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Optimal trajectories for hypervelocity flight

Lee, Woon Yung, Ph.D.

Rice University, 1989

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RICE UNIVERSITY

OPTIMAL TRAJECTORIES FOR HYPERVELOCITY FLIGHT

by

WOON YUNG LEE

A THESIS SUBMITTED
IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY

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April, 1989

Abstract

Optimal Trajectories for Hypervelocity Flight

by

Woon Yung Lee

This thesis discusses optimal trajectories for hypervelocity flight of interest in aeroassisted orbital transfer. Both coplanar orbital transfer and noncoplanar orbital transfer are studied. For these cases, the GEO-to-LEO transfer, the HEO-to-LEO transfer, and the LEO-to-LEO transfer are considered in connection with a spacecraft which is controlled during the atmospheric pass via the angle of attack (coplanar case) or via the angle of attack and the angle of bank (noncoplanar case).

For the noncoplanar case, three transfer maneuvers are studied. Type 1 involves four impulses and four space plane changes; Type 2 involves three impulses and three space plane changes; and Type 3 involves three impulses and no space plane change. In Type 1, the initial impulse directs the spacecraft away from Earth, and then is followed by an apogee impulse propelling the spacecraft toward Earth; in Types 2 and 3, the initial impulse directs the spacecraft toward Earth. A common element of these maneuvers is that they all include an atmospheric pass, with velocity depletion coupled with plane change.

Within the framework of classical optimal control, the following problems are studied: (P1) minimize the energy required for orbital transfer; (P2) maximize the time of flight during the atmospheric portion of the trajectory; and (P3) minimize the time integral of the square of the path inclination. Within the framework of minimax optimal control, the following problem is studied: (P4) minimize the peak heating rate.

Numerical solutions for Problems (P1), (P2), (P3), (P4) are obtained by

means of the sequential gradient–restoration algorithm for optimal control problems. The engineering implications of the results obtained are discussed. In particular, it is shown that the nearly–grazing solution [namely, the trajectory solving Problem (P3)] is a useful engineering compromise between energy requirements and aerodynamic heating requirements.

Key Words. Flight mechanics, astrodynamics, hypervelocity flight, aeroassisted orbital transfer, optimal trajectories, sequential gradient–restoration algorithm.

Acknowledgements

I would like to express my gratitude to Dr. Angelo Miele for his advice, guidance, and encouragement during my studies at Rice University. I also appreciate the advice and guidance given by Dr. Alan J. Chapman and Dr. John E. Merwin as Members of the Thesis Committee. I am indebted to the friends who have offered suggestions and support during the work. I would like to thank my parents and my parents-in-law for their love, prayers, and support for my education. Finally, I give thanks to God, who always led me to the right place. This research was supported by Jet Propulsion Laboratory and by Boeing Military Airplane Company.

Dedication

The author dedicates this work to his wife and his son with love and deep appreciation.

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1. Introduction

This thesis discusses optimal trajectories for hypervelocity flight of interest in aeroassisted orbital transfer. The main purpose of aeroassisted orbital transfer is to reduce the energy required for orbital transfer with respect to the energy needed in a Hohmann transfer maneuver. Aeroassisted orbital transfer can save propellant weight by comparison with the all-propulsive orbital transfer in space; in turn, the propellant weight saving implies an increase in the payload weight. It must be noted that aeroassisted orbital transfer involves aerodynamic heating of the spacecraft. Hence, a thermal protection system is needed, and this increases the weight of the spacecraft and decreases the payload weight. So, a compromise must be achieved between energy requirements and aerodynamic heating requirements.

Coplanar orbital transfer (problem without plane change) and noncoplanar orbital transfer (problem with plane change) are considered. For both the coplanar case and the noncoplanar case, the GEO-to-LEO transfer, the HEO-to-LEO transfer, and the LEO-to-LEO transfer are considered in connection with a spacecraft which is controlled during the atmospheric pass via the angle of attack (coplanar case) or via the angle of attack and the angle of bank (noncoplanar case).

For the noncoplanar case, three different transfer maneuvers are studied. Type 1 involves four impulses and four space plane changes; Type 2 involves three impulses and three space plane changes; and Type 3 involves three impulses and no space plane change. In Type 1, the initial impulse directs the spacecraft away from Earth, and then is followed by an apogee impulse propelling the spacecraft toward Earth; in Types 2 and 3, the initial impulse directs the spacecraft toward Earth. A common element of these maneuvers is that they all

include an atmospheric pass, with velocity depletion coupled with plane change.

Within the framework of classical optimal control, the following problems are studied: (P1) minimize the energy required for orbital transfer; (P2) maximize the time of flight during the atmospheric portion of the trajectory; and (P3) minimize the time integral of the square of the path inclination. Problems (P1), (P2), (P3) are Bolza problems of optimal control.

Within the framework of minimax optimal control, the following problem is studied: (P4) minimize the peak heating rate. Problem (P4) is Chebyshev problem of optimal control, which can be converted into a Bolza problem via suitable transformations.

Numerical solutions for Problems (P1), (P2), (P3), (P4) are obtained by means of the sequential gradient – restoration algorithm for optimal control problems. The engineering implications of the solutions are discussed in order to determine the most useful solutions in the light of energy requirements and heat transfer requirements. In particular, the merit of the nearly – grazing solution (P3) are discussed.

Previous Research. Previous research on the topics covered in this paper can be found in Refs. 1 – 43.

Since the initial paper by London (Ref. 1), considerable research has been done on aeroassisted orbital transfer (AOT), especially on the optimization and guidance of flight trajectories (Refs. 2 – 6). For a survey of the state of the art, see the paper by Walberg (Ref. 7). The survey paper by Mease presents an overview of what has been learned regarding the optimal control of aeroassisted orbital transfer vehicles (Ref. 8).

While most of the work on optimal trajectories has dealt with minimum – fuel transfers (namely, transfers minimizing the total characteristic

velocity), recent work performed at Rice University under JPL and Boeing sponsorship has dealt with transfers minimizing alternative performance indexes (for instance, the peak heating rate, the peak dynamic pressure, and the peak altitude drop during the atmospheric part of the AOT maneuver). See Refs. 9–15. In this connection, we note that an overview of theoretical calculus of variations and optimal control can be found in Refs. 16–18; an analysis of the performance of hypervelocity vehicles can be found in Refs. 19–20; a discussion of optimal atmospheric trajectories can be found in Ref. 21; and a discussion of optimal space trajectories can be found in Ref. 22.

Concerning the algorithms useful for solving optimal control problems of the Bolza type, see Refs. 23–28. In particular, first–order methods (gradient methods) are presented in Refs. 23–27 and second–order methods (quasilinearization methods) are presented in Ref. 28. Both first–order methods and second–order methods require the solution of a linear, two–point boundary–value problem at each iteration. In this connection, one of the most effective techniques is the method of particular solutions, discussed in Refs. 29–31.

Concerning optimal control problem of the Chebyshev type (minimax problems), see Refs. 32–41. While the Chebyshev problem is not covered by the Bolza formulation, it can be brought into the Bolza format by various transformations. These transformations are discussed in Refs. 32–41. Aerospace applications of minimax optimal control are presented in Refs. 9–15, Ref. 35, and Ref. 38.

Outline. In Section 2, the notations used in this thesis are listed. In Section 3, the equations of motion for aeroassisted flight are presented in general form.

In Section 4, we state the basic equations and boundary conditions to be

solved for coplanar, aeroassisted orbital transfer using lift modulation. Next, we discuss the performance indexes of interest, the algorithm employed, and the experimental data for coplanar AOT maneuvers. In addition, we present the numerical results obtained using the sequential gradient – restoration algorithm for optimal control problems and we discuss the implications of the results.

In Section 5, we state the basic equations and boundary conditions to be solved for noncoplanar, aeroassisted orbital transfer using lift modulation and bank angle modulation. Next, we discuss the performance indexes of interest, the algorithm employed, and the experimental data for noncoplanar AOT maneuvers. In addition, we present the numerical results obtained using the sequential gradient – restoration algorithm for optimal control problems and we discuss the implications of the results. Finally, in Section 6, we present the conclusions.

2. Notations

Throughout this thesis, these notations are employed:

C_D	= drag coefficient;
C_{D0}	= zero – lift drag coefficient;
C_L	= lift coefficient;
D	= drag, N;
f	= latitudinal component of the acceleration of gravity, m/sec^2
g	= radial component of the acceleration of gravity, m/sec^2 ;
h	= altitude, m;
H	= thickness of the atmosphere, m;
i	= total plane change, rad;
i_a	= atmospheric plane change, rad;
i_s	= space plane change, rad;
J_2	= characteristic constant of the Earth's gravitational field;
J_3	= characteristic constant of the Earth's gravitational field;
J_4	= characteristic constant of the Earth's gravitational field;
K	= induced drag factor;
L	= lift, N;
m	= mass, kg;
M	= Mach number;
r	= radial distance from the center of the Earth, m;
r_e	= radius of the Earth, m;
r_a	= radius of the outer edge of the atmosphere, m;
Re	= Reynolds number;
S	= reference surface area, m^2 ;
t	= T/τ = dimensionless time;

- T = running time, sec;
 V = velocity, m/sec;
 V_a = circular velocity at $r=r_a$, m/sec.

Greek Symbols

- α = angle of attack, rad;
 β = auxiliary control variable, rad;
 γ = path inclination, rad;
 ξ = auxiliary parameter, $(\text{rad})^{1/2}$;
 η = auxiliary parameter, $(\text{rad})^{1/2}$;
 θ = longitude, rad;
 μ = Earth's gravitational constant, m^3/sec^2 ;
 ξ = auxiliary parameter, $(\text{m}/\text{sec})^{1/2}$;
 ρ = air density, kg/m^3 ;
 σ = angle of bank, rad;
 τ = final time, sec;
 ϕ = latitude, rad;
 ψ = heading angle, rad;
 ω = angular velocity of the Earth, rad/sec;
 ΔV = characteristic velocity, m/sec.

Subscripts

- 0 = entry into the atmosphere;
 1 = exit from the atmosphere;
 00 = exit from the initial orbit;
 11 = entry into the final orbit;
 22 = apogee of the first elliptical transfer orbit for maneuvers of Type 1.

Superscripts

- = derivative with respect to dimensionless time;
- ~ = condition following the application of a velocity impulse.

Acronyms

- AFE = aeroassisted flight experiment;
- AFEV = aeroassisted flight experiment vehicle;
- AOT = aeroassisted orbital transfer;
- AOTV = aeroassisted orbital transfer vehicle;
- GEO = geosynchronous Earth orbit;
- HEO = high Earth orbit;
- LEO = low Earth orbit;
- LHR = local heating rate;
- PDP = peak dynamic pressure;
- PHR = peak heating rate;
- PTA = peak total acceleration.
- SGRA = sequential gradient – restoration algorithm.

3. General Equations of Motion

In this section, the atmospheric portion of the trajectory of a spacecraft is considered, and the equations of motion are first stated in general form, suitable for precision studies. One example is the study of the trajectory of the aeroassisted flight experiment vehicle (AFEV), to be flown in 1992 on board of the space shuttle. Because the AFEV must rendezvous with the space shuttle, the consideration of the terms due to the rotation and oblateness of the Earth is essential.

If various simplifications are incorporated, simplified equations of motion, suitable for feasibility studies, can be obtained. This refers to preliminary design studies of various types of aeroassisted orbital transfer vehicle (AOTV), performing either coplanar or noncoplanar orbital transfers. See Sections 4–5.

Differential System. Returning to precision studies, the following hypotheses are made: (i) the atmospheric pass is made with engine shut-off; hence, in this portion of the flight, the spacecraft behaves as a particle of constant mass; (ii) the aerodynamic forces are evaluated using the relative velocity; and (iii) the spacecraft flies from West to East. Upon normalizing the flight time to unity, the equations of motion are written as follows:

$$\dot{h} = \tau[V\sin\gamma], \quad (1)$$

$$\begin{aligned} \dot{V} = \tau[& -D/m - g\sin\gamma - f\cos\gamma\sin\psi \\ & + \omega^2 r \cos\phi(\sin\gamma\cos\phi - \cos\gamma\sin\phi\sin\psi)], \end{aligned} \quad (2)$$

$$\begin{aligned} \dot{\gamma} = \tau[& (L/mV)\cos\sigma + (V/r - g/V)\cos\gamma \\ & + (f/V)\sin\gamma\sin\psi + 2\omega\cos\phi\cos\psi \\ & + (\omega^2 r/V)\cos\phi(\cos\gamma\cos\phi + \sin\gamma\sin\phi\sin\psi)], \end{aligned} \quad (3)$$

$$\dot{\theta} = \tau[V\cos\gamma\cos\psi/r\cos\phi], \quad (4)$$

$$\dot{\phi} = \tau[V\cos\gamma\sin\psi/r], \quad (5)$$

$$\begin{aligned} \dot{\psi} = & \tau[(L/mV)\sin\sigma/\cos\gamma - (V/r)\cos\gamma\cos\psi\tan\phi \\ & - (f/V)\cos\psi/\cos\gamma + 2\omega(\tan\gamma\cos\phi\sin\psi - \sin\phi) \\ & - (\omega^2 r/V)\sin\phi\cos\phi\cos\psi/\cos\gamma], \end{aligned} \quad (6)$$

where $0 \leq t \leq 1$.

In the above equations, the following symbols and conventions are employed: h is the altitude, positive upward; V is the modulus of the relative velocity; γ is the path inclination, the angle between the local horizon and the relative velocity vector $\vec{V}$; γ is positive when $\vec{V}$ is directed upward with respect to the local horizon; θ is the longitude, measured along the Equator, starting from the fundamental meridian, the Greenwich meridian; θ is positive Eastward; ϕ is the latitude, measured along the local meridian, starting from the equatorial plane; ϕ is positive Northward; ψ is the heading angle, the angle between the local parallel and the projected velocity vector $\vec{V}_p$; in turn $\vec{V}_p$ is obtained by projecting $\vec{V}$ on the local horizon, and ψ is positive if $\vec{V}_p$ is directed inward with respect to the local parallel. Also, in the above equations, $\vec{D}$ is the drag, $\vec{L}$ is the lift, and σ is the bank angle, the angle between the lift vector $\vec{L}$ and the $(\vec{r}, \vec{V})$ plane; σ is positive if the spacecraft is banked to the left.

The symbol ω denotes the angular velocity of the Earth, $\omega = 0.72921116E-04$ rad/sec. Note that the terms linear in ω are due to the Coriolis acceleration, while the terms quadratic in ω are due to the transport acceleration. Also, r denotes the radial distance from the center of the Earth, g is the radial component of the acceleration of gravity, and f is the latitudinal component of the acceleration of gravity, given by

$$r = r_e + h, \quad (7)$$

$$g = (\mu/r^2)[1 + (3/2)J_2(r_e/r)^2(1 - 3\sin^2\phi) - 2J_3(r_e/r)^3(5\sin^3\phi - 3\sin\phi) - (5/8)J_4(r_e/r)^4(35\sin^4\phi - 30\sin^2\phi + 3)], \quad (8)$$

$$f = (\mu/r^2)[3J_2(r_e/r)^2\sin\phi\cos\phi + (3/2)J_3(r_e/r)^3(5\sin^2\phi\cos\phi - \cos\phi) + (5/2)J_4(r_e/r)^4(7\sin^3\phi\cos\phi - 3\sin\phi\cos\phi)]. \quad (9)$$

Here, $\mu = 0.3986E+15 \text{ m}^3/\text{sec}^2$ denotes the Earth's gravitational constant and J_2, J_3, J_4 denote the characteristic constants of the Earth's gravitational field, whose values are

$$J_2 = +0.1082E-03, \quad (10)$$

$$J_3 = -0.2560E-05, \quad (11)$$

$$J_4 = -0.1520E-05. \quad (12)$$

For engineering applications, gravitational terms of order higher than the fourth can be ignored.

