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RICE UNIVERSITY

OPTIMAL TRAJECTORIES FOR THE
AEROASSISTED FLIGHT EXPERIMENT

by

ZHIGAO ZHAO

A THESIS SUBMITTED
IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS OF THE DEGREE OF
DOCTOR OF PHILOSOPHY

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Abstract

Optimal Trajectories for the Aeroassisted Flight Experiment

by

Zhigao Zhao

The aeroassisted flight experiment (AFE) is to simulate a transfer from a geosynchronous Earth orbit (GEO) to a low Earth orbit (LEO). Specifically, the AFE spacecraft is released from the Space Shuttle and is accelerated by means of a solid rocket motor toward Earth, so as to achieve atmospheric entry conditions identical with those of a spacecraft returning from GEO. During the atmospheric pass, aerodynamic and thermal data are gathered for use in designing aeroassisted orbital transfer (AOT) vehicles. Following the atmospheric pass, the AFE spacecraft ascends to the specified LEO either directly or indirectly via an intermediate parking Earth orbit (PEO). The optimal trajectories are determined by minimizing the postatmospheric characteristic velocity if the entry path inclination is fixed or the total characteristic velocity if the entry path inclination is free. The final maneuver includes the rendezvous with and the capture by the Space Shuttle. The entry and exit orbital planes of the AFE spacecraft are identical with the orbital plane of the Space Shuttle.

Under following two assumptions: (i) the instantaneous orbital plane is nearly identical with the entry orbital plane; (ii) the Earth's angular velocity is relatively small, the motion of the AFE spacecraft of order six (combination method) can be decoupled into two subsystems of order three, the longitudinal motion and the lateral motion (decomposition method).

The results of optimal trajectories are nearly the same by using the combination method and the decomposition method. The optimal control is bang-bang if the entry path

inclination is fixed and is constant if the entry path inclination is free. Finally, the optimal trajectories are advantageous over a nonoptimal reference trajectory in terms of the main quantities of interest, namely, the characteristic velocity, peak dynamic pressure, peak heating rate, and peak wedge angle.

Key Words. Flight mechanics, aerodynamics, astrodynamics, aeroassisted orbital transfer, aeroassisted flight experiment, optimal trajectories, reference trajectory, combination method, decomposition method, longitudinal motion, lateral motion, sequential gradient-restoration algorithm.

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1. Introduction

The field of aeroassisted orbital transfer (AOT) has received considerable attention in recent years (Refs. 1-19 and Refs. 20-31). For a spacecraft that must be transferred from a high Earth orbit (HEO) to a low Earth orbit (LEO), considerable savings in characteristic velocity are possible if the AOT mode is employed instead of the all-propulsive mode. In turn, this leads to propellant savings and payload increases.

In the AOT mode or synergistic mode, the spacecraft deorbits from HEO with a decelerating propulsive impulse, undershoots LEO, enters an elliptical transfer orbit leading to atmospheric entry, and traverses the upper atmosphere using the aerodynamic forces to deplete excess velocity. Then, after exiting the atmosphere, it reaches LEO with near-circular velocity. The resulting savings in propellant mass are beneficial; however, they are partly offset by the detrimental effects due to the added thermal protection required to cope with the heating rates generated while the spacecraft traverses the upper atmosphere at hypervelocity speeds.

The aeroassisted flight experiment (AFE), originally planned by NASA for 1994, refers to a spacecraft to be launched and then recovered by the Space Shuttle (Refs. 12-13). This experiment simulates a real AOT transfer from a geosynchronous Earth orbit (GEO) to a low Earth orbit (LEO). The maneuver of the AFE vehicle is initiated by releasing the spacecraft from the Space Shuttle, which is flying at the altitude of 160 NM above the Earth surface. By means of a solid rocket motor, the AFE spacecraft is accelerated toward Earth, so as to achieve atmospheric entry conditions identical with those of a spacecraft returning from GEO. During the atmospheric pass, aerodynamic and thermal data are gathered for use in designing AOT vehicles. Following the atmospheric pass, the AFE spacecraft ascends to the specified LEO either directly or indirectly via an intermediate parking Earth orbit (PEO). The final maneuver includes the rendezvous with and the capture by the Space

Shuttle. Clearly, the entry and exit orbital planes of the AFE spacecraft are identical with the orbital plane of the Space Shuttle.

Concerning the AFE optimal trajectory, much work has been done at Rice University in recent years and previous studies were carried out in Refs. 27-31. This thesis continues the optimization studies initiated in Refs. 27-29, with the intent of arriving of AFE nominal trajectories useful for generation of AFE guidance trajectories.

In this thesis, optimal atmospheric pass trajectories are studied under following conditions: (i) the angle of attack is constant, $\alpha = 17$ deg; (ii) the spacecraft is controlled via the angle of bank; (iii) the entry path inclination is either fixed, identical with that of the nominal trajectory of Ref. 12, or free; (iv) two transfer maneuvers are considered: (DA) direct ascent to LEO; and (IA) indirect ascent to LEO via PEO; (v) the exit orbital plane is identical with the entry orbital plane; (vi) the propellant mass is minimized; this is the same as minimizing the postatmospheric characteristic velocity if the entry path inclination is fixed or the total characteristic velocity if the entry path inclination is free.

While the motion of the AFE spacecraft in a 3D-space is described by a system of six ordinary differential equations (combination method), substantial simplifications are possible if one exploits two facts: (i) the instantaneous orbital plane is nearly identical with the entry orbital plane; (ii) the Earth's angular velocity is relatively small. Under these assumptions, the complete system of order six can be decoupled into two subsystems of order three, one describing the longitudinal motion and one describing the lateral motion (decomposition method).

For both the combination method and the decomposition method, problems (DA) and (IA) are solved using the sequential gradient-restoration algorithm (SGRA). Then the resulting optimal trajectories are compared to determine whether the combination and decomposition methods yield consistent results, thereby simplifying the generation of guidance trajectories. The optimal trajectories are also compared with a nonoptimal

reference trajectory supplied by Ref. 12 in terms of the main quantities of interest, namely, the characteristic velocity, peak dynamic pressure, peak heating rate, and peak wedge angle.

2. Notations

Throughout this thesis, the following notations are employed:

- C_D = drag coefficient;
- C_L = lift coefficient;
- D = drag, N;
- DP = dynamic pressure, N / m^2 ;
- g = local acceleration of gravity, m / sec^2 ;
- h = altitude, m;
- HR = heating rate, W / m^2 ;
- i = orbital inclination, rad;
- L = lift, N;
- m = mass, kg;
- r = radial distance from the center of the Earth, m;
- r_e = radius of the Earth, m;
- r_a = radius of the outer edge of the atmosphere, m;
- S = reference surface area, m^2 ;
- T = running time, sec;
- t = dimensionless time;
- V = velocity, m/sec;
- V_a = circular velocity at $r = r_a$, m/sec;
- V_* = reference velocity, m/sec;
- α = angle of attack, rad;
- γ = path inclination, rad;
- η = wedge angle, rad;
- θ = longitude, rad;

- μ = bank angle, rad;
 μ_e = Earth's gravitational constant, $\text{m}^3 / \text{sec}^2$;
 ρ = air density, kg / m^3 ;
 ρ_* = reference air density, kg / m^3 ;
 τ = final time, sec;
 ϕ = latitude, rad;
 χ = heading angle, rad;
 ω = angular velocity of the Earth, rad/sec;
 Ω = longitude of the ascending node, rad;
 ΔV = characteristic velocity, m/sec.

Subscripts

- 0 = entry into the atmosphere;
 1 = exit from the atmosphere;
 00 = exit from the initial orbit;
 11 = perigee for Transfer (IA) or apogee for Transfer (DA);
 22 = apogee for Transfer (IA).

Superscripts

- = derivative with respect to time;
 ~ = quantity defined in an inertial system.

Acronyms

- AFE = aeroassisted flight experiment;
 AOT = aeroassisted orbital transfer;
 CM = combination method;
 DA = direct ascent;
 DM = decomposition method;
 GEO = geosynchronous Earth orbit;

HEO = high Earth orbit;
IA = indirect ascent;
LEO = low Earth orbit;
ODE = ordinary differential equation;
OT = optimal trajectory;
PEO = parking Earth orbit;
RT = reference trajectory;
SGRA = sequential gradient-restoration algorithm.

3. System Description

The motion of the AFE spacecraft takes place partly in the atmosphere and partly in space. For the purposes of this thesis, the trajectory begins at GEO ($h = 19323 \text{ NM} = 35786 \text{ km}$) and ends at LEO ($h = 178 \text{ NM} = 330 \text{ km}$). We consider two transfer maneuvers: (DA) direct ascent to LEO; and (IA) indirect ascent to LEO via an intermediate parking Earth orbit (PEO, $h = 197 \text{ NM} = 365 \text{ km}$).

For Transfer (DA), the key points of the maneuver are these: point 00, exit from GEO; point 0, atmospheric entry; point 1, atmospheric exit; point 11, entry into LEO. Point 00 is the apogee of the preatmospheric transfer orbit $00 \rightarrow 0$; point 11 is the apogee of the postatmospheric transfer orbit $1 \rightarrow 11$. Propulsive impulses are applied at two points: at point 00 to deorbit from GEO; at point 11 in order to circularize the motion into LEO.

