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RICE UNIVERSITY

Cometary Knots in the Helix Nebula

by

Kerry Dorinda Patrick Handron

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IN PARTIAL FULFILLMENT OF THE
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Abstract

Cometary Knots in the Helix Nebula

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The central star of the Helix Nebula, the closest planetary nebula, is ringed by hundreds of Cometary Knots. Presented here are images taken with WFPC2 of a field in the northern portion of the bright ring and some physical parameters and models derived from the data. The field of view contains more than thirty well formed Cometary Knots and reveals that the entire bright ring is composed of knot-like structures. The data is in three narrow emission line filters, $H\alpha$, [OIII] and [NII]. Optical thickness to ionizing photons for the knots is established, physical properties such as density in the ionized portions of the knots and dust mass are determined and the beginning of a model to explain the detailed structure set forth.

Contents

Abstract	ii
1 Introduction	1
1.1 A few basics of planetary nebulae	1
1.2 Basic parameters of the Helix Nebula	2
1.3 The cometary knots in the Helix Nebula	3
1.3.1 Other observations of the Cometary Knots	3
1.3.2 Possibilities for the origin and nature of the Cometary Knots	4
2 Data	5
2.1 WFPC2 data	5
2.2 Flux calibration	8
2.3 Discussion of the images	10
2.3.1 $H\alpha$	10
2.3.2 [OIII]	11
2.3.3 [NII]	12
2.3.4 Three color and ratio images	12
2.4 Selection of knots studied	13
3 Results and Models	17
3.1 Optical depth to the Lyman Continuum	17
3.1.1 Surface brightness and distance dependence	17
3.1.2 Absolute calibration from $H\beta$ total flux	18
3.1.3 Measurement of substellar $S_{H\alpha}$ and distance	19
3.1.4 Results	21
3.2 Densities	22
3.2.1 Nebular field	22
3.2.2 Ionized portions of the Cometary Knots	24
3.3 Optical depth	24
3.4 Masses of the Cometary Knots	27
3.5 Sizes of the Cometary Knots	28

3.6	Structure of cusps/exponential atmospheres	29
3.6.1	Measurement of the front edge of the Cometary Knots	29
3.6.2	Density to observed surface brightness relation	30
3.6.3	Explanation of exponential density distribution	31
3.6.4	The force	32
4	Conclusion	36
4.1	Summary of results	36
4.2	Continuing Observations	37
4.2.1	Hubble Space Telescope	37
4.2.2	Ground Based Telescopes	37
4.3	Possible Next Steps	38
4.4	Final Comments	38
A	Data	39
B	Color Figures	41
	Bibliography	4⁴

Chapter 1

Introduction

1.1 A few basics of planetary nebulae

“Planetary Nebula” is a misnomer. The main resemblance planetary nebulae have to planets is that, like the outer planets, they appear as fuzzy greenish patches in small telescopes. Sir William Herschel popularized the name when he cataloged nonstellar objects.

The planetary nebula is a stage near the end of the evolutionary cycle for intermediate mass stars. Stars with main sequence masses between one and eight solar masses follow similar evolutionary tracks. After contracting to the main sequence and spending most of their life there burning hydrogen to helium in the core, the core can no longer support hydrogen burning and the burning is forced into a shell around the core. The star moves out the red giant branch of the H-R diagram and the helium core steadily increases in mass until the central temperatures are such that the helium ignites. After the helium flash it moves back towards the main sequence to the horizontal branch until the helium burning is also forced into a shell. At this point the star moves up the asymptotic giant branch. A star at this point in its life, with hydrogen and helium burning in shells around a core of carbon, nitrogen and oxygen, is unstable to dynamic pulsations. Small pulsations increase in amplitude until the material has achieved escape velocity. Once material has started to escape and the stellar atmosphere is smaller, the atmosphere that is left is less stable and ejection continues until the hydrogen rich zone is completely ejected. (Osterbrock 1989)

Only stars in a narrow mass range become planetary nebulae. Stars with a core mass of $\approx 0.6M_{\odot}$ evolve at such a rate that the white dwarf left over after the mass ejection becomes hot enough to ionize the nebular material while it is still near the star. Larger stars evolve faster and are not visible very long as planetary nebulae while smaller stars do not evolve to the required temperature before the surrounding material is too widely dispersed (Kwok 1993). The lifetime for a long lived planetary nebula before is only about 10,000 years. This estimate comes from considering the expansion velocities, surface brightness and the sizes of planetary nebulae.

A model that explains many of the observations of planetary nebulae is the interacting wind model. The extended red giant ejects the hydrogen rich atmosphere which becomes the nebula. A fast wind from the remaining white dwarf, $\approx 2000 \text{ km sec}^{-1}$, runs into the nebular material that is flowing away from the central star at $\approx 20 \text{ kmsec}^{-1}$. As the fast wind sweeps up the nebular material it compresses it into a spherical shell. As there may be several episodes of loss by massive slow winds and subsequent sweeping by fast ones, systems with several rings may be formed. The various morphologies of planetary nebulae observed can be explained by assuming the mass loss rate and velocity are functions of latitude on the central star (Frank and Balick 1993).

As the central star evolves and increases in temperature the nebula may change from ionization bounded to density bounded. Since a density bounded nebula is entirely ionized, there are photons with energies greater than the 13.6 eV's required to ionize hydrogen that are never processed, and escape from the nebula. Conversely an ionization bounded nebula has a shell of neutral material around it and all ionizing photons are absorbed in the nebula. A planetary nebula may change from ionization to density bounded as it evolves.

1.2 Basic parameters of the Helix Nebula

The distance to the Helix Nebula is not well determined; it is certainly at the outer limits of accurate parallax measurements. A distance determination based on the Shklovsky method, which assumes the ionized mass and electron temperature are the same for all planetary nebulae, yields a distance of 110 pc (Cahn and Kaler 1971). The ionizing flux of the central star of a planetary nebula can change by several orders of magnitude over the life of the nebula indicating that the ionized mass can change. Daub assumed the ionized mass is related to observable radio and infrared quantities and found a distance of 150 pc (Daub 1982). Zhang took core mass, luminosity and surface gravity found by distance independent quantities and determined a distance of 160 pc (Zhang 1993). I have taken the distance to the Helix Nebula to be 150 pc in this paper. The central star's coordinates (J2000) are 22:29:38.4 and -20:50:11.4.

Temperature calculations performed by Bohlin employing a Zanstra method with a model atmosphere and standard helium abundance indicates a surface temperature of 130,000K for the central star (Bohlin et al. 1982). X-ray spectra indicate a stellar

temperature of 113,000K(Leahy 1993). Bohlin derives a luminosity for the central star of $60L_{\odot}$.

Other studies of the nebula have measured such properties as density of the outer ring, expansion velocity, molecular emission, x-ray emission and a large faint outer halo. The densities estimated for the halo have a wide range and are discussed in section 3.2.2. The [OIII]5007 Å emission line indicates that the expansion of the nebula is about 20 km sec^{-1} in the outer ring(Terrett 1978). Inner shells with expansion velocities of up to 60 km sec^{-1} have been found(Meaburn 1992). Molecular emission is detected in H_2 and CO and is discussed in section 3.1.2. Two temperature X-ray emission indicates both the temperature for the central star and the existence of a hot plasma, either in a stellar corona or in a hot bubble created by interacting winds(Leahy et al. 1994).

1.3 The cometary knots in the Helix Nebula

The Cometary Knots ring the central star and with low resolution look like filaments. There are more than 600 easily observable at $1.5''$ resolution and all have a similar form. The head is a bright cusp on the edge towards the central star and the tail trails radially away. The innermost Cometary Knot is about $88''$ from the central star and it appears that the bright nebula, which extends to a radius of $300''$, has Cometary Knots all through it.

The Helix is the only nebula in which Cometary Knots have been found. No other planetary nebula is close enough to have such structures resolved. It may be that this structure is common to all planetary nebulae at a certain phase in their evolution.

