RICE UNIVERSITY

LAYER-STRIPPING REVERSE-TIME MIGRATION

by

RUEY-CHYUAN SHIH

A THESIS SUBMITTED
IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS FOR THE DEGREE

DOCTOR OF PHILOSOPHY

APPROVED, THESIS COMMITTEE:

Alan R. Levander

Alan R. Levander, Chairman
Associate Professor of Geophysics

D. W. Bally

Albert W. Bally
Professor of Geology

Manik Talwani

Manik Talwani
Professor of Geophysics

William W. Symes

William W. Symes
Professor of Mathematical Sciences

Thesis
Geol.
1990
Shih

Houston, Texas
April, 1990

3 1272 00653 4182

AT Q6145

LAYER-STRIPPING REVERSE-TIME MIGRATION

Ruey-Chyuan Shih

ABSTRACT

Reverse-time migration has proven to be successful for structures with steep dips and strong velocity contrasts. Applying this algorithm to a large scale seismic model requires significant computational expense, particularly if strong velocity contrasts are present in the model. Here I present a layer-stripping migration technique, in which I use the reverse-time method to migrate seismic sections through constant or smoothly varying velocity layers, one layer at a time. As part of the migration in a given layer, the bottom boundary of the layer is defined and a seismic section is collected along it. This new section serves as the boundary condition for migration in the next layer. This procedure is repeated layer by layer. The final migration result is composited from the individual layers images. The layer-stripping migration algorithm can be summarized as three steps: 1) model definition, 2) wavefield extrapolation and imaging, and 3) boundary determination. The migration scheme posed in this way is similar to datuming with an imaging condition.

The advantages of the layer-stripping method are: 1) it preserves the benefits of the reverse-time method, i.e., it handles strong velocity contrasts between layers and steeply dipping structures; 2) it eliminates artificial interlayer multiples; 3) it reduces computational expense in high velocity layers; and 4) it allows interpretational constraint during image formation.

The method has been implemented with both an explicit 4th-order time, 10th-order space, finite-difference approximation to the scalar wave equation, and an implicit 2nd-order time, 4th-order space finite-difference scheme applied to the linearly transformed wave equation (Li, 1986). The capability of post-stack layer-stripping reverse-time migration is illustrated on a synthetic CMP data and a CMP data from a survey over a

faulted anticline in a fold and thrust belt. For pre-stack layer-stripping reverse-time migration, I present migrations of two synthetic data sets and a field data example from part of the marine seismic reflection profile RU-3, crossing the Hosgri fault offshore southern central California. The Hosgri fault appears as a northeast dipping high angle fault with a thrust component.

ACKNOWLEDGEMENT

I would like to express my sincere appreciation to my thesis advisor, Dr Alan R. Levander, for his invaluable advice and support regarding all aspects of my research and thesis writing. I want to thank Dr. William W. Symes for taking a special interest in my research and for his many helpful discussions in this study. I also thank Dr. Albert W. Bally and Dr. Manik Talwani for their time and effort in reviewing my dissertation. Your helpful suggestions made the final document much better than I would have done alone.

I specially thank Dr. Stuart Henry for the assistance and friendship he has constantly provided. I thank Dr. Bruce S. Gibson for many useful experiences he has shared with me.

I would like to thank the Geological Survey of Canada and Chris J. Harrison for letting me use the Melville Island line 1660 seismic data. I also appreciate Dr. Anne Meltzer for providing me the CMP stacked section and time migration of the seismic profile RU-3.

Most of all, I thank my family, especially my beloved mother, my wife Jean and my daughter Tracy. Words can not express the love, encouragement and support my mother has shown me throughout my education. I am deeply grateful to my wife Jean, who has provided unflagging support in these years. The happy and smiling face of my daughter Tracy made even the worst pressure seem to fade to insignificant.

This research was supported by NSF grants EAR-8817146 and EAR-8707466, fellowship funds from the W. M. Keck Foundation, and the Office of Naval Research under contract N00014-85-0725.

To My Mother

TABLE OF CONTENTS

Abstract.....	ii
Acknowledgements	iv
Table of Contents.....	vi
List of Tables	ix
List of Figures	x
 Chapter 1: Layer-Stripping Reverse-Time Migration - the Movie	 1
Review of Conventional Migration Methods.....	1
Reverse-Time Migration.....	4
Layer-Stripping Reverse-Time Migration	7
Layer Definition	7
Wavefield Extrapolation and Imaging.....	7
Reverse-time wavefield extrapolation.....	8
Imaging Formation.....	8
Post-stack Migration	9
Pre-stack common-shot gather migration.....	10
Boundary Determination.....	11
The Advantages of Layer-Stripping Reverse-Time Migration	12
Outline of Thesis	14
Figure Captions.....	15
 Chapter 2: Wave Equation Development for Layer-Stripping Reverse-Time Migration	 20
Introduction	20
2nd-order Accurate Explicit Scheme Reverse-Time Wavefield Extrapolation.....	22
Formulation	22

Grid Dispersion.....	25
10th-order Space, 4th-order Time, Explicit Scheme Reverse-Time Wavefield Extrapolation.....	26
Formulation	27
4th-order Implicit Scheme Reverse-Time Wavefield Propagation.....	31
Finite-Difference Formulation of the Linearly Transformed Wave Equation	32
Absorbing Boundary Conditions.....	34
Impulse Responses.....	35
Figure Captions.....	39
Chapter 3: Post-Stack Layer-Stripping Reverse-Time Migration.....	52
Introduction	52
Synthetic Data Example.....	52
Data Description	53
Layer-Stripping Reverse-Time Migration Sequence and Result	54
Fold and Thrust Belt Data Example	56
Field Data.....	57
Layer-Stripping Reverse-Time Migration Results.....	58
Conclusion	60
Figure Captions.....	61
Chapter 4: Pre-Stack Layer-Stripping Reverse-Time Migration	92
Introduction	92
Calculation of the Imaging Condition.....	94
Synthetic Data Example 1	95
Migration Sequence and Result.....	95
Synthetic Data Example 2	98

Field Data Example -- Hosgri Fault, RU-3, Central California.....	100
General Setting	100
Pre-Stack Migration.....	101
The Dip of the Hosgri Fault.....	103
Conclusion	104
Figure Captions.....	105
Chapter 5: Conclusions.....	152
References.....	155
Appendix A: Velocity Analyses on Seismic Profile RU-3 for the Pre-Stack migration...	159

LIST OF TABLES

Table 2-1.	Weights for a 10th-order accurate finite-difference scheme second derivative operator.....	38
------------	--	----

LIST OF FIGURES

Figure 1-1.	Two-dimensional earth model.	16
Figure 1-2.	Velocity model with strong velocity contrasts between layers.	17
Figure 1-3.	Exploding reflector model.	18
Figure 1-4.	Imaging condition of common-shot gather migration.	19
Figure 2-1.	Explicit scheme 2nd-order finite-difference operator.	41
Figure 2-2.	General setting of reverse-time migration	42
Figure 2-3.	Successive steps of reverse-time migration	43
Figure 2-4.	Explicit scheme 10th-order finite-difference operator.	44
Figure 2-5.	Implicit scheme 4th-order finite-difference operator.	45
Figure 2-6.	Finite-difference grid of the linearly transformed wave equation.	46
Figure 2-7.	Successive steps of solving the linearly transformed wave equation in reverse-time direction	47
Figure 2-8.	Impulse response of the 2nd-order scheme reverse-time migration	48
Figure 2-9.	Impulse response of the 10th-order scheme reverse-time migration	49
Figure 2-10.	Impulse response of the 45-degree depth migration.	50
Figure 2-11.	Impulse response of the implicit 4-order scheme reverse-time migration.	51
Figure 3-1.	Velocity model for generating synthetic shot records	65
Figure 3-2.	Synthetic CMP stack.	66
Figure 3-3.	Time migration.	67
Figure 3-4.	Velocity model for post-stack layer-stripping reverse-time migration.	68
Figure 3-5.	Velocity model for migration in the first layer	69
Figure 3-6.	Layer-stripping migration of the first layer.	70
Figure 3-7.	Boundary value for migration in the second layer.	71
Figure 3-8.	Velocity model for migration in the second layer.	72

Figure 3-9.	Layer-stripping migration of the first and second layer	73
Figure 3-10.	Boundary value for migration in the third layer.....	74
Figure 3-11.	Complete layer-stripping reverse-time migration	75
Figure 3-12.	45-degree finite-difference depth migration	77
Figure 3-13.	Location map of the seismic profile Melville Island 1660	79
Figure 3-14.	CMP stack of the Robertson Point anticline.....	80
Figure 3-15.	Time migration.....	81
Figure 3-16.	A 2-layer velocity model for migration	82
Figure 3-17.	The layer-stripping migration using the 2-layer velocity model.....	83
Figure 3-18.	The 45-degree depth migration using the 2-layer velocity model.....	84
Figure 3-19.	A 3-layer velocity model for migration	85
Figure 3-20.	The layer-stripping migration using the 3-layer velocity model.....	86
Figure 3-21.	The 45-degree depth migration using the 3-layer velocity model.....	87
Figure 3-22.	A 6-layer velocity model for migration	88
Figure 3-23.	The layer-stripping migration using the 6-layer velocity model.....	89
Figure 3-24.	The 45-degree depth migration using the 6-layer velocity model.....	90
Figure 3-25.	Migrations of the synthetic data using different migration operators.....	91
Figure 4-1.	Velocity model used to generate the synthetic shot gathers for example 1	110
Figure 4-2.	Synthetic common-shot records	111
Figure 4-3.	Synthetic common-shot records with the direct waves muted	114
Figure 4-4.	Partial migration images of the first layer.....	117
Figure 4-5.	Composited image of the first layer	120
Figure 4-6.	Boundary values for migration in the 2nd layer	121
Figure 4-7.	Partial migration images of the second layer	124
Figure 4-8.	Composited image of the second layer.....	127
Figure 4-9.	The complete pre-stack migration image for example 1.....	128

Figure 4-10.	Velocity model used to generate the synthetic shot records for example 2	129
Figure 4-11.	Synthetic common-shot records	130
Figure 4-12.	The complete pre-stack migration image for example 2.....	133
Figure 4-13.	Velocity model used for migration	134
Figure 4-14.	Location map of the marine seismic profile RU-3.....	135
Figure 4-15.	Data processing sequence for the post-stack migration	136
Figure 4-16.	CMP stacked section.....	137
Figure 4-17.	Time migration.....	138
Figure 4-18.	Common-shot records	139
Figure 4-19.	Pre-stack layer-stripping reverse-time migration	141
Figure 4-20.	Velocity model used for migration	142
Figure 4-21.	Data processing sequence for the pre-stack migration	143
Figure 4-22.	Seismic interpretation on the pre-stack migration.....	144
Figure 4-23.	Seismic interpretation on the post-stack migration	145
Figure 4-24.	Migrated common depth gathers.....	146
Figure 4-25.	The dips of the Hosgri fault.....	147

Chapter 1

Layer-Stripping Reverse-Time Migration - the Movie

REVIEW OF CONVENTIONAL MIGRATION METHODS

Seismic migration is an image reconstruction technique which transforms the recorded reflection seismic data into a representation of the earth's subsurface structure. The seismic data recorded at the earth's surface contain seismic waves reflecting from all possible directions in the earth's subsurface. Thus, the recorded seismic signals generally do not represent the geological formations directly below the receiver (see Figure 1-1). Processing of reflection seismic data is aimed at making the recorded data more comprehensible geologically. Seismic migration is one of these processes and along with CMP stacking and deconvolution form the main framework of standard seismic processing. Migration depropagates the recorded signals back to their correct subsurface spatial positions based on wave theory considerations, thus enhancing the lateral resolution.

The development of seismic migration method closely follows the progression of computer technology. Historically, seismic migration was begun by using graphical methods. This was followed by diffraction summation and wavefront migration based on ray theory. In the early 1970s, seismic migration became explicitly associated with the scalar wave equation following the pioneering work of Jon Claerbout (Claerbout, 1971; Claerbout and Doherty, 1972; and Claerbout, 1976). In the following decades migration has been described in terms of the numerical solution of some partial differential equations (Gazdag and Sguazzero, 1984).

Conceptually, seismic migration consists of two stages, a wavefield extrapolation step and an imaging constructing procedure. Conventional migration methods usually extrapolate the wavefield toward the increasing depth direction and form the migration image according to the imaging condition. The wavefield extrapolation can be interpreted as a simple downward continuation in z , by which the solution at $z+dz$ can be obtained from the solution at z . In the seismic experiment, the surface recording is known and the velocity function can be approximately determined. Unfortunately, the knowledge of the surface recording alone is insufficient to provide a unique solution to solve the scalar wave equation. In addition to the surface recording, its normal derivative is needed as well while solving the scalar wave equation. For use in seismic migration, the scalar wave equation has traditionally been approximated in some way. Claerbout applied the parabolic approximations to the wave equations as a wavefield extrapolation operator, which allows the migration to be performed using the surface recorded information. However, using the parabolic wave equation, the wavefield can be propagated accurately within a limited range of propagation angles only. The approximation can be improved with additional terms that allow accurate propagation at much wider angles.

For a heterogeneous medium, Claerbout separates the parabolic wave equations into two terms, the diffraction term and the lens term. The diffraction term, or migration term, collapses diffraction energy thus produces notably laterally coherent migration image. The lens term is a time shift factor that stretches and bends the image ray along the correct path. Migration with the diffraction term only neglects the effect of Snell's law and therefore excludes ray bending. This migration scheme is called time migration. Even if the subsurface velocity model is known exactly, in a laterally variable medium, time migration cannot position the migration image correctly. The wavefield extrapolation equations with the diffraction term and the lens term together are known as depth migrations. The equations collapse diffractions and retain Snell's law, and in addition, its output coordinate

is in depth instead of in time. Since the velocities are treated correctly in depth migration, this scheme is more sensitive to errors in the velocity model. Note that neglecting higher order terms in the parabolic wave equation, results in depth migration schemes limiting migration accuracy to certain angles. The parabolic approximations most commonly used are the first-order and second-order approximations, usually called the 15-degree equation and the 45-degree equation, respectively. The name 15-degree or 45-degree gives a guide to the range of angles that can be handled properly. In practice, the 15-degree equation can handle the maximum dip up to 35 degrees and the 45-degree equation can be accurate up to 65 degrees (Yilmaz, 1987).

In summary, seismic migration can be summarized as two types, one is time migration and the other one is called depth migration. Time migration collapses diffraction but doesn't allow ray bending. The migration output is in time domain and is somewhat insensitive to velocity error. Depth migration collapses diffraction also obeys Snell's law. The migration algorithm is highly sensitive to velocity error and the output coordinate is in depth.

Three of the major numerical techniques for depth migration are diffraction migration, finite-difference migration and frequency domain migration. Diffraction migration is also known as Kirchhoff migration when using higher order approximations. The major advantage of diffraction migration is its good performance for steeply dipping structures. Disadvantages are its poor performance for low signal-to-noise (S/N) ratio conditions and the risk of the migration operator becoming spatially aliased. The finite-difference approach is commonly known as wave equation migration. The finite-difference method is stable under a low S/N ratio. However, this method includes a relatively long computational time and difficulty in handling steeply dipping data. Claerbout's methods are finite-difference schemes. Frequency domain migration also may be referred to as frequency-wavenumber (f-k) migration or Fourier transform migration. The advantages of frequency domain

migration include fast computing time, good performance under low S/N ratio, and an excellent performance for steep dips. For constant velocity, f-k migration algorithm is clearly the most efficient; however, it has difficulties with any variation in velocity functions. In a rapidly changing velocity model, the velocity model is participated into several constant velocity fields, many small steps of wavefield extrapolation are then performed. Gazdag (1978 and 1980) and Gazdag and Sguazzero (1984) developed the use of Fourier transform for migration for rapid changes in velocity. They constructed the wavefield by phase shifting each frequency component of measured data from one z-level to the next z-level. For a general velocity distribution, the phase-shift method may prove more popular than the f-k method.

REVERSE-TIME MIGRATION

The depth migration methods described above are generally achieved by downward continuation of surface data, in which an input section is extrapolated toward increasing depth into an earth model. The final migrated result is given by the amplitude of the depth extrapolated field when the desired initial time is reached. At the 52nd SEG Meeting in Dallas (1982), Whitmore presented an alternative of wave equation migration, known as reverse-time migration. This method permits a different approach to migration that is based on reverse-time wavefield calculation instead of depth extrapolation. Shortly after that, McMechan (1982, 1983) described almost identical approaches to migration. With further development by Baysal et al (1983), Kosloff and Baysal (1983), and Whitmore (1983), this method has initiated much interest and discussion (Levin, 1984).

Consider a two dimensional earth with x horizontal and z vertical, for which the pressure field is $P=P(x,z,t)$ and the medium density is constant. The scalar wave equation is given by

$$\frac{\partial^2 P(x,z,t)}{\partial x^2} + \frac{\partial^2 P(x,z,t)}{\partial z^2} = \frac{1}{v(x,z)^2} \frac{\partial^2 P(x,z,t)}{\partial t^2}, \quad (1.1)$$

where $v(x,z)$ represents the wave velocity which is variable in two dimensions. Generally, the scalar wave equation used to propagate a wavefield can be driven either forward or backward in time simply by reversing the time axis. That is, if $p(x,z,t)$ is one solution to equation (1.1), then so is $p(x,z,t_0-t)$ or $p(x,z,t_0+t)$ for any fixed t_0 . Since the purpose of migration is to migrate seismic energy back to its correct position, it is natural to solve the scalar wave equation successively in steps backward in time until the desired initial time is reached. This approach treats the recorded seismic time section as a natural boundary condition along the surface of the earth, which is then extrapolated to the interior of the earth.