For Transfer (IA), the key points of the maneuver are these: point 00, exit from GEO; point 0, atmospheric entry; point 1, atmospheric exit; point 22, entry into PEO; and point 11, entry into LEO. Point 00 is the apogee of the preatmospheric transfer orbit $00 \rightarrow 0$; point 22 is the apogee of both the first postatmospheric transfer orbit $1 \rightarrow 22$ and the second postatmospheric transfer orbit $22 \rightarrow 11$; point 11 is the perigee of the second postatmospheric transfer orbit $22 \rightarrow 11$. Propulsive impulses are applied at three points: at point 00 to deorbit from GEO; at point 22 in order to raise the height of the perigee of the second postatmospheric transfer orbit; and at point 11 in order to circularize the motion into LEO.

For the atmospheric portion ($h \leq h_a$) of the trajectory of the AFE spacecraft, we employ an Earth-fixed system; for the space portion of the trajectory ($h \geq h_a$), we employ an inertial system; here, $h_a = 400,000 \text{ ft} \approx 122 \text{ km}$ denotes the thickness of the atmosphere. For $h \leq h_a$, we compute the air density using the US Standard Atmosphere, 1976 (Ref. 32); for $h \geq h_a$, we assume that the air density is zero. For both the

atmospheric portion and the space portion of the trajectory, we neglect the effects due to the oblateness of the Earth; we assume that the gravitational field is central and obeys the inverse square law.

3.1. Atmospheric Pass.

With reference to the atmospheric portion of the trajectory of the AFE vehicle, the following additional hypotheses are employed: (i) the atmospheric pass is made with engine shut-off; (ii) the angle of attack is constant, $\alpha = 17$ deg; (iii) under extreme hypersonic conditions, the dependence of the aerodynamic coefficients on the Mach number and the Reynolds number is disregarded; (iv) the sideslip angle is zero; hence, the side force component of the aerodynamic force is zero; (v) the spacecraft is controlled via the angle of bank μ .

3.2. Differential System.

With the above assumptions, upon using an Earth-fixed system, and upon normalizing the interval of integration to unity, the equations of motion include the kinematical equations for the longitude θ , latitude ϕ , and radius r (Ref. 26),

$$\dot{\theta} = \tau(V \cos \gamma \cos \chi / r \cos \phi), \quad (1a)$$

$$\dot{\phi} = \tau(-V \cos \gamma \sin \chi / r), \quad (1b)$$

$$\dot{r} = \tau(V \sin \gamma), \quad (1c)$$

and the dynamical equations for the velocity V , path inclination γ , and heading angle χ (Ref. 26),

$$\dot{V} = \tau[-D/m - g \sin \gamma + \omega^2 r (\sin \gamma \cos^2 \phi + \cos \gamma \sin \chi \cos \phi \sin \phi)], \quad (1d)$$

$$\dot{\gamma} = \tau[(L/mV)\cos\mu + (V/r - g/V)\cos\gamma + 2\omega\cos\chi\cos\phi + (\omega^2 r/V)(\cos\gamma\cos^2\phi - \sin\gamma\sin\chi\cos\phi\sin\phi)], \quad (1e)$$

$$\dot{\chi} = \tau[(L/mV)\sin\mu/\cos\gamma + (V/r)\cos\gamma\cos\chi\tan\phi + 2\omega(\sin\phi + \tan\gamma\sin\chi\cos\phi) + (\omega^2 r/V)\cos\chi\cos\phi\sin\phi/\cos\gamma]. \quad (1f)$$

In the dynamical equations, the symbol ω denotes the angular velocity of the Earth; terms linear in ω are due to the Coriolis acceleration; terms quadratic in ω are due to the transport acceleration. Also in the dynamical equations, the acceleration of gravity is given by

$$g = \mu_e / r^2, \quad (2)$$

where μ_e denotes the Earth's gravitational constant. In addition, the aerodynamic forces are given by

$$D = (1/2)C_D(\alpha)\rho(h)SV^2, \quad (3a)$$

$$L = (1/2)C_L(\alpha)\rho(h)SV^2, \quad (3b)$$

where the air density ρ depends on the altitude h , with

$$h = r - r_e. \quad (4)$$

Since α is constant, both the drag coefficient and the lift coefficient are constant. The control of the spacecraft is the angle of bank μ . For given bank angle time history $\mu(t)$, Equations (1) must be integrated, subject to (2)-(4), over the time interval $0 \leq t \leq 1$. Here, $t = T/\tau$ is the dimensionless time; the initial time $t = 0$ corresponds to atmospheric entry; and the final time $t = 1$ corresponds to atmospheric exit.

3.3. Transformation Relations.

General transformation relations allow one to pass from quantities computed in an Earth-fixed system to quantities computed in an inertial system (direct relations), and vice versa (inverse relations). See Refs. 27-28 for details.

For the kinematic variables, the direct relations are

$$\tilde{\theta} = \theta + \tau\omega t, \quad (5a)$$

$$\tilde{\phi} = \phi, \quad (5b)$$

$$\tilde{r} = r, \quad (5c)$$

and the inverse relations are

$$\theta = \tilde{\theta} - \tau\omega t, \quad (6a)$$

$$\phi = \tilde{\phi}, \quad (6b)$$

$$r = \tilde{r}. \quad (6c)$$

For the dynamic variables, the general transformation relations are

$$\tilde{V} \cos \tilde{\gamma} \cos \tilde{\chi} = V \cos \gamma \cos \chi + \omega r \cos \phi, \quad (7a)$$

$$\tilde{V} \cos \tilde{\gamma} \sin \tilde{\chi} = V \cos \gamma \sin \chi, \quad (7b)$$

$$\tilde{V} \sin \tilde{\gamma} = V \sin \gamma, \quad (7c)$$

and they imply the direct relations

$$\tilde{V} = \sqrt{[V^2 + 2\omega r V \cos \gamma \cos \chi \cos \phi + (\omega r \cos \phi)^2]}, \quad (8a)$$

$$\tan \tilde{\gamma} = V \sin \gamma / \sqrt{[(V \cos \gamma)^2 + 2\omega r V \cos \gamma \cos \chi \cos \phi + (\omega r \cos \phi)^2]}, \quad (8b)$$

$$\tan \tilde{\chi} = V \cos \gamma \sin \chi / (V \cos \gamma \cos \chi + \omega r \cos \phi), \quad (8c)$$

and the inverse relations

$$V = \sqrt{[\tilde{V}^2 - 2\omega \tilde{r} \tilde{V} \cos \tilde{\gamma} \cos \tilde{\chi} \cos \tilde{\phi} + (\omega \tilde{r} \cos \tilde{\phi})^2]}, \quad (9a)$$

$$\tan \gamma = \tilde{V} \sin \tilde{\gamma} / \sqrt{[(\tilde{V} \cos \tilde{\gamma})^2 - 2\omega \tilde{r} \tilde{V} \cos \tilde{\gamma} \cos \tilde{\chi} \cos \tilde{\phi} + (\omega \tilde{r} \cos \tilde{\phi})^2]}, \quad (9b)$$

$$\tan \chi = \tilde{V} \cos \tilde{\gamma} \sin \tilde{\chi} / (\tilde{V} \cos \tilde{\gamma} \cos \tilde{\chi} - \omega \tilde{r} \cos \tilde{\phi}). \quad (9c)$$

In Eqs. (5)-(9), the tilde denotes quantities computed in the inertial system.

3.4. Orbital Elements.

Once the state variables $\tilde{\theta}, \tilde{\phi}, \tilde{\chi}$ are known, one can compute some important quantities such as the orbital inclination, the longitude of the ascending node, and the wedge angle; the latter is the angle between the instantaneous orbital plane and the entry orbital plane. In the inertial system, these quantities are defined through the relations

$$\cos \tilde{i} = \cos \tilde{\phi} \cos \tilde{\chi}, \quad (10a)$$

$$\sin(\tilde{\theta} - \tilde{\Omega}) = \cot \tilde{i} \tan \tilde{\phi}, \quad (10b)$$

$$\cos \tilde{\eta} = \sin \tilde{i} \sin \tilde{i}_0 \cos(\tilde{\Omega} - \tilde{\Omega}_0) + \cos \tilde{i} \cos \tilde{i}_0. \quad (10c)$$

In the Earth-fixed system, relations analogous to (10) are defined below:

$$\cos i = \cos \phi \cos \chi, \quad (11a)$$

$$\sin(\theta - \Omega) = \cot i \tan \phi, \quad (11b)$$

$$\cos \eta = \sin i \sin i_0 \cos(\Omega - \Omega_0) + \cos i \cos i_0. \quad (11c)$$

3.5. Initial Conditions.

For the problems with entry path inclination fixed, the initial values of the state variables $\tilde{\theta}_0, \tilde{\phi}_0, \tilde{r}_0, \tilde{V}_0, \tilde{\gamma}_0, \tilde{\chi}_0$ are given in the inertial system; in particular,

$$\tilde{r}_0 = r_a. \quad (12a)$$

Because of the relations (10), the initial values of the orbital elements $\tilde{i}_0, \tilde{\Omega}_0, \tilde{\eta}_0$ are known.