1.3.1 Other observations of the Cometary Knots

The Cometary Knots were first noted by Baade as ‘filaments’ (Vorontsov-Velyaminov 1968). It was postulated that the entire bright ring was made of these filaments, and indeed it seems to be very crowded with Cometary Knots, although in most cases the tails, the parts that are seen as filaments, are short or nonexistent in the bright ring.

Emission from the Cometary Knots includes [OIII], $\text{H}\alpha$ and [NII]. The [OIII] is a high ionization state ion that requires photons of greater than 25.4 eV. These photons do not travel as far into the dense portions of the knots as the $\text{H}\alpha$ ionizing photons so this emission is confined to an outer halo, in a diffuse region around the cusp defined by $\text{H}\alpha$. $\text{H}\alpha$ emission is simply the product of photons with energies greater than

13.6eV ionizing hydrogen and the resulting HII recombining to any excited state and dropping from the $n=3$ to the $n=2$ level. The [NII] emission is from nitrogen that has been ionized by greater than 14.3 eV photons and excited by collisions.

Meaburn et al. (1992) published images of the Cometary Knots with resolution of .86" in $H\alpha$ + [NII] and [OIII] filters. Huggins et al (1992) found that CO emission was localized around clumps of Cometary Knots. Walsh published images from the New Technology Telescope of the middle 7.5' of the nebula which includes most of the knots. He counted about 600 Cometary Knots in the nebula (Walsh 1993).

1.3.2 Possibilities for the origin and nature of the Cometary Knots

It is not known what the Cometary Knots actually are. Cappriotti (1971) originally suggested they are results of Rayleigh-Taylor instabilities where the hot diffuse material hits the denser cooler ring. Dyson et al. (1992) have proposed that they are remnants of maser spots known to exist on red giant stars. A third, exciting possibility, that the Cometary Knots are similar to our Oort cloud (Menzel 1971, Gussie 1995).

Chapter 2

Data

This work is based on a set of Wide Field Planetary Camera 2(WFPC2) images of a single field in three filters which image the northern portion of the main ring of the Helix Nebula. Processing of the data includes pipeline processing, cosmic ray cleaning, mosaicing the four CCD frames into a single field and calibrating into units of surface brightnesses. This appears to be a straight forward process, but it is complicated as software programs are not always available or correct and understanding of the telescope is always evolving. The images are excellent, well exposed in each filter and with good pointing. Figures 1 and 2 in the appendix are three-color pictures of both the entire field and a blow up of CCD # 4 that includes most of the well defined Cometary Knots.

2.1 WFPC2 data

The WFPC2 camera on the Hubble Space Telescope has three wide field cameras (WFCs) with optical trains of F/12.9 and .0997" per pixel resolution, and one planetary camera (PC) with F/30 and .0455" per pixel resolution. These four fields mosaic together to give the distinctive shape of the WFPC2 field of view.

The data presented here were taken on August 7th, 1994 in three filters: the F502N filter which isolates the 5007 Å [OIII] emission; the F656N, 6563 Å H α emission; and the F658N, the [NII] doublet at 6583 and 6548 Å. Each exposure was split for cosmic ray rejection and totaled 4800, 6600 and 3200 seconds in [OIII], H α and [NII] respectively. The pointing was RA 22:29:37.7 and dec -20:47:58.4 using the WF4-FIX mode to get a very bright Cometary Knot in the center of CCD number four and a position angle (PA) of 287 deg to include as many Cometary Knots as possible. Figure 2.1 shows the entire bright nebula with the field of view outlined.

The data was then pipeline processed at the Space Telescope Science Institute (STScI). Pipeline processing indicates standard and automatic processing of the data from the telescope before it is sent to the observer. This processing included building headers and reformatting the data to Flexible Image Transfer System(FITS) format. Further pipeline processing included in the data reduction is that which is common

Figure 2.1 This is an image of the entire nebula with an overlay of the WFPC2 fields of view. The first field is on the right and the field on the left will be imaged soon. The image is courtesy of the Anglo-Australian Observatory and copyrighted by David Malin.

to most programs: the analog to digital conversion (ADC), the bias level removal, the bias image subtraction, the dark image scaling and subtraction and the flat field image correction. STScI is constantly building new flats, and at the time of an observation and the pipeline processing, the best flats for that observation may not yet exist. This was true for these observations and the information concerning which flats are best was not available until several months after the observations. The ADC and flat field correction files used then are the best available now. The flat field is a combination of thermal vacuum testing on the ground and 5 frames of on-orbit illumination patterns. The ADC file was created based on thermal vacuum tests. The recommended bias file was changed and updated to include only data taken between June 14 and August 26 rather than the previous file that was made between April 23 and June 14. The dark file was updated for data taken after the July 28th decontamination rather than the July 10th. Decontaminations involve warming the CCD's to about 20°C to boil off contaminants that build up on the cold surfaces and degrade the telescopes throughput, especially in the UV. They also serve to remove about 80% of the hot pixels that accumulate between decontaminations. I did not recalibrate the data to use these new dark and bias files as the changes are not significant for this analysis and I did not discover the new files until quite recently.

My processing of the data started with the Image Reduction and Analysis Facility (iraf) tasks cosmic ray reject (*crrej*), *wmosaic* and *seam*. *Crrej* is a Space Telescope Data Analysis System (stdas) task designed to remove cosmic rays. It starts with a stack of coaligned images that are identical except for the cosmic rays and makes an “initial guess” image by taking the mean value on a pixel-by-pixel basis. Then it compares every pixel in the initial images to its corresponding pixel in the mean image, rejects pixels that are more than the specified number of σ s deviations and builds a new mean image. I used four iterations with rejection levels of 4, 4, 3 and 2σ s, an expansion radius of one pixel and an expansion discriminant reduction factor of 0.5. This means that on the first pass pixels more than 4σ s deviation from the mean image were rejected as well as adjacent pixels more than 2σ s deviation. *Crrej* assumes that all the images in the stack were taken with the same telescope configuration and of the same duration. This is not true for the H α images, which are exposures of 1100, 1200, 2100 and 2200 seconds. I scaled the images by simply multiplying the first by 2, the second by 11/6 and the third by 22/21. This brings the intensities to the same level but does not change the statistics that are dependent on different exposure lengths. This does not present a problem because for studying larger structures perfect cosmic ray removal is not crucial. The output of *crrej* is a single set of the four separate chip images.

The *wmosaic* task combines the four separate CCD's into a single image. The PC frame must be scaled spatially as its pixels are only .0455" rather than .0997" like the WFC's. *Wmosaic* does this scaling, performs a geometric distortion correction, rotates the four fields, makes small shifts to align the fields and small corrections in intensity levels on a chip to chip basis. Then *crrej* combines them into a single image. Because light on the edges of the fields is split between CCD's and the *wmosaic* task only imperfectly corrects for this, there are shadows at the edges of the fields which are usually 3 pixel or less. The task *seam* evaluates pixels that are in good regions on both sides of the possibly bad pixels to determine what is bad, then does a linear interpolation over the bad pixels. I ran five iterations, then removed the few remaining bad pixels by hand. Photometry close to these regions would not be accurate but this does not affect this analysis as none of the Cometary Knots are in affected areas.

The CCD's in WFPC2 are not perfect. There is a ruling error that causes every thirty-fourth row to be 3% smaller than the rest. The flat fields correct for this except for photometric and astrometric errors much smaller than this project considers.

There are also hot and dark pixels and charge traps. Charge traps are pixels that do not efficiently move the charge along during the read-out process, but instead collect some electrons and let them out as an exponential function of time. This leads to bad columns, especially when a bright object is on, or needs to read out over a trap. Unfortunately the brightest Cometary Knot, the one on which the pointing was chosen, is right on a charge trap. This is the knot in the center of WF-4. Using the iraf task image-edit (*imedit*) I did an interpolation over a column right in the middle of it. For purely cosmetic reasons, I also fixed by hand a few other bad columns and bad pixels that were scattered through the field.