In addition, the drag and the lift are given by

$$D = (1/2)C_D\rho SV^2, \quad (13)$$

$$L = (1/2)C_L\rho SV^2. \quad (14)$$

Generally speaking, the coefficients of drag and lift have the functional form

$$C_D = C_D(\alpha, M, Re), \quad (15)$$

$$C_L = C_L(\alpha, M, Re), \quad (16)$$

where α is the angle of attack, M is the Mach number, and Re is the Reynolds

number.

4. Coplanar Orbital Transfer

4.1. System Description

In this section, we consider coplanar aeroassisted orbital transfer from high Earth orbit (HEO) to low Earth orbit (LEO). In particular, HEO can be GEO, a geosynchronous Earth orbit. We employ the following assumptions: (i) the initial and final orbits are circular; (ii) three impulses are employed, one at the exit from the initial orbit, one at the exit from the atmosphere, and one at the entry into the final orbit; and (iii) the gravitational field is central and is governed by the inverse square law. The four key points of the maneuver are these: point 00, exit from the initial orbit; point 0, entrance into the atmosphere; point 1, exit from the atmosphere; and point 11, entrance into the final orbit.

The maneuver starts with a tangential propulsive burn, having characteristic velocity ΔV_{00} , at point 00; here, the spacecraft enters into an elliptical transfer orbit, connecting the point 00 and 0; this elliptical transfer orbit is such that its apogee occurs at r_{00} . At point 0, the spacecraft enters into the atmosphere; after traversing the upper layers of the atmosphere, it exits from the atmosphere at point 1; during the atmospheric pass, the velocity of the spacecraft is reduced, due to the aerodynamic drag. At point 1, the maneuver continues with a tangential propulsive burn, having characteristic velocity ΔV_1 ; then, the spacecraft enters into an elliptical transfer orbit connecting the points 1 and 11; this elliptical transfer orbit is such that its apogee occurs at r_{11} . The maneuver ends with a tangential propulsive burn, having characteristic velocity ΔV_{11} , at point 11; here, the spacecraft enters into the low Earth orbit, in that the magnitude of ΔV_{11} is such that the desired circularization into LEO is achieved. See Fig. 1.

Atmospheric Pass. With reference to the atmospheric portion of the

trajectory of an AOT vehicle, the following hypotheses are employed: (a) the atmospheric pass is made with engine shut-off; hence, in this portion of the flight, the AOT vehicle behaves as a particle of constant mass; (b) the Earth's rotation is neglected; (c) the Earth's oblateness is neglected; (d) under extreme hypersonic conditions, the dependence of the aerodynamic coefficients on the Mach number and the Reynolds number is disregarded.

Differential System. With the above assumptions, and upon normalizing the flight time to unity, the equations of motion are given by

$$\dot{h} = \tau[V\sin\gamma], \quad (17)$$

$$\dot{V} = \tau[-D/m - g\sin\gamma], \quad (18)$$

$$\dot{\gamma} = \tau[L/mV + (V/r - g/V)\cos\gamma], \quad (19)$$

where $0 \leq t \leq 1$. In the above equations,

$$r = r_e + h, \quad (20)$$

$$g = \mu/r^2, \quad (21)$$

where μ denotes the Earth's gravitational constant. In addition,

$$D = (1/2)C_D\rho SV^2, \quad (22)$$

$$L = (1/2)C_L\rho SV^2. \quad (23)$$

In particular, if a parabolic polar is postulated, the relation between the drag coefficient and the lift coefficient is given by

$$C_D = C_{D0} + KC_L^2. \quad (24)$$

Control Inequality Constraint. To obtain realistic solutions, the presence of upper and lower bounds on the lift coefficient is necessary. Therefore, the two-sided inequality constraint

$$C_{La} \leq C_L \leq C_{Lb} \quad (25)$$

must be satisfied everywhere along the interval of integration. The two-sided inequality constraint (25) can be converted into an equality constraint by means of the trigonometric transformation

$$C_L = (1/2)[(C_{La} + C_{Lb}) + (C_{Lb} - C_{La})\sin\beta], \quad (26)$$

where $\beta(t)$ denotes an auxiliary control. Equation (26) can be employed to eliminate $C_L(t)$ from (17)–(24). Therefore, the old control $C_L(t)$ is replaced with the new control $\beta(t)$. After a solution is found for $\beta(t)$, then $C_L(t)$ is computed with (26).

Boundary Equality Constraints. At the entrance of the sensible atmosphere ($t = 0$) and at the exit of the sensible atmosphere ($t = 1$), certain static and dynamic boundary conditions must be satisfied. Specifically, at atmospheric entry, we have

$$h_0 = H, \quad (27)$$

$$r_{00}^2(2V_a^2 - V_0^2) - 2r_{00}r_a V_a^2 + r_a^2 V_0^2 \cos^2 \gamma_0 = 0, \quad (28)$$

where V_a is the circular velocity at $r = r_a$. In addition, at atmospheric exit, we have

$$h_1 = H, \quad (29)$$

$$r_{11}^2(2V_a^2 - \tilde{V}_1^2) - 2r_{11}r_a V_a^2 + r_a^2 \tilde{V}_1^2 \cos^2 \gamma_1 = 0. \quad (30)$$

Note that $\tilde{V}_1$ is the value of the exit velocity after application of the propellant burn. Also, note that Eq. (30) can be rewritten in the alternative form

$$\tilde{V}_1 = V_a \sqrt{[2r_{11}(r_{11} - r_a)/(r_{11}^2 - r_a^2 \cos^2 \gamma_1)]}. \quad (31)$$

Equations (28), (30) are derived from energy conservation and angular momentum conservation applied to the HEO-to-entry and exit-to-LEO transfer orbits, respectively.

Boundary Inequality Constraints. To ensure that the vehicle enters into the atmosphere at point 0 and exits from the atmosphere at point 1, the entry and exit path inclinations must satisfy the inequality constraints

$$\gamma_0 \leq 0, \quad (32)$$

$$\gamma_1 \geq 0, \quad (33)$$

which must be complimented by the exit velocity condition

$$\tilde{V}_1 - V_1 \geq 0, \quad (34)$$

where $\tilde{V}_1$ is given by Eq. (31). These inequality constraints can be converted into equality constraints by means of the Valentine transformations

$$\gamma_0 + \eta^2 = 0, \quad (35)$$

$$\gamma_1 - \xi^2 = 0, \quad (36)$$

$$\tilde{V}_1 - V_1 - \xi^2 = 0, \quad (37)$$

where η , ξ , ξ denote parameters to be determined.

Summary. To sum up, the equations governing the atmospheric pass include: the differential system (17)–(24); the control inequality constraint (25),

converted into a control equality constraint by means of the trigonometric transformation (26); the boundary equality constraints (27)–(31); and the boundary inequality constraints (32)–(34), converted into boundary equality constraints by means of the Valentine transformations (35)–(37). In this formulation, the independent variable is the time t , $0 \leq t \leq 1$. The dependent variables include three state variables $[h(t), V(t), \gamma(t)]$, one control variable $[\beta(t)]$, and four parameters $[\tau, \eta, \zeta, \xi]$.

4.2. Performance Indexes

Subject to the previous constraints, different AOT optimization problems can be formulated, depending upon the performance index chosen. The resulting optimal control problems are either of the Bolza type or the Chebyshev type.

Problem (P1). Minimum Energy. It is required to minimize the energy required for orbital transfer. A measure of this energy is the total characteristic velocity ΔV , the sum of the characteristic velocity ΔV_{00} associated with the propulsive burn from the initial orbit, the characteristic velocity ΔV_1 associated with the propulsive burn at the exit from the atmosphere, and the characteristic velocity ΔV_{11} associated with the propulsive burn into the final orbit. Clearly,

$$I = \Delta V = \Delta V_{00} + \Delta V_1 + \Delta V_{11}, \quad (38)$$

with

$$\Delta V_{00} = V_a \sqrt{(r_a/r_{00})} - V_0(r_a/r_{00}) \cos \gamma_0, \quad (39)$$

$$\Delta V_1 = \tilde{V}_1 - V_1, \quad (40)$$

$$\Delta V_{11} = V_a \sqrt{(r_a/r_{11})} - \tilde{V}_1(r_a/r_{11}) \cos \gamma_1, \quad (41)$$

and

$$\tilde{V}_1 = V_a \sqrt{[2r_{11}(r_{11} - r_a)/(r_{11}^2 - r_a^2 \cos^2 \gamma_1)]}. \quad (42)$$

Problem (P2). Maximum Time. It is required to maximize the flight time during the atmospheric pass. Here, the performance index is given by

$$I = -\tau. \quad (43)$$

Problem (P3). Nearly-Grazing Trajectory. It is required to minimize the time integral of the square of path inclination. Here, the performance index is given by

$$I = \int_0^1 \tau \gamma^2 dt. \quad (44)$$

Problem (P4). Minimum Peak Heating Rate. It is required to minimize the peak value of the heating rate at a particular point of the spacecraft, for instance, the stagnation point. Here, the performance index is given by

$$I = \text{PHR} = \max_t (C\sqrt{\rho}V^{3.08}), \quad 0 \leq t \leq 1, \quad (45)$$

where C is a dimensional constant, characteristic of the spacecraft. This problem can be reformulated as that of minimizing the integral performance index

$$J = \int_0^1 \tau [C\sqrt{\rho}V^{3.08}]^q dt, \quad (46)$$

for large values of the exponent q (for example, $q = 5, 10$, or 20).

Remark. Problems (P1), (P2), (P3) are standard problems of optimal control of the Bolza type. Problem (P4) is nonstandard problem of optimal control of the Chebyshev type, if treated in the format (45); however, Problem (P4) becomes a standard problem of optimal control of the Bolza type, if treated in the format (46).

4.3. Experimental Data

The following data were used in the numerical experiments for coplanar orbital transfer.

For the spacecraft, it is assumed that: the mass per unit reference surface is $m/S = 300 \text{ kg/m}^2$; the zero-lift drag coefficient is $C_{D0} = 0.1$; the induced drag factor is $K = 1.11$; the lift coefficient for maximum lift-to-drag ratio is $C_{LE} = 0.3$; the maximum lift-to-drag ratio is $E_{\max} = 1.5$; and the bounds on the lift coefficient are $C_{La} = -0.9$ and $C_{Lb} = +0.9$.

The major physical constants used in computations are as follows: the radius of the Earth is $r_e = 6378 \text{ km}$; the radius of the outer edge of the sensible atmosphere is $r_a = 6498 \text{ km}$; the height of the sensible atmosphere is $H = 120 \text{ km}$; and the Earth's gravitational constant is $\mu = 398600 \text{ km}^3/\text{sec}^2$.

The assumed terminal orbits are as follows: the GEO radius is $r_{\text{GEO}} = 42164 \text{ km}$; the HEO radius is $r_{\text{HEO}} = 12996 \text{ km}$; the LEO radius is $r_{\text{LEO}} = 6558 \text{ km}$; the change of orbital inclination is $i = 0 \text{ deg}$.

The assumed atmospheric model is that of the US Standard Atmosphere, 1976 (see Ref. 43). In this model, the values of the density are tabulated at discrete altitudes. For intermediate altitudes, the density is computed by assuming an exponential fit for the function $\rho(h)$.

Concerning the peak heating rate, the following assumption is made: the constant C is such that, at the reference altitude $h_R = H/3 = 40 \text{ km}$ and at the reference velocity $V_R = V_a = 7.832 \text{ km/sec}$, the corresponding local heating rate is

$$\text{LHR} = C\sqrt{\rho_R} V_R^{3.08} = 348.7 \text{ W/cm}^2, \quad (47)$$

where the reference density is $\rho_R = 0.3996\text{E}-02 \text{ kg/m}^3$. Equation (47) gives the

convective heating rate under the following assumptions: the flow has low Reynolds number; the boundary layer is laminar; and the distribution of the heating rate over the nose exhibits a maximum at the stagnation point.

4.4. Algorithm

Problems (P1), (P2), (P3), (P4) were solved employing the sequential gradient–restoration algorithm (SGRA, Refs. 23–27). This algorithm was programmed in FORTRAN; a VS FORTRAN compiler was used, and the numerical results were obtained in double–precision arithmetic. Computations were performed at Rice University using an NAS–AS–9000 computer.

The interval of integration was divided into 100 steps. The differential systems were integrated using Hamming’s modified predictor–corrector method, with a special Runge–Kutta starting procedure (Ref. 42). Definite integrals were computed using a modified Simpson’s rule. Linear algebraic systems were solved using a standard Gaussian elimination routine. For both the gradient phase and the restoration phase of SGRA, the linear two–point boundary–value problem was solved using the method of particular solutions.

For each problem, the functional being minimized was suitably scaled. The following stopping conditions were employed for the sequential gradient–restoration algorithm:

$$P \leq E-08, \quad (48)$$

$$Q \leq E-08, \quad (49)$$

where P is the norm squared of the constraint error and Q is the norm squared of the error in the optimality conditions. An upper bound was imposed on the total number of iterations, namely,

$$N \leq 100. \quad (50)$$

4.5. Numerical Solutions

Summary data are shown in Tables 1–2 and Fig. 2 for both the GEO–to–LEO transfer and the HEO–to–LEO transfer. The following comments refer to the minimum energy solution (P1), the maximum time solution (P2), the nearly–grazing solution (P3), and the minimum peak heating rate solution (P4).

4.5.1. GEO–to–LEO Transfer

Energy. The total characteristic velocity is $\Delta V = 1.514$ km/sec for the solution (P1), $\Delta V = 1.517$ km/sec for the solution (P2), $\Delta V = 1.528$ km/sec for the solution (P3), and $\Delta V = 1.526$ km/sec for the solution (P4); these values are considerably below the total characteristic velocity of the two–impulse Hohmann transfer, $\Delta V = 3.939$ km/sec. On a relative basis, with respect to the total characteristic velocity of (P1), that of (P2) is 0.2% higher; that of (P3) is 0.9% higher; that of (P4) is 0.8% higher; and that of the two–impulse Hohmann transfer is 160.2% higher.

Peak Heating Rate. The peak heating rate is $\text{PHR} = 69.45$ W/cm² for the solution (P1), $\text{PHR} = 124.4$ W/cm² for the solution (P2), $\text{PHR} = 66.82$ W/cm² for the solution (P3), and $\text{PHR} = 66.76$ W/cm² for the solution (P4). On a relative basis, with respect to the peak heating rate of (P1), that of (P2) is 79.1% higher; that of (P3) is 3.8% lower; and that of (P4) is 3.9% lower.

Peak Dynamic Pressure. The peak dynamic pressure is $\text{PDP} = 0.1825$ N/cm² for the solution (P1), $\text{PDP} = 0.5896$ N/cm² for the solution (P2), $\text{PDP} = 0.1656$ N/cm² for the solution (P3), and $\text{PDP} = 0.1666$ N/cm² for the solution (P4). On a relative basis, with respect to the peak dynamic pressure of (P1), that of (P2) is 223.1% higher; that of (P3) is 9.3% lower; and that of (P4) is 8.7%

lower.

Peak Total Acceleration. The peak total acceleration is $PTA = 6.976$ m/sec^2 for the solution (P1), $PTA = 7.383$ m/sec^2 for the solution (P2), $PTA = 6.954$ m/sec^2 for the solution (P3), and $PTA = 6.959$ m/sec^2 for the solution (P4). On a relative basis, with respect to the peak total acceleration of (P1), that of (P2) is 5.8% higher; that of (P3) is 0.3% lower; and that of (P4) is 0.2% lower.

Minimum Altitude. The minimum altitude is $h = 75.47$ km for the solution (P1), $h = 67.25$ km for the solution (P2), $h = 75.95$ km for the solution (P3), and $h = 75.95$ km for the solution (P4). On a relative basis, with respect to the minimum altitude of (P1), that of (P2) is 10.9% lower; that of (P3) is 0.6% higher; and that of (P4) is 0.6% higher.