In particular, by definition,

$$\tilde{\eta}_0 = 0. \quad (12b)$$

For the problems with entry path inclination free, the initial values of the state variables $\tilde{\theta}_0, \tilde{\phi}_0, \tilde{r}_0, \tilde{\chi}_0$ are given in the inertial system; in particular,

$$\tilde{r}_0 = r_a. \quad (13a)$$

The initial values of the state variables $\tilde{V}_0, \tilde{\gamma}_0$ must be consistent with the relation

$$r_{00}^2(2V_*^2 - \tilde{V}_0^2) - 2r_{00}r_a V_*^2 + r_a^2 \tilde{V}_0^2 \cos^2 \tilde{\gamma}_0 = 0, \quad (13b)$$

where $V_* = \tilde{V}_a = \sqrt{(\mu_e / r_a)} = 7.831 \text{ km/sec}$ is a reference velocity, i.e., the circular velocity at $\tilde{r} = r_a$. This relation arises from energy conservation and angular momentum conservation applied to the preatmospheric transfer orbit connecting GEO with atmospheric entry. Because of the relations (10), the initial values of the orbital elements $\tilde{i}_0, \tilde{\Omega}_0, \tilde{\eta}_0$ are known. In particular, by definition,

$$\tilde{\eta}_0 = 0. \quad (13c)$$

3.6. Final Conditions.

The final time τ is free and is to be determined indirectly as the time instant at which

the spacecraft exits the atmosphere; hence,

$$\tilde{r}_1 = r_a. \quad (14a)$$

For transfer (DA), application of energy conservation and angular momentum conservation to the postatmospheric transfer orbit connecting atmospheric exit and LEO yields the relation

$$(DA) \quad r_{11}^2 (2V_*^2 - \tilde{V}_1^2) - 2r_{11}r_a V_*^2 + r_a^2 \tilde{V}_1^2 \cos^2 \tilde{\gamma}_1 = 0; \quad (14b)$$

for transfer (IA), application of energy conservation and angular momentum conservation to the postatmospheric transfer orbit connecting atmospheric exit and PEO yields the relation

$$(IA) \quad r_{22}^2 (2V_*^2 - \tilde{V}_1^2) - 2r_{22}r_a V_*^2 + r_a^2 \tilde{V}_1^2 \cos^2 \tilde{\gamma}_1 = 0. \quad (14c)$$

These relations guarantee that, after exiting, the spacecraft ascends to the specified apogee. Finally, the orbital plane at atmospheric exit must be identical with the orbital plane at atmospheric entry; hence, the exit value of the wedge angle must vanish,

$$\tilde{\eta}_1 = 0. \quad (14d)$$

3.7. Summary.

To sum up, the equations governing the atmospheric pass include the differential system (1), the analytical and functional relations (2)-(4), and the boundary conditions (12)-(14). In the boundary conditions, the inertial quantities are related to the Earth-fixed quantities via the transformation relations (5)-(9); also, the inertial triplet $(\tilde{\theta}, \tilde{\phi}, \tilde{\chi})$ is related to the orbital element triplet $(\tilde{i}, \tilde{\Omega}, \tilde{\eta})$ via the spherical trigonometric relations (10)-(11).

4. Optimal Control Problems

Subject to the previous constraints, different optimal control problems can be formulated, depending on the performance index chosen and the type of transfer maneuver considered. Here, we focus attention on the minimization of the total characteristic velocity $\Delta\tilde{V}$, which is a measure of the propellant consumption required for orbital transfer.

4.1. Direct Ascent to LEO.

For the direct ascent to LEO, two subcases are considered.

(i) If the entry path inclination is fixed, the preatmospheric characteristic velocity is fixed. Hence, the functional to be minimized is the postatmospheric characteristic velocity, given by

$$(DA) \quad I = \Delta\tilde{V} = \Delta\tilde{V}_{11}, \quad (15a)$$

$$(DA) \quad \Delta\tilde{V}_{11} = V_* \sqrt{r_a / r_{11}} - (r_a / r_{11}) \tilde{V}_1 \cos \tilde{\gamma}_1. \quad (15b)$$

(ii) If the entry path inclination is free, the functional to be minimized is the total characteristic velocity, given by

$$(DA) \quad I = \Delta\tilde{V} = \Delta\tilde{V}_{00} + \Delta\tilde{V}_{11}, \quad (16a)$$

$$(DA) \quad \Delta\tilde{V}_{00} = V_* \sqrt{r_a / r_{00}} - (r_a / r_{00}) \tilde{V}_0 \cos \tilde{\gamma}_0, \quad (16b)$$

$$(DA) \quad \Delta\tilde{V}_{11} = V_* \sqrt{r_a / r_{11}} - (r_a / r_{11}) \tilde{V}_1 \cos \tilde{\gamma}_1. \quad (16c)$$

4.2. Indirect Ascent to LEO via PEO.

For the indirect ascent to LEO via PEO, two subcases are considered.

(iii) If the entry path inclination is fixed, the preatmospheric characteristic velocity is fixed. Hence, the functional to be minimized is the postatmospheric characteristic velocity, given by

$$(IA) \quad I = \Delta \tilde{V} = \Delta \tilde{V}_{22} + \Delta \tilde{V}_{11}, \quad (17a)$$

$$(IA) \quad \Delta \tilde{V}_{22} = V_* \sqrt{2r_a r_{11} / (r_{11} r_{22} + r_{22}^2)} - (r_a / r_{22}) \tilde{V}_1 \cos \tilde{\gamma}_1, \quad (17b)$$

$$(IA) \quad \Delta \tilde{V}_{11} = V_* \sqrt{2r_a r_{22} / (r_{11} r_{22} + r_{11}^2)} - V_* \sqrt{r_a / r_{11}}. \quad (17c)$$

(iv) If the entry path inclination is free, the functional to be minimized is the total characteristic velocity, given by

$$(IA) \quad I = \Delta \tilde{V} = \Delta \tilde{V}_{00} + \Delta \tilde{V}_{22} + \Delta \tilde{V}_{11}, \quad (18a)$$

$$(IA) \quad \Delta \tilde{V}_{00} = V_* \sqrt{r_a / r_{00}} - (r_a / r_{00}) \tilde{V}_0 \cos \tilde{\gamma}_0, \quad (18b)$$

$$(IA) \quad \Delta \tilde{V}_{22} = V_* \sqrt{2r_a r_{11} / (r_{11} r_{22} + r_{22}^2)} - (r_a / r_{22}) \tilde{V}_1 \cos \tilde{\gamma}_1, \quad (18c)$$

$$(IA) \quad \Delta \tilde{V}_{11} = V_* \sqrt{2r_a r_{22} / (r_{11} r_{22} + r_{11}^2)} - V_* \sqrt{r_a / r_{11}}. \quad (18d)$$

4.3. Alternative Formulations.

If the entry path inclination is fixed, inspection of the performance indexes (15) or (17) shows that they include a constant part and a part that is linear in the exit value of the horizontal component of the inertial velocity $\tilde{V}_h = \tilde{V} \cos \tilde{\gamma}$. Hence, the minimization of $\Delta \tilde{V}$ is equivalent to maximizing the functional

$$J = \tilde{V}_{h1} = \tilde{V}_1 \cos \tilde{\gamma}_1. \quad (19)$$

In turn, because of the final conditions (14), the maximization of the functional (19) implies either maximizing the functional

$$K = \tilde{V}_1 \tag{20}$$

or minimizing the functional

$$H = \tilde{\gamma}_1. \tag{21}$$

5. Decomposition Technique

The study of the optimal trajectories of an AFE vehicle can be simplified to a considerable degree if the complete system of differential equations and boundary conditions is decomposed into a subsystem governing the longitudinal motion [state variables $r(t)$, $V(t)$, $\gamma(t)$; control variable $\mu(t)$; unknown parameter τ] and a subsystem governing the lateral motion [state variables $\theta(t)$, $\phi(t)$, $\chi(t)$; known control variable $\mu(t)$ and parameter τ]. The decomposition is possible if the following approximations are introduced into the equations of motion:

$$i \equiv i_0, \quad (22a)$$

$$\omega^2 \equiv 0. \quad (22b)$$

Approximation (22a) means that the instantaneous orbital plane is nearly identical with the entry orbital plane. Approximation (22b) means that terms quadratic in ω are small with respect to terms linear in ω and terms not containing ω . The following relations arise as a consequence of the approximations (22):

$$\cos \phi \cos \chi \equiv \cos i_0, \quad (23a)$$

$$\tilde{V} \equiv \sqrt{V^2 + 2\omega r V \cos \gamma \cos i_0}, \quad (23b)$$

$$\tilde{V} \cos \tilde{\gamma} \equiv \sqrt{V^2 \cos^2 \gamma + 2\omega r V \cos \gamma \cos i_0}, \quad (23c)$$

$$\tilde{\eta} \equiv \tilde{\eta}_0 = 0. \quad (23d)$$

Relation (23a) is due to (11a) and (22a). Relation (23b) is obtained by combining (8a), (22b), and (23a). Relation (23c) arises from (7a), (7b), (22b), and (23a). Relation (23d) is due to (22a). As the subsequent analysis shows, the approximation (22b) and its

implications (23b) and (23c) are essential to a simplified study of the longitudinal motion (Section 6), but nonessential, hence optional, to the study of the lateral motion (Section 7). The importance of the decomposition technique is that it enables one to generate AFE nominal trajectories useful for the generation of AFE guidance trajectories.