2.2 Flux calibration

The intensities of the images are given in detector numbers after the pipeline processing. More physically meaningful units would be surface brightnesses. The conversion is simply a function of telescope parameters.

$$DN = S \times A \times \Omega \times s_\lambda \times \frac{1}{G} \times T_\lambda \times t \quad (2.1)$$

DN is the detector number in counts per pixel; S, the surface brightness in photons $\text{cm}^{-2} \text{sec}^{-1} \text{ster}^{-1}$; A, the effective collection area of the telescope; Ω , the size of a pixel in steradians; s_λ , the response of the system; G, the gain in electrons DN^{-1} ; T_λ , the filter transmission, and t, the length of the exposure in seconds. The Optical Telescope Assembly (OTA) handbook lists $A = 38,990 \text{ cm}^2$ and $\Omega = .0997''^2 \text{ pixel}^{-1} = 2.303 \times 10^{-13} \text{ ster pixel}^{-1}$, and has plots of s_λ and T_λ . The iraf task wmosaic scales the intensity of the PC frame to match the WFC frames. Gain and exposure length are listed in the headers. Defining S_{Filter} as the observed surface brightness in a filter and $T_{\text{filter}}^\lambda$ as the transmission of a filter at a certain wavelength yields:

$$S_{\text{Filter}} \equiv \frac{DN}{A \times \Omega \times \frac{1}{G} \times t} = S_\lambda \times s_\lambda \times T_\lambda^{\text{Filter}} = \frac{DN}{\text{constant}}. \quad (2.2)$$

The constants are 3.00×10^6 , 3.69×10^6 and 6.59×10^6 for the F656N, F658N, and F502N filters respectively and the units for eq. 2.2 are surface brightnesses, photons $\text{cm}^{-2} \text{sec}^{-1} \text{ster}^{-1}$.

The [NII] doublet at 6548 and 6583 Å straddles H α at 6564 Å and the approximate 25 Å width of the WFPC2 narrow filters is not narrow enough to completely separate the lines. There is contamination by [NII] emission in the F656N filter that

is designed to isolate $H\alpha$ and by $H\alpha$ in the F658N filter that is designed for [NII]. Fig 2.2 illustrates the filter shapes taken from the WFPC2 observers handbook.

Figure 2.2 The profiles of F656N and F658N are plotted along with the position of the relevant emission lines. The abscissa is wavelength in Å and the ordinate is percent transmission of the filter. The middle line is $H\alpha$ and the outer two the [NII] doublet. All three lines are in the transmission region of F656N and two in F658N.

The surface brightness observed in each filter can be expressed as a function of the actual surface brightness in each line in the transmission region of the filter, the actual surface brightness of the continuum and the shape of the filter. The observed surface brightness in a filter is the sum of all the lines in the transmission region of the filter each multiplied by the transmission of the filter at the wavelength of the line, plus the continuum contribution. Taking the continuum to be negligible,

$$S_{F656N} = S_{H\alpha} \times T_{H\alpha}^{F656} + S_{[NII]6548} \times T_{[NII]6548}^{F656N} + S_{[NII]6583} \times T_{[NII]6583}^{F656N} \quad (2.3)$$

and

$$S_{F658N} = S_{H\alpha} \times T_{H\alpha}^{F658} + S_{[NII]6548} \times T_{[NII]6548}^{F658N} + S_{[NII]6583} \times T_{[NII]6583}^{F658N} \quad (2.4)$$

$T_{H\alpha}^{F656N}$ denotes the transmission of $H\alpha$ through the F656N filter. S_{F656N} and S_{F658N} were computed in the previous section, the T's are plotted in the Instrument Handbook and $S_{[NII]6583} = 3 \times S_{[NII]6548}$ so the only two variables in this set of two equations are $S_{H\alpha}$ and $S_{[NII]6583}$. Including values for T, these reduce to:

$$S_{H\alpha} = S_{F656N} \times 1.21 \times (1 - .194 \frac{S_{F656N}}{S_{F658N}}) \quad (2.5)$$

$$S_{[NII]} = S_{F658N} \times 1.16 \times (1 - .0398 \frac{S_{F658N}}{S_{F656N}}) \quad (2.6)$$

The algebra was done with the iraf task image arithmetic (imarith) in a series of steps. The resulting images are flux calibrated in photons $\text{cm}^{-2} \text{sec}^{-1} \text{ster}^{-1}$ and represent the emission from $H\alpha$ and $[NII]_{6583,6548}$ doublet, assuming there is no continuum. With the assumption of no continuum the surface-brightness-calibrated image in the F502N filter is the surface-brightness-calibrated [OIII] image.

2.3 Discussion of the images

2.3.1 $H\alpha$

When observed in $H\alpha$ the Cometary Knots are cusps with trailing tails pointing radially away from the central star. The two streamers of most of the tails are parallel to each other and continue outwards until they are lost in other Cometary Knots or in the bright ring. A few curve back on themselves and end or flay out for a short way before fading into the nebular background, but these are the exceptions. Several have brighter, curly, short filaments embedded in the tails. The tails are often darker between the bright streamers than the nearby in the field of the nebula, especially near the head of the knot as seen in figure 2.3.

The heads of the knots also exhibit remarkable similarity of form. For the largest knots the brightest portion is an area about five pixels wide and three pixels deep for the largest knots. As the proportions do not vary much between the knots any easily defined measurement works as well as another. An easy measurement to make is the distance from brightest point to brightest point across the part where the head meets

Figure 2.3 This knot shows the typical shape in $H\alpha$ with a bright cusp facing the central star and parallel streamers as the tail.

the tail. This does not depend on the settings of the display device. I define this as the diameter of a Cometary Knot. Measured this way the largest knot in the field has a diameter of 20 pixels and the smallest, 8. Cuts made along the long axis of the knots plotting intensity vs pixel number show a definite exponential drop from the brightest pixel going towards the central star. The structure is more complex on the inside of the knots as seen in figure 2.4.

The bright ring of the nebula appears to have very many clusters of these knots, in general smaller than the knots closer to the central star and so crowded I am not able to analyze individual knots. They do not have prominent tails and often group together in larger 'knot shapes'. This is illustrated in figure 2.5.

2.3.2 [OIII]

The knots actually appear very dark in [OIII], showing as extinction against the bright background nebula as seen in figure 2.6. The different Cometary Knots show widely varying degrees of absorption. About a quarter of those that are sharp and clear in [NII]+ $H\alpha$ are not seen at all in [OIII]. For those that are seen, the darkest part is inside the bright cup region. The extinction extends into the tail region one or two times the diameter for most of the knots. For the knot in the center of CCD 4, the extinction can be seen down the tail a distance of several times the diameter of the

Figure 2.4 This is a surface brightness in $H\alpha$ plot from behind one of the knots. The central star is away and to the left.

Cometary Knot. The knots do not show up only in absorption. Some of the knots that are closer to the central star have diffuse bright halos in the $[OIII]$ emission.

2.3.3 $[NII]$

The Cometary Knots are well defined in $[NII]$ and are similar to, but sharper than their forms in $H\alpha$. There is no faint outer halo and the edges of the cusps are smooth and clear as seen in figure 2.7. The tails become faint quickly and are hard to identify beyond about twice the diameter of the knots in these images. The tails are smooth across the tail, without bright streamers on the edges or darker areas in the middle.

2.3.4 Three color and ratio images

The three-color image was made with the iraf task `rgbsun` (red, green, blue, `sunraster` file) with blue for $[OIII]$, green for $H\alpha$ and red for $[NII]$. While there is no new information in the image some things are easier to notice. Manipulation of the color balance emphasizes the very long streamers out in the bright ring as shown in fig. 3 of the appendix. At a different color balance the darkness of the inner part of the heads of the knots becomes striking.

Ratio images also contain no new information, but rather emphasize certain characteristics. A ratio image of $\frac{H\alpha}{[NII]}$ also clearly shows the radial structure of the many

Figure 2.5 Two sections of the $H\alpha$ image taken from the bright ring showing many knot-like structures clumped together.

tails as well as different tail structure for some of the knots. Some of the knots have pointy tails with an overall shape like tadpoles.

The $\frac{[OIII]}{H\alpha}$ ratio brings out the greatest number of knots on the edges of the bright ring as well as emphasizing the long tails.