Flight Time. The flight time is $\tau = 2465$ sec for the solution (P1), $\tau = 3376$ sec for the solution (P2), $\tau = 2728$ sec for the solution (P3), and $\tau = 2655$ sec for the solution (P4). On a relative basis, with respect to the flight time of (P1), that of (P2) is 37.0% higher; that of (P3) is 10.7% higher; and that of (P4) is 7.7% higher.

4.5.2. HEO-to-LEO Transfer

Energy. The total characteristic velocity is $\Delta V = 1.049$ km/sec for the solution (P1), $\Delta V = 1.053$ km/sec for the solution (P2), $\Delta V = 1.060$ km/sec for the solution (P3), and $\Delta V = 1.054$ km/sec for the solution (P4); these values are considerably below the total characteristic velocity of the two-impulse Hohmann transfer, $\Delta V = 2.194$ km/sec. On a relative basis, with respect to the total characteristic velocity of (P1), that of (P2) is 0.4% higher; that of (P3) is 1.0% higher; that of (P4) is 0.5% higher; and that of the two-impulse Hohmann transfer is 109.2% higher.

Peak Heating Rate. The peak heating rate is $PHR = 35.10$ W/cm^2 for the

solution (P1), $\text{PHR} = 64.36 \text{ W/cm}^2$ for the solution (P2), $\text{PHR} = 34.45 \text{ W/cm}^2$ for the solution (P3), and $\text{PHR} = 34.40 \text{ W/cm}^2$ for the solution (P4). On a relative basis, with respect to the peak heating rate of (P1), that of (P2) is 83.4% higher; that of (P3) is 1.9% lower; and that of (P4) is 2.0% lower.

Peak Dynamic Pressure. The peak dynamic pressure is $\text{PDP} = 0.07487 \text{ N/cm}^2$ for the solution (P1), $\text{PDP} = 0.2392 \text{ N/cm}^2$ for the solution (P2), $\text{PDP} = 0.07185 \text{ N/cm}^2$ for the solution (P3), and $\text{PDP} = 0.07197 \text{ N/cm}^2$ for the solution (P4). On a relative basis, with respect to the peak dynamic pressure of (P1), that of (P2) is 219.5% higher; that of (P3) is 4.0% lower; and that of (P4) is 3.9% lower.

Peak Total Acceleration. The peak total acceleration is $\text{PTA} = 3.182 \text{ m/sec}^2$ for the solution (P1), $\text{PTA} = 3.226 \text{ m/sec}^2$ for the solution (P2), $\text{PTA} = 3.180 \text{ m/sec}^2$ for the solution (P3), and $\text{PTA} = 3.180 \text{ m/sec}^2$ for the solution (P4). On a relative basis, with respect to the peak total acceleration of (P1), that of (P2) is 1.4% higher; that of (P3) is 0.1% lower; and that of (P4) is 0.1% lower.

Minimum Altitude. The minimum altitude is $h = 79.77 \text{ km}$ for the solution (P1), $h = 72.12 \text{ km}$ for the solution (P2), $h = 79.97 \text{ km}$ for the solution (P3), and $h = 79.97 \text{ km}$ for the solution (P4). On a relative basis, with respect to the minimum altitude of (P1), that of (P2) is 9.6% lower; that of (P3) is 0.3% higher; and that of (P4) is 0.3% higher.

Flight Time. The flight time is $\tau = 2263 \text{ sec}$ for the solution (P1), $\tau = 3046 \text{ sec}$ for the solution (P2), $\tau = 2422 \text{ sec}$ for the solution (P3), and $\tau = 2258 \text{ sec}$ for the solution (P4). On a relative basis, with respect to the flight time of (P1), that of (P2) is 34.6% higher; that of (P3) is 7.0% higher; and that of (P4) is 0.2% lower.

4.5.3. Summary

From the previous analysis of coplanar transfer, GEO-to-LEO and HEO-to-LEO, the following conclusions can be inferred:

- (i) from the energy viewpoint, the total characteristic velocities of the solutions (P1), (P2), (P3), (P4) are nearly the same; they are considerably below the total characteristic velocity of the two-impulse Hohmann transfer;
 - (ii) in terms of the main quantities of interest (namely, peak heating rate, peak dynamic pressure, and peak total acceleration), the solutions (P1), (P3), (P4) are nearly the same, indeed, almost undistinguishable from one another; they are considerably superior to the solution (P2);
 - (iii) comparison of the GEO-to-LEO transfer and the HEO-to-LEO transfer shows that, regardless of the solution type, the former involves higher energy requirements, higher peak heating rate, higher peak dynamic pressure, higher peak total acceleration, lower minimum altitude, and longer flight time.
- For the nearly-grazing solutions (P3), the time histories of the state variables and the control variable are shown in Fig. 3.

5. Noncoplanar Orbital Transfer

5.1. System Description

In this section, we consider noncoplanar aeroassisted orbital transfer. The GEO-to-LEO transfer, the HEO-to-LEO transfer, and the LEO-to-LEO transfer are studied under the following assumptions: (i) the initial and final orbits are circular; (ii) at most, four impulses are employed, one at the exit from the initial orbit, one at apogee, one at the exit from the atmosphere, and one at the entrance into the final orbit; (iii) the prescribed plane change is performed partly in space and partly in the atmosphere; (iv) the space plane change is performed partly at the exit from the initial orbit, partly at apogee, partly at atmospheric exit, and partly at the entrance into the final orbit; and (v) the gravitational field is central and is governed by the inverse square law.

The five key points of the maneuver are these: point 00, exit from the initial orbit; point 22, apogee of the first elliptical transfer orbit (maneuver of Type 1); point 0, entrance into the atmosphere; point 1, exit from the atmosphere; and point 11, entrance into the final orbit.

Maneuver of Type 1. This maneuver involves four impulses and four space plane changes plus one atmospheric plane change.

At point 00, the maneuver starts with a nontangential propulsive burn, having characteristic velocity ΔV_{00} . The spacecraft performs the plane change i_{s00} and enters into a transfer orbit, going away from Earth and connecting the points 00 and 22; the latter is the apogee of the transfer orbit.

At point 22, the maneuver continues with a nontangential propulsive burn, having characteristic velocity ΔV_{22} . The spacecraft performs the plane change i_{s22} and enters into a transfer orbit going toward Earth and connecting the points 22 and 0.

At point 0, the spacecraft enters into the atmosphere; after traversing the upper layers of the atmosphere, it exits from the atmosphere at point 1. During the atmospheric pass, the plane change i_a is performed gradually; in addition, the velocity of the spacecraft is depleted, due to the aerodynamic drag.

At point 1, the maneuver continues with a nontangential propulsive burn, having characteristic velocity ΔV_1 . The spacecraft performs the plane change i_{s1} and enters into a transfer orbit, going away from Earth and connecting the points 1 and 11; the latter is the apogee of the transfer orbit.

At point 11, the maneuver ends with a nontangential propulsive burn, having characteristic velocity ΔV_{11} . The spacecraft performs the plane change i_{s11} and achieves the desired circularization into LEO orbit by proper adjustment of ΔV_{11} . See Fig. 4.

Maneuver of Type 2. This maneuver involves three impulses and three space plane changes plus one atmospheric plane change. It is identical with a maneuver of Type 1, with the following exception: the initial propulsive burn directs the spacecraft toward Earth; hence, the subsequent propulsive burn at apogee is missing ($\Delta V_{22} = 0$). See Fig. 5.

Maneuver of Type 3. This maneuver involves three impulses and no space plane change; hence, the plane change is entirely atmospheric. It is identical with a maneuver of Type 2 ($\Delta V_{22} = 0$), with one exception: the propulsive burns at the exit from the initial orbit, atmospheric exit, and LEO entrance are entirely tangential ($i_{s00} = 0, i_{s1} = 0, i_{s11} = 0$). See Fig. 6.

Atmospheric Pass. With reference to the atmospheric portion of the trajectory of an AOT vehicle, the following hypotheses are employed: (a) the atmospheric pass is made with engine shut-off; hence, in this portion of the flight, the AOT vehicle behaves as a particle of constant mass; (b) the Earth's

rotation is neglected; (c) the Earth's oblateness is neglected; (d) under extreme hypersonic conditions, the dependence of the aerodynamic coefficients on the Mach number and the Reynolds number is disregarded.

Differential System. With the above assumptions, and upon normalizing the flight time to unity, the equations of motion are given by

$$\dot{h} = \tau[V\sin\gamma], \quad (51)$$

$$\dot{V} = \tau[-D/m - g\sin\gamma], \quad (52)$$

$$\dot{\gamma} = \tau[(L/mV)\cos\sigma + (V/r - g/V)\cos\gamma], \quad (53)$$

$$\dot{\theta} = \tau[V\cos\gamma\cos\psi/r\cos\phi], \quad (54)$$

$$\dot{\phi} = \tau[V\cos\gamma\sin\psi/r], \quad (55)$$

$$\dot{\psi} = \tau[(L/mV)\sin\sigma/\cos\gamma - (V/r)\cos\gamma\cos\psi\tan\phi], \quad (56)$$

where $0 \leq t \leq 1$. In the above equations,

$$r = r_e + h, \quad (57)$$

$$g = \mu/r^2, \quad (58)$$

where μ denotes the Earth's gravitational constant. In addition,

$$D = (1/2)C_D\rho SV^2, \quad (59)$$

$$L = (1/2)C_L\rho SV^2. \quad (60)$$

In particular, if a parabolic polar is postulated, the relation between the drag coefficient and the lift coefficient is given by

$$C_D = C_{D0} + KC_L^2. \quad (61)$$

Control Inequality Constraint. Once more, the presence of upper and lower bounds on the lift coefficient is necessary to obtain realistic solutions. Therefore, the two-sided inequality constraint

$$C_{La} \leq C_L \leq C_{Lb} \quad (62)$$

must be satisfied everywhere along the interval of integration. The two-sided inequality constraint (62) can be converted into an equality constraint by means of the trigonometric transformation of the type

$$C_L = (1/2)[(C_{La} + C_{Lb}) + (C_{Lb} - C_{La})\sin\beta], \quad (63)$$

where $\beta(t)$ denotes an auxiliary control. Equation (63) can be employed to eliminate $C_L(t)$ from (51)–(61). Therefore, the old control $C_L(t)$ is replaced with the new control $\beta(t)$. After a solution is found for $\beta(t)$, then $C_L(t)$ is computed with (63).

Boundary Equality Constraints. At the entrance of the sensible atmosphere ($t = 0$) and at the exit of the sensible atmosphere ($t = 1$), certain static and dynamic boundary conditions must be satisfied. These conditions are now described for maneuvers of Type 1, Type 2, and Type 3. Specifically, at atmospheric entry, we have

$$h_0 = H, \quad (64)$$

$$(T1) \quad r_{22}^2(2V_a^2 - V_0^2) - 2r_{22}r_a V_a^2 + r_a^2 V_0^2 \cos^2 \gamma_0 = 0, \quad (65)$$

$$(T2) \quad r_{00}^2(2V_a^2 - V_0^2) - 2r_{00}r_a V_a^2 + r_a^2 V_0^2 \cos^2 \gamma_0 = 0, \quad (66)$$

$$(T3) \quad r_{00}^2(2V_a^2 - V_0^2) - 2r_{00}r_a V_a^2 + r_a^2 V_0^2 \cos^2 \gamma_0 = 0, \quad (67)$$

$$\theta_0 = 0, \quad (68)$$

$$\phi_0 = 0, \quad (69)$$

$$\psi_0 = 0, \quad (70)$$

where V_a is the circular velocity at $r = r_a$. In addition, at atmospheric exit, we have

$$h_1 = H, \quad (71)$$

$$\cos\phi_1\cos\psi_1 - \cos i_a = 0, \quad (72)$$

$$r_{11}^2(2V_a^2 - \tilde{V}_1^2) - 2r_{11}r_a V_a^2 + r_a^2 \tilde{V}_1^2 \cos^2 \gamma_1 = 0. \quad (73)$$

Note that Eq. (73) can be rewritten in the alternative form

$$\tilde{V}_1 = V_a \sqrt{[2r_{11}(r_{11} - r_a)/(r_{11}^2 - r_a^2 \cos^2 \gamma_1)]}. \quad (74)$$

Also, note that Eqs. (65), (66), (67), (73) are obtained from energy conservation and angular momentum conservation applied to the apogee-to-entry, HEO-to-entry, HEO-to-entry, and exit-to-LEO transfer orbits, respectively.

Equation (72) is a relation connecting the atmospheric plane change to the exit values of the latitude and the heading angle.

Boundary Inequality Constraints. To ensure that the vehicle enters into the atmosphere at point 0 and exits from the atmosphere at point 1, the entry and exit path inclinations must satisfy the inequality constraints

$$\gamma_0 \leq 0, \quad (75)$$

$$\gamma_1 \geq 0, \quad (76)$$

which must be complimented by the exit velocity condition

$$\tilde{V}_1 - V_1 \geq 0, \quad (77)$$

where $\tilde{V}_1$ is given by Eq. (74). These inequality constraints can be converted into equality constraints by means of the Valentine transformations

$$\gamma_0 + \eta^2 = 0, \quad (78)$$

$$\gamma_1 - \xi^2 = 0, \quad (79)$$

$$\tilde{V}_1 - V_1 - \xi^2 = 0, \quad (80)$$

where η , ξ , ξ denote parameters to be determined.

Plane Change Condition. The following approximate relation holds between the total plane change i , the space plane change i_s , and the atmospheric plane change i_a :

$$i = i_s + i_a. \quad (81)$$

In turn, the space plane change is approximately given by

$$(T1) \quad i_s = i_{s00} + i_{s22} + i_{s1} + i_{s11}, \quad (82)$$

$$(T2) \quad i_s = i_{s00} + i_{s1} + i_{s11}, \quad (83)$$

$$(T3) \quad i_s = 0, \quad (84)$$

where i_{s00} is the initial plane change, i_{s22} is the apogee plane change, i_{s1} is the exit plane change, and i_{s11} is the final plane change. As a consequence, the total plane change can be rewritten in the form

$$(T1) \ i = i_{s00} + i_{s22} + i_{s1} + i_{s11} + i_a, \quad (85)$$

$$(T2) \ i = i_{s00} + i_{s1} + i_{s11} + i_a, \quad (86)$$

$$(T3) \ i = i_a. \quad (87)$$

The quantities appearing on the right-hand sides of (85)–(87), if free, must be regarded as parameters to be determined.

Remark. It is to be emphasized that the total plane change predicted with the linear relations (81)–(87) is approximate. Exact nonlinear relations can be derived for the computation of i . However, the relative differences are small, of the order of 1%, for the examples presented in this thesis. Since the main thrust of the paper is a feasibility study and an assessment of the relative merits of various optimal trajectories, the simpler linear relations (81)–(87) are employed in place of the more complicated exact nonlinear relations.

Summary. To sum up, the equations governing the atmospheric pass include: the differential system (51)–(61); the control inequality constraint (62), converted into a control equality constraint by means of the trigonometric transformation (63); the boundary equality constraints (64)–(74); the boundary inequality constraints (75)–(77), converted into boundary equality constraints by means of the Valentine transformations (78)–(80); and the plane change conditions (81)–(87).

In this formulation, the independent variable is the time t , $0 \leq t \leq 1$. The dependent variables include six state variables $[h(t), V(t), \gamma(t), \theta(t), \phi(t), \psi(t)]$, two control variables $[\beta(t), \sigma(t)]$, and p parameters, with $p = 10$ for maneuvers of Type 1, $p = 8$ for maneuvers of Type 2, and $p = 4$ for maneuvers of Type 3. More specifically, the parameters to be determined are as follows:

$$(T1) \quad \tau, \eta, \zeta, \xi, i_{s00}, i_{s22}, i_{s1}, i_{s11}, i_a, r_{22}; \quad (88)$$

$$(T2) \quad \tau, \eta, \zeta, \xi, i_{s00}, i_{s1}, i_{s11}, i_a; \quad (89)$$

$$(T3) \quad \tau, \eta, \zeta, \xi. \quad (90)$$

5.2. Performance Indexes

Subject to the previous constraints, different AOT optimization problems can be formulated, depending upon the performance index chosen. The resulting optimal control problems are either of the Bolza type or the Chebyshev type.