6. Longitudinal Motion

In the Earth-fixed system, the subsystem governing the longitudinal motion includes the differential equations for radius r , velocity V , and path inclination γ ,

$$\dot{r} = \tau[V \sin \gamma], \quad (24a)$$

$$\dot{V} = \tau[-D/m - g \sin \gamma], \quad (24b)$$

$$\dot{\gamma} = \tau[(L/mV) \cos \mu + (V/r - g/V) \cos \gamma + 2\omega \cos i_0]. \quad (24c)$$

6.1. Initial Conditions.

If the entry path inclination is fixed, the initial values of the state variables r_0, V_0, γ_0 can be obtained by using the transformation relations (6) and (9), in particular,

$$r_0 = r_a. \quad (25)$$

If the entry path inclination is free, upon transforming inertial quantities into Earth-fixed quantities, the initial conditions (13a) and (13b) are replaced by

$$r_0 = r_a, \quad (26a)$$

$$2V_*^2(r_{00}^2 - r_{00}r_a) + 2\omega r_a V_0 \cos \gamma_0 \cos i_0 (r_a^2 - r_{00}^2) + V_0^2(r_a^2 \cos^2 \gamma_0 - r_{00}^2) = 0. \quad (26b)$$

6.2. Final Conditions.

The final time τ is free and is to be determined indirectly by the time instant at which the spacecraft exits the atmosphere; hence,

$$r_1 = r_a. \quad (27a)$$

Upon transforming inertial quantities into Earth-fixed quantities, the final conditions (14b) and (14c) are replaced by

$$\begin{aligned} \text{(DA)} \quad & 2V_*^2(r_{11}^2 - r_{11}r_a) + 2\omega r_a V_1 \cos \gamma_1 \cos i_0 (r_a^2 - r_{11}^2) \\ & + V_1^2(r_a^2 \cos^2 \gamma_1 - r_{11}^2) = 0, \end{aligned} \quad (27b)$$

or

$$\begin{aligned} \text{(IA)} \quad & 2V_*^2(r_{22}^2 - r_{22}r_a) + 2\omega r_a V_1 \cos \gamma_1 \cos i_0 (r_a^2 - r_{22}^2) \\ & + V_1^2(r_a^2 \cos^2 \gamma_1 - r_{22}^2) = 0. \end{aligned} \quad (27c)$$

In deriving (25)-(27), one must account for the transformation relations (5), (7), the approximations (22), and the implications (23).

6.3. Performance Indexes.

Using (22)-(23), the performance indexes of the optimal control problems of Section 4 can be reformulated below.

(i) For the direct ascent to LEO, if the entry path inclination is fixed, Equations (15) are replaced by

$$\text{(DA)} \quad I = \Delta \tilde{V} = \Delta \tilde{V}_{11}, \quad (28a)$$

$$\text{(DA)} \quad \Delta \tilde{V}_{11} = V_* \sqrt{r_a / r_{11}} - (r_a / r_{11}) \sqrt{V_1^2 \cos^2 \gamma_1 + 2\omega r_a V_1 \cos \gamma_1 \cos i_0}. \quad (28b)$$

(ii) For the direct ascent to LEO, if the entry path inclination is free, Equations (16) are replaced by

$$\text{(DA)} \quad I = \Delta \tilde{V} = \Delta \tilde{V}_{00} + \Delta \tilde{V}_{11}, \quad (29a)$$

$$\text{(DA)} \quad \Delta \tilde{V}_{00} = V_* \sqrt{r_a / r_{00}} - (r_a / r_{00}) \sqrt{V_0^2 \cos^2 \gamma_0 + 2\omega r_a V_0 \cos \gamma_0 \cos i_0}, \quad (29b)$$

$$(DA) \quad \Delta \tilde{V}_{11} = V_* \sqrt{r_a / r_{11}} - (r_a / r_{11}) \sqrt{V_1^2 \cos^2 \gamma_1 + 2\omega r_a V_1 \cos \gamma_1 \cos i_0}. \quad (29c)$$

(iii) For the indirect ascent to LEO via PEO, if the entry path inclination is fixed, Equations (17) are replaced by

$$(IA) \quad I = \Delta \tilde{V} = \Delta \tilde{V}_{22} + \Delta \tilde{V}_{11}, \quad (30a)$$

$$(IA) \quad \Delta \tilde{V}_{22} = V_* \sqrt{2r_a r_{11} / (r_{11} r_{22} + r_{22}^2)} - (r_a / r_{22}) \sqrt{V_1^2 \cos^2 \gamma_1 + 2\omega r_a V_1 \cos \gamma_1 \cos i_0}, \quad (30b)$$

$$(IA) \quad \Delta \tilde{V}_{11} = V_* \sqrt{2r_a r_{22} / (r_{11} r_{22} + r_{11}^2)} - V_* \sqrt{r_a / r_{11}}. \quad (30c)$$

(iv) For the indirect ascent to LEO via PEO, if the entry path inclination is free, Equations (18) are replaced by

$$(IA) \quad I = \Delta \tilde{V} = \Delta \tilde{V}_{00} + \Delta \tilde{V}_{22} + \Delta \tilde{V}_{11}, \quad (31a)$$

$$(IA) \quad \Delta \tilde{V}_{00} = V_* \sqrt{r_a / r_{00}} - (r_a / r_{00}) \sqrt{V_0^2 \cos^2 \gamma_0 + 2\omega r_a V_0 \cos \gamma_0 \cos i_0}, \quad (31b)$$

$$(IA) \quad \Delta \tilde{V}_{22} = V_* \sqrt{2r_a r_{11} / (r_{11} r_{22} + r_{22}^2)} - (r_a / r_{22}) \sqrt{V_1^2 \cos^2 \gamma_1 + 2\omega r_a V_1 \cos \gamma_1 \cos i_0}, \quad (31c)$$

$$(IA) \quad \Delta \tilde{V}_{11} = V_* \sqrt{2r_a r_{22} / (r_{11} r_{22} + r_{11}^2)} - V_* \sqrt{r_a / r_{11}}. \quad (31d)$$

6.4. Summary.

The relations governing the longitudinal motion include the differential system (24) and the boundary conditions (25)-(27). In this formulation, the independent variable is the time t , $0 \leq t \leq 1$. The dependent variables include three state variables $[r(t), V(t), \gamma(t)]$, one control variable $[\mu(t)]$, and one parameter (τ) . These variables must be determined in such

a way that the performance index (28) or (29) is minimized in the direct ascent case and the performance index (30) or (31) is minimized in the indirect ascent case, depending on whether the entry path inclination is fixed or free.

7. Lateral Motion

In the Earth-fixed system, the subsystem governing the lateral motion includes the differential equations

$$\dot{\theta} = \tau(V \cos \gamma \cos \chi / r \cos \phi), \quad (32a)$$

$$\dot{\phi} = \tau(-V \cos \gamma \sin \chi / r), \quad (32b)$$

$$\begin{aligned} \dot{\chi} = & \tau[(L / mV) \sin \mu / \cos \gamma + (V / r) \cos \gamma \cos \chi \tan \phi \\ & + 2\omega(\sin \phi + \tan \gamma \sin \chi \cos \phi) \\ & + (\omega^2 r / V) \cos \chi \cos \phi \sin \phi / \cos \gamma], \end{aligned} \quad (32c)$$

and the initial conditions

$$\theta_0 = \text{given}, \quad (33a)$$

$$\phi_0 = \text{given}, \quad (33b)$$

$$\chi_0 = \text{given}. \quad (33c)$$

It must be noted that the final condition $\tilde{\eta}_1 = 0$ [Eq.(14d)] is not included here due to assumption (22a) and its implication (23d). Indeed, a posteriori, the numerical results on optimal trajectories obtained by using the combination method (Sections 10-11) support this point of view; namely, they show that the motion of the spacecraft is nearly planar in an inertial space so as to avoid any energy dissipation associated with the lateral motion.

Since the initial values of the state variables $\theta(t), \phi(t), \chi(t)$ are given, and since the functions $r(t), V(t), \gamma(t), \mu(t)$ and the parameter τ are known from the solution of the

optimal control problem associated with the subsystem governing the longitudinal motion (Section 6), it is obvious that the subsystem (32)-(33) governing the lateral motion can be integrated in forward time.

7.1. Remark

While the hypothesis (22b) is essential to a simplified study of the longitudinal motion, it is optional to a simplified study of the lateral motion. Indeed, the previous considerations stand unchanged regardless of whether the last line of (32c) is dropped or retained. If it is retained, then a slightly more precise solution is obtained for the lateral motion.

8. Experimental Data

The following data were used in the numerical experiments.

Physical Constants. The radius of the Earth is $r_e = 6378\text{km}$; the radius of the outer edge of the atmosphere is $r_a = 6500\text{km}$; the thickness of the atmosphere is $h_a = 122\text{km}$; the Earth's gravitational constant is $\mu_e = 0.3986\text{E}+15\text{m}^3/\text{sec}^2$; the circular velocity at $r=r_a$ is $V_* = \tilde{V}_a = 7.831\text{km/sec}$; the angular velocity of the Earth is $\omega = 0.7292\text{E}-04\text{rad/sec}$.