There are different reverse-time migration schemes that use different form of the wave equation for wavefield extrapolation. McMechan (1983) suggested the use of the two-way wave equation (1.1). The problem raised with the full wave equation is that it generates artificial interlayer reflections from interfaces where the velocity changes rapidly. This problem can be avoided if one uses a one-way wave equation instead of a two-way wave equation. Although the result is generally reliable, the one-way wave equation may not accurately migrate the seismic section when a structure has steep dips or strong velocity contrasts. Also, the one-way wave equation is incapable of preserving correct wave amplitudes (Kosloff and Baysal, 1983) that is particularly important in pre-stack migration.

Baysal et al (1983) and Loewenthal and Mufti (1983) used a version of the full wave equation designed to minimize the effect of upcoming reflected wave components. They started with the 90-degree dip wave equation presented by Gazdag (1981). The square root of the spatial derivative operators is handled in the wavenumber domain and then

transformed back to the spatial domain. Multiplying the result with the velocity function, the output is equal to the time derivative. The time derivative is then approximated by finite-difference operator. This equation applies to dips up to 90-degree and permits lateral velocity variation. Unfortunately, the equation they used cannot be expressed without the Fourier transform.

Baysal et al (1984) presented another approach to reduce artificial reflections using a two-way nonreflecting wave equation. Their approach begins with the general wave equation consisting of the density function. Since medium density does not play any role in migration, its value may be chosen to minimize reflection. This is done by setting density so that the acoustic impedance is constant over the entire object space.

Reverse-time migration is a depth migration technique. This migration technique has proven to be successful for structures with steep dips and strong velocity contrasts, and should have superior imaging capabilities over the conventional dip-limited finite-difference depth migration algorithms (Kosloff and Baysal, 1983; Loewenthal and Mufti, 1983). A comparison between reverse-time migration and conventional phase shift plus interpolation migration schemes has been published by Gazdag and Carrizo (1986). Their work shows the surpassing accuracy and robustness of reverse-time migration. However, the high expense of reverse-time migration prohibits its use as a general migration algorithm at this time, particularly if strong velocity contrasts are present in the model. Although supercomputers make this less of a problem, applying this algorithm to a large seismic model requires significant computational expense. The other drawback of reverse-time migration, a drawback of conventional migration methods as well, is that the user has no control over the final migration image after specifying the velocity field. Using conventional migration methods needs a complete velocity mode before migration.

LAYER-STRIPPING REVERSE-TIME MIGRATION

In order to increase the efficiency of reverse-time migration and make interpretation a step of migration, I have developed a layer-stripping reverse-time migration technique. The method I am describing uses the reverse-time algorithm to migrate the seismic section layer by layer while appropriately rescaling the finite-difference grid space in each layer. The final migrated image is then composited from the individual layer migrations. Layer-stripping reverse-time migration is a form of depth migration, and can be applied to seismic data before or after stack.

The layer-stripping reverse-time migration algorithm consists of three steps, 1) layer definition, 2) wavefield extrapolation and imaging, and 3) boundary determination. An outline of the migration algorithm is described in the following paragraphs.

(1) LAYER DEFINITION

Layer definition is the first step of layer-stripping reverse-time migration. For a velocity model with strong velocity contrasts between layers, the initial velocity model is subdivided into several discrete layers with approximate boundaries (see Figure 1-2). Each layer has a constant or slowly varying velocity. The boundaries can have any geometry, and are assumed to be interfaces separating layers with large velocity contrasts. Although I have predefined the layer, only the uppermost boundary is assumed to be known exactly. The bottom boundary of the layer is considered to be approximated.

(2) WAVEFIELD EXTRAPOLATION AND IMAGING

After defining the layer, I migrate the seismic data in each layer with the use of the reverse-time migration method. The migration application involves two steps: i), reverse-time wavefield extrapolation and ii), an imaging formation.

(i) Reverse-time wavefield extrapolation:

To extrapolate the wavefield in each layer, I use a 10th order space, 4th order time explicit finite-difference scheme to approximate the two way acoustic wave equation (Dablain, 1986). Also, I apply a sponge layer transmission boundary (Israeli and Orzag, 1981) around the computation region to prevent reflections from the edge of the grid. In order to maintain nondispersive propagation in the grid interior, the 10th order finite-difference scheme requires sampling at 3 grid points per wavelength. Based on the above criterion, the layer-stripping method allows the grid space to be scaled appropriately for the velocity of each layer. The grid space can be broadened in the high velocity layers, thus reducing the cost of the computation. In practice, I have only rescaled the vertical grid from layer to layer. To honor the original experiment geometry, I keep the horizontal grid equal to the CMP trace spacing in post-stack migration and equal to the receiver group spacing in the pre-stack common-shot-gather migration.

(ii) Imaging formation:

After propagating the surface recordings back to the earth medium, we need an imaging condition to form the migration image. The imaging condition for a post-stack migration is different from a pre-stack migration. The details are described as follows:

Post-stack migration imaging condition

CMP stacked section can be thought of as a wavefield recorded at the earth's surface, in which the shot and receiver are placed at the same location (zero-offset). The travelling path of the seismic wave from source to reflector is the same as that from reflector back to receiver. These two paths can be combined by using the exploding reflector model (Loewenthal et al, 1976).

Imaging the exploding sources are located at the reflector, and the receivers are positioned along the earth's surface at the CMP positions. The exploding reflector model states that, if we explode the sources simultaneously, the seismic section recorded along the earth's surface is largely identical with a zero-offset section, except the zero-offset section records two-way travel time, while the exploding reflector model records one-way travel time (see Figure 1-3). In practice, these two sections can be made compatible, when scaling down the propagation velocity in the exploding reflector model by a factor of two. In other words, a zero-offset section can be simulated by replacing reflectors with sources, exploding them at time $t=0$ simultaneously, recording the medium response with the propagation velocity halved.

At time $t=0$, since no propagation has occurred, the wavefront shape of the sources should be identical with the shape of reflector, which forms the imaging condition for post-stack migration. The conceptual basis of migrating a post-stack seismic section is the exploding reflector model. The amplitude of the wavefield at time equal to zero is picked as the migration result, while the velocity used for migration is one half the true material velocity.

Pre-stack common-shot gather migration imaging condition

In pre-stack common-shot gather migration, the imaging condition bases on Claerbout's (1971) idea that reflectors exist where upcoming and downgoing are time coincident (see Figure 1-4). This imaging idea constructs migration result at the reflector location, instead of time zero. To construct the migration image, we may correlate the downgoing and the upgoing wavefields or sample the upgoing wavefield at the downgoing wavefront position (de Faria et al, 1986). The upgoing wavefield for both methods is obtained by the reverse-time wavefield propagation. For the correlation method, forward modeling constructs the downgoing wavefield. The depth migration section will be the result of the summation, for all times, of the multiplication of the downgoing and upgoing wavefields. Using the correlation method, yet, requires the knowledge of the shape of the source wavelet and an expensive forward wavefield propagation.

An alternative producing the same result is sampling the reflected wavefield at the downgoing wavefront position. The sampling method only needs the common-shot gather extrapolation. Using a ray tracing method to calculate the travel time from the source to each finite-difference grid point, we can sample the upgoing wavefield at the downgoing front position. The final migrated section will be given by

$$R(x,z) = U(x,z,t(x,z)), \quad (1.2)$$

where R represents the reflectivity distribution, U denotes the upgoing wavefield, and $t(x,z)$ is the one-way travel time from the source to the finite-difference grid point (x,z) . In the study presented here, I use the sampling method to produce migration image. In practice, image of the common-shot gather migration consists of the amplitude of the

wavefield at the direct wave arrival time from the source to each finite-difference grid. The final migration result is the superposition of these collected amplitude values.

The above paragraphs described the idea of the pre-stack and the post-stack reverse-time migration. If the migration result is considered satisfactory, then we go to the last step of the layer-stripping migration algorithm, boundary determination; otherwise, we readjust the velocity, and migrate this layer again until a satisfactory image is obtained.

We may establish the initial velocity function for migration from standard CMP velocity analysis, from shallow refraction velocities, from sonic logs or and other means available. In post-stack migration, lateral coherency of the migrated image is one criterion for adjusting the velocity function. Usually, migration smiles suggest the need of a lower velocity for the next migration iteration. However, this indication may fail if truncated seismic events appear in the CMP stacked section. The truncated edges act as a point source and result in migration smiles no matter what migration velocity is used. In pre-stack migration, the postmigration common-depth gather described by Al-Yahya (1989) is found to be excellent way of judging the accuracy of the velocity function. This method states that if we choose the migration velocities correctly, migrated signals in the common-depth gather should be aligned horizontally. If the chosen migration velocity is too low, the migrated signals curve upward. The migrated common-depth gather provides an excellent velocity analysis tool for pre-stack migration, in addition, this method may be developed as an imaging focusing technique and adapt the layer-stripping method as part of a seismic inversion algorithm.

(3) BOUNDARY DETERMINATION

Although I have chosen the bottom boundary of each layer when defining the initial model, the initial locations of the bottom boundaries are assumed to be approximate. The

last step of layer-stripping reverse-time migration is to determine the proper position of the bottom boundary for a given layer. This boundary is also the top boundary for the next layer. To find the exact boundary at the bottom of each layer, I scan a zone of seismic data around the initial guess for the interface. Picking the maximum square value of the migrated wavefield for each trace determines the proper interface position. Where the migrated signal is weak or ambiguous, the user can guide the choice of the boundary location. In practice, the boundary definition procedure requires a small amount of user smoothing. After determining the bottom boundary of a layer, I collect a new seismic section along the interface. The collected seismic time section serves as the boundary value for migration in the next layer. The boundary determination step allows the user some control over the migration image.

Repeating the above procedures layer by layer, I composite the individual layer images to obtain the final migration result. The reverse-time scheme posed in this way is similar to the first step of wave equation datuming (Berryhill, 1979), but with an imaging condition.

THE ADVANTAGES OF LAYER-STRIPPING REVERSE-TIME MIGRATION

The advantages of the layer-stripping reverse-time migration algorithm are:

(1) This algorithm totally preserves the advantages of the reverse-time method. Because the use of the two-way scalar wave equation, the layer-stripping reverse-time method can handle structures with steep dips, lateral velocity variations and strong velocity contrasts between layers.

(2) Layer-stripping reverse-time migration prevents artificial interlayer multiples without any modification to the scalar wave equation. Since I execute migration one layer at a time, and choose a constant or smoothly varying velocity inside each layer, migration

generates no artificial interlayer multiples during the calculation. In addition, this algorithm avoids Fourier transform of the data set and operates in the time domain.

(3) Compared to the standard reverse-time migration, the layer-stripping scheme reduces computation expense, which is a particular advantage in time consuming pre-stack migration. In standard reverse-time migration, the computational efficiency of the finite-difference scheme is restricted by the stability and band-limiting criteria. The processing burden is especially severe when strong velocity contrasts exist in the velocity model. Using the layer-stripping method allows the vertical grid spacing to be scaled appropriately for the velocity of each layer, thus reducing the cost of the computation. For example, consider a two layer velocity model, in which the velocity of one layer is twice as large as that of the other one. By breaking the model into two constant velocity layers, and migrating through each layer individually, one can make the time step twice as large as that of the conventional reverse-time migration method in both high and low velocity layers. Moreover, the vertical grid space can be doubled in the high velocity layer. Compared to conventional reverse-time, calculation in the low velocity layer needs one half the computer time, and calculation in the high velocity layer only needs a quarter of the computer time. Since I execute migration one layer at a time, the layer-stripping migration method reduces total computer memory requirements.

(4) By using this layer-stripping technique, interpretation becomes a step of migration. The layer-stripping migration algorithm largely determines the boundary between layers computationally, with some user intervention in areas of ambiguous signal strength.

(5) The layer-stripping technique can be combined with inversion algorithms. In each migration layer, after the execution of a constant velocity migration, an imaging focusing technique can be added by smoothly varying the velocity field laterally to produce a better focused image.

The layer-stripping reverse-time migration algorithm is particularly useful for a structure with strong velocity contrasts between layers. For a gradually increasing velocity model, where no strong velocity contrast exists, the layer-stripping technique reduces to conventional one stage reverse-time migration.

OUTLINE OF THESIS

In Chapter two I present the theory of layer-stripping reverse-time migration. Three different finite-difference implementations of this migration scheme are given, a 2nd-order accurate explicit finite-difference scheme, a 10th-order space, 4th-order time explicit finite-difference approximation to the two way wave equation, and a 4th-order space, 2nd-order time implicit finite-difference scheme for the linearly transformed wave equation (Li, 1986). Attention is paid to the accuracy of these three different finite-difference schemes. Results of post-stack layer-stripping reverse-time migration are given in Chapter three. Presented are migrations of a synthetic CMP seismic section and a field data example from a fold and thrust belt in the Canadian Arctic to illustrate the capabilities of the migration algorithm. In Chapter four, pre-stack migration is discussed, again with two synthetic shot gather pre-stack migrations and with a field data example from offshore central California.

FIGURE CAPTIONS

- Figure 1-1. A two-dimensional earth model, in which seismic source is located at point S and receiver is at R. Seismic signal recorded at point R could be a reflection from different point A, B, or C of the subsurface structure.
- Figure 1-2. Velocity model with strong velocity contrasts between layers. The velocity model can be subdivided into several layers of constant or smoothly varying velocity. The approximate layer boundaries can have any geometry and are assumed to be interfaces separating layers with large velocity contrasts.
- Figure 1-3. A zero-offset section is a wavefield recorded at the earth's surface, in which the shot and receiver are placed at the same location. Imaging the exploding sources are located at the reflector, and the receivers are positioned along the earth's surface at the CMP positions. The exploding reflector model states that, if we explode the sources simultaneously, the seismic section recorded along the earth's surface is largely identical with a zero-offset section, except the zero-offset section records two-way travel time, while the exploding reflector model records one-way travel time.
- Figure 1-4. Consider a one-dimensional earth model, in which 4 receivers are located at different depths z_1 , z_2 , z_3 , and z_4 , respectively. At the shallowest depth z_1 , the downgoing wave is recorded much earlier than the upgoing wave. As the receiver goes deeper, at z_2 , the two events are recorded closer. At the reflector position z_3 , the downgoing and upgoing waves are recorded simultaneously. Below the reflector at z_4 , only the downgoing wave will be recorded.

Figure 1-1

Figure 1-2

Figure 1-3

Figure 1-4

Chapter 2

Wave Equation Development for Layer-Stripping Reverse-Time Migration

INTRODUCTION

The layer-stripping migration method is an iterative migration algorithm, which migrates a seismic section layer by layer, one layer at a time. The desired outputs from each layer include a migrated image and an unmigrated seismic time section extrapolated from the top boundary of that layer to its lower boundary. The individual migration images are composited to form the final result. The seismic time section along the lower boundary of that layer serves as a natural boundary condition for migration in the next layer. Assume that a two-dimensional velocity structure can be adequately represented as a series of layers, each of constant or smoothly varying velocity and having a laterally varying boundary. The layer-stripping migration technique involves numerically solving a boundary value problem for the source-free wave equation over a series of layers. In each layer the extrapolated wavefield $P(x,z,t)$ satisfies the scalar wave equation

$$\frac{\partial^2 P(x,z,t)}{\partial x^2} + \frac{\partial^2 P(x,z,t)}{\partial z^2} = \frac{1}{v_i(x,z)^2} \frac{\partial^2 P(x,z,t)}{\partial t^2}, \quad \text{for } (x,y) \notin R, \quad (2.1)$$

and

$$P(x,z,t) = P_R(x,z,t), \quad \text{for } (x,y) \in R. \quad (2.2)$$

Here R is the top boundary of the layer i , $v_i(x,z)$ is a constant or slowly changing velocity field of that layer with horizontal coordinate x and vertical coordinate z , and $P_R(x,z,t)$ is the recorded wavefield along the boundary R .

In principle, the observed wavefield $P_R(x,z,t)$ and its normal derivative are needed to calculate the extrapolated wavefield $P(x,z,t)$. From a practical point of view, the extrapolated $P(x,z,t)$ can be computed in a number of ways by using the observed boundary condition $P_R(x,z,t)$ only. Berryhill (1979), Wiggins (1984) and Yilmaz and Lucas (1986) used Kirchhoff technique to extrapolate data recorded on an irregular boundary to find out what would have been recorded at a different boundary. Hill and Wuenschel (1985) decomposed a wavefield recorded on an irregular boundary into plane wave (f - k) components. These plane wave components were then used to synthesize the wavefield at other points in space. Also, Clayton and McMechan (1981) used p -tau transform to downward continue refraction seismic data. The p -tau downward continuation was used to produce a depth dependent velocity function directly from the recorded data.