Problem (P1). Minimum Energy. It is required to minimize the energy required for orbital transfer. A measure of this energy is the total characteristic velocity ΔV , the sum of the characteristic velocity ΔV_{00} associated with the propulsive burn from the initial orbit, the characteristic velocity ΔV_{22} associated with the propulsive burn at apogee, the characteristic velocity ΔV_1 associated with the propulsive burn at the exit from the atmosphere, and the characteristic velocity ΔV_{11} associated with the propulsive burn into LEO.

Transfer Maneuvers of Type 1. For these maneuvers, we have

$$I = \Delta V = \Delta V_{00} + \Delta V_{22} + \Delta V_1 + \Delta V_{11}, \quad (91)$$

with

$$\Delta V_{00} = V_a \sqrt{[\delta^2 + 1 - 2\delta \cos i_{s00}]/(r_a/r_{00})}, \quad (92)$$

$$\begin{aligned} \Delta V_{22} &= \sqrt{[(\epsilon V_a)^2 + (V_0 \cos \gamma_0)^2 (r_a/r_{22}) \\ &\quad - 2\epsilon V_a V_0 \cos \gamma_0 \sqrt{(r_a/r_{22})} \cos i_{s22}]/(r_a/r_{22})}, \end{aligned} \quad (93)$$

$$\Delta V_1 = \sqrt{[\tilde{V}_1^2 + V_1^2 - 2\tilde{V}_1 V_1 \cos i_{s1}]}, \quad (94)$$

$$\Delta V_{11} = \sqrt{[V_a^2 + (\tilde{V}_1 \cos \gamma_1)^2 (r_a/r_{11})]}$$

$$- 2V_a \tilde{V}_1 \cos \gamma_1 \sqrt{(r_a/r_{11}) \cos i_{s11}} \sqrt{(r_a/r_{11})}, \quad (95)$$

and

$$\delta = \sqrt{[2r_{22}/(r_{00} + r_{22})]}, \quad (96)$$

$$\epsilon = \sqrt{[2r_{00}/(r_{00} + r_{22})]}, \quad (97)$$

$$\tilde{V}_1 = V_a \sqrt{[2r_{11}(r_{11} - r_a)/(r_{11}^2 - r_a^2 \cos^2 \gamma_1)]}. \quad (98)$$

Transfer Maneuvers of Type 2. For these maneuvers, we have

$$I = \Delta V = \Delta V_{00} + \Delta V_1 + \Delta V_{11}, \quad (99)$$

with

$$\begin{aligned} \Delta V_{00} &= \sqrt{[V_a^2 + (V_0 \cos \gamma_0)^2 (r_a/r_{00})} \\ &\quad - 2V_a V_0 \cos \gamma_0 \sqrt{(r_a/r_{00}) \cos i_{s00}}] \sqrt{(r_a/r_{00})}}, \end{aligned} \quad (100)$$

$$\Delta V_1 = \sqrt{[\tilde{V}_1^2 + V_1^2 - 2\tilde{V}_1 V_1 \cos i_{s1}]}, \quad (101)$$

$$\begin{aligned} \Delta V_{11} &= \sqrt{[V_a^2 + (\tilde{V}_1 \cos \gamma_1)^2 (r_a/r_{11})} \\ &\quad - 2V_a \tilde{V}_1 \cos \gamma_1 \sqrt{(r_a/r_{11}) \cos i_{s11}}] \sqrt{(r_a/r_{11})}}, \end{aligned} \quad (102)$$

and

$$\tilde{V}_1 = V_a \sqrt{[2r_{11}(r_{11} - r_a)/(r_{11}^2 - r_a^2 \cos^2 \gamma_1)]}. \quad (103)$$

Transfer Maneuvers of Type 3. For these maneuvers, we have

$$I = \Delta V = \Delta V_{00} + \Delta V_1 + \Delta V_{11}, \quad (104)$$

with

$$\Delta V_{00} = V_a \sqrt{(r_a/r_{00})} - V_0(r_a/r_{00}) \cos \gamma_0, \quad (105)$$

$$\Delta V_1 = \tilde{V}_1 - V_1, \quad (106)$$

$$\Delta V_{11} = V_a \sqrt{(r_a/r_{11})} - \tilde{V}_1(r_a/r_{11}) \cos \gamma_1, \quad (107)$$

and

$$\tilde{V}_1 = V_a \sqrt{[2r_{11}(r_{11} - r_a)/(r_{11}^2 - r_a^2 \cos^2 \gamma_1)]}. \quad (108)$$

Problem (P2). Maximum Time. It is required to maximize the flight time during the atmospheric pass. Here, the performance index is given by

$$I = -\tau. \quad (109)$$

Problem (P3). Nearly-Grazing Trajectory. It is required to minimize the time integral of the square of path inclination. Here, the performance index is given by

$$I = \int_0^1 \tau \gamma^2 dt. \quad (110)$$

Problem (P4). Minimum Peak Heating Rate. It is required to minimize the peak value of the heating rate at a particular point of the spacecraft, for instance, the stagnation point. Except for a proportionality constant, the performance index is given by

$$I = \text{PHR} = \max_t (C \sqrt{\rho} V^{3.08}), \quad 0 \leq t \leq 1, \quad (111)$$

where C is a dimensional constant, characteristic of the spacecraft. This problem can be reformulated as that of minimizing the integral performance index

$$J = \int_0^1 \tau [C \sqrt{\rho} V^{3.08}]^q dt, \quad (112)$$

for large values of the exponent q (for example, $q = 5, 10$, or 20).

Remark. Problems (P1), (P2), (P3) are standard problems of optimal control of the Bolza type. Problem (P4) is nonstandard problem of optimal control of the Chebyshev type, if treated in the format (111); however, Problem (P4) becomes a standard problem of optimal control of the Bolza type, if treated in the format (112).

5.3. Experimental Data

The following data were used in the numerical experiments for noncoplanar orbital transfer.

For the spacecraft, it is assumed that: the mass per unit reference surface is $m/S = 300 \text{ kg/m}^2$; the zero-lift drag coefficient is $C_{D0} = 0.1$; the induced drag factor is $K = 1.11$; the lift coefficient for maximum lift-to-drag ratio is $C_{LE} = 0.3$; the maximum lift-to-drag ratio is $E_{\max} = 1.5$; and the bounds on the lift coefficient are $C_{La} = -0.9$ and $C_{Lb} = +0.9$.

The major physical constants used in computations are as follows: the radius of the Earth is $r_e = 6378 \text{ km}$; the radius of the outer edge of the sensible atmosphere is $r_a = 6498 \text{ km}$; the height of the sensible atmosphere is $H = 120 \text{ km}$; and the Earth's gravitational constant is $\mu = 398600 \text{ km}^3/\text{sec}^2$.

The assumed terminal orbits are as follows: the GEO radius is $r_{\text{GEO}} = 42164 \text{ km}$; the HEO radius is $r_{\text{HEO}} = 12996 \text{ km}$; the LEO radius is $r_{\text{LEO}} = 6558 \text{ km}$; the change of orbital inclination is $i = 30 \text{ deg}$.

The assumed atmospheric model is that of the US Standard Atmosphere, 1976 (see Ref. 43). In this model, the values of the density are tabulated at discrete altitudes. For intermediate altitudes, the density is computed by assuming an exponential fit for the function $\rho(h)$.

Concerning the peak heating rate, the following assumption is made: the

constant C is such that, at the reference altitude $h_R = H/3 = 40$ km and at the reference velocity $V_R = V_a = 7.832$ km/sec, the corresponding local heating rate is

$$\text{LHR} = C/\rho_R V_R^{3.08} = 348.7 \text{ W/cm}^2, \quad (113)$$

where the reference density is $\rho_R = 0.3996\text{E}-02 \text{ kg/m}^3$. Equation (113) gives the convective heating rate under the following assumptions: the flow has low Reynolds number; the boundary layer is laminar; and the distribution of the heating rate over the nose exhibits a maximum at the stagnation point.

5.4. Algorithm

Problems (P1), (P2), (P3), (P4) were solved employing the sequential gradient-restoration algorithm (SGRA, Refs. 23-27). This algorithm was programmed in FORTRAN; a VS FORTRAN compiler was used, and the numerical results were obtained in double-precision arithmetic. Computations were performed at Rice University using an NAS-AS-9000 computer.

The interval of integration was divided into 100 steps. The differential systems were integrated using Hamming's modified predictor-corrector method, with a special Runge-Kutta starting procedure (Ref. 42). Definite integrals were computed using a modified Simpson's rule. Linear algebraic systems were solved using a standard Gaussian elimination routine. For both the gradient phase and the restoration phase of SGRA, the linear two-point boundary-value problem was solved using the method of particular solutions.

For each problem, the functional being minimized was suitably scaled. The following stopping conditions were employed for the sequential gradient-restoration algorithm:

$$P \leq E-08, \quad (114)$$

$$Q \leq E-08, \quad (115)$$

where P is the norm squared of the constraint error and Q is the norm squared of the error in the optimality conditions. An upper bound was imposed on the total number of iterations, namely,

$$N \leq 100. \quad (116)$$

5.5. Numerical Solutions for GEO-to-LEO Transfer

Summary data are shown in Tables 3–5 and Fig. 7 for maneuvers of Type 1, Type 2, and Type 3. The following comments refer to the minimum energy solution (P1), the maximum time solution (P2), the nearly-grazing solution (P3), and the minimum peak heating rate solution (P4). In computing numerical solutions for Problems (P1), (P2), (P3), (P4), the following procedure was adopted: for Problem (P1), the plane change components and the apogee radius were optimized; for Problems (P2), (P3), (P4), the plane change components and the apogee radius were kept at the same levels determined for Problem (P1); this was done in order to impart good energy characteristics to the solutions of Problems (P2), (P3), (P4).

5.5.1. Maneuver of Type 1

Energy. The total characteristic velocity is $\Delta V = 1.569$ km/sec for the solution (P1), $\Delta V = 1.569$ km/sec for the solution (P2), $\Delta V = 1.615$ km/sec for the solution (P3), and $\Delta V = 3.212$ km/sec for the solution (P4); these values are considerably below the total characteristic velocity of the two-impulse Hohmann transfer, $\Delta V = 4.306$ km/sec. On a relative basis, with respect to the total characteristic velocity of (P1), that of (P2) is nearly the same; that of (P3) is 2.9%

higher; that of (P4) is 104.7% higher; and that of the two-impulse Hohmann transfer is 174.4% higher.

Peak Heating Rate. The peak heating rate is $\text{PHR} = 249.4 \text{ W/cm}^2$ for the solution (P1), $\text{PHR} = 247.3 \text{ W/cm}^2$ for the solution (P2), $\text{PHR} = 182.2 \text{ W/cm}^2$ for the solution (P3), and $\text{PHR} = 99.31 \text{ W/cm}^2$ for the solution (P4). On a relative basis, with respect to the peak heating rate of (P1), that of (P2) is 0.8% lower; that of (P3) is 26.9% lower; and that of (P4) is 60.2% lower.

Peak Dynamic Pressure. The peak dynamic pressure is $\text{PDP} = 2.580 \text{ N/cm}^2$ for the solution (P1), $\text{PDP} = 2.586 \text{ N/cm}^2$ for the solution (P2), $\text{PDP} = 1.495 \text{ N/cm}^2$ for the solution (P3), and $\text{PDP} = 1.422 \text{ N/cm}^2$ for the solution (P4). On a relative basis, with respect to the peak dynamic pressure of (P1), that of (P2) is 0.2% higher; that of (P3) is 42.1% lower; and that of (P4) is 44.9% lower.

Peak Total Acceleration. The peak total acceleration is $\text{PTA} = 36.69 \text{ m/sec}^2$ for the solution (P1), $\text{PTA} = 34.82 \text{ m/sec}^2$ for the solution (P2), $\text{PTA} = 22.80 \text{ m/sec}^2$ for the solution (P3), and $\text{PTA} = 48.20 \text{ m/sec}^2$ for the solution (P4). On a relative basis, with respect to the peak total acceleration of (P1), that of (P2) is 2.4% lower; that of (P3) is 36.1% lower; and that of (P4) is 35.1% higher.

Minimum Altitude. The minimum altitude is $h = 54.60 \text{ km}$ for the solution (P1), $h = 55.19 \text{ km}$ for the solution (P2), $h = 59.05 \text{ km}$ for the solution (P3), and $h = 55.09 \text{ km}$ for the solution (P4). On a relative basis, with respect to the minimum altitude of (P1), that of (P2) is 1.1% higher; that of (P3) is 8.2% higher; and that of (P4) is 0.9% higher.

Flight Time. The flight time is $\tau = 2229 \text{ sec}$ for the solution (P1), $\tau = 2284 \text{ sec}$ for the solution (P2), $\tau = 1342 \text{ sec}$ for the solution (P3), and $\tau = 559.7 \text{ sec}$ for the solution (P4). On a relative basis, with respect to the flight time of (P1), that of (P2) is 2.5% higher; that of (P3) is 39.8% lower; and that of (P4) is

74.9% lower.

5.5.2. Maneuver of Type 2

Energy. The total characteristic velocity is $\Delta V = 1.569$ km/sec for the solution (P1), $\Delta V = 1.570$ km/sec for the solution (P2), $\Delta V = 1.617$ km/sec for the solution (P3), and $\Delta V = 3.150$ km/sec for the solution (P4); these values are considerably below the total characteristic velocity of the two-impulse Hohmann transfer, $\Delta V = 4.306$ km/sec. On a relative basis, with respect to the total characteristic velocity of (P1), that of (P2) is 0.1% higher; that of (P3) is 3.1% higher; that of (P4) is 100.8% higher; and that of the two-impulse Hohmann transfer is 174.4% higher.

Peak Heating Rate. The peak heating rate is $\text{PHR} = 250.4$ W/cm² for the solution (P1), $\text{PHR} = 248.6$ W/cm² for the solution (P2), $\text{PHR} = 182.5$ W/cm² for the solution (P3), and $\text{PHR} = 99.93$ W/cm² for the solution (P4). On a relative basis, with respect to the peak heating rate of (P1), that of (P2) is 0.7% lower; that of (P3) is 27.1% lower; and that of (P4) is 60.1% lower.

Peak Dynamic Pressure. The peak dynamic pressure is $\text{PDP} = 2.586$ N/cm² for the solution (P1), $\text{PDP} = 2.571$ N/cm² for the solution (P2), $\text{PDP} = 1.499$ N/cm² for the solution (P3), and $\text{PDP} = 1.420$ N/cm² for the solution (P4). On a relative basis, with respect to the peak dynamic pressure of (P1), that of (P2) is 0.6% lower; that of (P3) is 42.0% lower; and that of (P4) is 45.1% lower.

Peak Total Acceleration. The peak total acceleration is $\text{PTA} = 35.53$ m/sec² for the solution (P1), $\text{PTA} = 35.50$ m/sec² for the solution (P2), $\text{PTA} = 22.94$ m/sec² for the solution (P3), and $\text{PTA} = 42.30$ m/sec² for the solution (P4). On a relative basis, with respect to the peak total acceleration of (P1), that of (P2) is 0.1% lower; that of (P3) is 35.4% lower; and that of (P4) is 19.1% higher.

Minimum Altitude. The minimum altitude is $h = 54.77$ km for the

solution (P1), $h = 55.03$ km for the solution (P2), $h = 59.03$ km for the solution (P3), and $h = 55.26$ m/sec² for the solution (P4). On a relative basis, with respect to the minimum altitude of (P1), that of (P2) is 0.5% higher; that of (P3) is 7.8% higher; and that of (P4) is 0.9% higher.