Spacecraft Data. The mass of the spacecraft is $m = 1678\text{kg}$; the reference surface area is $S = 14.31\text{m}^2$; the spacecraft flies at constant angle of attack, $\alpha = 17.0\text{deg}$; the lift coefficient is constant, $C_L = -0.3707$; the drag coefficient is constant, $C_D = 1.315$.

Atmospheric Model. The assumed atmospheric model is that of the US Standard Atmosphere, 1976 (Ref. 32). In this model, the values of the density are tabulated at discrete altitudes. For intermediate altitudes, the density is computed by assuming an exponential fit for the function $\rho(h)$.

Dynamic Pressure. The dynamic pressure is computed with the formula

$$DP = 0.5\rho V^2. \quad (34)$$

Heating Rate. The stagnation point heating rate is computed with the formula

$$HR = C\sqrt{(\rho/\rho_*)}(V/V_*)^{3.07}. \quad (35)$$

Here, $\rho_* = 0.3097\text{E}-03\text{kg/m}^3$ is a reference density, the density at $h_* = 60\text{km}$; $V_* = \tilde{V}_a = 7.831\text{km/sec}$ is a reference velocity; the constant $C = 282.3\text{W/cm}^2$ represents the stagnation point heating rate at $\rho = \rho_*$ and $V = V_*$, based on a nose radius of 1.0 ft.

Atmospheric Entry. In the inertial system, the initial conditions are as follows: the longitude is $\tilde{\theta}_0 = -134.52$ deg; the latitude is $\tilde{\phi}_0 = -4.49$ deg; the altitude is $h_0 = 122$ km, corresponding to the radius $\tilde{r}_0 = 6500$ km; the heading angle is $\tilde{\chi}_0 = -28.13$ deg; the orbital inclination is $\tilde{i}_0 = 28.45$ deg; the longitude of the ascending node is $\tilde{\Omega}_0 = -126.19$ deg; the wedge angle is $\tilde{\eta}_0 = 0.0$ deg.

For problems where the entry path inclination is fixed, the velocity is set at $\tilde{V}_0 = 10.308$ km/sec and the path inclination is set at $\tilde{\gamma}_0 = -4.49$ deg. These values are consistent with (13b).

For problems where the entry path inclination is free, the values of $\tilde{V}_0$ and $\tilde{\gamma}_0$ must be determined consistently with (13b) in the combination method. For the decomposition method, the values of V_0 and γ_0 must be determined consistently with (26b).

Atmospheric Exit. In the inertial system, the desired final conditions are as follows: the altitude is $h_1 = 122$ km, corresponding to the radius $\tilde{r}_1 = 6500$ km; the orbital inclination is $\tilde{i}_1 = 28.45$ deg; the longitude of the ascending node is $\tilde{\Omega}_1 = -126.19$ deg; the wedge angle is $\tilde{\eta}_1 = 0.0$ deg.

Transfer (DA). This is the direct ascent to LEO. The GEO conditions are as follows: the altitude is $h_{00} = 35786$ km, corresponding to the radius $\tilde{r}_{00} = 42164$ km; the path inclination is $\tilde{\gamma}_{00} = 0.0$ deg. The LEO conditions are as follows: the altitude is $h_{11} = 330$ km, corresponding to the radius $\tilde{r}_{11} = 6708$ km; the path inclination is $\tilde{\gamma}_{11} = 0.0$ deg.

Transfer (IA). This is the indirect ascent to LEO via PEO. The GEO conditions are as follows: the altitude is $h_{00} = 35786$ km, corresponding to the radius $\tilde{r}_{00} = 42164$ km; the path inclination is $\tilde{\gamma}_{00} = 0.0$ deg. The PEO conditions are as follows: the altitude is $h_{22} = 365$ km, corresponding to the radius $\tilde{r}_{22} = 6743$ km; the path inclination is

$\tilde{\gamma}_{22}=0.0\text{deg}$. The LEO conditions are as follows: the altitude is $h_{11}=330\text{km}$, corresponding to the radius $\tilde{r}_{11}=6708\text{km}$; the path inclination is $\tilde{\gamma}_{11}=0.0\text{deg}$.

9. Numerical Experiments

For the AFE spacecraft, both the combination method (CM) of Sections 3-4 and the decomposition method (DM) of Sections 5-7 were implemented in connection with the experimental data of Section 8. Optimal trajectories were computed for both Problems (DA) and (IA) under two scenarios: entry path inclination either fixed or free.

For (CM), the system of six ordinary differential equations, describing the motion of the AFE spacecraft in a 3D-space, was solved with respect to the state functions $\theta(t), \phi(t), r(t), V(t), \gamma(t), \chi(t)$, the control function $\mu(t)$, and the parameter τ which minimize either the postatmospheric characteristic velocity or the total characteristic velocity, while directly ascending to LEO or indirectly ascending to LEO via PEO.

For (DM), the longitudinal motion subsystem was solved with respect to the state functions $r(t), V(t), \gamma(t)$, the control function $\mu(t)$, and the parameter τ which minimize either the postatmospheric characteristic velocity or the total characteristic velocity, while directly ascending to LEO or indirectly ascending to LEO via PEO. Then, with the above functions and parameter known, the lateral motion subsystem was solved with respect to the functions $\theta(t), \phi(t), \chi(t)$ by integration in forward time.

9.1. Algorithm.

For both the complete differential system of the combination method and the longitudinal motion subsystem of the decomposition method, Problems (DA) and (IA) are optimal control problems of the Bolza type. They can be solved using the sequential gradient-restoration algorithm (SGRA), employed in conjunction with the primal formulation (Refs. 33-39).

We recall that the Bolza problem consists in the minimization of a functional whose value depends on the state vector $x(t)$, the control vector $u(t)$, and the parameter vector π .

The state, control, and parameter vectors are required to satisfy a system of differential equations, initial conditions, and final conditions. The solution of the Bolza problem is governed by the system of constraint equations and optimality conditions. Hence, the convergence conditions are represented by the relations

$$P \leq \varepsilon_1, \quad (36a)$$

$$Q \leq \varepsilon_2. \quad (36b)$$

Here, P is the norm squared of the error in the constraints; Q is the norm squared of the error in the optimality conditions; and $\varepsilon_1, \varepsilon_2$ are preselected small, positive numbers.

We also recall that the sequential gradient-restoration algorithm is an iterative technique which ultimately leads to the satisfaction of (36). Such technique involves a sequence of cycles, each cycle including two phases: the gradient phase and the restoration phase. A gradient phase is undertaken whenever Ineq. (36a) is satisfied; a restoration phase is undertaken whenever Ineq. (36a) is violated. In the gradient phase, the value of the augmented functional is decreased, while avoiding excessive constraint violation. In the restoration phase, the value of the constraint error is decreased, while avoiding excessive change in the value of the functional. In a complete gradient-restoration cycle, the value of the functional is decreased, while the constraints are satisfied to a preselected degree of accuracy. Thus, a succession of suboptimal solutions is generated, each new solution being an improvement over the previous one from the point of view of the value of the functional being minimized. SGRA stops whenever Ineqs. (36) are simultaneously satisfied. For details, see Refs. 33-39.

9.2. Continuous vs Discrete Control.

For both the combination method (CM) and the decomposition method (DM), for path inclination either fixed or free, and for both Problems (DA) and (IA), two cases are

investigated: continuous control and discrete control.

(i) Continuous Control. Here, the bank angle is continuously variable. Hence, $\mu(t)$ is treated as a control variable.

(ii) Discrete Control. Here, the trajectory is divided into s segments along which the bank angle is constant; hence, for each segment, the bank angle μ_i is treated as a parameter. Because the time duration τ_i of each segment is also a parameter to be optimized, the number of parameters is twice the number of segments.

For the combination method (CM) and for both Problems (DA) and (IA) with entry path inclination fixed, both the continuous case and the discrete case with $s = 2, 3, 4, 5$ were studied. The discrete case with $s = 1$ was not considered in view of the fact that Ineq. (36a) cannot be satisfied: there is no feasible solution for $s = 1$.

For the combination method (CM) and for both Problems (DA) and (IA) with entry path inclination free, both the continuous case and the discrete case with $s = 1, 2, 3, 4, 5$ were studied.

Finally, for comparison purposes, and for Problem (IA) with entry path inclination either fixed or free, a reference trajectory (RT) was also computed. This is the nominal trajectory of Ref. 12, which is flown with $\alpha=17.0\text{deg}$ and $s = 5$, using five constant values of the bank angle, specifically:

$$\mu_1 = -180.0, \quad \mu_2 = -43.9, \quad \mu_3 = 89.9, \quad \mu_4 = -89.9, \quad \mu_5 = 0.0\text{deg}. \quad (37)$$

10. Numerical Results: Entry Path Inclination Fixed

AFE optimal trajectories with entry path inclination fixed were computed for both Problems (DA) and (IA), both the combination and decomposition methods, and both continuous control and discrete control. The numerical results are shown in Tables 1-4 and Figs. 1-2.