Here I use the reverse-time method to extrapolate the wavefield. The reverse-time method is implemented by running a finite-difference algorithm backward in time and using the time reversed observed wavefield as boundary value at the receivers. The wavefield extrapolated to the redefined boundary is represented by the amplitude of the wavefield along that boundary at every computational time step. Let E_i denote a wavefield extrapolation operator

$$E_i = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} \right) - \frac{1}{v_i(x,z)^2} \frac{\partial^2}{\partial t^2}. \quad (2.3)$$

Wavefield extrapolated layer by layer in the layer-stripping reverse-time migration scheme can be expressed as

$$E_i P(x,z,t) = 0, \quad \text{for } i = 1, \dots, N, \quad (2.4)$$

where N is the number of layers.

To illustrate the layer-stripping reverse-time method, I start with a 2nd-order finite-difference scheme. However, in practice, an explicit scheme 10th-order space, 4th-order time finite-difference operator is used to obtain a stable and accurate migration result. Also, I have implemented the layer-stripping reverse-time migration technique using a 4th-order space, 2nd-order time approximation to the linearly transformed wave equation (Li, 1986).

2ND-ORDER ACCURATE EXPLICIT SCHEME REVERSE-TIME WAVEFIELD EXTRAPOLATION

FORMULATION

The reverse-time method used to develop the layer-stripping migration technique assumes propagation of energy is governed by the scalar wave equation (2.1). I introduce a finite-difference approximation for $\partial^2 P / \partial t^2$ in Equation (2.1) using the Taylor series expansion

$$f(x) = f_0 + (x-x_0) f_0^{(1)} + \frac{(x-x_0)^2}{2!} f_0^{(2)} + \dots + \frac{(x-x_0)^n}{n!} f_0^{(n)} + R_n, \quad (2.5)$$

the value of the point $P(x,z)$ at time $t+\Delta t$ and $t-\Delta t$ can be expressed as below:

$$P(x,z,t+\Delta t) = P(x,z,t) + \frac{\partial P(x,z,t)}{\partial t} \Delta t + \frac{\partial^2 P(x,z,t)}{\partial t^2} \frac{\Delta t^2}{2} + \theta(\Delta t^3), \quad (2.6)$$

and

$$P(x,z,t-\Delta t) = P(x,z,t) - \frac{\partial P(x,z,t)}{\partial t} \Delta t + \frac{\partial^2 P(x,z,t)}{\partial t^2} \frac{\Delta t^2}{2} - \theta(\Delta t^3), \quad (2.7)$$

where $\theta(\Delta t^3)$ represents the high-order terms. Summing Equations (2.6) and (2.7) gives 3rd-order error terms cancel

$$P(x,z,t+\Delta t) + P(x,z,t-\Delta t) - 2P(x,z,t) = \frac{\partial^2 P(x,z,t)}{\partial t^2} \Delta t^2 + \theta(\Delta t^4). \quad (2.8)$$

Dividing by Δt^2 and neglecting the high-order term $\theta(\Delta t^2)$, Equation (2.8) becomes

$$\frac{\partial^2 P(x,z,t)}{\partial t^2} = \frac{P(x,z,t+\Delta t) + P(x,z,t-\Delta t) - 2P(x,z,t)}{\Delta t^2}. \quad (2.9)$$

Equation (2.9) is a 2nd-order accurate central-difference approximation for the 2nd time derivative in the scalar wave equation. Similarly, the spatial derivatives in the scalar wave equation can be expressed as

$$\frac{\partial^2 P(x,z,t)}{\partial x^2} = \frac{P(x+\Delta x,z,t) + P(x-\Delta x,z,t) - 2P(x,z,t)}{\Delta x^2}, \quad (2.10)$$

and

$$\frac{\partial^2 P(x,z,t)}{\partial z^2} = \frac{P(x,z+\Delta z,t) + P(x,z-\Delta z,t) - 2P(x,z,t)}{\Delta z^2}. \quad (2.11)$$

Suppose the finite-difference spatial and temporal grid are defined as follows:

$$\begin{aligned} x_m &= m\Delta x, \quad m=0,1,\dots,M, \\ z_n &= n\Delta z, \quad n=0,1,\dots,N, \text{ and} \\ t_l &= l\Delta t, \quad l=0,1,\dots,L, \end{aligned} \tag{2.12}$$

where $\Delta x = \Delta z = \Delta h$ is spatial grid spacing and Δt is time step. Based on the 2nd-order central-difference approximation, wave equation (2.1) can be written as

$$\begin{aligned} P(m,n,l+1) &= 2P(m,n,l) - P(m,n,l-1) - \frac{v(m,n)^2 \Delta t^2}{\Delta h^2} [P(m+1,n,l) + P(m-1,n,l) \\ &\quad + P(m,n+1,l) + P(m,n-1,l) - 4P(m,n,l)]. \end{aligned} \tag{2.13}$$

The 2nd-order finite-difference operator is shown in Figure 2-1. Equation (2.13) suggests that the knowledge of wavefield $P(x,z,t)$ at time t and $t-\Delta t$ enables extrapolation of $P(x,z,t+\Delta t)$. Solution of Equation (2.13) in forward modeling generates a seismic time response from an initial set of conditions in a velocity model. Simply by reversing the time axis, Equation (2.13) can be used for migration with the boundary values recorded at the earth's surface. For migration, the surface recording $P(x,z=0,t)$, for $0 < t < T_L$, is used as the boundary condition and is inserted at the physical boundary $z=0$, and extrapolated backward in time (see Figure 2-2). At each time step the finite-difference operator carries all the energy at the previous time step into the earth model. The calculation starts from the last time sample backward until time zero is reached. The boundary values at T_L and T_{L+1} are also needed in order to solve Equation (2.13) in the reverse-time problem, where T_L is the last recorded time sample. In reverse-time migration it is assumed that $P(x,z,t)=0$ for all $t > T_L$. After time T_L , the wave has traveled away from the medium, and the earth model

below the last seismic arrival is assumed to be homogeneous and infinite. Figure 2-3 shows the intermediate reverse-time computational steps.

Equation (2.13) is formulated from the exact two-way wave equation. Solutions of this equation allow energy to be propagated in all directions. In this case, artificial interlayer multiples are generated from where the velocity varies rapidly. Since only a constant or smoothly varying velocity function is assigned to each layer migration, the layer-stripping method prevents artificial interlayer multiples.

GRID DISPERSION

Grid dispersion limits the usefulness of the discretization schemes for the wave equation. In order to avoid grid dispersion in the 2nd-order accurate finite-difference scheme, a minimum of 10 points per wavelength sampling are required in the calculation. That is,

$$\Delta h = \frac{v_{\min}}{10f_{\max}}, \quad (2.14)$$

where Δh is the spatial grid spacing, $f_{\max}$ is the highest frequency desired, and $v_{\min}$ is the minimum velocity in the model. The stability criterion for the 2nd-order finite-difference scheme requires that the time step Δt be

$$\Delta t < \frac{\Delta h}{\sqrt{n} v_{\max}}, \quad (2.15)$$

where $v_{\max}$ is the maximum velocity in the model and n represents the number of spatial dimensions of the scalar wave equation (Alford et al, 1974, Levander, 1989).

In the forward modeling problem, the spatial grid spacing is determined according to the accuracy criterion stated in Equation (2.14). In migration, the horizontal grid spacing is kept equal the data trace separation for convenience. To reduce the spatial sampling requirements we can use high-order finite-difference scheme. For example, a 4th-order accurate finite-difference scheme requires 5 points per wavelength (Alford et al, 1974, Levander, 1989), and a 10th-order accurate finite-difference scheme requires 3 points per wavelength to prevent grid dispersion (Dablain, 1986).

In order to further decrease grid dispersion I reduce the vertical grid spacing in the calculation. This approach assumes that most of the reflection waves travel vertically. In practice, I have let the vertical grid size be one half of the horizontal one. The time step is determined according to the vertical grid spacing. If Δx is not equal to Δz , Equation (2.13) becomes

$$\begin{aligned}
 P(m,n,l-1) = & 2P(m,n,l) - P(m,n,l+1) - \frac{v(m,n)^2 \Delta t^2}{\Delta x^2} [P(m+1,n,l) + P(m-1,n,l) \\
 & - 2P(m,n,l)] - \frac{v(m,n)^2 \Delta t^2}{\Delta z^2} [P(m,n+1,l) + P(m,n-1,l) - 2P(m,n,l)]. \quad (2.16)
 \end{aligned}$$

10TH-ORDER SPACE, 4TH-ORDER TIME, EXPLICIT SCHEME REVERSE-TIME WAVEFIELD EXTRAPOLATION

A comparison between high-order and low-order finite-difference schemes for forward modeling has been published by Dablain (1986). Using high-order differencing operators require fewer grid points per wavelength to prevent temporal and spatial dispersion. In the forward modeling problem, high-order finite-difference schemes have practical advantages over low-order approaches by reducing CPU memory requirement and computational time. In migration, high-order operators relieve the grid dispersion problem.

I have used a 10th-order space, 4th-order time finite-difference scheme to the scalar wave equation for the reverse-time migration algorithm. Following Dablain (1986), the 10th-order space, 4th-order time, finite-difference scalar wave equation (2.1) is derived as follows.

FORMULATION

A temporal 4th-order accurate differencing operator is derived by expanding the value of the points $p(x,z,t+\Delta t)$ and $p(x,z,t-\Delta t)$ to high-order power series:

$$\begin{aligned} P(x,z,t+\Delta t) &= P(x,z,t) + \frac{\partial P(x,z,t)}{\partial t} \Delta t + \frac{\partial^2 P(x,z,t)}{\partial t^2} \frac{\Delta t^2}{2} + \frac{\partial^3 P(x,z,t)}{\partial t^3} \frac{\Delta t^3}{6} \\ &\quad + \frac{\partial^4 P(x,z,t)}{\partial t^4} \frac{\Delta t^4}{24} + \dots \end{aligned} \quad (2.17)$$

and

$$\begin{aligned} P(x,z,t-\Delta t) &= P(x,z,t) - \frac{\partial P(x,z,t)}{\partial t} \Delta t + \frac{\partial^2 P(x,z,t)}{\partial t^2} \frac{\Delta t^2}{2} - \frac{\partial^3 P(x,z,t)}{\partial t^3} \frac{\Delta t^3}{6} \\ &\quad + \frac{\partial^4 P(x,z,t)}{\partial t^4} \frac{\Delta t^4}{24} + \dots \end{aligned} \quad (2.18)$$

Summing Equations (2.17) and (2.18) results in

$$P(x,z,t+\Delta t) + P(x,z,t-\Delta t) - 2P(x,z,t) = \frac{\partial^2 P(x,z,t)}{\partial t^2} \Delta t^2 + \frac{\partial^4 P(x,z,t)}{\partial t^4} \frac{\Delta t^4}{12} + \dots \quad (2.19)$$

Since we are interested in the 4th-order accurate time expression, we drop the higher-order terms in Equation (2.19), giving

$$\frac{\partial^2 P(x,z,t)}{\partial t^2} = \frac{1}{\Delta t^2} [P(x,z,t+\Delta t) + P(x,z,t-\Delta t) - 2P(x,z,t)] - \frac{\partial^4 P(x,z,t)}{\partial t^4} \frac{\Delta t^4}{12}, \quad (2.20)$$

Equation (2.20) is a 4th-order accurate expression for $\partial^2 P / \partial t^2$. In order to express Equation (2.20) in an explicit finite-difference form, $\partial^4 P / \partial t^4$ must be approximated in some way. One means of doing this is to express $\partial^4 P / \partial t^4$ using the ordinary central-difference scheme directly; however, this requires more initial conditions for the wavefield. Another way to estimate $\partial^4 P / \partial t^4$ is to substitute $\partial^4 P / \partial t^4$ with spatial derivatives. Assume constant velocity, then $\partial^4 P / \partial t^4$ can be expressed as

$$\frac{\partial^4 P(x,z,t)}{\partial t^4} = \frac{\partial^2}{\partial t^2} \left(\frac{\partial^2 P(x,z,t)}{\partial t^2} \right). \quad (2.21)$$

Since

$$\frac{\partial^2 P(x,z,t)}{\partial t^2} = v^2 \left(\frac{\partial^2 P(x,z,t)}{\partial x^2} + \frac{\partial^2 P(x,z,t)}{\partial z^2} \right), \quad (2.22)$$

Equation (2.21) becomes

$$\frac{\partial^4 P(x,z,t)}{\partial t^4} = v^2 \frac{\partial^2}{\partial t^2} \left(\frac{\partial^2 P(x,z,t)}{\partial x^2} + \frac{\partial^2 P(x,z,t)}{\partial z^2} \right). \quad (2.23)$$

Equation (2.23) can be rewritten as

$$\frac{\partial^4 P(x,z,t)}{\partial t^4} = v^2 \left(\frac{\partial^2}{\partial x^2} \frac{\partial^2 P(x,z,t)}{\partial t^2} + \frac{\partial^2}{\partial z^2} \frac{\partial^2 P(x,z,t)}{\partial t^2} \right). \quad (2.24)$$

Following the same substitutions, the spatial derivatives of the scalar wave equation can be substituted for the time derivatives as

$$\frac{\partial^4 P(x,z,t)}{\partial t^4} = v^4 \left[\frac{\partial^2}{\partial x^2} \left(\frac{\partial^2 P(x,z,t)}{\partial x^2} + \frac{\partial^2 P(x,z,t)}{\partial z^2} \right) + \frac{\partial^2}{\partial z^2} \left(\frac{\partial^2 P(x,z,t)}{\partial x^2} + \frac{\partial^2 P(x,z,t)}{\partial z^2} \right) \right]. \quad (2.25)$$

Rewriting Equation (2.25) gives

$$\frac{\partial^4 P(x,z,t)}{\partial t^4} = v^4 \left(\frac{\partial^4 P(x,z,t)}{\partial x^4} + 2 \frac{\partial^4 P(x,z,t)}{\partial x^2 \partial z^2} + \frac{\partial^4 P(x,z,t)}{\partial z^4} \right). \quad (2.26)$$

In Equation (2.26), the 4th time derivative is approximated entirely with the spatial derivatives. The expression for the updated time scheme is

$$\begin{aligned} P(x,z,t+\Delta t) &= 2P(x,z,t) - P(x,z,t-\Delta t) + v^2 \Delta t^2 \left(\frac{\partial^2 P(x,z,t)}{\partial x^2} + \frac{\partial^2 P(x,z,t)}{\partial z^2} \right) \\ &+ \frac{v^4 \Delta t^4}{12} \left(\frac{\partial^4 P(x,z,t)}{\partial x^4} + 2 \frac{\partial^4 P(x,z,t)}{\partial x^2 \partial z^2} + \frac{\partial^4 P(x,z,t)}{\partial z^4} \right) \end{aligned} \quad (2.27)$$

The derivatives are then expanded to 10th-order accurate finite-difference expression, as

$$\frac{\partial^2 P(x,z,t)}{\partial x^2} = \frac{1}{\Delta x^2} \left[\omega_0 P(x,z,t) + \sum_{k=1}^5 \omega_k (P(x+k\Delta x,z,t) + P(x-k\Delta x,z,t)) \right], \quad (2.28)$$

in which the derived coefficients ω_k , $k=0,5$ are listed in Table 2-1.

The 4th-order spatial derivatives in Equation (2.27) are computed with the ordinary central-difference of the 2nd-order spatial derivatives:

$$\frac{\partial^4 P(x,z,t)}{\partial x^4} = \frac{1}{\Delta x^2} \left[\frac{\partial^2 P(x+\Delta x,z,t)}{\partial x^2} + \frac{\partial^2 P(x-\Delta x,z,t)}{\partial x^2} - 2 \frac{\partial^2 P(x,z,t)}{\partial x^2} \right], \quad (2.29)$$

$$\frac{\partial^4 P(x,z,t)}{\partial z^4} = \frac{1}{\Delta z^2} \left[\frac{\partial^2 P(x,z+\Delta z,t)}{\partial z^2} + \frac{\partial^2 P(x,z-\Delta z,t)}{\partial z^2} - 2 \frac{\partial^2 P(x,z,t)}{\partial z^2} \right], \text{ and} \quad (2.30)$$

$$\frac{\partial^4 P(x,z,t)}{\partial x^2 \partial z^2} = \frac{1}{\Delta x^2} \left[\frac{\partial^2 P(x+\Delta x,z,t)}{\partial z^2} + \frac{\partial^2 P(x-\Delta x,z,t)}{\partial z^2} - 2 \frac{\partial^2 P(x,z,t)}{\partial z^2} \right]. \quad (2.31)$$

The final 10th-order space, 4th-order time different grid size finite-different approximation to the scalar wave equation for reverse-time migration is

$$\begin{aligned} & P(m,n,l-1) \\ &= 2P(m,n,l) - P(m,n,l+1) \\ &+ v^2 \Delta t^2 \left\{ \frac{1}{\Delta x^2} \left[\omega_0 P(m,n,l) + \sum_{k=1}^5 \omega_k (P(m+k,n,l) + P(m-k,n,l)) \right] \right. \\ &+ \frac{1}{\Delta z^2} \left[\omega_0 P(m,n,l) + \sum_{k=1}^5 \omega_k (P(m,n+k,l) + P(m,n-k,l)) \right] \left. \right\} \\ &+ \frac{v^4 \Delta t^4}{12} \left\{ \frac{1}{\Delta x^4} \left[\omega_0 P(m+1,n,l) + \sum_{k=1}^5 \omega_k (P(m+1+k,n,l) + P(m+1-k,n,l)) \right] \right. \\ &+ \omega_0 P(m-1,n,l) + \sum_{k=1}^5 \omega_k (P(m-1+k,n,l) + P(m-1-k,n,l)) \\ &- 2 \omega_0 P(m,n,l) + \sum_{k=1}^5 \omega_k (P(m+k,n,l) + P(m-k,n,l)) \left. \right] \\ &+ \frac{1}{\Delta z^4} \left[\omega_0 P(m,n+1,l) + \sum_{k=1}^5 \omega_k (P(m,n+1+k,l) + P(m,n+1-k,l)) \right. \\ &+ \omega_0 P(m,n-1,l) + \sum_{k=1}^5 \omega_k (P(m,n-1+k,l) + P(m,n-1-k,l)) \left. \right\} \end{aligned}$$

$$\begin{aligned}
& - 2 \omega_0 P(m,n,l) + \sum_{k=1}^5 \omega_k (P(m,n+k,l) + P(m,n-k,l)))] \\
& + 2 \left[\frac{1}{\Delta x^2 \Delta z^2} (\omega_0 P(m+1,n,l) + \sum_{k=1}^5 \omega_k (P(m+1,n+k,l) + P(m+1,n-k,l))) \right. \\
& + \frac{1}{\Delta x^2 \Delta z^2} (\omega_0 P(m-1,n,l) + \sum_{k=1}^5 \omega_k (P(m-1,n+k,l) + P(m-1,n-k,l))) \\
& \left. - \frac{2}{\Delta x^2 \Delta z^2} (\omega_0 P(m,n,l) + \sum_{k=1}^5 \omega_k (P(m,n+k,l) + P(m,n-k,l))) \right]. \tag{2.32}
\end{aligned}$$

Equation (2.32) is correct for a constant velocity layer. When the velocity is smoothly varying, there will be error introduced. More works are needed to determine how the wavefield is sensitive to the error. The operator for the 10th-order space, 4th-order time finite-difference operator is shown in Figure 2-4. The 4th-order time, 10th-order space operator is replaced with 2nd-order time, 2nd-order space and 2nd-order time, 4th-order space operators near the grid boundaries.

