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RICE UNIVERSITY

**PHASE SHIFT ANALYSIS IN THE ELASTIC
SCATTERING OF PROTONS BY OXYGEN**

By

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**A THESIS SUBMITTED
IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY**

**Houston, Texas
May 1961**

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I. INTRODUCTION

The main task of nuclear spectroscopy in the past thirty years has been the experimental determination of the properties of nuclear energy levels. These properties include the level position, total angular momentum, parity, and isotopic spin. To be totally satisfactory, any theory of nuclear structure must be able to predict correctly the values of these experimentally determined level parameters for all levels of all nuclei. To date no theory has met this stringent requirement.

However, the independent particle model of the nucleus has had considerable success in predicting the properties of nuclear ground states.¹ In this model the individual nucleons move independently within a fixed potential; the interaction between individual pairs of nucleons is not taken into account.² Also it is assumed that the potential involves a strong spin-orbit coupling between the spin of each nucleon and its orbital angular momentum. This coupling is such that for two single particle states which have the same radial and orbital quantum numbers, the one with the larger total angular momentum will be lower in energy.³ With this type of spin-orbit coupling, and with a radial potential intermediate between a square well potential and a harmonic oscillator potential, the independent particle model is

¹M. G. Mayer and J. H. D. Jensen, Elementary Theory of Nuclear Shell Structure (John Wiley and Sons, Inc., 1955).

²Blatt and Weisskopf, Theoretical Nuclear Physics (John Wiley and Sons, Inc., 1952), p. 767.

³Eugene Feenberg, Shell Theory of the Nucleus (Princeton University Press, 1955), p. 22.

able to predict correctly the spins and parities of many nuclear ground states. It also can fit the so-called "magic numbers," which are numbers of neutrons or protons for which nuclei exhibit special stability.⁴ Magic number nuclei are viewed in this model as nuclei having closed shells of atomic structure. The independent particles model assumes that the nucleons in a closed shell couple to give zero total angular momentum.⁵

The nucleus O^{16} is a double magic number nucleus; i. e. it has a magic number of protons and also a magic number of neutrons. Thus the mirror nuclei O^{17} and F^{17} would in this model both be viewed as consisting of a central core about which an extra nucleon moves. The total angular momentum of either of these nuclei would be just the total angular momentum of the extra nucleon. However it is not clear whether this picture of a single particle moving about a central core would apply to the excited states of these nuclei. The independent particle model as described above has only recently been applied to excited states, and there is much evidence concerning excited states of nuclei which indicates that the concept of independent motion of nucleons in a fixed potential cannot explain the properties of these states. There are, for example, excited states which give every indication of being formed by the relative motion of clusters of

⁴R. D. Evans, The Atomic Nucleus (McGraw Hill, 1955), p. 144.

⁵Blatt and Weisskopf, op. cit., p. 769.

29 December 1958

Fig. 36 Energy levels of F^{17} . for notation, see fig. 1, p. 7.

Figure 1

nucleons.^{6,7} Obviously this is not independent motion of nucleons. For these reasons the properties of the excited states of the mirror nuclei O^{17} and F^{17} are of great interest for nuclear theory.

The excited states of the nucleus F^{17} have been studied through the $O^{16}(p, p)O^{16}$ reaction by Laubenstein and Laubenstein at the University of Wisconsin,⁸ by Eppling at the University of Wisconsin,⁹ by R.R. Henry at Rice Institute,¹⁰ and by Sempert at Zurich.¹¹ The first three of these investigations were done using Van de Graaff accelerators as proton sources, while Sempert used a proton beam obtained from a cyclotron. The energy resolution of the proton beam was nearly two orders of magnitude better in the Van de Graaff experiments than in the cyclotron experiment.

The latest energy level diagram of F^{17} is shown in Fig. 1.¹² The analyses of Laubenstein, Eppling and Henry agreed in the total angular momentum and parity assigned to the 3.10 Mev. state ($E_p = 2.66$ Mev.), and indicated the assignment shown for the 3.86 Mev.

⁶C.M. Jones, G.C. Phillips, and Philip D. Miller, Phys. Rev. 117, 525 (1960).

⁷T.A. Tombrello and G.C. Phillips. Nuclear Physics 20, 648 (1960).

⁸R.A. Laubenstein and M.J.W. Laubenstein. Phys. Rev. 84, 12 (1951).

⁹Eppling, Cameron, Davis, et. al. Phys. Rev. 91, 438 (A), (1953).

¹⁰R.R. Henry. M.A. Thesis, The Rice Institute (1954).

¹¹Sempert, Schneider and Martin. Helv. Phys. Acta 27, 313 (1954).

¹²F. Ajzenberg-Selove, Energy Levels of Light Nuclei. VI.

state ($E_p = 3.47$ Mev.). There is no conclusive assignment for the positions, angular momenta and parities of the next two states, shown at excitations of 4.4 Mev. and 4.7 Mev., which are very broad and have a considerable overlap. The level shown at an excitation of 5.1 Mev. is based on Sempert's analysis, and it is not supported by the Van de Graaff experiments.

In view of the uncertain and contradictory nature of the information available concerning these levels, it was felt that a further investigation of the $O^{16}(p, p)O^{16}$ reaction would be of interest. If cross section data are available at a sufficient number of angles a phase shift analysis can in principle determine the positions, angular momenta and parities of the two broad overlapping states. This thesis will describe the measurement of the elastic scattering cross sections, the phase shift analysis, and the conclusions that may be drawn from the results.

II. EXPERIMENTAL EQUIPMENT

The proton source used in this experiment was the 6.0 Mev. Van de Graaff accelerator. The beam from the accelerator passes through a 90° analyzing magnet, which selects particles of a rather sharply defined energy. The energy resolution of this system has been determined through the use of a 180° magnetic spectrometer¹³ to be .06%. The bombarding energy is determined through the standard technique of nuclear magnetic resonance,¹⁴ using either a lithium or a hydrogen probe. The absolute calibration is done with the 180° magnetic spectrometer.¹³ The bombarding energy can be determined to within 10 kilovolts.¹³

The scattering chamber used was the large volume gas target scattering chamber constructed by John Russell and Charles Reich,^{15, 16} and described in their M.A. theses. In this chamber the separation of the target gas from the accelerator high vacuum is accomplished by means of a differentially pumped tube containing nearly seventy two millimeter slits, rather than through the use of a foil. With this system it is possible to maintain pressures as high as one centimeter of mercury in the scattering chamber without significantly affecting the Van de Graaff vacuum. At opposite ends of this differential pumping

¹³Kendall Famularo, PH.D. Thesis, The Rice Institute, (1952).

¹⁴R.D. Jones, M.A. Thesis, The Rice Institute, (1950).

¹⁵Russell, Phillips and Reich, Phys. Rev. 104 (1956) 135.

¹⁶C.W. Reich, M.A. Thesis, The Rice Institute, (1954).

Figure 2

system are two 1.5 mm slits which define the beam direction to an accuracy of 0.1° .

The target gas is admitted to the scattering chamber through a slow leak valve, by means of which the flow rate can be accurately controlled, so as to produce a nearly constant chamber pressure. The chamber is equipped with two oil manometers (using butyl phthalate), which are used to read the chamber pressure as well as the pressure in the first stage of the differential pumping tube.

The chamber as originally constructed by Russell and Reich was modified by the addition of a second particle detector,¹⁰ and both detectors are scintillation counters using thallium activated cesium iodide crystals. The detectors are shown in the photograph (Fig. 2). Under optimum conditions pulse height resolutions of 5% may be obtained with these detectors. In both detectors the photomultiplier tubes and associated electronics are separated from the vacuum system by "O" ring vacuum seals.

The Faraday cup which collects the beam current is described in its latest form in the PH.D. thesis of Russell.¹⁷ The cup is separated from the target gas by a thin aluminum foil, and kept under a high vacuum, to prevent gas ions from conducting charge away from the cup. Since beam particles passing through the foil tend to knock electrons into the cup, thus neutralizing part of the beam current, the

¹⁷J.L. Russell, PH.D. Thesis, The Rice Institute, (1956).

cup is provided with an electro-static suppressor ring at a potential of about -200 volts, as well as a small permanent magnet which deflects the electrons without appreciably deflecting the heavier beam particles.

The number of incident beam particles is determined through the use of a beam current integrator built by Phillip Miller and described in his PH.D. thesis.¹⁸ In this current integrator the beam current is used to discharge a known capacitor which has first been charged to a known voltage. From the initial and final voltages and the value of the capacitance the integrated charge may be determined. The calibration of this current integrator is described in the section on experimental procedure.

The electronic equipment used in connection with the experiment included two 5 channel integral pulse height analyzers and two 5 channel scalers, as well as a 256 channel differential analyzer used from time to time to take pulse height distributions as a check on counter resolution.

¹⁸P.D. Miller, PH.D. Thesis, The Rice Institute, (1959).

III. DETAILS OF PROCEDURE

A. Preparation for the Experiments

As in most experimental work in nuclear physics, the greater part of the time spent in connection with the experiments was spent in preparing the equipment for the taking of data. The first part of this preparation was the leak testing of the scattering chamber vacuum system, to insure that the target gas would not be contaminated with atmospheric nitrogen. The Faraday cup vacuum system had to be checked with special care, since this system had to be maintained at very low pressure ($< 2 \times 10^{-5}$ mm Hg) during the experiments. To find very small leaks in this system it was sometimes necessary to use a helium leak detector.

The next step in the preparations was the checking out of the particle detectors and preamplifiers. The most common difficulty encountered here was electronic noise in the preamplifier circuits. These circuits were re-built with entirely new parts from time to time in order to reduce the noise problem. The pulse height resolution of the detectors was checked using a thorium α particle source. The resolution was gradually improved by polishing the surfaces of the scintillation crystals, but the largest single improvement was made by choosing the smallest slit sizes which would give a large enough counting rate for the experiment. This indicates that the largest effect tending to produce poor resolution was inhomogeneity in the

crystals, rather than surface effects. For the different experimental runs, the detector resolution ranged from 7% to 12%.