2.4 Selection of knots studied

The selection of which knots to include was simple. I used all the knots that are alone in the field with no overlap of other knots. This includes twenty-two knots. There are also seven that overlap with a single other knot or tail for less than half of their width. Because the knots appear symmetric I could consider just the clear half of each of these knots. Fig 2.10 shows the least clear knot that I considered. I did not include any knots from the crowded bright ring.

Figure 2.6 The heads of the Cometary Knots are dark in [OIII] with the extinction extending back into the tails. Some of the knots also have bright, diffuse halos.

Figure 2.7 In [NII] the tails are less striking while the heads have linear form to $H\alpha$.

Figure 2.8 Two ratio images of [NII] over $H\alpha$ show a very different form for the tails. Compare these with the $H\alpha$ image in figure 2.3.

Figure 2.9 Ratio images of [OIII] over $H\alpha$ emphasize the long tails where there is extinction in 5007 and emission in $H\alpha$.

Figure 2.10 This is the least well defined Cometary Knot I measured.

Chapter 3

Results and Models

This chapter explores some of the properties of the Cometary Knots that can be determined from the images, along with some models to fit the data. Properties of the Cometary Knots to be considered include the dependence of their surface brightness on their distance from the central star, their absolute surface brightness, their exact shape and the amount of extinction. These properties are used to derive such information as their optical depths, bounds on the luminosity of the central star, the magnitude and nature of the force on the knots and their masses.

3.1 Optical depth to the Lyman Continuum

The optical depth of the Cometary Knots to the Lyman continuum can be investigated by determining the surface brightness and distance from the central star of individual knots. The photon flux through a set area, in this case represented by the Cometary Knots, falls off as $\frac{1}{r^2}$ if there is no intervening material. There is very little material in the case of the Helix as is described in section 3.2.1. The portions of the nebula that are filled with Cometary Knots are the exception, containing much material and absorbing many if not all of the ionizing photons. Whether or not the Cometary Knots do stop all the ionizing photons can be tested by studying the relation between their surface brightness and the distance from the central star. If the Cometary Knots are optically thick to the ionizing radiation and if observations were made of the substellar point from the perspective of the star, the surface brightness would fall off exactly as the inverse square of the distance from the star.

3.1.1 Surface brightness and distance dependence

Baldwin et al, (1991) originally pointed out that in the case of a physically thin, optically thick ionized zone illuminated by a single star, the flux of ionizing photons at some point in the nebula, $F_{Ly\alpha}$, must be equal to the total number of photoionizations per unit time in the overlying column. The number of photoionizations must exactly equal the number of recombinations for conditions of ionization equilibrium. Therefore number of ionizing photons at a particular point in the nebula, defined

by its distance from the central star, is related to the number of Lyman continuum photons from the star by simple geometry. This means that

$$F_{Lyc} = \frac{Q}{4\pi r^2} = \int N_e N_{H^+} \alpha_B dr \quad (3.1)$$

where Q is the number of Lyman continuum photons emitted by the star per unit time, α_B , the sum of all recombinations leading to the $n=2$ state, N_e and N_{H^+} are the number densities of electrons and protons and dr is an element of distance to the central star from the point in the nebula.

The rate of recombinations that produce an observable $H\alpha$ photons is $\alpha_{H\alpha}^{eff}$, and the surface brightness is given by

$$S_{H\alpha} = \int \frac{N_e N_{H^+} \alpha_{H\alpha}^{eff}}{4\pi} dr. \quad (3.2)$$

Eliminating the density from these equations yields

$$S_{H\alpha} = \frac{\alpha_{H\alpha}^{eff}}{\alpha_B} \frac{1}{4\pi} \frac{Q}{4\pi r^2}. \quad (3.3)$$

It is assumed that Lyman continuum photons from direct recombinations to the ground state immediately ionize an nearby atom and the entire process has no net effect. This equation assumes that the ratio of $\alpha_{H\alpha}^{eff}$ to α_B is constant through the nebula. This is a good assumption as they are both weakly dependent on temperature and density in the same manner.

3.1.2 Absolute calibration from $H\beta$ total flux

Equation 3.3 contains two uncertain quantities. Neither the distance to the nebula which is implicit in r nor the total flux from the central star, Q , is well determined. A measurement of the total flux in $H\beta$ from the entire nebula allows both of the unknown quantities to be eliminated from the expected surface brightness equations, if it is assumed that the entire nebula is optically thick to ionizing photons. This may be consistent with the observation of H_2 emission from the nebula (Storey 1982). It is not clear from that study if the H_2 emission is from dense optically thick knots, or an outer shell. CO mapping indicates an incomplete shell structure with an inner edge at about $360''$ from the central star on an East-West line. This is near the outer edge of the bright optical ring, and the CO emission peaks $40''$ further out at what appears to be an ionization front (Healy et al. 1990). A molecular shell outside the ionized portion of the nebula indicates that the nebula is thick to ionizing photons.

An independent observation is the total flux in $H\beta$ from the entire nebula. The observed flux from the entire nebula, $F_{H\beta}$, is a function of the luminosity of the entire nebula in $H\beta$, $L_{H\beta}$, and distance to the star, D . If the nebula is thick to ionizing photons then $L_{H\beta}$ is a simple function of the total number of ionizing photons from the star, Q . With the recombination coefficients $\alpha_{H\beta}$, the effective recombination coefficient for $H\beta$ and α_B the luminosity can be related to the ionizing flux of the star, according to

$$F_{H\beta} = \frac{L_{H\beta}}{4\pi D^2} = \frac{Q}{4\pi D^2} \frac{\alpha_{H\beta}}{\alpha_B}. \quad (3.4)$$

Solving this for Q yields

$$Q = F_{H\beta} \frac{\alpha_B}{\alpha_{H\beta}} 4\pi D^2, \quad (3.5)$$

and combining this with equations 3.3 and 3.4 yields

$$S_{H\alpha} = \frac{\alpha_{H\alpha}}{\alpha_B} \frac{1}{4\pi} \frac{Q}{4\pi r^2} \quad (3.6)$$

$$= \frac{\alpha_{H\alpha}}{\alpha_B} \frac{1}{4\pi} \frac{1}{4\pi r^2} F_{H\beta} \frac{\alpha_B}{\alpha_{H\beta}} 4\pi D^2 \quad (3.7)$$

$$= \frac{\alpha_{H\alpha}}{\alpha_{H\beta}} \frac{F_{H\beta}}{4\pi} \left(\frac{D}{r} \right)^2. \quad (3.8)$$

As $\left(\frac{D}{r} \right)^2$ is just the inverse projected distance on the sky in arcseconds squared, this equation is independent of the distance to the nebula. The total observed flux from the nebula in $H\beta$ is 4.46×10^{-10} ergs $\text{sec}^{-1} \text{cm}^{-2}$ which is 109 photons $\text{sec}^{-1} \text{cm}^{-2}$. (O'Dell 1962) With the ratio of $\frac{\alpha_{H\alpha}^{eff}}{\alpha_{H\beta}^{eff}} = 3.86$ and $\left(\frac{D}{r} \right)$ defined as ϕ , the surface brightness in $H\alpha$ predicted from the total flux from the nebula in $H\beta$ is given by:

$$S_{H\alpha} = \frac{1.43 \times 10^{12}}{\phi^2} \frac{\text{photons}}{\text{sec ster cm}^2}. \quad (3.9)$$

3.1.3 Measurement of substellar $S_{H\alpha}$ and distance

The measurement of the surface brightness in $H\alpha$ is not completely straightforward. The surface brightness indicated in equation 3.9 is the surface brightness measured along a line of sight from the central star. Figure 3.1 is a sample Cometary Knot, with the thin vertical box representing the emission included in the substellar surface brightness. The surface brightness that is measured in these observations is the sum

of all the emission along a skewed line of sight indicated by the thin skewed box. As I did not know the complete geometry for every knot, I made no correction for this. The knots have a 'brightest region', which for the larger ones is about three pixels deep and five pixels wide and represented by the black box. The brightest pixels of each knot are often scattered over an area this large. I took the mean surface brightness of this box to be my measured surface brightness. For the smaller Cometary Knots I took smaller boxes.