An interesting conclusion is that, for each transfer, the optimal continuous control trajectory and the optimal discrete control trajectories coalesce into a single trajectory: a two-subarc trajectory flown with constant bank angle in each subarc. This is the same as stating that the optimal control is bang-bang.

Table 1 refers to the combination method and compares the summary results for three trajectories: the direct ascent optimal trajectory (DAOT), the indirect ascent optimal trajectory (IAOT), and the indirect ascent reference trajectory (IART) of Ref. 12. Shown in Table 1 are the following quantities: the number of bank angle segments, the angle of bank sequence, the flight time; the minimum altitude, the peak dynamic pressure, the peak heating rate; the peak change of orbital inclination, the peak change of longitude of the ascending node, the peak wedge angle; the postatmospheric characteristic velocity and its components. The preatmospheric characteristic velocity is not shown, since it has the same value for DAOT, IAOT, and IART. The following comments are pertinent:

(i) Trajectories DAOT and IAOT involve two subarcs flown with constant bank angle: a short entry subarc along which $\mu_1 = 176.68^\circ$ and a long exit subarc along which $\mu_2 = 5.48^\circ$. The fact that $\mu_1 \neq 180^\circ$ and $\mu_2 \neq 0^\circ$ means that the spacecraft is compensating for the effects due to the Earth rotation so as to exit the atmosphere in the correct orbital plane, that of the Space Shuttle.

(ii) The postatmospheric characteristic velocity of trajectory DAOT (about 72 m/sec) is less than that of trajectory IAOT (about 82 m/sec), which in turn is less than that of

trajectory IART (about 98 m/sec).

(iii) While trajectories DAOT and IAOT are flown in approximately 820 sec and 796 sec respectively, trajectory IART is flown in a much shorter time, approximately 488 sec.

(iv) Compared with trajectory IART, trajectories DAOT and IAOT are characterized by higher minimum altitude (by about 2.9 km), hence lower peak dynamic pressure and lower peak heating rate.

(v) Compared with trajectory IART, trajectories DAOT and IAOT involve much smaller values of the peak change of orbital inclination, peak change of longitude of the ascending node, and peak wedge angle. Indeed, for trajectories DAOT and IAOT, these quantities are nearly zero, of order $1/100$ deg; this means that, for optimal flight, the motion of the AFE spacecraft must be nearly planar in an inertial space. On the other hand, for trajectory IART, the same quantities are about 100 times larger, hence of order 1 deg.

Table 2 compares the combination and decomposition methods and presents summary results for trajectories DAOT and IAOT. Shown in Table 2 are the same quantities appearing in Table 1.

The main conclusion which can be derived from Table 2 is that, for both the direct ascent to LEO and the indirect ascent to LEO via PEO, the results of the decomposition method are in close agreement with those of the combination method. In turn, this conclusion has important implications for the development of effective guidance schemes (Ref. 26).

Table 3 refers to the combination and decomposition methods and presents the entry conditions for trajectories DAOT and IAOT. Shown in Table 3 are the entry values (in both the inertial system and the Earth-fixed system) of the following quantities : longitude, latitude, altitude, radius, velocity, path inclination, heading angle; orbital inclination, longitude of the ascending node, wedge angle.

No conclusion can be derived from Table 3 in that the entry values of the physical

quantities under consideration are obviously the same for both the combination method and the decomposition method.

Table 4 compares the combination and decomposition methods and presents the exit conditions for trajectories DAOT and IAOT. Shown in Table 4 are the same quantities appearing in Table 3, albeit computed at atmospheric exit.

The main conclusion which can be derived from Table 4 is that the exit values predicted with the decomposition method are quite close to those predicted with the combination method. Again, this conclusion has important implications for the development of effective guidance schemes.

Figures 1 and 2 compare the combination and decomposition methods with reference to trajectory DAOT (Fig. 1) and trajectory IAOT (Fig. 2). Each figure contains 12 parts showing the time histories of the following quantities: altitude, relative velocity; relative path inclination, relative longitude; latitude, relative heading angle; bank angle, dynamic pressure; heating rate, inertial orbital inclination; inertial longitude of the ascending node, inertial wedge angle.

The main conclusion which can be derived from Figs. 1-2 is that, in the scale of the drawings, the trajectories obtained via the combination method are identical with the trajectories obtained via the decomposition method. Again, this result has important implications for the development of effective guidance schemes.

11. Numerical Results: Entry Path Inclination Free

AFE optimal trajectories with entry path inclination free were computed for both Problems (DA) and (IA), both the combination and decomposition methods, and both continuous control and discrete control. The numerical results are shown in Tables 5-8 and Figs. 3-4.

An interesting conclusion is that, for each transfer, the optimal continuous control trajectory and the optimal discrete control trajectories coalesce into a single trajectory: a one-subarc trajectory flown with constant bank angle. This is the same as stating that the optimal control is constant.

Table 5 refers to the combination method and compares the summary results for three trajectories: the direct ascent optimal trajectory (DAOT), the indirect ascent optimal trajectory (IAOT), and the indirect ascent reference trajectory (IART) of Ref. 12. Shown in Table 5 are the following quantities: the number of bank angle segments, the angle of bank sequence, the flight time; the minimum altitude, the peak dynamic pressure, the peak heating rate; the peak change of orbital inclination, the peak change of longitude of the ascending node, the peak wedge angle; the total characteristic velocity and its components. The following comments are pertinent:

(i) Trajectories DAOT and IAOT involve a single subarc flown with constant bank angle $\mu = 5.14^\circ$. The fact that $\mu \neq 0^\circ$ means that the spacecraft is compensating for the effects due to the Earth rotation so as to exit the atmosphere in the correct orbital plane, that of the Space Shuttle.

(ii) The total characteristic velocity of trajectory DAOT is less than that of trajectory IAOT, which in turn is less than that of trajectory IART.

(iii) While trajectories DAOT and IAOT are flown in approximately 845 sec and 821 sec respectively, trajectory IART is flown in a much shorter time, approximately 488 sec.

(iv) Compared with trajectory IART, trajectories DAOT and IAOT are characterized by higher minimum altitude (by about 3.4 km), hence lower peak dynamic pressure and lower peak heating rate.

(v) Compared with trajectory IART, trajectories DAOT and IAOT involve much smaller values of the peak change of orbital inclination, peak change of longitude of the ascending node, and peak wedge angle. Indeed, for trajectories DAOT and IAOT, these quantities are nearly zero, of order 1/100 deg; this means that, for optimal flight, the motion of the AFE spacecraft must be nearly planar in an inertial space. On the other hand, for trajectory IART, the same quantities are about 100 times larger, hence of order 1 deg.

(vi) Comparing trajectories DAOT and IAOT of Table 5 with the corresponding trajectories of Table 1, we see that the effect of freeing the entry path inclination is to produce flatter trajectories, hence trajectories characterized by higher minimum altitude (by about 0.5 km), lower peak dynamic pressure, and lower peak heating rate. The total characteristic velocity is also lower in Table 5, even though the corresponding quantity is not shown in Table 1.

Table 6 compares the combination and decomposition methods and presents summary results for trajectories DAOT and IAOT. Shown in Table 6 are the same quantities appearing in Table 5.

The main conclusion which can be derived from Table 6 is that, for both the direct ascent to LEO and the indirect ascent to LEO via PEO, the results of the decomposition method are in close agreement with those of the combination method. In turn, this conclusion has important implications for the development of effective guidance schemes (Ref. 26).

Table 7 refers to the combination and decomposition methods and presents the entry conditions for trajectories DAOT and IAOT. Shown in Table 7 are the entry values (in both the inertial system and the Earth-fixed system) of the following quantities : longitude,

latitude, altitude, radius, velocity, path inclination, heading angle; orbital inclination, longitude of the ascending node, wedge angle.

The main conclusion which can be derived from Table 7 is that the entry values of the velocity and the path inclination predicted with the decomposition method are quite close to those predicted with the combination method.

Upon comparing trajectories DAOT and IAOT of Table 7 with the corresponding trajectories of Table 3, we see that the effect of freeing the entry path inclination is to produce flatter trajectories, hence, trajectories characterized by a smaller modulus of the entry path inclination (by about 0.4 deg).

Table 8 compares the combination and decomposition methods and presents the exit conditions for trajectories DAOT and IAOT. Shown in Table 8 are the same quantities appearing in Table 7, albeit computed at atmospheric exit.

The main conclusion which can be derived from Table 8 is that the exit values predicted with the decomposition method are quite close to those predicted with the combination method. Again, this conclusion has important implications for the development of effective guidance schemes.

Figures 3 and 4 compare the combination and decomposition methods with reference to trajectory DAOT (Fig. 3) and trajectory IAOT (Fig. 4). Each figure contains 12 parts showing the time histories of the following quantities: altitude, relative velocity; relative path inclination, relative longitude; latitude, relative heading angle; bank angle, dynamic pressure; heating rate, inertial orbital inclination; inertial longitude of the ascending node, inertial wedge angle.

The main conclusion which can be derived from Figs. 3-4 is that, in the scale of the drawings, the trajectories obtained via the combination method are identical with the trajectories obtained via the decomposition method. Again, this result has important implications for the development of effective guidance schemes.