The target gas was procured from the Houston Oxygen Company and was commercial grade oxygen which has been tested by them to be 99.7% pure. In this connection the natural abundances of the various isotopes (by atoms) are as follows:¹⁹

Isotope	Natural Abundance
O ¹⁶	99.758%
O ¹⁷	.037
O ¹⁸	.204

Several calculations had to be done before the experiments. It was necessary to calculate the laboratory scattering angles corresponding to the desired angles in the center-of-mass coordinate system. The relation between the angles in the two systems is (for elastic scattering):²⁰

$$\cot \mathcal{J} = \frac{M_1}{M_2} \csc (\textcircled{H}) + \cot (\textcircled{H}) ,$$

where $\mathcal{J}$ = laboratory angle at which incident particle is scattered, degrees

$(\textcircled{H})$ = center-of-mass angle at which incident particle is scattered, degrees

M_1 = mass of incident particle, a. m. u.

M_2 = mass of target particle, a. m. u.

¹⁹Evans, op. cit., p. 252

²⁰Houston, Principles of Quantum Mechanics (Dover, 1959), p. 178.

It was also necessary to calculate the relation between the laboratory cross section and the center-of-mass cross section. This relation is obtained from the requirement that the number of particles scattered into the detector in a given time will be the same in either coordinate system. That is,

$$I_{LAB}(\vartheta) \Delta\Omega_{LAB} = I_{C.M.}(\Theta) \Delta\Omega_{C.M.},$$

where

$$I_{LAB}(\vartheta) = \text{cross section in the laboratory system, barns per steradian}$$

$$I_{C.M.}(\Theta) = \text{cross section in the center-of-mass system, barns per steradian}$$

$$\Delta\Omega_{LAB} = \text{solid angle of detector in the laboratory system, steradians}$$

$$\Delta\Omega_{C.M.} = \text{solid angle of detector in the center-of-mass system, steradians}$$

this gives the relation²¹

$$I_{C.M.}(\Theta) = \frac{\sin^3 \Theta}{\sin^3 \vartheta} \frac{1}{1 + (M_1/M_2) \cos \Theta} I_{LAB}(\vartheta),$$

where the angles and masses are as defined above. The values of the conversion factor were taken from a table,²² and were checked by hand calculation.

²¹Evans, op.cit., p. 850.

²²Tables for the Transformation of Angular Distribution Data from the Laboratory System to the Center of Mass System, Jerry B. Marion, Arthur S. Ginzborg, Shell Development Company.

Fig. 3

SLIT SYSTEM

Before each experiment the geometrical factors involved in the cross section determination were measured. These factors are illustrated in Fig. 3. The long axis of each slit is perpendicular to the diagram, and the counter rotates about an axis perpendicular to the plane of the diagram. As shown in the diagram, the solid angle of the detector is determined by the rear slit area A and the distance R from the scattering center to the rear slit. The target length T is determined by the front slit width W , the slit separation S and the laboratory angle Θ . The target length varies inversely as $\sin \Theta$. The slit dimensions A and W were measured each time with a travelling microscope. The other dimensions R and S were measured in the following way. A meter stick was attached to the top of the scattering chamber with one edge passing directly over the center of the chamber. The counter was rotated to a position directly under the straight edge, and using a plumb bob the position of each slit was noted on the meter stick. The difference between these positions is the slit separation S . The Faraday cup assembly was removed so that the counter could then be rotated through 180° , and the new position of the rear slit was noted on the meter stick. The distance from the scattering center to the rear slit is half the difference between the two readings of the rear slit position.

It was also necessary to perform an alignment procedure to make certain that the beam axis intersected the counter axis of rotation, for if it did not, the distance R would vary with angle. The alignment

procedure was as follows. The Faraday cup assembly, which is shown in Fig. 2 opposite the nozzle of the beam inlet tube, was removed. A strong light was directed down the beam inlet tube, and a telescope was focused on the light, through the Faraday cup mount. A vertical straight edge mounted on one of the counter housings was made to intercept each edge of the light beam, and the position of the counter corresponding to the center of the beam was noted from the azimuth circle below the chamber. Then the counter was rotated through a half circle to the opposite side of the chamber, and once again the position of the counter corresponding to the straight edge being in the beam center was determined. If the two readings were found to differ by exactly 180° , the alignment was true. Otherwise the orientation of the beam inlet tube was adjusted to correct the error.

The azimuth circle reading corresponding to the scattering angle zero was also determined using the light beam. An angle reading was taken with the light beam falling on each edge of the front slit. The mean value of these two readings was noted. Then the front slit was removed, and the zero reading for the rear slit was determined. In general the two zeroes would not agree, and the slit housing was adjusted to produce agreement. In this way the zero correction was determined for each counter.

The next step in the preparations was the calibration of the current integrator described in the previous section. The condenser to

be calibrated was charged to a known multiple of a standard cell voltage. The value of the multiple was determined by a resistance ratio, and the value of the ratio was measured using a Leeds and Northrup Type "K" potentiometer. The condenser voltage was then allowed to decay exponentially through an accurately known resistance, until its voltage reached the standard cell voltage. The time required for the decay was measured using a 60 cycle scaler. The ratio of the initial to the final voltage was

$$V_{\text{FINAL}} / V_{\text{INITIAL}} = e^{-\frac{t}{RC}},$$

where

t = time of decay, seconds

R = resistance, ohms

c = capacitance, farads

The value of the capacitance was then given by

$$C = + \frac{t}{R} \left(\ln V_{\text{INITIAL}} / V_{\text{FINAL}} \right)^{-1}.$$

A nominal 5 μfd condenser was used, and in the calibrations done before the different experiments its values were found to be as follows:

Time	Capacity
Jan. 1960	5.2304 μfd
April 1960	5.2707
Sept. 1960	5.2191

The maximum variation observed was thus just under 1%.

Figure 4

B. Procedure in Taking Data

In order to take data with reasonable efficiency it was necessary to have at least two people present, and three was the usual number. While one person controlled the Van de Graaff the second person could record data, and the third could make pressure and temperature readings.

For each cross section determination the quantities required included the following:

1. The number of protons counted
2. The corresponding integrated beam charge
3. The laboratory scattering angle
4. The gas pressure and temperature
5. The analyzing magnet nuclear magnetic resonance oscillator frequency

These points will be discussed below.

1) To determine the number of proton counts a 5 channel bias curve was taken for each data point; the best number could then be taken from the curve after the experiment. A typical bias curve is shown in Fig. 4. In taking the excitation curves it was necessary to occasionally adjust the amplifier gain settings in order to maintain the sharp drop-off in the last channel of the bias curve. In addition the bias settings were checked every 3 to 4 hours, and it was usually necessary to reset them all, as they would tend to drift as much as 2 volts in that time. The pulse height spectra from the detectors were examined from time to time with the 256 channel differential

pulse height analyzer, as a check on counter resolution and noise. The signal to noise ratio, defined as the ratio of the peak height to the noise immediately below the peak, ranged between 30 and 40 to one in these experiments.

2) The capacitor used in integrating the beam current was the same 5μ farad condenser throughout the experiments, and this capacitor was charged to about 10 V., giving a charge of about 50μ coul. Using this charge value, the maximum statistical error in the number of protons counted was 3%, although in most cases it was of the order of 1 or 2%.

3) The greater part of the data obtained consisted of excitation curves, so that the angle settings needed to be changed only occasionally. These settings were read to an accuracy of about 3 minutes of arc.

4) The target gas pressure was read using the oil manometer mentioned previously, to an accuracy of about .3%. A reading would be taken for each data point except during periods when the pressure was exceptionally stable, when a reading would be taken only for every second or third data point. Once every few hours the pressure in the first stage of the differential pumping system was read, since this was needed in the determination of energy loss of the protons in reaching the center of the chamber. The gas temperature was determined from a mercury thermometer in thermal contact with the lid of the scattering chamber.

5) The nuclear magnetic resonance frequency of the hydrogen probe exposed to the 90° analyzing magnet field was measured with a Hewlett Packard frequency meter, to an accuracy of about one part in ten thousand.

C. The Determination of the Energies and Cross Sections from the Data

The energy calibration was based on an experiment performed by Mr. Beckner²³ with the 180° magnetic spectrometer studying the $B^{11}(p, \alpha)Be^8$ reaction. He measured the energy of the outgoing α particles corresponding to the ground state of Be^8 , and from this it was possible to calculate the energy of the incident protons. The manner in which the bombarding energy for any energy was calculated from this calibration, and the corrections which were considered will now be described.

As mentioned previously, the particles from the accelerator pass through a 90° analyzing magnet. The momentum of a proton which passes through the magnet is²⁴

$$p = e B \rho ,$$

in the M.K.S. system of units. Here e is the charge of the proton, B is the magnetic induction and ρ is the radius of curvature. It is important to note that this is the relativistically correct momentum.²⁴ The non-relativistic approximation to the kinetic energy of the proton

²³E.H. Beckner, PH.D. Thesis, Rice University, (1961).

²⁴Panofsky and Phillips, Classical Electricity and Magnetism (Addison-Wesley, 1955), chapter XXII.

is

$$E_{K,}^{N.R.} = \frac{p^2}{2m_0} \quad)$$

where m_0 is the rest mass of the proton. From the previous expression, this will be proportional to the square of the magnetic induction. However, the magnetic induction is proportional to the nuclear magnetic resonance frequency, which is measured. Thus the non-relativistic energy at the magnet can be written

$$E_K^{NR} = \alpha F^2,$$

where α is a constant and F is the measured resonance frequency.

The relativistically correct expression for the kinetic energy in terms of momentum is

$$E_K^R = ((pc)^2 + (m_0c^2)^2)^{1/2} - m_0c^2.$$

This can be re-written

$$E_K^R = m_0c^2 \left(\left(1 + \frac{(pc)^2}{(m_0c^2)^2} \right)^{1/2} - 1 \right).$$

Expanding the square root, and neglecting terms of the sixth order in PC/m_0C^2 , one obtains

$$E_K^R = \frac{p^2}{2m_0} \left(1 - \frac{1}{2} \frac{(p^2/2m_0)}{m_0c^2} \right).$$

In terms of the non-relativistic energy,

$$E_K^R = E_K^{NR} \left(1 - \frac{1}{2} \frac{E_K^{NR}}{m_0c^2} \right),$$

or in terms of the frequency

$$\bar{E}_K^R = \alpha F^2 \left(1 - \frac{1}{2} \frac{\alpha F^2}{m_0 c^2} \right).$$

The energy calibration described above determines a relativistically correct energy E_0 for a particular frequency F_0 . For other frequencies the first approximation to the energy is

$$\Sigma = \bar{E}_0 \left(\frac{F}{F_0} \right)^2 = \alpha' F^2.$$

This expression is in general equal to neither the relativistically correct nor the non-relativistic energy, since it is equal to the relativistic energy only for $F = F_0$. For $F = F_0$,

$$\alpha' F_0^2 = \alpha F_0^2 \left(1 - \frac{1}{2} \frac{\alpha F_0^2}{m_0 c^2} \right).$$

This gives

$$\alpha = \alpha' \left(1 + \frac{1}{2} \frac{E_0}{m_0 c^2} \right),$$

to the first order in $E_0/m_0 c^2$.