Flight Time. The flight time is $\tau = 2228$ sec for the solution (P1), $\tau = 2236$ sec for the solution (P2), $\tau = 1333$ sec for the solution (P3), and $\tau = 531.3$ sec for the solution (P4). On a relative basis, with respect to the flight time of (P1), that of (P2) is 0.4% higher; that of (P3) is 40.2% lower; and that of (P4) is 76.2% lower.

5.5.3. Maneuver of Type 3

Energy. The total characteristic velocity is $\Delta V = 2.177$ km/sec for the solution (P1), $\Delta V = 2.340$ km/sec for the solution (P2), $\Delta V = 2.281$ km/sec for the solution (P3), and $\Delta V = 4.103$ km/sec for the solution (P4); these values are considerably below the total characteristic velocity of the two-impulse Hohmann transfer, $\Delta V = 4.306$ km/sec. On a relative basis, with respect to the total characteristic velocity of (P1), that of (P2) is 7.5% higher; that of (P3) is 4.8% higher; that of (P4) is 88.5% higher; and that of the two-impulse Hohmann transfer is 97.8% higher.

Peak Heating Rate. The peak heating rate is $\text{PHR} = 341.1$ W/cm² for the solution (P1), $\text{PHR} = 196.0$ W/cm² for the solution (P2), $\text{PHR} = 224.2$ W/cm² for the solution (P3), and $\text{PHR} = 125.4$ W/cm² for the solution (P4). On a relative basis, with respect to the peak heating rate of (P1), that of (P2) is 42.5% lower; that of (P3) is 34.3% lower; and that of (P4) is 63.2% lower.

Peak Dynamic Pressure. The peak dynamic pressure is $\text{PDP} = 6.154$ N/cm² for the solution (P1), $\text{PDP} = 2.899$ N/cm² for the solution (P2), $\text{PDP} = 2.854$ N/cm² for the solution (P3), and $\text{PDP} = 3.234$ N/cm² for the solution (P4).

On a relative basis, with respect to the peak dynamic pressure of (P1), that of (P2) is 52.9% lower; that of (P3) is 53.6% lower; and that of (P4) is 47.4% lower.

Peak Total Acceleration. The peak total acceleration is $PTA = 75.73 \text{ m/sec}^2$ for the solution (P1), $PTA = 28.02 \text{ m/sec}^2$ for the solution (P2), $PTA = 37.52 \text{ m/sec}^2$ for the solution (P3), and $PTA = 87.07 \text{ m/sec}^2$ for the solution (P4). On a relative basis, with respect to the peak total acceleration of (P1), that of (P2) is 63.0% lower; that of (P3) is 50.5% lower; and that of (P4) is 15.0% higher.

Minimum Altitude. The minimum altitude is $h = 46.49 \text{ km}$ for the solution (P1), $h = 50.65 \text{ km}$ for the solution (P2), $h = 52.04 \text{ km}$ for the solution (P3), and $h = 46.82 \text{ km}$ for the solution (P4). On a relative basis, with respect to the minimum altitude of (P1), that of (P2) is 8.9% higher; that of (P3) is 11.9% higher; and that of (P4) is 0.7% higher.

Flight Time. The flight time is $\tau = 481.7 \text{ sec}$ for the solution (P1), $\tau = 638.4 \text{ sec}$ for the solution (P2), $\tau = 574.1 \text{ sec}$ for the solution (P3), and $\tau = 381.7 \text{ sec}$ for the solution (P4). On a relative basis, with respect to the flight time of (P1), that of (P2) is 32.5% higher; that of (P3) is 19.2% higher; and that of (P4) is 20.8% lower.

5.5.4. Summary

From the previous analysis of noncoplanar transfer, GEO-to-LEO, the following conclusions can be inferred:

(i) from the energy viewpoint, the total characteristic velocities of the solutions (P1), (P2), (P3) are nearly the same; they are considerably below the total characteristic velocity of the solution (P4) as well as the total characteristic velocity of the two-impulse Hohmann transfer;

(ii) also from the energy viewpoint, maneuvers of Type 1 and Type 2 are superior to maneuvers of Type 3; in addition, they are almost undistinguishable

from one another in terms of the main quantities of interest, namely, total characteristic velocity, peak heating rate, peak dynamic pressure, and peak total acceleration; therefore, maneuvers of Type 2 must be preferred to maneuvers of Type 1, since they are simpler in execution;

(iii) for all maneuver types, the nearly-grazing solution (P3) appears to be an excellent compromise between energy requirements and aerodynamic heating requirements;

(iv) overall, it appears that the nearly-grazing solution (P3), coupled with a transfer maneuver of Type 2 (namely, three impulses, three space plane changes, plus one atmospheric plane change), is the preferred solution.

For the GEO-to-LEO nearly-grazing solutions (P3), the time histories of the state variables and the control variables are shown in Fig. 8.

5.6. Numerical Solutions for HEO-to-LEO Transfer

Summary data are shown in Tables 6–8 and Fig. 9 for maneuvers of Type 1, Type 2, and Type 3. The following comments refer to the minimum energy solution (P1), the maximum time solution (P2), the nearly-grazing solution (P3), and the minimum peak heating rate solution (P4). In computing numerical solutions for Problems (P1), (P2), (P3), (P4), the following procedure was adopted: for Problem (P1), the plane change components and the apogee radius were optimized; for Problems (P2), (P3), (P4), the plane change components and the apogee radius were kept at the same levels determined for Problem (P1); this was done in order to impart good energy characteristics to the solutions of Problems (P2), (P3), (P4).

5.6.1. Maneuver of Type 1

Energy. The total characteristic velocity is $\Delta V = 1.724$ km/sec for the solution (P1), $\Delta V = 1.724$ km/sec for the solution (P2), $\Delta V = 1.769$ km/sec for the

solution (P3), and $\Delta V = 1.953$ km/sec for the solution (P4); these values are considerably below the total characteristic velocity of the two-impulse Hohmann transfer, $\Delta V = 3.788$ km/sec. On a relative basis, with respect to the total characteristic velocity of (P1), that of (P2) is nearly the same; that of (P3) is 2.6% higher; that of (P4) is 13.3% higher; and that of the two-impulse Hohmann transfer is 119.7% higher.

Peak Heating Rate. The peak heating rate is $\text{PHR} = 172.1$ W/cm² for the solution (P1), $\text{PHR} = 167.4$ W/cm² for the solution (P2), $\text{PHR} = 130.1$ W/cm² for the solution (P3), and $\text{PHR} = 92.62$ W/cm² for the solution (P4). On a relative basis, with respect to the peak heating rate of (P1), that of (P2) is 2.7% lower; that of (P3) is 24.4% lower; and that of (P4) is 46.2% lower.

Peak Dynamic Pressure. The peak dynamic pressure is $\text{PDP} = 1.633$ N/cm² for the solution (P1), $\text{PDP} = 1.526$ N/cm² for the solution (P2), $\text{PDP} = 0.9739$ N/cm² for the solution (P3), and $\text{PDP} = 0.7577$ N/cm² for the solution (P4). On a relative basis, with respect to the peak dynamic pressure of (P1), that of (P2) is 6.6% lower; that of (P3) is 40.4% lower; and that of (P4) is 53.6% lower.

Peak Total Acceleration. The peak total acceleration is $\text{PTA} = 22.37$ m/sec² for the solution (P1), $\text{PTA} = 21.11$ m/sec² for the solution (P2), $\text{PTA} = 14.99$ m/sec² for the solution (P3), and $\text{PTA} = 13.14$ m/sec² for the solution (P4). On a relative basis, with respect to the peak total acceleration of (P1), that of (P2) is 5.6% lower; that of (P3) is 33.0% lower; and that of (P4) is 41.3% lower.

Minimum Altitude. The minimum altitude is $h = 57.70$ km for the solution (P1), $h = 58.30$ km for the solution (P2), $h = 61.69$ km for the solution (P3), and $h = 62.21$ km for the solution (P4). On a relative basis, with respect to the minimum altitude of (P1), that of (P2) is 1.0% higher; that of (P3) is 6.9% higher; and that of (P4) is 7.8% higher.

Flight Time. The flight time is $\tau = 1486$ sec for the solution (P1), $\tau = 1494$ sec for the solution (P2), $\tau = 1202$ sec for the solution (P3), and $\tau = 843.7$ sec for the solution (P4). On a relative basis, with respect to the flight time of (P1), that of (P2) is 0.5% higher; that of (P3) is 19.1% lower; and that of (P4) is 43.2% lower.

5.6.2. Maneuver of Type 2

Energy. The total characteristic velocity is $\Delta V = 1.907$ km/sec for the solution (P1), $\Delta V = 1.910$ km/sec for the solution (P2), $\Delta V = 1.945$ km/sec for the solution (P3), and $\Delta V = 2.077$ km/sec for the solution (P4); these values are considerably below the total characteristic velocity of the two-impulse Hohmann transfer, $\Delta V = 3.788$ km/sec. On a relative basis, with respect to the total characteristic velocity of (P1), that of (P2) is 0.2% higher; that of (P3) is 2.0% higher; that of (P4) is 8.9% higher; and that of the two-impulse Hohmann transfer is 98.6% higher.

Peak Heating Rate. The peak heating rate is $\text{PHR} = 123.5$ W/cm² for the solution (P1), $\text{PHR} = 119.1$ W/cm² for the solution (P2), $\text{PHR} = 94.89$ W/cm² for the solution (P3), and $\text{PHR} = 72.27$ W/cm² for the solution (P4). On a relative basis, with respect to the peak heating rate of (P1), that of (P2) is 3.6% lower; that of (P3) is 23.2% lower; and that of (P4) is 41.5% lower.

Peak Dynamic Pressure. The peak dynamic pressure is $\text{PDP} = 1.009$ N/cm² for the solution (P1), $\text{PDP} = 0.9269$ N/cm² for the solution (P2), $\text{PDP} = 0.6206$ N/cm² for the solution (P3), and $\text{PDP} = 0.4697$ N/cm² for the solution (P4). On a relative basis, with respect to the peak dynamic pressure of (P1), that of (P2) is 8.1% lower; that of (P3) is 38.5% lower; and that of (P4) is 53.4% lower.

Peak Total Acceleration. The peak total acceleration is $\text{PTA} = 14.05$ m/sec² for the solution (P1), $\text{PTA} = 13.03$ m/sec² for the solution (P2), $\text{PTA} =$

9.545 m/sec² for the solution (P3), and PTA = 9.196 m/sec² for the solution (P4).

On a relative basis, with respect to the peak total acceleration of (P1), that of (P2) is 7.3% lower; that of (P3) is 32.1% lower; and that of (P4) is 34.5% lower.

Minimum Altitude. The minimum altitude is $h = 61.03$ km for the solution (P1), $h = 61.70$ km for the solution (P2), $h = 64.60$ km for the solution (P3), and $h = 65.80$ km for the solution (P4). On a relative basis, with respect to the minimum altitude of (P1), that of (P2) is 1.1% higher; that of (P3) is 5.8% higher; and that of (P4) is 7.8% higher.

Flight Time. The flight time is $\tau = 1424$ sec for the solution (P1), $\tau = 1432$ sec for the solution (P2), $\tau = 1219$ sec for the solution (P3), and $\tau = 907.0$ sec for the solution (P4). On a relative basis, with respect to the flight time of (P1), that of (P2) is 0.6% higher; that of (P3) is 14.4% lower; and that of (P4) is 36.3% lower.

5.6.3. Maneuver of Type 3

Energy. The total characteristic velocity is $\Delta V = 2.766$ km/sec for the solution (P1), $\Delta V = 2.859$ km/sec for the solution (P2), $\Delta V = 2.833$ km/sec for the solution (P3), and $\Delta V = 5.003$ km/sec for the solution (P4); these values are considerably below the total characteristic velocity of the two-impulse Hohmann transfer, $\Delta V = 3.788$ km/sec. On a relative basis, with respect to the total characteristic velocity of (P1), that of (P2) is 3.4% higher; that of (P3) is 2.4% higher; that of (P4) is 80.9% higher; and that of the two-impulse Hohmann transfer is 36.9% higher.

Peak Heating Rate. The peak heating rate is $\text{PHR} = 308.4$ W/cm² for the solution (P1), $\text{PHR} = 240.6$ W/cm² for the solution (P2), $\text{PHR} = 220.4$ W/cm² for the solution (P3), and $\text{PHR} = 110.0$ W/cm² for the solution (P4). On a relative basis, with respect to the peak heating rate of (P1), that of (P2) is 22.0% lower;

that of (P3) is 28.5% lower; and that of (P4) is 64.3% lower.

Peak Dynamic Pressure. The peak dynamic pressure is $PDP = 9.367 \text{ N/cm}^2$ for the solution (P1), $PDP = 7.324 \text{ N/cm}^2$ for the solution (P2), $PDP = 5.325 \text{ N/cm}^2$ for the solution (P3), and $PDP = 5.455 \text{ N/cm}^2$ for the solution (P4). On a relative basis, with respect to the peak dynamic pressure of (P1), that of (P2) is 21.8% lower; that of (P3) is 43.2% lower; and that of (P4) is 41.8% lower.

Peak Total Acceleration. The peak total acceleration is $PTA = 110.4 \text{ m/sec}^2$ for the solution (P1), $PTA = 69.61 \text{ m/sec}^2$ for the solution (P2), $PTA = 65.04 \text{ m/sec}^2$ for the solution (P3), and $PTA = 116.6 \text{ m/sec}^2$ for the solution (P4). On a relative basis, with respect to the peak total acceleration of (P1), that of (P2) is 36.9% lower; that of (P3) is 41.1% lower; and that of (P4) is 5.6% higher.

Minimum Altitude. The minimum altitude is $h = 41.36 \text{ km}$ for the solution (P1), $h = 42.48 \text{ km}$ for the solution (P2), $h = 45.01 \text{ km}$ for the solution (P3), and $h = 40.34 \text{ km}$ for the solution (P4). On a relative basis, with respect to the minimum altitude of (P1), that of (P2) is 2.7% higher; that of (P3) is 8.8% higher; and that of (P4) is 2.5% lower.

Flight Time. The flight time is $\tau = 356.9 \text{ sec}$ for the solution (P1), $\tau = 461.2 \text{ sec}$ for the solution (P2), $\tau = 426.7 \text{ sec}$ for the solution (P3), and $\tau = 324.7 \text{ sec}$ for the solution (P4). On a relative basis, with respect to the flight time of (P1), that of (P2) is 29.2% higher; that of (P3) is 19.6% higher; and that of (P4) is 9.0% lower.

5.6.4. Summary

From the previous analysis of noncoplanar transfer, HEO-to-LEO, the following conclusions can be inferred:

(i) from the energy viewpoint, the total characteristic velocities of the solutions (P1), (P2), (P3) are nearly the same; they are considerably below the

total characteristic velocity of the solution (P4) as well as the total characteristic velocity of the two-impulse Hohmann transfer;

(ii) also from the energy viewpoint, maneuvers of Type 1 are slightly superior to maneuvers of Type 2, which in turn are much superior to maneuvers of Type 3; from the point of view of the remaining quantities of interest (namely, peak heating rate, peak dynamic pressure, and peak total acceleration), maneuvers of Type 2 are the best; therefore, maneuvers of Type 2 must be preferred to maneuvers of Type 1, since they are simpler in execution;

(iii) for all maneuver types, the nearly-grazing solution (P3) appears to be an excellent compromise between energy requirements and aerodynamic heating requirements;

(iv) overall, it appears that the nearly-grazing solution (P3), coupled with a transfer maneuver of Type 2 (namely, three impulses, three space plane changes, plus one atmospheric plane change), is the preferred solution.

