U(1) + k^4 U(2) + \dots$$

And if $2 \int_0^{\infty} (V_0^2 - U_0^2) dr$ is defined as the effective range of the potential, r_0 , and the higher order terms unde-

fined, then:

$$K \equiv \frac{\pi \cot \delta_0}{e^{2i\eta} - 1} + h(\eta) = D \left[-\frac{1}{2} + \frac{5}{2} k^2 - P_0^3 k^4 + \dots \right]$$

The degree of approximation in the variation of K with energy hinges on the expansion of v and u in powers of k^2 . That the first term is u_0 or v_0 involves no approximation. However, whether u can be expanded in terms of k^2 depends on the potential shape. Therefore the series representation of the energy dependence of K is exact only to the k^2 term. However, if the error in assuming u to be expandible in k^2 involves only an error of order k^4 , then K would have a corresponding error of order k^6 .

The significance of the K function is that it relates the phase shift at different energies to a power series in the energy. It is a powerful technique, when applicable. Applied to the low energy proton-proton data it was found that only the first two terms of the energy expansion $(-\frac{1}{2} + \frac{5}{2} k^2)$ were necessary to fit the experimental data up to 20 MEV.¹¹ Other than facilitating the comparison of scattering data at different energies, this confirmed the fact that no potential shape information was available in the low energy proton-proton scattering data. Expanding K in energy inherently assumes that the range of the potential is short compared with the particle de Broglie wave length. An alpha particle has twice the momentum and hence half the wave length of a proton of the same energy. The maximum range of the potential between two alpha particles, as shown in the previous section, is about four times the range of forces be-

tween two protons. This means that the region of validity of the K function expansion is about 2 MEV for alpha-alpha scattering as compared with 20 MEV for proton-proton scattering.

The first limitation of the expansion of the K function in a power series of the energy is that an experimental zero phase shift at some energy other than zero cannot be represented because the cotangent becomes infinite as δ_0 goes to zero. It is obvious that if the experimental phase shift at some energy crosses zero or $n\pi$, where n is an integer, as the S-wave alpha-alpha phase shift does at 20 MEV, then K has a pole at that energy. A power series in the energy is not sufficiently general to describe the experimental K function for all energies of alpha-alpha scattering. There are, of course, many possible energy expansions that could be developed for K in which the expansion coefficients would be in terms of integrals over the wave function. These expansions, as the one derived above would yield no direct information about the potential shape. As an example, the effective range of the potential is defined to be $2 \int_0^\infty (v_0^2 - u_0^2) dr$, a term that is related to the range of nuclear force only indirectly. The data can be expanded in terms of some type of range expansion, but it is questionable whether any potential shape information could be obtained from such an expansion.

In spite of the infeasibility of fitting all the alpha-alpha data with a range expansion it is still true that K should be a smoothly varying function of the energy at distances far from its poles. Since the wave functions used in the derivation were expanded about the energy origin, the

theory should apply near zero energy, provided there are no poles in K near the origin. The advantage of this technique is that the low energy scattering data can be extrapolated to zero energy making possible examination of the unbound ground state of ${}^8\text{Be}$. This is the only technique, so far proposed, that has yielded precise information about the width of the ${}^8\text{Be}$ ground state.

For the theory to be applied, it was assumed that a two body description of ${}^8\text{Be}$ is complete, and that below 3 MEV the shape of K does not depend appreciably on the existence of a pole at 20 MEV, so that the usual energy expansion would apply. It was found that only terms up to k^4 in equation (7) were necessary to fit the data below 3 Mev. The coefficients obtained made possible an explicit expression for the S-wave phase shift as a function of energy. Using this expression the change in energy required to change the phase from 45° to 135° , across the ground state at 192 ± 4 KEV, can be computed giving the width of the ground state of ${}^8\text{Be}$. The number obtained is 17 electron volts (LAB) or 8.5 ± 3 electron volts (CM). The error is due primarily to the uncertainty in the energy of the ground state. If the assumed alpha binding energy of 96 KEV is in error by 6 KEV, the width is in error by a factor of 1.7. Fig. 30 shows the K function and its energy expansion. The data begins to deviate from this expansion in the vicinity of 5 MEV. Fig. 31 is an enlarged view to show the intersection of $h(\eta)$ and the energy expansion which occurs at the resonant energy, since $\cot(\delta_0 = 90^\circ)$ is zero. The reason for the small width obtained is that the penetrability