Figure 3.1 The vertical box represents the emission summed in the surface brightness measured from the central star; the skewed box the surface brightness measured from outside the nebula and the black box the brightest portion of the knot.

The projected distance to the central star is straightforward to measure. The *stdas* task *metric* uses header information, does a geometric correction and returns sky coordinates for pixel coordinates. A check by hand of the position of the bright star in CCD # 2 and one lower right of CCD # 4 along with a few knots shows that the header information is good. The easiest definite point to identify is the brightest point in the [NII] cusp. The [NII] images are a little sharper than the H α ones so this is the 'position' of the knot I measured. The projected distance is a minimum estimate of the true distance. The closest Cometary Knot in this field is 118 arc seconds from the central star. At a distance of 150 pc this would correspond to 2.6×10^{17} cm. The furthest extent of this field is 290". Images of the entire nebula show that the Cometary Knots closest to the central star are about 88" from it.

3.1.4 Results

The expected surface brightness for regions optically thick to ionizing radiation (defined by eq. 3.9) and the measured surface brightness for 32 knots are plotted in fig. 3.2. The ϕ_s^{-1} dependence becomes a slope of -2 on the log-log plot.

Figure 3.2 The average of the brightest region is plotted against the projected distance to the central star. The line is the surface brightness in $H\alpha$ predicted by the $H\beta$ flux of the entire nebula. The +’s indicate medium extinction, the circles, no extinction and the the squares a large amount.

There are at least three major effects on this plot. The first is that the measured distance is only the distance projected onto the plane of the sky, and is a minimum value of the true distance. Therefore the line defined by the predicted surface brightness can only define a maximum for the data points. This is certainly true, but assuming the knots are distributed in a shell, some of them should have projected distances almost equal to true distances and have surface brightnesses close to the predicted values. The second effect on the plot is the difference between the line of sight for measuring surface brightness and the long axis of the knot. Only when the two are the same will the measured surface brightness be the same as the substellar surface brightness in eq. 3.9. (Refer to fig 3.1 and the previous section.) The third

effect concerns the optical depth in ionizing photons of the entire nebula. This only affects the scaling of the line predicted by the $H\beta$ emission, which assumes the nebula is optically thick to the Lyman continuum. If it is not, this line will be too low on the graph, underestimating the luminosity of the star. As the predicted line is actually too high by a factor of 1.6, the second effect is evidently much stronger.

3.2 Densities

The density of an emission line object can be estimated from the observed surface brightness and an assumed geometry. The images suggest a geometry of a thick shell for the material that is in the background, and cylindrically symmetric cusps for the Cometary Knots.

The observed surface brightness, $S_{H\alpha}$, is the integral along the line of sight of the emission per unit volume:

$$S_{H\alpha} = \int_{back}^{front} \frac{N_e N_p \alpha_{H\alpha}^{eff}}{4\pi} dl \quad (3.10)$$

where N_e and N_p are number densities of protons and electrons and $\alpha_{H\alpha}^{eff}$ is the effective recombination coefficient that yields an $H\alpha$ photon in the appropriate temperature and approximate density regimes. Assuming the simple case that the temperature is constant and that the density is constant in the object and zero elsewhere, the integral over dl is simply L , and one has

$$N_e N_p = \frac{4\pi S_{H\alpha}}{L \alpha_{H\alpha}^{eff}} \quad (3.11)$$

3.2.1 Nebular field

One simple model for the Helix is that the material is uniformly concentrated in a thick shell with an inner radius of 150" and an outer of 250".

This is not unreasonable as a plot of surface brightness versus distance from the central star for such a geometry matches the observed surface brightness over the field of observation. Fig. 3.4 is a plot of L versus distance from the star assuming the thick shell model. This agrees fairly well with the observed surface brightness.

Further support for a thick shell model is that the helical shape seen in $H\alpha + [NII]$ light almost completely disappears in $[OIII]$ and the nebula appears to be a thick shell (Warner and Rubin 1975). Assuming this model, as depicted in fig. 3.3, at

Figure 3.3 Assuming a thick shell model, the line of sight depth, L , is $275''$ at a distance $130''$ from the central star.

Figure 3.4 This plot is of L , the line of sight distance, through the thick shell described in fig. 3.3. Both axes are in arcseconds.

a point $130''$ from the central star, $L=275''$. At a distance of 150 pc, this corresponds to 6.08×10^{17} cm.

The observed $S_{H\alpha} = 5 \times 10^7$ photons $\text{sec}^{-1} \text{cm}^{-2} \text{ster}^{-1}$ and a value for $\alpha_{H\alpha}^{eff}$ of $4.18 \times 10^{-13} \text{cm}^3 \text{sec}^{-1}$ then lead to a value for the nebular density of 50 protons cm^{-3} .

Other more direct determination of nebular densities in the Helix are those of S.Hawley(1978) from the [SII] doublet spectra made with a spatial resolution of about $24''$. He measured densities in the outer ring from less than 100 to as much as 1200 protons cm^{-3} . The doublet ratio is probably not accurate at the low end of his density range and the high values may result from the inclusion of emission from

several cometary knots. Warner and Ruben took spectra at 12 points across the nebula and found values with error estimated at 40% or better from 3727 to 4101Å and 4861 to 7135Å. Their ratios for [NII] lines indicate densities of 10^6 cm^{-3} or so while their [SII] doublet data predicts the same the low densities (≤ 300) as those found by Hawley. Their interpretation of this is that the Cometary Knots provided the high densities and the field the low. They also noted that the [OI] auroral line at 5575 Å is weak and there is an unmeasured effect on their derived density from the night sky (Warner and Rubin 1975).

3.2.2 Ionized portions of the Cometary Knots

This same method yields densities for the ionized portions of the Cometary Knots. If they are cylindrically symmetric the measured width across a feature is the same as its depth for the knots that are lying in the plane of the sky. This depth is L and for most knots is 5 pixels, or .455". There is little contribution to the total emission in the line of sight from the regions outside this center box as can be seen by taking a cut across the knot and assuming that such a cut is the same as a cut along the line of sight. A value of $S_{H\alpha} = 1.0 \times 10^8 \text{ photons cm}^{-2} \text{ sec}^{-1} \text{ ster}^{-1}$ is representative of the brightest parts of the brightest Cometary Knots and with a path length of seven pixels yields a value of $1400 \text{ protons cm}^{-3}$. Seven pixels is just a little wider than the usual width of the brightest part of the knot. This width was chosen to make some allowance for the emission from the less dense edges of the knot that are in the line of sight. Values of the densities inside the Cometary Knots have been estimated to be 10^6 cm^{-3} by assuming they are in pressure equilibrium with the surrounding nebula (Dyson et al. 1989). This is not in contradiction with the density of the ionized portions being 1400 cm^{-3} . The Cometary Knots have different physical properties from the rest of the nebula and the intermediary region, the bright ionized head, could well have intermediary physical properties.

3.3 Optical depth

The Cometary Knots are generally dark in the 5007 Å emission line of [OIII] with only a faint outer halo. The dusty interiors of the knots are scattering and absorbing the visible photons that are emitted behind the knots along our line of sight. There is some extinction in $H\alpha$ seen in some of the bigger, darker knots, but it is much less because the knots are surrounded by a skin of emission in $H\alpha$, and the longer

photons at this wavelength are less affected. It is straightforward and useful to find the optical depths of the knots in the [OIII] 5007 Å emission line. The background nebular emission near each Cometary Knot is quite smooth in [OIII] so that small sections are easy to fit with simple functions which then approximate the image without the knot. The optical depth τ is defined by:

$$\tau = -\ln \frac{S}{S_0} \quad (3.12)$$

where S_0 is the surface brightness of the fitted function and S is the observed surface brightness. The iraf task *imsurfit* was used to fit a surface to chosen background sections of an image then output a 'response' image or the original image divided by the fitted image. Taking the natural log of this image with *imfunction* yields a map of the optical depths in the image. Fig 3.5 is a surface map of the optical depth of two knots.

Figure 3.5 Map of τ shows the optical depth for the two large dark knots in the lower right of CCD 4.