12. Conclusions

In this thesis, the GEO-to-LEO transfer of an AFE spacecraft is considered, and optimal trajectories are determined by minimizing the postatmospheric characteristic velocity if the entry path inclination is fixed and the total characteristic velocity if the entry path inclination is free. It is assumed that the angle of attack is constant and that the spacecraft is controlled via the angle of bank. The optimization is performed with respect to the time history of the angle of bank and the flight time for both the case of a continuous control and the case of a discrete control. Two transfer maneuvers are considered: (DA) direct ascent to LEO; and (IA) indirect ascent to LEO via PEO. For comparison purposes and only for Transfer (IA), a reference trajectory is also considered: this is a five-segment trajectory, identical to the nominal trajectory given in Ref. 12.

While the motion of the AFE spacecraft in a 3D-space is described by a system of six ordinary differential equations (combination method), substantial simplifications are possible if one exploits the smallness of two key quantities: the change of orbital inclination and the Earth's angular velocity. Indeed, the complete system of order six can be decoupled into two subsystems of order three, one describing the longitudinal motion and one describing the lateral motion (decomposition method).

In the decomposition method, the angle of bank history and the flight time are determined via the longitudinal motion subsystem. In this subsystem, subject to the specified LEO requirements, the following functional is minimized: the postatmospheric characteristic velocity if the entry path inclination is fixed or the total characteristic velocity if the entry path inclination is free. Then, the lateral motion subsystem can be integrated in forward time.

Systematic numerical computations are performed for both the combination and decomposition methods, both transfers (DA) and (IA), entry path inclination either fixed or

free, and both continuous control and discrete control. From the totality of results, the following main conclusions emerge:

(i) The optimal trajectories obtained with the decomposition method are nearly identical with those obtained with the combination method. This result has important guidance implications.

(ii) If the entry path inclination is fixed, the optimal continuous control trajectories and the optimal discrete control trajectories coalesce into a single trajectory: a two-subarc trajectory flown with bank angle constant along each subarc. Thus, the optimal control is bang-bang. The values of the bank angle are $\mu_1 = 176.68 \text{ deg}$ for the entry subarc and $\mu_2 = 5.48 \text{ deg}$ for the exit subarc.

(iii) If the entry path inclination is free, the optimal continuous control trajectories and the optimal discrete control trajectories coalesce into a single trajectory: a one-subarc trajectory along which the bank angle is constant, $\mu = 5.14 \text{ deg}$.

(iv) From the point of view of the characteristic velocity, the optimal trajectory of transfer (DA) is advantageous over the optimal trajectory of transfer (IA), and both are advantageous over the nonoptimal reference trajectory of Ref. 12. The advantages of the optimal trajectories over the reference trajectory of Ref. 12 extend to other quantities of interest such as the peak dynamic pressure, the peak heating rate, and the peak wedge angle.

(v) The optimal trajectories with entry path inclination free enter the atmosphere at a flatter angle (by about 0.4 deg) than the optimal trajectories with entry path inclination fixed at the value preselected in Ref. 12. In turn, this results in lower characteristic velocity, higher minimum altitude, lower peak dynamic pressure, and lower peak heating rate.

(vi) The vertical component of the lift during the atmospheric pass ensures exit conditions compatible with the desired LEO requirements. The lateral component of the lift during the atmospheric pass is directed in such a way that it nearly offsets the effects due to

the Earth rotation. In this way, the instantaneous orbital plane is almost identical with the entry orbital plane, meaning that the wedge angle is nearly zero during the atmospheric pass. This means that, for efficient flight, the motion of the AFE spacecraft is nearly planar in an inertial space.

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Table 1. Summary results,
AFE optimal and reference trajectories,
combination method,
entry path inclination fixed.

Quantity	DAOT	IAOT	IART	Units
s	2	2	5	—
μ_1	176.68	176.68	-180.0	deg
μ_2	5.48	5.48	-43.9	deg
μ_3			89.9	deg
μ_4			-89.9	deg
μ_5			0.0	deg
τ	819.5	795.7	487.6	sec
min(h)	77.5	77.5	74.6	km
max(DP)	1174	1174	1629	N/m ²
max(HR)	148	148	158	W/cm ²
max($\Delta \tilde{i}$)	0.01	0.01	1.08	deg
max($\Delta \tilde{\Omega}$)	0.00	0.00	0.32	deg
max($\tilde{\eta}$)	0.01	0.01	1.08	deg
$\Delta \tilde{V}_{22}$	0.00	71.78	87.89	m/sec
$\Delta \tilde{V}_{11}$	72.03	10.08	10.08	m/sec
$\Delta \tilde{V}$	72.03	81.86	97.97	m/s

Table 2. Summary results,
AFE optimal trajectories,
combination and decomposition methods,
entry path inclination fixed.

Quantity	DAOT		IAOT		Units
	Combination method	Decomposition method	Combination method	Decomposition method	
s	2	2	2	2	—
μ_1	176.68	176.68	176.68	176.68	deg
μ_2	5.48	5.48	5.48	5.48	deg
τ_1	96.5	97.4	96.5	97.4	sec
τ_2	723.0	719.4	699.2	695.6	sec
τ	819.5	816.8	795.7	793.0	sec
min(h)	77.53	77.51	77.53	77.51	km
max(DP)	1175	1183	1175	1184	N/m ²
max(HR)	148	149	148	149	W/cm ²
max($\Delta\tilde{i}$)	0.01	0.01	0.01	0.01	deg
max($\Delta\tilde{\Omega}$)	0.00	0.00	0.00	0.00	deg
max($\tilde{\eta}$)	0.01	0.01	0.01	0.01	deg
$\Delta\tilde{V}_{22}$	0.00	0.00	71.78	71.80	m/sec
$\Delta\tilde{V}_{11}$	72.03	72.04	10.08	10.07	m/sec
$\Delta\tilde{V}$	72.03	72.04	81.86	81.87	m/sec

Table 3. Entry conditions,
AFE optimal trajectories,
combination and decomposition methods,
entry path inclination fixed.

Quantity	DAOT		IAOT		Units
	Combination method	Decomposition method	Combination method	Decomposition method	
θ_0	-134.52	-134.52	-134.52	-134.52	deg
$\tilde{\theta}_0$	-134.52	-134.52	-134.52	-134.52	deg
ϕ_0	-4.49	-4.49	-4.49	-4.49	deg
h_0	122	122	122	122	km
r_0	6500	6500	6500	6500	km
V_0	9.895	9.895	9.895	9.895	km/sec
$\tilde{V}_0$	10.308	10.308	10.308	10.308	km/sec
γ_0	-4.68	-4.68	-4.68	-4.68	deg
$\tilde{\gamma}_0$	-4.49	-4.49	-4.49	-4.49	deg
χ_0	-29.42	-29.42	-29.42	-29.42	deg
$\tilde{\chi}_0$	-28.13	-28.13	-28.13	-28.13	deg
$\tilde{i}_0$	28.46	28.46	28.46	28.46	deg
$\tilde{\Omega}_0$	-126.19	-126.19	-126.19	-126.19	deg
$\tilde{\eta}_0$	0.00	0.00	0.00	0.00	deg

Table 4. Exit conditions,
AFE optimal trajectories,
combination and decomposition methods,
entry path inclination fixed.

Quantity	DAOT		IAOT		Units
	Combination method	Decomposition method	Combination method	Decomposition method	
θ_1	-81.21	-81.34	-82.77	-82.91	deg
$\tilde{\theta}_1$	-77.78	-77.93	-79.45	-79.60	deg
ϕ_1	22.06	22.02	21.54	21.49	deg
h_1	122	122	122	122	km
r_1	6500	6500	6500	6500	km
V_1	7.466	7.479	7.476	7.490	km/sec
$\tilde{V}_1$	7.881	7.895	7.892	7.905	km/sec
γ_1	0.81	0.81	0.87	0.86	deg
$\tilde{\gamma}_1$	0.77	0.76	0.82	0.82	deg
χ_1	-19.51	-19.56	-20.16	-20.22	deg
$\tilde{\chi}_1$	-18.44	-18.49	-19.06	-19.11	deg
$\tilde{l}_1$	28.45	28.45	28.45	28.45	deg
$\tilde{\Omega}_1$	-126.19	-126.19	-126.19	-126.19	deg
$\tilde{\eta}_1$	0.00	0.00	0.00	0.00	deg

Table 5. Summary results,
AFE optimal and reference trajectories,
combination method,
entry path inclination free.

Quantity	DAOT	IAOT	IART	Units
s	1	1	5	—
μ_1	5.14	5.14	-180.0	deg
μ_2			-43.9	deg
μ_3			89.9	deg
μ_4			-89.9	deg
μ_5			0.0	deg
τ	844.6	820.8	487.6	sec
min(h)	78.0	78.0	74.6	km
max(DP)	1069	1069	1629	N/m ²
max(HR)	137	137	158	W/cm ²
max($\Delta \tilde{i}$)	0.01	0.01	1.08	deg
max($\Delta \tilde{\Omega}$)	0.01	0.01	0.32	deg
max($\tilde{\eta}$)	0.01	0.01	1.08	deg
$\Delta \tilde{V}_{00}$	1.490	1.490	1.490	km/sec
$\Delta \tilde{V}_{22}$	0.000	0.072	0.088	km/sec
$\Delta \tilde{V}_{11}$	0.072	0.010	0.010	km/sec
$\Delta \tilde{V}$	1.562	1.572	1.588	km/sec

Table 6. Summary results,
AFE optimal trajectories,
combination and decomposition methods,
entry path inclination free.

