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A unified approach to complex seismic imaging problems

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Rice University, 1991

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A UNIFIED APPROACH TO COMPLEX SEISMIC IMAGING PROBLEMS

by

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A THESIS SUBMITTED
IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS FOR THE DEGREE
DOCTOR OF PHILOSOPHY

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A UNIFIED APPROACH TO COMPLEX SEISMIC IMAGING PROBLEMS

Claude F. Lafond

Abstract

Two current challenges in seismic imaging are to obtain more detailed images of complex structures from reflection data and to constrain the regional structure of the Earth using wide-angle data. These are complex problems for which traditional methods fail because they are based on too many simplifying assumptions. I develop a unified approach which addresses these tasks by starting with a fundamental problem formulation, leading to a practical numerical solution which converges rapidly. It is based on pre-stack depth migration and cell-stripping tomography in heterogeneous media, which allow layer-stripping and retain all the information from the data, incorporates a depth focusing technique for improved image resolution and utilizes user-interaction and geologic input to guide and constrain the imaging process.

I first describe a fast and accurate dynamic ray-tracing scheme in heterogeneous media which allows complex model definition and rapid two-point ray tracing. This ray tracing method is then used to compute Green functions in a layer-stripping pre-stack depth migration algorithm. The algorithm itself is based on a Kirchhoff integral in heterogeneous media using exact weighting factors and specialized to 2.5 D migration. I examine the migration results with a depth-focusing technique which analyzes common image panels for horizontal alignment, relating the degree of non alignment, or Migration Moveout (MMO) to corrections in the velocity model along the raypaths. Finally, I develop a cell-stripping tomography (CST) algorithm which distributes velocity residuals only to the

relevant cells, allowing resolution of both horizontal and vertical discontinuities and providing starting models for migration.

Although computer-intensive, this unified approach is successful both in synthetic tests and for obtaining local and regional images of the edge of the Santa Maria basin in central California. It is more faithful to the velocity and dip information contained in the data, allows more control over the imaging process and with available computing power promises to be routinely applicable.

Acknowledgements

I would like to express my sincere gratitude to my advisor, Alan R. Levander, for his constant support and guidance during my years at Rice. His dynamic presence, his drive to try out new ideas, as well as his personal friendship have made this work possible. I would also like to thank Bruce Gibson, who was of great help during the first part of this project, both for his insights and his assistance with our first computer. In the later part of this project, many discussions with Stuart Henrys were enlightening, and he was responsible for a great and reliable computer system. Finally, I would like to thank all the professors, students and staff of the Department of Geology & Geophysics who have made life much easier by their friendly, dynamic and relaxed attitude.

This research was supported by NSF Grant EAR 8817146. I was supported by a W. M. Keck Foundation fellowship. The computations were performed on the ELXSI 6400, Stardent GS2000 and SUN 4/390 of the Rice Geophysical Computing Center.

*'Beaucoup de problèmes peuvent être résolus si l'on n'en
fait pas des problèmes'*

René Descartes

(A lot of problems can be solved if one does not consider them as
problems).

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Introduction

Imaging techniques are designed to obtain an image of the interior of an object based on indirect observations made at or near its surface. Often, these observations are measurements or recordings of the properties of waves going through the interior of the object. *Seismic imaging* is a technique which uses the propagation of seismic waves through the Earth to find out about its interior. It was originally applied to earthquake arrival times to obtain a velocity structure for the Earth; later on the idea of using human-triggered sources appeared when oil prospectors recorded refracted waves from dynamite shots in order to delineate gross features of the subsurface favorable to oil traps, such as salt domes.

Since then exploration seismology has grown considerably both in terms of research and economic importance, being the cornerstone of oil and gas exploration. Data acquisition techniques have become very sophisticated, the frequencies involved now ranging from about 1 to 100 Hz and scale lengths from a few meters to tens of kilometers. Seismic imaging techniques have evolved accordingly during the last thirty years: NMO analysis and stacking (1960's), finite-difference and integral migration techniques (1970's), DMO correction and tomography (1980's) have been developed successfully and made routinely applicable by the corresponding increases in computing power.

The modern tasks of seismic imaging, however, require new approaches. Their scope is to obtain far more detailed images of the Earth interior on very small to very large scales: conventional methods fail because they oversimplify the problem. Although there are some attempts to adapt these methods to handle new problems, such as Pre-Stack Imaging (PSI), kinematic imaging and reflection tomography, the constant increase in computing power and decrease in cost justify more realistic approaches.

I have chosen a fundamental approach. I start by redescribing each problem in its most basic form, ignoring traditional simplifications. To each aspect of problem-solving, I respond with a solution which addresses the basic issue, but is still made practical for reasonable implementation and routine application. This approach therefore stands half-way between conventional simplifying methods and pure numerical or statistical schemes.

I explain in detail what the new challenges of seismic imaging are and why traditional methods do not work in chapter 1. I present the new philosophy and introduce the four main methods on which I base my approach. Chapter 2 describes the ray-tracing method, chapter 3 the pre-stack migration algorithm, chapter 4 the depth focusing technique and chapter 5 the tomography scheme. These chapters are in this order because every chapter builds on or uses the results of all previous chapters. Chapters 2 and 3 are more mathematical and algorithmic in nature, while chapters 4 and 5 talk more about methods and contain field data studies.

Chapter 1

New Challenges in Seismic Imaging

1 - The traditional approach

Routine seismic imaging nowadays consists largely of *post-stack techniques* applied to reflection data. These techniques are collectively referred to as post-stack time migration. They imply extensive pre-processing of the data: statics corrections, deconvolution, multiple elimination, velocity analysis, filtering and spherical divergence corrections are but a few steps common to standard processing. The field data, naturally sorted into shot gathers, must also be re-sorted into CDP or Common Offset gathers, a very long procedure for large data sets.

Once the data are processed, the next step is the application of Dip Moveout corrections (DMO) in order to eliminate the dependence of moveouts on reflector dips. It is followed by Normal Moveout (NMO) analysis to eliminate the dependence of moveouts on offset, so that events align coherently on CDP gathers, as well as to obtain migration velocity models. It is important to note that NMO analysis is a focusing procedure theoretically valid only for flat reflectors and constant velocity media. The data are then stacked to obtain a first image and only then is post-stack time migration applied to obtain a

picture of the Earth interior. This image must be converted to a depth section using the velocities obtained for migration.

2 - Why it is not enough

The current need in seismic imaging is to obtain more detailed pictures of the Earth interior. For the *oil industry*, this is becoming essential as drilling is more expensive, since obvious oil reservoirs have been found and when economic and environmental concerns justify exploiting existing reservoirs to their fullest potential. This means obtaining very detailed pictures of complex structures in order to make accurate evaluations and predictions. At the same time, *crustal studies* designed to constrain the regional structure of the earth are becoming more widespread and of higher quality (Mooney & Brocher, 1987). They are analogous to exploration experiments but on a much larger scale: source to receiver offsets of 10 to 200 km, with target depths of 30 km and more. They are often characterized by larger shots but much smaller folds.

These new imaging tasks require new approaches. Traditional methods do not work here because they are based on the simplifying assumption that the velocity field is smooth inside the Earth. This is true to a certain degree, since the Earth is a physical system with a limited range for its parameters, so that pre-processing, stacking and post-stack migration often produce coherent results. This is why they have been used and are still used with great success. But this approach breaks down as soon as one is interested in the base of a salt dome, the exact pattern of a complicated fault zone, the shape of the downgoing slab in a subduction zone or the exact depth of the Moho. In those cases we need as much detail in the velocity model as possible, so that stacking and averaging methods must be ruled out.

3 - New philosophy

Detailed seismic imaging, whether in the case of complex structures or crustal studies, requires several new ways of thinking.

3.1 - One approach which takes into account the heterogeneity of the Earth is *layer stripping*. To obtain an accurate image of some part of the subsurface, we must already have a good knowledge of what stands between our observation surface and that particular area. Layer-stripping allows us to eliminate the effect of this intermediate layer and concentrate on the target. It can be achieved in two ways: by actually propagating the whole wavefield through the earth and collecting new seismic sections along a new datum, which is the datuming technique (Berryhill, 1979; Shih & Levander, 1991). I have chosen the other way, which explicitly includes the known intermediate layer in the velocity model so that its effects are accounted for when acting on the target. This approach eliminates computer-intensive datuming and aliasing problems, but requires an accurate propagation model for arbitrary media.

This layer-stripping approach in effect divides the model into several submodels; this set of submodels is commonly called a *macro model* of the Earth. The next step is to perform detailed analysis on those submodels in order to achieve good resolution. Refining a submodel then yields a *micro model* of the Earth.

3.2 - One key point in detailed imaging is to *keep as much information as possible* about the original data. This excludes conventional stacking, because stacking N traces in a CDP gather effectively reduces the information content by N . Similarly, averaging N velocity residuals in one cell in standard tomography reduces the velocity information by

N. Without keeping all the original information it is not possible to recover detailed structure. This is why, for the macro models, I use *pre-stack migration*, which produces a wealth of information, and a *cell-stripping tomographic approach* which strives to distribute velocity residuals only in those cells where they are most likely to apply, taking into account cells already modified.

3.3 - For obtaining micro models, it is essential to have a *focusing technique* which, based on the macro model, actually gives a detailed picture of the subsurface. Although the macro velocity model is close to lithologic velocities, layer-stripping and tomographic techniques only yield gross features of the subsurface: this is not a problem, since migration is sensitive to variations in the velocity model over 4 to 10 wavelengths only (Versteeg, 1990). However, to increase resolution of individual layers, we need a focusing step analogous to the standard focusing steps of DMO and NMO plus stack. The result of migration is then to bring up detailed reflectors: this is when the velocity model is actually replaced by the migrated section. To do this I develop a pre-stack focusing analysis which I call MMO (Migration Moveout) analysis, which allows iterative updating of the velocity model with very fast convergence.

3.4 - *Parameter constraint* must be central to these imaging algorithms. When imaging is viewed as an inversion technique, the ideas developed above can be formulated in terms of the resolution versus variance trade-off principle central to inversion (Stolt & Weglein, 1985). The traditional approach to processing produces images with little variance: they are sharp and relatively insensitive to the input parameters. The resolution, however, in terms of how much detail one can see in the image, is poor. Complex imaging techniques, on the other hand, produce highly detailed (high resolution) images. However, they are very sensitive to the input parameters such as velocities and layer boundaries (they

have a high variance). This is why they require strong constraints on those parameters. This I achieve through user-interaction, which includes *geologic input and correlation*.

4 - A unified approach: thesis outline

I have articulated my approach around four central algorithms.

In chapter 2, I present a fast and accurate dynamic ray tracing scheme in heterogeneous media. This scheme is faster than most non-finite difference methods. It allows forward modeling in complex and large models with little user effort. Benchmarks against an established algorithm prove its accuracy. Its flexibility is ideal for computing the Green functions of migration and the traveltimes and raypaths needed for tomography.

A complete derivation of the Kirchhoff integral in heterogeneous media is given in chapter 3. I develop the two-way integral formula in 3-D and specialize the formula to the case of 2-D migration of a point source (2.5-D approximation) using the asymptotic method of stationary phase. I describe in detail the algorithm for its implementation and its restrictions, based on the ray tracing described previously, as well as demonstrate it with a synthetic test. The reason for using the integral approach is again its flexibility: it allows explicit and fast storage of partial quantities (ray angles, traveltimes, amplitudes), determination of specular source-receiver pairs, and has no dip limitation.

I develop an approach to MMO analysis (depth focusing analysis) in chapter 4. The pre-stack migrated data are arranged to compare the images obtained at one point from migrating constant offset sections. The degree of non-horizontality is related to corrections

in the velocity model, which are in turn used to update the velocity model for re-migration. On synthetic tests this analysis yields very fast convergence even for complicated waveform patterns. I apply the method to a field data set from central California and obtain a detailed picture of the edge of the Santa Maria basin and Hosgri Fault zone.

Finally, chapter 5 describes a cell-stripping tomographic approach (CST) for inverting wide-angle data and obtaining macro models for pre-stack migration. This approach allows good resolution of velocity models even for sparse crustal data and is based on a one-pass principle. On a synthetic test with small horizontal ray coverage, it is successful in recovering vertical and horizontal velocity contrasts. Coupled with the pre-stack migration algorithm and MMO analysis, I apply it on another set from central California and obtain an image of the upper crust in the Santa Maria Basin.

5 - Is it really justified ?

Currently, the approach I have outlined is expensive even for 2-D data and traditional methods still work very well in a lot of cases. It is tempting to reject new ideas on the basis of impracticality. However, in addition to those essential motivations mentioned above (complex structures and crustal imaging), there are a number of practical incentives for trying these methods.

The multiplicity of 3-D data acquisition in the industry is a good reason to perform this unified approach, since velocities are likely to be heterogeneous and structures complicated. It is remarkable that the techniques I present here can be adapted to 3-D

without any conceptual difficulty, whereas traditional methods are much more of a problem.

The constant increase in computing power makes it likely that computers of the 21st century will handle this kind of imaging as easily as those of today handle traditional processing. Furthermore, a number of supercomputers already exist which allow us to try out these techniques today. The emergence of very high quality wide-angle data sets also allows us to try out these new methods on valuable data.