FIG. 30

$E \text{ (LAB) MEV}$

3 4 5

FIG. 30

$$a = -1.76 \times 10^{-10} \text{ cm.}$$

$$r_0 = 1.096 \times 10^{-13} \text{ cm.}$$

$$P = .314 \text{ cm.}$$

FIG. 31

E(LAB) MEV

$$K' = \frac{\pi \cot \delta_0}{e^{2\pi\eta} - 1} + h(\eta)$$

$$K' = \frac{\hbar^2}{zz'Me^2} \left[-\frac{1}{a} + \frac{r_0}{2} k^2 - P r_0^3 k^4 + \dots \right]$$

$$a = -1.76 \times 10^{-10} \text{ cm.}$$

$$r_0 = 1.096 \times 10^{-13} \text{ cm.}$$

$$P = .314 \text{ cm.}$$

FIG. 31

B) MEV

for alpha-alpha scattering at 192 KEV is about 10^{-4} .

The above analysis determines three expansion parameters which in turn determine the width of the ground state. The zero energy scattering length is a relatively large negative number which, by the usual interpretation, means that there is a slightly unbound state in Be^8 . This is not a derived result since the expansion was forced to cross $h(\eta)$ at 192 KEV, the accepted ground state energy. The effective range, r_0 , of the expansion is about a factor of three smaller than the range of potentials tried in the previous section that give reasonable results. The shape parameter P is undefined, but its size hints at the great amount of potential shape information available in the scattering data. The ratio of the reduced width to the Wigner limit is 0.2 (assuming a hard sphere radius of $5.7 \times 10^{-13} \text{cm}$) for the ground state as compared with 0.7 for the D-state and 0.9 for the G-state as determined in the section on dispersion theory.

The width determination assumes that the S-wave phase shift does not go to zero or $n\pi$ (except at zero energy) in the unexplored region below 400 KEV, where the lowest data point occurs. The work of P. Swan⁴¹ shows that the S-wave phase shift must be zero at zero energy since Be^8 has no bound states. This means that approximately one-quarter cycle of the wave function is inside the nuclear radius at the ground state energy. At resonance, the solution at the nuclear surface is approximately $\sin \frac{\pi}{2}$. Changing the energy by 200 KEV amounts to a change in the argument by approximately $\frac{1}{2}(4k)R \sim 2$. The solution to the wave equation at the nuclear radius is,

then, approximately $\sin(\frac{\pi}{2} \pm 0.2)$ so that the interior solution at the surface is decreased by about 5% by changing the bombarding energy by 200 KEV. But for the phase shift to go to zero, the wave function must go to the order of 10^{-4} in this region (in order to match the regular coulomb wave function at the nuclear radius). It would seem that no reasonable potential could produce such a rapid variation in the wave function as would be required to produce a zero in the S-wave phase shift for low energy alpha-alpha scattering. If the range expansion at low energy does not apply, it is because the two body scalar interaction description of Be^8 does not apply. Another way of stating the above result is that as long as the wave length of the incident alpha particles is long compared with the range of nuclear forces the K function cannot be a rapidly varying function of energy. This also might be taken as proof of the obvious fact that particles of long de Broglie wave length cannot be used to investigate the shape of short range potentials.

Direct measurement of the lifetime of the ground state has been attempted by several investigators by measuring the length of travel of the residual Be^8 nucleus produced by a number of reactions that leave Be^8 in a state of motion. The reaction $\text{B}^{11}(\text{p}, \alpha) \text{Be}^8$ was observed by Treacy⁴² and a lifetime of less than 4×10^{-15} seconds was determined for the ground state. This requires the width to be greater than 0.1 electron volt.

The shape independent formalism yields another piece of information. It is clear from Fig. 31 that K must cross $h(\gamma)$

near zero energy. This confirms the well established fact that the ground state of Be^8 is an S-state.⁴²

V. CONCLUSIONS

Reliable alpha-alpha scattering data is now available from 0.4 to 6 MEV and 12 to 23 MEV. The unexplored region from 6 to 12 MEV is of considerable interest not only because precise phase shift information is desirable for potential calculations, but also to determine the status of proposed states in this region. The slopes of the S-wave phase shifts at 6 and 12 MEV are consistent with a smooth variation in the unexplored region. This places an upper limit of about 1 MEV for an undetected S-state in this region. The D-wave phase shift data cannot rule out undetected D-states in the unexplored region because of its rapid change in slope due to the resonance at 6 MEV. The level structure of ${}^8\text{Be}$ given by the present alpha-alpha scattering data is:

	Spin	Excitation	$\sqrt{\lambda_L}$	δ_λ^2
Ground State	0	0	8.5 MEV(CM)	0.2 MEV(CM)
First Excited	2	2.9 MEV	2.0 MEV(CM)	0.9 MEV(CM)
Second Excited	4	11.6 MEV	6.7 MEV(CM)	1.5 MEV(CM)

The strongest evidence to date against the existence of any other low lying states, excepting the three listed above, is due the ${}^6\text{Li}({}^3\text{He}, p){}^8\text{Be}$ experiment of Moak and Wisseman.²⁸

It has been shown in this thesis that there is a considerable amount of potential shape information inherent in the data. It is clear that the potential must have a moderately repulsive core and be attractive in the vicinity of 5×10^{-13} cm. Nothing in the data is inconsistent with the two body concept

of Be^8 . The S-wave phase shift is probably known over an energy range wide enough to permit explicit extraction of a potential of interaction. Whether a potential so derived can describe the scattering for all partial waves must await further investigation. It has been shown that the potential so derived must qualitatively be of the form shown in Fig. 32.

FIG. 32

VI. DEFINITION OF SYMBOLS

a = Alpha-alpha zero energy scattering length.

A = Area of the rear slit of the detector slit system.

$$A_L = (P_L^2 + G_L^2)^{\frac{1}{2}}$$

b = Well depth parameter.

C_L = Coulomb penetrability defined by:

$$C_L = \left\{ \left[1 + \frac{a^2}{L^2} \right] \left[1 + \frac{a^2}{(L-1)^2} \right] \dots \left[1 + \frac{a^2}{1^2} \right] \right\}^{\frac{1}{2}} \left[\frac{2\pi\eta}{e^{\frac{1}{2}\pi\eta}} \right]^{\frac{1}{2}} / [1.3.5 \dots (2L+1)]$$

d = Number density of the target gas.

$$D = \hbar^2 / 4e^2 M$$

e = Electronic charge.

E_L = Energy of the incident alpha particle in the laboratory system.

E_{CM} = Energy of the center of mass system.

$E_{\lambda L}$ = Constant of Wigner-Eisenbud resonance expansion, units of energy.

$f(\theta)$ = Coefficient of the outgoing wave of plane wave scattered from a scattering center. It is normalized such that:

$$\left(\frac{d\sigma}{d\Omega} \right)_{CM} = |f(\theta)|^2$$

F_L = Regular coulomb wave function specified by:

$$\lim_{\rho \rightarrow \infty} F_L = \sin \left[\rho - \frac{L\pi}{2} - \eta \ln 2\rho + \arg \Gamma(L+1+i\eta) \right]$$

G_L = Irregular coulomb wave function specified by:

$$\lim_{\rho \rightarrow \infty} G_L = \cos \left[\rho - \frac{L\pi}{2} - \eta \ln 2\rho + \arg \Gamma(L+1+i\eta) \right]$$

G = Geometry factor of the detector slit system, $G = Aw/Rs$.

$$\mathcal{E}_L = \frac{e}{\hbar^2} \left[\frac{\partial F_L}{\partial \rho} - \frac{G_L}{A_L} \right]_{\rho=kR}$$

$\hbar$ = $\frac{1}{2\pi}$ times Planck's constant.

$$h(\eta) = \operatorname{Re} \left\{ \Gamma'(i\eta) / \Gamma(i\eta) \right\} - \ln \eta$$

I = Number of beam particles passing through target per data point.

$$k = (2\mu E_{cm}/\hbar^2)^{1/2} = (4.788 E_L)^{1/2} \times 10^{12} \text{ cm}^{-1}$$

$$K = \pi \cot \delta_0 / e^{2\pi i \eta_{-1}}$$

$$K_w = \left[\frac{M}{\hbar^2} (E_{cm} + b) \right]^{1/2}$$

l = Target thickness.

L = Angular momentum quantum number.

M = Mass of alpha particle.

N = Number of detected alpha particles for a particular data point.

$d(r^n)$ of order r^n .

P = Shape parameter in effective range formalism.

r = Distance between particles.

r_0 = Effective range as determined by the shape independent formalism.

R = Nuclear radius, or maximum extension of nuclear potential.