Quantity	DAOT		IAOT		Units
	Combination method	Decomposition method	Combination method	Decomposition method	
s	1	1	1	1	—
μ	5.14	5.14	5.14	5.14	deg
τ	844.6	843.6	820.8	819.8	sec
min(h)	78.0	78.0	78.0	78.0	km
max(DP)	1069	1069	1069	1069	N/m ²
max(HR)	137	137	137	137	W/cm ²
max($\Delta\tilde{i}$)	0.01	0.01	0.01	0.01	deg
max($\Delta\tilde{\Omega}$)	0.01	0.01	0.01	0.01	deg
max($\tilde{\eta}$)	0.01	0.01	0.01	0.01	deg
$\Delta\tilde{V}_{00}$	1.490	1.490	1.490	1.490	km/sec
$\Delta\tilde{V}_{22}$	0.000	0.000	0.072	0.072	km/sec
$\Delta\tilde{V}_{11}$	0.072	0.072	0.010	0.010	km/sec
$\Delta\tilde{V}$	1.562	1.562	1.572	1.572	km/sec

Table 7. Entry conditions,
AFE optimal trajectories,
combination and decomposition methods,
entry path inclination free.

Quantity	DAOT		IAOT		Units
	Combination method	Decomposition method	Combination method	Decomposition method	
θ_0	-134.52	-134.52	-134.52	-134.52	deg
$\tilde{\theta}_0$	-134.52	-134.52	-134.52	-134.52	deg
ϕ_0	-4.49	-4.49	-4.49	-4.49	deg
h_0	122	122	122	122	km
r_0	6500	6500	6500	6500	km
V_0	9.895	9.906	9.895	9.906	km/sec
$\tilde{V}_0$	10.308	10.320	10.308	10.319	km/sec
γ_0	-4.311	-4.306	-4.311	-4.306	deg
$\tilde{\gamma}_0$	-4.138	-4.133	-4.138	-4.133	deg
χ_0	-29.42	-29.42	-29.42	-29.42	deg
$\tilde{\chi}_0$	-28.13	-28.13	-28.13	-28.13	deg
$\tilde{i}_0$	28.46	28.46	28.46	28.46	deg
$\tilde{\Omega}_0$	-126.19	-126.19	-126.19	-126.19	deg
$\tilde{\eta}_0$	0.00	0.00	0.00	0.00	deg

Table 8. Exit conditions,
AFE optimal trajectories,
combination and decomposition methods,
entry path inclination free.

Quantity	DAOT		IAOT		Units
	Combination method	Decomposition method	Combination method	Decomposition method	
θ_1	-79.04	-79.02	-80.63	-80.60	deg
$\tilde{\theta}_1$	-75.51	-75.49	-77.20	-77.18	deg
ϕ_1	22.75	22.75	22.24	22.25	deg
h_1	122	122	122	122	km
r_1	6500	6500	6500	6500	km
V_1	7.466	7.481	7.476	7.491	km/sec
$\tilde{V}_1$	7.881	7.896	7.892	7.906	km/sec
γ_1	0.807	0.806	0.864	0.863	deg
$\tilde{\gamma}_1$	0.764	0.763	0.819	0.818	deg
χ_1	-18.58	-18.57	-19.27	-19.26	deg
$\tilde{\chi}_1$	-17.57	-17.56	-18.22	-18.21	deg
$\tilde{i}_1$	28.45	28.46	28.45	28.46	deg
$\tilde{\Omega}_1$	-126.20	-126.20	-126.20	-126.20	deg
$\tilde{\eta}_1$	0.00	0.00	0.00	0.00	deg

Fig. 1A. AFE optimal trajectories,
entry path inclination fixed,
Transfer(DA), altitude.

Fig. 1B. AFE optimal trajectories,
entry path inclination fixed,
Transfer(DA), relative velocity.

Fig. 1C. AFE optimal trajectories,
entry path inclination fixed,
Transfer(DA), relative path inclination.

Fig. 1D. AFE optimal trajectories,
entry path inclination fixed,
Transfer(DA), relative longitude.

Fig. 1E. AFE optimal trajectories,
entry path inclination fixed,
Transfer(DA), latitude.

Fig. 1F. AFE optimal trajectories,
entry path inclination fixed,
Transfer(DA), relative heading angle.

Fig. 1G. AFE optimal trajectories,
entry path inclination fixed,
Transfer(DA), bank angle.

Fig. 1H. AFE optimal trajectories,
entry path inclination fixed,
Transfer(DA), dynamic pressure.

Fig. 1I. AFE optimal trajectories,
entry path inclination fixed,
Transfer(DA), heating rate.

Fig. 1J. AFE optimal trajectories,
entry path inclination fixed,
Transfer(DA), inertial orbital inclination.

Fig. 1K. AFE optimal trajectories,
entry path inclination fixed,
Transfer(DA), inertial longitude of the ascending node.

Fig. 1L. AFE optimal trajectories,
entry path inclination fixed,
Transfer(DA), inertial wedge angle.

Fig. 2A. AFE optimal trajectories,
entry path inclination fixed,
Transfer (IA), altitude.

Fig. 2B. AFE optimal trajectories,
entry path inclination fixed,
Transfer (IA), relative velocity.

Fig. 2C. AFE optimal trajectories,
entry path inclination fixed,
Transfer (IA), relative path inclination.

Fig. 2D. AFE optimal trajectories,
entry path inclination fixed,
Transfer (IA), relative longitude.

Fig. 2E. AFE optimal trajectories,
entry path inclination fixed,
Transfer (IA), latitude.

Fig. 2F. AFE optimal trajectories,
entry path inclination fixed,
Transfer (IA), relative heading angle.

Fig. 2G. AFE optimal trajectories,
entry path inclination fixed,
Transfer (IA), bank angle.

Fig. 2H. AFE optimal trajectories,
entry path inclination fixed,
Transfer (IA), dynamic pressure.

Fig. 2I. AFE optimal trajectories,
entry path inclination fixed,
Transfer (IA), heating rate.

Fig. 2J. AFE optimal trajectories,
entry path inclination fixed,
Transfer (IA), inertial orbital inclination.

Fig. 2K. AFE optimal trajectories,
entry path inclination fixed,
Transfer (IA), inertial longitude of the ascending node.

Fig. 2L. AFE optimal trajectories,
entry path inclination fixed,
Transfer (IA), inertial wedge angle.

Fig. 3A. AFE optimal trajectories,
entry path inclination free,
Transfer(DA), altitude.

Fig. 3B. AFE optimal trajectories,
entry path inclination free,
Transfer(DA), relative velocity.

Fig. 3C. AFE optimal trajectories,
entry path inclination free,
Transfer(DA), relative path inclination.

Fig. 3D. AFE optimal trajectories,
entry path inclination free,
Transfer(DA), relative longitude.

Fig. 3E. AFE optimal trajectories,
entry path inclination free,
Transfer(DA), latitude.

Fig. 3F. AFE optimal trajectories,
entry path inclination free,
Transfer(DA), relative heading angle.

Fig. 3G. AFE optimal trajectories,
entry path inclination free,
Transfer(DA), bank angle.

Fig. 3H. AFE optimal trajectories,
entry path inclination free,
Transfer(DA), dynamic pressure.

Fig. 3I. AFE optimal trajectories,
entry path inclination free,
Transfer(DA), heating rate.

Fig. 3J. AFE optimal trajectories,
entry path inclination free,
Transfer(DA), inertial orbital inclination.

Fig. 3K. AFE optimal trajectories,
entry path inclination free,
Transfer(DA), inertial longitude of the ascending node.

Fig. 3L. AFE optimal trajectories,
entry path inclination free,
Transfer(DA), inertial wedge angle.

Fig. 4A. AFE optimal trajectories,
entry path inclination free,
Transfer (IA), altitude.

Fig. 4B. AFE optimal trajectories,
entry path inclination free,
Transfer (IA), relative velocity.

Fig. 4C. AFE optimal trajectories,
entry path inclination free,
Transfer (IA), relative path inclination.

Fig. 4D. AFE optimal trajectories,
entry path inclination free,
Transfer (IA), relative longitude.

Fig. 4E. AFE optimal trajectories,
entry path inclination free,
Transfer (IA), latitude.

Fig. 4F. AFE optimal trajectories,
entry path inclination free,
Transfer (IA), relative heading angle.

Fig. 4G. AFE optimal trajectories,
entry path inclination free,
Transfer (IA), bank angle.

Fig. 4H. AFE optimal trajectories,
entry path inclination free,
Transfer (IA), dynamic pressure.

Fig. 4I. AFE optimal trajectories,
entry path inclination free,
Transfer (IA), heating rate.

Fig. 4J. AFE optimal trajectories,
entry path inclination free,
Transfer (IA), inertial orbital inclination.

Fig. 4K. AFE optimal trajectories,
entry path inclination free,
Transfer (IA), inertial longitude of the ascending node.

Fig. 4L. AFE optimal trajectories,
entry path inclination free,
Transfer (IA), inertial wedge angle.