Another practical advantage of detailed imaging is that it requires little pre-processing of the data. Statics corrections are accounted for automatically; amplitudes should be preserved; the data can be kept in field format; multiples should be eliminated automatically; direct waves can be used for velocity information automatically. Time spent imaging is often recovered in terms of the processing time and human labor saved.

Since detailed imaging retains all the information of the original data set, it is not necessary to have as dense a receiver array and shot spacing as is customary. Images of comparable quality can be obtained with fewer traces, which can offset a large part of the cost of detailed imaging. Also, detailed imaging can be performed on a subset of the data only, around the target, thanks to the Kirchhoff algorithm.

Finally, this approach unifies geophysicists and geologists as well, since physical theory and numerical methods are used simultaneously with geological constraints and interpretation.

Chapter 2

Fast and Accurate Dynamic Ray Tracing in Complex Models

Abstract

I develop a fast and accurate dynamic ray tracing method for 2.5-D heterogeneous media based on the kinematic algorithm proposed by Langan et al. (1985). This algorithm divides the model into cells of constant slowness gradient and the positions, directions and traveltimes of the rays are expressed as polynomials of the travel path length, accurate to the second order in the gradient. This method is efficient because of the use of simple polynomials at each ray tracing step.

I derive similar polynomial expressions for the dynamic ray tracing quantities by integrating the ray tracing system and expanding the solutions to the second order in the slowness gradient. This new algorithm efficiently computes the geometrical spreading, amplitude and wavefront curvature on individual rays. The two-point ray tracing problem is solved by the shooting method using the geometrical spreading. Paraxial corrections based on the wavefront curvature improve the accuracy of the traveltimes and amplitude at a given receiver.

The computational results for two simple velocity models are compared with those obtained with the SEIS83 seismic modeling package (Cerveny and Psencik, 1984): this new method is accurate for both traveltimes and amplitudes while being significantly faster. I present a complex velocity model which shows that the algorithm allows for realistic models and easily computes rays in structures that pose difficulties for conventional methods. The method can be extended to ray tracing in 3-D heterogeneous media and can be used as a support for a Gaussian beam algorithm. In the next chapters I will use it for computing the Green functions and imaging condition needed for prestack depth migration, as well as for transmission tomography on wide-angle data.

1 - Introduction

Recent efforts to invert seismic data for the Earth structure and to adapt Kirchhoff migration techniques to pre-stack migration in heterogeneous media demonstrate the need for a fast and accurate method to compute the traveltimes and amplitudes. Fast ray tracing as a forward modeling tool is also required for the interpretation of reflection and wide-angle seismic surveys. Full-blown asymptotic ray theory (e.g. Cerveny and Ravindra, 1971) becomes too expensive in this case because the number of rays needed is considerable and the dimensions of the model may be very large. Other methods which have been proposed (McMechan and Mooney, 1980; Cassel, 1982; Spence et al., 1984) are generally faster but depend on somewhat cruder approximations of either the velocity model or the amplitude variations. Finally, the input of the model and the generation of rays must be simple enough to allow a number of successive runs to be done efficiently.

I base my method on a fast kinematic algorithm described by Langan et al. (1985), in which the velocity model is divided into cells of constant slowness gradient. All the quantities involved in ray tracing are expanded as polynomials of the travel path, s , accurate to the second order in the slowness gradient. Using the same approach, I derive similar polynomial expressions for the *dynamic* quantities by integrating the ray tracing system and expanding the solutions to the second order in the gradient, for the case of *2-D velocity models*. At little additional cost compared to the kinematic algorithm, one can then obtain the geometrical spreading, amplitude and wavefront curvature on individual rays. These in turn can be used to solve the two-point ray tracing problem by the shooting method and to perform paraxial corrections to improve the accuracy of the traveltimes and amplitudes at each receiver.

After reviewing the method and the kinematic results of Langan et al., I derive the polynomial expressions for the dynamic ray tracing quantities. The algorithm is described in detail, showing how to incorporate these formulae and use them for two-point ray tracing and paraxial corrections. The analysis of two simple models then shows that this algorithm produces results which compare favorably with those obtained with the SEIS83 seismic package (Cerveny and Psencik, 1984). I conclude with a model of a structurally complex continental margin to show the versatility of this ray tracing method.

2 - Kinematic ray tracing

Let us first review the method of Langan et al. One starts with the ray tracing system

$$\frac{d}{ds} [S(\mathbf{r}) \frac{d\mathbf{r}}{ds}] = \nabla_{\mathbf{r}} [S(\mathbf{r})] \quad (2.1)$$

which can be derived from the eikonal equation or directly from Fermat's integral. $S(\mathbf{r})$ is the slowness at position $\mathbf{r}$, s is the travel path length. Equation (2.1) can be integrated twice to yield

$$\mathbf{r}(s) = \mathbf{r}_0 + \mathbf{n}_0 \int_0^s \frac{S(\mathbf{r}_0)}{S(\mathbf{r})} ds' + \int_0^s \frac{1}{S(\mathbf{r})} \int_0^{s'} \nabla_{\mathbf{r}''} S(\mathbf{r}'') ds'' ds' \quad (2.2)$$

where $\mathbf{r}_0$ and $\mathbf{n}_0$ are the position and direction of the ray at $s=0$, respectively.

Defining a *constant* slowness gradient $\mathbf{G}$ as

$$\mathbf{G} = \frac{\nabla_{\mathbf{r}} S(\mathbf{r})}{S(\mathbf{r})}, \quad (2.3)$$

one can integrate (2.2) approximately by substituting the formula (2.3) and keeping terms to the second order in $\mathbf{G}$ only, obtaining

$$r_i(s) = r_{0i} + n_{0i} \left[s + \frac{s^2}{2} (\mu_i - k) + \frac{s^3}{6} (3k^2 - G^2 - 2k\mu_i) \right], \quad (2.4)$$

$$\text{with } k = G_i n_{0i} \quad \text{and} \quad \mu_i = \frac{G_i}{n_{0i}}.$$

The index i runs over the three coordinates (x_1, x_2, x_3) or, interchangeably, (x, y, z) . The direction $\mathbf{n}$ of the ray is obtained by taking the derivative of (2.4) with respect to the travel path s :

$$n_i(s) = n_{0i} \left[1 + s(\mu_i - k) + \frac{s^2}{2}(3k^2 - G^2 - 2k\mu_i) \right]. \quad (2.5)$$

Finally, the traveltime $t(s)$ is obtained by integrating the product of the slowness and ray direction along the travel path:

$$t(s) = sS_0 \left(1 + \frac{sk}{2} + \frac{s^2}{6}(G^2 - k^2) \right). \quad (2.6)$$

The velocity, or slowness model is first divided into cells of constant slowness gradient. The geometry for the whole model and an individual cell are shown in Figure 2.1. Through each cell, the left hand side of (2.4) is set equal to a position on the cell boundaries where the ray is most likely to exit. This boundary and the initial raypath value are determined by assuming a straight ray. The exact raypath s is then solved for using Newton's method, which converges very quickly. The new ray direction and traveltime are computed using (2.5) and (2.6), and these updated quantities are input to the next cell. Interfaces are interpolated with bicubic splines and the ray direction across them is changed according to Snell's law.

3 - Dynamic ray tracing

I now derive similar polynomial expressions for the dynamic ray tracing quantities which can be accumulated through successive cells, so as to compute the geometrical spreading, amplitude and wavefront curvature for each ray at any point along the travel path.

The basic 3-D dynamic ray tracing system, which results from integration of the eikonal equation, can be expressed in terms of the ray declination ϑ and azimuth φ (Cerveny et al., 1974, p. 11):

$$\begin{aligned}\frac{dx_1}{ds} &= \sin \vartheta \cos \varphi, \\ \frac{dx_2}{ds} &= \sin \vartheta \sin \varphi, \\ \frac{dx_3}{ds} &= \cos \vartheta, \\ \frac{d\vartheta}{ds} &= (G_x \cos \varphi + G_y \sin \varphi) \cos \vartheta - G_z \sin \vartheta, \\ \frac{d\varphi}{ds} &= \frac{G_y \cos \varphi - G_x \sin \varphi}{\sin \vartheta}.\end{aligned}\tag{2.7}$$

Differentiating with respect to the ray declination ϑ_0 and azimuth φ_0 at the source, one obtains a system of ten equations. Here I consider the case where the model properties are invariant in the out-of plane direction (along the y axis):

$$\varphi = \varphi_0 = \frac{\partial \varphi}{\partial \vartheta_0} = \frac{\partial \vartheta}{\partial \varphi_0} = 0,$$

which corresponds to the 2.5-D approximation. This implies that although the velocity model is two-dimensional, I am still modeling a point source with a 3-D wavefront.

This gives the system of five equations

$$\frac{dg_1}{ds} = - (G_x g_1 + G_z g_3) \sin \vartheta + \vartheta' \cos \vartheta,$$

$$\frac{dg_3}{ds} = - (G_x g_1 + G_z g_3) \cos \vartheta - \vartheta' \sin \vartheta,$$

$$\frac{d\vartheta'}{ds} = - (G_z \cos \vartheta + G_x \sin \vartheta) \vartheta' \quad (2.8a - e)$$

$$\frac{dh_2}{ds} = \varphi' \sin \vartheta,$$

$$\frac{d\varphi'}{ds} = - \frac{G_x \varphi'}{\sin \vartheta},$$

where

$$g_i = \frac{\partial x_i}{\partial \vartheta_0}, \quad h_i = \frac{\partial x_i}{\partial \varphi_0}, \quad \vartheta' = \frac{\partial \vartheta}{\partial \vartheta_0}, \quad \varphi' = \frac{\partial \varphi}{\partial \varphi_0}.$$

For the *out-of-plane* term, first integrate (2.8e), substitute the algebraic expression (2.5) for the sine and expand to the second order in G , giving

$$\varphi'(s) = \varphi'(0) \left[1 - \mu_x s + \frac{\mu_x s^2}{2} (2\mu_x - k) \right]. \quad (2.9)$$

This gives the variation in ray azimuth at a travel path s when the azimuth at the source varies. It is equal to 1 in homogeneous media.

Inserting (2.9) into (2.8d) and integrating yields

$$h_2(s) = h_2(0) + \varphi'(0) n_{0x} s [1 - \frac{sk}{2} + \frac{s^2}{6}(3k^2 - G^2 - \mu_x k)]. \quad (2.10)$$

The quantity h_2 corresponds to the derivative of the y position with respect to the ray azimuth at the source. Although the rays are confined to the x-z plane, this term is non-zero and allows for spreading in the y-direction. In homogeneous media, this term is simply the distance between the source and the point at the surface directly above the ray.

For the *in-plane* term, solve (2.8c) in the same way as (2.8e), giving

$$\vartheta'(s) = \vartheta'(0) [1 - sk + \frac{s^2}{2}(2k^2 - G^2)]. \quad (2.11)$$

This is the variation of the ray declination (angle from the vertical) when the declination at the source varies. It is equal to 1 in homogeneous media. In heterogeneous media, the first-order term shows that this variation is directly related to the relative change in slowness accumulated along the ray.

Solving the inhomogeneous system of equations (2.8a-b) by substitution and differentiation, I obtain an inhomogeneous, second-order linear differential equation in g_1 , for which I assume solutions of the form

$$g_1(s) = g_1(0) + \int_0^s \lambda_1(s') \vartheta'(s') ds'. \quad (2.12)$$

After some manipulations,

$$\left[\frac{\lambda_1}{\sin \vartheta} \right]_0^s = \left[\frac{\cos \vartheta}{\sin \vartheta} \right]_0^s + \int_0^s (G_z \sin \vartheta - G_x \cos \vartheta) ds'. \quad (2.13)$$

Using the explicit formula (2.5), first find a polynomial solution to (2.13), then multiply by (2.11) and integrate (2.12), obtaining

$$g_1(s) = g_1(0) + \vartheta'(0) n_{0z} s \left[1 + \frac{s}{2} (2\mu_z - 3k) + \frac{s^2}{6} (12k^2 - 4G^2 - 7k\mu_z) \right], \quad (2.14)$$

$$g_3(s) = g_3(0) - \vartheta'(0) n_{0x} s \left[1 + \frac{s}{2} (2\mu_x - 3k) + \frac{s^2}{6} (12k^2 - 4G^2 - 7k\mu_x) \right]. \quad (2.15)$$

The quantities g_1 and g_3 are analogous to h_2 : they give the derivatives of the x and z positions with respect to the declination at the source. In homogeneous media, they are just the projections of the ray path length upon the z and x axis, respectively.

4 - Ray tracing algorithm

I have incorporated these formulae into an interactive ray tracing program (Appendix A). The model input has been made as simple as possible. An arbitrary number of continuous or discontinuous interfaces are interpolated with bicubic splines. They can start or end anywhere in the model. These define regions of constant slowness gradient both in x and in z. Diffraction points, pinchouts and intersection of interfaces can be eliminated by subdividing the appropriate cells into smaller cells of constant velocity in a semiautomatic fashion. This model formulation yields fast computations and retains the advantages of the cell method while accomodating reasonably complex models.

