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Rice University, 1987

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CONFINEMENT OF PRESTRESSED CONCRETE COLUMNS

by

HANI ELBAYADI ELIAS

A THESIS SUBMITTED
IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS FOR THE DEGREE

DOCTOR OF PHILOSOPHY

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ABSTRACT

CONFINEMENT OF PRESTRESSED CONCRETE COLUMNS

BY

HANI ELBAYADI ELIAS

Prestressed concrete columns generally have a much smaller amount of longitudinal reinforcement compared to reinforced concrete columns and the confinement of the core is thus provided mainly by the lateral reinforcement. The main objective of this study was to evaluate the need for lateral reinforcement and its influence on the confinement in short prestressed concrete columns.

In the experimental part of this investigation, twenty-three specimens were tested under axial loading. Primary variables were the type and amount of lateral reinforcement. Three types of lateral reinforcement were investigated: conventional ties, continuous square spiral and the welded wire mesh. The volumetric ratio of lateral reinforcement to concrete core was varied between 1.09% and 2.18%. Experimental results showed that lateral reinforcement was much more effective in increasing the ductility than the strength.

It was also found that the currently available analytical confinement models did not accurately represent the behavior of prestressed concrete columns and considerably overestimated the strength in the post-peak loading region. Therefore, based on the observed behavior, an analytical model for predicting the stress-strain relation of confined concrete in axially loaded precast prestressed concrete columns with rectilinear lateral reinforcement is proposed. The ascending part of the model consists of a second degree parabola followed by

a linear segment and an exponentially decreasing curve. This model takes into account any increase in strength and ductility resulting from lateral reinforcement, and more realistically represents the post-peak behavior. Finally, some design implications are suggested and the concept of strain ductility factor as a measure of ductility is proposed.

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LIST OF SYMBOLS

A_g	= Gross section area
A_o	= Cross sectional area of the lateral reinforcement
A_{ps}	= Cross sectional area of the prestressing reinforcement
b	= Side dimension of the concrete core
C	= Center to center distance between the longitudinal bars
C_1	= Parameter controlling the shape of the descending branch of the proposed stress-strain curve
D	= Parameter controlling the descending branch of the stress-strain curve
d	= Diameter of the lateral reinforcement
d_ℓ	= Diameter of the longitudinal reinforcement
E	= Modulus of elasticity of steel
E_c	= Modulus of elasticity of concrete
E_{ci}	= Modulus of elasticity of concrete at transfer
E_i	= Initial modulus of elasticity of concrete
E_t	= Tangent modulus of elasticity of steel
f_1	= Stress at the peak of the stress-strain curve
f_c	= Concrete stress
f'_c	= Concrete compressive strength
f'_{ci}	= Concrete compressive strength at transfer
f_{pe}	= Compressive stress in concrete due to effective prestress
f_{pi}	= Initial stress in the prestressing reinforcement
f_{pu}	= Tensile strength of the prestressing reinforcement
f_{py}	= Yield strength of the prestressing reinforcement
f_r	= Average confining pressure in the concrete at peak stress
f_s	= Stress in the lateral reinforcement

f_u	= Ultimate strength of the nonprestressing reinforcement
f_y	= Yield strength of the nonprestressing reinforcement
I	= Moment of inertia of a steel bar
K	= Effective length factor
K	= Stiffness factor
K, K_3, K_s	= Ratio of maximum stress in the confined core to the strength of plain concrete
K_{exp}	= Experimental maximum stress ratio
K_p	= Predicted maximum stress ratio
k_1	= Equivalent stiffness of the lateral reinforcement
ℓ_o	= Length of one leg of the lateral reinforcement
M	= Bending moment
P_{cr}	= Critical buckling load of a prestressing bar
r	= Carry over factor
S	= Spacing of the lateral reinforcement
$SDF_{(80\%)}$	= Strain ductility factor
V	= Shearing force
w_c	= Unit weight of concrete in lb/ft^3
Z	= Slope of the descending branch of the stress-strain curve
α_1, α_2	= Constants used in the development of the equation of the maximum stress ratio
β_1, β_2	= Constants used in the development of the equation of the maximum strain corresponding to the maximum stress
ϵ_o	= Strain corresponding to the maximum stress
ϵ_1	= Minimum strain corresponding to the maximum core stress
ϵ_2	= Maximum strain corresponding to the maximum core stress
ϵ_{50}	= Strain corresponding to 50% of the maximum stress
ϵ_{80}	= Strain corresponding to 80% of the maximum load on the load-strain curve

ϵ_l	= Longitudinal strain
ϵ_{max}	= Strain corresponding to the maximum load on the load-strain curve
ϵ_u	= Ultimate concrete strain
θ	= Angle caused by spring translation
ρ	= Volumetric ratio of lateral reinforcement to the concrete core
ρ_l	= Volumetric ratio of longitudinal reinforcement to the concrete core
ϕ	= Angle caused by joint rotation

Chapter 1

INTRODUCTION

1.1 General

Precast prestressed concrete columns offer an economical and efficient alternative to reinforced concrete columns and thus are being increasingly used in precast construction. Compared to reinforced concrete columns, they have a much smaller amount of longitudinal reinforcement, which is generally provided in the form of small diameter bars or strands of high strength steel having an ultimate strength in the range of 250-270 ksi. Since columns play a very important role in the safety of buildings against collapse, a certain amount of lateral reinforcement is also provided to ensure adequate ductility during overload conditions.

The precast columns are also generally prestressed. However, the precompression is meant only to resist tensile stresses during transportation, handling and erection, and to resist loads from small eccentricities. The ACI code [2] specifies a minimum effective prestress over the gross section of not less than 225 psi which indirectly sets a minimum limit on longitudinal steel ratio. In practice, the precast columns are generally prestressed to an average stress of about 500 psi.

1.2 Background

Most of the past research on the behavior of prestressed concrete columns has been aimed at developing rational methods for predicting the column strength taking into consideration the eccentricity, level

of prestress, slenderness, and strength of steel and concrete [5,7,16,33]. However, studies on the effects of lateral confining reinforcement on the strength and ductility of prestressed columns are relatively fewer. Presently, there is no rational basis for evaluating the lateral reinforcement contribution [25]. Limited tests on prestressed columns [9,15] have indicated that the presence of lateral ties affected only the mode of failure rather than the ultimate capacity. Recognizing that the brittle mode of failure is undesirable, even though the ultimate capacity can be attained, the PCI (Prestressed Concrete Institute) recommendations [23] allow the design of columns without lateral reinforcement provided the nominal capacity is reduced by 15%.

Lateral reinforcement in prestressed concrete columns is usually provided in the form of closed ties. The use of such ties, however, is labor intensive and thus uneconomical. Moreover, the current provisions [2] on the amount of lateral reinforcement for prestressed concrete columns are also believed to be excessively conservative [9]. In an effort to reduce production costs, the precast manufacturers have shown interest in the use of continuous square spiral and the welded wire mesh as lateral reinforcement for prestressed columns. However, the test data on the performance of these types of reinforcement is severely limited and there is a need to study the relative merits in their use as confining reinforcement.

The analytical models for predicting the load-deformation behavior of axially loaded columns have been developed on the basis of tests on nonprestressed columns which had relatively large amounts of lateral

and longitudinal reinforcement. These models, therefore do not accurately represent the behavior of prestressed columns. For the same reason, the confinement requirements for nonprestressed columns may also not be directly applicable to prestressed concrete columns. It may thus be desirable to develop an analytical model which realistically represents the behavior of prestressed columns.

1.3 Objectives and Scope

This study was intended to investigate the confinement effects of lateral reinforcement on the load deformation behavior of short precast prestressed concrete columns subjected to concentric loading. The principal objectives of this study were: (1) to investigate experimentally the effect of lateral reinforcement on the strength and ductility of prestressed concrete columns, (2) to compare the relative performance of single ties, continuous square spiral, and the welded wire mesh as lateral reinforcement, and (3) to analytically model the confinement provided by the lateral reinforcement.

To achieve these objectives, twenty-three columns of 5.25 in. x 5.25 in. cross section and 26 inches in length were tested under monotonic axial compression to failure. The primary variables were the type and amount of lateral reinforcement. The types of lateral reinforcement investigated were conventional ties, continuous square spiral and the welded wire mesh. The volumetric ratio of lateral reinforcement to concrete core was varied between 1.09% and 2.18%. The observed load-strain curves were analyzed and a model is proposed to simulate the stress-strain behavior of confined concrete in prestressed columns.

1.4 Contents

A review of the different models proposed in the literature to simulate the behavior of confined concrete in reinforced columns is presented in Chapter 2. Then, the experimental program is described in Chapter 3. Further, Chapter 4 summarizes the experimental results obtained during this investigation. In Chapter 5, the effect of different variables on the behavior of prestressed concrete columns is discussed. In addition, the results obtained from the application of some previous models are compared with the experimental results. Chapter 6 is devoted to the development of a model to predict the stress-strain of the confined concrete. In Chapter 7 the principle of strain ductility factor is introduced as a measure of ductility for concrete columns. Finally, conclusions and recommendations for further studies are listed in Chapter 8.

Chapter 2

CONCRETE CONFINEMENT

2.1 General

Concrete as a structural material is capable of developing large compressive strength but it lacks sufficient ductility when used alone. To compensate for such a deficiency under compressive stresses, it is necessary to confine the concrete. In practice, this is achieved by providing lateral reinforcement in the form of closely spaced steel ties or spirals. Such confinement may increase strength but generally improves ductility.

2.2 Unconfined Concrete

The stress-strain behavior of unconfined concrete is obtained from standard cylinders (6 in. x 12 in.) loaded in uniaxial compression. Figure 2.1 represents such behavior of concrete of different strengths. The stress-strain curves, which have a somewhat similar character, consist of an initial almost linear portion up to about 40% of the compressive strength, a nonlinear portion up to the peak stress, and a final descending part. It can be noticed that the higher the concrete strength, the higher the strain at maximum stress. At strains higher than the strain corresponding to the maximum stress, a certain stress can still be carried.

The descending portion of the curve is of particular interest for understanding the behavior at relatively high strain. It is usually difficult to define unless a relatively stiff testing machine is used in obtaining the stress-strain relationship. The use of a flexible

Fig. 2.1 Stress-Strain Curves for Concrete Cylinders Loaded in Uniaxial Compression (Adapted from Ref. 20)

machine results in an uncontrolled sudden release of energy which fails the specimen instantaneously in an explosive manner.

2.3 Concrete Confinement by Lateral Reinforcement

Concrete can be confined by providing lateral reinforcement in the form of closely spaced steel ties or spirals. At low levels of stress in the concrete, the lateral reinforcement is hardly stressed; hence, the concrete may be considered as unconfined. At stresses approaching the uniaxial strength, the transverse strains become very high and the concrete bears out against the lateral reinforcement, which then applies a confining reaction to the concrete. Thus, the lateral reinforcement provides a kind of passive confinement. If the confinement becomes effective before the concrete reaches its maximum stress, then the strength gain may be substantial. But if the confinement does not become effective until after the peak stress is reached, its contribution to strength increase may be small or sometimes negligible. However, in such a case an improvement in ductility is generally observed. The ductility here is defined as the ability of the concrete to sustain large strain beyond the peak stress without any substantial loss of strength.

2.3.1 Confinement Mechanism

Previous tests [13] have demonstrated that circular spirals confine concrete much more effectively than square ties. The reason for this difference between the confinement by circular spirals and square ties is illustrated in Fig. 2.2. Circular spirals, because of their shape, are in axial hoop tension and provide a continuous confining

pressure around the circumference. In the case of square ties, because the lateral concrete pressure is resisted by the bending of ties, the confinement is provided in the form of reactions near the corners of the ties. The bending resistance of ties is usually small and will require a relatively large outward deflection before effectively restraining the concrete. Because of the internal arching action between the corners, the concrete is confined effectively only in the corners and central region of the core, and a portion of the concrete core, as shown in Fig. 2.2, remains unconfined. In regions between the ties, the confined area of the concrete may even be further reduced due to arching of concrete between the lateral reinforcement as illustrated in Fig. 2.3.

2.3.2 Factors Affecting Confinement by Rectilinear Ties

Confinement appears to have little effect on the stress-strain behavior until the concrete strength is approached. Beyond that, the behavior of confined concrete is a function of many variables, the major ones being the following:

(1) Amount of lateral reinforcement: This amount is measured by the volumetric ratio of lateral reinforcement to the concrete core. A high lateral reinforcement content means a high confining pressure resulting in a slight increase in strength and a significant improvement in ductility of the confined concrete.

(2) Characteristics of lateral reinforcement: When the stress-strain relationship of the lateral reinforcement has a yield plateau, its yield strength sets an upper limit to the confining pressure. As soon as such lateral reinforcement yields, it permits the

Fig. 2.2 Confinement by Square Hoops and Circular Spiral (Adapted from Ref. 21)

Fig. 2.3 Arch Formation in the Vertical Direction of a Laterally Reinforced Column (Adapted from Ref. 29)

concrete to expand without providing additional confining pressure. On the other hand, lateral steel without a well defined yield plateau, continues to restrain lateral expansion of concrete until either the internal cracking has gradually progressed to a point where the concrete cannot carry any further load or the lateral steel fails in tension.