Considering the model that the Helix is a ball of [OIII] emission and the Cometary Knots are blocking the emission behind them, the optical depth should indicate a knot's position in the nebula. The very dark ones should be near the front if they are

all of similar optical depths, those that are not observed in [OIII] in the very back, and those with medium optical depths should have their long axes aligned with the plane of the sky. The darkest knots have optical depths of 0.7 and the lightest can not be seen in [OIII]. Fig. 3.6 is three panels of the same field of view in the the three filters. Note that the small knots near the star in the middle are very bright in the $H\alpha$ and [NII] strips and nonexistent in the [OIII] strip while the two on the far right are striking in [OIII] but do not particularly stand out in other wavelengths.

Figure 3.6 These three strips are of the same field of view but in different filters. [OIII] is on the bottom, $H\alpha$ in the middle and [NII] is on top. Note how the three small knots near the star in the middle look about the same in $H\alpha$, but vastly different in [OIII].

The [OIII] emission is not from a ball, but a more complex shape. Fig. 3.7 shows the [OIII] and $H\alpha$ surface brightnesses along a cut across the field of view on a vector radial to the central star. The [OIII] emission drops well below its interior value by the outer edge of the field while to $H\alpha$ is still bright. The argument presented for the [OIII] being a ball still holds but the effect will be less significant. This could explain why there seems to be a maximum optical depth.

Figure 3.7 This plot is an average of 10 vectors radial to the central star across the field of view. The upper line is [OIII] surface brightness and the lower, H α . The abscissa is simple pixel number from the starting point. The central star is to the right. Note how the [OIII] emission has dropped to or below its central value by the outer edge of the field while the H α is still above its central value. The [OIII] shell structure is interior to H α .

3.4 Masses of the Cometary Knots

The masses of the knots can be calculated from the measured optical depths,

$$\tau = N \times \pi a^2 \times Q\left(\frac{2\pi a}{\lambda}\right) \quad (3.13)$$

where N is the number of particles in the line of sight, a is the radius and $Q(\frac{2\pi a}{\lambda})$ is the efficiency of the grains at scattering photons.

Assuming the dust particles are spherical, with density $\rho = \frac{m}{\frac{4}{3}\pi a^3}$, the mass per column is as follows:

$$\frac{mass}{column} = \frac{\tau}{\pi a^2} \rho \frac{4}{3} \pi a^3 \frac{1}{Q(\frac{2\pi a}{\lambda})} \quad (3.14)$$

$$= \frac{4\tau \rho a}{3Q(\frac{2\pi a}{\lambda})} \quad (3.15)$$

Using *imstat* on the area around a knot on the τ maps described in sec. 3.3 yields the mean τ in that region. The 'total extinction' for a knot is obtained by multiplying the number of pixels in the region that just includes the knot by the mean τ in that region. The two large knots in the lower right of CCD 4 and the one in the center

of CCD 4 are the darkest and largest, and therefore have the largest derived masses. Note that because there probably is nebular emission in front of all the knots all of the τ s are too small and the derived masses are lower limits. Taking $\frac{\rho}{Q(\frac{2\pi a}{\lambda})} = 1 \text{ gm cm}^{-3}$ and $a=200 \text{ nm}$ following O'Dell and Wen(1994), the minimum dust mass for the upper knot of the dark pair is $2.6 \times 10^{26} \text{ gms}$ or $1.3 \times 10^{-7} M_{\odot}$. With a value for the the gas to dust mass ratio of 150 this yields a total mass of the knot of $2.0 \times 10^{-5} M_{\odot}$ (Sodroski et al. 1994). The next two most massive knots, the center one and the lower of the dark pair have dust masses of 1.2×10^{-7} and $8.9 \times 10^{-8} M_{\odot}$ respectively. With the estimated total number of large Cometary Knots in the entire nebula, the total mass is about $10^{-2} M_{\odot}$.

At least two other authors have computed the mass of the Cometary Knots. Meaburn used a formula relating the optical depth in visual wavelengths to mass based on the masses derived from radio data. (Bohlin et al. 1978) (Meaburn 1992) Instead of assuming characteristics about the dust he assumed an interstellar reddening law to get $E(B-V)$ that is the measured quantity in Bohlin's relation. Meaburn's numbers for the size and optical depth of a knot are quite similar to those presented here, and using our method predict a mass within the range of masses presented here. The mass that he predicts using Bohlin's formula is $1 \times 10^{-5} M_{\odot}$, also within the mass range presented here.

Huggins et al (1992) mapped the Helix in CO and with a series of assumptions found the mass of the Cometary Knots. They assumed that the CO emission is optically thin and that all the oxygen is bound in CO. This gives them a measure of the amount of oxygen in the knots and, with an O/H ratio of 3×10^{-4} from the interstellar medium, yields a gas mass. They find the mass of a typical knot to be $\geq 5 \times 10^{-6} M_{\odot}$.

3.5 Sizes of the Cometary Knots

Figure 3.8 shows linear size of the knot versus projected distance from the central star. Knots showing no extinction are circles(o), those with medium extinction are pluses(+) and those with high extinction are squares($\square$). The size of the knots is generally decreasing with increasing distance. The knots which show negligible extinction are generally small for their projected distance indicating their true distance is farther and they are in the back of the nebula. Knots with very large maximum extinction should also be small for their projected distance, assuming they are near

the front of the nebula, but that is less clear from this plot. More Cometary Knots are needed to investigate this trend.

Figure 3.8 This plot shows the width of Cometary Knots and their distance from the central star. The pluses represent knots with middle values for the maximum extinction, the squares those with significant extinction and the circles those with no extinction on this diameter to distance from the central star plot.

3.6 Structure of cusps/exponential atmospheres

The resolution of these images is such that the structure of individual knots can be closely examined. The large knots are about 20 pixels across the bright heads and longer along the axis pointing towards the central star. Fig. 3.9 details the $H\alpha$ structure along the long axis of two typical knots. This section only considers the region between the brightest part of a knot and the central star along the long axis.

3.6.1 Measurement of the front edge of the Cometary Knots

Going towards the central star the surface brightness along the long axis of a knot falls off exponentially from the brightest point. Figure 3.9, made by normalizing the

Figure 3.9 The surface brightness in H α was normalized and averaged and plotted as a function of the distance from the brightest point going towards the central star.

surface brightness of five knots and plotting the natural log of this against the distance from the brightest point, shows this clearly. The measured slope is -2.5. Other knots are smaller or are viewed from a greater angle and the projection is greater leading to a much steeper slope.

3.6.2 Density to observed surface brightness relation

Observations are of surface brightness rather than density, but the two are directly related. Surface brightness is dependent on density such that if an exponential density structure exists in a single direction from a half sphere an exponential surface brightness will be observed. This situation is sketched in figure 3.10.

The surface brightness is the sum of all the emission along a line of sight:

$$S = 2 \times \int_0^{r_o} j dy = 2 \times \int n^2 dy. \quad (3.16)$$

Taking $n = n_o e^{-\frac{h}{H}}$ and $h \equiv x - x_s$ yields

$$S = 2 \times n_o^2 \int e^{-2\frac{h}{H}} dy = 2 \times n_o^2 \int e^{-\frac{2}{H}(x-x_s)} dy. \quad (3.17)$$

The figure indicates $x_s^2 + y_s^2 = r_o^2$, therefore:

$$S = 2 \times n_o^2 \int e^{-\frac{2}{H}(x-\sqrt{r_o^2-y^2})} dy = 2 \times n_o^2 e^{-2\frac{x}{H}} \int_{y_{min}}^{y_{max}} e^{\frac{2}{H} \times (r_o^2-y^2)^{\frac{1}{2}}} dy. \quad (3.18)$$

Figure 3.10 Taking the surface of a knot to be a half sphere, and assuming an exponential density distribution along the long axis from all points on the surface, it can be shown that the observed surface brightness will also be exponential.

The integral is simply a constant that relates the density to surface brightness on a knot by knot basis as $y_{\min}$ and $y_{\max}$ are constant over a single knot. Other assumptions implicit in this, besides a semi-circular ionization front, are that there are no lateral flows and that an exponential column constrained by the same force exists in each spot in the direction of the central star.