Through each cell, the ray is first traced using the kinematic algorithm outlined above. Once the travel path s is known, the quantities (2.9), (2.10), (2.11), (2.14) and (2.15) can be updated for that cell. The *Jacobian* J of a ray at point P , which gives the ratio of the ray tube elementary area to the solid angle at the source, is given by:

$$J = \sqrt{(g_1^2 + g_3^2)} h_2 \quad (2.16)$$

for a point source. In homogeneous media, J becomes the square of the travel path length, the familiar geometrical spreading term. For a line source, h_2 is equal to 1.

To compute the total amplitude U of a ray arriving at point P_n from point P_0 , I use the formula given by Cerveny and Ravindra (1971):

$$\frac{U(P_n)}{U(P_0)} = \frac{VQ}{L}, \quad (2.17)$$

where V , Q and L are defined as

$$\begin{aligned} V &= \left(\frac{v_0 \rho_0}{v_n \rho_n} \right)^{\frac{1}{2}} \left(\prod_1^m \frac{v'_i \rho'_i}{v_i \rho_i} \right)^{\frac{1}{2}}, \\ Q &= \prod_1^m R_i, \\ L &= \left[\frac{J(P_n)}{J(P_0)} \prod_1^m \frac{\cos \vartheta_i}{\cos \vartheta'_i} \right]^{\frac{1}{2}}. \end{aligned} \quad (2.18)$$

for a ray that has crossed m interfaces. The velocity is v , ρ the density, ϕ the angle between the ray and the normal to an interface, and R_i the complex Zoeppritz's elastic reflection/transmission coefficient for interface i . The unprimed quantities are computed just before an interface, the primed ones just after. Since all the quantities appearing in (2.18) are readily accessible, the *total amplitude* can be computed on individual rays.

In the expression (2.18) above, L corresponds to the geometrical spreading of the ray. Since it is directly related to the width of the ray tube, it can be used to obtain a regular interval between ray arrivals at the surface, or to search efficiently for a given arrival, making two-point ray tracing using the shooting method very fast.

Furthermore, *paraxial corrections* may be used to improve the accuracy of the traveltime and amplitude at a receiver. To perform these corrections, one needs the wavefront curvature at the end of the ray. The curvature, C , is related to the Jacobian J as given above by the relation

$$C = \frac{1}{J} \frac{dJ}{ds}, \quad (2.19)$$

which gives

$$C = \frac{g_1 g'_1 + g_3 g'_3}{g_1^2 + g_3^2}. \quad (2.20)$$

The wavefront curvature C can thus be calculated at little additional cost. If one then knows the position, traveltime and amplitude of a ray at a position $\mathbf{r}$, one can compute

those quantities at a point $\mathbf{r}'$ nearby using paraxial correction formulae such as given by Cerveny et al. (1984).

Finally, it is important to note that the dynamic quantities calculated above are discontinuous across interfaces: the wavefront and interface curvatures as well as the local velocity variations affect the rate of change in ray tube area. To transform these quantities, I use the results of Cerveny et al. (1974, p. 18). The correspondence between their variables and mine is

$$\begin{aligned}
 P_i &= \frac{n_i}{V}, \\
 Y_{11} &= g_1, \quad Y_{13} = g_3, \quad Y_{22} = h_2, \\
 Z_{11} &= \frac{1}{V}(n_z \vartheta' + n_x g_k G_k), \quad Z_{13} = -\frac{1}{V}(n_x \vartheta' - n_z g_k G_k), \quad Z_{22} = \frac{n_x}{V} \varphi',
 \end{aligned} \tag{2.21}$$

all other quantities being zero.

5 - Two benchmark models

The first velocity model (Figure 2.2a) was used as a test case by Cerveny (1985). It consists of a high-velocity lens embeded in low-velocity material. The velocity is constant within each layer. There are 51 receivers spaced every 200 meters along the top surface. Figure 2.2b and 2.2c show synthetic seismograms obtained with the Cerveny and Psencik program and the present method, respectively: they are virtually identical. Figure 2.3 shows (a) the rays to the three major interfaces obtained with the present method, (b) the

difference between the traveltimes computed by SEIS83 and the present algorithm at each receiver for each interface, and (c) the difference between the magnitude of the amplitudes computed by the two methods, at each receiver and for each interface.

The traveltime agreement is excellent. Errors as large as 0.2 % are due to slightly different source parameters (zero offset) and a poor interpolation scheme which I use at the end of the traveltime curve. These errors are negligible. The amplitude agreement is also excellent. Isolated peaks occur for the same reasons as above. The overall amplitude difference is higher than for traveltimes, as amplitudes are more sensitive to the definition of the velocity model in general.

The second model (Figure 2.4a) consists of high velocity gradients both in the x direction (up to 0.01 km/s/km) and the z direction (up to 0.17 km/s/km). There are 51 receivers spaced every 0.5 km along the top surface. The two synthetic seismograms shown in 2.4b and 2.4c again indicate excellent agreement between the SEIS83 algorithm and the present method. Figure 2.5 shows traveltime and amplitude comparisons between SEIS83 and this method. Again, the traveltime agreement is excellent, especially at the largest offsets. The amplitude agreement is also very good, since this model contains velocity gradients higher than allowed by the second-order approximation. The high peaks occur near the critical angles, and are due to the linear interpolation of the amplitude between rays straddling a receiver. When the amplitude curve varies rapidly (i.e. at critical angles), a more accurate interpolation scheme, such as the one described above, must be implemented. For both models, the present algorithm was significantly faster than SEIS83.

6 - A complex model

Figure 2.6a shows a complex velocity model of crustal structure in the Central California margin (Levander and Putzig, 1991). The source is located at (68 km, 1.2 km). There are 101 receivers spaced 0.75 km along the Earth surface. The range of velocity variation within each layer is indicated in km/s.

I chose this model because it presents several elements which cause difficulties in conventional ray-tracing algorithms: 1) there are thin layers and pinchouts near the surface; 2) some interfaces have very little velocity contrast, while others have a strong velocity contrast; 3) in the upper part of the model, the velocities vary laterally as the material changes from ocean water to sediment to upper crust; 4) there are large vertical boundaries; and 5) in the deeper part, there are strong, localized velocity gradients.

The synthetic record section shown in 2.6b was obtained by convolving the complex amplitudes obtained with this algorithm with an analytical Ricker wavelet. It is plotted with a reducing velocity of 6 km/s and the whole section is normalized to peak amplitude.

The overall result demonstrates that the algorithm dealt efficiently with the complexity of the model. The pinchouts did not generate very large amplitude anomalies. Rays were traced through the vertical boundaries and the result is a slight bulge upward of the arrivals (longer traveltimes) for offsets of 20 to 25 km (model coordinates 90 to 95 km). The continuity of the arrival curves is remarkable despite the heterogeneity of the structure (isolated discontinuities in the upper branches are again due to inadequate interpolation between rays straddling a receiver). Note that the reflection off the top of the lower crustal

thin layer is not seen because of its extremely low amplitude. Total computation time for this shot record was about 30 seconds on an ELXSI 6400.

7 - Conclusion

I have expanded a particularly efficient kinematic ray tracing procedure described by Langan et al. (1985) for the computation of the quantities needed in the dynamic ray tracing system for 2.5-D heterogeneous media. In addition to raypaths and traveltimes computations, the present algorithm permits fast and accurate calculations of geometrical spreading, total amplitude and wavefront curvature in complex velocity structures.

Two-point ray tracing by the shooting method becomes very fast, and paraxial corrections improve the accuracy of the traveltime and amplitude at a receiver. Numerical comparisons with SEIS83 (Cerveny and Psencik, 1984) have shown the new scheme to be very accurate while being significantly faster. Although this method is slower than finite-difference kinematic calculations when not vectorized, its flexibility allows it to be extended to ray tracing in 3-D heterogeneous media, to be used in a Gaussian beam algorithm and, most importantly, to compute dynamic Green functions in heterogeneous media, as I shall demonstrate in the next chapters.

(a)

(b)

Fig. 2.1 - Geometry of the ray tracing algorithm

Fig. 2.2 - Benchmark 1: model IP1

Fig. 2.3 - Benchmark 1: traveltimes and amplitudes comparisons

Fig. 2.4 - Benchmark 2: the Crust model

Fig. 2.5 - Benchmark 2: traveltimes and amplitudes comparisons

Fig. 2.6 - Complex ray tracing model and synthetic seismogram

Chapter 3

Pre-Stack Kirchhoff Migration in Heterogeneous Media

Abstract

The integral method has long been used for wavefield extrapolation and migration of seismic data. It is successful for models with constant or slowly depth-varying velocities, or as a pure kinematic imaging technique. However, for the application of pre-stack depth migration to complex structures and wide-angle data, it is desirable to obtain a rigorous implementation valid for strongly heterogeneous media, arbitrary recording geometries and large velocity models.

I present here a formal derivation of the Kirchhoff integral in heterogeneous media which uses exact weighting factors. I specialize it to point-source migration of 2-D data, the so-called 2.5-D approximation, using the asymptotic method of stationary phase. I then describe in detail the algorithm which implements this formula, using the dynamic ray tracing scheme described in the previous chapter to compute the Green functions. For a benchmark test, I migrate a complex synthetic data set and obtain a good quality image despite very low shot coverage.

1 - Introduction

The Kirchhoff-Helmholtz integral method of constructing or reconstructing a wavefield from surface data has long been known in wave optics (Helmholtz 1860, Kirchhoff 1883, Born & Wolf 1964). It is the wave-equation formulation of the summation methods used for migration in the 1960's. The integral method was revived in 1970 when applied to forward modeling in constant velocity media with zero source-to-receiver offset (Trorey 1970, Hilterman 1970). The method was later extended to non-zero separation of source and receiver, and various Green functions for the problem examined (Hilterman 1975, Trorey 1977, Kuhn & Alhilali 1977, Berryhill 1977). More recently, earthquake seismologists have used it to model waves in a laterally inhomogeneous Earth using various techniques (Haddon & Buchen 1981, Frazer & Sinton 1984, Frazer & Sen 1985).

The integral method has been used for migration in the mid-1970's (Gardner et al. 1974, French 1975). Schneider (1978) and Berryhill (1979) gave formal derivations of the equation for constant velocity media and zero offset sections. They demonstrated the success of the exact, wave-equation formulation of the summation principle when applied to constant velocity (or slowly depth-varying velocity) in 2.5 and 3-D sections. Wiggins (1984) applied the idea to non-zero-offset sections using his KRK integral formulation, strictly valid only for constant-velocity media. Carter & Frazer (1984) derived a more general, albeit more complicated, formula for depth-varying velocity and zero offset, correcting for lateral velocity variations using Fermat's principle.

Finally, another approach to migration which does not use the wave-equation formulation is the so-called 'kinematic imaging technique' or ray-equation migration

(Milkereit 1987, McMechan & Fuis 1987, Hu & McMechan 1986, Wen & McMechan 1984). In this approach, amplitudes are incorporated only in the form of weighting factors obtained through simple geometrical optics considerations. These methods emphasize the imaging condition (i.e. traveltimes computations) but do not incorporate exact amplitude terms.

For pre-stack depth migration of complex structures or wide-angle data, it is desirable to obtain an exact integral method formulation, valid for heterogeneous media and arbitrary recording geometries. This implies that horizontal and vertical velocity variations within each layer may be large enough to justify extensive dynamic ray tracing to compute the Green function. Furthermore, since most pre-stack depth migrations are performed in 2-D only due to limited computer resources, we need a 2.5 scheme to approximate the 3-D experiment (point source).

Additional requirements for the algorithm are that it account for elasticity and anisotropy, since three-component seismic data are becoming increasingly available. Although some work has been done in this direction (Kuo & Dai 1984, Dai & Kuo 1986), it is far more complex. It seems appropriate to first examine the acoustic isotropic case to evaluate the potential of the method on familiar grounds. However, it is important to note that although I only use P-waves for imaging, their amplitudes as they appear in the integral kernel incorporate the complex reflection/transmission coefficients of an elastic medium. Furthermore, the method can be used for migrating shear wave data by replacing the velocities and P-wave reflection/transmission coefficients with the appropriate S-wave quantities.

2 - The 3-D, one-way Kirchhoff integral

The Kirchhoff integral can be derived from the wave equation using Green's theorem (Schneider, 1978). It can be written in frequency form as:

$$U(\mathbf{r}, \omega) = \frac{1}{4\pi} \int_{S_0} \left[G(\mathbf{r}_0, \mathbf{r}, \omega) \frac{\partial U}{\partial n}(\mathbf{r}_0, \omega) - U(\mathbf{r}_0, \omega) \frac{\partial G}{\partial n}(\mathbf{r}, \mathbf{r}_0, \omega) \right] dS_0, \quad (3.1)$$

where U is the wavefield as a function of position and frequency, G is the appropriate Green function, $\mathbf{r}$ is a point in the medium, and $\mathbf{r}_0$ is a point on the surface S_0 of observations, with unit normal $\mathbf{n}$. This equation is valid for zero source-receiver separation.