4TH-ORDER IMPLICIT SCHEME REVERSE-TIME WAVEFIELD PROPAGATION

Finite-difference implementation of the scalar wave equation can be either in explicit or implicit scheme. Explicit formulations express the value of a point at future time in terms of the value of that point and its neighboring points at present time and past time. On the other hand, implicit methods express the value of points at future time in terms of the value of those points at future time, present time and past time. Finite-difference solutions of the scalar wave equation have traditionally used explicit formulations. The problem with explicit finite-difference formulation of the scalar wave equation is that they are limited by the stability criterion, which requires taking a small time step for wavefield propagation.

Implicit finite-difference equations have been shown to have greater stability ranges than explicit formulations (Lindemuth and Killeen, 1973; Mitchell and Griffiths, 1980; and Emerman et al, 1982). Time step in implicit differencing schemes can be broadened to reduce the calculational time.

FINITE-DIFFERENCE FORMULATION OF THE LINEARLY TRANSFORMED WAVE EQUATION

I have formulated an implicit scheme 4th-order space, 2nd-order time reverse-time migration scheme based on the linearly transformed wave equation (Li, 1986)

$$\frac{\partial^2 P}{\partial x^2} + \frac{2}{v} \frac{\partial^2 P}{\partial t' \partial z'} = 0. \quad (2.33)$$

For a constant velocity medium, by using the transformations

$$x' = x, \quad (2.34)$$

$$z' = \frac{1}{\sqrt{2}}(z - vt), \text{ and} \quad (2.35)$$

$$t' = \frac{1}{\sqrt{2}}\left(\frac{z}{v} + t\right), \quad (2.36)$$

the linearly transformed wave equation is an exact transformation of the scalar wave equation (2.1), therefore, there is no dip limitation when Equation (2.33) is used for wavefield extrapolation. For consideration of stability, Equation (2.33) is implemented in implicit scheme.

Using the 4th-order central-difference finite-difference expression

$$\frac{\partial^2 P(x,z,t)}{\partial x^2} = \frac{1}{12\Delta x^2} \{-P(x+2\Delta x,z,t) + 16P(x+\Delta x,z,t) - 30P(x,z,t) + 16P(x-\Delta x,z,t) - P(x-2\Delta x,z,t)\}, \quad (2.37)$$

the linearly transformed wave equation (2.33) can be rewritten as

$$\frac{\partial^2 P}{\partial t' dz'} = \frac{v}{24\Delta x^2} \{-P(x+2\Delta x,z,t) + 16P(x+\Delta x,z,t) - 30P(x,z,t) + 16P(x-\Delta x,z,t) - P(x-2\Delta x,z,t)\}. \quad (2.38)$$

Let $P_{t',z'}$ denote a vector along the x-axis and T a pentadiagonal matrix for the $\partial^2/\partial x^2$ operator with coefficients $(-1,16,-30,16,-1)$ along its diagonal equation. Equation (2.38) becomes

$$\frac{\partial^2 P_{t',z'}}{\partial t' dz'} = \frac{v}{24\Delta x^2} T P_{t',z'}. \quad (2.39)$$

To solve Equation (2.39) using the Crank-Nicolson scheme, we first do the centered time differencing

$$\frac{\partial P_{t',z'}}{dz'} (P_{t'+\Delta t',z'} - P_{t',z'}) = \frac{v\Delta t'}{48\Delta x^2} T (P_{t'+\Delta t',z'} + P_{t',z'}), \quad (2.40)$$

then we do the centered space differencing

$$(P_{t'+\Delta t',z'+\Delta z'} - P_{t',z'+\Delta z'}) - (P_{t'+\Delta t',z'} - P_{t',z'})$$

$$= \frac{v\Delta z'\Delta t'}{96\Delta x'^2} T (P_{t'+\Delta t', z'+\Delta z'} + P_{t', z'+\Delta z'} + P_{t'+\Delta t', z'} + P_{t', z'}). \quad (2.41)$$

Let $\alpha = v\Delta z'\Delta t'/96\Delta x'^2$ and I be an identity matrix. The resulting finite-differencing representation of Equation (2.41) becomes

$$(I + \alpha T)P_{t', z'+\Delta z'} = (I - \alpha T)(P_{t', z'} + P_{t'+\Delta t', z'+\Delta z'}) - (I + \alpha T)P_{t'+\Delta t', z'} \quad (2.42)$$

Equation (2.42) shows that the elements of the unknown vector on the left-hand side are given implicitly by the elements of the three known vectors on the right-hand side. The 4th-order accurate finite-difference operator is shown in Figure 2-5. Wave propagation in the transformed system can be characterized by the scalar wave equation, if the coordinates (z', t') is rotated by 45 degrees. Figure 2-6 shows the finite-difference grid of the linearly transformed wave equation in both coordinates (z, t) and (z', t') . Surface data $P(x, z=0, t)$ corresponds to $P(x, z'=-vt/\sqrt{2}, t'=t/\sqrt{2})$, and the post-stack migration image $P(x, z, t=0)$ corresponds to $P(z'=z/\sqrt{2}, x, t'=z/(v\sqrt{2}))$ in the transformed system. Equation (2.42) is an exact transformation from a two-way wave equation, but is transformed to a parabolic equation which has the same numerical efficiency as the one-way 15-degree wave equation. Equation (2.42) can be implemented for reverse-time migration. Figure 2-7 displays the successive step of solving Equation (2.42) in the reverse-time direction. For each step in the $z'-t'$ plane, a banded-matrix solution is used to compute the wavefield along the whole x -axis. Generally, implicit finite-difference schemes have the advantage of greater stability over explicit schemes. Using an implicit finite-difference method allows us to enlarge the time step, thus reducing the computational cost.

ABSORBING BOUNDARY CONDITIONS

To simulate an infinite earth structure in a finite velocity model, it is necessary to apply boundary conditions that make the edges of the computational grid appear transparent. In this way, artificial reflections introduced by the edges of the computational region can be minimized.

For the 2nd-order accurate finite-difference scheme, I have used the A-2 absorbing boundary condition (Clayton and Engquist, 1977) to prevent artificial boundary reflections. The A-2 absorbing boundary condition is a 2nd-order paraxial approximation of the scalar wave equation, which propagates waves in one direction within a limited propagation angle. This equation can be used to model an outgoing traveling wave; thus, artificial reflections from the finite-difference grid boundary are reduced.

To eliminate the boundary edge reflections, I have applied a sponge layer transmission boundary condition (Israeli and Orszag, 1981; Dablain, 1986) to the explicit 10th-order space, 4th-order time finite-difference scheme. The sponge layer boundary condition radiates the outgoing seismic waves, and uses a damping term to attenuate the incoming residual boundary reflections.

Clayton and Engquist (1980) designed the B3 absorbing boundary condition for wave equation migration. For the implicit 4th-order space, 2nd-order time finite-difference algorithm, I have used the B3 boundary condition on both ends of the x-axis to minimize boundary reflections.

IMPULSE RESPONSES

Migration of a single isolated wavelet on a single trace through a constant velocity medium provides an impulse response of the migration algorithm. The desired output in the

zero-offset section is a semicircle, which indicates the dip limitations of migration algorithm. Figure 2-8 shows the impulse response of the 2nd-order reverse-time migration algorithm. The velocity model used for this migration is a 1750 m/sec constant velocity medium, the horizontal grid spacings is 20 meters and the vertical grid spacing is 10 m. The dominant frequency for the input wavelet is 35 Hz. The result shown in Figure 2-8 is a semicircle with very noisy background. Since the finite-difference formula is approximated directly from the scalar wave equation, the semicircle is expected. Because the wavefield is undersampled, grid dispersion appears, which allows seismic wave propagating accurately up to 23 degrees.

Figure 2-9 shows impulse response of the 10th-order reverse-time migration scheme. To maintain the most stable migration result, a time step $\Delta t < \Delta h / (2v_{\max})$ suggested by Dablain (1986) is used in the calculation. For the same parameters used to generate Figure 2-8, the response in Figure 2-9 shows a clear semicircle, which indicates that the 10th-order scheme is valid for events with any propagating angle. The impulse response was migrated on a ELXSI 6400 minisuper computer. It required 2.2 hours CPU time to produce the impulse response in Figure 2-9 when the 10th-order scheme is used. For comparison, an impulse response of the 45-degree depth migration algorithm is generated and displayed in Figure 2-10. The 45-degree depth migration program is part of a commercial seismic data processing package (DISCO) installed in ELXSI 6400. Without using the array processor, the 45-degree migration used 0.5 hours CPU time to finish the impulse response in Figure 2-10. The computational efficiency for the 45-degree migration is 4 times faster than the 10th-order scheme. However, the heart shape impulse response of the 45-degree migration scheme shows that its migration dip is limited to 65 degrees, which is much less than the all dips 10th-order finite-difference reverse-time migration.

Figure 2-11 displays the impulse response of the implicit 4th-order differencing reverse-time migration scheme when the same parameters as in Figure 2-8 are used. Grid

dispersion now appears inside the semicircle. The dip limitation of the implicit 4th-order reverse-time migration is less than 35 degree. Compared with the impulse responses shown in 2-8 and 2-9, the 4th-order implicit scheme is more accurate than the 2nd-order scheme, but much less accurate than the 10th-order explicit scheme. 0.6 hours CPU time was used to generate the response shown in Figure 2-11, which is about a quarter of the 10th-order scheme. The efficiency of the implicit scheme 4th-order reverse-time migration is similar to the 45-degree depth migration, but the 4th-order scheme is less accurate. In practice, the 4th-order implicit scheme has an accuracy limitation and is not used.

k	w
0	-2.927222222
1	1.666666667
2	-0.238095238
3	0.039682539
4	-0.004960317
5	0.000317460

Table 2-1

FIGURE CAPTIONS

- Figure 2-1. Explicit scheme 2nd-order finite-difference operator. The finite-difference spatial and temporal grid are defined as follows: $x_m = m\Delta x$, $z_n = n\Delta z$, and $t_l = l\Delta t$, where Δx and Δz are spatial grid spacing and Δt is time step.
- Figure 2-2. For reverse-time migration, the surface recording $P(x, z=0, t)$, for $0 < t < T_L$, is used as the boundary condition and is inserted at the physical boundary $z=0$, and extrapolated backward in time. The calculation starts from the last time sample backward until time zero is reached.
- Figure 2-3. Successive steps of reverse-time migration. At each time step the finite-difference operator carries all the energy at the previous time step into the earth model. In this figure, d represents the surface recordings.
- Figure 2-4. Explicit scheme 10th-order space, 4th-order time finite-difference operator, in which the 4th time derivative is approximated entirely with the spatial derivatives. The finite-difference spatial and temporal grid are defined the same as that in Figure 2-1.
- Figure 2-5. Implicit scheme 4th-order space, 2nd-order time finite-difference operator. The finite-difference spatial and temporal grid are defined the same as that in Figure 2-1.
- Figure 2-6. Finite-difference grid of the linearly transformed wave equation in both coordinates of before and after transformation. In this figure, d denotes the surface recording, and i represents the post-stack migration image. Surface data $P(x, z=0, t)$ corresponds to $P(x, z'=-vt/\sqrt{2}, t'=t/\sqrt{2})$, and the post-stack migration image $P(x, z, t=0)$ corresponds to $P(z'=z/\sqrt{2}, x, t'=z/(v\sqrt{2}))$ in the transformed system.
- Figure 2-7. Successive steps of solving the linearly transformed wave equation in the reverse-time direction. At each time step the finite-difference operator carries all the energy at the previous time step into the earth model. In this figure, d denotes the surface recording, and i represents the post-stack migration image.
- Figure 2-8. Impulse response of the 2nd-order scheme reverse-time migration. The velocity model used for this migration is a 1750 m/sec constant velocity medium, the horizontal grid spacings is 20 meters and the vertical grid spacing is 10 m. The dominant frequency for the input wavelet is 35 Hz. Since the finite-difference formula is approximated directly from the scalar wave equation, the semicircle is expected. Because the wavefield is undersampled, grid dispersion appears, which allows seismic wave propagating accurately up to 23 degrees.
- Figure 2-9. Impulse response of the 10th-order accurate reverse-time migration. The parameters used to generate this figure are the same as that used to generate the response in Figure 2-6. The clear semicircle shown in the figure

indicates that the 10th-order scheme is valid for events with any propagating angle. It took 2.2 hours CPU time to migrate the point response by using a ELXSI 6400 minisuper computer.

Figure 2-10. Impulse response of the 45-degree depth migration. The heart shape impulse response of the 45-degree migration scheme shows that its migration dip is limited to 65 degrees, which is much less than the all dips 10th-order finite-difference reverse-time migration. The 45-degree depth migration program is part of a commercial seismic data processing package (DISCO) installed in ELXSI 6400. Without using the array processor, the 45-degree migration used 0.5 hours CPU time to finish this impulse response.

Figure 2-11. Impulse response of the implicit 4th-order differencing reverse-time migration scheme when the same parameters as in Figure 2-8 are used. The dip limitation of the implicit 4th-order reverse-time migration is less than 35 degree. The 4th-order implicit scheme is more accurate than the 2nd-order scheme, but much less accurate than the 10th-order explicit scheme. 0.6 hours CPU time was used to generate the response shown here.

Figure 2-1

Initial condition: $\phi(z, x, t > t_{\text{last}}) = 0$

Boundary value: $\phi(z=0, x, t) = \text{surface data}, 0 < t < t_{\text{last}}$

Figure 2-2

Figure 2-3

Figure 2-5

Figure 2-6

Figure 2-7

Figure 2-8

Figure 2-9

Figure 2-10

Figure 2-11

Chapter 3

Post-Stack Layer-Stripping Reverse-Time Migration

INTRODUCTION

CMP stacking is the most widely used means of increasing the S/N ratio in the standard seismic data processing, it also attenuates coherent noise such as multiples. Since the NMO correction before CMP stacking is implemented by using the velocities of the primary reflections, the stacking velocity of multiples are different from primaries, multiples become attenuated when stacked. With the help of the exploding reflector model, the migration of a CMP stacked section can be considered as propagating the surface recordings back to their initial time $t=0$, while the propagation velocity is one half of the true medium velocity. At time $t=0$, the amplitudes of the wavefield are picked as the migration image. Layer-stripping reverse-time migration can be applied to post-stack or pre-stack seismic data. In this chapter, I present migrations of a synthetic 28-fold CMP seismic section and a 6-fold CMP section from a fold and thrust belt in the Canadian Arctic Islands to demonstrate the capability of the layer-stripping reverse-time migration. The results of a 45-degree finite-difference depth migration are also displayed for comparison.

SYNTHETIC DATA EXAMPLE

In this example, the synthetic CMP-stacked section is obtained from synthetic shot records, which are processed with a standard CMP survey procedure to form the stacked section. Synthetic shot records are generated using a 4th-order accurate space, 2nd-order accurate time two-dimensional finite-difference forward modeling program. In the forward

modeling program, a band-limited source is inserted in the velocity model according to a source insertion principle described in Alterman and Karal (1968), Kelly et al (1976), Aki and Richards (1980), and Levander (1989). The Clayton and Engquist (1977) A2 absorbing boundary condition is applied at the finite-difference grid edge to minimize boundary reflections.