3.6.3 Explanation of exponential density distribution

A constant force acting on a mass of gas can create exponential density structure. Hydrostatic equilibrium occurs when convection is not an important process:

$$a\rho = -\frac{dP}{dx} \quad (3.19)$$

where a is an effective acceleration from the constant force and ρ is the gas density. Combining this with the perfect gas law $P = \frac{\rho k T}{\mu m_h}$ where k is boltzman's constant and m_h is the mass of a proton, yields

$$-\int \frac{a\mu m_h}{kT} dx = \int \frac{d\rho}{\rho} \quad (3.20)$$

$$e^{-(\frac{a\mu m_h}{kT}x)} = \frac{\rho}{\rho_o} \equiv e^{-\frac{x}{H}}. \quad (3.21)$$

The force that produces the acceleration is in the radial direction from the central star and could be radiation pressure, wind from the central star or a slow nebular flow. Each of these would be constant over the small change in radii over which the exponential distribution is observed.

3.6.4 The force

The central star is very hot and luminous and radiation pressure could be the constraining force. The star does not appear to have a wind, but this was determined by studying ultraviolet lines and in such a hot star these lines become weak. A slow nebular flow driven by the expansion of the nebula could also constrain the knots. The measured scale height constrains either the luminosity for the central star, its mass loss rate, or a density, depending on which mechanism is dominant.

The force per unit area due to radiation pressure is

$$f_{rp} = ma_{rp} \quad (3.22)$$

where a_{rp} is the effective acceleration felt by the ionized material in the knot due to radiation pressure. The mass per unit area, m is just the density times dr where dr is along the radial line to the central star and the long axis of the knot. The flux from the central star, F , that is intercepted is related to the force by

$$f_{rp} = \frac{\Delta F}{c} \quad (3.23)$$

where ΔF is the change in flux due to photons hitting the Cometary Knot material and c is the speed of light. Thus,

$$d\tau = \frac{\Delta F}{F} = n_p \sigma dr \quad (3.24)$$

where τ is the optical depth to ionizing photons; n_p the number density of protons and σ the effective cross section per proton. Combining these yields

$$a_{rp} = \frac{\Delta F}{c\rho dr} = \frac{n_p \sigma F}{c\rho} \quad (3.25)$$

where n_p is the number density of protons and σ the effective cross section per proton. Taking $\mathcal{R} \equiv \frac{n_p \sigma}{\rho}$ to give an effective cross section due to the dust for the protons. Osterbrock(1989) gives a value for $\mathcal{R}$ of $5 \times 10^{-22} \text{ cm}^2$. The ultraviolet photons will kick electrons off the surface of the dust grains creating a charge on the grains. With

this charge they will be coupled with the protons and the dust's larger cross sections will be dominant, so eq. 3.25 becomes

$$a_{rp} = \frac{\mathcal{R}}{c} F. \quad (3.26)$$

Combining equations 3.26 and 3.21 yields

$$\frac{1}{H} = \frac{\mathcal{R}F}{ckT} \quad (3.27)$$

A typical value for several of the best Cometary Knots is $H = 6.6 \times 10^{14}$ cm. This requires an F of 3.5×10^5 ergs $\text{cm}^{-2} \text{sec}^{-1}$ at the knots, at a distance $\approx 140''$ from the central star. Assuming the nebula is 150 pc away this leads to a luminosity of 60,000 $L_\odot$. The luminosity for the central star is not well determined with estimates ranging from a lower limit of 60 $L_\odot$ based on a Zanstra method temperature calculation to an uncertain value of 1700 $L_\odot$ based on surface gravity calculations (Bohlin et al. 1992) (Zhang and Kwok 1993). The derived 60,000 $L_\odot$ clearly indicates pressure from the photons is not determining the shape of the Cometary Knots.

A wind from the central star could provide the force that maintains the exponential structure of the ionized portions of the cometary knots. The mass loss rate and the wind velocity determine the force. The calculation parallels that for radiation pressure but instead of an effective cross section due to dust for photons, the proton-proton cross section must be calculated. The cross section for the transfer of all the momentum from one proton to another as a function of the relative energy between the two is $\mathcal{E}_{rel}$ (Jackson, cpt. 13)

$$\sigma(\mathcal{E}_{rel}) = 4\pi \left(\frac{e^2}{8\pi\epsilon_o\mathcal{E}_{rel}} \right)^2 \ln\Lambda \quad (3.28)$$

The assumption that the difference in energy between the incoming protons and those in the knot is $\mathcal{E}_{rel}$ is determined with just with the velocities as its assumed the temperatures are equal, $\mathcal{E}_{rel} = \frac{1}{2}mv^2$ and $\Lambda = \frac{8\pi\epsilon_oL_D\mathcal{E}_{rel}}{Z_aZ_b\epsilon^2}$ where L_D is the Debye length in the plasma, $\left(\frac{\epsilon_o kT}{e^2 n} \right)^{\frac{1}{2}}$. Taking the density, n to be 10^3 as per section 3.2.2 and the temperature, T to be 10^4 , $L_D \approx 20$ cm. The velocity of the incoming protons is the only contribution to the $\mathcal{E}_{rel}$ term, so $\mathcal{E}_{rel} = \frac{1}{2}m_p v^2$. A wind speed of 2000 km s^{-1} is reasonable and yields a $\ln\Lambda$ of 30 while the slowest wind considered, 800 km sec^{-1} yields a $\ln\Lambda$ of 28 (Kwok 1993). With all the appropriate constants this yields

$$\sigma = 7.52 \times 10^{-8} \frac{1}{v^4} \text{cm}^2 \quad (3.29)$$

where v is in cm sec^{-1} . Replacing the $\mathcal{R}$ from equation 3.27 with this quantity and replacing c with v yields:

$$H = \frac{vkT}{\sigma F} \quad (3.30)$$

The energy flux due to the stellar wind, F , is given by $F = \frac{\dot{M}}{4\pi r^2} v^2$ where r is the distance from the star to the knot. With the measured H , solving for $\dot{M}$ yields

$$\dot{M} = \frac{vkT}{H\sigma} \frac{4\pi r^2}{v^2} = 1.5 \times 10^{-14} v^3 \frac{M_{\odot}}{\text{year}}. \quad (3.31)$$

For a 2000 km sec^{-1} wind the required mass loss rate would be $1.2 \times 10^{-4} M_{\odot} \text{ year}^{-1}$ and is unlikely. Cerruti-Sola and Perinotto (1984) studied IUE spectra for evidence of wind in the central stars of planetary nebula. They found no evidence of a wind for the central star of the Helix. They determined that they might miss winds with velocities less than 1200 km sec^{-1} . A wind of 800 km s^{-1} would only require a mass loss rate of $7 \times 10^{-6} \frac{M_{\odot}}{\text{year}}$. The X-ray emission measured by Leahy et al(1994) indicates a velocity of about 1000 km sec^{-1} and a mass loss rate of about $10^{-8} M_{\odot} \text{ year}^{-1}$. There is still a discrepancy of more than two orders of magnitude between the wind required to maintain the observed density structure and the wind derived from other observational evidence. It does not appear that a wind from the central star is capable of producing the observed scale height.