It can be shown that for a planar surface, the first term of the integral is equal to the negative of the second term. This is still true to a good approximation if the surface is not exactly flat, provided the wavelength of its topography is large compared to the wavelength of the incident field. In the time domain, this is known as Kirchhoff's approximation (see Wiggins 1984, Hill & Wuenschel 1985).

Equation (3.1) then reduces to

$$U(\mathbf{r}, \omega) = - \frac{1}{2\pi} \int_{S_0} U(\mathbf{r}_0, \omega) \frac{\partial G}{\partial n}(\mathbf{r}, \mathbf{r}_0, \omega) dS_0 \quad (3.2)$$

which eliminates the space derivative of the observed field, not usually a measured quantity. I use a Green function which is the free-space geometrical optics solution to the wavefront from a point source. In the frequency domain, this function takes the form:

$$G(\mathbf{r}, \mathbf{r}_0, \omega) = A(\mathbf{r}, \mathbf{r}_0) e^{i\omega\tau}, \quad (3.3)$$

where A is the amplitude term and τ the travel time. Differentiation of (3.3) with respect to n yields:

$$\frac{\partial G}{\partial n} = \frac{\partial A}{\partial n} e^{i\omega\tau} + i\omega A \frac{\partial \tau}{\partial n} e^{i\omega\tau}. \quad (3.4)$$

I shall ignore the first term, under the assumption that at far offsets, the amplitude variation is much slower than the travelttime variation, equivalent to saying that we are many wavelengths away from the source. This far-offset approximation reduces computational expenses. Using the eikonal equation, the derivative in the second term of (3.4) can be written as

$$\frac{\partial \tau}{\partial n} = \nabla \tau \cdot \mathbf{n} = \frac{1}{v} \mathbf{t} \cdot \mathbf{n} = \frac{\cos \alpha}{v}, \quad (3.5)$$

where α is the angle between the normals of the wavefront and the observation surface, and v the velocity at the observation surface. Inserting (3.5) into (3.4) and substituting into (3.2) yields the Rayleigh-Sommerfeld diffraction formula in heterogeneous media:

$$U(\mathbf{r}, \omega) = -\frac{i\omega}{2\pi} \int_{S_0} \frac{A(\mathbf{r}, \mathbf{r}_0) \cos \alpha_0}{v_0} e^{i\omega\tau} U(\mathbf{r}_0, \omega) dS_0. \quad (3.6)$$

3 - The 2.5-D, one-way Kirchhoff integral

In the case of a line survey or a 2-D migration of a portion of a 3-D data set, one is faced with the problem that the Kirchhoff integral is defined over a 2-D surface, whereas

the data are non-zero on a line only. If the datuming surface has an a priori unknown geometry, a new derivation of the integral with a line source (2-D Green's function), or integration over y must be done (Wiggins 1984, Carter & Frazer 1984).

Here, I use the asymptotic method method of stationary phase to perform the integration over the y -axis. This method has been used for inversion by Sullivan & Cohen (1987). Given an integral of the form

$$I(\alpha) = \int f(x) \exp[i\alpha\varphi(x)] dx, \quad (3.7)$$

where x is an element of an Euclidian space of dimension m , and given the two conditions

$$\begin{aligned} \alpha > > 1, \\ \exists x_0, \nabla_x \varphi|_{x_0} = 0, \end{aligned} \quad (3.8)$$

then an asymptotic expression for (3.7) is

$$I(\alpha) \equiv \left(\frac{2\pi}{\alpha}\right)^{\frac{m}{2}} f(x_0) \frac{\exp\left[i\alpha\varphi(x_0) + i\operatorname{sgn}(A)\frac{\pi}{4}\right]}{\sqrt{|\det(A)|}}, \quad (3.9)$$

where

$$A = (A_{ij}) = \left(\frac{\partial^2 \varphi}{\partial x_i \partial x_j}\right)_{x_0} \quad (3.10)$$

is an $m \times m$ matrix and $\text{sgn}(A)$ is the number of positive eigenvalues of A minus the number of negative eigenvalues of A .

To apply these expressions to y -integration of the Kirchhoff integral, I set $m=1$ with the following correspondances:

$$\begin{aligned}
 x &\rightarrow y, \\
 \varphi(x) &\rightarrow \tau(y), \\
 \alpha &\rightarrow \omega, \\
 f(x) &\rightarrow -\frac{i\omega}{2\pi} \int dx_0 \frac{A \cos \alpha_0}{v_0} U(x_0, y_0, \omega).
 \end{aligned} \tag{3.11}$$

The phase of the kernel, i.e. the traveltime, is stationary at $y=0$ for rays which propagate within the (x,z) plane, according to Fermat's principle. Furthermore, I have assumed infinite frequency when selecting a geometrical Green function. The two conditions for the asymptotic expansion are therefore satisfied.

The paraxial approximation of the travel time along the y -axis is (see Cerveny et al., 1984):

$$\tau(y) = \tau(0) + \frac{1}{2v_0} (C_y) y^2, \tag{3.12}$$

where C_y is the out-of-plane, or secondary curvature of the wavefront. The matrix A has become a scalar and we have, using (3.10):

$$\det(A) = \frac{\partial^2 \tau}{\partial y^2} = \frac{C_y}{v_0},$$

$$\text{sgn}(A) = 1.$$
(3.13)

I now substitute these into the general expression (3.9), obtaining:

$$U(\mathbf{r}, \omega) \equiv \left(\frac{2\pi}{\omega} \right)^{\frac{1}{2}} \frac{-i\omega}{2\pi} \int dx_0 \frac{A(\mathbf{r}, \mathbf{r}_0) \cos \alpha_0}{v_0} \exp \left(i\omega\tau + i\frac{\pi}{4} \right) \left(\frac{v_0}{C_y} \right)^{\frac{1}{2}} U(\mathbf{r}_0, \omega),$$
(3.14)

where $\mathbf{r}$ and $\mathbf{r}_0$ are now confined to the x-z plane. Rearranging (3.14) yields the 2.5-D Kirchhoff integral in heterogeneous media:

$$U(\mathbf{r}, \omega) = \frac{-ie^{i\pi/4}}{\sqrt{2\pi}} \frac{\omega}{\sqrt{\omega}} \int \frac{A(\mathbf{r}, \mathbf{r}_0) \cos \alpha_0}{\sqrt{v_0 C_y}} e^{i\omega\tau} U(\mathbf{r}_0, \omega) dx_0.$$
(3.15)

4 - The 2.5-D two-way Kirchhoff integral

To apply equation (3.15) to non-zero separation of sources and receivers, I follow the principle described by Wiggins (1984). The geometry of the problem is shown in figure (3.1). I first apply (3.15) once to backpropagate the signal recorded at the receivers $\mathbf{r}_0$ (our original data) to the sources $\mathbf{r}'$ on the new datum. I then apply (3.15) again to backpropagate the signal from the sources $\mathbf{r}_0'$ to the receivers $\mathbf{r}$. Although we can not back propagate seismic waves from sources directly, this last step can be justified by conceptually interchanging sources and receivers, using the reciprocity principle. This gives, in the frequency domain:

$$U(\mathbf{r}, \mathbf{r}', \omega) = -\frac{i\omega}{2\pi} \int dx \int dx' \frac{AA' \cos \alpha_0 \cos \alpha'_0}{\sqrt{v_0 v'_0} C_y C'_y} e^{i\omega(\tau + \tau')} U(\mathbf{r}_0, \mathbf{r}'_0, \omega), \quad (3.16)$$

where subscripts 0 correspond to the observation surface, unprimed quantities correspond to receivers, primed ones to sources. A is the geometrical spreading from $\mathbf{r}_0$ to $\mathbf{r}$, A' from $\mathbf{r}'_0$ to $\mathbf{r}'$.

I finally transform back to the time domain:

$$U(\mathbf{r}, \mathbf{r}', t) = -\frac{1}{2\pi} \int dx \int dx' \frac{AA' \cos \alpha_0 \cos \alpha'_0}{\sqrt{v_0 v'_0} C_y C'_y} \frac{\partial U}{\partial t}(\mathbf{r}_0, \mathbf{r}'_0, t + \tau + \tau'), \quad (3.17)$$

which gives the final imaging formula by taking coincident new sources and receivers and zero time:

$$U(\mathbf{r}) = -\frac{1}{2\pi} \int dx \int dx' \frac{AA' \cos \alpha_0 \cos \alpha'_0}{\sqrt{v_0 v'_0} C_y C'_y} \frac{\partial U}{\partial t}(\mathbf{r}_0, \mathbf{r}'_0, \tau + \tau'). \quad (3.18)$$

For constant velocity, one has the following correspondences:

$$\begin{aligned} \frac{1}{\sqrt{v_0 v'_0}} &\Rightarrow \frac{1}{v}, \\ \cos \alpha_0 \cos \alpha'_0 &\Rightarrow \frac{zz'}{RR'}, \\ \frac{AA'}{\sqrt{C_y C'_y}} &\Rightarrow \frac{1}{\sqrt{RR'}}. \end{aligned} \quad (3.19)$$

Substitution of these terms into expression (3.18) yields

$$U(\mathbf{r}) = -\frac{1}{2\pi v} \int dx \int dx' \frac{zz'}{(RR')^{3/2}} \frac{\partial U}{\partial t} (\mathbf{r}_o \mathbf{r}'_o \tau + \tau'), \quad (3.20)$$

so that in constant velocity media the formulae degenerate to the usual Kirchhoff integral. Figure 3.2 shows the shape of the operator for constant velocity (5 km/s) and depth-varying velocity (4 to 7.2 km/s).

5 - Algorithm

The algorithm is divided into four parts: data set preparation, model definition, ray tracing and imaging (Appendix B). The first two parts condition the data set and set up an initial velocity model for the migration. The ray tracing routine then computes the Green function everywhere in an imaging area specified by the user. The result is written onto disk or tape and passed to the imaging routine.

Pre-stack migration in general requires very little data pre-processing compared to post-stack migration. A typical pre-processing sequence includes debiasing, trace editing and frequency filtering, as well as deconvolution in the case of strong water multiples. Otherwise, pre-stack migration implicitly performs correct static corrections. It also tends to eliminate weak multiples because they are not included in the imaging condition (we do not ray trace multiples). No amplitude balancing should be applied, since one of the advantages of this algorithm is to preserve true amplitude in the image, which allows some quantitative estimation of reflector strengths.

The only unconventional step in the pre-processing sequence is to take the first derivative of the data set, as indicated by the kernel of equation (3.18). Although seismic recordings are not actually displacements, this step has the effect of increasing the frequency content of the wavelet, which results in a sharper image and less depth distortion (see below).

Finally, the data set is read in once to extract the header information concerning the positions of the shots and receivers. This information is used to produce a file listing all ground locations having seen shots or receivers. Ray tracing is done from these locations, which eliminates a lot of redundancy in the case of roll-along and marine surveys.

The second step, model definition, consists in defining a macro velocity model which includes interfaces that have been resolved so far, setting up the ray tracing grid and mapping it to the imaging grid, and defining the image area. The velocity model can be arbitrarily complicated, but smooth enough for the ray tracing to give consistent results. Note that the image area can be limited to a small part of the whole model to reduce computation times.

The horizontal grid spacing corresponds to the group or half-group spacing, while the vertical grid spacing should be such that a typical wavelet is sampled at least 5 or 6 times. If the vertical sampling is Δz and the local velocity v , then the two-way traveltime between two gridpoints at normal incidence is $2\Delta z/v$. For a wavelet of duration Δt , the mapping from time to depth is therefore $N=v\Delta t/2\Delta z$, where N is the number of vertical depth points in which a whole time wavelet is mapped. This stretching effect, typical of pre-stack migration, increases with the velocity and the ratio of offset to target depth. We shall see some examples below of this stretching effect.

The third step, ray tracing, is done with the algorithm which I described in the previous chapter. As a check, I wrote a different version of the forward modeling program to visualize the ray sets used to compute the Green functions, in order to check for even coverage, shadow zones, or errors in the velocity model. This algorithm is faster than the forward modeling one because of the absence of reflection surfaces. Rays are traced from the array of ground locations at the surface corresponding to the shots and receivers, down through the model, which is more efficient than from every image point (Gray, 1986). Geometrical spreading of the rays is used to obtain an even illumination of the grid of imaging points, in the same way that it was used for interval ray tracing in forward modeling. Since rays arbitrarily cross cell borders, paraxial time corrections are applied to compute traveltimes exactly at the cell corners (the amplitudes are less sensitive to this error).

Since the Kirchhoff kernel (3.18) is symmetric with respect to shots and receivers, I actually store not the ray amplitude, but the full weighting factor at each image point: this value can be used indifferently for a shot or a receiver. This greatly reduces disk storage and improves the speed of the imaging loop.

Finally, the imaging routine computes the image at each point of the grid using the integral (3.18) and the dynamic ray tracing results. It can read traces in any order, although it uses the sorting order of the data to avoid redundancy of the primary header. Several passes of the data set are made, each time loading a portion of the Green function data from disk. This allows each trace read in to be either discarded or immediately found in the Green function table.