Thus

$$\bar{E}_K^R = \left(1 + \frac{1}{2} \frac{E_0}{m_0 c^2} \right) \alpha' F^2 \left(1 - \frac{1}{2} \frac{\alpha' F^2 \left(1 + \frac{1}{2} \frac{E_0}{m_0 c^2} \right)}{m_0 c^2} \right).$$

Again to first order in $E_0/m_0 c^2$ this gives

$$\bar{E}_K^R = \alpha' F^2 \left(1 - \frac{1}{2} \frac{(\alpha' F^2 - E_0)}{m_0 c^2} \right),$$

or

$$E_K^R = \Sigma \left(1 - \frac{1}{2} \frac{\Sigma - E_0}{m_0 c^2} \right).$$

Thus the correction term changes sign at $\Sigma = E_0$. This relativistic correction was never more than 5 Kev., this being its value at a bombarding energy of 5.75 Mev. The fact that it was so small justifies the approximations made above.

Another correction applied to the bombarding energy was a correction due to the inhomogeneity of the field of the 90° magnet. The momentum of a proton passing through the magnet is proportional to the average field seen by the proton, and since the field is not perfectly homogeneous, this will not be the same as the field seen by the nuclear magnetic resonance probe. Evidently this correction is a function only of field strength. The percentage correction as a function of field strength is given by Richard Chapman,²⁵ and in this experiment it ranged from 1.6 Kev. ($E_p=3.5$ Mev.) to 5.8 Kev. ($E_p=5.75$ Mev.).

The bombarding energy was also corrected for the energy loss in the target gas to get the correct energy at the center of the scattering chamber. In this calculation it was assumed that the chamber pressure was uniform, and that the pressure gradient was uniform across the slits leading from the chamber to the first stage of the differential pumping tube. The energy loss in the first stage of the differential pumping tube was calculated assuming a uniform pressure equal to the pressure read on the oil manometer. The energy loss

²⁵Richard Chapman, PH.D. Thesis, The Rice Institute, (1954).

in the second and third stages was neglected. This was justified, since the pressure in the second stage was less than a hundredth the pressure in the first stage, which itself was less than 5% of the chamber pressure. The total energy loss in the gas was 15 to 20 Kev. throughout the experiments, while the target thickness, or energy loss in the part of the beam seen by the detector, was never more than 2.5 Kev.

This completes the description of the corrections applied in determining the bombarding energy at the center of the chamber. A check on these calculations was provided by the very narrow state in F^{17} which is observed at a bombarding energy of 3.47 Mev., 8, 9, 10 and which has a width less than 3.5 Kev. On this basis the bombarding energy is believed to be known to an accuracy of ± 10 Kev.

The formula for the differential scattering cross section in the laboratory coordinate system is

$$I_{LA3}(\theta) = \frac{N}{N_0} \frac{R^2}{T n A}$$

Here N is the number of protons counted, N_0 is the corresponding number of integrated beam particles ($N_0 = Q/e$), R is the distance from the scattering center to the rear slit of the detector, T is the length of beam seen by the detector, n is the number of target nuclei per unit volume and A is the area of the rear slit. In terms of the slit system diagram given previously,

$$T = \frac{WR}{S \sin \theta} \quad)$$

so that

$$I_{LAB}(\theta) = \frac{N}{N_0} \frac{\sin \theta}{m G} ,$$

where

$$G = \frac{AW}{RS} .$$

The values of the geometrical factor G were of the order of 10^{-4} cm. for both detectors throughout the experiments.

The number of counts N for each data point was taken from the 5 channel integral bias curve. This number was taken to be the number corresponding to the drop-off in the bias curve. It was felt that this was a better number than the number which would be obtained by extrapolating to zero bias, since the extrapolation to zero bias would count particles inelastically scattered from the slit edges.

The number of target nuclei per unit volume was calculated from the measured pressure and temperature, assuming the validity of the ideal gas law. The pressures used were only about 1 cm. of Hg, so the ideal gas law was quite valid.

IV. EXPERIMENTAL RESULTS AND ESTIMATED ERRORS

12 excitation curves and 4 angular distributions were taken in the 3.5 to 5.75 Mev. range of bombarding energy. The data is shown in Figs. 5 through 20, together with some of the data points of Henry and Eppling. For reasons which will become clear in the next section, the excitation curves were taken mostly at angles corresponding to the zeroes of various Legendre functions, as tabulated below:

Θ (C.M.)	
39.23°	$P_3 = 0$
54.75	$P_2 = 0$
63.50	$\frac{dP_3}{d\cos\theta} = 0$
90	$P_1 = P_3 = \frac{dP_2}{d\cos\theta} = 0$
103.46	No Zero
114.66	No Zero
125.27	$P_2 = 0$
131.38	$\frac{dP_4}{d\cos\theta} = 0$
140.77	$P_3 = 0$
149.43	$P_4 = 0$
161.28	No Zero
166.75	Most backward angle

The angular distributions also show points taken from the excitation curves, for comparison.

The main source of error was in the choice of the number of

counts from the 5 channel bias curves. This error is estimated to be 3%. The overall error due to geometrical factors is estimated to be $\pm .4\%$. The error in the determination of the gas pressure is estimated to be $\pm .3\%$. The error in the determination of the gas temperature involves the question of heating effect due to the beam, which is not taken account of, since the temperature of the gas is assumed to be the same as that of the wall of the scattering chamber. It is reasonable to assume that if this introduced a significant error, measurements of cross sections as a function of beam current with the other quantities held fixed would detect it. Since the effect has not been detected it is assumed to be very small. The temperature of the wall of the scattering chamber is read to an accuracy of $\pm .3\%$. The error in the determination of the integrated beam charge is estimated to be $\pm 1\%$.

In addition to these errors there is the error due to imperfect detection efficiency, which is distinguished from the other errors in that it is always in the same direction. This error is estimated to be less than 2% .¹⁵

Excluding the error due to imperfect detector efficiency, the R.M.S. error is 3.2% . The errors are summarized below.

Choice of Number of Counts

3%

Geometrical Factors

area of rear slit	. 2%
slit separation	. 2%
radius	. 2%
front slit width	. 2%
calculational approximations	. 2%
uncertainty in angle	5 min.
<hr/>	
Total Geometrical	. 4%
Gas Pressure	. 3%
Gas Temperature	. 3%
Charge Integration	1. 0%
<hr/>	
R. M. S. ERROR	3. 2%

The comparison of the present data with that of Henry shows discrepancies of a few percent at some angles, although at other angles agreement is good. A semi-complete reproduction of the experimental data of Henry is given in figs. 31 through 36.

FIG. 5

FIG. 6

FIG. 8

MENT

$$\theta_{\text{CM}} = 103.46$$

FIG. 9

$E_{\text{LAB}}, \text{MEV.}$

FIG. 10

FIG. 11

FIG. 19

FIG. 19

ANGULAR DISTRIBUTION

○ NUMBER 1 COUNTER

x NUMBER 2 COUNTER

• FROM EXCITATION CURVES

— PHASES,

$$\delta_0 = 101^\circ$$

$$\delta_1^+ = 131^\circ$$

$$\delta_1^- = -16^\circ$$

$$\delta_2^+ = -8^\circ$$

$$\delta_2^- = 56^\circ$$

ANGULAR DISTRIBUTION
 NUMBER 1 COUNTER
 NUMBER 2 COUNTER
 FROM EXCITATION CURVES
 PHASES,

$$\begin{aligned}\phi_0 &= 101^\circ \\ \phi_1^+ &= 131^\circ \\ \phi_1^- &= -16^\circ \\ \phi_2^+ &= -8^\circ \\ \phi_2^- &= 56^\circ\end{aligned}$$

FIG. 20

FIG. 20

ANGULAR DISTRIBUTION

x NUMBER 1 COUNTER

o NUMBER 2 COUNTER

• FROM EXCITATION CURVES

— PHASES,

$$\delta_0 = 92.5^\circ$$

$$\delta_1^+ = 154^\circ$$

$$\delta_1^- = -19^\circ$$

$$\delta_2^+ = -7^\circ$$

$$\delta_2^- = 71^\circ$$

ANGULAR DISTRIBUTION
 NUMBER 1 COUNTER
 NUMBER 2 COUNTER
 FROM EXCITATION CURVES

BASES,

$\theta_0 = 92.5^\circ$

$\theta_1^+ = 154^\circ$

$\theta_1^- = -19^\circ$

$\theta_2^+ = -7^\circ$

$\theta_2^- = 71^\circ$

V. ANALYSIS OF THE DATA

A. General Discussion of the Data

It is evident immediately from the excitation curves that the large anomaly in the 4 to 5 Mev. range of bombarding energy involves the interference of at least two states of the compound nucleus F^{17} . At angles such as 54.75° and 125.27° the cross section shows a maximum at a bombarding energy of about 4.3 Mev., whereas at the most backward angle of 166.75° the cross section shows a minimum at the same energy. Thus there appears to be constructive interference at the middle range angles and destructive interference at the backward angles. Another point of significance which may be noticed at once is that the total variation of the cross section at 90° throughout this anomalous energy region is considerably less than at the other angles. For a reason which will become clear in what follows, this suggests that at least one of the states of F^{17} involved is formed by protons of odd orbital angular momentum, and hence that at least one of the states is of odd parity.

It was decided to assume that the data in the 4 to 5 Mev. range of bombarding energy could be explained in terms of the interference of only two states of F^{17} . There was of course no way of knowing that only two states were involved, but it was natural to consider the simplest possibility first. Even assuming that there were only two states in this region, it would still be necessary to take some account of the levels above 5.0 Mev. to fit the data in the 4.0 to 5.0 Mev. region.