Velocity studies made in the inner regions show line splitting corresponding to nebular expansion of up to 60 km sec^{-1} (Meaburn and White 1981). This expansion is another mechanism for maintaining the observed structure of the knots. The method is very similar to that for a stellar wind. The $\mathcal{E}_{rel}$ is much smaller, $2.9 \times 10^{-11} \frac{\text{g cm cm}^2}{\text{sec}^2}$ and $\ln \Lambda = 15$. These numbers yield a cross section of

$$\sigma = 4\pi \left(\frac{2.56 \times 10^{-38}}{8\pi 8.8 \times 10^{-21} 2.9 \times 10^{-11}} \right)^2 15 = 4.1 \times 10^{-15} \text{ cm}^2. \quad (3.32)$$

The density of the nebular flow is hidden in F as $F = \rho v^3$ and solving for it with equation 3.30 yields:

$$\rho = \frac{vkT}{\sigma} \frac{1}{v^3 H} = 9.6 \times 10^{-26} \frac{\text{grams}}{\text{cm}^3} = 10^{-2} \frac{\text{protons}}{\text{cm}^3} \quad (3.33)$$

This is an incredibly small density. It does not seem plausible that such a small density would be smooth so that knots show such regular shapes. Also there is the question of where the [OIII] emission is coming from. The Cometary Knots could

be moving radially outwards with the same velocity as the nebular flow so that the velocity and therefore the force in eq. 3.33 drops to zero. Huggins et al (1992) give some evidence for this effect by showing that the CO emission, which is localized near some of the larger clumps of knots, has kinematics similar to the envelope. Another problem may be that the assumption that the temperatures of the flowing material and the ionized portions of the knots are the same, is not good. The X-ray data indicate a temperature of 0.43 KeV (Leahy 1993) which would be a significant contribution to the $\mathcal{E}_{rel}$ when considering the nebular flow possibility. The other possibility is that this is not the correct way of describing the interaction of the nebular flow with the Cometary Knots.

Chapter 4

Conclusion

This paper provides some of the analysis of the WFPC2 observations of the Helix taken in August 1994. Physical properties of the Cometary Knots, including optical thickness to the Lyman continuum, mass and density of the ionized portions are determined. This project is growing with more observations already made, more planned and a lot more work to do with the data we have.

4.1 Summary of results

The optical thickness to ionizing photons of the Cometary Knots is shown in Fig. 3.2. The expected $\frac{1}{r^2}$ dependence is well illustrated though the absolute scaling predicted by $H\beta$ is much higher than observed. The discrepancy may be explained by geometry and different viewing angles.

Densities in the nebular ring and in the ionized portions of the Cometary Knots are calculated with measured surface brightness and assumed geometries. The low density of the nebular ring, ≈ 50 protons cm^{-3} is not inconsistent with values derived from measured doublets. The density of the densest, ionized, optically thin portion of the Cometary Knots is $\approx 1400 \text{ cm}^{-3}$ is reasonable considering the densities in the regions bordering it, but there are no other observations to compare it with.

The Cometary Knots are dark in the [OIII] image so optical depths can be measured and masses for the knots derived. The optical depths range from zero, indicating the Cometary Knot is in the back of the nebula with most of the emission in front of it, to 0.7. The dust mass, derived assuming properties including density, size and shape for the dust particles, ranges up to $1.3 \times 10^{-7} M_{\odot}$.

The side of the Cometary Knots facing the central star shows an exponential decay in surface brightness. This corresponds to an exponential decay in density. This can be explained in the steady state situation by a constant force acting on the gas. The central star is not luminous enough for that force to be photon pressure and mass loss rate required to explain by a stellar wind is too high. On the other hand, a slow nebular flow could easily create enough force and in fact would crush the ionized

portions. Possible explanations are that the knots are moving with the gas or that they are not in steady state.

The large total mass of the objects seems to indicate they are not solid bodies that formed with the star, as there is no evidence for such a mass in our solar system. The shape of the dark images does not correspond well to the expected shape of dusty clouds. The nature of these objects is not yet determined.

4.2 Continuing Observations

There have been few observations published about the Cometary Knots. The Meaburn and Walsh papers present ground based optical images, and Huggins's CO maps are correlated with the knots. There have been observations made this year with the VLA, but nothing yet published.

4.2.1 Hubble Space Telescope

The Cycle 5 program to expand these observations is in progress. On July 4, 1995 the WFPC2 camera imaged the same field in the [OI] and the continuum through the F631N and F547M filters. The fields are very closely aligned with the original observations. The Cometary Knots are bright in [OI] and the continuum shows no structures except for the field stars.

Observations of an adjacent field with a very different position angle must wait for a satellite roll. Many more Cometary Knots will be imaged in this field including one much closer to the central star. The field will be imaged in all five filters.

4.2.2 Ground Based Telescopes

Both high and low resolution spectra of individual Cometary Knots will contain important data. Low resolution spectra can yield line ratios from which densities and temperatures can be derived. They might also detect emission from the OH ion at 3080 Å, a significant clue to the nature of the Cometary Knots. The presence of OH would indicate H₂O and a composition that is similar to the material in our Oort cloud. High resolution spectra can detect flows from the Cometary Knots and motion along the line of sight. After an encouraging response to our first proposal at CTIO, indicating we should try in the fall semester when the Helix is visible longer, we were rejected. We will apply next year.

4.3 Possible Next Steps

My continuing work will certainly attempt to resolve some the inconsistencies indicated here. The predicted surface brightness in $H\alpha$ of the knots by the total flux from the nebula in $H\beta$ might be reconciled at least partly by better modeling of the emission structure of the Cometary Knots. The modeling of the interaction between slowly expanding nebular material and the knots indicates that either the knots are comoving, the interior of the nebula has densities of 10^{-2} proton cm^{-3} , there is some balancing force, or the modeling not correct.

The number of Cometary Knots should more than double with the adjacent field of view. There may be a correlation between distance from the central star and scale height in the ionized portions of the Cometary Knots. More Cometary Knots are needed to consider this and the possible size to position and optical depth dependence discussed in section 3.5. Comparison of the surface brightness in the various emission lines will build a picture of the ionization structure.

4.4 Final Comments

The very basic question concerning the origin and nature of the Cometary Knots is not yet answered. Detailed models based on observations of the Cometary Knots provide clues. The Helix is unique in its proximity to Earth, allowing resolution of fine structure, yet it is in an evolutionary state that is common to many stars, near the ends of their lives. If the Cometary Knots are related to material like the Oort cloud, that would be strong evidence that the collapse that created our sun, planetary system and Oort cloud is not unique.

Appendix A

Data

RA	dec	distance	brightness	tau	chord
337.396796	-20.804285	2.09	7.71	m	17
337.396465	-20.800507	2.13	7.64	m	16
337.397445	-20.801251	2.12	7.63	m	19
337.399253	-20.800078	2.13	7.41	m	13
337.396601	-20.797317	2.17	7.57	m	11
337.400118	-20.797708	2.16	7.38	m	14
337.401314	-20.794433	2.19	7.50	m	14
337.407390	-20.800279	2.12	7.60	m	19
337.410791	-20.797227	2.14	7.42	m	15
337.415583	-20.797919	2.15	7.41	m	12
337.418632	-20.796762	2.16	7.41	l	17
337.418635	-20.795325	2.18	7.32	l	15
337.405054	-20.790285	2.22	7.40	m	16
337.398767	-20.793636	2.20	7.51	m	11
337.412208	-20.795215	2.17	7.37	m	15
337.413113	-20.795226	2.17	7.40	n	7
337.408888	-20.790539	2.21	7.48	m	13
337.405901	-20.792430	2.20	7.56	m	11
337.410631	-20.793398	2.19	7.26	l	11
337.412084	-20.794397	2.18	7.41	m	10
337.423004	-20.789613	2.24	7.32	l	11
337.423169	-20.788418	2.25	7.22	n	6
337.423389	-20.789330	2.24	7.41	l	7
337.421595	-20.787282	2.25	7.15	m	9
337.422962	-20.784761	2.28	7.40	m	10
337.418805	-20.789293	2.23	7.16	l	7
337.418274	-20.787350	2.25	7.36	n	8
337.412160	-20.788816	2.23	7.35	n	7
337.414982	-20.792767	2.20	7.26	n	7
337.412614	-20.791091	2.21	7.21	n	6
337.395814	-20.797503	2.17	7.30	n	6

Table A.1 The distance is in $\log(\text{arcseconds})$ and the brightness $\log(\text{surface brightness})$ as described in the text. The optical depth is coded as medium(m), none(n) and large(l). The chord size is in pixels.

Appendix B

Color Figures

Figure B.1 This three color image depicts [OIII] in blue, $H\alpha$ in green and [NII] in red. It is a slightly trimmed mosaic of the four WFPC2 CCD's and about 155 arcseconds across.

Figure B.2 Most of the Cometary Knots that are clear and isolated fall on CCD 4. The scale here is about 75 arcseconds across.

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