The output is a 3-D volume of migrated data whose axes are surface location, migrated source-receiver offset and depth. The fourth dimension, midpoint, does not appear because I stack all the contributions obtained at one point from migrating source-receiver pairs of a given offset. Although it would be interesting to look at it, storage restrictions force us to discard it and keep specular data only. One can then look at common offset images (COI) or common image gathers (CIG) on a graphics workstation. The former simulate migration of common offset data and show the variation in image character as the offset varies. The latter can be analyzed for horizontal alignment in order to optimize the velocity model and the stack for the final image. I describe this in the next chapter.

This procedure is fast and reliable even for very large models. For a given surface position, ray tracing through a 700*400 point model to compute and store the traveltimes and the amplitude factors takes one minute on the Stardent GS 2000 (10 scalar Mflops). For a large number of shots, this accounts for most of the execution time; the imaging loop itself (computing the image at each grid point for a given source-receiver pair) is a vectorized procedure. Execution times therefore depend mostly on the size of the data set and the amount of ray tracing to be done.

6 - Synthetic Example

I test the validity of the algorithm on the model shown in figure 3.3, which features two anticlines and one syncline, a 45 degree dipping layer and a velocity reversal in the central layer. The synthetic seismograms were generated using acoustic wave finite-difference forward modeling. There are 8 shots distributed every 800 meters along the

surface. Every shot is recorded by the same fixed 201-receiver spread at the surface ($\Delta g = 40$ m). The shot gather obtained near the center of the model is shown in figure 3.4: as expected, it exhibits complex arrival patterns, including diffractions from the tops of the anticlines, linear reverse-moveout events corresponding to the steeply dipping segment and a bowtie due to the syncline. Note that the direct wave has been muted.

The result of migrating the first layer, shown at the top of figure 3.5, is excellent. The image is free from migration artifacts despite the low shot coverage. The steeply dipping segment is imaged correctly, as is the anticline, even though its right side is less bright because of sparse illumination from the recording geometry. The imaging grid spacings are 40 m horizontally (the group spacing) and 8 m horizontally (5 samples per wavelet).

Having obtained an image for the first interface, I include it in the velocity model for migrating the second layer. This layer-by-layer approach allows progressive resolution of the image with depth. It is similar to the layer-stripping method (see Shih & Levander, 1991), except that actual datuming is unnecessary due to the ray tracing algorithm.

The result of the second migration, shown at the bottom of figure 3.5, is also very good. The image is plotted with reverse polarity for clarity. Both the syncline and the anticline have been imaged properly. Figure 3.6 shows the final image composited from the two layer migrations. The amplitude is boosted appropriately to compensate for transmission energy loss through the first interface; for field data this loss can be compensated for by applying AGC to the section.

The importance of the first derivative operator mentioned above is illustrated in figure 3.7. I obtained the top image by migrating the original data *without* the first derivative taken. It is much lower frequency than the correct image (figure 3.5), because of wavelet stretching effects. If I take the first derivative of this image, however (bottom of figure 3.7), the result is comparable to the correct image, indicating that the derivative operator commutes to some degree with the migration operator. The main difference is that taking the derivative after migration picks up the image area boundaries, which are artificially defined. However, this may be an important time-saving consideration for large data sets.

7 - Conclusion

I have presented a derivation of a Kirchhoff integral formula for pre-stack depth migration in heterogeneous media and specialized to point-source migration for 2-D velocity models (the 2.5-D approximation). The method allows rapid imaging of structures with steep dips and strong lateral velocity variations due to a fast dynamic ray tracing algorithm to calculate the Green functions. The migrated data are displayed as a 3-D volume from which velocity adjustments in the model and optimal stacking for the image will be derived. The algorithm is successful in imaging a complex synthetic model with sharp folds, steep dips and strong velocity variations. The method can be easily applied to 3-D data.

Fig. 3.1 - Migration extrapolation geometry.

Fig. 3.2 - Pre-stack Kirchhoff operator in constant and variable velocities.

Fig. 3.3 - WGC model used to test the migration algorithm.

Fig. 3.4 - Typical shot gather for the WGC model

Fig. 3.5 - Migration result for the first layer and second layers.

Fig. 3.6 - Composited migration result for the WGC model.

Fig. 3.7 - First derivatives of the WGC data before and after migration.

Chapter 4

MMO Analysis and Depth Focusing in Complex Structures

Abstract

Pre-stack depth migration is essential to obtaining detailed images of complex structures and crustal features. Now that computer speed makes it more routinely applicable, its main drawbacks are its sensitivity to the migration velocity model, as well as its lack of a focusing step such as NMO-correction plus stack before migration. The migrated pre-stack data, however, can be sorted into Common Image Gathers (CIG): a CIG contains the images obtained at one Earth location from migrating source-receivers pairs of various offsets. If the migration velocity model is correct, all events in a CIG are flat.

Based on the pre-stack Kirchhoff algorithm in heterogenous media and the fast dynamic ray tracing scheme described in the two previous chapters, I develop a method for analyzing CIG's for horizontal alignment. A formula relating the extent of non-alignment to velocity corrections along the raypaths allows iterative updating of the velocity model and depth focusing of the reflection events. This type of analysis, which I call Migration

MoveOut analysis (MMO), is the post-migration analog of NMO. I demonstrate the success of the technique using synthetic data from a model with strong velocity variations, and apply it to a marine data set from central California, producing a high-quality image of the edge of the Santa Maria basin.

1 - Introduction

Pre-stack depth migration, which holds the promise of imaging complex structures, has become more feasible with the advent of very fast computers. Since it is applied to the original data without any simplifying assumptions about the dips or the velocities, it allows us in principle to recover a more faithful image of the earth's interior. The difficulty, though, is that it is very sensitive to the velocity model and often produces sections lacking the crispness of post-stack migrated sections: conventional processing (NMO analysis) tends by definition to align the reflection signals and therefore improve apparent resolution. Here, I introduce a focusing method which uses the wealth of information produced by pre-stack migration in order to obtain an optimal stack.

The depth migrated data can be sorted into a three-dimensional set with axes corresponding to surface-coordinate, migrated source-receiver offset, and depth (figure 4.1). One can look at common offset images (COI), which simulate migration of common offset data, to relate the variation in reflector positioning and focusing to velocity corrections (Van Trier, 1990). Here, I choose to look at common image gathers (CIG), i.e. gathers at a final image location containing all the images obtained from migrating source-receiver pairs of different offsets, in order to make quantitative statements about the velocity errors. My approach is similar to the one suggested by Al-Yahya (1989), albeit

more general. The advantage of looking at CIG's is that it is easy to identify the event of interest based on horizontal alignment.

Regardless of the shooting geometry, if the migration velocity model were correct and the algorithm perfect, all the traces in a given CIG should be identical. Based on this principle, I propose an approach very similar to NMO analysis, which I call MMO analysis (Migration MoveOut analysis). I derive a method to analyze each CIG for migration moveout (i.e. non-horizontal alignment of the image), and a formula for relating the degree of non-alignment to the error in velocity. I can thus correct the velocity model and re-migrate to obtain better focusing of the image, regardless of the complexity of the structure. In the end each horizontally aligned CIG is stacked to form the final image.

MMO analysis is a focusing technique in the sense that an interface must be fairly well identified in order to analyze it, in the same way that an event must be fairly coherent on a CMP gather for an NMO analysis to work well. Only in the limit of many iterations does it become a true inversion algorithm. However, in migration the initial velocities are often quite well known (forward modeling, well logs, conventional processing) and it is only small adjustments in velocities that are needed to actually focus a blurred section.

After reviewing the migration algorithm, I discuss the various ways that migration moveout can be picked on individual CIG's. I derive the formulae relating MMO to velocity corrections and describe how they are applied. Several migrations of a synthetic data set for a model with strong velocity variations demonstrate the success of this approach. Finally, I apply the method to a marine data set from central California and obtain a high-quality image of the Hosgri fault zone and the edge of the Santa Maria basin.

2 - Migration Algorithm

The migration itself is described in detail in the preceding chapter. The Kirchhoff method allows explicit determination of raypaths and specular reflections, subsequently used for backprojecting migration moveouts into the velocity model. Since the algorithm operates layer by layer, from the top down, it allows resolution of the velocity in individual layers: as we improve the velocity model in the top layers, we increase our chances of better focusing in the next layers. This approach is similar to the layer-stripping method of Shih & Levander (1991) but does not require actual datuming of the data at each step.

The added features of the migration algorithm, compared to standard migration as described earlier, are twofold. First, during ray tracing I keep track not only of the traveltime and the Kirchhoff kernel at each grid point, but also of the ray angle, traveltime and raypath since the last interface. These quantities appear in the formulae that are derived below. Second, during imaging I actually keep track of which source-receiver pair gives the strongest contribution at each image point. This allows building a table of specular source-receiver pairs for each image point which can be used for relating migration moveouts to the proper seismic traces. This reinforces the intuitive aspect of the Kirchhoff approach.

3 - Picking a CIG

There are two ways to pick an event on a CIG. For synthetic data and high quality, normal incidence field data, I use an automatic picking algorithm. For lower quality field data or data which contain strong amplitude variations and phase shifts, e.g. from shallow-

layer reflections, I use a graphics workstation to display selected CIG's and digitize the events (appendix D). Cubic splines are fitted to the user-defined curves to smooth out small fluctuations. The first method is used for the synthetic data and the second method for the California data set presented below.

For automatic event picking, the reflector is digitized from a stacked image to provide each CIG with a reference depth z_0 . For each CIG, one then compute the maximum power along a set of curves defined as second-order polynomials with their origin at z_0 :

$$P = \sum_{i=1}^N \left(z_0 + ax_i + bx_i^2 \right)^2 \quad (4.1)$$

where N is the number of active traces in the CIG, and x_i the x-coordinate of trace i . I am not trying to search for hyperbolic or ellipsoidal moveout curves, since I do not assume constant velocity and do not know the history of the rays in the preceding layers. The second-order polynomial with two degrees of freedom provides us with a good searching range and is well adapted for estimating the typically small, generally linear migration moveouts.

This results in a two-dimensional plot of the power for each (a,b) pair which can be contoured in order to find the pair corresponding to the maximum power. If there are several peaks, as may occur if a migration artifact or noise burst is picked, the values closer to zero (horizontal alignment) are preferred. This approach is preferable to computing cross-correlation of the traces, which works poorly if traces are missing (as often occurs in migration: not all offsets contribute to a given image point), tends to include and blend migration artifacts, and becomes distorted across phase shifts.

Interactive picking of the CIG's is comparatively more robust but more labor intensive. Whereas automatic picking yields an event on all CIG's in a very short time, interactive picking can be performed on a limited number of CIG's only. If the velocity structure is fairly well known before migration, as is often the case, a number of CIG's will be aligned and need not be picked at all. One can then concentrate on those parts of the model where MMO is largest and quickly discard aligned CIG's by visual inspection.

4 - Solving for the velocity

Having picked the migrated event depths on several or all CIG's, I want to correct the velocity model so that each CIG event aligns horizontally at a new depth. I do not a priori know these new depths; in particular, they are not given by the zero-offset or near-offset traces because of the usual velocity-depth ambiguity in migration (Yilmaz & Chambers 1980).

I first need a relationship between a change of velocity dv and the change in imaging depth dz for a given trace. In order to satisfy the imaging condition, I must have

$$\text{time}(S,Q) + \text{time}(Q,R) = \text{time}(S,Q') + \text{time}(Q',R)$$

where S designates the shot, R the receiver, and my initial image point Q has moved vertically to Q' (figure 4.2). In terms of travel path and velocity, this translates into

$$\frac{L^s}{v_m} + \frac{L^r}{v_m} = \frac{L^s + dL^s}{v_n} + \frac{L^r + dL^r}{v_n} \quad (4.2)$$

where L is the original travel path length since the last interface, dL the change in path length, v_m the migration velocity, v_n the new velocity. Quantities with superscript s correspond to the source, r to the receiver. In terms of the original traveltimes, I have

$$t^s + t^r = (t^s + t^r) \frac{v_m}{v_n} + \frac{dL^s + dL^r}{v_n}, \quad (4.3)$$

which yields

$$v_n = v_m + \frac{dL^s + dL^r}{t^s + t^r}, \quad (4.4)$$

so that

$$dv = \frac{dL^s + dL^r}{t^s + t^r}. \quad (4.5)$$

I can now use paraxial approximations to evaluate the change in travel path, dL , as the position of the rays endpoints changes vertically by an amount dz .

The travel time change is (see Cerveny et al., 1984)

$$\delta t = \frac{\cos \alpha}{v} \delta z + \frac{\sin^2 \alpha}{2vC} \delta z^2 + O(\delta z^3), \quad (4.6)$$

where C is the radius of curvature of the wavefront and α the angle of the ray with the vertical at the imaging point. For *most focusing* applications, the second term in (4.6) will be very small and I can neglect it to save on storage. I now multiply by the velocity to obtain dL and insert into (4.5):

$$dv = \frac{\cos \alpha^s + \cos \alpha^r}{t^s + t^r} dz. \quad (4.7)$$

When dealing with one CIG then, I must find a depth adjustment dz_i for each trace so that

$$z_i + dz_i = z_c \quad (4.8 a)$$

$$\frac{\cos \alpha_i^s + \cos \alpha_i^r}{t_i^s + t_i^r} dz_i = dv_i \quad (4.8 b)$$

with the index i ranging over the number of traces in the CIG. The values z_c and dv_i are unknown. I have picked the z_i 's, and the cosines and traveltimes are stored during ray tracing.