There appear to be 3 such states in the energy range covered, and from the data at 90° they appear to be located at bombarding energies of 5.2 Mev., 5.4 Mev., and 5.54 Mev. As will be discussed below, the shapes of these three resonances at 90° indicate that the first two levels are levels of odd ℓ , and that the last is a level of even ℓ . The formalism and methods of the analysis will now be discussed.

B. Introduction to the Expressions Used in the Analysis

The wave equation for the elastic scattering of spin zero particles by a spherically symmetric potential is²⁶

$$-\frac{\hbar^2}{2\mu} \nabla^2 u(r, \theta) + V(r) u(r, \theta) = E u(r, \theta),$$

where

μ = reduced mass

$u(r, \theta)$ = wave function

$V(r)$ = interaction potential

E = energy in the center-of-mass coordinate system, Mev.

$\hbar$ = Planck's constant

Considering for the moment the case of a limited range potential (which excludes the case of the coulomb potential), the wave function for large r will have the form

$$u(r, \theta) \xrightarrow{r \rightarrow \infty} e^{ikz} + \frac{f(\theta) e^{ikr}}{kr}$$

²⁶Schiff, Quantum Mechanics (McGraw-Hill, 1955), p. 70.

where

$$K = \frac{\mu v}{\hbar} = \text{wave number}$$

v = initial lab velocity of projectile (target at rest in lab).

Here the first term is a plane wave representing the incident beam, and the second term is an outgoing spherical wave representing the scattered particles (In the case in which a coulomb potential is also present we would have instead of the plane wave e^{iKz} an incoming wave which is coulomb distorted even at infinite separation, but this does not alter the point to be made here). In terms of the previous expression the differential scattering cross section is given by

$$I(\theta) = \frac{1}{K^2} |f(\theta)|^2.$$

For the case of scattering by a spherically symmetric potential the wave function can be expanded in terms of Legendre polynomials for any r :

$$u(r, \theta) = \sum_{l=0}^{\infty} R_l(r) P_l(\cos \theta).$$

It is natural to consider two separate regions, the interior and exterior of the nucleus, and to require that the values and slopes of the interior and exterior wave functions must match at the nuclear surface. It can be shown that the most general form for the exterior radial wave function is²⁶

$$R_l(r) = A_l (\cos \delta_l f_l(kr) - \sin \delta_l n_l(kr)),$$

where

$$A_l = \text{const.},$$

$$f_l(kr) \equiv \left(\frac{\pi}{2kr}\right)^{1/2} J_{l+1/2}(kr),$$

$$m_l(kr) \equiv (-1)^{l+1} \left(\frac{\pi}{2kr} \right)^{1/2} J_{-(l+\frac{1}{2})}(kr),$$

and where the numbers δ_l are real functions of the center-of-mass energy only, known as the phase shifts. Then if γ_l is the ratio of slope to value of the interior radial wave function at $r=a$, the continuity requirement may be written

$$\gamma_l = k \frac{\cos \delta_l f_l'(ka) - \sin \delta_l m_l'(ka)}{\cos \delta_l f_l(ka) - \sin \delta_l m_l(ka)}.$$

This may be re-written

$$\tan \delta_l = \frac{k f_l'(ka) - \gamma_l f_l(ka)}{k m_l'(ka) - \gamma_l m_l(ka)}$$

This establishes the connection between the phase shifts and the conditions at the surface of the nucleus. It may be easily seen that for a given radius a , and for a given phase shift δ_l , there is only one interior radial function $R_{l_{IN}}(r)$ which will satisfy the continuity conditions at the surface. For if we suppose that there are two, $R_{l_{IN}}^{(1)}(r)$ and $R_{l_{IN}}^{(2)}(r)$, and if we consider the difference between them

$$\Delta_l(r) \equiv R_{l_{IN}}^{(1)}(r) - R_{l_{IN}}^{(2)}(r),$$

then the fact that both wave functions satisfy the continuity conditions at $r=a$ requires that

$$\Delta_l(a) = 0.$$

and

$$\left. \frac{d\Delta_l}{dn} \right|_a = 0.$$

But Δ_l must also be a solution of the radial wave equation

$$\frac{1}{n^2} \frac{d}{dn} \left(n^2 \frac{d\Delta_l}{dn} \right) + \left\{ \frac{2\mu}{\hbar^2} [E - V(n)] - \frac{l(l+1)}{n^2} \right\} \Delta_l = 0,$$

or

$$\frac{d^2 \Delta_l}{dn^2} + \frac{2}{n} \frac{d\Delta_l}{dn} + \left\{ \frac{2\mu}{\hbar^2} [E - V(n)] - \frac{l(l+1)}{n^2} \right\} \Delta_l = 0.$$

Thus it follows that

$$\left. \frac{d^2 \Delta_l}{dn^2} \right|_a = 0.$$

By differentiating the wave equation it can be shown that all the radial derivatives of Δ_l are zero at $r=a$, so that $\Delta_l(r)$ is identically zero. This shows that the radius and the phase shift uniquely determine the interior radial wave function.

For large r it can be shown that the exterior wave function has the form

$$u(r, \theta) \xrightarrow{r \rightarrow \infty} \frac{1}{kr} \sum_{l=0}^{\infty} (2l+1) i^l e^{i\delta_l} \sin(kr - \frac{l\pi}{2} + \delta_l) P_l,$$

and that the scattering amplitude $f(\theta)$ introduced previously is given

by

$$f(\theta) = \sum_{l=0}^{\infty} (2l+1) e^{i\delta_l} \sin \delta_l P_l(\cos \theta).$$

To see something of the physical significance of the phase shifts, it

is of interest to consider the asymptotic form of the expansion of the plane wave:

$$e^{ikz} \xrightarrow{n \rightarrow \infty} \frac{1}{kn} \sum_{l=0}^{\infty} (2l+1) i^l \sin(kn - \frac{l\pi}{2}) P_l(\cos\theta).$$

Comparing this with the expression for $u(r, \Theta)$, we see that the case

$\delta_l = 0$ (or any multiple of π) is the case of no interaction. Considering the radial wave function $\sin(Kr - \frac{l\pi}{2} + \delta_l)$, if δ_l increases from zero in the positive direction the effect is to shift the sine curve inward, since for a particular r the sine function will have the value that it had for a larger r in the case $\delta_l = 0$. Similarly if δ_l changes from zero in the negative direction the curve $\sin(Kr - \frac{l\pi}{2} + \delta_l)$ is shifted outward. This argument indicates that if $\frac{d\delta_l}{dE} > 0$, the effective force for the l^{th} partial wave is becoming more attractive with increasing energy, whereas if $\frac{d\delta_l}{dE} < 0$ the effective force for that wave is becoming more repulsive with increasing energy. The energy derivative of the phase shift has been related to the effective collision time by F. Smith.²⁷ He defines the collision lifetime as the difference between the time the particles spend within a distance R of one another and the time they would have spent within this distance of one another in the case of no interaction, taken in the limit as $R \rightarrow \infty$. He shows that for elastic scattering this effective collision time is

²⁷F. Smith, Phys. Rev. 118, 349 (1960).

$$\tau_l = \hbar \frac{d\delta_l}{dE} .$$

The same result was reached from a different definition by Wigner.²⁸

From this expression we see that if $\frac{d\delta_l}{dE}$ is positive the particles spend a longer time near each other than in the case of no interaction, while if $\frac{d\delta_l}{dE}$ is negative they spend less time near each other. This is consistent with the previous argument, since for an attractive force one would expect the particles to spend a longer time near each other. The quantity $\frac{d\delta_l}{dE}$ has also been interpreted in terms of a "density of states" function for nuclear states.²⁹

Whenever the detailed nature of the interaction permits the existence of a definite state of the compound system, the requirement that the total angular momentum and parity of the state be good quantum numbers (i.e. have definite values) requires that one and only one of the radial wave functions R_l will become large in the interior region. This means that the associated exterior wave function gets "pulled in," or that the associated phase shift δ_l moves sharply in the positive direction, producing a resonance in the scattering cross section. In terms of the collision lifetime derived by Smith we see at once that since narrow resonances correspond to large positive values of $\frac{d\delta_l}{dE}$ we have the important result that narrow resonances correspond to long lived states of the compound system, and broad resonances to

²⁸E. P. Wigner, Phys. Rev. 98, 145, (1955).

²⁹G. C. Phillips, T. A. Griffy and L. C. Biedenharn, Nuclear Physics 21, 327, (1960).

short lived states.

It can be shown that for the case in which a coulomb field is also present (but there are no spins) the scattering amplitude may be written³⁰

$$f(\theta) = -\frac{\eta}{2} \csc^2 \frac{\theta}{2} e^{i\eta \ln \csc^2 \frac{\theta}{2}} + \sum_{l=0}^{\infty} (2l+1) e^{i\alpha_l} \sin \delta_l P_l(\cos \theta).$$

where

$$\eta = \frac{z_1 z_2 e^2}{\hbar v},$$

$$\alpha_0 = 0,$$

$$\alpha_l = 2 \sum_{s=1}^l \tan^{-1} \left(\frac{\eta}{s} \right), \quad l \neq 0.$$

The previous formula may be obtained from this by setting $\eta = 0$.

The first term in the above expression is the Rutherford scattering term; i. e. if the second term is set equal to zero then $\frac{1}{k^2} |f(\theta)|^2$ will be just the Rutherford cross section.

There is an important difference to be noted between the case in which a coulomb field is present and the previous case. If we consider the case in which only S wave ($l = 0$) scattering is significant, then for the case in which there is no coulomb field,

³⁰C. L. Critchfield and D. C. Dodder, Phys. Rev. 76, 602 (1949).

$$I(\theta) = \frac{1}{K^2} \sin^2 \delta_0 ,$$

whereas in the case in which a coulomb field is present,

$$I(\theta) = \frac{1}{K^2} \left| -\frac{\eta}{2} \cot^2 \frac{\theta}{2} e^{i\eta \ln \cot^2 \frac{\theta}{2}} + e^{i\delta_0} \sin \delta_0 \right|^2 .$$

The point of interest is that in the first case the cross section is invariant to the sign of the phase shift δ_0 , while in the second case it is not, because of the presence of the Rutherford term in the scattering amplitude. Thus the Rutherford term acts as a probe which in principle makes it possible to extract more information from the scattering cross section than could otherwise be obtained.