For the HEO-to-LEO nearly-grazing solutions (P3), the time histories of the state variables and the control variables are shown in Fig. 10.

5.7. Numerical Solutions for LEO-to-LEO Transfer

Summary data are shown in Tables 9–11 and Fig. 11 for maneuvers of Type 1, Type 2, and Type 3. The following comments refer to the minimum energy solution (P1), the maximum time solution (P2), the nearly-grazing solution (P3), and the minimum peak heating rate solution (P4). In computing numerical solutions for Problems (P1), (P2), (P3), (P4), the following procedure was adopted: for Problem (P1), the plane change components and the apogee radius were optimized; for Problems (P2), (P3), (P4), the plane change components and the apogee radius were kept at the same levels determined for Problem (P1); this was done in order to impart good energy characteristics to the solutions of Problems

(P2), (P3), (P4).

5.7.1. Maneuver of Type 1

Energy. The total characteristic velocity is $\Delta V = 2.441$ km/sec for the solution (P1), $\Delta V = 2.586$ km/sec for the solution (P2), $\Delta V = 2.490$ km/sec for the solution (P3), and $\Delta V = 3.845$ km/sec for the solution (P4); some of these values are considerably below the total characteristic velocity of the three-impulse Hohmann transfer, $\Delta V = 3.899$ km/sec. On a relative basis, with respect to the total characteristic velocity of (P1), that of (P2) is 5.9% higher; that of (P3) is 2.0% higher; that of (P4) is 57.5% higher; and that of the three-impulse Hohmann transfer is 59.7% higher.

Peak Heating Rate. The peak heating rate is $\text{PHR} = 212.9$ W/cm² for the solution (P1), $\text{PHR} = 159.5$ W/cm² for the solution (P2), $\text{PHR} = 159.5$ W/cm² for the solution (P3), and $\text{PHR} = 111.4$ W/cm² for the solution (P4). On a relative basis, with respect to the peak heating rate of (P1), that of (P2) is 25.1% lower; that of (P3) is 25.1% lower; and that of (P4) is 47.7% lower.

Peak Dynamic Pressure. The peak dynamic pressure is $\text{PDP} = 4.544$ N/cm² for the solution (P1), $\text{PDP} = 3.287$ N/cm² for the solution (P2), $\text{PDP} = 2.698$ N/cm² for the solution (P3), and $\text{PDP} = 2.752$ N/cm² for the solution (P4). On a relative basis, with respect to the peak dynamic pressure of (P1), that of (P2) is 27.7% lower; that of (P3) is 40.6% lower; and that of (P4) is 39.4% lower.

Peak Total Acceleration. The peak total acceleration is $\text{PTA} = 54.27$ m/sec² for the solution (P1), $\text{PTA} = 34.83$ m/sec² for the solution (P2), $\text{PTA} = 36.08$ m/sec² for the solution (P3), and $\text{PTA} = 85.85$ m/sec² for the solution (P4). On a relative basis, with respect to the peak total acceleration of (P1), that of (P2) is 35.8% lower; that of (P3) is 33.5% lower; and that of (P4) is 58.2% higher.

Minimum Altitude. The minimum altitude is $h = 46.76$ km for the

solution (P1), $h = 48.60$ km for the solution (P2), $h = 50.67$ km for the solution (P3), and $h = 47.81$ km for the solution (P4). On a relative basis, with respect to the minimum altitude of (P1), that of (P2) is 3.9% higher; that of (P3) is 8.4% higher; and that of (P4) is 2.2% higher.

Flight Time. The flight time is $\tau = 452.8$ sec for the solution (P1), $\tau = 672.4$ sec for the solution (P2), $\tau = 509.1$ sec for the solution (P3), and $\tau = 321.8$ sec for the solution (P4). On a relative basis, with respect to the flight time of (P1), that of (P2) is 48.5% higher; that of (P3) is 12.4% higher; and that of (P4) is 28.9% lower.

5.7.2. Maneuver of Type 2

Energy. The total characteristic velocity is $\Delta V = 2.682$ km/sec for the solution (P1), $\Delta V = 2.726$ km/sec for the solution (P2), $\Delta V = 2.687$ km/sec for the solution (P3), and $\Delta V = 6.616$ km/sec for the solution (P4); some of these values are considerably below the total characteristic velocity of the three-impulse Hohmann transfer, $\Delta V = 3.899$ km/sec. On a relative basis, with respect to the total characteristic velocity of (P1), that of (P2) is 1.6% higher; that of (P3) is 0.2% higher; that of (P4) is 146.7% higher; and that of the three-impulse Hohmann transfer is 45.4% higher.

Peak Heating Rate. The peak heating rate is $\text{PHR} = 171.3$ W/cm² for the solution (P1), $\text{PHR} = 184.8$ W/cm² for the solution (P2), $\text{PHR} = 151.6$ W/cm² for the solution (P3), and $\text{PHR} = 103.3$ W/cm² for the solution (P4). On a relative basis, with respect to the peak heating rate of (P1), that of (P2) is 7.9% higher; that of (P3) is 11.5% lower; and that of (P4) is 39.7% lower.

Peak Dynamic Pressure. The peak dynamic pressure is $\text{PDP} = 4.753$ N/cm² for the solution (P1), $\text{PDP} = 5.740$ N/cm² for the solution (P2), $\text{PDP} = 3.631$ N/cm² for the solution (P3), and $\text{PDP} = 8.663$ N/cm² for the solution (P4).

On a relative basis, with respect to the peak dynamic pressure of (P1), that of (P2) is 20.8% higher; that of (P3) is 23.6% lower; and that of (P4) is 82.3% higher.

Peak Total Acceleration. The peak total acceleration is $PTA = 57.47$ m/sec^2 for the solution (P1), $PTA = 67.05$ m/sec^2 for the solution (P2), $PTA = 46.23$ m/sec^2 for the solution (P3), and $PTA = 175.9$ m/sec^2 for the solution (P4).

On a relative basis, with respect to the peak total acceleration of (P1), that of (P2) is 16.7% higher; that of (P3) is 19.6% lower; and that of (P4) is 206.1% higher.

Minimum Altitude. The minimum altitude is $h = 44.76$ km for the solution (P1), $h = 43.44$ km for the solution (P2), $h = 46.82$ km for the solution (P3), and $h = 35.11$ km for the solution (P4). On a relative basis, with respect to the minimum altitude of (P1), that of (P2) is 2.9% lower; that of (P3) is 4.6% higher; and that of (P4) is 21.6% lower.

Flight Time. The flight time is $\tau = 479.4$ sec for the solution (P1), $\tau = 504.8$ sec for the solution (P2), $\tau = 492.3$ sec for the solution (P3), and $\tau = 266.0$ sec for the solution (P4). On a relative basis, with respect to the flight time of (P1), that of (P2) is 5.3% higher; that of (P3) is 2.7% higher; and that of (P4) is 44.5% lower.

5.7.3. Maneuver of Type 3

Energy. The total characteristic velocity is $\Delta V = 3.086$ km/sec for the solution (P1), $\Delta V = 3.132$ km/sec for the solution (P2), $\Delta V = 3.129$ km/sec for the solution (P3), and $\Delta V = 7.406$ km/sec for the solution (P4); these values are somewhat below the total characteristic velocity of the three-impulse Hohmann transfer, $\Delta V = 3.899$ km/sec. On a relative basis, with respect to the total characteristic velocity of (P1), that of (P2) is 1.5% higher; that of (P3) is 1.4% higher; that of (P4) is 140.0% higher; and that of the three-impulse Hohmann

transfer is 26.3% higher.

Peak Heating Rate. The peak heating rate is $\text{PHR} = 229.2 \text{ W/cm}^2$ for the solution (P1), $\text{PHR} = 224.6 \text{ W/cm}^2$ for the solution (P2), $\text{PHR} = 186.9 \text{ W/cm}^2$ for the solution (P3), and $\text{PHR} = 92.09 \text{ W/cm}^2$ for the solution (P4). On a relative basis, with respect to the peak heating rate of (P1), that of (P2) is 2.0% lower; that of (P3) is 18.5% lower; and that of (P4) is 59.8% lower.

Peak Dynamic Pressure. The peak dynamic pressure is $\text{PDP} = 10.44 \text{ N/cm}^2$ for the solution (P1), $\text{PDP} = 11.48 \text{ N/cm}^2$ for the solution (P2), $\text{PDP} = 7.419 \text{ N/cm}^2$ for the solution (P3), and $\text{PDP} = 11.51 \text{ N/cm}^2$ for the solution (P4). On a relative basis, with respect to the peak dynamic pressure of (P1), that of (P2) is 10.0% higher; that of (P3) is 28.9% lower; and that of (P4) is 10.2% higher.

Peak Total Acceleration. The peak total acceleration is $\text{PTA} = 141.2 \text{ m/sec}^2$ for the solution (P1), $\text{PTA} = 108.0 \text{ m/sec}^2$ for the solution (P2), $\text{PTA} = 87.05 \text{ m/sec}^2$ for the solution (P3), and $\text{PTA} = 189.4 \text{ m/sec}^2$ for the solution (P4). On a relative basis, with respect to the peak total acceleration of (P1), that of (P2) is 23.5% lower; that of (P3) is 38.3% lower; and that of (P4) is 34.1% higher.

Minimum Altitude. The minimum altitude is $h = 37.88 \text{ km}$ for the solution (P1), $h = 37.49 \text{ km}$ for the solution (P2), $h = 40.50 \text{ km}$ for the solution (P3), and $h = 31.09 \text{ km}$ for the solution (P4). On a relative basis, with respect to the minimum altitude of (P1), that of (P2) is 1.0% lower; that of (P3) is 6.9% higher; and that of (P4) is 17.9% lower.

Flight Time. The flight time is $\tau = 485.0 \text{ sec}$ for the solution (P1), $\tau = 513.8 \text{ sec}$ for the solution (P2), $\tau = 484.9 \text{ sec}$ for the solution (P3), and $\tau = 257.7 \text{ sec}$ for the solution (P4). On a relative basis, with respect to the flight time of (P1), that of (P2) is 5.9% higher; that of (P3) is nearly the same; and that of (P4) is 46.9% lower.

5.7.4. Summary

From the previous analysis of noncoplanar transfer, LEO-to-LEO, the following conclusions can be inferred:

(i) from the energy viewpoint, the total characteristic velocities of the solutions (P1), (P2), (P3) are nearly the same; they are considerably below the total characteristic velocity of the solution (P4) as well as the total characteristic velocity of the three-impulse Hohmann transfer;

(ii) maneuvers of Type 1 are superior to maneuvers of Type 2 from the viewpoints of total characteristic velocity, peak dynamic pressure, and peak total acceleration; however, the opposite is true from the viewpoint of peak heating rate; therefore, for the LEO-to-LEO transfer, the conclusions are not as firm as for the GEO-to-LEO transfer and the HEO-to-LEO transfer; nevertheless, the fact that maneuvers of Type 2 are simpler in execution than maneuvers of Type 1 must be kept in mind;

(iii) for all maneuver types, the nearly-grazing solution (P3) appears to be an excellent compromise between energy requirements and aerodynamic heating requirements;

(iv) overall, it appears that the nearly-grazing solution (P3), coupled with either a transfer maneuver of Type 1 (namely, four impulses, four space plane changes, plus one atmospheric plane change) or a transfer maneuver of Type 2 (namely, three impulses, three space plane changes, plus one atmospheric plane change), is the preferred solution.

For the LEO-to-LEO nearly-grazing solutions (P3), the time histories of the state variables and the control variables are shown in Fig. 12.

6. Conclusions

This thesis discusses optimal trajectories for hypervelocity flight of interest in aeroassisted orbital transfer. Both coplanar orbital transfer (problem without plane change) and noncoplanar orbital transfer (problem with plane change) are considered. For these cases, the GEO-to-LEO transfer, the HEO-to-LEO transfer, and the LEO-to-LEO transfer are considered in connection with a spacecraft which is controlled during the atmospheric pass via the angle of attack (coplanar case) or via the angle of attack and the angle of bank (noncoplanar case).

For the noncoplanar case, three different transfer maneuvers are studied. Type 1 involves four impulses and four space plane changes; Type 2 involves three impulses and three space plane changes; and Type 3 involves three impulses and no space plane change. In Type 1, the initial impulse directs the spacecraft away from Earth, and then is followed by an apogee impulse propelling the spacecraft toward Earth; in Types 2 and 3, the initial impulse directs the spacecraft toward Earth. A common element of these maneuvers is that they all include an atmospheric pass, with velocity depletion coupled with plane change.

Within the framework of classical optimal control, the following problems are studied: (P1) minimize the energy required for orbital transfer; (P2) maximize the time of flight during the atmospheric portion of the trajectory; and (P3) minimize the time integral of the square of the path inclination. Problems (P1), (P2), (P3) are Bolza problems of optimal control.

Within the framework of minimax optimal control, the following problem is studied: (P4) minimize the peak heating rate. Problem (P4) is a Chebyshev problem of optimal control, which can be converted into a Bolza problem via suitable transformations.

Numerical solutions for Problems (P1), (P2), (P3), (P4) are obtained by means of the sequential gradient–restoration algorithm for optimal control problems, leading to the conclusions below:

(a) for coplanar transfer, either GEO–to–LEO or HEO–to–LEO, the solutions (P1), (P3), (P4) are nearly the same in terms of the main quantities of interest, namely, total characteristic velocity, peak heating rate, peak dynamic pressure, and peak total acceleration;

(b) for noncoplanar transfer, either GEO–to–LEO or HEO–to–LEO, the nearly–grazing solution (P3) appears to be an excellent compromise between energy requirements and aerodynamic heating requirements; also, maneuvers of Type 1 and Type 2 are superior to maneuvers of Type 3; overall, it appears that the nearly–grazing solution (P3), coupled with a transfer maneuver of Type 2 (three impulses, three space plane changes, plus one atmospheric plane change), is the preferred solution;

(c) for noncoplanar transfer, LEO–to–LEO, the nearly–grazing solution (P3) appears to be an excellent engineering compromise between energy requirements and aerodynamic heating requirements; also, maneuvers of Type 1 and Type 2 are superior to maneuvers of Type 3; overall, it appears the nearly–grazing solution (P3), coupled with either a transfer maneuver of Type 1 (four impulses, four space plane changes, plus one atmospheric plane change) or a transfer maneuver of Type 2 (three impulses, three space plane changes, plus one atmospheric plane change), is the preferred solution.