The system (4.8) represents an underdetermined problem: if there are 100 active traces in a CIG, it consists of 200 equations for 201 unknowns. In transmission tomography, knowing the *observed* traveltimes allows one to distribute velocity corrections along the ray paths; here we do not have an 'observed' focusing depth. We would therefore need to run numerous tomographic analyses, each time with a different set of initial conditions (i.e. depth correction for each CIG). Although this is feasible, it is a long procedure and there are no obvious ways to guide the search towards convergence.

For practicality and stability, I prefer instead to assume a constant velocity correction dv for each CIG. System (4.8) then becomes an overdetermined problem, with 200 equations and 102 unknowns for a CIG with 100 traces. I solve it by trying out a range of values of z_c , close to the original image; for each such value, I compute the

corresponding distribution of dv 's, its mean and covariance. I then select the mean value of the distribution with the least covariance as the constant velocity correction for that CIG. This imposed velocity condition guarantees the stability of the algorithm.

5 - Correcting the Velocity Model

Each velocity correction for a CIG is distributed in the current layer over the ray paths contributing to the image in this CIG, using the ray information and the specular table described above. If a fair number of CIG's have been picked as in automatic picking, I average the velocity corrections in each cell, each correction being weighted by the number of active traces in the corresponding CIG.

At this stage, I obtain a corrected velocity (or slowness) model. To use it directly for the next migration, I only have to smooth out small velocity fluctuations so that the next ray tracing step gives consistent traveltimes and amplitudes. These fluctuations are due to the inaccuracy in event picking, irregular number of traces in each CIG, non-perfect recording geometry and discretization of the model.

On the other hand, one can tie these velocity corrections to other velocity information from well log data or geologic interpretation. In this case the algorithm becomes an interactive process allowing the geophysicist and geologist to guide the migration. Since pre-stack migration is sensitive to the details of the velocity structure over 4 to 10 wavelengths only (Versteeg, 1990), one can apply large scale gradients or constant velocity corrections to selected parts of each layer.

Once I have converged to a model which gives a satisfactory image of a given layer, I proceed to the next layer. The velocity in the new layer, if not known approximately from other sources, can be arrived at by examining the amplitude and phase behaviour of the previous CIG's, or by transmission/refraction tomography. I continue the whole process layer by layer to the bottom of the imaging area.

6 - Synthetic Test

The synthetic test is the crust model described in chapter 2 and shown again in figure 4.3a. It contains large velocity gradients both vertically and horizontally. There are 11 sources at the surface, spaced every 2.5 kms from 0 to 25 kms. Each shot is recorded by the same 101 receivers spread at the surface every 0.25 kms, from 0 to 25 km. Figure 4.3b shows the ray sets for the middle shot gather ($x=12.5$ km) and figure 4.3c the corresponding synthetic seismogram with the direct wave muted.

I first migrate the top layer using the correct velocity model. In the image shown at the top of figure 4.4, the reflector is well positioned as expected, but is not well focused. This can be understood by looking at the CIG from the center of the model, shown at the bottom of figure 4.4: the event is well aligned horizontally, but the phase shift early in the event produces destructive interference when the gather is stacked. I obtain a better image by stacking only the first 25 traces on each CIG (figure 4.5), by muting the individual CIG's in a way analogous to choosing NMO mutes. However, I shall show below that the focusing analysis can do this automatically.

My second trial is to migrate with an initial constant velocity of 3.0 km/s in the first layer. The result is shown in figure 4.6 with the central CIG. Because the velocity is too low, the interface is not well focused and is positioned above its true position. On the CIG, I see the event curving upward as I go to larger offsets, which indicates that the migration velocity is too low (Al-Yahya, 1989). NMO analysis on the CIG's yields an average velocity correction of 0.08 km/s. I re-migrate with a new velocity of 3.08 km/s and the final image, shown in figure 4.7, is excellent.

The algorithm gives a velocity correction optimal for stacking all the traces in the gathers, including post-critical reflections. It therefore produces the best image by automatically adjusting the velocity to account for phase shifts and other types of wavelet distortion. Although this may prevent the algorithm from giving exact velocities, I feel this approach is justified because I avoid having to work explicitly with unknown wavelet variations, while obtaining velocities closer to lithologic velocities than those obtained from NMO analysis.

In my third trial, I start out with an initial velocity of 2.5 km/s in the top layer. The first image (figure 4.8a), is very poor. Automatic NMO analysis yields successive velocity corrections of 0.211 km/s, 0.242 km/s and 0.120 km/s. The final image (figure 4.8b), obtained after 4 migrations, is excellent. It was migrated with a corrected velocity of 3.073 km/s.

I repeat this procedure for the second layer. For the first layer, I use my best estimate of the velocity, 3.08 km/s. I first migrate with the correct velocity model. The image, shown in figure 4.9, is excellent: all the signals on the CIG's are aligned in phase, because at this depth (8 km) we are in the pre-critical reflection regime. I then migrate with

an initial velocity of 3.5 km/s. The image, shown in figure 4.10a, is very poor. After three iterations of automatic MMO analyses, I reach a final velocity value of 4.18 km/s. The image, shown in figure 4.10b, is excellent. Figure 4.10c shows the complete migrated image.

Depth picking is a crucial point in the analysis. At small offsets, if the velocity is nearly correct, the adjustment in depth becomes comparable to the wavelet width and the system of equations (4.8) becomes unstable for small values of dz . At large offsets this problem is less crucial, but if there are phase shifts and migration artifacts, picking the right event becomes difficult.

7 - Field Data Example: The Hosgri Fault Zone in the Santa Maria Basin

The marine seismic profile RU3 was collected in 1986 by Rice University in conjunction with the Houston Area Research Center and Pacific Gas and Electric's Offshore Deep Crustal Geophysical Survey. Airgun shots fired every 50 m were recorded by a 180 channel streamer with group spacing of 25 m. With a near trace offset of 255 m, the maximum recording offset was 4730 m. Figure 4.11 shows a typical shot gather, from the center of the line. I concentrate here on 12 km of the line closest to the coast. Water depth ranges from about 110 m near the coast to about 330 m offshore. Deformed sediments extend to a depth of about 2 km and overlay Franciscan basement rocks. The Hosgri fault zone near the coast is the major structural discontinuity, separating low-velocity sediments from strongly heterogeneous, high velocity Franciscan material.

It is clear from the post-stack migrated section shown in figure 4.12 (Meltzer, 1988) that in this area conventional processing does not produce satisfactory results. The sedimentary layers are very discontinuous and a detailed stratigraphic interpretation is difficult. Both the image of the fault zone and the resolution below two seconds are poor.

My first approach is to migrate the section without accounting for the fault, using the velocities obtained from CMP semblance analysis as my starting velocities. The resulting image (figure 4.13) is an improvement over the conventional result. A number of prominent sedimentary reflectors can be picked almost continuously across the image and the resolution below 2 km depth has also increased.

Having obtained a general velocity model, I now concentrate on the Hosgri fault zone itself and run several migrations with different velocity models with and without the fault. When I include the fault, the velocities to the southwest are those obtained from the general migration, and to the northeast I start with a constant velocity of 3.5 km/s, obtained from refraction analysis. In the final partial image (figure 4.14), the fault is clearly delineated as dipping about 75 degrees to the Northeast. Several tests done by moving the fault one way or the other do not change the dip. The incoherency of the reflections northeast of the fault is due to the heterogeneous nature of the Franciscan rocks.

It must be noted that large offsets create migration artifacts on the northeast side of the fault, as shown in figure 4.15. The top image in figure 4.15 was obtained by summing offsets of 250 to 650 m: the reflectors are very coherent and their terminations are sharp. The bottom image, however, obtained with offsets of 2000 to 2500 m, contains migration smiles. Although these artifacts are of much lower amplitude than the true reflectors, they may be misinterpreted for reflector continuations on an image with AGC applied. One

advantage of the CIG analysis technique is that I could identify these artifacts and mute them before stacking.

The composite image of both the general and the partial migrations is shown in figure 4.16. The initial and final velocity models are shown at the top and bottom of figure 4.17, respectively. In addition to giving a sharper image of the fault, it also yields a new interpretation of the deepest reflectors, which can be seen as gently undulating from the southwest to come up against the Hosgri fault itself. Figure 4.18, which shows a CIG before and after MMO analysis on the second layer below the water, demonstrates that small adjustments in the velocity can greatly increase the quality of the image: the velocity correction derived from MMO in this case was only 0.08 km/s on average. However, this correction was enough to align the main event horizontally on the CIG.

8 - Conclusion

Pre-stack migration produces a wealth of data from which one can extract velocity corrections, which in turn can be used to improve the focusing of the reflectors. Using my pre-stack Kirchhoff algorithm in heterogenous media with the fast dynamic ray tracing scheme, I have developed a method for analyzing common image gathers (CIG) for horizontal alignment. A formula relating the extent of migration moveout (MMO) to velocity corrections along the raypaths allows iterative updating of the velocity model and depth focusing of the interfaces. On synthetic data with strong velocity variations, this approach proves to be fast and very successful. On a marine data set from central California, the method yields a high-quality image of the Hosgri fault zone and the edge of the Santa Maria basin.

Fig. 4.1 - The 3-D migrated pre-stack data volume

Fig. 4.2 - Migration velocity-depth problem and MMO geometry

Fig. 4.3 - The Crust model

Fig. 4.4 - Top layer migration with the correct velocity model, all traces stacked.

Fig. 4.5 - Top layer migration with the correct velocity model, selective stacking.

Fig. 4.6 - Top layer migration with initial velocity of 3 km/s, first iteration.

Fig. 4.7 - Top layer migration with initial velocity of 3 km/s, second iteration.

Fig. 4.8 - Top layer migration with initial velocity of 2.5 km/s.

Fig. 4.9 - Second layer migration with the correct velocity model.

Fig. 4.10 - Second layer migration with initial velocity of 3.5 km/s.

Fig. 4.11 - Shot gather from line RU3

Fig. 4.12 - Post-stack time migration of the Hosgri fault zone

Fig. 4.13 - Pre-stack migration with MMO analysis, without the Hosgri fault

Fig. 4.14 - Pre-stack migration with MMO analysis of the fault itself

Fig. 4.15 - Partial image of the Hosgri fault

Fig. 4.16 - Final image obtained with this approach

Fig. 4.17 - Final velocity model after MMO analysis and focusing

CIG before MMO

CIG after MMO

Fig. 4.18 - Comparison of CIG's before and after MMO analysis

Chapter 5

Cell-Stripping Tomography and Wide-angle Imaging

Abstract

Imaging wide-angle data is a complex task because of low fold and sparse ray coverage. I have developed migration tools which can handle those difficulties. Of crucial importance is the starting velocity model, which can not be obtained by conventional seismic reflection velocity analysis. Traditional tomography methods like SIRT do not work very well because there is insufficient cell sampling to recover sharp velocity boundaries in the model and they require many iterations, which can be very expensive in the case of large models.

I develop a cell-stripping tomography algorithm which distributes velocity residuals only in cells where they are most likely to apply. The rays are sorted in order of increasing path length and shorter rays are processed first, constraining the near-surface velocities. For longer rays, traveltimes residuals are adjusted so as to eliminate the effects of velocity errors in previously resolved cells. The algorithm is based on the ray tracing scheme of chapter 2. On a synthetic wide-angle test the method is able to recover the correct velocity gradient, a vertical and a horizontal boundary. SIRT smears these boundaries.

I use this method on the Continuous Offset Profile (COP) data set from central California and obtain an accurate velocity model to a depth of 10 km. This model is used as a starting model for the pre-stack depth migration algorithm described in chapter 3. Using this method and the depth focusing technique of chapter 4, I obtain an image of the upper crust in the Santa Maria basin over a range of 30 km.

1 - Introduction

Tomography has been widely applied as a form of kinematic inversion of transmission and sometimes reflection data (Scales, 1987; Bishop et al., 1985; Fawcett & Clayton, 1984). Among the various methods, SIRT (Simultaneous Iterative Reconstruction Technique) is one of the most robust and accurate (McMechan, 1983). In SIRT the velocity correction to a cell is the average of many corrections obtained from many rays traversing that cell, weighted by some factor such as incremental or total path length. For cross-hole tomography, for example, averaging the corrections for many rays sampling a given cell usually yields a good updating of the velocity for this cell (McMechan et al., 1987). For wide-angle data, however, this is not the case because we usually have low ray coverage, or can not practically divide a crustal model into very small cells. Often we must be content to have at most one ray in a given cell. The cell-stripping tomography method is designed with this restriction in mind.

The underlying idea of cell-stripping tomography (CST) is that velocity corrections should be assigned only to the cells where they are most likely to apply. For instance, if a cell is crossed by a very short ray which has some time residual, the corresponding velocity correction is very likely to belong to that cell. If a long ray crosses the same cell, its

velocity residual could belong to this cell or any other cells along its raypath. If there is a zone with a velocity far from the correct velocity deep into the model, the long ray will yield a high velocity residual which will be spread over the whole raypath in standard tomography, including in the near-surface cells. It is therefore desirable to distribute velocity corrections according to the sampling characteristic of each ray.