The corresponding expression for the scattering cross section in the case of the scattering of charged particles of spin 1/2 by charged particles of spin zero is³¹

$$I(\theta) = \frac{1}{K^2} \left\{ |f_{\text{coh}}(\theta)|^2 + |f_{\text{inc}}(\theta)|^2 \right\} ,$$

where.

$$f_{\text{coh}}(\theta) = -\frac{\eta}{2} \cot^2 \frac{\theta}{2} e^{i\eta \ln \cot^2 \frac{\theta}{2}} + \sum_{\ell=0}^{\infty} e^{i\alpha_{\ell}} \left[(\ell+1) e^{i\delta_{\ell}^{+}} \sin \delta_{\ell}^{+} + \ell e^{i\delta_{\ell}^{-}} \sin \delta_{\ell}^{-} \right] P_{\ell}(\cos \theta) ,$$

³¹Laubenstein and Laubenstein, Phys. Rev. 84, 18, (1951).

and

$$f_{INC}(\theta) = \sum_{\ell=1}^{\infty} e^{i\alpha_{\ell}} \left[e^{i\delta_{\ell}^{-}} \sin \delta_{\ell}^{-} - e^{i\delta_{\ell}^{+}} \sin \delta_{\ell}^{+} \right] \sin \theta \frac{dP_{\ell}}{d\cos \theta}.$$

In these expressions δ_{ℓ}^{+} refers to states of the compound system having total angular momentum $J = \ell + 1/2$ and δ_{ℓ}^{-} refers to states having total angular momentum $J = \ell - 1/2$. Just as in the spin zero case, each phase shift is associated with a particular value of J and of the parity π , so that only one of them will vary anomalously across an isolated resonance.

The first term in the cross section formula is associated with those particles whose spins are not reversed in the scattering process, while the second is associated with those whose spins are reversed. This spin reversal is due to effective spin-orbit forces and will in general produce a spin polarization, which may be shown to be¹⁸

$$S_p = 2 \frac{\text{Re}(f_c) \text{Im}(f_i) - \text{Im}(f_c) \text{Re}(f_i)}{|f_c|^2 + |f_i|^2}.$$

This spin polarization is in a direction perpendicular to the scattering plane, with positive S_p being taken as in the direction $\vec{K}_{IN} \times \vec{K}_{OUT}$.

The spin reversal occurs across the scattering plane, and cannot occur for $\Theta = 0$ or 180° , since the scattering plane is undefined for those angles; thus the incoherent amplitude f_{INC} approaches zero as Θ approaches zero or 180° , as may be seen from the $\sin \Theta$

factor in the above equation.

C. The Relevant Expression from Dispersion Theory

The single level dispersion formula for the behavior of a resonating phase shift near an isolated resonance may be written³²

$$\delta_{J,\pi} = -T_{an}^{-1} \left(\frac{F_\ell(\eta, \rho)}{G_\ell(\eta, \rho)} \right)_{\rho=KR} + T_{an}^{-1} \left(\frac{\frac{1}{2} \Gamma_{J,\pi}}{E_{J,\pi} - E - \Delta_{J,\pi}} \right),$$

$$\frac{1}{2} \Gamma_{J,\pi} = \gamma_{J,\pi}^2 \left[\frac{\rho}{F_\ell^2 + G_\ell^2} \right]_{\rho=KR},$$

$$\Delta_{J,\pi} = -\gamma_{J,\pi}^2 (1 + q_\ell)_{\rho=KR},$$

$$q_\ell = KR \left[\frac{1}{F_\ell} \frac{\partial F_\ell}{\partial \rho} - \frac{1}{F_\ell^2 + G_\ell^2} \frac{G_\ell}{F_\ell} \right]_{\rho=KR},$$

$$\eta = \frac{z' z e^2}{\hbar v}$$

Where

F_ℓ = Regular coulomb wave function

G_ℓ = Irregular coulomb wave function

K = wave number, as before

R = effective nuclear radius, fermis

$\frac{1}{2} \Gamma_{J,\pi}$ = resonance half-width, Mev.

³²E. P. Wigner and L. Eisenbud, Phys. Rev. 72, 29, (1947).

$$\gamma_{J,\pi}^2 = \text{reduced width, a constant, Mev.}$$

$$E_{J,\pi} = \text{constant expansion parameter, Mev.}$$

$$E = \text{Energy in the center-of-mass system, Mev.}$$

$$\Delta_{J,\pi} = \text{level shift, Mev.}$$

The first term is mathematically equivalent to the phase shift resulting from scattering by a charged, hard sphere of radius R , while the second represents the effect of the particular resonant state of the compound system. In the expression for the width the quantity $\frac{KR}{F\ell^2 + G\ell^2}$ is the coulomb barrier penetration factor. It increases with increasing energy, and decreases rapidly with increasing ℓ , due to the effect of the centrifugal barrier, which is proportional to $\ell(\ell+1)$.

D. Graphical Method of Analysis

The various terms in the two amplitudes involved in the scattering cross section may be represented as vectors in the complex plane⁸ and this method is often useful in obtaining a qualitative picture of the energy dependence of the cross section for various situations involving different level parameters.

The Rutherford term in the coherent amplitude is a vector of magnitude

$$|R| = \frac{\gamma}{2} \csc^2 \frac{\theta}{2}$$

and with orientation

$$\arg R = \pi + \gamma \ln \csc^2 \frac{\theta}{2}$$

COHERENT AND INCOHERENT
 AMPLITUDES FOR $\theta = 90^\circ$
 NOTE THAT $P_1 = P_3 = P'_2 = P'_4 = 0$
 P_2 AND P'_3 ARE NEGATIVE

Figure 21

with respect to the positive x direction.

One can easily show that the locus of the quantity $e^{i\delta} \sin \delta$ is a circle of diameter 1 with its center located at the point $(0, \frac{i}{2})$. Similarly the locus of $(\ell + 1)P_\ell e^{i\delta_\ell} \sin \delta_\ell e^{i\alpha_\ell}$ is a circle of diameter $(\ell + 1)P_\ell (\cos \Theta)$, tangent at the origin to a line whose inclination with respect to the positive x axis is α_ℓ . Which side of the line the circle is on is determined by the sign of $P_\ell (\cos \Theta)$. These points are illustrated in Fig. 21.

E. Phase Shift Analysis in 4 to 5 Mev. Region

It was decided to assume that the data in the 4.3 Mev. region could be explained in terms of the interference of two broad states in this region. There was of course no way of knowing that only two states were involved, but it was natural to consider the simpler possibility first. Thus there was the problem of determining the angular momenta, parities, positions and widths of two broad overlapping levels which interfere in a complicated way.

To determine the spins and parities of the two levels involved it was decided to try all possibilities through $\ell = 3$. The argument against a higher angular momentum state being involved is based on the fact that the states are evidently very broad. The Wigner limit on the half width of a state is³³

³³R.G. Thomas, Phys. Rev. 81, 148 (1951).

$$\frac{1}{2} \Gamma_{\ell_w} = \frac{3\hbar^2}{2\mu a^2} \frac{Ka}{F_\ell^2 + G_\ell^2}$$

where a is the nuclear radius, and the other quantities are as defined previously. For a given energy and radius this width decreases rapidly with increasing orbital angular momentum ℓ . The reason for this is that the states of higher ℓ are longer lived, since the particles must penetrate the centrifugal barrier in order to escape, so that the Heisenberg uncertainty principle requires the states of higher ℓ to have smaller widths. At a bombarding energy of 4 Mev. (which corresponds to an entry in the table used³⁴), with the radius

$$a = 1.4 (A_1^{1/3} + A_2^{1/3}) \text{ Fermis} = 5.03 \text{ Fermis},$$

$$\frac{3\hbar^2}{2\mu a^2} = 2.64 \text{ Mev.},$$

$$Ka = 2.08,$$

$$\log_{10} \gamma = -.2,$$

$$F_0^2 + G_0^2 = 1.44,$$

$$F_1^2 + G_1^2 = 1.98,$$

$$F_2^2 + G_2^2 = 4.60,$$

$$F_3^2 + G_3^2 = 22,$$

$$F_4^2 + G_4^2 = 260.$$

³⁴Graphs of Coulomb Functions, Sharp, Gore and Paul.
 A. E. C. L. No. 268.

This gives the following Wigner limit half widths:

$$\frac{1}{2} \Gamma_{0w} = 3.82 \text{ Mev.}$$

$$\frac{1}{2} \Gamma_{1w} = 2.78 \text{ Mev.}$$

$$\frac{1}{2} \Gamma_{2w} = 1.20 \text{ Mev.}$$

$$\frac{1}{2} \Gamma_{3w} = .250 \text{ Mev.}$$

$$\frac{1}{2} \Gamma_{4w} = .021 \text{ Mev.}$$

Thus, from the excitation curves, a state with $\ell = 4$ is conclusively ruled out, but a state with $\ell = 3$ is not. With this restriction, the possible total angular momenta and parities involved in these two states are as follows:

J^π	Resonant Phase Shift
$\frac{1}{2}^+$	δ_0
$\frac{3}{2}^-$	δ_1^+
$\frac{5}{2}^-$	δ_1^-
$\frac{5}{2}^+$	δ_2^+
$\frac{3}{2}^+$	δ_2^-
$\frac{7}{2}^-$	δ_3^+
$\frac{5}{2}^-$	δ_3^-

Here J is the total angular momentum and π is the parity. Assuming that the levels do not have the same J^π values, this means that there are 42 possible combinations.

The method of graphical analysis described above was used to determine the correct combination. For this purpose ink drawings

were made showing the various resonance circles in the two scattering amplitudes, for 5 angles and for energies of 4.1 Mev. and 4.3 Mev. The Rutherford vector R and the coulomb angles χ_l which determine the orientation of the resonance circles change rather slowly with energy, so that for approximate calculations of trial excitation curves it is possible to use the same diagram of the resonance circles over a range of several hundred kilovolts. By placing tracing paper over the ink drawings it was possible to make calculations with fair speed, using the same resonance circles over and over again.

In performing these calculations it was of course necessary to assume values for the non-resonant phase shifts. The values used for the S wave phase shift δ_0 were based on the phase shift analysis of Henry, which extended up to a bombarding energy of 3.4 Mev. His curve, shown in Fig. 22, was extended linearly up to the 4.3 Mev. area to give trial values of δ_0 . The other non-resonant phase shifts were at first neglected, since the purpose in these calculations was to determine the shapes which the excitation curves would have for various combinations of spin and parity.