DATA DESCRIPTION

Synthetic shot records are generated from the complex velocity model shown in Figure 3-1. The model is 8 km wide and 3 km deep, and is composed of three irregular layers with velocities $v_1 = 3,500$ m/sec, $v_2 = 5,000$ m/sec, and $v_3 = 3,250$ m/sec. The principal structural features in the model are an anticline, a syncline, and a 45-degree dipping layer. The finite-difference program was used to generate 78 synthetic shot records, each 2 seconds in length with a 4 ms sampling interval. The first shot is at 920 m surface position, the shot interval is 80 m. Receivers are evenly distributed at 40 m intervals. This geometry simulates a 20 m midpoint interval CMP survey.

Figure 3-2 shows the CMP stacked section for this model, average fold is 28. Prominent features on the CMP section are: diffraction hyperbolas which originates from the tip of the anticlines and the intersections of two events, a bowtie from the syncline, and velocity pull-up below the 45-degree dipping event. Because of the negative reflection coefficient at the second interface, the lower event has reverse polarity.

Migration moves seismic events back to their correct positions, and therefore, has the effects of tightening anticlines, untieing bowties from synclines, and moving dipping events in the updip directions. Time migration result of the CMP section is shown in Figure 3-3, in which the velocity model for migration is obtained from the CMP stacking velocity analysis. As can be seen, the first interface has been correctly imaged. However, the time

migration has failed to position the image of the second layer correctly. The shape of the anticline is distorted, and residual velocity pull-up is visible below the 45-degree dipping event. The imperfections of the migration image of the second interface are due to the use of time migration, which can not handle lateral velocity variations. It is necessary to apply a depth migration algorithm to achieve a correct migration.

LAYER-STRIPPING REVERSE-TIME MIGRATION SEQUENCE AND RESULT

The success of a migration algorithm directly depends on the correct choice of migration velocity. To demonstrate the capability of the layer-stripping reverse-time migration scheme, I have used the correct velocity function in Figure 3-1 to migrate the CMP stacked section shown in Figure 3-2. As the first step of the layer-stripping migration algorithm, I divide the velocity model into three migration layers according to the velocity model (see Figure 3-4). At this moment, only the position of the uppermost boundary, i.e. the surface, is considered to be known exactly. Positions of the boundaries separating the first layer from the second layer are approximate.

The computation region for migrating the first layer is shown in Figure 3-5. The horizontal grid spacing is chosen to be 20 m, equivalent to the midpoint interval of the stack section. A 10 m vertical grid spacing is used in calculation. In order to minimize grid dispersion in the 10th-order finite-difference scheme, the shortest wavelength propagated on the grid must be at least three times the grid dimension. Based on the vertical grid size (10 m) and the propagation velocity (1750 m/sec), I have applied a 58Hz low-pass filter to the CMP stack before migration. The finite-difference time step for the calculation is 2.857 ms. The CMP stack is inserted along the dotted line on the top boundary of the region. The shaded zone in Figure 3-5 represents the initial guess region for the bottom boundary of the first layer. At each time step, seismic data within this zone are saved. The migration image

inside this zone is scanned to define the proper position for the bottom boundary of the first layer.

The migration image for the first layer is displayed in Figure 3-6. The seismic events, including the dipping events and the anticline, are correctly migrated. Figure 3-7 shows the seismic time section collected along the newly defined boundary. This time section is used for migration in the second layer after it is scaled by the interface transmission coefficient. In a zero-offset section, downward continuation of a wavefield from one boundary to another can be thought of as moving sources and receivers together from one boundary to another. Figure 3-7 represents an unmigrated time section along the top boundary of the second layer.

Following the same procedure, the second layer is migrated and its bottom boundary is determined. Figure 3-8 shows the computation region for migration in the second layer. In this migration, a constant velocity 2500 m/sec is assigned to every grid point in the computation region. The newly collected unmigrated time section (Figure 3-7) is applied along the nonplanar interface (dotted line) and propagated through the computation region.

In the migration of the second layer, the horizontal grid spacing is kept the same as that in the first layer, but the vertical grid spacing is rescaled to 14 m based on the frequency 58 Hz and the velocity 2500 m/sec. The use of this grid spacing sets the time step for the calculation in this layer at 2.8 ms. The enlarged vertical grid spacing saves some computation time in the second layer.

Figure 3-9 shows the composited migration result of the 1st layer and the 2nd. Images of the first two layers are correctly migrated. Because of the negative reflection coefficient, the second interface appears with reversed polarity. Figure 3-10 shows the new boundary value for the migration in the 3rd layer. Since there is no reflector beneath the second interface, no further migration is performed. The final layer-stripping reverse-time migration result is shown in Figure 3-11, displayed twice; once with normal and reverse

polarity. The structures for the velocity model in Figure 3-1 have been correctly migrated; the dipping event, the anticline and the syncline are properly positioned.

For comparison, I have migrated the same CMP stacked section using Claerbout's 45-degree finite-difference migration. Figure 3-12 shows the 45-degree depth migration result when the same input and model as those in the above example are used. The 45-degree depth migration result is also displayed with normal and reverse-time polarity. The two algorithms produce comparable results.

FOLD AND THRUST BELT DATA EXAMPLE

As a practical illustration, I have migrated a CMP stacked section from a complex fold and thrust belt on Melville Island in the Canadian Arctic Islands. The reflection seismic line 1660 collected by Panarctic Oils Limited was processed at Rice with careful attention paid to muting, velocity analysis, and elevation and residual static corrections (Harrison, unpublished report, 1987). Figure 3-13 shows the general location of this reflection seismic line. In this part of the Arctic Islands, the geologic structures are dominated by long continuous folds with upright, steep-limbed anticlines, and wide, flat-bottomed synclines (Tozer and Thorsteinsson, 1964). Individual anticlines are paired with buried thrusts (Fox, 1985). Intermediate detachment levels are required to reconcile the differences between surface and subsurface structures (Harrison and Bally, 1988). Line 1660 crosses the Robertson Point Anticline nearly perpendicular to the regional tectonic fabric. Strong velocity contrasts between layers are observed from velocity analysis and sonic logs from nearby wells. The interval velocity ranges from approximately 3.0 to 4.3 km/sec in the near surface foredeep siliciclastic formations to 6.1 to 6.4 km/sec in the deep carbonates. The intervening bedded halite and dolostone of the Bay Fiord Formation (an important

decoupling level) have interval velocities of about 5.2 to 5.5 km/sec.(Harrison, unpublished report, 1987).

FIELD DATA

Figure 3-14 shows the 6-fold CMP stacked section in Robertson Point Anticline with a 30 Hz low-pass filter applied. The midpoint interval of this section is 33.5 m, 4-second of data are shown. Apparent in the data is an anticline which is cored by blind thrust faults. Inside the anticline the image is poor. Much detail in the CMP stacked section is missing below the steep-limbed anticlines. High angle and low angle diffractions appear associated with the faults. Geologic information suggests deformations occurred above the interface at 2600 ms; reflectors below 2600 ms should be flat (Harrison and Bally, 1988). Due to the complicated structure inside the core of the anticline, the quality of the CMP stack is poor; most importantly several truncated reflection segments appear beneath the anticline. No matter what migration algorithm is used, migration smiles will be produced from the truncated reflections. Reducing the migration velocity can decrease the smiles from these truncated reflections but can not solve the problem.

A time migration of the CMP stack is displayed in Figure 3-15, most of the diffraction energy is collapsed, and the truncated reflections are brought closer together. The expected overmigration smiles are visible in the time migration results. However, time migration is inadequate in a region of complex velocity, since it does not obey Snell's law making it incapable of handling the seismic waves transmitted through a complex overburden (Larner et al, 1981). A depth migration is required to produce a reliable image. Since steep dips, strong velocity contrasts and lateral velocity variations exist in this region, migration of the data set can demonstrate the feasibility of the layer-stripping reverse-time migration algorithm.

LAYER-STRIPPING REVERSE-TIME MIGRATION RESULTS

As a preliminary migration result, the CMP section is migrated using the 2-grid velocity model shown in Figure 3-15. The first guess velocity model is obtained from sonic velocities from the nearby well. The success of the migration (Figure 3-17) is shown by the steep events on the flanks of the anticline, the collapse of diffractions in the core of the anticline, and the ability to trace the fault planes and the sub-horizontal reflectors beneath the anticline. However, the image at the interior of the anticline is still confused. Undermigrated diffraction tails can be seen at the faults. Increase the velocity in the second layer may improve the undermigrated image; however, doing so will enhance the migration smiles beneath the anticline, thus diminishing the capability of tracing the sub-horizontal reflectors.

In this migration example, the boundary between layers is largely determined by computer program, with some manual intervention in areas of ambiguous signal strength. For instance, above the core of the anticline, where a clear interface can not be traced, a smooth curve is chosen as the boundary. Although the boundary position is determined automatically, it may zigzag, requiring a small amount of lateral smoothing to obtain a smooth surface for migration in the next layer.

For comparison, the CMP section is migrated with the same velocity model using a 45-degree finite-difference depth migration algorithm (Figure 3-18). The depth migration produces a result comparable to the layer-stripping reverse-time migration. To further improve the image of this section, a more complicated velocity model must be devised.

The CMP section was migrated using a 3 layers velocity model (see Figure 3-19). Velocities were obtained from velocity analyses on a number of neighboring CMP traces. Figure 3-20 shows the layer-stripping migration result. Compared with the migration result

using the 2-grid model, the diffracted energy is better collapsed. Two thrust faults on the left-hand side of the anticline around 6 km deep can be better traced, and the detachment surface is aligned better, although still accompanied with migration smiles. The core of the anticline is still not clear, particularly note the crossing events present. The conventional depth migration is displayed in Figure 3-21. The two migrations produce comparable results.

A more complicated velocity model with 6 layers is used to further improve the image. Figure 3-22 shows the 6-layer velocity model, in which velocities from the semblance analysis is combined with the sonic velocities from the nearby well. To flatten the detachment surface and the interfaces below it, I have chosen a low velocity layer inside the anticline. The velocity, 4.2 km/sec, is an imaging velocity, which doesn't represent the true lithological velocity. The layer-stripping migration is displayed in Figure 3-23. The image inside the anticline is clearer and the interfaces beneath the detachment surface are better flattened than with migration using the 3 layers model.

Figure 3-24 shows the migration result using the 45-degree depth migration scheme. It produces a comparable result, but smoother image in the core of the anticline. The smoother appearance of the migration can be explained with the point diffraction responses shown in Figure 2-10. Above 65 degrees, the impulse response of the 45-degree wave equation is dominated by nonpropagating evanescent waves inside the semicircle (Claerbout, 1985), the high dip energy is smeared throughout the image, giving it a smoother appearance.

To further improve the image requires more accurate velocity model and a better stack or else pre-stack depth migration. Since the area is very complicated, a more accurate velocity function can be obtained from forward modeling.

CONCLUSION

Layer-stripping reverse-time migration preserves the advantages of reverse-time migration while avoiding interlayer multiples and somewhat reducing computational expense in high velocity layers. I have demonstrated that this algorithm is useful for post-stack migration in structural models with strong velocity contrasts between layers. In addition it gives the user some interpretational control over the final image. However, the computational efficiency is restricted by the stability and band-limiting criteria of finite-difference scheme. The efficiency of the layer-stripping reverse-time migration scheme is expected to increase considerably by the use of the linearly transformed wave equation (Li, 1986) and an implicit finite-difference scheme. Figure 3-25 shows the synthetic data migration results, when the 10th-order explicit reverse-time scheme, the 4th-order implicit reverse-time scheme, and the 45-degree depth migration are used to migrate the CMP stack (Figure 3-2) using the velocity model shown in Figure 3-5. Although the 4th-order implicit scheme and the 45-degree depth migration are 4 times faster than the explicit 10th-order scheme; the 4th-order implicit operator generates unacceptable grid dispersion along the dipping events.

FIGURE CAPTIONS

- Figure 3-1. Two-dimensional velocity model used to generate synthetic shot records. The model is 8 km wide and 3 km deep, and is composed of three irregular layers with velocities $v_1 = 3,500$ m/sec, $v_2 = 5,000$ m/sec, and $v_3 = 3,250$ m/sec. The principal structural features in the model are an anticline, a syncline, and a 45-degree dipping layer.
- Figure 3-2. The CMP stacked section for the model shown in Figure 3-1, average fold is 28. Prominent features on the CMP section are: diffraction hyperbolas which originates from the tip of the anticlines and the intersections of two events, a bowtie from the syncline, and velocity pull-up below the 45-degree dipping event. Because of the negative reflection coefficient at the second interface, the lower event has reverse polarity. The CMP stack is obtained from 78, 2-second, synthetic shot records, which are processed with a standard CMP survey procedure to form the stacked section. Synthetic shot records are generated using a 4th-order accurate space, 2nd-order accurate time two-dimensional finite-difference forward modeling program. The shot interval is 80 m, and the receiver interval 40 m intervals. This geometry simulates a 20 m midpoint interval CMP survey.
- Figure 3-3. Time migration of the CMP stack shown in Figure 3-3. The velocity model for migration is obtained from the CMP stacking velocity analysis. In the figure, the first interface has been correctly imaged; however, the second layer is not correctly imaged. Below the first interface, the shape of the anticline is distorted. Residual velocity pull-up is visible below the 45-degree dipping event.
- Figure 3-4. Velocity model for layer-stripping reverse-time migration. The velocity model is divided into three migration layers according to the velocity model in Figure 3-4. At this moment, only the position of the uppermost boundary, ie the surface, is considered to be known exactly. Positions of the boundaries separating the first layer from the second layer are approximate.
- Figure 3-5. The computation region for migrating the first layer. The CMP stack is inserted along the dotted line on the top boundary of the region. The shaded zone represents the initial guess region for the bottom boundary of the first layer. At each time step, seismic data within this zone are saved. The migration image inside this zone is scanned to define the proper position for the bottom boundary of the first layer. A sponge-layer transmission boundary is applied around the computation region to prevent boundary reflections.
- Figure 3-6. The migration image for the first layer. The seismic events, including the dipping events and the anticline, are correctly migrated. The horizontal grid spacing for reverse-time calculation is 20 m, equivalent to the midpoint interval of the stack section. A 10 m vertical grid spacing is used. Based on the vertical grid size (10 m) and the propagation velocity (1750 m/sec), I

have applied a 58Hz low-pass filter to the CMP stack in Figure 3-2 before migration. The finite-difference time step is 2.857 ms.

- Figure 3-7. The unmigrated seismic time section collected along the newly defined lower boundary of the first layer. This time section is used for migration in the second layer after it is scaled by the interface transmission coefficient.
- Figure 3-8. The computation region for migration in the second layer. A constant velocity 2500 m/sec is assigned to every grid point in the computation region. The newly collected time section in Figure 3-7 is applied along the nonplanar interface (dotted line) and propagated through the computation region. The shaded zone represents the initial guess region for the bottom boundary of the second layer. A sponge-layer transmission boundary is applied around the computation region to prevent boundary reflections.
- Figure 3-9. The composited migration result of the 1st layer and the 2nd. Images of the first two layers are correctly migrated. Because of the negative reflection coefficient, the second interface appears with reversed polarity. In migration, the horizontal grid spacing is kept the same as that in the first layer, but the vertical grid spacing is rescaled to 14 m based on the frequency 58 Hz and the velocity 2500 m/sec. The time step for the calculation in this layer is 2.8 ms.
- Figure 3-10. The seismic time section collected along the lower boundary of the second layer. This time section is the new boundary value for the migration in the 3rd layer.
- Figure 3-11a. The final layer-stripping reverse-time migration result displayed with normal polarity. The structures for the velocity model in Figure 3-1 have been correctly migrated; the dipping event, the anticline and the syncline are properly positioned.
- Figure 3-11b. The final layer-stripping reverse-time migration result displayed with reverse polarity.
- Figure 3-12a. The 45-degree depth migration result displayed with normal polarity. The same input and model as those in the layer-stripping migration are used. The structures have been correctly migrated.
- Figure 3-12b. The 45-degree depth migration result displayed with reverse polarity.
- Figure 3-13. Maps showing the location of the reflection seismic line Melville Island 1660. The * sign shown in the upper figure and the veavy solid line shown in the bottom figure indicate the location of the seismic line. Line 1660 is located on Melville Island in the Canadian Arctic Islands. In this part of the Arctic Islands, the geologic structures are dominated by long continuous folds with upright, steep-limbed anticlines, and wide, flat-bottomed synclines. Individual anticlines are paired with buried thrusts. Line 1660 nearly perpendicular to the regional tectonic fabric.