This approach is similar to a focusing method such as the one I described in the previous chapter, in the sense that it is essentially a one-pass process. By using an accurate distribution of velocity residuals, one must be able to converge very quickly to an acceptable solution. The drawback of this method is that it does not produce a smooth updated model as in standard SIRT, since it eliminates averaging steps. This tradeoff is analogous to the one between post-stack and pre-stack migration. However, as in the migration case, it is preferable to obtain well-resolved images, even if they are sensitive to the model parameters, than to obtain smooth, low-resolution images.

2 - Cell-stripping tomography

The first step is to trace all the rays for the starting model and to compute the time residuals. I do this with the ray tracing algorithm described in chapter 2. It is very important to do accurate two-point ray tracing: paraxial corrections to the traveltimes are not enough because I also need exact raypaths, since a single ray is enough to contribute an important correction to a given cell. The ray tracing is therefore slower than for forward modeling or migration, although I do use geometrical spreading on the rays to converge to a receiver. On average, 3 to 4 iterations on the ray take-off angle are needed to obtain the ray arrival point within 1 % of the receiver position, relative to the receivers spacing.

When all the rays are traced, they are sorted in order of increasing number of cells crossed. The idea is then to start with the shortest rays, since their velocity corrections are best constrained to a limited number of cells. They also give corrections to the near-surface cells, through which all the rays pass, so that it is important to get accurate corrections for them. Furthermore, near-surface velocities are usually well known from other sources such as well logs and refraction arrivals, so that by including them in the starting velocity model, no corrections will be done and the domain of velocity corrections will be more restricted.

Once I have the near-surface velocities, I work my way down with increasingly longer rays. For each such ray, a check is made for cells which have already been corrected. Assume there are M cells which have already been corrected, each with velocity V_i and travel path dL_i , and N cells which have not been corrected, with velocity V_i^c and travel path dL_i^c (figure 5.1). The computed traveltimes is approximated by

$$t_c = \sum_M \frac{dL_i}{V_i} + \sum_N \frac{dL_i^c}{V_i^c}. \quad (5.1)$$

If the ray has a traveltimes residual $dt = t_{\text{obs}} - t_c$ and the M cells have a computed velocity correction dV_i , I need a velocity correction dV_i^c in each of the remaining N cells so that

$$\sum_M \frac{dL_i}{(V_i + dV_i)} + \sum_N \frac{dL_i^c}{(V_i^c + dV_i^c)} = t_c + dt. \quad (5.2)$$

Inserting (5.1) into (5.2), I obtain

$$\sum_N \frac{dL_i^c}{(V_i^c + dV_i^c)} - \sum_N \frac{dL_i^c}{V_i^c} = dt_a \quad (5.3)$$

where

$$dt_a = dt + \sum_M \frac{dL_i}{V_i} - \sum_M \frac{dL_i}{(V_i + dV_i)} \quad (5.4)$$

is the *adjusted* traveltime residual, obtained by stripping the effects of velocity errors in the cells which have already been resolved. Equation (5.3) can be reduced by factorization of the left-hand side term:

$$- \sum_N \frac{dV_i^c dL_i^c}{V_i^c (V_i^c + dV_i^c)} = dt_a \quad (5.5)$$

and simplified further by assuming that the velocity corrections are much smaller than the velocities themselves, which is the basis of any focusing technique:

$$- \sum_N \frac{dV_i^c dL_i^c}{V_i^{c^2}} \equiv dt_a. \quad (5.6)$$

I now assume a constant velocity correction ΔV for these N cells. If the ray tracing is dense enough, each new ray will sample one unresolved cell at a time, so that this assumption does not degrade the resolution too much. The final formula for the velocity correction is therefore:

$$\Delta V = - \frac{dt_a}{\left(\sum_N \frac{dL_i^c}{V_i^{c^2}} \right)}. \quad (5.7)$$

If there are no cells already corrected and only one new cell ($N=1$ and $M=0$), then (5.7) becomes

$$\Delta V = - dt \frac{V^2}{L} = - dt \frac{L}{t^2} = \Delta \left(\frac{L}{t} \right), \quad (5.8)$$

the standard formula for relating a change in velocity to a change in traveltime.

All the quantities appearing in (5.7) are readily available from ray tracing so that implementing the CST algorithm is straightforward. The only difficult part is its instability: if there is a very erroneous velocity correction in one of the top cells, it will affect all other rays passing through that cell. I remedy this problem in three ways. First, if there are more than one ray traversing the same suite of cells, which can occur if the grid is dense enough, the corrections for these rays are averaged like in standard SIRT. Second, I avoid using very short path lengths in a given cell, since the path length appears in the denominator of equation (5.7). This is to say that a ray which barely samples a cell should not be used for correcting that cell. Third, time residuals smaller than a user-given threshold are eliminated, so as not to propagate small numerical errors by using equation (5.7). This threshold is typically on the order of 1ms or less and is an absolute number: since I work with adjusted time residuals, longer rays do not necessarily have larger residuals than shorter rays.

3 - Synthetic test

To test the validity of the method, I choose a synthetic model which simulates a wide-angle experiment. It contains vertical velocity gradients of up to 0.1 km/s per km in the left part of the model, a horizontal interface with a 1.5:1 velocity contrast and a vertical boundary with a velocity contrast varying between 2.3:1 at the surface and 1:1 at a depth of 6 km (figure 5.2). There are 11 shots distributed from $x=0$ every 2.5 km and the same 101 receivers spread evenly along the surface for each shot. Offsets range continuously from 0 to 25 km. Figure 5.2 also shows the ray diagram and synthetic seismogram for the middle shot ($x=12.5$ km), for direct arrivals only. These were generated by Tracit (appendix A). The rays are strongly deflected by the vertical boundary and this translates into a kink in the arrival times curve which can often be observed on field shot gathers.

A total of 605 traveltimes were picked using SVS (appendix D). I first use the correct velocity model as input to the tomography algorithm: no velocity corrections are done as the RMS time residual is 12 ms only. I then use a starting model which is laterally homogeneous and contains a high uniform vertical gradient of 0.5 km/s per km (top of figure 5.3). Ray tracing this model and matching the computed to the observed traveltimes yields an RMS time residual of 497 ms. The updated model after one iteration of CST is shown in the middle of figure 5.3. Both the velocity gradient in the left part of the model and the high velocity zone in the right part have been recovered. There is also some evidence for the horizontal discontinuity at 3 km depth, although the velocities below it are too high. These velocities were in fact obtained using rays not realizable in the exact model.

By comparison, the updated model obtained with SIRT is shown at the bottom of figure 5.3. The velocities on the left part of the model are too low. There is some indication of a high velocity zone in the right part of the model, but the velocity is still too low and the vertical boundary has been spread over the lower part of the model so as not to be recognizable as such. Although my algorithm gives a more quantitative image, it is able to converge to the right velocities and resolve both the vertical and horizontal transitions better and more quickly than with the SIRT method.

4 - Application to the COP data

The COP (Continuous Offset Profile) seismic data set was acquired in 1986 by Rice University in conjunction with the Houston Advanced Research Center and Pacific Gas and Electric's Offshore Deep Crustal Geophysical Survey (Levander & Putzig, 1990). A total of 406 airgun shots (10,000 cubic inches) were fired every 155 m as the boat moved toward the coast. They were recorded by a fixed array of 154 receivers spaced at about 325 m on land. The shots therefore span about 60 km and the receivers 50 km. Offsets range from about 2.5 to 120 km. The data were re-sorted to receiver gathers and reduced with a velocity of 6 km/s.

The data vary greatly in quality with some exceptionally good records and some unusable records due to cultural and instrument noise. The initial data volume of 60,000 traces was reduced to 22,000 traces. Figure 5.4 shows one of the best receiver gathers, with the receiver located 5 kms from the coast. The shallow event is interpreted as a direct arrival: it has a velocity lower than 6 km/s.

For the tomographic analysis I concentrate on these direct arrivals. Since the velocity gradient is relatively high in this region (1 km/s per 5 km), direct waves have limited horizontal moveouts, so that only gathers for receivers close to the coast have direct arrivals which can be clearly identified. Overall, I picked twenty-eight receiver gathers for a total of about 2500 picks. These picks were done using SVS (Appendix D). The average error in doing those picks is estimated at plus or minus 50 ms, which represents a relative error in the traveltimes of about 0.3 %.

The starting model for tomography consists of near-surface layers overlaying a laterally homogeneous zone with a constant vertical gradient of 1km/s per 7.5 km (top of figure 5.5). The near-surface structure and velocities were obtained through carefully iterated forward modeling and were therefore not changed during tomography. This is where it is important to constrain the starting model using as much a priori information as possible. There are 200 ray tracing cells across the model (600 m wide), which corresponds to 4 shots or 2 receivers per box, and 200 vertical cells (150 m high). Ray tracing this model and comparing the computed to the picked traveltimes, the RMS value of the time residuals is 1050 ms.

The updated velocity model obtained after one iteration of my algorithm indicates a high velocity gradient from about 4.4 km/s just below the constrained layers to about 6.5 km/s at a depth of 10 km. A smoothed version of this model is used for the second iteration. The RMS time residual is 600 ms. However, all rays travelling to the left of the 90 km mark give very small traveltime residuals, whereas those travelling to the right have arrivals too slow.

The final interpreted model, shown at the bottom of figure 5.5, incorporates a near-vertical boundary at $x=90$ km. The RMS time residual for this model is now 200 ms, which corresponds to a relative velocity error of about 1 %. I kept this model for the migration algorithm.

5 - Imaging the COP data

The COP data set is typical of a lot of wide-angle data sets in that it has a very low fold. This means that a very high signal to noise ratio must be kept because a given depth point is not imaged many times. This is particularly important when looking at the CIG's. I therefore kept only those gathers with very clear reflections and low ambient noise. These are all located away from the coast, since rays reflecting off deep layers have very large horizontal moveouts. Overall I kept 32 receiver gathers. Figure 5.6 shows the best such gather. The only pre-processing done is debiasing, 4 to 24 Hz band filtering and taking the first derivative.

The experiment geometry (figure 5.7) shows that with this kind of shot and receiver coverage, the fairly high vertical gradient and an assumed target depth between 20 and 30 km, the portion of the crust that can be imaged is about 30 kms in length. I therefore concentrate the imaging part of my algorithm on this area, although the ray tracing model has to include the full 120 km of the line.

I obtain the first image (figure 5.8) using the velocity model obtained from CST in the previous section. This image is plotted at true amplitude. It shows dipping reflectors from the SW at a depth of 20 to 25 kms, which turn around to become flat at 25 to 28 km

to the NE. These reflectors are well identified and correspond in character to the events appearing on the receiver gathers. On two CIG's taken at $x=10$ and $x=20$ km (figure 5.9), these events are well identified and appear to be dipping down generally, indicating a velocity too high. MMO analysis and depth focusing yield an average velocity correction of minus 0.3 km/s. I apply this correction to the velocity model and obtain the improved image shown in figure 5.10. The most prominent, dipping reflector appears clearly on the corresponding CIG's (figure 5.11) and is identified as the Moho. Comparison with a model obtained for the same area through labor-intensive forward modeling (figure 5.12, plotted at the same scale as figure 5.10) indicates good general agreement despite a difference in depth of about 2 km for the Moho. Figure 5.13 shows the final velocity model obtained with my algorithm.

6 - Conclusion

I developed a cell-stripping tomography technique which distributes velocity corrections only to those cells where they are most likely to belong. Rays are processed from the top down, and any time residual for a given ray is adjusted so as to eliminate the effects of velocity errors in the cells previously resolved. The method is well suited for wide-angle data applications which commonly have sparse ray coverage and allows detailed resolution of vertical boundaries even with horizontally traveling rays.

I successfully applied the technique on a synthetic model with strong vertical gradients and a vertical velocity discontinuity. Although the updated model is more quantitative than the one obtained with SIRT, it is better resolved. On the COP data set, I

used three iterations to obtain a preliminary velocity model for migration. The final relative error in velocity is about 1 % and the model is well constrained to a depth of 10 km.

I migrated the COP data using the pre-stack Kirchhoff algorithm and the velocity model derived for tomography analysis. After performing depth focusing analysis, I obtained a clear image of the upper crust in the Santa Maria basin over a length of about 30 kms, with the Moho at a depth of about 21 to 26 km.

Fig. 5.1 - Geometry of the cell-stripping tomography algorithm

Fig. 5.2 - Synthetic test for CST: description

Fig. 5.3 - Synthetic test for CST: results

Fig. 5.4 - COP receiver gather showing the direct arrival

Fig. 5.5 - Results of CST on the COP data

Fig. 5.6 - COP receiver gather showing the Moho reflections

Fig. 5.7 - COP experiment geometry

Fig. 5.8 - Initial image for the COP data

Fig. 5.9 - Initial CIG's for the COP data

Fig. 5.10 - Final image for the COP data

Fig. 5.11 - Final CIG's for the COP data

Fig. 5.12 - Model obtained through forward-modeling

Fig. 5.13 - Final velocity model obtained with the present algorithm

Conclusion

This unified approach to complex seismic imaging problems has proven successful in handling complex structures and wide-angle data. The ray tracing method is fast, versatile and reliable. I have easily implemented it as the basis of all other techniques. The pre-stack Kirchhoff depth migration method is very efficient and accurate, while the focusing analysis allowed much better imaging of the Hosgri fault zone in the Santa Maria basin. Integrating these three imaging tools with the cell-stripping tomography technique produced a good quality image of the Moho in the same area.