(3) Tie spacing: This variable is generally expressed as the ratio of tie spacing to the depth of the concrete core. As shown in Fig. 2.3, the concrete is confined by arching between the ties and, if the spacing is large, a considerable volume of concrete remains unconfined and tends to spall off. As such, a smaller spacing leads to a more effective confinement. The tie spacing also controls the buckling of the longitudinal bars.

(4) Size of lateral reinforcement: In the case of rectilinear ties, the confinement depends upon the flexural stiffness of the tie bar which, in turn, depends upon the bar size and the unsupported length. A larger diameter bar used as lateral reinforcement leads generally to more effective confinement for a given unsupported length of tie. If the flexural stiffness of the tie bar is small, concrete pressure can easily push the ties outward and ties will not effectively confine the concrete.

(5) Content and distribution of longitudinal reinforcement: A large content of well distributed longitudinal reinforcement provides better confinement of the concrete core. However, because the transverse steel provides the confining reaction to the longitudinal bars, the longitudinal bars must be placed tightly against the lateral steel.

In addition to the factors discussed above, several other variables, such as the rate of loading, the lateral steel configuration, Poisson's ratio, shrinkage, creep, etc., also affect the behavior of the confined concrete.

2.4 Confinement Models

Several models have been proposed in the past to predict the stress-strain behavior of concrete confined by rectilinear ties. Most of these models, however, are based on tests on reinforced concrete columns. In this section, previous studies on the behavior of concrete columns confined by rectilinear ties are summarized.

2.4.1 Chan and Blume et al.

Chan [10] suggested a trilinear curve for unconfined and confined concrete as shown in Fig. 2.4. In unconfined concrete, the slope of BC is negative and ϵ_u , the ultimate concrete strain, is small. In concrete confined with suitable lateral reinforcement, the slope of BC can be positive with ϵ_u attaining much greater value than for unconfined concrete. According to Chan, the segment BC is only a function of the volumetric ratio of the lateral reinforcement.

The effect of yield stress of the lateral steel was added by Blume et al. [6]. They adopted a similar trilinear curve in which OA approximated the curve for unconfined concrete up to $0.85 f'_c$ where f'_c is the 28 day concrete compressive strength. The shape of ABC depended on the content and yield stress of the lateral reinforcement. However, this model did not predict the drop in strength at higher strain values.

Fig. 2.4 Stress-Strain Curve for Concrete Confined by Rectilinear Ties as Proposed by Chan [10] and Blume et al. [6]

2.4.2 Roy and Sozen

Roy and Sozen [24] conducted tests on 5 in. x 5 in. x 25 in. concentrically loaded specimens. They concluded that the confinement provided by rectilinear ties did not enhance the strength of the confined concrete. An idealized bilinear stress-strain relationship for laterally reinforced concrete was proposed as shown in Fig. 2.5. To define the descending branch, an equation for the strain value corresponding to 50% of the maximum stress, ϵ_{50} , was suggested as follows:

$$\epsilon_{50} = \frac{3}{4} \frac{\rho h}{S} \quad (2.1)$$

where ρ = volumetric lateral reinforcement ratio,

h = shorter dimension of the concrete section, and

S = tie spacing.

2.4.3 Soliman and Yu

After testing sixteen eccentrically loaded specimens, 4 in. x 6 in. x 52 in., Soliman and Yu [30] proposed the stress-strain curve shown in Fig. 2.6 for confined concrete. It consists of an ascending parabola followed by two straight lines. The stresses and strains at the critical points were related to the lateral steel content and tie spacing.

2.4.4 Sargin

Sargin [26] tested nineteen plain and thirty-two laterally reinforced specimens under concentric loading in a study on the effects of rectilinear confinement on the behavior of concentrically loaded

Fig. 2.5 Stress-Strain Model for Concrete Confined by Rectilinear Ties as Proposed by Roy and Sozen [24]

Fig. 2.6 Stress-Strain Curve for Concrete Confined by Rectilinear Ties as Proposed by Soliman and Yu [30]

concrete columns. The specimens tested were 5 in. x 5 in. x 20 in. without any longitudinal reinforcement. During the tests, the strains were measured over a ten inch gage length. Forty-five specimens were cast in a vertical position and the remaining six in a horizontal one. Test results indicated that the average concrete strength of the plain concrete specimens was about 98% of the corresponding cylinder strength and the strain corresponding to the maximum stress was about 0.0024. The stress-strain relationship was modelled as a continuous curve, as shown in Fig. 2.7, having an equation:

$$f_c = K_3 f_c' \frac{Ax + (D-1)x^2}{1 + (A-2)x + Dx^2} \quad (2.2)$$

where $x = \frac{\epsilon_l}{\epsilon_0}$

$$A = \frac{E_i \epsilon_0}{K_3 f_c'}$$

$$D = 8 - 5 \times 10^{-5} f_c'$$

where f_c = concrete stress,

K_3 = ratio of maximum stress in the confined concrete core to cylinder strength,

f_c' = 28 day concrete cylinder compressive strength in psi,

ϵ_l = longitudinal strain,

ϵ_0 = strain corresponding to the maximum stress,

E_i = initial modulus of elasticity of concrete ($72000 \sqrt{f_c'}$),

and

D = parameter controlling the slope of the descending branch.

Based on a regression analysis of the test results, the following empirical equations were suggested to predict the maximum stress ratio

Fig. 2.7 Stress-Strain Curve for Concrete Confined by Rectilinear Ties as Proposed by Sargin [26]

and the strain at maximum stress:

$$K_3 = 1 + 0.0146 \left(1 - 0.245 \frac{S}{b_e}\right) \frac{\rho f_y}{\sqrt{f_c}} \quad (2.3)$$

$$\epsilon_0 = 0.0024 + 0.0000374 \left(1 - 0.734 \frac{S}{b_e}\right) \frac{\rho f_y}{\sqrt{f_c}} \quad (2.4)$$

where S = tie spacing,

b_e = width of the core (inside of ties),

ρ = volumetric ratio of tie steel to concrete core, and

f_y = yield stress of lateral reinforcement in psi.

The effect of confinement on the slope of the descending branch seems to have been neglected in Sargin's equation since the parameter D , controlling the descending branch, is a function of f_c' only.

2.4.5 Vallenas, Bertero and Popov

Vallenas et al. [32] reported the results of fourteen columns tested under axial loading. Two of them were made of plain concrete. The other twelve columns, which were laterally reinforced with ties, were divided into two series of six columns each. In the first series, columns had longitudinal reinforcement while in the second series, no longitudinal reinforcement was used. Half of the columns of each series were cast without any cover. The core, measured from outside to outside of the lateral reinforcement, was 9 in. x 9 in. The rate of loading was 25 μ strain/sec. and the longitudinal strain was measured over a gage length of eight inches. The test results indicated an average increase of 13% in the maximum concrete stress due to lateral reinforcement. Also, the longitudinal reinforcement resulted in an additional increase in the concrete stress of about 7%. Columns without

cover failed in a symmetrical mode while those with cover failed in an asymmetrical mode due to uneven failure of the cover. The stress-strain relationship, shown in Fig. 2.8, was proposed to predict the test results. It is divided into three regions:

1. Region AB: $\epsilon_l \leq \epsilon_0$

$$\frac{f_c}{f_c'} = \frac{\frac{E_c \epsilon_0}{f_c'} \left(\frac{\epsilon_l}{\epsilon_0}\right) - K \left(\frac{\epsilon_l}{\epsilon_0}\right)^2}{1 + \left[\frac{E_c \epsilon_0}{K f_c'} - 2\right] \left(\frac{\epsilon_l}{\epsilon_0}\right)} \quad (2.5)$$

2. Region BC: $\epsilon_0 \leq \epsilon_l \leq \epsilon_{.3K}$

$$\frac{f_c}{f_c'} = K \left[1 - Z \epsilon_0 \left(\frac{\epsilon_l}{\epsilon_0} - 1\right)\right] \quad (2.6)$$

3. Region CD: $\epsilon_l > \epsilon_{.3K}$

$$\frac{f_c}{f_c'} = 0.3 K \quad (2.7)$$

where

$$\epsilon_0 = 0.0024 + 0.0005 \left(1 - 0.734 \frac{S}{b_e}\right) \frac{\rho f_y}{\sqrt{f_c'}} \quad (2.8)$$

$$K = 1 + 0.0091 \left(1 - 0.245 \frac{S}{b_e}\right) \frac{\left(\rho + \frac{d}{d_l} \rho_l\right) f_y}{\sqrt{f_c'}} \quad (2.9)$$

$$Z = \frac{0.5}{\frac{3}{4} \rho \sqrt{\frac{b_e}{S}} + \frac{3 + 0.002 f_c'}{f_c' - 1000} - 0.002} \quad (2.10)$$

where d = diameter of the lateral reinforcement,

d_l = diameter of the longitudinal reinforcement,

Fig. 2.8 Stress-Strain Relationship of Confined Concrete as Proposed by Vallenat et al. [32]

E_c = modulus of elasticity of concrete,

K = ratio of maximum stress in the confined concrete to the cylinder strength,

ρ_l = volumetric ratio of the longitudinal reinforcement to the concrete core,

ρ = volumetric ratio of the lateral reinforcement to the concrete core, and

Z = slope of segment BC.

In the above equations, K , ϵ_0 and the equation for the ascending portion of the curve were similar to the ones proposed by Sargin with some modifications to account for the confinement provided by the longitudinal reinforcement. The equation of the descending branch was adapted from Kent and Park's model.

2.4.6 Sheikh and Uzumeri

Sheikh and Uzumeri [29] reported the results of twenty-four reinforced concrete columns, 12 in. x 12 in. x 77 in., tested under concentric loading. The gage length for longitudinal strain measurements was 21 inches for eighteen specimens and 19 inches for the remaining six specimens. The core was considered as the area bounded by the centerline of the lateral reinforcement. The test results showed that, in addition to the variables discussed earlier which affected the behavior of confined core, the distribution of the longitudinal steel around the core and the tie configuration were also important. Based on their results, Sheikh and Uzumeri [28] suggested an analytical model for predicting the behavior of concrete confined by rectilinear ties. This model, illustrated in Fig. 2.9,

Fig. 2.9 Stress-Strain Model for Concrete Confined by Rectilinear Ties as Proposed by Sheikh and Uzumeri [29]

consists of an ascending second degree parabola followed by three straight lines. The strain values at point A, B and C are given by the following equations:

$$\epsilon_{s1} = 0.55 K_s f_c' \times 10^{-6} \quad (2.11)$$

$$\frac{\epsilon_{s2}}{\epsilon_0} = 1 + \frac{0.81}{C} \left[1 - 5 \left(\frac{S}{b} \right)^2 \right] \frac{\rho f_y}{\sqrt{f_c'}} \quad (2.12)$$

$$\epsilon_{s85} = 0.225 \rho \sqrt{\frac{b}{S}} + \epsilon_{s2} \quad (2.13)$$

K_s , the ratio of confined concrete strength to the strength of plain concrete specimens of similar size and shape is given by:

$$K_s = 1 + \frac{1}{P_{occ}} \left[\left(1 - \frac{nC^2}{5.5b^2} \right) \left(1 - 0.5 \frac{S}{b} \right)^2 b^2 \right] 2.73 \sqrt{\rho f_y} \quad (2.14)$$

where P_{occ} = theoretical capacity of column concrete core

$$= f_{cp} \times A_{cc},$$

f_{cp} = strength of plain concrete specimens taken as $0.85 f_c'$,

A_{cc} = area of concrete in the core,

n = number of longitudinal bars,

C = center to center distance between the longitudinal bars,

b = one side dimension of the core,

S = tie spacing,

ρ = volumetric ratio of lateral reinforcement to the concrete core, and

ϵ_0 = average longitudinal strain corresponding to the maximum stress in plain concrete.

In the development of this model, the strength of the confined concrete was calculated by using the concept of effective confined area within the nominal concrete core which is defined as:

$$A_{\text{effective}} = (1 - \frac{nc^2}{5.5b^2})(b - 0.5S)^2 \quad (2.15)$$

It can be noticed that equation 2.12, for the strain ϵ_{s2} at point b, gives negative values when tie spacing is greater than 0.45 of the core dimension.

2.4.7 Modified Kent and Park

Based on the test data from previous investigators, Kent and Park [14] suggested a stress-strain model for confined concrete. This model was later modified [22] to include the strength and strain increase at maximum concrete stress due to confinement. Figure 2.10 shows the details of the modified Kent and Park model. This model consists of an ascending second degree parabola followed by two straight lines. The characteristics of this model are as follows:

1. Region AB: $\epsilon_l \leq .002 K$

$$f_c = K f_c' [2 (\frac{\epsilon_l}{.002 K}) - (\frac{\epsilon_l}{.002 K})^2] \quad (2.16)$$

$$\text{where } K = 1 + \frac{\rho f_y}{f_c'}$$

2. Region BC: $.002 K \leq \epsilon_l \leq \epsilon_{20K}$

$$f_c = K f_c' [1 - Z (\epsilon_l - .002 K)] \quad (2.17)$$

where Z, the slope of BC, is given by:

Fig. 2.10 Modified Kent and Park Model [22]

$$Z = \frac{0.5}{\frac{3 + .002 f_c'}{f_c' - 1000} + \frac{3}{4} \rho \sqrt{\frac{b''}{s}} - .002 K} \quad (2.18)$$

where ρ = volumetric ratio of transverse reinforcement to concrete core, and

b'' = width of the confined core measured from the outside of the lateral reinforcement.