R = Radius of rotation of rear slit of detector slit system.

s = Separation of front and rear slit of detector slit system.

u_L = Wave function, for partial wave L , for case of nuclear and coulomb forces.

u_a, u_b = Wave functions at particular energies E_a and E_b .
The $L = 0$ subscript is understood.

v_a, v_b = Defined by:

$$v = \frac{1}{C_0} [G_0 + F_0 \cot \delta_0] \quad \text{evaluated at } E_a \text{ or } E_b.$$

$V(r)$ = Short range, radially dependent, central, nuclear potential.

$$V = MV(r)/\hbar^2$$

w = Width of front slit of detector slit system.

$$\alpha_L = \sum_{j=1}^L \arctan \frac{\eta}{s}$$

η^2 = Reduced width, MEV.

$\Gamma_{\lambda L}$ = Laboratory width. Defined by:

$$\frac{\Gamma_{\lambda L}}{2} = \left[\frac{\rho \gamma_L^2}{A_L^2} \right]_{\rho = KR} \quad \text{evaluated at the resonance energy.}$$

δ_0 = Nuclear phase shift of partial wave of order L.

$\delta_{\lambda R}$ = Contribution to the nuclear phase shift of order L of resonance term of Wigner-Eisenbud expansion.

$$\Delta_{\lambda L} = -\gamma^2 [g_L + L]_{\rho = KR}$$

$$\eta = 4e^2/\hbar v = \frac{1.259}{\sqrt{E_L}}$$

$$\Phi_L = \arctan \left[\frac{F_L}{G_L} \right]_{\rho = KR}$$

Θ_L = Laboratory scattering angle.

Θ_{CM} = Center of mass scattering angle.

Θ Also refers to center of mass scattering angle.

ψ = Total wave function.

$$\rho = kr$$

μ = Reduced mass of the alpha particles, $\frac{1}{2}M$.

$\left(\frac{d\sigma}{d\Omega} \right)_{LAB}$ = Differential scattering cross section in the laboratory system.

$\left(\frac{d\sigma}{d\Omega} \right)_{CM}$ = Differential scattering cross section in the center of mass system.

APPENDIX

Definition: The usual definition of the phase shift, δ_L , of partial wave L , caused by a central scalar potential, $V_T(r)$, that differs from a comparison potential, $V_c(r)$, only in the region $r < a$, is,

$$(1.) \quad [X_L]_{r=a} = \frac{F_L' + G_L' \tan \delta_L}{F_L + G_L \tan \delta_L}$$

$$(\cdot)' = \left(\frac{d}{dr} \right)$$

Where F_L and G_L are the regular and irregular solutions of,

$$(2.) \quad F_L'' + \left(1 - \frac{V_c(r)}{E} - \frac{\hbar^2 L(L+1)}{2\mu E r^2} \right) F_L = 0$$

The regular solution is specified by,

$$F_L = 0, r = 0$$

$$\lim_{r \rightarrow \infty} F_L \rightarrow \sin[\sigma_L(kr)]$$

Where the form of $\sigma_L(kr)$ depends on $V_c(r)$.

The irregular solution is specified by,

$$\lim_{r \rightarrow \infty} G_L \rightarrow \cos[\sigma_L(kr)]$$

$V_c(r)$ is the comparison potential. (For the coulomb problem, $V_c(r) = \frac{ZZ'e^2}{r}$ for the neutral particle problem, $V_c(r) = 0$.) If $V_T(r) = V_c(r)$, the phase shifts, δ_L , are identically zero.

X_L is defined as u_L'/u_L , where u_L is defined by,

$$(3.) \quad u_L'' + \left(1 - \frac{V_T(r)}{E} - \frac{\hbar^2 L(L+1)}{2\mu E r^2} \right) u_L = 0$$

u_L is the regular solution specified by,

$$u_L = 0, r = 0$$

$$\lim_{r \rightarrow \infty} u_L \rightarrow \sin[\sigma_L(kr) + \delta_L]$$

$V_T(r)$ is the true potential of interaction under study, and δ_L is a measure of the difference between $V_T(r)$ and $V_C(r)$.

The well known Riccati form of the Schroedinger wave equation is obtained by substituting $\chi_L = u'_L/u_L$ in equation (3.)

$$(4.) \quad \chi'_L + \chi_L^2 + 1 - \frac{V_T(r)}{E} - \frac{\hbar^2}{2\mu} \frac{L(L+1)}{Er^2}$$

The further substitution in equation (4.) of $\cot \Phi_L = \chi_L$, gives,

$$(5.) \quad \Phi'_L = 1 - \frac{1}{E} \left[V_T(r) + \frac{\hbar^2}{2\mu} \frac{L(L+1)}{r^2} \right] \sin^2 \Phi_L$$

So that a definition of δ_L that is entirely equivalent to equation (1.) is,

$$(6.) \quad [\Phi_L]_{r=a} = \arccot \left[\frac{F'_L + G'_L \tan \delta_L}{F_L + G_L \tan \delta_L} \right]_{r=a}$$

Theorem: If the experimental phase shift, δ_L , of any partial wave is negative at any bombarding energy, then $[V_T(r) - V_C(r)]$ must be positive (repulsive) in some spatial region provided $V_T(r)$ exists and is real.