- Figure 3-14. The 6-fold CMP stacked section in Robertson Point Anticline with a 30 Hz low-pass filter applied. The midpoint interval of this section is 33.5 m, 4-second of data are shown. Apparent in the data is an anticline which is cored by blind thrust faults. Inside the anticline the image is poor. Much detail in the CMP stacked section is missing below the steep-limbed anticlines. High angle and low angle diffractions appear associated with the faults. Geologic information suggests deformations occurred above the interface at 2600 ms; reflectors below 2600 ms should be flat. Due to the complicated structure inside the core of the anticline, the quality of the CMP stack is poor; most importantly several truncated reflection segments appear beneath the anticline. No matter what migration algorithm is used, migration smiles will be produced from the truncated reflections.
- Figure 3-15. A time migration of the CMP stack displayed in Figure 3-14. In the migration result, most of the diffraction energy is collapsed, and the truncated reflections are brought closer together. The expected overmigration smiles are visible in the time migration results.
- Figure 3-16. A 2-layer velocity model used for layer-stripping reverse-time migration. The first guess velocity model is obtained from sonic velocities from the nearby well. The boundary between layers is largely determined by the automatic procedure, with some manual intervention in areas of ambiguous signal strength. For instance, above the core of the anticline, where a clear interface can not be traced, a smooth curve is chosen as the boundary. A small amount of lateral smoothing is required to obtain a smooth surface for migration in the next layer.
- Figure 3-17. Layer-stripping migration result using the 2-layer velocity model in Figure 3-16. The success of the migration is shown by the steep events on the flanks of the anticline, the collapse of diffractions in the core of the anticline, and the ability to trace the fault planes and the sub-horizontal reflectors beneath the anticline. However, the image at the interior of the anticline is still confused. Undermigrated diffraction tails can be seen at the faults.
- Figure 3-18. The 45-degree finite-difference depth migration using the CMP section in Figure 3-14 and the velocity model in Figure 3-16. The depth migration produces a result comparable to the layer-stripping reverse-time migration shown in Figure 3-17.
- Figure 3-19. A 3-grid velocity model used for layer-stripping reverse-time migration. Velocities were obtained from velocity analyses on a number of neighboring CMP traces.
- Figure 3-20. The layer-stripping migration result using the 3-layer velocity model in Figure 3-19. The diffraction energy is better collapsed, two thrust faults on the left-hand side of the anticline around 6 km deep can be better traced, and the detachment surface is aligned better, although still accompanied with migration smiles. The core of the anticline is still not clear, note the crossing events present.

- Figure 3-21. The 45-degree finite-difference depth migration using the CMP section in Figure 3-14 and the 3-layer velocity model in Figure 3-19. The depth migration produces a result comparable to the layer-stripping reverse-time migration shown in Figure 3-20.
- Figure 3-22. A 6 layers velocity model used to further improve the migration image. Velocities are obtained from the semblance analysis combining with the sonic velocities from the nearby well.
- Figure 3-23. The layer-stripping migration result using the 6-layer velocity model in Figure 3-22. The image inside the anticline is clearer and the interfaces beneath the detachment surface are better flattened than with migration using the 3-layer and 2-layer models.
- Figure 3-24. The 45-degree depth migration result using the CMP section in Figure 3-14 and the 6-layer velocity model in Figure 3-22. Compared to the layer-stripping reverse-time migration in Figure 3-23, the 45-degree depth migration produces a comparable result, but smoother image in the core of the anticline. Above 65 degrees, the impulse response of the 45-degree wave equation is dominated by nonpropagating evanescent waves inside the semicircle, the high dip energy is smeared throughout the image, giving it a smoother appearance.
- Figure 3-25. The synthetic data migration results, when the 10th-order explicit reverse-time scheme, the 4th-order implicit reverse-time scheme, and the 45-degree depth migration are used to migrate the CMP stack in Figure 3-2 using the velocity model shown in Figure 3-5. The 4th-order implicit operator generates unacceptable grid dispersion along the dipping events.

Figure 3-1

Figure 3-2

Figure 3-3

Figure 3-4

Figure 3-5

Figure 3-6

Figure 3-7

Figure 3-8

Figure 3-9

Figure 3-10

Figure 3-11.a

Figure 3-11.b

Figure 3-12.a

Figure 3-12.b

MI 1660

Figure 3-14

Figure 3-15

Figure 3-16

Figure 3-17

Figure 3-18

Figure 3-19

Figure 3-20

Figure 3-21

Figure 3-22

Figure 3-23

Figure 3-24

Figure 3-25

Chapter 4

Pre-Stack Layer-Stripping Reverse-Time Migration

INTRODUCTION

It has been long recognized that CMP stacking is exact only for horizontal layer media. In areas of complex structures, the stacking procedure may discriminate against steeply dipping events, because dipping events have higher apparent stacking velocities (Levin, 1971). Some of the stacking effects can be corrected by applying dip moveout (DMO) correction to common-offset section after it has been NMO corrected (Yilmaz and Claerbout, 1980; Deregowski and Rocca, 1981; Bolondi et al, 1982; Hale, 1984). The DMO processing enhances the quality of the CMP stacked section and provides a dip-corrected gather for the subsequent post-stack migration. However, the use of DMO correction is restricted either by the assumption of small-offset (Yilmaz and Claerbout, 1980) or by the assumption of constant velocity medium (Hale, 1984). The most important drawback of the DMO technique is that it still can not perform accurately in areas having lateral velocity variations. Under these circumstances, pre-stack depth migration becomes the ultimate tool for imaging subsurface structures correctly.

Pre-stack depth migration avoids NMO correction and CMP stacking procedures. The pre-stack migration result can be approached by two means, one is full pre-stack migration (Schultz and Sherwood, 1980) and the other is common-shot gather migration followed by stacking (Temme, 1984; de Faria et al, 1986; Reshef and Kosloff, 1986; Milkereit, 1987; Chang and McMechan, 1989). Full pre-stack migration extrapolates a multiple-offset seismic data set to a zero-offset section, and then forms the migration image at time $t=0$. NO further stacking is needed. The zero-offset section can be obtained by any wavefield

extrapolation method. In principle, the zero-offset data is obtained by extrapolating the common-shot gather from z to $z+\Delta z$, sorting the extrapolated wavefield to the common-receiver gather, and then extrapolating the new wavefield downward by Δz . The wavefield extrapolation posed in this way can be thought of as downward continuation of both sources and receivers from one boundary to another. The same procedure is repeated until the velocity model is migrated. The full pre-stack migration is not very attractive from a practical point of view because it requires a large amount of data transposition (from common-shot gathers to common-receiver gathers and vice versa) at every boundary. In addition, the image will be diminished if there are many fewer shot points than receiver positions.

The same pre-stack migration result can be obtained by common-shot gather migration followed by stacking. This migration scheme migrates shot-records one at a time. In each common-shot gather migration, only the wavefield representing the receivers are extrapolated; therefore the migration image is formed at the reflector rather than at time zero. The individual image of the common-shot gather migration consists of only a partial image covered by a single shot experiment. The complete migration result is a superposition of these partial migration images. The final migration result achieved by using the common-shot gather migration can be considered as being true common depth point stacking.

Pre-stack layer-stripping reverse-time migration can be implemented either as a full pre-stack migration or as a common-shot gather migration. In this study, the migration is implemented as common-shot gather migration. The layer-stripping migration scheme is illustrated by migration of two synthetic examples and a marine field data set. The illustration focuses on obtaining reflector position and shape rather than detailed stratigraphic structural interpretation.

Synthetic shot records are generated from the 4th-order accurate finite-difference forward modeling program described in chapter 3. Velocity models have complex structures, strong velocity contrasts, steeply dipping layers, and lateral velocity variations. In the field data example, part of a marine seismic profile, RU-3, offshore central California is migrated.

CALCULATION OF THE IMAGING CONDITION

As mentioned in Chapter 1, common-shot gather migration requires an imaging condition different from time zero. The imaging condition is the one-way travel time from the shot point to each finite-difference grid points and can be obtained by ray tracing. Because constant velocity layers are used in the migrations presented here, Fermat's principle is expressed as the simple square-root equation

$$t = \frac{\sqrt{x^2 + z^2}}{v_0}, \quad (4.2)$$

which is used to calculate the one-way travel time from the shot point to each finite-difference grid point.

For the grid points in the first layer, the one-way travel time is simply computed by using Equation (4.2). Suppose point $(i\Delta x, j\Delta z)$ is the grid point along the newly defined layer boundary. To calculate the one-way travel time at grid point $(m\Delta x, n\Delta z)$ in the next layer, first, use the square-root method to compute the travel time between every point on the boundary and the point $(m\Delta x, n\Delta z)$. The desired one-way travel time from the shot point to point $(m\Delta x, n\Delta z)$ is determined by Fermat's principle as

$$T_s(m,n) = \min [T_s(i,j) + T_{i,j}^{m,n}], \quad (4.3)$$

where $T_s(m,n)$ represents the one-way travel time from the shot point to point $(m\Delta x, n\Delta z)$ in the next layer, $T_s(i,j)$ is the one-way travel time from the shot point to point $(i\Delta x, j\Delta z)$ on the layer boundary, and $T_{i,j}^{m,n}$ is the travel time between boundary point $(i\Delta x, j\Delta z)$ and point $(m\Delta x, n\Delta z)$. The same method is repeated for each layer.

SYNTHETIC DATA EXAMPLE 1

In the first example, 78 synthetic shot records were generated from the velocity model shown in Figure 4-1. The geometry of the synthetic seismic survey and the collected seismic data have been described in the previous chapter. Figure 4-2 shows three synthetic shot records with the shot points located at points A, B and C in the velocity model. Direct waves and postcritical reflections are recorded in the shot record. We wish to reduce all non-reflecting energy before migration; therefore, direct waves have been muted in the preliminary data processing. Figure 4-3 displays the processed shot records with the source located at points A, B and C.

MIGRATION SEQUENCE AND RESULT

In addition to muting direct waves, a spherical divergence correction and a 58 Hz low-pass filter have been applied to each shot record before migration. Using the layer-stripping migration scheme, the velocity model is partitioned into three layers (see Figure 4-1). The migration is performed using a rectangular grid. The horizontal grid size is 40 m, the receiver interval, and the vertical grid spacing for migration in the first layer is 20 m.

The time step is 2.857 ms. In migration, when the calculated imaging time lies between two computational time steps, a linear interpolation is used to determine the amplitude of the wavefield at that specific time.

Figure 4-4 shows migration images from the individual shot gathers with the source points at A, B and C, when the correct velocity 3500 m/sec is used. Figure 4-4a is the partial migration image with the source at point A, in which the 45-degree dipping layer and the horizontal event beneath the shot point are correctly migrated. The right flank of the anticline is invisible in this figure because of the source position. The migration image between the 45-degree event and the apex of the anticline is not clear. One important reason of such a confused image is formed because reflections from different interfaces arrive almost simultaneously in the shot record (see Figure 4-3a). Another important reason is that the large amplitude and phase shifted postcritical reflections become part of the image and twist the true image. Migration smiles from the intersections between two events are also visible. Figure 4-4b shows the image with the source at point B. As the source moves to the center of the velocity model, most of the seismic events, including the horizontal event, the 45-degree dipping event, and the left-flank of the anticline, are focused. The right flank of the anticline is still invisible. Because the direct wave is not completely removed in the input shot record, a diffraction from the unmuted direct wave appears on the upper left corner in Figure 4-4b. The migration image with the source at shot point C is shown in Figure 4-4c. Here the right flank of the anticline is imaged; however, the left dipping events are missing.

Observation from a single shot point provides a partial image. To obtain a complete image, we composite all shots. However, as seen in the partial migration results, images from the direct waves and postcritical reflections mask the true migration image. The composited image will be strongly distorted if these large amplitude images are also added. To obtain a clear migration result, I have muted images beyond the critical angle before

stacking. The same mute is applied to each partial image. Although the critical angle is determined from the velocity distribution, the choice of the muting range is not critical. A number of mute patterns are applied to the individual migration images until the best image is obtained. Figure 4-5 shows the composited image for the first layer from 78 shot records. The shape and the position of the reflector is correctly imaged. Since the surface recordings are migrated to their midpoint position in the spatial domain, interfaces on both ends of the velocity model are invisible. Because most of the shot points are to the left of the anticline velocity model, the right flank of the anticline has smaller amplitude.

Following the same method I have described in the previous chapter, I choose the maximum power within a first guess zone of the migration image to determine the location of the lower boundary of the layer. The event representing the redefined boundary is treated as part of the migration image for this layer. For migration in the next layer, seismic signals prior to the one-way travel time from the source to the newly defined boundary are muted in the new boundary condition. A cosine taper is applied to the end of the mute pattern. Figure 4-6 displays the seismic time sections extrapolated from the surface to the first interface with the shot points at A, B and C. After being scaled with the transmission coefficient, the extrapolated time sections are used as the boundary values for migration in the next layer. Although the muting procedure is adequate to exclude the useless signals, it is structure dependent. For instance, because the direct wave travel time from the source to the right flank of the anticline is incorrect in Figure 4-6a and 4-6b, muting does not correctly exclude the undesired signals. In Figure 4-6c, the muting is not proper either, due to the incorrect one-way travel time from the source to the event left of the apex of the anticline.

Repeating the same migration procedure, Figure 4-7 shows the partial migration images for the second layer with the source at points A, B and C when velocity 5000 m/sec is used. The horizontal grid spacing used for migration in this layer is kept the same as 40

m, but the vertical grid size is enlarged to 28 m. In Figure 4-7a, the partial images of the second interface are visible. Because of the source location, the left flank of the syncline is invisible. Since reflection from the second interface cannot be isolated on the right hand side of the input time section (see Figure 4-6a), a noisy background is seen in the second layer. Noises originated from the unmuted signals are also seen in the partial image. In Figure 4-7b, part of the second layer covered by this shot are correctly migrated. The right flanks of the syncline and the anticline are invisible. Noises from the unmuted signals are also seen along the top of this layer. In Figure 4-7c, since most of the one-way travel times in the second layer are incorrect, only the horizontal events on the right hand side of the velocity model are correctly migrated. Lots of noises are seen in the upper part of this layer. Figure 4-8 shows the superimposed migration result for the second layer. Since most of the destructive noises are cancelled out, the second interface, including the horizontal events, the syncline, and the anticline, is correctly focused. Figure 4-9 is a true amplitude display of the complete migration image for the whole model. The two interfaces are both correctly migrated.

SYNTHETIC DATA EXAMPLE 2

In the previous example, I have demonstrated that the pre-stack layer-stripping reverse-time migration algorithm is capable of producing correct migration image in structurally complex areas. Next I apply this migration algorithm to a synthetic data set which simulates the field marine reflection survey. The velocity model in this example is shown in Figure 4-10; it is 11.8 km wide and 2 km deep. This model consists of a water column, in which water depth varies from 110 m to 250 m, and a sequence of constant velocity layers. On the right hand side of the velocity model the layered structure is faulted and has an uplifted region. The interval velocities in this model increase from 1500 m/sec

(water velocity) to 3500 m/sec. Strong velocity contrasts exist between layers, except for boundaries between the two thin layers where the velocity contrasts are only about 2%. 67 shot records were generated from $x=4830$ m to $x=11430$ m with a source interval of 100 meters. The cable is 4730 m long, extending to the left of the shot with a 25 m receiver interval. The maximum source-receiver offset is 4730 m, and the near offset is 370 m. 2 seconds of data are generated, and the time sample is resampled to 4 ms. Both source and receiver are located at depth of 40 m deep. This geometry duplicates that of the marine survey shown in the next section.

Figure 4-11 shows three synthetic records with the shot position at points A, B and C in the velocity model Figure 4-10. In Figure 4-11a, the primary reflection is quickly followed by the free surface reflection and the water bottom reflection. Because the primary reflection can not be isolated, the combined wavelet is treated as a primary reflection in this example. In Figure 4-11b only shallow reflections are visible, because the subsurface structure covered by this shot is complicated. Shot point C is located above the high velocity uplift, and refractions are visible in Figure 4-11c. Because direct waves, refractions, and postcritical reflections can not be removed easily in shot records, preliminary processing includes only a spherical divergence correction and a 40 Hz low-pass filter. In both the forward modeling and the migration, the calculation window was extended a half-cable length from the shot point to the right to provide a proper coverage of dipping subsurface structures.

Using the same migration sequence described in the previous section, a migration composited from 67 shots is displayed in Figure 4-12. The velocity model is shown in Figure 4-13. Since there is no interesting target beneath the deepest interface, the migration is stopped after the deepest interface is imaged. In Figure 4-12, the overall image is correctly migrated; however, the two interfaces with small velocity contrast are invisible in the final result. One reason is that the reflections from these two interfaces are invisible in

the migration input, and the other reason is probably that these interfaces are masked by the long wavelength primaries in the migration image. The noise inside the high velocity uplift is an artifact.

FIELD DATA EXAMPLE -- HOSGRI FAULT, RU-3, CENTRAL CALIFORNIA

GENERAL SETTING

This example is a pre-stack migration of part of the marine seismic profile RU-3. Line RU-3 was collected from offshore central California by Rice University and the Houston Area Research Center in conjunction with Pacific Gas and Electric Company's Offshore Deep Crustal Geophysical Survey. The location of the line RU-3 is shown in Figure 4-14. This line extends 110 km from the coast seaward. Profile RU-3 crosses the offshore Santa Maria basin in a dip direction near Point San Luis. In this illustration, the 12 km of the profile nearest to the coast was migrated. Water depth along this part of the profile varies from 110 m to 300 m. In this part of RU-3, mildly deformed sediments are deposited on top of Franciscan basement rocks. At the northeast end of the profile the Hosgri fault is a major structural discontinuity. The fault zone was interpreted as a linear band of faults 3-5 km wide on a post-stack time migration (Meltzer, 1988). An abrupt change in sediment thickness across the fault results in a sharp velocity contrast across the fault.

Profile RU-3 was recorded with a 180 channel system, a receiver group spacing of 25 m, and a maximum offset of 4730 m. The shot interval was 50 m. 16-second data were recorded with a sample rate of 4 ms. The data processing sequence is shown in Figure 4-15. Figure 4-16 shows the 45-fold CMP stacked section for the part of the profile of interest, in which 3-second of data are displayed. The dominant features of this CMP section are a deformed layering sequence to the southwest and adjacent to the Hosgri fault.

Diffractions associated with faults are visible. Coherent noise also can be seen across the sedimentary column. The post-stack time migration of this CMP section is displayed in Figure 4-17; most of the diffractions are collapsed. Interfaces can be better traced in the post-stack time migration than in the CMP section. However, the image of the Hosgri fault is unclear.