3. Region CD: $\epsilon_\ell \geq \epsilon_{20K}$

$$f_c = .2 K f_c' \quad (2.19)$$

This model assumes that rectilinear ties cause an increase in strength equal to ρf_y . However, the spacing of the lateral reinforcement was not taken into consideration. Moreover, the strain at peak stress, as given by this model, is not as high as has actually been observed in recent tests [27].

2.4.8 Fafitis and Shah

Based on tests on 3 in. x 6 in. cylinders laterally confined with circular spiral reinforcement, Fafitis and Shah [12] proposed a model, shown in Fig. 2.11, to predict the stress-strain relationship of spirally reinforced concrete columns. The ascending branch of the curve is expressed by the following mathematical expression:

$$f_c = f_1 \left[1 - \left(1 - \frac{\epsilon_\ell}{\epsilon_1} \right)^A \right] \quad (2.20)$$

The descending branch of the curve is an exponentially decreasing curve defined by:

$$f_c = f_1 \exp [-D(\epsilon_\ell - \epsilon_1)^{1.15}] \quad (2.21)$$

Fig. 2.11 Stress-Strain Model for Concrete Confined by Circular Spiral as Proposed by Fafitis and Shah [12]

where f_1 and ϵ_1 are the stress and the corresponding strain at the peak of the stress-strain curve. The parameter A is given by:

$$A = \frac{E_c \epsilon_1}{f_1} \quad (2.22)$$

where E_c = modulus of elasticity of plain concrete in psi

$$= 33 w_c^{1.5} \sqrt{f_c'}, \text{ as defined by the ACI code}$$

where w_c is the unit weight of concrete in lb/ft³.

The parameter D was calculated from the experimental results as:

$$D = 0.17 f_c' \exp (-0.01 f_r) \quad (2.23)$$

where f_r represents the average confining pressure in the concrete at peak stress which, from equilibrium condition, can be derived as $\frac{\rho f_y}{2}$. The peak stress f_1 and the corresponding peak strain ϵ_1 were also calculated from the experimental results as follows:

$$f_1 = f_c' + (1.15 + \frac{3048}{f_c'}) f_r \quad (2.24)$$

$$\epsilon_1 = 1.027 \times 10^{-7} f_c' + 0.0296 \frac{f_r}{f_c'} + 0.00195 \quad (2.25)$$

To apply this model to columns with square ties, Fafitis and Shah suggested that a square core may be considered equivalent to a circular column having an equivalent diameter that is equal to the length of the side of the confined square core. The application of this model to rectangular or square columns is only approximate and is primarily intended for columns with heavy lateral reinforcement. However, the exponentially decreasing descending curve in this model seems to realistically represent the change in curvature of the descending branch.

Chapter 3

EXPERIMENTAL INVESTIGATION**3.1 General**

An experimental investigation was carried out to study the confinement effectiveness of lateral and longitudinal reinforcement in prestressed concrete columns. The lateral reinforcement studied during this investigation included single ties, continuous square spiral and the welded wire mesh. The test columns were subjected to axial compression under controlled loading to determine the stress-strain behavior up to a strain level as high as 0.04.

3.2 Test Specimens

A total of twenty-three specimens, which had a cross section of 5.25 in. x 5.25 in. and were 26 inches long, were tested under concentric loading. With these dimensions, they can be regarded as one-third-scale model of short prestressed concrete columns. They were divided into test series A, B, C and D. Specimens in series A did not have any lateral reinforcement. In series B, C and D, the lateral reinforcement consisted of single ties, continuous square spiral, and welded wire mesh, respectively. The details of the specimens tested during this investigation are given in Table 3.1. The letter in the specimen designation, e.g. B32, refers to the type of lateral reinforcement. The first number represents the spacing and the second one represents the particular specimen number within a group of similar specimens.

Within each series, three different spacings of lateral

Table 3.1 Details of Test Specimens

Series	Type of Lateral Reinforcement	Specimens	Spacing of Lateral Reinforcement (in)
A	None	A11, A12	--
B	Single Ties	B11, B12, B13	4
		B21, B22, B23	3
		B31, B32, B33	2
C	Continuous Square Spiral	C11, C12	4
		C21, C22	3
		C31, C32	2
D	Welded Wire Mesh	D11, D12	4
		D21, D22	3
		D31, D32	2

reinforcement were studied. These were 4, 3 and 2 inches with a corresponding volumetric ratio of 1.09, 1.46 and 2.18 percent of the concrete core respectively. The core, which is defined as the area enclosed by the center line of the lateral reinforcement, was kept constant and had a cross section of 4.25 in. x 4.25 in. for all the specimens. The ratio of the core to gross area was 65.5% which represents typical prototype proportions. To ensure consistency of test results, at least two identical specimens were tested within each category. Each specimen was prestressed to an average effective stress over the gross area of 478 psi using high strength smooth wires. Since smooth wires are not very effective in transferring prestress, external anchorages were added to ensure uniform prestressing force along the length of the specimen. To prevent any premature failure outside the instrumentation zone, the spacing of the lateral reinforcement was reduced to 1 inch in the top and bottom end regions of the specimens. As a further precaution against failure in the end regions, two steel collars were attached to the top and bottom ends of the specimens. The cross sectional details and the dimensions of a typical specimen are shown in Fig. 3.1.

3.3 Prestressing and Fabrication

All specimens were cast in a horizontal position in the prestressing bed shown in Fig. 3.2. To ensure uniformity of the concrete mix three specimens were cast in series simultaneously. After assembling and placing the reinforcement in the prestressing bed, the wires were pretensioned using a hydraulic jack. The prestressing force was controlled by monitoring the strain gages attached to the

Fig. 3.1 Dimensions of a Typical Test Specimen

Fig. 3.2 Three Specimens in Series in the Prestressing Bed

prestressing wires and also by a calibrated pressure dial gage connected to the hydraulic jack. During prestressing, the elongation of the prestressing wires was also measured.

At the initial jacking, the prestressing wires were stressed to about 150 ksi, which is equivalent to $0.61 f_{pu}$ and $0.70 f_{py}$ compared with the code allowable values of $0.85 f_{pu}$ and $0.94 f_{py}$ respectively, where f_{pu} is the tensile strength of the prestressing wires and f_{py} is the yield strength. After releasing the jack, the stress level in the wires dropped to about 135 ksi. The prestressing wires were carefully placed relative to the center line of the specimens to avoid any eccentricity of the prestressing force. Figure 3.3 shows the details of a specimen with continuous square spiral before casting.

After assembling the formwork, concrete was placed and several representative cylinders were cast. The specimens and cylinders were then covered with plastic sheets. Two to three days later, the specimens were stripped off the formwork and the cylinders removed from their molds. The prestressing force was then transferred to the concrete by cutting the wires. During this time, the strain gages attached to the prestressing reinforcement were monitored to record the prestress loss at transfer. The specimens were then cured in the curing room for two weeks before placing them in the laboratory environment.

The average loss of prestress at transfer, mainly due to elastic shortening and anchorage slip, was 17% of the initial prestressing force and caused the stress level to drop to an average value of 111.5 ksi, which is within the maximum allowable values specified by the

Fig. 3.3 Details of a Specimen with Continuous Square Spiral before Casting

ACI code of $0.74 f_{pu}$ and $0.82 f_{py}$. The calculations of the prestress losses at transfer are given in Appendix B. The calculated losses were found to have a reasonably good agreement with the measured losses. The effective prestress, f_{pe} , at transfer for each of the specimens is shown in Table 3.2.

3.4 Material Properties

3.4.1 Concrete

The concrete used in all the specimens was made with well graded crushed stone with three quarter inch maximum size aggregate and a well graded river sand. To facilitate proper placement and compaction of concrete, the water to cement ratio was selected to result in a slump of 3 to 4 inches. Details of the concrete mix design are provided in Appendix A. Six control cylinders (6 in. x 12 in.) were cast during each casting operation. Three of them were tested at transfer of prestress and the remaining three at the time of testing of the specimens. The compressive strength of concrete, f'_c , for each specimen at testing is shown in Table 3.2. The average concrete compressive strength was 5600 psi.

3.4.2 Reinforcement

The prestressing reinforcement consisted of 0.192 inch diameter smooth wires conforming to ASTM A421, Type WA. It had an ultimate tensile strength, f_{pu} , of 245 ksi and a yield strength, f_{py} , of 215 ksi. Figure 3.4 shows a typical stress-strain curve for the prestressing reinforcement.

In series B, the lateral reinforcement which consisted of single

Table 3.2 Concrete Strength at Testing and Effective Prestress at Transfer

Series	Specimen	f'_c (psi)	f_{pe}^* (psi)
A Plain Concrete	A11	5620	570
	A12	5620	570
B Single Ties	B11	5630	525
	B12	5630	525
	B13	5630	525
	B21	5600	468
	B22	5600	468
	B23	5600	468
	B31	5550	443
	B32	5550	443
	B33	5550	443
C Continuous Square Spiral	C11	5790	500
	C12	5790	466
	C21	5410	487
	C22	5410	335
	C31	5500	525
	C32	5500	471
D Welded Wire Mesh	D11	5180	434
	D12	5180	468
	D21	5140	401
	D22	6100	486
	D31	6100	448
	D32	6100	523

*Measured effective prestress

Fig. 3.4 Typical Stress-Strain Curve for the Prestressing and Lateral Reinforcement

ties, was made from # 2 plain cold rolled reinforcing steel which had an ultimate tensile strength, f_u , of 100 ksi and a yield strength of 90 ksi. These bars had an actual diameter of 0.243 inches which was used in all the calculations. Their stress-strain relationship is typically as shown in Fig. 3.4. Premature anchorage failure of ties was prevented by a 135 degree bend extending ten bar diameter into the confined core.

The lateral reinforcement in series C consisted of continuous square spiral made from #2 reinforcing steel similar to single ties.

Welded wire fabric was used as lateral reinforcement in series D. The wires in the transverse direction were smooth W4.5 wires, similar to the ones used in series B and C, and the longitudinal wires were smooth W1.1 wires. They were spot welded at the intersections of the mesh. Table 3.3 shows the welded wire fabric designations of the specimens in series D.

Table 3.3 Welded Wire Mesh Designation in Series D

Specimens	Designation
D11, D12	4.25 x 4 - W1.1 x W4.5
D21, D22	4.25 x 3 - W1.1 x W4.5
D31, D32	4.25 x 2 - W1.1 x W4.5

The designation of 4.25 x 4 - W1.1 x W4.5 means size W1.1 longitudinal wires spaced at 4.25 inches and size W4.5 transverse wires spaced at 4 inches. The number following the W represents the cross sectional area of the wire in square inches multiplied by 100.

The mesh was wrapped around the prestressing reinforcement such that the transverse wires were on the inside of the specimens. The

outside end of the mesh overlapped a distance of 3 inches while the internal end of the mesh had a 135 degree bend anchored in the core as shown in Fig. 3.5.

3.5 Test Setup and Instrumentation

Specimens were loaded in an upright position in a 400 kip Tinius-Olsen machine with a spherical head. They were tested in a strain control mode using a deflectometer to ensure accurate measurement of the load-strain curves. To ensure the uniform distribution of the applied load, thin layers of plaster of Paris paste were used at the top and the bottom surfaces of the specimens. The symmetry of the specimens with respect to the testing machine was also checked using a plumb bob. A general view of the test set-up is shown in Fig. 3.6.

Strain in the reinforcement was measured through a set of electrical resistance strain gages which were attached both to the prestressing wires and the lateral reinforcement. The axial deformation in the test region of the specimens was measured with two LVDTs (Linear Variable Differential Transducer) placed on the opposite faces of the specimens. They were attached to two 5/16 threaded rods embedded in the concrete at a gage length of 10 in. This arrangement kept them operational beyond the strain level corresponding to the spalling of the cover. Before testing, both LVDTs were calibrated and checked for linearity. In addition, two external strain gages were bonded to the concrete surface at mid-height of each specimen on the remaining two faces. The main purpose of the external gages was to assist in checking the symmetry of the specimen with respect to the testing

Fig. 3.5 Anchorage of Welded Wire Mesh Used as Lateral Reinforcement in Series D

Fig. 3.6 Test Setup

machine. However, as soon as the concrete cover began to spall, these strain gages were damaged and their readings were not reliable any more.

3.6 Testing Procedure

Before testing, specimens were painted with a thin white latex paint to make crack observation easier and to improve the photographing of the cracks. Then, the external strain gages and the LVDT's were installed. After an initial alignment of the specimen in the testing machine, a load of 30-40 kips was applied. The readings from the external strain gages, the LVDTs and the strain gages attached to the prestressing reinforcement were checked for uniform load distribution. If an eccentricity was noted, the specimen was unloaded and its position was readjusted as necessary to get a uniform strain on all four sides. During testing, the load was applied monotonically at a strain rate of 160μ strain/min. The strain gages and transducers were read at load increments of 10 kips up to the peak load and after that at about 5 kips loading intervals. The data was recorded on a magnetic cassette and printed simultaneously. Each of the two LVDTs was hooked to an X-Y plotter to obtain continuous load-strain plots during the test. Cracks in the cover were marked as they appeared and photos were taken at appropriate loading intervals. Each test took approximately two to three hours from the time of initial load application to the termination of the test. The test was terminated either when the specimen showed signs of sudden collapse or when the strain gages were damaged such that no further readings could be taken.