The first combinations tried were those involving $l = 1$ and $l = 3$. These combinations were eliminated one by one, on the basis that they could not be made to give the correct shape for the excitation curve at the most backward angle ($\Theta_{c.m.} = 166.75$).

It was then discovered that there was a simple argument, based

on the excitation curve at $\Theta_{c.m.} = 90^\circ$, which eliminated all $\ell = 2$, $\ell = 3$ combinations at one stroke. This argument is based on the magnitude of the experimental cross section, but it is independent of the choice of the non-resonant phase shifts. The combinations concerned are those involving the following spins and parities:

$$\begin{array}{l} \tilde{J} = \begin{array}{l} \frac{5}{2}^-, \frac{7}{2}^- \\ \frac{5}{2}^+, \frac{5}{2}^- \\ \frac{3}{2}^+, \frac{7}{2}^- \\ \frac{3}{2}^+, \frac{5}{2}^- \end{array} \quad \text{involving the phase shifts} \quad \begin{array}{l} \delta_2^+, \delta_3^+ \\ \delta_2^+, \delta_3^- \\ \delta_2^-, \delta_3^- \\ \delta_2^-, \delta_3^+ \end{array} \end{array}$$

Considering the formula for the differential cross section given previously, the incoherent amplitude is

$$f_{INC} = \sin \theta \sum_{\ell=1}^{\infty} e^{i\chi_{\ell}} \left\{ e^{i\delta_{\ell}^-} \sin \hat{\epsilon}_{\ell}^- - e^{i\delta_{\ell}^+} \sin \hat{\epsilon}_{\ell}^+ \right\} \frac{dP_{\ell}}{d\cos \theta}.$$

At 90° ,

$$\frac{d^2\sigma}{d\cos \theta} = 3 \cos \theta = 0.$$

Therefore, since the two states are assumed to be formed by particles with $\ell = 2$ and $\ell = 3$, only the $\ell = 3$ resonance will contribute to the incoherent amplitude at 90° . Furthermore by hypothesis there is no $\ell = 1$ resonance in this energy region, so that the $\ell = 1$ phase shifts

δ_1^+ and δ_1^- will be essentially equal. Thus there will be very little $\lambda = 1$ contribution to the incoherent amplitude at 90° , so that

$$f_{INC}(90^\circ) = P_3'(0) \{ e^{i\delta_3^-} \sin \delta_3^- - e^{i\delta_3^+} \sin \delta_3^+ \} e^{i\alpha_3}.$$

This may be expressed in a more useful form as follows:

$$e^{i\delta_3^-} \sin \delta_3^- - e^{i\delta_3^+} \sin \delta_3^+ = e^{i(\delta_3^- + \delta_3^+)} \left\{ e^{-i\delta_3^+} \sin \delta_3^- - e^{-i\delta_3^-} \sin \delta_3^+ \right\}.$$

Expressing $\sin \delta_3^\pm$ exponentially,

$$e^{i\delta_3^-} \sin \delta_3^- - e^{i\delta_3^+} \sin \delta_3^+ = e^{i(\delta_3^- + \delta_3^+)} \left\{ \frac{e^{i(\delta_3^- - \delta_3^+)} - e^{-i(\delta_3^- + \delta_3^+)}}{2i} - \frac{e^{i(\delta_3^+ - \delta_3^-)} - e^{-i(\delta_3^+ + \delta_3^-)}}{2i} \right\}$$

Two of the terms cancel, leaving

$$e^{i\delta_3^-} \sin \delta_3^- - e^{i\delta_3^+} \sin \delta_3^+ = e^{i(\delta_3^+ + \delta_3^-)} \sin(\delta_3^- - \delta_3^+).$$

Thus

$$f_{INC}(90^\circ) = P_3'(0) e^{i(\alpha_3 + \delta_3^+ + \delta_3^-)} \sin(\delta_3^- - \delta_3^+).$$

So the incoherent contribution to the cross section at 90° is

$$\frac{d\sigma_{INC}}{d\Omega}(90^\circ) = \frac{1}{k^2} P_3'(0)^2 \sin^2(\delta_3^- - \delta_3^+).$$

The difference between the two $\ell = 3$ phase shifts will be the resonant part of whichever one of them is resonating, and at resonance this difference will be 90° . Thus at the position of the $\ell = 3$ resonance,

$$\frac{d\sigma_{INC}}{d\Omega}(90^\circ) = \frac{1}{K^2} P_3'(0)^2,$$

and at the same energy

$$\frac{d\sigma}{d\Omega}(90^\circ) \geq \frac{1}{K^2} P_3'(0)^2.$$

Now $P_3'(0) = -1.500$, and at a bombarding energy of 4.3 Mev.,

$$K = 4.28 \times 10^{+12} \text{ cm}^{-1}.$$

It follows that at the position of the $\ell = 3$ resonance,

$$\frac{d\sigma}{d\Omega}(90^\circ) \geq .12 \text{ barns.}$$

Experimentally, however,

$$\frac{d\sigma}{d\Omega}(90^\circ) < .06 \text{ barns}$$

throughout the entire range of bombarding energy from 3.7 Mev. to 4.7 Mev. It follows that all $\ell = 2$, $\ell = 3$ combinations are ruled out. Again, this result is independent of the choice of the non-resonant phase shifts (except in the assumption that the phase shifts δ_1^+ and δ_1^- are not significantly different away from $\ell = 1$ resonances).

The next combinations considered were those involving $\ell = 1$ and $\ell = 2$. It was possible to use the same type of argument which eliminated the $\ell = 2, \ell = 3$ combinations to get an estimate of the position of the $\ell = 1$ resonance. Just as in the previous case, there will be no $\ell = 2$ contribution to the incoherent amplitude at 90° , and the amplitude will be

$$f_{IN-}(90^\circ) = e^{i(\alpha_1 + \delta_1^+ + \delta_1^-)} \sin(\delta_1^- - \delta_1^+),$$

since

$$\frac{dP_i}{d\Omega} = 1.$$

It follows that

$$\frac{d\sigma}{d\Omega}(90^\circ) \geq \frac{1}{K^2} \sin^2(\delta_1^- - \delta_1^+)$$

At the position of the $\ell = 1$ resonance the difference between δ_1^+ and δ_1^- will be 90° , so that, at resonance

$$\frac{d\sigma}{d\Omega}(90^\circ) \geq \frac{1}{K^2}$$

Putting in the value of K for a bombarding energy of 4.3 Mev., as before, one gets

$$\frac{d\sigma}{d\Omega}(90^\circ) \geq .054 \text{ barns}$$

From the 90° excitation curve (Fig. 8) it follows that the position of the $\ell = 1$ resonance lies between 4.2 and 4.3 Mev.

Using this information trial excitation curves were calculated as before, for various assumptions of the position of the $\ell = 2$ resonance. Of the four $\ell = 1$, $\ell = 2$ combinations the one which showed the most promise in these rough calculations was the combination involving the phase shifts δ_1^+ and δ_2^- , that is, the J^π combination $\frac{3}{2}^-$, $\frac{3}{2}^+$. This agreed with the previous tentative assignment of the spins and parities.

An attempt was begun to calculate the exact values of δ_1^+ and δ_2^- throughout the 4.0 to 5.0 Mev. range of bombarding energy, assuming the correctness of the assignment. As before, the values used for δ_0 were based on an extrapolation of the results of Henry, but for these more careful calculations values were also assumed for the other non-resonant phase shifts. They were taken to be the hard sphere phase shifts

$$- \tan^{-1} \left. \frac{F_\ell}{G_\ell} \right|_{r=a}$$

defined by the radius

$$a = 1.4 (A_1^{1/3} + A_2^{1/3}) \text{ Fermis} = 5.03 \text{ Fermis.}$$

Since two phase shifts are assumed to be resonating in this region, it was decided to apply a special technique to determine their values. This technique is as follows:

For a particular set of the non-resonant phase shifts at a particular energy, the set of all pairs of values of δ_1^+ and δ_2^- which

will fit the observed cross section at a given center-of-mass angle determines a curve in the δ_1^+, δ_2^- plane. For a different center-of-mass angle there will be another such curve in the δ_1^+, δ_2^- plane. Any point of intersection of these two curves represents a pair δ_1^+, δ_2^- which fits the data at both angles. For the correct set of the non-resonant phase shifts, all of these curves for various scattering angles will meet at a point, and this point will be the correct pair δ_1^+, δ_2^- .

Such curves were originally calculated by hand, using the graphical method outlined above, but as this was very time consuming, the problem of the calculation of these curves was programmed for an I.B.M. 709 computer. The program, as well as calculating the curves corresponding to perfect fits, calculates the curves corresponding to 10% misses in each direction; this procedure establishes the sensitivity of the cross section to each phase shift at the various scattering angles. Such a graph for four scattering angles is shown in Fig. 23.

Starting at 4.00 Mev. with the non-resonant phases described above, such graphs were calculated for seven energies in this range (4, 4.1, 4.2, 4.3, 4.4, 4.5, 4.6 Mev.). It was found that with the initial choice of the non-resonant phases the data could be fit within 5% at all four angles in the 4.3 Mev. region but that the theoretical curves missed the experimental results considerably at 4.00 Mev. Accordingly the non-resonant phase shifts were adjusted at 4.00 Mev.

to produce a better fit. It was found that a reduction of the magnitude of the non-resonant P wave phase shift δ_1^- by 10° produced a considerable improvement. With this change a new set of graphs was calculated throughout the energy range. From each graph a best pair δ_1^+, δ_2^- was taken, and smooth curves were drawn passing through these values.

In order to fit the data at as many angles as possible, and to extend the analysis to lower and higher energies, it was decided to go over to the method of least squares analysis. In this method the criterion for goodness of fit is the error expression

$$\chi^2 = \sum_{\theta} \left(\frac{\sigma_{\text{CALC}}(\theta) - \sigma_{\text{EXP}}(\theta)}{\sigma_{\text{EXP}}(\theta)} \right)^2$$

where σ_{CALC} is the theoretical differential cross section and σ_{EXP} is the experimental cross section.

The minimization of this error expression was also programmed for the I.B.M. 709. The method used was that in which the fit is improved by iterating the phase shifts one at a time.