PRE-STACK MIGRATION

In this example, 225 shot records were migrated using the pre-stack layer-stripping reverse-time migration algorithm. Figure 4-18a shows one shot gather with shot position at point A in the CMP stacked section. In Figure 4-18a, because of the shallow water depth, the resolution of the pack of early arrivals in each trace is poor. No data area appears below 3-second in this shot record, except for the large offset traces. Figure 4-18b shows another shot record with shot point at point B in the CMP stack. In this shot, shallow refractions can be easily identified because of the uplifted high velocity Franciscan rocks. Diffractions associated with the Hosgri fault are also visible in Figure 4-18b.

Pre-migration processing of the 4-second shot records have included spherical divergence correction, predictive deconvolution, and a 35 Hz low-pass filtering. Direct waves were not muted in the shot records, since the standard seismic processing cannot easily remove the direct waves, multiples, and refractions. Velocity analysis was done independently before migration. Different velocity analysis methods were used to establish the initial migration velocities. Standard CMP velocity analysis was performed in the sedimentary column. Above the Franciscan uplift, where CMP velocity analysis is not available, the velocity was obtained from shallow refractions (see appendix A).

Pre-stack migration began with migrating the processed shot records using the water velocity, 1500 m/sec. In the individual migration results, migrated images beyond the

critical angle were muted. The muted migration images were then composited to form the image of the water bottom. To determine the precise position of the water bottom, I scanned a range of seismic data around the hypothesized boundary. For each trace, the amplitude square of each point inside the given range was calculated. Picking the maximum value from each trace determined the layer boundary. After the layer boundary is defined, shot records were downward continued using the same reverse-time method. Repeating the same migration procedure, I migrated the seismic time section through the following layers. Since the initial interval velocities converted from stacking velocities didn't always produce a satisfactory image, the initial velocities were adjusted to produce the best migration images.

Figure 4-19 shows the pre-stack migration results from a 7-layer velocity model shown in Figure 4-20. The velocity model shown in Figure 4-20 was developed from the initial velocity model and from iterative migration. In the pre-stack migration, the Hosgri fault appears as a northeast dipping high angle fault with a thrust component. Thrusts are also visible on the west of the fault.

The processing sequence for producing the pre-stack migration is shown in Figure 4-21. It is a different processing sequence from that for the post-stack time migration. Compared to the processing sequence for the post-stack time migration, the proposed pre-stack migration scheme greatly reduces the labor required for the data processing before migration. The seismic interpretations of Figures 4-19 and 4-17 are displayed in Figures 4-22 and 4-23, respectively. Compare the pre-stack depth migration in Figure 4-22 to the post-stack time migration in Figure 4-23. These two migration results show similar images in the sedimentary column, although the time migration is clearer overall. However, the pre-stack layer-stripping reverse-time migration provides a better image of the fault zone than the post-stack time migration does. In addition, the pre-stack depth migration displayed the fault image in true depth. Although the layer-stripping technique gives a

satisfactory image, migration with constant velocities in each layer is roughly equivalent to a brute stack, which results in some choppy features in the migration. A better image could be obtained by using a focusing technique to better determine migration velocities.

Figures 4-24a and 4-24b show two common depth gathers, which are stacked to form the images at positions A and B in Figure 4-22. If the velocities for positions A and B are chosen correctly, the seismic signal for the same event should be aligned horizontally. As we have seen, the velocities used for our first pass migration are not quite correct. The common depth gather for migration velocity analysis as described in Al-Yahya (1989) could be used to produce a better focused image.

THE DIP OF THE HOSGRI FAULT

To evaluate how different velocity structures chosen for the Hosgri fault affect the migration result, I have migrated the seismic data with three different fault structures. Figure 4-25a shows the three models used in migration. The migration results from model A : a westward dipping normal fault, model B : a vertical fault, and model C : an eastward dipping high-angle fault, are displayed in Figures 4-25b, 4-25c, and 4-2d, respectively. The appearance of the fault in the pre-stack migration image depends critically on the velocity model. A distinct fault plane is visible because of the strong velocity contrast along the fault. In Figure 4-25a, the coherent images of the sedimentary column are stopped by the westward dipping normal fault. Images to the right of the fault are incoherent. In the migration result using the vertical fault model, Figure 4-25c, the zone of the coherent images is further extended to the right. Similarly, using the normal fault model, the coherent images are terminated by the fault plane. Compared to the image shown in Figure 4-25b; however, seismic events adjacent to the right of the fault plane show better coherency, and the images below 2-second appear differently. The best image is achieved

with the eastward dipping high-angle fault model, Figure 4-25d, in which the layers west of the fault have the best image. Thrusts are also visible in the westward dipping sedimentary column between one and 2-second. Also, a reflection at 2.8 km is recovered beneath the thrust fault but is not recovered for either the normal or vertical faults. For comparison, a post-stack time migration is shown in Figure 4-25e. Compared the coherent images below the sedimentary column, only the images using the eastward dipping high-angle fault model, in Figure 4-25d, are comparable to the post-stack migration result.

CONCLUSION

Pre-stack layer-stripping reverse-time migration is capable of imaging geologically complex areas. The layer-stripping method is powerful for imaging structures with strong velocity contrasts. This migration algorithm makes interpretation a step of migration. The migration image critically depends on the chosen velocity model. Postcritical reflections are always a problem for wide-angle migration (McMechan and Fuis, 1987; Milkereit, 1990). With careful muting, the pre-stack layer-stripping reverse-time migration method can be used to migrate wide-angle precritical seismic data. The quality of the migration image depends strongly on the survey geometry. A correct migration depends on a correct velocity function. Using the method described in this chapter to calculate the image condition strongly restricts the quality of the migration image. The imaging condition should be obtained by using the more accurate methods, such as that described by Langan et al (1985), Reshef and Kosloff (1986), and Vidale (1988). Following constant velocity migration, an image focusing technique can be added by smoothly varying the velocity field to obtain a better image. Incorporating with the migrated common-depth gather described by Al-Yahya (1989), the layer-stripping reverse-time method can be expanded as an iterative velocity analysis tool.

FIGURE CAPTIONS

- Figure 4-1. Two-dimensional velocity model used to generate the synthetic shot records for example 1. The model is 8 km wide and 3 km deep, and is composed of three irregular layers with velocities $v_1 = 3,500$ m/sec, $v_2 = 5,000$ m/sec, and $v_3 = 3,250$ m/sec. The principal structural features in the model are an anticline, a syncline, and a 45-degree dipping layer. 78 shot records were generated from this model. The first shot is at $x=980$ m. The shot interval is 80 m, and the receiver interval 40 m intervals.
- Figure 4-2a. Synthetic shot record for the velocity model in Figure 4-1 with the source at point A in the velocity model. Synthetic shot records are generated using a 4th-order accurate space, 2nd-order accurate time two-dimensional finite-difference forward modeling program. Direct waves and postcritical reflections are also recorded. Because of the negative reflection coefficient on the second interface, which has reversed polarity.
- Figure 4-2b. Synthetic shot record for the velocity model in Figure 4-1 with the source at point B in the middle of the velocity model.
- Figure 4-2c. Synthetic shot record for the velocity model in Figure 4-1 with the source at point C in the velocity model.
- Figure 4-3a. Synthetic shot record for the velocity model in Figure 4-1 with the source at point A in the velocity model. The direct waves have been muted. This section is used as the boundary value for common-shot gather migration.
- Figure 4-3b. Synthetic shot record for the velocity model in Figure 4-1 with the source at point B in the velocity model. The direct waves have been muted. This section is used as the boundary value for common-shot gather migration. Note the unmuted direct waves on the left of the velocity model.
- Figure 4-3c. Synthetic shot record for the velocity model in Figure 4-1 with the source at point C in the velocity model. The direct waves have been muted. This section is used as the boundary value for common-shot gather migration. Direct waves are not completely removed, but part of the first interface are muted.
- Figure 4-4a. Partial migration images from the individual shot gathers in Figure 4-3a, when the correct velocity 3500 m/sec is used. The 45-degree dipping layer and the horizontal event beneath the shot point are correctly migrated. The right flank of the anticline is invisible in this figure because of the source position. The migration image between the 45-degree event and the apex of the anticline is not clear, because reflections from different interfaces arrive almost simultaneously in the shot record, and the phase shifted postcritical reflections twist the true image. Migration smiles from the intersections between two events are also visible.

- Figure 4-4b. Partial images from the individual shot gathers in Figure 4-3b, when the correct velocity 3500 m/sec is used. Because the source is at the center of the velocity model, most of the seismic events, including the horizontal event, the 45-degree dipping event, and the left-flank of the anticline, are focused. The right flank of the anticline is still invisible. Because the direct wave is not completely removed in the input shot record, a diffraction from the unmuted direct wave appears on the upper left corner.
- Figure 4-4c. Partial images from the individual shot gathers in Figure 4-3c, when the correct velocity 3500 m/sec is used. Here the right flank of the anticline is imaged; however, the left dipping events are missing.
- Figure 4-5. The composited image for the first layer from 78 shot records. The shape and the position of the reflector is correctly imaged. Since the surface recordings are migrated to their midpoint position in the spatial domain, interfaces on both ends of the velocity model are invisible. Because most of the shot points are to the left of the anticline velocity model, the right flank of the anticline has smaller amplitude.
- Figure 4-6a. The seismic time sections extrapolated from the surface to the first interface with the shot points at A. Seismic signals prior to the one-way travel time from the source to the newly defined boundary are muted. A cosine taper is applied to the end of the mute pattern. After being scaled with the transmission coefficient, this time section is used as the boundary values for migration in the second layer. Because the direct wave travel time from the source to the right flank of the anticline is incorrect in , the undesired signals are not muted correctly.
- Figure 4-6b. The seismic time sections extrapolated to the first interface, using the time section in Figure 4-3b . Seismic signals prior to the one-way travel time from the source to the newly defined boundary are muted. This time section is used for migration in the second layer. The same reason as that in Figure 4-6b, the undesired signals are not muted correctly.
- Figure 4-6c. The seismic time sections extrapolated from the surface to the first interface, when the time section in Figure 4-3c is used. Seismic signals prior to the one-way travel time from the source to the newly defined boundary are muted. This time section is used as for migration in the second layer. Due to the incorrect one-way travel time from the source to the event left of the apex of the anticline, where the muting is not properly.
- Figure 4-7a. The partial migration images for the second layer with the source at points A when velocity 5000 m/sec is used. The horizontal grid spacing used for migration in this layer is kept the same as 40 m, and the vertical grid size is to 28 m. Because of the source location, the left flank of the syncline is invisible. A noisy background is seen because reflection from the second interface cannot be isolated on the right of the input time section. Noises originated from the unmuted signals also become part of the image.
- Figure 4-7b. The partial migration images for the second layer with the source at points B when velocity 5000 m/sec is used. The right flanks of the syncline and the

anticline are invisible. Noises from the unmuted signals are also seen along the top of this layer.

- Figure 4-7c. The partial migration images for the second layer with the source at points C when velocity 5000 m/sec is used. Since most of the one-way travel times are incorrect, only the horizontal events on the right hand side of the velocity model are correctly migrated. Lots of noises are visible in the upper part of the layer.
- Figure 4-8. The superimposed migration result for the second layer from 78 shot records. The second interface, including the horizontal events, the syncline, and the anticline, is correctly focused.
- Figure 4-9. The true amplitude display of the complete migration image for the whole model. The first interface and the second are both correctly migrated. Because most of the shot points are to the left of the anticline velocity model, the right flank of the anticline has smaller amplitude.
- Figure 10. 67 shots records were generated from this velocity model. It is 11.8 km wide and 2 km deep. This model has a water column, in which water depth varies from 110 m to 250 m, and a sequence of constant velocity layers. On the right hand side of the velocity model the layered structure is faulted and has an uplifted region. The shots are located from $x=4830$ m to $x=11430$ m with a source interval of 100 meters. The cable is 5100 m long, extending to the left of the shot with a 25 m receiver interval. The maximum source-receiver offset is 4730 m, and the near offset is 370 m. Both source and receiver are located at depth of 40 m deep.
- Figure 4-11a. The synthetic record with the shot position at points A in the velocity model Figure 4-10. Here the primary reflection is quickly followed by the free surface reflection and the water bottom reflection. Because the primary reflection can not be isolated, the combined wavelet is treated as a primary reflection in this example. Preliminary processing includes only a spherical divergence correction and a 40 Hz low-pass filter.
- Figure 4-11b. The synthetic record with the shot position at points B in the velocity model Figure 4-10. Because the subsurface structure covered by this shot is complicated, only shallow reflections are visible here.
- Figure 4-11c. The synthetic record with the shot position at points C in the velocity model Figure 4-10. Shot point C is located above the high velocity uplift, and refractions are visible.
- Figure 4-12. The migration composited from 67 shots. The velocity model used is shown in Figure 4-13. The overall image is correctly migrated; however, the two interfaces with small velocity contrast are invisible in the final result. One reason is that the reflections from these two interfaces are invisible in the migration input, and the other reason is probably that these interfaces are masked by the long wavelength primaries in the migration image. Since there is no interesting target beneath the deepest interface, the migration is

stopped after the deepest interface is imaged. The noise inside the high velocity uplift is an artifact.

- Figure 4-13. The velocity model used for migration in Figure 4-12. The migration is stopped after the deepest interface is imaged.
- Figure 4-14. Map showing location of the marine profile RU-3, offshore central California. The heavy solid line marks the 12 km of the profile nearest to the coast migrated. Line RU-3 extends 110 km from the coast seaward, which crosses the offshore Santa Maria basin in a dip direction near point San Luis. Water depth along this part of the profile varies from 110 m to 300 m.
- Figure 4-15. Data processing sequence for the post-stack time migration shown in Figure 4-17. This sequence is designed to maximize the resolution of the sedimentary column.
- Figure 4-16. The 45-fold CMP stacked section for the part of the profile interested, in which 3-second of data are displayed. Profile RU-3 was recorded with a 180 channel system, a receiver group spacing of 25 m, and a maximum offset of 4730 m. The shot interval was 50 m. 16-second data were recorded with a sample rate of 4 ms. The dominant features of this CMP section are a deformed layering sequence to the southwest and adjacent to the Hosgri fault. Diffractions associated with faults are visible. Coherent noise also can be seen across the sedimentary column.
- Figure 4-17. The post-stack time migration of this CMP section is displayed in Figure 4-17; most of the diffractions are collapsed. Interfaces can be better traced in the post-stack time migration than in the CMP section. However, the image of the Hosgri fault is unclear. The data processing sequence for this migration is displayed in Figure 4-15.
- Figure 4-18a. Common-shot record with the source at point A in the CMP stack in Figure 4-16. Because of the shallow water depth, the resolution of the pack of early arrivals in each trace is poor. The standard seismic processing cannot remove the direct waves, multiples, and refractions from the shot record. No data area appears below 3-second in this gather, except for the large offset trace.
- Figure 4-18b. Common-shot record with the source at point B in the CMP stack in Figure 4-16. Because of the high velocity of the Franciscan rocks, refractions are easily seen.
- Figure 4-19. The pre-stack migration results from a 7-layer velocity model shown in Figure 4-21. The velocity model was developed from standard CMP velocity analysis in the sedimentary column, from shallow refraction velocities above the Franciscan uplift where CMP velocity analysis is poor, and from iterative migration. In the post-stack migration, the Hosgri fault appears as a northeast dipping high angle fault with a thrust component. Thrusts are also visible on the west of the fault.

- Figure 4-20. The 7-layer velocity model used for migration in Figure 4-20. The solid lines represent the layer boundaries. The dashed line on the right of the velocity model is a false one used to separate two different velocities.
- Figure 4-21. Data processing sequence for the pre-stack layer-stripping migration.
- Figure 4-22. Seismic interpretation on the pre-stack migration result. The faults are interpreted based on the visible discontinuities.
- Figure 4-23. Seismic interpretation on the post-stack time migration. The fault zone was interpreted as a linear band of faults 3-5 km wide on a post-stack time migration (Meltzer, 1988).
- Figure 4-24a. The migrated common-depth gather, which are stacked to form the images at position A in Figure 4-22. The bars correspond to the interpreted layer boundary in Figure 4-22. If the velocities for positions A are chosen correctly, the seismic signal for the same event should be aligned horizontally. As we have seen, the velocities used for our first pass migration are not quite correct.
- Figure 4-24a. The migrated common-depth gather at position A in Figure 4-22. Since the seismic signal for the same event is not aligned horizontally. The velocities used for our first pass migration are not quite correct.
- Figure 4-25a. The three models used in migration. Model A : a westward dipping normal fault. Model B : a vertical fault. Model C : an eastward dipping high-angle fault. Velocities used for these three models are the same, but the positions of the fault are different.
- Figure 4-25b. The migration results from model A : a westward dipping normal fault. The coherent images of the sedimentary column are stopped by the westward dipping normal fault. Images to the right of the fault are incoherent.
- Figure 4-25c. The migration results from model B : a vertical fault. The zone of the coherent images is further extended to the right, and then is terminated by the fault plane. Compared to the image shown in Figure 4-25b; however, seismic events adjacent to the right of the fault plane show better coherency, and the images below 2-second appear differently.
- Figure 4-25d. The migration results from model C : an eastward dipping high-angle fault. The layers west of the fault have the best image. Thrusts are also visible in the westward dipping sedimentary column between one and 2-second. Also, a reflection at 2.8 km is recovered beneath the thrust fault but is not recovered for either the normal or vertical faults.
- Figure 4-25e. A post-stack time migration. Compare the image coherency below the sedimentary column in this figure to the three Figure 4-25b, 4-25c, 4-25d. Only the images using the eastward dipping high-angle fault model, in Figure 4-25d, are comparable to this migration result.