Chapter 4

EXPERIMENTAL RESULTS**4.1 General**

During the tests, the behavior of the specimens was observed in terms of axial deformation, crack propagation and failure mode, and strain in the lateral confining reinforcement. Each of these behavior aspects for the four series of specimens is presented in this chapter.

4.2 General Behavior

(a) Specimens without lateral reinforcement: Two of the specimens tested did not have any lateral reinforcement. However, the amount of prestressing reinforcement and the level of prestress were identical to the remaining specimens. Under axial loading, both of these specimens exhibited a brittle type of failure in which a diagonal crack opened suddenly and the specimen was unable to sustain any further load.

(b) Specimens with ties: Specimens with single ties as lateral reinforcement did not experience any visible cracks up until the peak load when cracks parallel to the longitudinal axis of the specimen began to develop. With the increase in axial strain, these cracks became wider and led to the spalling of the concrete cover. As the strain increased further, more concrete was ejected from the specimen core. During the test, separation of the cover from the core could be detected by lightly tapping the surface, where a hollow sound indicated a region of separation. In all specimens of this series, the

spalling of the concrete cover was completed at a longitudinal strain between 0.0045 and 0.005. The buckling of the prestressing reinforcement occurred thereafter and, as expected, depended on the tie spacing. In specimens which had the smallest tie spacing of two inches, the prestressing steel buckled at an average longitudinal strain of about 0.008. In specimens with tie spacing of three inches and four inches, buckling of the prestressing bars occurred at an average longitudinal strain of about 0.007 and 0.006, respectively.

(c) Specimens with continuous square spiral: As in specimens with single ties, vertical cracks on the surface of the specimens with continuous square spiral did not appear until the maximum load was reached. With the increase in axial strain, these cracks became wider, and several concrete pieces were ejected out from the concrete between the lateral reinforcement until the cover was completely lost, which occurred at a longitudinal strain between 0.004 and 0.005. Furthermore, buckling of the prestressing reinforcement in the specimens with four inch spacing was observed at a longitudinal strain close to 0.006. In specimens with three and two inch spacing, the buckling of the prestressing bars occurred at a longitudinal strain of about 0.007 and 0.008, respectively.

(d) Specimens with welded wire mesh: In specimens with welded wire mesh as lateral reinforcement, fine cracks appeared on the cover soon after the peak load was reached. These cracks started spreading and widening rather rapidly. Several longitudinal cracks at locations along the vertical wires of the mesh were also observed suggesting that these vertical wires participated in resisting the lateral strain.

As the axial strain increased, some concrete pieces were ejected out from the core while other pieces were restrained by the vertical wires of the mesh. These restrained concrete pieces exerted pressure on the vertical wires causing them to bend outwards. They also resulted in some of the welds, which were still embedded in concrete, to snap with an audible noise. As the outward concrete pressure increased, the horizontal wires at the free end of the mesh started to pull out but were restrained by the vertical wires. The lateral confinement was totally lost when the weld failure caused the horizontal wires to separate from the vertical wires of the mesh.

4.3 Axial Load vs. Strain

During each test, axial strain was measured by two LVDTs. These strains represented an average value of the axial strain over a gage length of ten inches and were continuously plotted on X-Y recorders up to a strain value of about 3 to 4%. These plots were then averaged along the strain axis to obtain the load versus average longitudinal strain relationship. For the purpose of comparing different specimens, the load in these plots was normalized by the product of the gross area and the concrete compressive strength at testing.

The ascending branches of the load-strain curves, in all specimens were essentially linear up to about 50% of the maximum load. Beyond that, the strain increased more rapidly than the load. After the peak load was reached, the slope of the descending portion of the curve was observed to be very much dependent upon the amount of lateral reinforcement. For smaller amounts of lateral reinforcement, the descending part of the load-strain curve was quite steep in the

beginning becoming less steep as the strain increased. At about 15% of the maximum load, the curve became nearly flat. For larger amounts of lateral reinforcement, the rate of change of the descending part was gradual, becoming flat at about 30% of the maximum load.

(a) Specimens without lateral reinforcement: The load vs. average longitudinal strain curves of the specimens without lateral reinforcement (series A) are shown in Fig. 4.1. The maximum load occurred at an average longitudinal strain of 0.0022 and 0.002 for specimens A11 and A12, respectively. The maximum load, the prestressing force corresponding to that load and the ratio of the concrete stress at the maximum load to the corresponding cylinder strength are given in Table 4.1. To calculate the concrete stress at the maximum load, the peak load was added to the corresponding prestressing force, and the result was then divided by the gross area. In specimen A12, the maximum concrete stress was almost equal to the corresponding cylinder strength. In specimen A11, this value was about 7% higher. On the average, the maximum concrete stress in specimens without lateral reinforcement was 3% higher than the corresponding cylinder strength.

The ACI code specified factor, 0.85, which represents the ratio between the concrete stress at the maximum load and the cylinder strength, does not seem to hold in this case. The possible reasons for this behavior are the size of the specimens being close to the size of the cylinders and the horizontal casting position of the specimens.

(b) Specimens with ties: Figure 4.2 shows the load vs. average longitudinal strain curves for two of the specimens with four inch

Fig. 4.1 Load vs. Strain Curves for Specimens without Lateral Reinforcement

Table 4.1 Maximum Load and Ratio of Concrete Stress at the Maximum Load to Concrete Strength in Specimens without Lateral Reinforcement

Specimen	f_c' at Testing (psi)	P_{max} (kips)	Prestressing Force at the Maximum Load (kips)	Ratio of Concrete Stress at the Maximum Load to Concrete Strength
A11	5620	157.8	7.9	1.07
A12	5620	144.0	7.9	.99

spacing. During the testing of the third specimen, B13, one of the LVDTs malfunctioned and it was not possible to record the load vs. strain curve. The peak load, however, was recorded. The load vs. average longitudinal strain curves of the remaining specimens in this series are shown in Figs. 4.3 and 4.4. For most of the specimens in this series, the maximum load occurred at an average longitudinal strain between 0.0021 and 0.0024. The maximum load, the prestressing force corresponding to that load, the ratio of the concrete stress at the maximum load to the corresponding cylinder strength, and the failure mode are shown in Table 4.2.

(c) Specimens with continuous square spiral: The load vs. average longitudinal strain curves of these specimens are shown in Figs. 4.5, 4.6 and 4.7. The maximum load, the ratio of the concrete stress at the maximum load to the corresponding cylinder strength, and the failure mode of each specimen are presented in Table 4.3. As noticed, the concrete stress at the maximum load did not reach the corresponding cylinder strength. The peak load was reached at an average longitudinal strain between 0.0021 and 0.0025.

(d) Specimens with welded wire mesh: Experimental results of these specimens are presented in Table 4.4 and the load vs. average longitudinal strain curves are shown in Figs. 4.8, 4.9 and 4.10. These specimens failed by anchorage failure causing the load to drop rather quickly.

4.4 Stress in the Lateral Reinforcement

(a) Specimens with ties: Strain gages attached to the lateral reinforcement indicated that the yielding of ties did not occur at

Fig. 4.2 Load vs. Strain Curves for Specimens Laterally Reinforced with Ties at 4 in. Spacing

Fig. 4.3 Load vs. Strain Curves for Specimens Laterally Reinforced with Ties at 3 in. Spacing

Fig. 4.4 Load vs. Strain Curves for Specimens Laterally Reinforced with Ties at 2 in. Spacing

Table 4.2 Maximum Load, Ratio of Concrete Stress at the Maximum Load to Concrete Strength and Mode of Failure in Specimens with Single Ties

Specimen	f_c' at Testing (psi)	P_{max} (kips)	Prestressing Force at the Maximum Load (kips)	Ratio of Concrete Stress at the Maximum Load to Concrete Strength	Mode of Failure
B11	5630	139	6.7	0.94	1
B12	5630	139	6.7	0.94	1
B13	5630	152	6.7	0.99	1
B21	5600	152.2	5.1	1.02	1
B22	5600	151.2	5.1	1.02	1
B23	5600	144.3	5.1	0.97	2
B31	5500	155.1	4.4	1.05	2
B32	5500	158.2	4.4	1.07	1
B33	5500	144.5	4.4	0.98	1

- (1) diagonal shear plane
(2) shear cone pattern

Fig. 4.5 Load vs. Strain Curves for Specimens Laterally Reinforced with Square Spiral at 4 in. Spacing

Fig. 4.6 Load vs. Strain Curves for Specimens Laterally Reinforced with Square Spiral at 3 in. Spacing

Fig. 4.7 Load vs. Strain Curves for Specimens Laterally Reinforced with Continuous Square Spiral at 2 in. Spacing

Table 4.3 Maximum Load, Ratio of Concrete Stress at the Maximum Load to Concrete Strength and Mode of Failure in Specimens with Continuous Square Spiral

Specimen	f_c' at Testing (psi)	P_{max} (kips)	Prestressing Force at the Maximum Load (kips)	Ratio of Concrete Stress at the Maximum Load to Concrete Strength	Mode of Failure
C11	5790	146.2	6.0	0.96	2
C12	5790	148.6	5.1	0.97	1
C21	5410	134.6	5.6	0.95	1
C22	5410	143.8	1.5	0.98	1
C31	5500	135.8	6.7	0.94	1
C32	5500	132	5.2	0.91	1

- (1) diagonal shear plane
(2) shear cone pattern

Table 4.4 Maximum Load and Ratio of Concrete Stress at the Maximum Load to Concrete Strength in Specimens with Welded Wire Mesh

Specimen	f_c' at Testing (psi)	P_{max} (kips)	Prestressing Force at Maximum Load (kips)	Ratio of Concrete Stress at the Maximum Load to Concrete Strength
D11	5180	126.8	4.2	0.92
D12	5180	121.1	5.1	0.88
D21	5140	120.0	3.3	0.88
D22	6100	162.7	5.6	1.00
D31	6100	170.6	4.6	1.05
D32	6100	163.2	6.6	1.01

Fig. 4.8 Load vs. Strain Curves for Specimens Laterally Reinforced with Welded Wire Mesh 4.25 x 4 - W1.1 x W4.5

Fig. 4.9 Load vs. Strain Curves for Specimens Laterally Reinforced with Welded Wire Mesh 4.25 x 3 - W1.1 x W4.5

Fig. 4.10 Load vs. Strain Curves for Specimens Laterally Reinforced with Welded Wire Mesh 4.25 x 2 - W1.1 x W4.5

maximum load. The recorded stresses in the ties were between 30 and 60 ksi, while the tie yield stress was 90 ksi. Figure 4.11 shows the tie strain versus the longitudinal strain for three strain gages in specimen B21. As noticed, the strain in the ties was very small at early stage of loading. After the cover was lost, the strain increased drastically.

(b) Specimens with continuous square spiral: As in the case of specimens with single ties, the yielding of the lateral reinforcement did not coincide with the peak load. Figure 4.12 shows the strain in the lateral reinforcement versus the longitudinal strain for the strain gages in specimen C21. As observed, strains in the lateral reinforcement were very small at the beginning of the test. At the maximum load, which occurred at a longitudinal strain between 0.0021 and 0.0025, the lateral reinforcement experienced stresses between 20 and 40 ksi. At a longitudinal strain of 0.004 to 0.005, which corresponded to the complete spalling of the cover, yielding was observed in few segments of the continuous square spiral.

(c) Specimens with welded wire mesh: Similar to single ties and continuous square spiral, the stress level in the horizontal wires of the mesh at the maximum load was between 30 and 40 ksi. After the cover spalled off, the stresses were observed to be between 45 and 60 ksi.

4.5 Mode of Failure

(a) Specimens without lateral reinforcement: The specimens without lateral reinforcement failed in a relatively brittle mode of failure. As shown in Fig. 4.13, the failure resulted from a diagonal crack

Fig. 4.11 Tie Strain vs. Longitudinal Strain for the Strain Gages in Specimen B21

Fig. 4.12 Lateral Reinforcement Strain vs. Longitudinal Strain for the Strain Gages in Specimen C21

Fig. 4.13 Specimen without Lateral Reinforcement after Failure

which opened suddenly at a 24 to 28 deg. angle, with respect to the longitudinal axis of the specimen.

(b) Specimens with ties: Specimens with single ties failed in one of the following modes: (1) by formation of a diagonal shear plane at an angle of 35 to 40 deg. with the longitudinal axis, as shown in Fig. 4.14; or (2) by formation of a shear cone pattern, as shown in Fig. 4.15. The difference between these two failure modes can be attributed to the unsymmetrical loss of the cover. A uniform spalling of the cover is expected to result in a shear cone type of failure but, in practical terms, it is a rare occurrence.