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nearly-grazing trajectories,
lift coefficient versus time.
- Fig. 8H. Noncoplanar transfer, GEO-to-LEO
nearly-grazing trajectories,
bank angle (deg) versus time.
- Fig. 9A. Noncoplanar transfer, HEO-to-LEO,
total characteristic velocity (km/sec).
- Fig. 9B. Noncoplanar transfer, HEO-to-LEO,
peak heating rate (W/cm^2).
- Fig. 9C. Noncoplanar transfer, HEO-to-LEO,
peak dynamic pressure (N/cm^2).
- Fig. 9D. Noncoplanar transfer, HEO-to-LEO,
peak total acceleration (m/sec^2).
- Fig. 9E. Noncoplanar transfer, HEO-to-LEO,
minimum altitude (km).
- Fig. 9F. Noncoplanar transfer, HEO-to-LEO,
flight time (sec).
- Fig. 10A. Noncoplanar transfer, HEO-to-LEO

- nearly-grazing trajectories,
altitude (km) versus time.
- Fig. 10B. Noncoplanar transfer, HEO-to-LEO
nearly-grazing trajectories,
velocity (km/sec) versus time.
- Fig. 10C. Noncoplanar transfer, HEO-to-LEO
nearly-grazing trajectories,
path inclination (deg) versus time.
- Fig. 10D. Noncoplanar transfer, HEO-to-LEO
nearly-grazing trajectories,
longitude (deg) versus time.
- Fig. 10E. Noncoplanar transfer, HEO-to-LEO
nearly-grazing trajectories,
latitude (deg) versus time.
- Fig. 10F. Noncoplanar transfer, HEO-to-LEO
nearly-grazing trajectories,
heading angle(deg) versus time.
- Fig. 10G. Noncoplanar transfer, HEO-to-LEO
nearly-grazing trajectories,
lift coefficient versus time.
- Fig. 10H. Noncoplanar transfer, HEO-to-LEO
nearly-grazing trajectories,
bank angle (deg) versus time.
- Fig. 11A. Noncoplanar transfer, LEO-to-LEO,
total characteristic velocity (km/sec).
- Fig. 11B. Noncoplanar transfer, LEO-to-LEO,

- peak heating rate (W/cm^2).
- Fig. 11C. Noncoplanar transfer, LEO-to-LEO,
peak dynamic pressure (N/cm^2).
- Fig. 11D. Noncoplanar transfer, LEO-to-LEO,
peak total acceleration (m/sec^2).
- Fig. 11E. Noncoplanar transfer, LEO-to-LEO,
minimum altitude (km).
- Fig. 11F. Noncoplanar transfer, LEO-to-LEO,
flight time (sec).
- Fig. 12A. Noncoplanar transfer, LEO-to-LEO
nearly-grazing trajectories,
altitude (km) versus time.
- Fig. 12B. Noncoplanar transfer, LEO-to-LEO
nearly-grazing trajectories,
velocity (km/sec) versus time.
- Fig. 12C. Noncoplanar transfer, LEO-to-LEO
nearly-grazing trajectories,
path inclination (deg) versus time.
- Fig. 12D. Noncoplanar transfer, LEO-to-LEO
nearly-grazing trajectories,
longitude (deg) versus time.
- Fig. 12E. Noncoplanar transfer, LEO-to-LEO
nearly-grazing trajectories,
latitude (deg) versus time.
- Fig. 12F. Noncoplanar transfer, LEO-to-LEO
nearly-grazing trajectories,

heading angle(deg) versus time.

Fig. 12G. Noncoplanar transfer, LEO-to-LEO
nearly-grazing trajectories,
lift coefficient versus time.

Fig. 12H. Noncoplanar transfer, LEO-to-LEO
nearly-grazing trajectories,
bank angle (deg) versus time.

Table 1A. Coplanar transfer, GEO-to-LEO,
summary results.

Quantity	(P1)	(P2)	(P3)	(P4)	Units
ΔV	0.1514E+01	0.1517E+01	0.1528E+01	0.1526E+01	km/sec
PHR	0.6945E+02	0.1244E+03	0.6682E+02	0.6676E+02	W/cm ²
PDP	0.1825E+00	0.5896E+00	0.1656E+00	0.1666E+00	N/cm ²
PTA	0.6976E+01	0.7383E+01	0.6954E+01	0.6959E+01	m/sec ²
$h_{\min}$	0.7547E+02	0.6725E+02	0.7595E+02	0.7595E+02	km
τ	0.2465E+04	0.3376E+04	0.2728E+04	0.2655E+04	sec

Table 1B. Coplanar transfer, GEO-to-LEO,
characteristic velocity components.

Quantity	(P1)	(P2)	(P3)	(P4)	Units
ΔV_{00}	0.1490E+01	0.1491E+01	0.1490E+01	0.1490E+01	km/sec
ΔV_1	0.3946E-11	0.1216E-05	0.1896E-01	0.1642E-01	km/sec
ΔV_{11}	0.2448E-01	0.2651E-01	0.1946E-01	0.2007E-01	km/sec
ΔV	0.1514E+01	0.1517E+01	0.1528E+01	0.1526E+01	km/sec

Table 2A. Coplanar transfer, HEO-to-LEO,
summary results.

Quantity	(P1)	(P2)	(P3)	(P4)	Units
ΔV	0.1049E+01	0.1053E+01	0.1060E+01	0.1054E+01	km/sec
PHR	0.3510E+02	0.6436E+02	0.3445E+02	0.3440E+02	W/cm ²
PDP	0.7487E-01	0.2392E+00	0.7185E-01	0.7197E-01	N/cm ²
PTA	0.3182E+01	0.3226E+01	0.3180E+01	0.3180E+01	m/sec ²
$h_{\min}$	0.7977E+02	0.7212E+02	0.7997E+02	0.7997E+02	km
τ	0.2263E+04	0.3046E+04	0.2422E+04	0.2258E+04	sec

Table 2B. Coplanar transfer, HEO-to-LEO,
characteristic velocity components.

Quantity	(P1)	(P2)	(P3)	(P4)	Units
ΔV_{00}	0.1024E+01	0.1026E+01	0.1025E+01	0.1025E+01	km/sec
ΔV_1	0.1430E-11	0.2604E-06	0.1489E-01	0.6531E-02	km/sec
ΔV_{11}	0.2453E-01	0.2653E-01	0.2045E-01	0.2264E-01	km/sec
ΔV	0.1049E+01	0.1053E+01	0.1060E+01	0.1054E+01	km/sec

Table 3A. Noncoplanar transfer, GEO-to-LEO, Type 1, summary results.

Quantity	(P1)	(P2)	(P3)	(P4)	Units
ΔV	0.1569E+01	0.1569E+01	0.1615E+01	0.3212E+01	km/sec
PHR	0.2494E+03	0.2473E+03	0.1822E+03	0.9931E+02	W/cm ²
PDP	0.2580E+01	0.2586E+01	0.1495E+01	0.1422E+01	N/cm ²
PTA	0.3569E+02	0.3482E+02	0.2280E+02	0.4820E+02	m/sec ²
$h_{\min}$	0.5460E+02	0.5519E+02	0.5905E+02	0.5509E+02	km
τ	0.2229E+04	0.2284E+04	0.1342E+04	0.5597E+03	sec

Table 3B. Noncoplanar transfer, GEO-to-LEO, Type 1, characteristic velocity components.

Quantity	(P1)	(P2)	(P3)	(P4)	Units
ΔV_{00}	0.2009E-02	0.2009E-02	0.2009E-02	0.2009E-02	km/sec
ΔV_{22}	0.1515E+01	0.1515E+01	0.1514E+01	0.1514E+01	km/sec
ΔV_1	0.3349E-01	0.3432E-01	0.8164E-01	0.1678E+01	km/sec
ΔV_{11}	0.1795E-01	0.1795E-01	0.1795E-01	0.1795E-01	km/sec
ΔV	0.1569E+01	0.1569E+01	0.1615E+01	0.3212E+01	km/sec

Table 3C. Noncoplanar transfer, GEO-to-LEO, Type 1, plane change components and apogee radius.

Quantity	(P1)	(P2)	(P3)	(P4)	Units
i_{s00}	0.8781E-02	0.8781E-02	0.8781E-02	0.8781E-02	deg
i_{s22}	0.7114E+01	0.7114E+01	0.7114E+01	0.7114E+01	deg
i_{s1}	0.1158E-01	0.1158E-01	0.1158E-01	0.1158E-01	deg
i_{s11}	0.4613E-02	0.4613E-02	0.4613E-02	0.4613E-02	deg
i_s	0.7139E+01	0.7139E+01	0.7139E+01	0.7139E+01	deg
i_a	0.2286E+02	0.2286E+02	0.2286E+02	0.2286E+02	deg
i	0.3000E+02	0.3000E+02	0.3000E+02	0.3000E+02	deg
r_{22}	0.4227E+05	0.4227E+05	0.4227E+05	0.4227E+05	km

Table 4A. Noncoplanar transfer, GEO-to-LEO, Type 2, summary results.

Quantity	(P1)	(P2)	(P3)	(P4)	Units
ΔV	0.1569E+01	0.1570E+01	0.1617E+01	0.3150E+01	km/sec
PHR	0.2504E+03	0.2486E+03	0.1825E+03	0.9993E+02	W/cm ²
PDP	0.2586E+01	0.2571E+01	0.1499E+01	0.1420E+01	N/cm ²
PTA	0.3553E+02	0.3550E+02	0.2294E+02	0.4230E+02	m/sec ²
$h_{\min}$	0.5477E+02	0.5503E+02	0.5903E+02	0.5526E+02	km
τ	0.2228E+04	0.2236E+04	0.1333E+04	0.5313E+03	sec

Table 4B. Noncoplanar transfer, GEO-to-LEO, Type 2, characteristic velocity components.

Quantity	(P1)	(P2)	(P3)	(P4)	Units
ΔV_{00}	0.1517E+01	0.1517E+01	0.1516E+01	0.1516E+01	km/sec
ΔV_1	0.3370E-01	0.3488E-01	0.8275E-01	0.1616E+01	km/sec
ΔV_{11}	0.1797E-01	0.1797E-01	0.1797E-01	0.1797E-01	km/sec
ΔV^{11}	0.1569E+01	0.1570E+01	0.1617E+01	0.3150E+01	km/sec

Table 4C. Noncoplanar transfer, GEO-to-LEO, Type 2, plane change components.

Quantity	(P1)	(P2)	(P3)	(P4)	Units
i_{s00}	0.7107E+01	0.7107E+01	0.7107E+01	0.7107E+01	deg
i_{s1}	0.1192E-01	0.1192E-01	0.1192E-01	0.1192E-01	deg
i_{s11}	0.7952E-02	0.7952E-02	0.7952E-02	0.7952E-02	deg
i_s	0.7127E+01	0.7127E+01	0.7127E+01	0.7127E+01	deg
i_a	0.2287E+02	0.2287E+02	0.2287E+02	0.2287E+02	deg
i	0.3000E+02	0.3000E+02	0.3000E+02	0.3000E+02	deg

Table 5A. Noncoplanar transfer, GEO-to-LEO, Type 3, summary results.

Quantity	(P1)	(P2)	(P3)	(P4)	Units
ΔV	0.2177E+01	0.2340E+01	0.2281E+01	0.4103E+01	km/sec
PHR	0.3411E+03	0.1960E+03	0.2242E+03	0.1254E+03	W/cm ²
PDP	0.6154E+01	0.2899E+01	0.2854E+01	0.3234E+01	N/cm ²
PTA	0.7573E+02	0.2802E+02	0.3752E+02	0.8707E+02	m/sec ²
$h_{\min}$	0.4649E+02	0.5065E+02	0.5204E+02	0.4682E+02	km
τ	0.4817E+03	0.6384E+03	0.5741E+03	0.3817E+03	sec

Table 5B. Noncoplanar transfer, GEO-to-LEO, Type 3, characteristic velocity components.

Quantity	(P1)	(P2)	(P3)	(P4)	Units
ΔV_{00}	0.1494E+01	0.1492E+01	0.1491E+01	0.1492E+01	km/sec
ΔV_1	0.6658E+00	0.8301E+00	0.7721E+00	0.2592E+01	km/sec
ΔV_{11}	0.1793E-01	0.1793E-01	0.1793E-01	0.1793E-01	km/sec
ΔV	0.2177E+01	0.2340E+01	0.2281E+01	0.4103E+01	km/sec

Table 5C. Noncoplanar transfer, GEO-to-LEO, Type 3, plane change components.

Quantity	(P1)	(P2)	(P3)	(P4)	Units
i_s	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	deg
i_a	0.3000E+02	0.3000E+02	0.3000E+02	0.3000E+02	deg
i	0.3000E+02	0.3000E+02	0.3000E+02	0.3000E+02	deg

Table 6A. Noncoplanar transfer, HEO-to-LEO, Type 1, summary results.

Quantity	(P1)	(P2)	(P3)	(P4)	Units
ΔV	0.1724E+01	0.1724E+01	0.1769E+01	0.1953E+01	km/sec
PHR	0.1721E+03	0.1674E+03	0.1301E+03	0.9262E+02	W/cm ²
PDP	0.1633E+01	0.1526E+01	0.9739E+00	0.7577E+00	N/cm ²
PTA	0.2237E+02	0.2111E+02	0.1499E+02	0.1314E+02	m/sec ²
$h_{\min}$	0.5770E+02	0.5830E+02	0.6169E+02	0.6221E+02	km
τ	0.1486E+04	0.1494E+04	0.1202E+04	0.8437E+03	sec

Table 6B. Noncoplanar transfer, HEO-to-LEO, Type 1, characteristic velocity components.

Quantity	(P1)	(P2)	(P3)	(P4)	Units
ΔV_{00}	0.5341E+00	0.5341E+00	0.5341E+00	0.5341E+00	km/sec
ΔV_{22}	0.1111E+01	0.1111E+01	0.1110E+01	0.1110E+01	km/sec
ΔV_1	0.5963E-01	0.5980E-01	0.1060E+00	0.2898E+00	km/sec
ΔV_{11}	0.1869E-01	0.1869E-01	0.1869E-01	0.1869E-01	km/sec
ΔV^{11}	0.1724E+01	0.1724E+01	0.1769E+01	0.1953E+01	km/sec

Table 6C. Noncoplanar transfer, HEO-to-LEO, Type 1, plane change components and apogee radius.

Quantity	(P1)	(P2)	(P3)	(P4)	Units
i_{s00}	0.2057E+01	0.2057E+01	0.2057E+01	0.2057E+01	deg
i_{s22}	0.1075E+02	0.1075E+02	0.1075E+02	0.1075E+02	deg
i_{s1}	0.1267E+00	0.1267E+00	0.1267E+00	0.1267E+00	deg
i_{s11}	0.3861E-01	0.3861E-01	0.3861E-01	0.3861E-01	deg
i_s	0.1298E+02	0.1298E+02	0.1298E+02	0.1298E+02	deg
i_a	0.1702E+02	0.1702E+02	0.1702E+02	0.1702E+02	deg
i	0.3000E+02	0.3000E+02	0.3000E+02	0.3000E+02	deg
r_{22}	0.1892E+05	0.1892E+05	0.1892E+05	0.1892E+05	km

Table 7A. Noncoplanar transfer, HEO-to-LEO, Type 2, summary results.

Quantity	(P1)	(P2)	(P3)	(P4)	Units
ΔV	0.1907E+01	0.1910E+01	0.1945E+01	0.2077E+01	km/sec
PHR	0.1235E+03	0.1191E+03	0.9489E+02	0.7227E+02	W/cm ²
PDP	0.1009E+01	0.9269E+00	0.6206E+00	0.4697E+00	N/cm ²
PTA	0.1405E+02	0.1303E+02	0.9545E+01	0.9196E+01	m/sec ²
$h_{\min}$	0.6103E+02	0.6170E+02	0.6460E+02	0.6580E+02	km
τ	0.1424E+04	0.1432E+04	0.1219E+04	0.9070E+03	sec

Table 7B. Noncoplanar transfer, HEO-to-LEO, Type 2, characteristic velocity components.

Quantity	(P1)	(P2)	(P3)	(P4)	Units
ΔV_{00}	0.1815E+01	0.1815E+01	0.1814E+01	0.1814E+01	km/sec
ΔV_1	0.7091E-01	0.7404E-01	0.1102E+00	0.2424E+00	km/sec
ΔV_{11}	0.2098E-01	0.2098E-01	0.2098E-01	0.2098E-01	km/sec
ΔV^{11}	0.1907E+01	0.1910E+01	0.1945E+01	0.2077E+01	km/sec

Table 7C. Noncoplanar transfer, HEO-to-LEO, Type 2, plane change components.

Quantity	(P1)	(P2)	(P3)	(P4)	Units
i_{s00}	0.1720E+02	0.1720E+02	0.1720E+02	0.1720E+02	deg
i_{s1}	0.2703E+00	0.2703E+00	0.2703E+00	0.2703E+00	deg
i_{s11}	0.8008E-01	0.8008E-01	0.8008E-01	0.8008E-01	deg
i_s	0.1755E+02	0.1755E+02	0.1755E+02	0.1755E+02	deg
i_a	0.1245E+02	0.1245E+02	0.1245E+02	0.1245E+02	deg
i_i	0.3000E+02	0.3000E+02	0.3000E+02	0.3000E+02	deg

Table 8A. Noncoplanar transfer, HEO-to-LEO, Type 3, summary results.