Such calculations were done initially in the 3.8 to 5.0 Mev. range of bombarding energy, using as starting phase shifts the values obtained from the phase shift contours described above. These calculations yielded phase shifts which did not lie on perfectly smooth curves, but when smooth curves were drawn to best fit these results it was found that the resultant set of phase shifts gave a good fit to the data. The cross sections calculated from these phase shifts are

shown in figs. 5 through 20, while the phase shifts themselves are shown in fig. 24. Also shown in fig. 24 are phase shifts obtained by Henry at lower energy, where only δ_0 and δ_1^- were large. The spin polarization predicted from these phase shifts is shown in fig. 25. As is evident from fig. 24, the phase shift δ_1^- begins to move in the positive direction as the bombarding energy approaches 5 Mev. Presumably this is due to the effect of an energy level above 5 Mev.

The reduced widths of the states in the 4 to 5 Mev. region have been calculated from the dispersion theory formula given above, and are

$$\gamma^2 = .11 \text{ Mev.}, \quad \frac{3}{2}^- \text{ level}$$

and

$$\gamma^2 = .91 \text{ Mev.}, \quad \frac{3}{2}^+ \text{ level}.$$

An attempt has also been made to determine the level structure above 5 Mev. There appear to be 3 levels up to the highest energy reached, and the 90° excitation curve (Fig. 8) indicates that they are located at 5.20, 5.40, and 5.54 Mev. The general shape of the 90° excitation curve indicates that the first two of these levels are formed by protons of odd orbital angular momentum ℓ , and that the 5.54 Mev. state is formed by protons of even ℓ ; the reason for this is that states of even ℓ interfere with the Rutherford amplitude at 90° , whereas states of odd ℓ do not, as is evident from Fig. 21.

Various combinations of the total angular momentum J and parity π were assumed for these three levels, and for each combina-

tion theoretical excitation curves were calculated from 5.0 to 5.6 Mev., using the single level formula. The best qualitative results were obtained with the following combination

E lab	J π	Phase Shift
5.20	$3/2^-$	δ_1^+
5.40	$7/2^-$	δ_3^+
5.54	$3/2^+$	δ_2^-

The theoretical excitation curves for this combination are shown in figs. 25 through 29, for four scattering angles. The results are not at all conclusive as far as the total angular momenta are concerned but do indicate that the assumed ℓ values are correct.

An attempt was made to obtain a better fit to the first of these three levels, assuming that the phase shift δ_1^+ was the one resonating, and the results are shown in figures 5 through 16. The poor fit at 54° shows that the assignment is not conclusive.

VI CONCLUSIONS

The results of the analysis in the 3.8 to 5.0 Mev. region indicates the following level structure:

Elab	F^{17*}	$J \pi$	$\frac{\Gamma}{2}$	γ^2	$\gamma^2 / \left(\frac{3\pi^2}{2\mu a^2} \right)$
4.26 Mev.	4.60 Mev.	$3/2^-$	.110 Mev.	.11 Mev.	.042
4.65 Mev.	4.97 Mev.	$3/2^+$	.500 Mev.	.91 Mev.	.345

Here the resonance energy is the energy at which the resonant part of the corresponding phase shift is 90° . The half widths given are only average widths, since the width is a function of energy. The quantity $\frac{3\pi^2}{2\mu a^2}$, known as the Wigner limit, is the theoretical upper limit of a reduced width, although the theory is not at all exact. A reduced width approaching the Wigner limit implies that the state is mainly of a single particle configuration.³⁵ For this experiment, for the radius chosen, the Wigner limit is 2.64 Mev.

Thus the results imply that in the $3/2^+$ state the system spends an appreciable part of the time in the configuration of an O^{16} core and a single proton, while in the $3/2^-$ state it spends a considerably smaller fraction of the time in such a configuration.

In fig. 30 these levels are compared with the corresponding levels of the mirror nucleus O^{17} , and the agreement is seen to be good. This indicates that the coulomb forces in these nuclei have

³⁵T. Teichman and E.P. Wigner, Phys. Rev. 87, 123 (1952).

FIG. 30

COMPARISON OF
PROPOSED LEVELS
WITH LEVELS IN O^{17}

only a small effect on the energy levels.

It is of interest to compare the lifetimes of these two states in F^{17} , using the collision lifetime developed by Smith, which was described above. This lifetime is

$$\tau_J^\pi = \hbar \frac{d\delta_J^\pi}{dE}$$

For the broad $3/2^+$ state, the resonating phase shift δ_2^- changes by one radian over an energy range of 700 Kev. In the a.m.u. - Mev. fermi system of units,

$$\hbar = 6.4698 \text{ Mev. - Nuclear Seconds.}$$

where $1 \text{ Nuclear Second} = 1.0173 \times 10^{-22} \text{ Seconds.}$

This gives a collision lifetime

$$\tau = \frac{6.47}{.7} = 9.3 \text{ nuclear seconds}$$

On the other hand the $3/2^-$ state is considerably narrower; the resonating phase shift δ_1^+ changes by 80° or 1.4 radians over an energy range of 200 Kev. This gives a collision time

$$t = 45.5 \text{ Nuclear Seconds.}$$

By contrast, the $7/2^-$ state at a bombarding energy of 3.47 Mev. has a width less than 3 Kev., so that the resonating phase shift must change by one radian in less than .003 Mev. This gives a collision time of

$$t = 2200 \text{ Nuclear Seconds.}$$

These comparisons illustrate the fact that the broad $3/2^+$ state cannot be considered a state of the compound system in the same

sense that the $7/2^-$ state may be so considered. The collision lifetime for the state is roughly equal to the time required for the proton to travel a nuclear diameter. Evidently the configuration does not last long enough to make it meaningful to describe it as a state of the compound nucleus in the sense of Bohr's assumption.³⁶

To summarize, the following table shows the spin and parity assignments of the states of F^{17} up to the highest energy reached in this experiment. Assignments which are not conclusive are shown in parentheses.

F^{17*}	E_{lab}	J^π
3.10 MEV	2.66 MEV	$1/2^-$
3.86	3.47	$(7/2^-)$
4.60	4.26	$3/2^-$
4.97	4.65	$3/2^+$
5.47	5.20	$(3/2^-)$
5.67	5.40	$(7/2^-)$
5.84	5.54	$(3/2^+)$

³⁶N. Bohr, Nature 137, 344 (1936).

Fig. 33

R.R. Henry

APPENDIX A

Geometrical Calculations for the Rice University Small Volume Scattering Chamber

A small volume scattering chamber, usable either with gas targets or solid targets, has recently been constructed by C.M. Jones, and is described in his Ph.D. thesis.³⁷ A schematic diagram of the scattering chamber is shown in figure A1. As shown in the diagram, there are ports in the top and bottom pieces for the particle detectors. The top and bottom pieces may be totated, so that the angle of each counter axis with respect to the incident beam may be changed. In order to determine a center-of-mass scattering cross section it is necessary to know the angle between the counter axis and the beam axis, as well as the orientation of the counter slit system with respect to the beam axis. These points are illustrated in figure A2.

In the figure $\hat{K}$ is a unit vector in the direction of the beam. $\hat{K}$ and $\hat{J}$ are perpendicular to $\hat{K}$ and to each other. The vector $\vec{n}$ is in the direction of the counter axis, and the angle φ is the orientation of the top piece of the scattering chamber. The angle Θ is the angle between the counter axis $\vec{n}$ and the vector $\hat{K}$. When the top piece is rotated, the detector moves in the direction of the unit vector

$$\hat{u} = \frac{1}{\sin \Theta} \hat{n} \times \hat{K}$$

³⁷C.M. Jones, PH.D. Thesis, Rice University, (1961).

NOTE: BOLT HOLES NOT SHOWN

FIGURE A1
FULL SECTION OF
ASSEMBLED CHAMBER

SCALE
1 INCH

FIGURE A2

FIGURE A3

or else in the direction $-\hat{\mu}$. The orientation of the detector slit system is fixed with respect to the top piece, and consequently if the long axes of the detector slits are perpendicular to the beam for one orientation of the top piece they will not be perpendicular to the beam for other orientations of the top piece. In order to take this into account it is useful to define a unit vector $\hat{\lambda}$ which is perpendicular both to the beam axis $\hat{k}$ and the counter axis $\hat{n}$. Since

$$|\hat{\lambda} \times \hat{n}| = |\sin(\hat{\lambda}, \hat{n})| = \sqrt{1 - \sin^2 \theta \cos^2 \psi},$$

We define

$$\hat{\lambda} = [1 - \sin^2 \theta \cos^2 \psi]^{-1/2} \hat{k} \times \hat{n}$$

It is also useful to introduce a unit vector $\hat{A}$ which represents the direction of the long axes of the detector slits. The unit vectors $\hat{\mu}$, $\hat{\lambda}$ and $\hat{A}$ are all perpendicular to the counter axis $\hat{n}$. The relation between them is shown in figure A3. In this figure the counter axis is perpendicular to the diagram. α is the angle between $\hat{A}$ and $\hat{\mu}$, and is fixed. η is the angle between $\hat{\lambda}$ and $\hat{\mu}$, and γ is the angle between $\hat{\lambda}$ and $\hat{A}$.

$$\cos \eta = \hat{\lambda} \cdot \hat{\mu} = \frac{(\hat{\lambda} \times \hat{n}) \cdot (\hat{\mu} \times \hat{n})}{\sin \theta [1 - \sin^2 \theta \cos^2 \psi]^{1/2}}$$

But

$$\begin{aligned} (\hat{\lambda} \times \hat{n}) \cdot (\hat{\mu} \times \hat{n}) &= \hat{n} \cdot (\hat{n} \times \{\hat{\lambda} \times \hat{\mu}\}) \\ &= \hat{n} \cdot ((\hat{n} \cdot \hat{\mu}) \hat{\lambda} - (\hat{n} \cdot \hat{\lambda}) \hat{\mu}) \end{aligned}$$

$$\begin{aligned}
 &= (\hat{k} \cdot \hat{n})(\hat{l} \cdot \hat{n}) \\
 &= \sin \theta \cos \theta \cos \varphi
 \end{aligned}$$

Thus

$$\cos \eta = \frac{\cos \theta \cos \varphi}{[1 - \sin^2 \theta \cos^2 \varphi]^{1/2}}$$

Let l_0 denote the length of beam seen by the detector when the detector slit axes are perpendicular to the beam. Then in the general case the length of beam seen by the detector is

$$l = \frac{l_0}{\cos \gamma}$$

From the diagram, (figure A3),

$$\cos \gamma = \cos (\eta - \alpha)$$

From the formula given above for $\cos \eta$,

$$\sin \eta = \left(1 - \frac{\cos^2 \theta \cos^2 \varphi}{1 - \sin^2 \theta \cos^2 \varphi} \right)^{1/2} = \frac{\sin \varphi}{(1 - \sin^2 \theta \cos^2 \varphi)^{1/2}}.$$

As discussed previously, the geometrical factor which determines the counting rate for a given cross section and target density is

$$G = \frac{A l}{R^2} ,$$

where A is the area of the rear slit, l is the length of beam seen by the detector, as before and R is the distance from the scattering center to the rear slit.