When the failure occurred by formation of a diagonal shear plane, the length of the failure zone was about 7 to 8 inches. On the other hand, when the failure was in the form of a shear cone pattern, the length of the failure zone was about 6 to 6.5 inches. None of these specimens showed anchorage failure of ties, thus indicating that the ten bar diameter extension performed satisfactorily.

(c) Specimens with continuous square spiral: Specimens laterally reinforced with continuous square spiral also failed in one of the two modes described above for the specimens with ties. As in the case of single ties, the continuous spiral did not break during the testing of any of these specimens.

(d) Specimens with welded wire mesh: Specimens in this series failed primarily due to the anchorage failure of the welded wire mesh. Unlike the ties, it is not practical to anchor both ends of the mesh in the column core. Once the cover was lost, the free end of the mesh opened as shown in Fig. 4.16. The length of the failure zone in this case was about 8 to 9 inches.

Fig. 4.14 Diagonal Shear Plane Formation

Fig. 4.15 Shear Cone Type of Failure

Fig. 4.16 Opening of the Lateral Mesh in Specimen D22

Chapter 5

DISCUSSION OF TEST RESULTS

5.1 General

In this chapter, the effect of confining reinforcement on the strength and ductility of prestressed concrete columns is discussed. In particular, the effect of different types and amounts of lateral reinforcement on the strength and ductility of prestressed concrete columns is examined. Finally, the observed load-strain behavior is compared with that predicted using previously proposed models for reinforced concrete columns.

5.2 Effect of Confining Steel on Concrete Stress at Maximum Load

For different amounts and types of confining reinforcement, the concrete stress in the specimens at maximum load, as presented in Tables 4.2 through 4.4, varied between $0.88 f_c'$ and $1.07 f_c'$ with the average being $0.97 f_c'$ which indicates that the lateral reinforcement did not have a noticeable effect on concrete stress at the peak load. Although data on prestressed concrete columns is somewhat limited, similar observations have been made by Carinci and Halvorsen [9]. Previous studies on reinforced concrete columns, with single ties and continuous square spiral as lateral reinforcement [11], also confirm this observation.

As shown in Fig. 4.11 and 4.12, the strain in the lateral reinforcement is very small up to an axial strain of about 0.0022 which corresponds to the peak load. This indicates that the lateral reinforcement is not active in confining the core until after the

maximum load is reached. There are two possible reasons for this behavior. Firstly, the concrete during curing tends to shrink due to the loss of water. Shrinkage strains, calculated by the method proposed in reference [4] and presented in Appendix C, have been found to be equal to about 76μ strains. Therefore, significant longitudinal strain should develop for the transverse Poisson's strain to overcome this initial shrinkage strain and for the concrete to start bearing against the lateral reinforcement.

Secondly, for specimens with rectilinear ties, each leg of the lateral reinforcement is analogous to a string in that it can take stresses most effectively in tension only. Therefore, in order for the reinforcement to be effective in confining the concrete, it must first undergo bending deformation in its own plane, as shown in Fig. 5.1. Such deformation does not seem to take place until the cover begins to crack which, for the specimens tested in this program, occurred at or just after the maximum load was reached.

It was also observed that the lateral reinforcement did not yield when the specimens reached their peak load. A similar behavior was also observed in reinforced concrete specimens tested by Burdette and Hilsdorf [8].

5.3 Effect of Lateral Reinforcement on Strength and Ductility

The load-strain curves for series B specimens, which are laterally reinforced with single ties, and for series C specimens, which are laterally reinforced with continuous square spiral, are presented in Fig. 5.2 and Fig. 5.3 respectively. In each case, the amount of lateral reinforcement does not appear to affect the slope of the

Fig. 5.1 The In-Plane Deformation of the Lateral Reinforcement

Fig. 5.2 Comparison between Load vs. Strain Curves for Specimens Laterally Reinforced with Single Ties (Series B)

Fig. 5.3 Comparison between Load vs. Strain Curves for Specimens Laterally Reinforced with Continuous Square Spiral (Series C)

ascending branches of the load-strain curves. Furthermore, the gain in strength at maximum load for the specimens having single ties or continuous square spiral as lateral reinforcement is very small. The most significant effect of the lateral reinforcement is on the descending branch, the slope of which gets less steep as the amount of lateral reinforcement is increased. If we define ductility of columns as the ability to sustain axial load at higher axial deformation, then increasing the amount of lateral reinforcement clearly improves the ductility. The specimens are able to carry higher loads at strains beyond the peak load when the amount of lateral reinforcement is increased. A better understanding of the effect of lateral reinforcement can be obtained by analyzing the behavior of the cover and the core which is discussed in sections 6.2 and 6.3.

5.4 Effect of Type of Lateral Reinforcement

The axial load-strain behavior of specimens with three different types of lateral reinforcement is shown in Fig. 5.4. The amount and spacing of lateral reinforcement in all three specimens are the same. The ascending branches of the load-strain curves for the three specimens are almost identical. However, comparing the post-peak behavior of the specimens, it is observed that the specimen with single ties was able to carry slightly more load at higher strains than the specimen with continuous square spiral. In the case of specimen reinforced with welded wire mesh, the strength was close to that of the specimen with single ties up to a longitudinal strain of about 0.006. Thereafter, the strength of the specimen with welded wire fabric dropped drastically, primarily due to the opening of the welded wire mesh.

Fig. 5.4 Load vs. Strain Curves for Three Specimens Laterally Reinforced with Three Types of Lateral Reinforcement

The relatively better behavior of the specimens with single ties compared to those with continuous square spiral may be attributed to the mechanism of confining forces which develop because of the Poisson's effect that causes concrete to exert lateral pressure on the transverse reinforcement. As shown in Fig. 5.5, the confining forces in the case of single ties act in one horizontal plane. When continuous square spiral is used as lateral reinforcement, the confining forces, as shown in Fig. 5.6, are horizontal but do not act in one plane.

The relative better performance of the single ties seems to contradict the conclusions drawn by Iyengar et al. [13] in which they found the continuous square spiral to be more effective in improving the strength than the single ties. This conclusion was reached by comparing their test results on concrete prisms reinforced with continuous square spiral to the results of Szulczynski and Sozen [31] on specimens with single ties.

5.5 Observed Behavior vs. Analytical Models

In order to compare the behavior of columns, observed during these tests, to the behavior predicted by some of the previous models developed from tests on reinforced concrete columns, the load-strain curves need to be modified to exclude the contribution of longitudinal reinforcement. This is required because the models predict the behavior in terms of the concrete load-strain curves. As such, the net load-strain curves are obtained by subtracting the appropriate prestressing reinforcement contribution from the experimental load-strain curve.

Fig. 5.5 Confining Forces in Case of Single Ties

Fig. 5.6 Confining Forces in Case of Continuous Square Spiral

5.5.1 Prestressing Reinforcement Load-Strain Curve

Under axial compression, a longitudinal steel bar in a concrete column tends to buckle and, in the absence of the cover, is restrained by the lateral reinforcement. The buckling load of such bar is calculated from Euler's theory as:

$$P_{cr} = \frac{\pi^2 EI}{(K S)^2} \quad (5.1)$$

where I = the moment of inertia of the steel bar,

S = the spacing between the lateral reinforcement, and

K = the effective length factor.

The longitudinal reinforcement, in general, has been considered to have hinged condition at the points of contact with the lateral reinforcement, and the value of K in the above equation has been taken equal to one. More realistically, however, the restraint provided by the lateral reinforcement has a finite stiffness which depends upon the axial stiffness of the lateral reinforcement. If this support condition as well as the continuity effect of the longitudinal reinforcement are taken into consideration, the value of K in equation 5.1 can be calculated more accurately.

As discussed in section 4.5, test results indicated that the inelastic deformation in the specimens generally occurred in a limited region in which the cover was completely destroyed and the longitudinal reinforcement became exposed as shown in Fig. 5.7. Outside this region, no damage was observed in the specimens. Table 5.1 shows the length of the exposed bar, within the failure region, in each corner of the specimens tested in series B and C, and the average value for each specimen. The average length of the exposed bar varies between 5.5

Fig. 5.7 Length of Exposed Bar

Table 5.1 Length of the Exposed Longitudinal Reinforcement

Specimen	Bar Location				Average (in.)
	N E	S E	S W	N W	
B11	6.5	7.0	6.0	7.0	6.6
B12	5.5	7.0	6.0	7.0	6.4
B13	6.5	7.0	6.0	7.0	6.6
B21	8.0	6.5	7.0	7.0	7.1
B22	6.0	6.0	6.5	7.0	6.4
B23	7.0	6.0	6.0	6.5	6.4
B31	5.0	6.0	6.5	5.5	5.8
B32	6.0	6.0	6.0	6.0	6.0
B33	6.5	6.0	5.5	6.0	6.0
C11	6.0	8.0	8.0	7.0	7.3
C12	5.0	6.0	5.0	6.0	5.5
C21	6.0	6.0	6.5	5.5	6.0
C22	6.0	9.0	8.0	6.0	7.3
C31	7.0	7.0	6.0	6.0	6.5
C32	7.5	5.0	6.0	6.0	6.1

in. and 7.3 in. with the mean being 6.39 in. The latter value is close to 1.5 times the core dimension (6.38 in.). Therefore, the length of the exposed longitudinal reinforcement can be considered as one and half times the core dimension.

In the undamaged part where the cover remains intact, the restraint provided by the lateral reinforcement to the longitudinal steel can be represented by a support having translation free and rotation fixed conditions. Therefore, to calculate the buckling load of a longitudinal bar, the lateral restraints can be more accurately modelled as shown in Fig. 5.8. The number of springs is calculated by dividing the proposed length of the exposed longitudinal reinforcement by the spacing. This results in two springs in the case of four and three inch spacing and three springs in the case of two inch spacing. The spring stiffness k_1 is calculated as shown in Fig. 5.9. Based on the observations made during these tests, the anchorage of single ties is assumed to be perfect.

Using the moment distribution method modified for axial load [17,18], the critical buckling load is calculated for cases shown in Fig. 5.10 (a) and (b). The critical load, for case (a), is found to be:

$$P_{cr} = \frac{1.507 \pi^2 E_t I}{S^2} \quad (5.2a)$$

$$= \frac{\pi^2 E_t I}{(0.82 S)^2} \quad (5.2b)$$

and for case (b), it is given by:

$$P_{cr} = \frac{2.046 \pi^2 E_t I}{S^2} \quad (5.3a)$$

Fig. 5.8 Suggested Model for the Calculation of the Buckling Load

$\equiv$

$$k = \frac{E A_0}{.5 \ell_0}$$

$$k_1 = \frac{2\sqrt{2} E A_0}{\ell_0}$$

Fig. 5.9 Equivalent Spring Stiffness

(a)

$$P_{cr} = \frac{1.507 \pi^2 E_t I}{S^2}$$

$$= \frac{\pi^2 E_t I}{(0.82 S)^2}$$

(b)

$$P_{cr} = \frac{2.046 \pi^2 E_t I}{S^2}$$

$$= \frac{\pi^2 E_t I}{(0.70 S)^2}$$

Fig. 5.10 Buckling Load for Case (a) Two Springs and Case (b) Three Springs

$$= \frac{\pi^2 E_t I}{(0.70 S)^2} \quad (5.3b)$$

where E_t is the tangent modulus of the prestressing reinforcement. The calculation details of equations 5.2 and 5.3 are shown in Appendix D. Table 5.2 shows the buckling load for different lateral reinforcement spacing calculated using the above mentioned procedure. The longitudinal strain corresponding to the calculated buckling load and the experimentally observed longitudinal strain at buckling are also given in Table 5.3. The calculated and measured strains agree very well. After the prestressing bar reaches its buckling load, the decrease in its load carrying capacity is assumed to be negligible.

During the period between transfer of prestress and testing, prestress losses were very small and can be considered negligible. Therefore, the effective prestress at testing is considered equal to that at transfer. Knowing the effective prestress at testing and the buckling load, the prestressing reinforcement load-strain curve can be calculated. A typical load-strain curve for the prestressing steel, calculated using the above procedure, is shown in Fig. 5.11.

5.5.2 Experimental vs. Analytical Load-Strain Curves

The observed load-strain behavior of prestressed columns is now compared with the load-strain behavior predicted by various analytical models. Figure 5.12 shows such comparison for a specimen laterally reinforced with single ties having three inch spacing. The vertical axis in this figure represents the load carrying capacity of the concrete in the core and the cover, normalized by the product of the gross area and the concrete compressive strength.

Table 5.2 Buckling Load of a Prestressing Steel Bar

Spacing (in.)	Buckling Load (kip)
4	1.83
3	3.25
2	5.07

Table 5.3 Calculated and Observed Longitudinal Strain at Buckling

Specimen	Calculated Strain	Observed Strain
B11	0.0062	0.006
B12	0.0062	0.006
B21	0.0075	0.007
B22	0.0075	0.007
B23	0.0075	0.007
B31	0.0089	0.008
B32	0.0089	0.008
B33	0.0089	0.008
C11	0.006	0.006
C12	0.0057	0.006
C21	0.0076	0.007
C22	0.0065	0.007
C31	0.0091	0.008
C32	0.0087	0.008

Fig. 5.11 Prestressing Reinforcement Load vs. Strain Curve
(Specimen B22)

Fig. 5.12 Comparison between Experimental and Analytical Net Load vs. Strain Curves

It is noticed that the ascending branch is well simulated by the different models. The peak load predicted by various models varies between +10 and -15% of the observed value. Sargin's equation, which crosses the strain axis, is obviously not very realistic. Compared to Park's model and Sheikh's model, the descending branch of the experimental curve typically has a valley which represents a rapid initial drop in the load carrying capacity. This valley may be attributed to the fact that prestressed concrete columns are lightly reinforced in the longitudinal direction and confinement is provided mainly by the lateral reinforcement. Furthermore, the application of the recently proposed model by Fafitis and Shah [12] leads to a considerable overestimation of the concrete carrying capacity in the post-peak loading zone.