Quantity	(P1)	(P2)	(P3)	(P4)	Units
ΔV	0.2766E+01	0.2859E+01	0.2833E+01	0.5003E+01	km/sec
PHR	0.3084E+03	0.2406E+03	0.2204E+03	0.1100E+03	W/cm ²
PDP	0.9367E+01	0.7324E+01	0.5325E+01	0.5455E+01	N/cm ²
PTA	0.1104E+03	0.6961E+02	0.6504E+02	0.1166E+03	m/sec ²
$h_{\min}$	0.4136E+02	0.4248E+02	0.4501E+02	0.4034E+02	km
τ	0.3569E+03	0.4612E+03	0.4267E+03	0.3247E+03	sec

Table 8B. Noncoplanar transfer, HEO-to-LEO, Type 3, characteristic velocity components.

Quantity	(P1)	(P2)	(P3)	(P4)	Units
ΔV_{00}	0.1041E+01	0.1030E+01	0.1032E+01	0.1043E+01	km/sec
ΔV_1	0.1707E+01	0.1811E+01	0.1783E+01	0.3942E+01	km/sec
ΔV_{11}	0.1793E-01	0.1793E-01	0.1793E-01	0.1793E-01	km/sec
ΔV	0.2766E+01	0.2859E+01	0.2833E+01	0.5003E+01	km/sec

Table 8C. Noncoplanar transfer, HEO-to-LEO, Type 3, plane change components.

Quantity	(P1)	(P2)	(P3)	(P4)	Units
i_s	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	deg
i_a	0.3000E+02	0.3000E+02	0.3000E+02	0.3000E+02	deg
i	0.3000E+02	0.3000E+02	0.3000E+02	0.3000E+02	deg

Table 9A. Noncoplanar transfer, LEO-to-LEO, Type 1, summary results.

Quantity	(P1)	(P2)	(P3)	(P4)	Units
ΔV	0.2441E+01	0.2586E+01	0.2490E+01	0.3845E+01	km/sec
PHR	0.2129E+03	0.1595E+03	0.1595E+03	0.1114E+03	W/cm ²
PDP	0.4544E+01	0.3287E+01	0.2698E+01	0.2752E+01	N/cm ²
PTA	0.5427E+02	0.3483E+02	0.3608E+02	0.8585E+02	m/sec ²
$h_{\min}$	0.4676E+02	0.4860E+02	0.5067E+02	0.4781E+02	km
τ	0.4528E+03	0.6724E+03	0.5091E+03	0.3218E+03	sec

Table 9B. Noncoplanar transfer, LEO-to-LEO, Type 1, characteristic velocity components.

Quantity	(P1)	(P2)	(P3)	(P4)	Units
ΔV_{00}	0.9222E+00	0.9222E+00	0.9222E+00	0.9222E+00	km/sec
ΔV_{22}	0.6200E-01	0.5489E-01	0.5648E-01	0.6856E-01	km/sec
ΔV_1	0.1412E+01	0.1577E+01	0.1479E+01	0.2822E+01	km/sec
ΔV_{11}	0.4562E-01	0.3185E-01	0.3185E-01	0.3185E-01	km/sec
ΔV	0.2441E+01	0.2586E+01	0.2490E+01	0.3845E+01	km/sec

Table 9C. Noncoplanar transfer, LEO-to-LEO, Type 1, plane change components and apogee radius.

Quantity	(P1)	(P2)	(P3)	(P4)	Units
i_{s00}	0.3415E+01	0.3415E+01	0.3415E+01	0.3415E+01	deg
i_{s22}	0.4828E+00	0.4828E+00	0.4828E+00	0.4828E+00	deg
i_{s1}	0.6444E+01	0.6444E+01	0.6444E+01	0.6444E+01	deg
i_{s11}	0.1937E+00	0.1937E+00	0.1937E+00	0.1937E+00	deg
i_s	0.1054E+02	0.1054E+02	0.1054E+02	0.1054E+02	deg
i_a	0.1946E+02	0.1946E+02	0.1946E+02	0.1946E+02	deg
i	0.3000E+02	0.3000E+02	0.3000E+02	0.3000E+02	deg
r_{22}	0.1006E+05	0.1006E+05	0.1006E+05	0.1006E+05	km

Table 10A. Noncoplanar transfer, LEO-to-LEO, Type 2, summary results.

Quantity	(P1)	(P2)	(P3)	(P4)	Units
ΔV	0.2682E+01	0.2726E+01	0.2687E+01	0.6616E+01	km/sec
PHR	0.1713E+03	0.1848E+03	0.1516E+03	0.1033E+03	W/cm ²
PDP	0.4753E+01	0.5740E+01	0.3631E+01	0.8663E+01	N/cm ²
PTA	0.5747E+02	0.6705E+02	0.4623E+02	0.1759E+03	m/sec ²
$h_{\min}$	0.4476E+02	0.4344E+02	0.4682E+02	0.3511E+02	km
τ	0.4794E+03	0.5048E+03	0.4923E+03	0.2660E+03	sec

Table 10B. Noncoplanar transfer, LEO-to-LEO, Type 2, characteristic velocity components.

Quantity	(P1)	(P2)	(P3)	(P4)	Units
ΔV_{00}	0.2326E+00	0.2097E+00	0.2186E+00	0.1505E+01	km/sec
ΔV_1	0.2431E+01	0.2497E+01	0.2450E+01	0.5092E+01	km/sec
ΔV_{11}	0.1913E-01	0.1913E-01	0.1913E-01	0.1913E-01	km/sec
ΔV	0.2682E+01	0.2726E+01	0.2687E+01	0.6616E+01	km/sec

Table 10C. Noncoplanar transfer, LEO-to-LEO, Type 2, plane change components.

Quantity	(P1)	(P2)	(P3)	(P4)	Units
i_{s00}	0.9087E+00	0.9087E+00	0.9087E+00	0.9087E+00	deg
i_{s1}	0.1228E+02	0.1228E+02	0.1228E+02	0.1228E+02	deg
i_{s11}	0.4892E-01	0.4892E-01	0.4892E-01	0.4892E-01	deg
i_s	0.1324E+02	0.1324E+02	0.1324E+02	0.1324E+02	deg
i_a	0.1676E+02	0.1676E+02	0.1676E+02	0.1676E+02	deg
i	0.3000E+02	0.3000E+02	0.3000E+02	0.3000E+02	deg

Table 11A. Noncoplanar transfer, LEO-to-LEO, Type 3, summary results.

Quantity	(P1)	(P2)	(P3)	(P4)	Units
ΔV	0.3086E+01	0.3132E+01	0.3129E+01	0.7406E+01	km/sec
PHR	0.2292E+03	0.2246E+03	0.1869E+03	0.9209E+02	W/cm ²
PDP	0.1044E+02	0.1148E+02	0.7419E+01	0.1151E+02	N/cm ²
PTA	0.1412E+03	0.1080E+03	0.8705E+02	0.1894E+03	m/sec ²
$h_{\min}$	0.3788E+02	0.3749E+02	0.4050E+02	0.3109E+02	km
τ	0.4850E+03	0.5138E+03	0.4849E+03	0.2577E+03	sec

Table 11B. Noncoplanar transfer, LEO-to-LEO, Type 3, characteristic velocity components.

Quantity	(P1)	(P2)	(P3)	(P4)	Units
ΔV_{00}	0.1505E+00	0.1331E+00	0.1705E+00	0.1699E+01	km/sec
ΔV_1	0.2918E+01	0.2981E+01	0.2941E+01	0.5689E+01	km/sec
ΔV_{11}	0.1793E-01	0.1793E-01	0.1793E-01	0.1793E-01	km/sec
ΔV	0.3086E+01	0.3132E+01	0.3129E+01	0.7406E+01	km/sec

Table 11C. Noncoplanar transfer, LEO-to-LEO, Type 3, plane change components.

Quantity	(P1)	(P2)	(P3)	(P4)	Units
i_s	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	deg
i_a	0.3000E+02	0.3000E+02	0.3000E+02	0.3000E+02	deg
i	0.3000E+02	0.3000E+02	0.3000E+02	0.3000E+02	deg

FIG. 1. COPLANAR, AERODASSISTED ORBITAL TRANSFER.

Fig. 2A. Coplanar transfer, total characteristic velocity (km/sec).

Fig. 2B. Coplanar transfer, peak heating rate (W/cm^2).

Fig. 2C. Coplanar transfer, peak dynamic pressure (N/cm^2).

Fig. 2D. Coplanar transfer, peak total acceleration (m/sec^2).

Fig. 2E. Coplanar transfer, minimum altitude (km).

Fig. 2F. Coplanar transfer, flight time (sec).

FIG. 3A. COPLANAR TRANSFER,
NEARLY-GRAZING TRAJECTORIES,
ALTITUDE(KM) VS TIME.

FIG. 3B. COPLANAR TRANSFER,
NEARLY-GRAZING TRAJECTORIES,
VELOCITY(KM/SEC) VS TIME.

FIG. 3C. COPLANAR TRANSFER,
NEARLY-GRAZING TRAJECTORIES,
PATH INCLINATION(OE) VS TIME.

FIG. 3D. COPLANAR TRANSFER,
NEARLY-GRAZING TRAJECTORIES,
LIFT COEFFICIENT VS TIME.

FIG. 4. NONCOPLANAR, AERASSISTED ORBITAL TRANSFER, TYPE 1.

FIG. 5. NONCOPLANAR, AEROASSISTED ORBITAL TRANSFER, TYPE 2.

FIG. 6. NONCOPLANAR, AEROGASSISTED ORBITAL TRANSFER, TYPE 3.

Fig. 7A. Noncoplanar transfer, GEO-to-LEO, total characteristic velocity (km/sec).

Fig. 7B. Noncoplanar transfer, GEO-to-LEO, peak heating rate (W/cm^2).

Fig. 7C. Noncoplanar transfer, GEO-to-LEO, peak dynamic pressure (N/cm^2).

Fig. 7D. Noncoplanar transfer, GEO-to-LEO, peak total acceleration (m/sec^2).

Fig. 7E. Noncoplanar transfer, GEO-to-LEO, minimum altitude (km).

Fig. 7F. Noncoplanar transfer, GEO-to-LEO, flight time (sec).

FIG. 8A. NONCOPLANAR TRANSFER, GEO-TO-LEO, NEARLY-GRAZING TRAJECTORIES, ALTITUDE(KM) VS TIME.

FIG. 8B. NONCOPLANAR TRANSFER, GEO-TO-LEO, NEARLY-GRAZING TRAJECTORIES, VELOCITY(KM/SEC) VS TIME.

FIG. 8C. NONCOPLANAR TRANSFER, GEO-TO-LEO, NEARLY-GRAZING TRAJECTORIES, PATH INCLINATION(DEG) VS TIME.

FIG. 8D. NONCOPLANAR TRANSFER, GEO-TO-LEO, NEARLY-GRAZING TRAJECTORIES, LONGITUDE(DEG) VS TIME.

FIG. 8E. NONCOPLANAR TRANSFER, GEO-TO-LEO, NEARLY-GRAZING TRAJECTORIES, LATITUDE(DEG) VS TIME.

FIG. 8F. NONCOPLANAR TRANSFER, GEO-TO-LEO, NEARLY-GRAZING TRAJECTORIES, HEADING ANGLE(DEG) VS TIME.

FIG. 8G. NONCOPLANAR TRANSFER, GEO-TO-LEO, NEARLY-GRAZING TRAJECTORIES, LIFT COEFFICIENT VS TIME.

FIG. 8H. NONCOPLANAR TRANSFER, GEO-TO-LEO, NEARLY-GRAZING TRAJECTORIES, BANK ANGLE(DEG) VS TIME.

Fig. 9A. Noncoplanar transfer, HEO-to-LEO, total characteristic velocity (km/sec).

Fig. 7B. Noncoplanar transfer, GEO-to-LEO, peak heating rate (W/cm^2).

Fig. 9C. Noncoplanar transfer, HEO-to-LEO, peak dynamic pressure (N/cm^2).

Fig. 9D. Noncoplanar transfer, HEO-to-LEO, peak total acceleration (m/sec^2).

Fig. 9E. Noncoplanar transfer, HEO-to-LEO, minimum altitude (km).

Fig. 9F. Noncoplanar transfer, HEO-to-LEO, flight time (sec).

FIG. 10A. NONCOPLANAR TRANSFER, HEO-TO-LEO, NEARLY-GRAZING TRAJECTORIES, ALTITUDE(KM) VS TIME.

FIG. 10B. NONCOPLANAR TRANSFER, HEO-TO-LEO, NEARLY-GRAZING TRAJECTORIES, VELOCITY(KM/SEC) VS TIME.

FIG. 10C. NONCOPLANAR TRANSFER, HEO-TO-LEO, NEARLY-GRAZING TRAJECTORIES, PATH INCLINATION(DEG) VS TIME.

FIG. 10D. NONCOPLANAR TRANSFER, HEO-TO-LEO, NEARLY-GRAZING TRAJECTORIES, LONGITUDE(DEG) VS TIME.

FIG. 10E. NONCOPLANAR TRANSFER, HEO-T0-LE0, NEARLY-GRAZING TRAJECTORIES, LATITUDE(DEG) VS TIME.

FIG. 100. NONCOPLANAR TRANSFER, HEO-T0-LE0, NEARLY-GRAZING TRAJECTORIES, LIFT COEFFICIENT VS TIME.

FIG. 10F. NONCOPLANAR TRANSFER, HEO-T0-LE0, NEARLY-GRAZING TRAJECTORIES, HEADING ANGLE(DEG) VS TIME.

FIG. 10H. NONCOPLANAR TRANSFER, HEO-T0-LE0, NEARLY-GRAZING TRAJECTORIES, BANK ANGLE(DEG) VS TIME.

Fig. 11A. Noncoplanar transfer, LEO-to-LEO, total characteristic velocity (km/sec).

Fig. 11B. Noncoplanar transfer, LEO-to-LEO, peak heating rate (W/cm^2).

Fig. 11C. Noncoplanar transfer, LEO-to-LEO, peak dynamic pressure (N/cm^2).

Fig. 11D. Noncoplanar transfer, LEO-to-LEO, peak total acceleration (m/sec^2).

Fig. 11E. Noncoplanar transfer, minimum altitude (km).

Fig. 11F. Noncoplanar transfer, LEO-to-LEO, flight time (sec).

FIG. 12A. NONCOPLANAR TRANSFER, LEO-TO-LEO, NEARLY-GRAZING TRAJECTORIES, ALTITUDE(KM) VS TIME.

FIG. 12B. NONCOPLANAR TRANSFER, LEO-TO-LEO, NEARLY-GRAZING TRAJECTORIES, VELOCITY(KM/SEC) VS TIME.

FIG. 12C. NONCOPLANAR TRANSFER, LEO-TO-LEO, NEARLY-GRAZING TRAJECTORIES, PATH INCLINATION(DEG) VS TIME.

FIG. 12D. NONCOPLANAR TRANSFER, LEO-TO-LEO, NEARLY-GRAZING TRAJECTORIES, LONGITUDE(DEG) VS TIME.

FIG. 12E. NONCOPLANAR TRANSFER, LEO-TO-LEO, NEARLY-GRAZING TRAJECTORIES, LATITUDE(DEG) VS TIME.

FIG. 12F. NONCOPLANAR TRANSFER, LEO-TO-LEO, NEARLY-GRAZING TRAJECTORIES, HEADING ANGLE(DEG) VS TIME.

FIG. 120. NONCOPLANAR TRANSFER, LEO-TO-LEO, NEARLY-GRAZING TRAJECTORIES, LIFT COEFFICIENT VS TIME.

FIG. 12H. NONCOPLANAR TRANSFER, LEO-TO-LEO, NEARLY-GRAZING TRAJECTORIES, BANK ANGLE(DEG) VS TIME.