Since $l = \frac{l_0}{\omega_2 \gamma}$,
 $G = \frac{G_0}{\omega_2 \gamma}$

where $G_0 = \frac{A l_0}{R^2}$

Thus

$$G = G_0 \frac{(1 - \sin^2 \theta \cos^2 \psi)^{1/2}}{\cos \alpha \cos \theta \cos \psi + \sin \alpha \sin \psi}$$

In this expression θ and α are constants determined by the construction of the scattering chamber; ψ is the angle representing the orientation of the top (or bottom) piece, and may be read from an azimuth circle attached to the scattering chamber.

The laboratory scattering angle δ is given by

$$\cos \delta = \hat{n} \cdot \hat{l} = \sin \theta \cos \psi.$$

It is necessary to consider the effect of errors in the geometry of the scattering chamber. The previous discussion assumed that the beam axis and the counter axis intersect at a point. A coordinate system was set up with its origin at the point of intersection, and with the unit vector $\hat{l}$ in the direction of the beam axis. It was assumed that the angle θ between the counter axis and the unit vector $\hat{l}$ was constant (i.e. independent of the orientation of the top piece).

Let us consider the case in which the beam direction is in error. Let us define a coordinate system $\hat{i}, \hat{j}, \hat{k}$ whose origin lies on

the counter axis for all orientations of the top piece. Then in this coordinate system the beam axis does not pass through the origin; further more the beam direction will be slightly different from the direction of the unit vector $\hat{l}$. However the angle Θ between the counter axis and the unit vector $\hat{k}$ will be constant, as before.

Let $\hat{b}$ be a unit vector in the direction of the beam. Then

$$\hat{b} = \hat{l} + \epsilon_y \hat{j} + \epsilon_z \hat{k},$$

where

$$\epsilon_y \ll 1, \quad \epsilon_z \ll 1.$$

Proceeding as before, the unit vector $\hat{t}$ perpendicular both to $\hat{b}$ and the counter axis $\hat{n}$ is

$$\hat{t} = [1 - \sin^2 \Theta \cos^2 \varphi]^{-1/2} \hat{b} \times \hat{n} + O(\epsilon^2),$$

or

$$\hat{t} = \hat{t}_0 + [1 - \sin^2 \Theta \cos^2 \varphi]^{-1/2} \bar{\epsilon} \times \hat{n},$$

where

$$\bar{\epsilon} = \epsilon_y \hat{j} + \epsilon_z \hat{k}$$

and $\hat{t}_0$ is the vector for the case $\epsilon = 0$, derived previously. As before, the direction in which the detector moves when the top piece is rotated is

$$\hat{u} = \frac{1}{\sin \theta} \hat{n} \times \hat{k}.$$

the angle γ between $\hat{t}$ and $\hat{u}$ is now given by

$$\cos \gamma = \left(\hat{t}_0 + \frac{\vec{E} \times \hat{n}}{\sqrt{1 - \sin^2 \theta \cos^2 \varphi}} \right) \cdot \hat{u},$$

or letting

$$\vec{E} = E (\cos \beta \hat{j} + \sin \beta \hat{k}),$$

$$\cos \gamma = \frac{\cos \theta \cos \varphi}{\sqrt{1 - \sin^2 \theta \cos^2 \varphi}} + E \frac{\sin \theta \sin \varphi \cos \beta + (\cos \theta - \frac{1}{\sin \theta}) \sin \beta}{\sqrt{1 - \sin^2 \theta \cos^2 \varphi}}$$

since

$$\begin{aligned} (\vec{E} \times \hat{n}) \cdot \hat{u} &= (\vec{E} \times \hat{n}) \cdot \left(\frac{\hat{n} \times \hat{k}}{\sin \theta} \right) = \frac{1}{\sin \theta} \hat{n} \cdot (\hat{k} \times \{\vec{E} \times \hat{n}\}) \\ &= \frac{1}{\sin \theta} \hat{n} \cdot (\vec{E} (\hat{k} \cdot \hat{n}) - (\hat{k} \cdot \vec{E}) \hat{n}) \\ &= \frac{1}{\sin \theta} ((\hat{n} \cdot \vec{E}) \sin \theta - (\hat{k} \cdot \vec{E})) \\ &= E \left(\sin \theta \sin \varphi \cos \beta + \left(\cos \theta - \frac{1}{\sin \theta} \right) \sin \beta \right). \end{aligned}$$

As before,

$$\cos \gamma = \cos \gamma \cos \alpha + \sin \gamma \sin \alpha$$

and

$$G = \frac{G_0}{\cos \gamma}$$

The true scattering angle δ is given by

$$\cos \delta = \hat{n} \cdot \hat{b} = \cos \delta_0 + \hat{n} \cdot \vec{E}$$

where δ_0 is the value of δ for $E = 0$.

This gives

$$\cos \delta = \cos \delta_0 + E \{ \sin \theta \sin \varphi \cos \beta + \cos \theta \sin \beta \}.$$

The values of the error parameters E and β were measured and were found to be

$$E = 2.444 \times 10^{-4},$$

$$\beta = 90^\circ$$

Using these values the calculation of the geometrical factor G and the true scattering angle δ was programmed for an I.B.M. 650 computer. The program calculated the values of the geometrical factor and the scattering angle as a function of the angle φ , which measures the orientation of the top (or bottom) piece of the chamber. This angle may be read from an azimuth circle attached to the scattering chamber. These results were calculated for all four particle detectors.

APPENDIX B

A. Program for the Calculation of Coulomb Wave Functions

The presently available tables of coulomb wave functions are unsatisfactory in that the application of these tables to a particular problem involves a large amount of double interpolation, which is very time consuming and leads to results of dubious accuracy.

Furthermore for $\ell = 3$ or 4 the tables have large gaps which often include the region of interest.

What is needed is a program which will calculate coulomb wave functions for a particular target nucleus and a particular incident particle, as a function of bombarding energy, for various radii. Such a program has been developed, for an I.B.M. 709 computer.

The program uses subroutines developed by Griffy,³⁸ which calculate the regular coulomb wave function F_ℓ and the irregular coulomb wave function G_ℓ . The details concerning these subroutines may be found elsewhere,³⁸ but the method is as follows:

F_ℓ and G_ℓ are the regular and irregular solutions, respectively, of the equation³⁹

$$\frac{d^2 u_\ell}{dn^2} + \left[k^2 - \frac{\ell(\ell+1)}{n^2} - \frac{2\pi}{k^2} \frac{z_1 z_2 e^2}{n} \right] u_\ell(n) = 0$$

³⁸T.A. Griffy, PH.D. Thesis, Rice University, (1961).

³⁹L.I. Schiff, op.cit., p. 115.

Here K is wave number, μ is the reduced mass, and Z_1 and Z_2 are the charge numbers of the incident and target nucleus. F_l and G_l are each functions of both the variables $\rho = Kr$ and $\eta = \frac{Z_1 Z_2 e^2}{\hbar v}$. For large ρ they have asymptotic forms given by the expansion⁴⁰

$$G_l + iF_l \xrightarrow{\rho \rightarrow \infty} e^{i\theta_l} \left\{ 1 + \frac{(\eta-l)(\eta+l+1)}{(1!)(2\rho)} + \frac{(\eta-l)(\eta-l+1)(\eta+l+1)(\eta+l+2)}{(2!)(2\rho)^2} + \dots \right\}$$

where

$$\theta_l = \rho - \eta \ln \rho - \frac{l\pi}{2} + \arg \Gamma(l+1+i\eta)$$

while for $\rho \rightarrow 0$ the regular function F_l may be calculated from the series⁴⁰

$$F_l = C_l(\eta) \rho^{l+1} \sum_{n=l+1}^{\infty} A_n^l(\eta) \rho^{n-l-1}$$

where C_l is a normalizing factor given by

$$C_0(\eta) = \left(\frac{2\pi\eta}{e^{2\pi\eta} - 1} \right)^{1/2}$$

$$C_l(\eta) = \frac{(\eta^2 + l^2)^{1/2}}{l(2l+1)} C_{l-1}(\eta)$$

and $A_n^l(\eta)$ is given by

⁴⁰Tables of Coulomb Wave Functions, National Bureau of Standards, Applied Mathematics Series, V. 17.

$$A_{l+1}^l = 1, \quad A_{l+2}^l = \frac{\eta}{l+1},$$

$$(n+l)(n-l-1) A_n^l = 2\eta A_{n-1}^l - A_{n-2}^l$$

$$\text{for } n > l + 2.$$

The program used in calculating F_l and G_l starts by calculating F_l and G_l at a distance of about 50 fermis (1 fermi = 10^{-13} cm.) from the nucleus, using the asymptotic series. Also initial values of the derivatives F_l' and G_l' are determined. The program then works its way in toward the nucleus in steps of .1 fermi, using the differential equation to determine the values of F_l and G_l for smaller and smaller radii. At the conclusion of this process the computer will have in its memory values of F_l and G_l for a large number of radii, at spacings of .1 fermi.

Using this subroutine the main program calculates the following functions:

$$A_l^2 = F_l^2 + G_l^2$$

$$\phi_l = \tan^{-1} \frac{F_l}{G_l}$$

$$P_l = \frac{KR}{F_l^2 + G_l^2}$$

$$l + g_l = l + KR \left[\frac{1}{F_l} \frac{\partial F_l}{\partial \rho} - \frac{1}{A_l^2} \frac{G_l}{F_l} \right]_{\rho=KR}$$

These functions were discussed previously. The results are printed out for as many radii as desired, as a function of bombarding energy, for $l = 0, 1, 2, 3$, and 4 . The results for the functions A_l^2 and ϕ_l for the elastic scattering of protons by oxygen are shown in B1 and B2. The results are seen to agree with values calculated from the table of Sharp.³⁴

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