As such, most of these models which have been developed from experimental tests on reinforced concrete columns having, in general, a relatively large amount of reinforcement in the lateral and longitudinal directions does not appear to represent the behavior of prestressed concrete columns. There is, therefore, a need for an analytical model which is based on tests of prestressed columns and thus represents the behavior of such columns more accurately.

Chapter 6

PROPOSED STRESS-STRAIN RELATIONSHIP FOR CONFINED CONCRETE**6.1 General**

Based on the test results, an analytical model is proposed to simulate the stress-strain behavior of confined concrete core in prestressed concrete columns. The model is developed on the basis of observed behavior of the concrete cover and the confined core which are briefly discussed in the following sections.

6.2 Behavior of Concrete Cover

During this study, vertical cracks in the cover were observed at a longitudinal strain between 0.0021 and 0.0026. Initiation of similar cracks in reinforced concrete columns [26,29,31] has been observed at a longitudinal strain between 0.0015 and 0.002. One possible reason for such difference in strain values is the horizontal casting position of the test specimens which results in better compaction of concrete in the cover and less likelihood of air and water concentration under the lateral reinforcement.

After the concrete cover cracks, its contribution to the load carrying capacity diminishes quickly and, at a certain point, it ceases. This point corresponds to the complete spalling of the cover. Experimental observations in this study indicated that cover had spalled off at a longitudinal strain between 0.004 and 0.005. This agrees well with previous suggestion [21] that cover can be assumed to have spalled at a strain value equal to that at which the stress dropped to 50% of the ultimate strength in the concrete cylinder. For the

concrete used in this program, this value was about 0.0045. Table 6.1 shows the average longitudinal strain at first cracking and complete spalling of the cover for specimens in series B and C.

6.3 Observed Stress-Strain Behavior of Core

The stress-strain behavior of concrete in the core is obtained by subtracting the load carried by the prestressing reinforcement from the total load and dividing the net load by the appropriate effective area of concrete. Initially, in region I of the stress-strain curve shown in Fig. 6.1, the entire cross section of the specimen is effective and the concrete stress is obtained by dividing the net load by the gross area. After the cover spalls off, the entire load is carried by the concrete core. At this stage which is represented by region III of the curve, the stress in the core is obtained by dividing the net load by the core area. Region II represents a transition of area from the gross section to the area of the confined core. It has been suggested in the past [32] that the cover, in reinforced concrete columns, can be assumed to decrease linearly between initial cracking and the complete spalling of the cover. The use of this assumption was found inappropriate as it resulted in stress-strain relation with two peaks which does not seem to be correct. In the absence of any other valid assumption, a transition curve was fitted between the two ends of region II. As shown in Fig. 6.2, the shape of the transition curve depends upon the relative values of ordinates at the two ends of the transition region. The stress-strain relationships of concrete in the core for specimens B21, B31, C11 and C31 obtained using the above procedure are shown in Figs. 6.3

Table 6.1 Strain at Initial Cracking and Complete Loss of Cover

Specimen	Strain at Initial Cracking	Strain at Complete Loss of Cover
B11	0.0023	0.0045 - 0.005
B12	0.0024	0.0045
B21	0.0022	0.0045
B22	0.0024	0.005
B23	0.0021	0.005
B31	0.0022	0.005
B32	0.0026	0.005
B33	0.0024	0.0045 - 0.005
C11	0.0021	0.004 - 0.0045
C12	0.0021	0.004 - 0.0045
C21	0.0024	0.0045
C22	0.0021	0.0045
C31	0.0026	0.0045 - 0.005
C32	0.0021	0.005

Fig. 6.1 Observed Stress vs. Strain Curve of the Concrete Core

Fig. 6.2 Possible Conditions for the Transition Curve in Region II

through 6.6. For the purpose of comparing these curves, the stress is normalized by the corresponding concrete compressive strength.

6.4 Analytical Model of Stress-Strain Behavior

The stress-strain curves of the concrete core obtained from the experimental results appear to have a common general shape. The ascending branch in each case resembles a parabolic curve, followed by a small region of relatively constant stress. Finally, a descending branch which is characterized by a rapid change in curvature and can be best approximated by an exponentially decreasing curve. Given these attributes of the stress-strain curve, a model consisting of an ascending parabola followed by a straight line and an exponentially decreasing curve was deemed appropriate.

The proposed model for the stress-strain behavior of the confined concrete core in prestressed concrete columns under axial compression is shown in Fig. 6.7. The ascending branch OA of the model consists of a second degree parabola with peak at point A ($\epsilon_1, K f'_c$), where ϵ_1 represents the minimum strain corresponding to the maximum stress in the core, f'_c the concrete strength, and K the maximum stress ratio representing the strength gain of the concrete core due to lateral confinement. This is followed by a horizontal line, AB, which represents the region where the change in stress is relatively small. The strain, ϵ_2 , at point B represents the maximum strain corresponding to the maximum stress in the core. The descending part is expressed as an exponentially decreasing curve which, compared to a linear representation, reproduces more realistically the change in curvature of the descending branch.

Fig. 6.3 Stress vs. Strain Curve for the Concrete Core in Specimen B21

Fig. 6.4 Stress vs. Strain Curve for the Concrete Core in Specimen B31

Fig. 6.5 Stress vs. Strain Curve for the Concrete Core in Specimen C11

Fig. 6.6 Stress vs. Strain Curve for the Concrete Core in Specimen C31

Fig. 6.7 Proposed Stress-Strain Relationship

In calculating the parameters K , ϵ_1 and ϵ_2 , required to define the proposed model, the concrete strength in the specimens is taken as f_c' . However, when this model is applied to a full scale column, f_c' should be substituted by $k_c f_c'$, where k_c is a coefficient which accounts for the difference between concrete strength in a column and that in a test cylinder. In the absence of test data on full scale prestressed columns cast horizontally, the value of k_c can be taken, conservatively, 0.85 as specified by the ACI code.

6.4.1 Ascending Branch

The ascending branch of the proposed stress-strain curve, modeled as a second degree parabola passing through the origin and the point A with coordinates $(\epsilon_1, K f_c')$, is defined by the following equation:

$$f_c = K f_c' \left[2 \left(\frac{\epsilon_\ell}{\epsilon_1} \right) - \left(\frac{\epsilon_\ell}{\epsilon_1} \right)^2 \right] \quad (6.1)$$

where f_c = concrete stress,

K = maximum stress ratio which represents the ratio of the concrete strength in the core to the strength of plain concrete,

ϵ_ℓ = longitudinal strain in the concrete, and

ϵ_1 = minimum strain corresponding to the maximum stress.

6.4.2 Determination of the Maximum Stress Ratio, K

The increase in strength in reinforced concrete columns, which is expressed by the maximum stress ratio, K , is due to a combination of lateral and longitudinal reinforcement. In prestressed concrete columns, the longitudinal reinforcement does not seem to contribute to the confinement of the concrete core, and the factor, K , thus repre-

sents the strength increase of the core due to the lateral reinforcement only. Table 6.2 shows different mathematical forms of K proposed by several investigators. The factor K is generally assumed to be directly proportional to ρ , the lateral reinforcement ratio, f_s' , the stress in the lateral reinforcement at maximum concrete stress, and inversely proportional to f_c' or $\sqrt{f_c'}$. Sargin [26] and Scott et al. [27] indicated that for a given volumetric ratio of lateral reinforcement, larger spacing of ties drastically reduced the confinement of the core. Therefore, the spacing of ties, S , which is generally expressed in the non-dimensional form of $\frac{S}{b}$, is also incorporated in the calculation of the maximum stress ratio. By examining various expressions for K given in Table 6.2, it is observed that the stress ratio factor, K , can be expressed in a generalized form by the following equation:

$$K = 1 + \alpha_1 \left(1 - \alpha_2 \frac{S}{b}\right) \frac{(\rho f_s')^m}{(f_c')^n} \quad (6.2)$$

where α_1 , α_2 , m and n are constants.

In the case of circular columns with continuous circular spiral as lateral reinforcement, Iyengar et al. [13] and Ahmad and Shah [1] suggested the following general expression for calculating K :

$$K = 1 + \alpha \left(1 - \frac{S}{d_c}\right) \frac{\rho f_y}{f_c'} \quad (6.3)$$

where the value of α ranges between 2.0 and 2.3, and d_c represents the core diameter measured from outside to outside of the lateral reinforcement.

In most of the confinement studies involving either rectilinear or circular lateral reinforcement, the parameters m and n , in equation

Table 6.2 Mathematical Expressions for the Maximum Stress Ratio

Investigator	K
Roy and Sozen	1.0
Somes	$1 + 1.025 \frac{\rho f'_s}{f'_c}$
Sargin	$1 + .0146(1 - .245 \frac{S}{b}) \frac{\rho f_y}{\sqrt{f'_c}}$
Vallenas et al.	$1 + .0091(1 - .245 \frac{S}{b}) (\rho + \frac{d}{d_l} \rho_l) \frac{f_y}{\sqrt{f'_c}}$
Sheikh and Uzumeri	$1 + (1 - \frac{4c^2}{5.5b^2})(1 - 0.5 \frac{S}{b})^2 2.73 \frac{\sqrt{\rho f_y}}{f'_c}$
Modified Kent and Park	$1 + \frac{\rho f_y}{f'_c}$
Fafitis and Shah*	$1 + (1.15 + \frac{3048}{f'_c}) \frac{\rho f_y}{2f'_c}$

*for circular spiral lateral reinforcement

6.2, have been selected as one. In this study also, the values of these parameters are chosen as one and the equation 6.2 becomes

$$K = 1 + \alpha_1 (1 - \alpha_2 \frac{S}{b}) \frac{\rho f_s}{f_c} \quad (6.4)$$

In almost all the specimens tested in this study, when the maximum concrete stress was reached, the stress in the lateral reinforcement was close to 60 ksi. Therefore, f_s in the expression for K can be replaced by f_y , with a maximum of 60 ksi.

The above equation was tested against the experimental results to determine the values of α_1 and α_2 . The following error criteria was used to determine the value of the constants in equation 6.4:

$$\text{Error 1} = \sum \text{Abs} \left(\frac{K_{\text{exp}} - K_p}{K_{\text{exp}}} \right) \quad (6.5)$$

$$\text{Error 2} = \sum \left(\frac{K_{\text{exp}} - K_p}{K_{\text{exp}}} \right) \quad (6.6)$$

where K_{exp} and K_p represent the experimental and the predicted maximum stress ratios, respectively. The first equation gives the summation over all the specimens of the absolute error, while the second one gives the total cumulative error of all the specimens.

In the case of single ties, the experimental results indicated that when the vertical spacing was close to the core dimension, the increase in strength was negligible; thus suggesting that, for this spacing of ties, K should assume a value of 1.0. This is possible only if, for $\frac{S}{b} = 1.0$, the factor $(1 - \alpha_2 \frac{S}{b})$ becomes zero which yields a value of $\alpha_2 = 1.0$.

After a large number of iterations to minimize the error, it was found that a value of $\alpha_1 = 1.5$ gave an average absolute error,

Error 1, of 2.9% per specimen and a mean error, Error 2, of 0.4% per specimen, which is quite an acceptable range of error. With these values of constants, the equation 6.4 becomes:

$$K = 1 + 1.5 \left(1 - \frac{S}{b}\right) \frac{\rho f_y}{f_c} \geq 1 \quad (6.7)$$

The same procedure was also used for continuous square spiral. For reasons similar to the case of single ties, the value of α_2 was also chosen as 1.0. It was found that, in this case, $\alpha_1 = 1.0$ gave an acceptable range of error for the predicted value of K. The average absolute error, Error 1, was 2.7% and the error per specimen varied between -0.5 and -5%. For continuous square spiral, the equation 6.4 then becomes:

$$K = 1 + \left(1 - \frac{S}{b}\right) \frac{\rho f_y}{f_c} \geq 1 \quad (6.8)$$

Table 6.3 shows a comparison between the experimental and predicted values of K as well as the ratio between them. As noticed, a good agreement exists between the predicted and the experimental values.