This approach is easy to use and allows a lot of user interaction and control during the imaging process. The images obtained are sometimes less smooth than their post-stack counterparts, but they are much better constrained. The updated velocities obtained through focusing or tomography are very close to lithologic velocities, while the CIG's provide an excellent check on the validity of some of the reflectors. However, although I obtained good images, there is still room for improvement: the ray tracing scheme could perhaps be vectorized, at least for the computation of Green functions, and full tomographic-style MMO analysis might be implemented for field data given enough computing power. The next logical step is of course to implement this approach for 3-D data sets.

As computing power increases and very good quality wide-angle data sets become more available, I expect these kinds of techniques to be used almost routinely in the near future. They appear as necessary to deal with complex seismic imaging problems. If computer power were unlimited, then perhaps a full-blown statistical inversion could be done, which would combine iterative modeling, migration and inversion in one single job. However, apart from the fact that unlimited power will never be available, one has to remember that our seismic theory and experiments are never as perfect as we would like them to be. This is why it is important to look into this kind of unified approach, since it addresses these complex problems in a fundamental way while remaining practical and geologically oriented.

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Appendix A

Tracit

Tracit is a graphics interactive forward-modeling program which allows rapid calculation and visualization of raypaths, amplitudes and synthetic seismograms for reflection, transmission and Green function rays. It is based on the algorithm described in chapter 2. It consists in two parts: model building and ray tracing.

1 - Model building

1.1 - The first step is to create a file with the interface coordinates. An interface is a bi-continuous line across which there is a zero or first order velocity discontinuity. It can start or end anywhere in the model, but can not be vertical because of the spline interpolations. If there is some topography at the surface and $z=0$ is set to the water level, the z coordinates of the top interface (the Earth surface) can be set to negative values. There must be one continuous interface (or set of joining interfaces) across the top of the model, and one across the bottom of the model. This is necessary for gradient calculations.

If the model name is *test*, the digitizer file, of arbitrary format, must be called *test.dig*. The first five lines are reserved for text. Each interface is preceded by a ***EVENT** on one line followed by an (x,z) pair on each line (all coordinates in km). The end of the file is then marked by a ***STOP**. The utility **MODPLOT** allows plotting of the model with the splined interfaces. It only requires this file for input and should be used regularly to check if the model entered is correct.

1.2 - The second step is to define an input file containing information about the model. It is created by running **INTRACIT**, which will create the file *test.inp*. This module is self-explanatory. It allows the user to specify the number of boxes used in x and in z: normally the box size should correspond to the typical length scale in the model. The user also specifies the distance between two successive ray arrivals at the surface: the smaller the distance, the longer the computations, but the better the chances of obtaining head waves and boundary rays.

1.3 - The third step is to run **MODEL** to compute the velocity, the slowness gradient and interface position (if any) in each cell. It creates the file *test.mod*. This module requires an X-window environment: if there are boxes which contain more than one interface, it will display them on the screen one after the other, each time showing the various interfaces (figure A.1). The user presses any mouse button and by moving the cursor can position a cross to divide the cell into smaller cells. When the position is satisfactory, 'C' is pressed for 'confirm', and the user can assign specific interface numbers to specific subcells if there still are ambiguities (in the case of a pinchout, for instance, it will be necessary to force one interface at the detriment of the other). Between each cell displayed the whole model is recalculated.

2 - Ray tracing

Once the model is built, **TRACIT** will perform standard ray tracing (figure A.2). The source is specified interactively. Each 'run', or set of rays, corresponds to the number of interfaces encountered before a reflection, plus one. For instance, all refracted/turning rays, which have no reflections, are in run # 1, the next run computes all the rays transmitting through the first interface and reflecting off a second interface, and so on. The 'print' option allows plotting of raypaths and traveltimes on a laser printer (the postscript files *Trmodel.#*), as well as writing of synthetic seismograms in RICE format (files *synseis.#*) for further display or plotting.

Module **GREEN** has the same model-building procedure as **TRACIT** and the same graphics display. It requires an extra file, *test.ctl*, described in Appendix B. Otherwise, it traces only transmitted rays and stops them at any of the four boundaries of the model, allowing visualization of the Green function. The rays coverage is the same as used in migration: this information is read from *test.ctl*, and the tolerance specified at the surface in *test.inp* has no meaning. This also means that the rays are not divided into 'runs' or sets, and that no synthetic seismograms are computed.

Fig. A.1 - Graphics display of Model

Fig. A.2 - Graphics display of Tracit

Appendix B

Kiwi

Kiwi (Kirchhoff Inversion & Wide-angle Imaging) is a package for pre-stack depth migration and depth focusing (MMO analysis). It is based on the methods and algorithms described in chapters 3 and 4. There are five parts: data conditioning, model building, ray tracing, imaging and focusing analysis.

1 - Data conditioning

The data set must be in RICE format. If it is too large to fit onto the disk at one time, it is subdivided into a number of smaller data sets. If the model name is *test*, the name of the data set or sets must be *test.data.1*, *test.data.2*, etc... (this order must be respected later on).

The first step is to take the first derivative of the data. This is done by running module **DER10**. The second step is to run **KIWISSET** to build a file containing the list of all ground locations having seen a shot or receiver, as well as an index table for relating these locations to actual traces. The name of this output file is *test.data.set*.

2 - Model building

The file containing the positions of the interfaces, called *test.dig*, must be created first. It is identical in format to the ones used for TRACIT and can therefore be checked with MODPLOT as well. Then a control file named *test.ctl* is created by running **KIWICTL**. This module is self explanatory. It builds information about the ray tracing model in a way identical to INTRACIT, as well as information about the imaging area, the data set and the CIG's.

The top of the imaging area is defined by a number of interfaces, with actual imaging starting five depth samples below them to avoid re-imaging of those interfaces. This first imaging sample is set to one on each trace. The bottom of the area is defined by a user-specified constant number of samples below the first sample. If the top interface or interfaces are not flat, this means that the individual image traces do not start at the same absolute depth sample. This avoids overlapping layers and allows better storage efficiency because each image trace has the same number of samples. The traces in a given CIG are still referenced to the same relative depth, of course. However, to form a final image the CIG's relative depths must be converted to absolute depths (see below). Note also that it is important to define an imaging area which is entirely contained in the ray tracing model.

Once these files are created, module **KIWIMOD** is run to actually build the model. It is used in the same way as the program MODEL for ray tracing. If there are never more than one interface per cell, so that the user will not have to divide cells interactively, module **KIWIMODT** can be run instead, which does not require an X-window environment. Both programs create the file *test.mod*, which is slightly different from its TRACIT counterpart.

3 - Ray tracing

Ray tracing for the Green functions is done by executing **KIWIRTR**. This module requires almost no user-interaction. The output files are *test.layer#.rtr*, which contains traveltimes and Kirchhoff weighting factors, and *test.layer#.rtt*, which is optional and contains incremental traveltimes since the top of the imaging area and angle cosines at each imaging point. These files are unformatted and can be quite large. Their size in number of bytes is determined by the formula $nrt \cdot npx \cdot npz \cdot 6$ for *rtr* and $nrt \cdot npx \cdot npz \cdot 5$ for *rtt*, where *nrt* is the number of raytraces and *npx*, *npz* the number of imaging points horizontally and vertically. For instance, a model with 200*100 points and 400 ray traces will store 90 Mb of information. If no quantitative MMO analysis is to be performed, one should not request the creation of the *rtt* file.

4 - Imaging

Imaging is also an automatic procedure with no user interaction. Two programs can be run.

KIWIMIG directly produces the CIG file, called *test.layer#.cig*. It does not compute auxilliary quantities and is very fast. The size in bytes of the *cig* file is given by $npx \cdot nof \cdot (168 + npz \cdot 4)$, where *nof* is the number of traces in a CIG.

KIWICIG produces two files: the *cig* file and an auxilliary file containing specular source-receiver pairs information, called *test.layer#.spc*. It is slower than the purely migration module. The size in bytes of the *spc* file is given by $npx \cdot nof \cdot npz \cdot 4$.

The image of the layer is obtained from the *cig* file with module **ADD**, which allows selective stacking of traces in CIG's.

KIWITRAN is then run to correctly position the image of the layer within the entire model. It does not require any user-interaction except for the total number of samples. Once an image has been obtained, it is wise to delete the *rtr* file from memory, since it is very large and is not used subsequently.

5 - Focusing analysis

If **KIWICIG** has been run, one can look at the *cig* file, which is in RICE format, to perform a focusing analysis. To perform automatic picking of the CIG's, module **KIWISMB** can be run. It only requires specification of a window for each CIG, which can be determined by looking at a preliminary image. If only a few CIG's need correction, or if event picking is difficult, one can pick CIG's individually (see Appendix D), producing a file called *test.layer#.pck*. Module **KIWIMMO** is then run to perform the depth focusing analysis. It will list the velocity corrections for each CIG and produce a new file with the corrected velocity model, called *test.layer#.modc*. The user can then use this information and tie it to any other velocity or geologic information to produce the best estimate of a new velocity model for migration. A new run is initiated by starting again at step 2.

Appendix C

Tomo

This package performs standard SIRT as well as cell-stripping tomography as described in chapter 5. It consists of three steps: model building, traveltime picking and tomography.

1 - Model building

The model building part is very similar to those of Tracit and Kiwi. The interface file is first created and called *test.dig*. Module **TOMODEF** is then run to build the file *test.def*. In addition to the standard model information, it allows the user to specify the spacing of the rays as well as the minimum absolute time residual to take into account for tomography. The model file, *test.mod*, is then built using **TOMOMOD** if X-window is required, or **TOMOMODT** if not.

2 - Traveltime picking

This can be done using the SVS module (appendix D) or other schemes. The output file, named *test.pck*, must be in the SVS picks format.

3 - Tomography

There are two modules: **TOMO1** for cell-stripping tomography and **TOMO2** for SIRT with inverse path length weighting. Other than the tomographic method, the two modules work in the same way (figure C.1). The model is displayed with color coded velocities, with a color bar on the right-hand side. The picked traveltimes are plotted as blue square boxes and the computed traveltimes as green crosses. The user can load any individual ensemble (a set of picks), and perform iterative ray tracing and tomographic analysis. Each time a new ensemble is selected, the initial model is read in. An AUTO option allows automatic ray tracing of all ensembles with simultaneous tomographic analysis. The model can be smoothed iteratively at any point in the process. When the program is exited, the last corrected model is written to the file *test.modm*.

Fig. C.1 - Graphics display of Tomo

Appendix D

Interactive Seismic Processing

This appendix describes a number of stand alone modules which I have developed to complement the three main packages. Based on RICE format, they allow easy manipulation and visualization of data and results.

1 - SVS

SVS (Seismic Visualization System) allows visualization, picking and editing of seismic data interactively. It loads the whole data set and displays one gather at a time on the screen (figure D.1). If there are more than 250 traces in a gather or more than 750 samples per trace, it automatically resamples the gather to make it fit. Since all the data are loaded, the user can quickly go to the previous or next gather, or jump to a specified gather using the appropriate keys. If no option is entered on the command line, the data are plotted in color or using a variable area gray scale (the WG/VA key allows switching between the two). If 'bw' is typed on the command line, the data are displayed in black & white using variable area or wiggle traces (using the same switch). The user can specify a gain and an AGC gate to be applied interactively.

To edit traces, the 'edit' key is pressed, and the user positions the mouse on each trace to edit and clicks the mouse button. When it is done, the user can reject the traces selected by typing 'd', or write their identification to a file called *editout* by pressing 'enter'. To pick traces, on CIG's for instance, the user selects the 'pick' key, positions the cursor on the first sample to pick, presses the mouse button and hold it down while moving the cursor across the picks. When the button is released, the picks are splined and displayed as a thick black line. They can be rejected by pressing 'd', or written to a file called *pickout* by pressing enter. This file can be used directly in the KIWIMMO module.

2 - Velplot

VELPLOT allows color plotting of velocity models on the screen. It requires no user -interaction. Each cell of the model is plotted as a rectangle of maximum size so that the whole model will fill in the screen. The velocity range is determined automatically and each velocity is shown with a given color, the lowest velocities being more red and highest more blue. The input file can be in finite-difference format, Tracit format, Kiwi format, Tomo format or RICE format.

3 - Processing tools

ADD has been described in appendix B. **COMBINE** allows combining of several data sets vertically and can be used to form a final image from individual layer migrations. **MERGE** will merge several data sets by assigning them to different locations. **MUTE** can be used to apply mute patterns, for instance to CIG's. **TABLE** produces a listing of header values.

Gain	Agc	uG/vA	Prev	Jump	Next	Edit	Pick	Exit
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Fig. D.1 - Graphics display of SVS