Proof: A function, $\Delta_L(R)$, may be defined by,

$$(7.) \quad [\Phi_L]_{r=R} = \arccot \left[\frac{F'_L + G'_L \tan \Delta_L(R)}{F_L + G_L \tan \Delta_L(R)} \right]_{r=R}$$

Where R is a variable boundary position as shown in Fig.

33. $\Delta_L(R)$ may be thought of as the phase shift that would obtain if the potential for $r < R$ were $V_T(r)$, and for $r > R$ were $V_C(r)$. By this definition, $\delta_L = \Delta_L(R)$, for $R > a$. Equation (7.) must not be construed to imply that the solution to the wave equation, in the region where $[V_T(r) - V_C(r)]$ is non-zero, is $F_L + G_L \tan \Delta_L(R)$. The differential equation for $\Delta_L(R)$ may be

obtained by differentiating equation (7.) with respect to the position of the boundary R,

$$(8.) \quad \frac{1}{k} \frac{d\Phi_L(R)}{dR} = \left[\frac{1}{(F_L + G_L \tan \Delta_L(R))^2 + (F_L' + G_L' \tan \Delta_L(R))^2} \left\{ \dots \right. \right. \\ \left. \left. - (F_L + G_L \tan \Delta_L(R))(F_L'' + G_L'' \tan \Delta_L(R)) - (F_L + G_L \tan \Delta_L(R)) \sec^2 \Delta_L(R) G_L' \frac{d\Delta_L(R)}{k dR} \right. \right. \\ \left. \left. + (F_L' + G_L' \tan \Delta_L(R))^2 + (F_L' + G_L' \tan \Delta_L(R)) \sec^2 \Delta_L(R) G_L \frac{d\Delta_L(R)}{k dR} \right\} \right]_{r=R}$$

From equation (2.):

$$(9.) \quad [F_L'' + G_L'' \tan \Delta_L(R)] = [F_L + G_L \tan \Delta_L(R)] \left[\frac{V_L(R)}{E} + \frac{\hbar^2}{2\mu} \frac{L(L+1)}{Er^2} - 1 \right]$$

From the Wronskian relation:

$$(10.) \quad F_L' G_L - F_L G_L' = 1$$

From equation (7.):

$$(11.) \quad \sin^2 \Phi_L = \frac{(F_L + G_L \tan \Delta_L(R))^2}{(F_L' + G_L' \tan \Delta_L(R))^2 + (F_L + G_L \tan \Delta_L(R))^2}$$

From equation (5.):

$$(12.) \quad \frac{d\Phi_L(R)}{k dR} = \left\{ 1 - \frac{1}{E} \left[V_T(r) + \frac{\hbar^2}{2\mu} \frac{L(L+1)}{r^2} \right] \sin^2 \Phi_L \right\}_{r=R}$$

Using equations (9.), (10.), (11.), (12.) equation (8.)

may be reduced to:

$$(13.) \quad \frac{d\Delta_L(R)}{k dR} = -\frac{1}{E} [V_T(R) - V_L(R)] [F_L \cos \Delta_L(R) + G_L \sin \Delta_L(R)]^2$$

This first order differential equation for the phase shift contains the boundary conditions of the wave functions at the origin and at infinity.

This equation may be written in integral form.

$$(14.) \quad \delta_L = - \int_0^{\infty} \frac{k}{E} [V_T(r) - V_L(r)] [F_L \cos \Delta_L(r) + G_L \sin \Delta_L(r)]^2 dr$$

The constant of integration must be zero since for $[V_T(r) - V_C(r)]$ of vanishing strength, the phase shift at infinity must be zero. The coefficient of $[V_T(r) - V_C(r)]$ in equation (14.) is negative real. Therefore, if an experimental phase shift is negative, at any energy, the actual potential, $V_T(r)$, is more repulsive than the comparison potential, $V_C(r)$, in some spatial region.

Q. E. D.

Fig. 33

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