6.4.3 Minimum Strain ϵ_1 at Peak Stress

To calculate ϵ_1 , the minimum strain corresponding to the maximum concrete stress in the core, the equation proposed by Park et al. [22] was modified to the following form:

$$\epsilon_1 = 1.3 \epsilon_0 K \quad (6.9)$$

where ϵ_0 is the strain corresponding to the maximum stress in plain concrete which was found to be 0.0022. The values of ϵ_1 , for the different specimens, using the above equation are presented in Table 6.5

Table 6.3 Experimental and Predicted Maximum Stress Ratio

Specimen	K experimental	K predicted	$\frac{K \text{ predicted}}{K \text{ experimental}}$
B11	0.95*	1.01	1.01
B12	0.95*	1.01	1.01
B21	1.08	1.07	0.99
B22	1.03	1.07	1.04
B23	1.02	1.07	1.05
B31	1.23	1.19	0.97
B32	1.31	1.19	0.91
B33	1.14	1.19	1.04
C11	0.96*	1.003	1.003
C12	0.97*	1.003	1.003
C21	1.00	1.05	1.05
C22	1.03	1.05	1.02
C31	1.08	1.12	1.04
C32	1.08	1.12	1.04

*Rounded to 1

6.4.4 Maximum Strain ϵ_2 at Peak Stress

Several mathematical expressions have been proposed for calculating ϵ_2 , the maximum strain corresponding to the maximum stress, in reinforced concrete columns, some of which are presented in Table 6.4. All of these equations can be expressed in the following general form

$$\frac{\epsilon_2}{\epsilon_0} = \frac{\epsilon_1}{\epsilon_0} + \beta_1 \left(1 - \beta_2 \frac{S}{b}\right) \frac{\rho f_y}{\sqrt{f'_c}} \quad (6.10)$$

where β_1 and β_2 are constants. The strains ϵ_2 and ϵ_1 , in the above equation were divided by ϵ_0 to amplify the value of β_1 .

The experimental value of ϵ_2 was obtained from the observed stress-strain curve as follows. A point, with calculated coordinates $(\epsilon_1, 0)$, was first plotted on the experimental curve and a vertical line was drawn. At its intersection with the curve, a horizontal line was then drawn. The intersection of this horizontal line with the descending branch of the curve indicated the point with horizontal coordinate ϵ_2 .

As discussed previously, for a tie spacing equal to the core dimension, the confinement effect tended to be negligible and ϵ_2 will have the same value as ϵ_1 . Thus the value of β_2 in equation 6.10 ought to be 1.0. By using the error criteria described earlier, it was found that a value of $\beta_1 = 0.06$ gave an average absolute error, Error 1, of 14.1% per specimen and a mean error, Error 2, of 7.6% per specimen. Thus, the equation 6.10 becomes:

$$\frac{\epsilon_2}{\epsilon_0} = \frac{\epsilon_1}{\epsilon_0} + 0.06 \left(1 - \frac{S}{b}\right) \frac{\rho f_y}{\sqrt{f'_c}} \quad (6.11a)$$

or

Table 6.4 Mathematical Expressions for ϵ_2

Investigator	Proposed Expression
Sargin	$0.0024 + 0.0000374(1 - 0.734 \frac{S}{b}) \frac{\rho f_y}{\sqrt{f'_c}}$
Vallenas et al.	$0.0024 + 0.0005(1 - .734 \frac{S}{b}) \frac{\rho f_y}{\sqrt{f'_c}}$
Sheikh and Uzumeri	$\epsilon_o \{ 1 + \frac{0.81}{C} [1 - 5(\frac{S}{b})^2] \frac{\rho f_y}{\sqrt{f'_c}} \}$

$$\epsilon_2 = \epsilon_1 + 0.06 \epsilon_0 \left(1 - \frac{S}{b}\right) \frac{\rho f_y}{\sqrt{f'_c}} \quad (6.11b)$$

where f_y and f'_c are in psi units.

In the case of continuous square spiral, the above equation was also found to be valid. The mean error, Error 2, in this case was -6.3% per specimen. The values of ϵ_2 obtained from equation 6.11b are shown in Table 6.5.

6.4.5 Descending Branch

The test results indicate that the descending branch of the stress-strain curve initially has a small region where the stress drops rather rapidly followed by a region of quick change in curvature. An exponential equation, similar to the one suggested by Fafitis and Shah [12], was found to best represent this portion of the curve and is expressed as:

$$f_c = K f'_c \exp [-C_1(\epsilon_\ell - \epsilon_2)] \quad (6.12)$$

where C_1 is the parameter governing the shape of the descending branch of the curve. It was found that this parameter in Fafitis and Shah equation was not sensitive to the variation in the amount of lateral reinforcement. Therefore, it was modified to the following form:

$$C_1 = 0.17 f'_c \exp(-0.075 \sqrt{\rho f_y}) \quad (6.13)$$

The test results also show that even at axial strains as high as 3%, confined concrete can still carry a certain minimum load. Similar observations have been made in previous tests [22,32] on reinforced concrete columns. Therefore, a base limit on the descending

Table 6.5 Predicted Values for K , ϵ_1 and ϵ_2

Specimens	K	ϵ_1	ϵ_2
B11, B12	1.01	0.0029	0.0030
B21, B22, B23	1.07	0.0031	0.0035
B31, B32, B33	1.19	0.0034	0.0046
C11, C12	1.003	0.0029	0.0030
C21, C22	1.05	0.0030	0.0035
C31, C32	1.12	0.0032	0.0045

branch of the proposed stress-strain model was considered appropriate. Based on the observed stress-strain data, a base value of $0.2 K f_c'$ is proposed. The equation 6.12 then becomes:

$$f_c = K f_c' \exp [-C_1(\epsilon_l - \epsilon_2)] \geq 0.2 K f_c' \quad (6.14)$$

6.5 Comparison between Experimental and Proposed Load-Strain Curves

Experimental load-strain curves for specimens of series B are compared with the analytical load-strain curves in Figs. 6.8 through 6.15. For specimens of series C, a similar comparison is made in Figs. 6.16 through 6.21. In these figures, the total load represents the load carrying capacity of the core, the cover and the prestressing reinforcement. It is noted that the ascending branches of the experimental curves for all specimens are well reproduced by the analytical model. The maximum load carrying capacity is predicted within a maximum error of $\pm 5\%$. Similarly, the strain at maximum load is also reasonably well predicted for all specimens with the exception of B32.

The descending branches of specimens B11, B12, B21, B22, B31 and B32 which were reinforced laterally with single ties, are reasonably well reproduced by the proposed curve. The maximum error is within $\pm 15\%$. However, the predicted curves of specimens B23 and B33 overestimate the experimental load-strain curves.

In the case of specimens reinforced laterally with continuous square spiral, the experimental descending branches also show a good agreement with the behavior predicted by the proposed model.

Fig. 6.8 Comparison between Experimental and Analytical Load-Strain Curves for Specimen B11

ig. 6.9 Comparison between Experimental and Analytical Load-Strain Curves for Specimen B12

Fig. 6.10 Comparison between Experimental and Analytical Load-Strain Curves for Specimen B21

Fig. 6.11 Comparison between Experimental and Analytical Load-Strain Curves for Specimen B22

Fig. 6.12 Comparison between Experimental and Analytical Load-Strain Curves for Specimen B23

Fig. 6.13 Comparison between Experimental and Analytical Load-Strain Curves for Specimen B31

Fig. 6.14 Comparison between Experimental and Analytical load-Strain Curves for Specimen B32

Fig. 6.15 Comparison between Experimental and Analytical Load-Strain Curves for Specimen B33

Fig. 6.16 Comparison between Experimental and Analytical Load-Strain Curves for Specimen C11

Fig. 6.17 Comparison between Experimental and Analytical Load-Strain Curves for Specimen C12

Fig. 6.18 Comparison between Experimental and Analytical Load-Strain Curves for Specimen C21

Fig. 6.19 Comparison between Experimental and Analytical Load-Strain Curves for Specimen C22

Fig. 6.20 Comparison between Experimental and Analytical Load-Strain Curves for Specimen C31

Fig. 6.21 Comparison between Experimental and Analytical Load-Strain Curves for Specimen C32

Chapter 7

DESIGN IMPLICATIONS**7.1 General**

Lateral reinforcement in prestressed concrete columns performs several functions: (a) it provides confinement to concrete, thus increasing ductility, (b) it prevents buckling of longitudinal reinforcement, and (c) it increases the transverse shear strength. Lateral reinforcement also helps in controlling splitting stresses in the end regions of columns. However, a clear distinction should be made between the role of lateral reinforcement in columns designed for strength and in columns where ductility is an important design consideration.

7.2 Strength Requirements

The ACI building code 318-83[2] requires that, in prestressed concrete columns with an average effective prestress equal to or greater than 225 psi, the lateral reinforcement be made of at least #3 bars and have a spacing not to exceed 48 tie bar diameter or least dimension of the column cross-section. However, the code also permits the waiver of these requirements provided tests and structural analysis show adequate strength and feasibility of construction. The PCI recommendations [23] allow the elimination of column ties, provided the nominal capacity is reduced by 15%.

In the specimens tested during this study, the lateral reinforcement spacing was less than the maximum spacing allowed by the code. The test results showed that the increase in concrete strength due

to lateral reinforcement was very small and generally negligible. Moreover, the lateral reinforcement was not effective in confining the concrete until the maximum load was reached and the cover started to crack. Thus, the maximum load carrying capacity of the prestressed concrete columns is not affected by the amount and spacing of the lateral reinforcement.

The test results showed also that the maximum load in columns was reached at an axial strain of 0.0022 - 0.0024. If initial strain in the prestressing reinforcement is larger than this value, then the prestressing reinforcement will remain in tension until the maximum column capacity is reached. The function of lateral reinforcement in preventing buckling of the longitudinal reinforcement in these situations thus becomes redundant.

The function of lateral reinforcement in resisting shear is obvious and the design procedure is clearly specified by the code. Moreover, the use of confining reinforcement in the end regions of the prestressed columns to control splitting stresses is very much desirable. However, the prestress level in columns is generally low and the required end zone reinforcement will not be substantial.

7.3 Ductility Requirements

Prestressed concrete columns may also be required to have some ductility, as in seismic regions. Ductility is regarded as the ability of a column to undergo axial deformation without suffering significant loss of strength. Limiting the spacing of the lateral reinforcement indirectly assures a certain minimum ductility of columns. However, no attempt has been made to relate quantitatively the amount of lateral

confining reinforcement to the deformation capacity of columns.

In the absence of any definitive criteria to measure the ductility of prestressed concrete columns with rectilinear ties, a strain ductility factor is proposed which gives a measure of the deformation capacity while assuring at least eighty percent of the maximum strength. It is defined as:

$$SDF_{(80\%)} = \frac{\epsilon_{80} - \epsilon_{\max}}{\epsilon_{\max}} \quad (7.1)$$

where $\epsilon_{\max}$ is the strain corresponding to the maximum load on the load-strain curve and ϵ_{80} is the strain at 80% of the maximum load on the descending branch of the load-strain curve as shown in Fig. 7.1. $SDF_{(80\%)}$ represents a measure of the axial strain, expressed as a percentage of the column strain at peak load, that a column can sustain beyond the peak load while maintaining 80% of the load capacity.

$SDF_{(80\%)}$ for each of the test specimens computed from the experimental load-strain curves, the average value for each group of identical specimens and the $SDF_{(80\%)}$ calculated using the proposed model are shown in Table 7.1. The specimens in series B and C were designed to satisfy the requirements of the ACI code for prestressed concrete columns. In addition, the specimens with two inch spacing satisfied the requirements of appendix A of the code for regions of moderate seismic risk except for the spacing limitation of $8d_l$, where d_l is the diameter of the longitudinal reinforcement. This provision would have severely limited the spacing of the lateral reinforcement because of the small size of the prestressing reinforcement.

As shown in Table 7.1, $SDF_{(80\%)}$ for the specimens in series B with four inch spacing did not increase appreciably over that of the

Fig. 7.1. Strain at Eighty Percent of the Maximum Load

Table 7.1 Strain Ductility Factors at 80% of the Maximum Load

Specimen	Amount of Lateral Reinforcement (ρ)	Experimental SDF (80%)	Average	Calculated SDF (80%)
A11	----	26%	31%	----
A12	----	35%		
B11	1.09	36%	38%	33%
B12	1.09	40%		
B21	1.46	59%	53%	49%
B22	1.46	55%		
B23	1.46	45%		
B31	2.18	100%	83%	79%
B32	2.18	76%		
B33	2.18	74%		
C11	1.09	32%	32%	30%
C12	1.09	32%		
C21	1.46	33%	35%	45%
C22	1.46	37%		
C31	2.18	53%	80%	76%
C32	2.18	107%		

specimens without lateral reinforcement. However, the ductility increased more rapidly as the spacing was decreased to three and two inches. In series C, $SDF_{(80\%)}$ for specimens with four inch spacing was almost equal to that of the specimens without lateral reinforcement. The ductility of specimens with three inch spacing was surprisingly low, but those with two inch spacing showed appreciable increase in $SDF_{(80\%)}$.

If a column with a strain ductility factor of 75% or more can be considered acceptable in regions of moderate seismic risk, then the columns with two inch spacing tested in this study would be acceptable. The lateral reinforcement spacing of these columns is slightly above $10d_\ell$. This suggests that the spacing limitation of $8d_\ell$ in the ACI code, may be increased to $10d_\ell$ for prestressed concrete columns.

Chapter 8

SUMMARY AND CONCLUSIONS**8.1 Summary**

The primary objective of this research was to investigate the confinement effect of lateral reinforcement on the load-deformation behavior of prestressed concrete columns under axial compression. This study was divided into two parts: an experimental investigation in which twenty-three columns were tested and an analytical study which dealt with stress-strain modelling of confined prestressed concrete.