Figure 4-1

Figure 4-2a

Figure 4-2b

Figure 4-2c

Figure 4-3a

Figure 4-3b

Figure 4-3c

Figure 4-4a

Figure 4-4b

Figure 4-4c

Figure 4-5

Figure 4-6a

Figure 4-6b

Figure 4-6c

Figure 4-7a

Figure 4-7b

Figure 4-7c

Figure 4-8

Figure 4-9

Figure 4-10

Figure 4-11a

Figure 4-11b

Figure 4-11c

2 km

Figure 4-12

Figure 4-13

Figure 4-14

Figure 4-15

Figure 4-16

Figure 4-17

Figure 4-18a

Figure 4-18b

Figure 4-19

Figure 4-20

Figure 4-21

Figure 4-22

Figure 4-23

Figure 4-24b

Figure 4-24a

Figure 4-25a

Figure 4-25b

Figure 4-25c

Figure 4-25d

Figure 4-25e

Chapter 5

Conclusions

The examples of post-stack and pre-stack migrations have illustrated that layer-stripping reverse-time migration is a useful migration method, particularly for velocity models with strong velocity contrasts between layers. Layer-stripping reverse-time migration handles steeply dipping structures, eliminates artificial interlayer multiples, and allows the user some interpretational control over the final migration image. In addition, compared to the conventional one-stage reverse-time migration, the layer-stripping scheme greatly reduces the memory requirements and somewhat saves computer time. The layer-stripping method can be applied to a velocity model with gradually increasing velocity, but in this case the migration technique reduces to regular reverse-time migration.

Any wavefield extrapolation technique can be incorporated in the layer-stripping migration method. Due to the robustness of reverse-time migration, the layer-stripping migration algorithm presented here uses the reverse-time method to extrapolate the wavefield. Unfortunately, the band-limiting and stability criteria limit the computational efficiency of the finite-difference scheme. In the forward modeling problem, using a high-order accurate finite-difference scheme increases the calculation efficiency. From the other point of view, since the trace spacing is fixed, reverse-time migration needs a high-order finite-difference scheme to prevent grid dispersion. Because of the long operator length, using a high-order finite-difference scheme makes migration become more time consuming.

I have used a 10th-order accurate space, 4th-order accurate time finite-difference scheme to migrate band limited seismic time sections. The migrations produce encouraging results; however, the computational cost is still high compared to the 45-degree finite-difference depth migration. The implicit 4th-order accurate finite-difference scheme

provides an alternative which reduces the computational expense; however, the 4th-order implicit finite-difference scheme has dip limitations.

To provide the boundary values for migration in the following layer, datuming is incorporated as part of the post-stack migration procedure. Saving boundary values in a region around the first guess for the interface avoids repeating the reverse-time calculation. However, doing so involves lots of data transposition. If the data set is large, such as in pre-stack migration, the transpose operation can be undesirable from a computational viewpoint. Considering the cost, I have extrapolated the wavefield to the new boundary after it is defined in migration.

Although pre-stack migration is a time consuming data processing procedure, the labor in preliminary data processing is vastly less than that required for CMP stacking plus migration. The pre-stack migration processing consists of only spherical divergence correction, predictive deconvolution, low-pass filtering, migration and stacking. Of course, velocity estimation is done independently.

A correct velocity model is essential for any migration algorithm to produce a correct result. Using the conventional migration methods requires prior knowledge of the shape and interval velocity of each layer. The layer-stripping migration algorithm only needs a correct interval velocity. The shape of each layer is determined as part of the migration algorithm. The results of the layer-stripping migration indicate that the choice of a velocity model strongly affects the migration output. An accurate velocity analysis method is important to avoid an improper interpretation. For post-stack migration, a more detailed velocity function can be obtained from forward modeling. For pre-stack migration, the Al-Yahya's (1989) post-migration common-depth gather supplies an excellent tool for velocity analysis. Examining the migrated common-depth gather layer by layer, the layer-stripping migration scheme can be developed as a powerful iterative velocity analysis technique. The method also eliminates the need for identification of events on the shot record, which is not

always possible. The proposed iterative velocity analysis method can also be used as an image focusing technique. It can be implemented by first varying the velocity field to obtain a roughly horizontally alignment of events on the migrated common-depth gather, and then can be followed by a fine tuning of the local velocity variations.

The one-way travel time calculation method described in the previous chapter is adequate for an initial study; however, it requires a model having constant velocity layers. To further improve the migration image or develop the migration algorithm as a velocity analysis tool, this migration scheme requires a more general travel time calculation method, such as the fast ray tracing method described by Langan et al (1985), the finite-difference method by Vidale (1987), or the direct solution of the eikonal equation proposed by Reshef and Koslof (1986).

The layer-stripping reverse-time migration method could be improved with further work. The linear interpolation between time samples could be replaced by a more accurate method. Also, incorporating a hybrid computational method into the scheme would enhance its power, which is valuable for migrating wide-angle seismic reflection data sets which generally have sparse receiver points. For instance, the Kirchhoff integral can compute the passage of seismic energy through the low velocity surface layers, while the reverse-time method can be used to compute the propagation in the structure beneath the low velocity medium. A hybrid method will prevent the band limiting problem caused by shallow low velocity layers and sparse receiver points.

References

- Al-Yahya, K., 1989, Velocity analysis by iterative profile migration: *Geophysics*, v. 54, p. 718-729.
- Alford, R. M., Kelly, K. R., and Boore, D. M., 1974, Accuracy of finite-difference modeling of the acoustic wave equation: *Geophysics*, v. 39, p. 834-842.
- Alterman, Z., and Karal, Jr. F. C., 1968, Propagation of elastic waves in layered media by finite difference methods: *Bull. Seism. Soc. Am.*, v. 58, p. 367-398.
- Baysal, E., Kosloff, D. D., and Sherwood, J. W. C., 1983, Reverse time migration: *Geophysics*, v. 48, p. 1514-1524.
- Baysal, E., Kosloff, D. D., and Sherwood, J. W. C., 1984, A two way nonreflecting wave equation: *Geophysics*, v. 49, p. 132-141.
- Berryhill, J. R., 1979, Wave-equation datuming: *Geophysics*, v. 44, p. 1329-1343.
- Bolondi, G., Loinger, E., and Rocca, F., 1982, Offset continuation of seismic sections: *Geophys. Prosp.*, v. 30, p. 813-828.
- Chang, W. F., and McMechan, G. A., 1989, Wave field processing of data from the 1986 program for array seismic studies of the continental lithosphere Ouachita experiment: *J. Geophys. Res.*, v. 94, p. 17781-17792.
- Claerbout, J. F., 1971, Toward a unified theory of reflector mapping: *Geophysics*, v. 36, p. 467-481.
- Claerbout, J. F., 1976, *Fundamentals of geophysical data processing*, McGraw-Hill, N.Y.
- Claerbout, J. F., 1985, *Imaging the earth's interior*, Blackwell Scientific publications, Oxford.
- Claerbout, J. F., and Doherty, S. M., 1972, Downward continuation of moveout-corrected seismograms: *Geophysics*, v. 37, p. 741-768.
- Clayton, R. W., and Engquist, B., 1977, Absorbing boundary conditions for acoustic and elastic wave equations: *Bull. Seis. Soc. Am.*, v. 67, p. 1529-1540.
- Clayton, R. W., and Engquist, B., 1980, Absorbing boundary conditions for wave-equation migration: *Geophysics*, v. 45, p. 895-904.
- Clayton, R. W., and McMechan, G., 1981, Inversion of refraction data by wave-field extrapolation: *Geophysics*, v. 46, p. 860-868.
- Dablain, M. A., 1986, The application of higher-order differencing to the scalar wave equation: *Geophysics*, v. 51, p.54-66.
- de Faria, E. L., Stoffa, P. L., and Loewenthal, D., 1986, Migration before stack using reverse time propagation: Presented at the 1986 SEG Annual Meeting, Houston, TX.

- Deregowski, S., and Rocca, F., 1981, Geometrical optics and wave theory for constant-offset sections in layer media: *Geophys. Prosp.*, v. 29, p. 374-387.
- Emerman, S. H., Schmidt, W., and Stephen, R. A., 1982, An implicit finite-difference formulation of the elastic wave equation: *Geophysics*, v. 47, p. 1521-1526.
- Engquist, B., and Majda, A., 1977, Absorbing boundary conditions for the numerical simulation of waves: *Mathematics of computation*, v. 31, p. 629-651.
- Fox, F. G., 1985, Structural geology of the Parry Islands fold belt: *Bulletin of Canadian Petroleum Geology*, v. 33, p. 306-340.
- Gazdag, J., 1978, Wave equation migration with the phase-shift method: *Geophysics*, v. 43, p. 1342-1351.
- Gazdag, J., 1980, Wave equation migration with the accurate space derivative method: *Geophysical Prospecting*, v. 28, p. 60-70.
- Gazdag, J., 1981, Modeling of the acoustic wave equation with transform methods: *Geophysics*, v. 46, p. 854-859.
- Gazdag, J., and Carrizo, E., 1986, On reverse-time migration: *Geophysical Prospecting*, v. 34, p. 822-832.
- Gazdag, J., and Sguazzero, P., 1984, Migration of seismic data: *Proceedings of the IEEE*, v. 72, p. 1302-1315.
- Hale, D., 1984, Dip moveout by Fourier transform: *Geophysics*, v. 49, p. 741-757.
- Harrison, J. C., and Bally, A. W., 1988, Cross-sections of the Parry Islands fold belt on Melville Island, Canadian Arctic Islands: implications for the timing and kinematic history of some thin-skinned decollement systems: *Bulletin of Canadian Petroleum Geology*, v. 36, p. 311-332.
- Hill, N. R., and Wuenschel, P. C., 1985, Numerical modeling of refraction arrivals in complex areas: *Geophysics*, v. 50, p. 90-98.
- Israeli, M., and Orszag, S. A., 1981, Approximation of radiation boundary conditions: *J. Comp. Phys.*, v. 41, p. 115-135.
- Kelly, K. R., Ward, R. W., Treitel, S. and Alford, R. M., 1976, Synthetic seismograms: a finite-difference approach: *Geophysics*, v. 41, p. 2-27.
- Kosloff, D., D., and Baysal, E., 1983, Migration with the full acoustic wave equation: *Geophysics*, v. 48, p. 677-687.
- Langan, R. T., Lerche, I., and Cutler, R. T., 1985, Tracing of rays through heterogeneous media: An accurate and efficient procedure: *Geophysics*, v. 50, p. 1456-1465.
- Larner, K., Hatton, L., and Gibson, B. S., 1981, Depth migration of imaged time sections: *Geophysics*, v. 46, p. 734-750.

- Levander, A. R., 1989, Finite-difference forward modeling in seismology: The Encyclopedia of Geophysics, Van Nostrand Reinhold.
- Levin, F. K., 1971, Apparent velocity from dipping interface reflections: *Geophysics*, 36, p. 510-516.
- Levin, S. A., 1984, Principle of reverse-time migration: *Geophysics*, v. 49, p. 581-583.
- Li, Z., 1986, Wave-field extrapolation by the linear transformed wave equation operator: *Geophysics*, v. 51, p. 1538-1551.
- Lindemuth, I., and Killeen, J., 1973, Alternating-direction implicit techniques for two-dimensional magnetohydrodynamic calculations: *J. Comp. Physics*, v. 13, p. 181-208.
- Loewenthal, D., and Mufti, I. R., 1983, Reversed time migration in spatial frequency domain: *Geophysics*, v. 48, p. 627-635.
- Loewenthal, D., Lu, L., Roberson, R., and Sherwood, J., 1976, The wave equation applied to migration: *Geophysical Prospecting*, v. 24, p. 380-399.
- McMechan, G. A., 1982, Determination of source parameters by wavefield extrapolation: *Geophysical Journal of the Royal Astronomical Society*, v. 71, p. 613-628.
- McMechan, G. A., 1983, Migration by extrapolation of time independent boundary values: *Geophysical Prospecting*, v. 31, p. 413-420.
- McMechan, G. A., and Fuis, G. S., 1987, Ray equation migration of wide-angle reflections from southern Alaska: *J. Geophys. Res.*, v. 92, p. 407-420.
- Meltzer, A. S., 1988, Crustal structure and tectonic evolution: central California: PhD thesis, Department of Geology and Geophysics, Rice University, Houston, TX.
- Milkereit, B., 1987, Migration of noisy crustal seismic data: *J. Geophys. Res.*, v. 92, p. 7916-7930.
- Milkereit, B., Epili, D., Green, A. G., Mereu, R. F., and Morel-a-l'Huissier, P., 1990, Migration of wide-angle seismic reflection data from the Grenville Front in Lake Huron. (submitted to *J. Geophys. Res.*)
- Mitchell, A., and Griffiths, D., 1980, The finite-difference methods in partial differential equations: John Wiley and Sons, Inc.
- Reshef, M., and Koslof, D., 1986, Migration of common-shot gathers, *Geophysics*, v. 51, p. 324-331.
- Schultz, P. S., and Sherwood, J. W. C., 1980, Depth migration before stack: *Geophysics*, v. 45, p. 376-393.
- Temme, P., 1984, A comparison of common-midpoint, single-shot, and plane-wave depth migration: *Geophysics*, v. 49, p. 1896-1907.

- Tozer, E. T., and Thorsteinsson, R., 1964, Western Queen Elizabeth islands, arctic archipelago: Geological Survey of Canada, Memoir 332, 242 p.
- Vidale, J., 1988, Finite-difference calculation of travel times: Bull. Seis. Soc. Am., v. 78, p. 2062-2076.
- Whitmore, N. D., 1983, Iterative depth migration by backward time propagation: Presented at the 53rd Annual International SEG Meeting in Las Vegas.
- Wiggins, J. W., 1984, Kirchhoff integral extrapolation and migration of nonplanar data: Geophysics, v. 49, p. 1239-1248.
- Yilmaz, O., 1987, Seismic data processing: Investigations in geophysics No. 2: Society of Exploration Geophysicists.
- Yilmaz, O., and Claerbout, J. F., 1980, Prestack partial migration: Geophysics, v. 45, p. 1753-1779.
- Yilmaz, O., and Lucas, D., 1986, Prestack layer replacement: Geophysics, v. 51, p. 1355-1369.

Appendix A

Velocity Analyses on Seismic Profile RU-3 for the Pre-Stack Migration

In this appendix, I describe the velocity analysis methods used to obtain the initial velocity function for the pre-stack migration of line RU-3. For most of the profile, velocity analyses was done by using the standard CMP velocity analysis program VELEX of the DISCO seismic data processing package. The CMP velocity analyses were applied to seismic data at every 500-m interval. Each time, eleven different velocity functions were used to compute the normal moveout, and stack the seismic data to form eleven CMP traces. An example is shown in Figure A-1, in which the figure to the left shows the eleven CMP stacks using the different velocity functions shown on the right-hand side figure. Suppose an earth model has horizontally layered subsurface structures. If the stacking velocities are chosen correctly, the stacked events should be aligned horizontally. The stacking velocities correspond to the picked events are then marked by the $\oplus$ sign on the velocity functions. Figure A-1 represents a typical example with the CMP points on top of the sedimentary sequence. The star signs in Figure A-1 mark the events and the stacking velocity values I picked for this particular CMP position. The stacking velocities are then converted to the interval velocities for migration.

As the shot point moves to the right of the Hosgri fault, the standard CMP velocity analysis technique is no longer available. Figure A-2 shows a typical example with the CMP points located to the right of the Hosgri fault, in which only the velocity of the water layer can be obtained. To obtain the velocity of the Franciscan rocks, I measured the shallow refractions directly from the shot records. Figure A-3 displayed a typical shot record with shot point on top of the high speed Franciscan uplift. The star sign shown in Figure A-3 marks the refraction waves used to measure the velocity. To obtain the accurate velocity, I have also sorted the shot records to the receiver gather, and measured the

reversed velocity. Figure A-4 shows the receiver gather corresponding to the shot record shown in Figure A-3. Again, the star sign marks the refracted waves measured. The calculated average velocity for the Franciscan rocks is 3000 m/sec, and the surface of the Franciscan rocks is dipping at 12-degrees seaward.

Figure A-1. Velocity analysis using the standard velocity analysis program VELEX of the DISCO seismic data processing package. The figure to the left shows the eleven CMP stacks using the different velocity functions shown on the right-hand side figure. This figure represents a typical example with the CMP points on top of the sedimentary sequence. The star signs in the figure mark the events and the stacking velocity values I picked for this particular CMP position.

Figure A-2. This figure represents a typical example with the CMP points on top of the high speed Franciscan uplift. The star signs in the figure mark the events and the stacking velocity values I picked for this particular CMP position.

Figure A-3. A typical shot record with shot point on top of the high speed Franciscan uplift. Shallow refractions, marked by the star sign, are easily seen on the data.

Figure A-4. The receiver gather corresponds to the shot record shown in Figure A-3. Shallow refractions are marked by the star sign.