8.1.1 Experimental Investigation

The main purpose of the experimental investigation was: (1) to investigate the effects of lateral reinforcement on the strength and ductility of prestressed concrete columns, (2) to examine the use of continuous square spiral and the welded wire mesh as an acceptable alternative to single ties, and (3) to study the behavior of prestressed concrete columns under large axial strain.

To accomplish these objectives, twenty-three prestressed concrete columns 5.25 in. x 5.25 in. x 26 in. were tested under monotonic axial compression. They were prestressed to an average effective stress over the cross section of 478 psi. The ratio of core to gross area was kept constant at 65.5% in all specimens. Variable for the test specimens included the volumetric ratio of lateral reinforcement to concrete core ($\rho = 1.09, 1.45$ and 2.18%) and the type of lateral reinforcement (single ties, continuous square spiral and welded wire

mesh). The specimens were tested in a vertical position under a strain control mode. Strain in the longitudinal and lateral reinforcement was measured with electrical resistance strain gages and axial deformation was measured with two LVDTs attached over a ten inch gage length. In order to prevent failure outside the instrumented zone, both top and bottom end regions of the specimens were stiffened by reducing the lateral reinforcement spacing to one inch and attaching two external steel collars. During each test, strain gages and LVDTs were read by a scanner unit at discrete intervals. The crack development in the cover was carefully recorded and marked on the surface of the specimens. At the end of the test, the mode of failure identified was observed and documented.

8.1.2 Analytical Study

The experimental investigation showed that the available confinement models did not accurately represent the behavior of prestressed concrete columns and considerably overestimated the strength in the post-peak loading region. Therefore, an analytical model was developed based on the experimental test results for predicting the stress-strain behavior of concrete confined by rectilinear reinforcement in axially loaded precast prestressed concrete columns. In developing this model, the buckling load of the prestressing bars was calculated using the moment distribution method modified for axial load. The ascending part of the model consists of a second degree parabola followed by a linear segment. The post-peak behavior is represented by an exponentially decreasing curve. This model takes into account any increase in strength and ductility resulting from the lateral

reinforcement and more realistically represents the post-peak behavior. Finally, the concept of strain ductility factor is proposed as a measure of ductility in prestressed concrete columns.

8.2 Conclusions

The following conclusions are drawn from the results and observations made during this investigation:

(1) Lateral reinforcement in prestressed concrete columns does not increase the column capacity at maximum load.

(2) Closely spaced lateral reinforcement increases effectively the ductility in prestressed concrete columns. Its degree of effectiveness depends upon the volumetric ratio, spacing, properties of the lateral reinforcement as well as the ratio of core to gross area.

(3) Continuous square spiral has been observed to be slightly less effective in increasing the strength of the confined core than single ties. Its effect on increasing the ductility was almost similar to single ties.

(4) Specimens with welded wire mesh as lateral reinforcement exhibited similar strength increase in the core as specimens with single ties. Due to anchorage failure, the ductility was reduced drastically.

(5) The lateral reinforcement did not affect the ascending branches of the load-strain curves up until the peak load.

(6) The lateral reinforcement did not yield at maximum load.

(7) At peak load, the plain concrete strength was approximately equal to the cylinder strength, f_c' , and not $0.85 f_c'$, as usually

assumed in column design. This is due to the size effect and may be also due to the horizontal casting position.

(8) The presence of lateral reinforcement in prestressed concrete columns decreased the length of the failure zone. After the concrete reached its maximum stress, most of the inelastic deformation occurred over a length approximately equal to one and half times the core dimension.

(9) Test results suggested that the tie spacing limitation of $8d_l$ in appendix A of the ACI code, may be increased to $10d_l$ for prestressed concrete columns in regions of moderate seismic risk.

(10) Most of the previous confinement models have been found to overestimate the post-peak behavior of prestressed concrete columns. An analytical model is proposed to predict the stress-strain behavior of the confined core in axially loaded short precast prestressed concrete columns laterally reinforced with rectilinear lateral reinforcement. The proposed model has been found to predict the experimental results with a reasonable accuracy.

8.3 Suggestions for Future Research

(1) Effect of strain gradient on the behavior of prestressed concrete columns.

(2) Behavior of medium and long prestressed columns under biaxial loading.

(3) Slenderness effect in medium and long prestressed concrete columns.

(4) Confinement of light weight concrete columns.

(5) Confinement effectiveness of prestressed concrete columns

with circular spiral reinforcement.

(6) Lateral reinforcement requirements in medium prestressed concrete columns.

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APPENDIX A

Material Properties

A.1 Concrete

Each set of specimens was cast using laboratory mixed concrete. The concrete mix was designed according to the Standard Practice for Selecting Proportions for Normal, Heavyweight and Mass Concrete (ACI 211.1-81). Several trial batches were tested before the final proportions of concrete mix were decided. Type III portland cement was used to achieve a 14 day concrete strength of 5500 psi. The coarse aggregate was well graded crushed stone of three-quarter of an inch maximum size and river sand was used as fine aggregate. The amount of water was adjusted to give a slump of 3 to 4 inches to facilitate proper compaction and placement of concrete.

Quantities for one cubic yard of concrete are as follows:

Material	lb/cu. yd.
Coarse aggregate	1728
Fine aggregate	1323
Cement	486
Water	338
Water/Cement ratio	0.70

Six 6 in. x 12 in. cylinders were cast in plastic molds. They were removed from the molds and cured along with the corresponding specimens in a curing room. For each set of specimens, three cylinders were capped and tested for compression strength at transfer of prestress. The remaining three were tested when the specimens were

tested. Table A.1 gives the casting and testing schedule, and the average compressive strength of concrete at transfer and at testing.

A.2 Reinforcing Steel

A minimum of three coupons, randomly selected from each type of reinforcement, were tested in tension to obtain the stress-strain relationship. Tests were carried out in a 110 kip MTS machine. An MTS extensometer, having a gage length of one inch, was used to measure the elongation. It was detached shortly before the ultimate load was reached. The load was then increased to cause fracture of the steel bar. During each test, the load-strain readings were plotted. Typical stress-strain curves of prestressing and lateral reinforcement are shown in Fig. 3.4.

Table A-1 Concrete Strength at Transfer of Prestress and at Testing of Columns

Specimen	Age at Transfer (Days)	Age at Testing (Days)	Concrete Strength at Transfer (psi)	Concrete Strength at Testing (psi)
A11	4	19	4150	5620
A12	4	20	4150	5620
B11	2	16	3900	5630
B12	2	17	3900	5630
B13	2	20	3900	5630
B21	2	17	3820	5600
B22	2	21	3820	5600
B23	2	22	3820	5600
B31	3	24	3780	5550
B32	3	25	3780	5550
B33	3	24	3780	5550
C11	2	22	3800	5790
C12	2	23	3800	5790
C21	2	15	3850	5410
C22	2	16	3850	5410
C31	3	23	3830	5500
C32	3	22	3830	5500
D11	2	19	3490	5180
D12	2	20	3490	5180
D21	2	25	3490	5140
D22	2	16	4400	6100
D31	2	14	4400	6100
D32	2	15	4400	6100

Appendix B

Prestress Losses

The prestress losses at transfer were calculated as follows:

1. Elastic Shortening ES:

$$\begin{aligned}
 E_{ci} &= 57000 \sqrt{f'_{ci}} \\
 &= 57000 \sqrt{4000} = 3600 \text{ ksi} \\
 n &= \frac{E_s}{E_{ci}} = \frac{29500}{3600} = 8.19 \\
 F_i &= f_{pi} \times A_{ps} \\
 &= 135 \times 0.1158 = 15.63 \text{ kips} \\
 ES &= \frac{nF_i}{A_t} = \frac{8.19 \times 15.63}{28.26} = 4.53 \text{ ksi}
 \end{aligned}$$

2. Anchorage losses ANC:

$$ANC = \frac{\Delta E_s}{\ell}$$

Assuming $\Delta = 0.01"$, then

$$\begin{aligned}
 ANC &= \frac{0.01 \times 29500}{26} \\
 &= 11.35 \text{ ksi}
 \end{aligned}$$

3. Relaxation RE:

$$\begin{aligned}
 f_{ps}(t) &= f_{pi} (1 - 0.1 \log(t) \left(\frac{f_{pi}}{f_{py}} - 0.55 \right)) \\
 f_{ps}(3 \text{ days}) &= 135 (1 - 0.1 \times \log(3 \times 24) \times \left(\frac{135}{215} - 0.55 \right)) \\
 &= 135 (1 - 0.0145) \\
 &= 133.05 \text{ ksi}
 \end{aligned}$$

$$RE = f_{pi} - f_{ps}(3 \text{ days})$$

$$= 135 - 133.05$$

$$= 1.95 \text{ ksi}$$

4. Total calculated losses at transfer TL:

$$TL = 4.53 + 11.35 + 1.95 = 17.83 \text{ ksi}$$

(13.2% of the initial prestress)

Appendix C

Shrinkage Strain Calculations

According to the recommendations of ACI committee 209 [4], the shrinkage strain is calculated as follows:

1. Time of shrinkage coefficient S_t :

$$\begin{aligned} S_t &= \frac{t}{35 + t} \\ &= \frac{7}{35 + 7} \\ &= 0.167 \end{aligned}$$

2. Relative humidity coefficient S_h :

$$S_h = 1.4 - 0.01 \times H$$

Assuming a relative humidity, H, of 75%, then:

$$\begin{aligned} S_h &= 1.4 - 0.01 \times 75 \\ &= 0.65 \end{aligned}$$

3. Minimum thickness of member coefficient S_{th} :

$$S_{th} = 1.0$$

4. Slump of concrete coefficient S_s :

For a slump of 3 to 4 inches,

$$S_s = 1.03$$

5. Fines coefficient S_f :

For 46.5% fines content,

$$S_f = 0.95$$

6. Air content S_e :

Assuming 4% air content,

$$S_e = 0.98$$

7. Cement content factor S_c :

$$S_c = 0.917$$

As recommended by ACI committee 209 for moist-cured concrete, the ultimate shrinkage strain, ϵ_{shu} can be assumed as:

$$\epsilon_{shu} = 0.0008$$

Therefore the shrinkage strain is calculated as:

$$\begin{aligned}\epsilon_{sh} &= \epsilon_{shu} \cdot S_t \cdot S_h \cdot S_{th} \cdot S_s \cdot S_f \cdot S_e \cdot S_c \\ &= 0.0008 \times 0.167 \times 0.65 \times 1.0 \times 1.03 \times 0.95 \times 0.98 \times 0.917 \\ &= 0.000076 \text{ in./in.}\end{aligned}$$

$$\epsilon_{sh} = 76 \mu \text{ strain}$$

Appendix D

Calculation of the Buckling Load of a Prestressing Bar

Case (a): Two lateral springs

The moment distribution method modified for axial load [17] is used for the calculation of the buckling load. With reference to Fig. D.1, the following equations can be written:

$$M_{BA} = K \phi - K(1 + r)\theta$$

$$M_{BB}' = K(1 - r) \phi$$

where K = stiffness factor,

r = carry over factor,

ϕ = angle caused by joint rotation, and

θ = angle caused by spring translation.

$$V_{BA} = \frac{K(1 + r)}{S} (2\theta - \phi) - P\theta$$

$$F = k_1 S \theta$$

$$V_{BB}' = 0$$

The spring constant k_1 is calculated from Fig. 5.9 as follows:

$$\begin{aligned} k_1 &= \frac{2\sqrt{2} E A_0}{\ell_0} \\ &= \frac{2\sqrt{2} \times 29500 \times 0.04638}{4.25} \\ &= 910 \text{ kip/in} \end{aligned}$$

$$\sum M_B = 0$$

$$(2K - rK)\phi - K(1 + r)\theta = 0 \quad (D.1)$$

Fig. D.1 Deflected Shape in Case of Two Lateral Springs

$$= 63.2 \text{ ksi}$$

$$P_{cr} = 1.83 \text{ kips}$$

$$(2). \text{ For } S = 3 \text{ in}$$

$$\sigma = \frac{1.507 \pi^2 \times 29500 \times 0.000066708}{(3)^2 \times 0.02895}$$

$$= 112.3 \text{ ksi}$$

$$P_{cr} = 3.25 \text{ kips}$$

Case (b): Three lateral springs

With reference to Fig. D.2, the following equations can be written:

$$M_{BA} = K\phi_1 - K(1+r)\theta_1$$

$$V_{BA} = \frac{K(1+r)}{S} (2\theta_1 - \phi_1) - P\theta_1$$

$$M_{BB'} = K\phi_1 - K(1+r)\theta_2$$

$$V_{BB'} = \frac{K(1+r)}{S} (2\theta_2 - \phi_1) - P\theta_2$$

$$V_{B'B} = \frac{K(1+r)}{S} (2\theta_2 - \phi_1) - P\theta_2$$

$$V_{B'B''} = \frac{K(1+r)}{S} (-2\theta_2 + \phi_1) + P\theta_2$$

$$F_B = k_1 S \theta_1$$

$$F_{B'} = k_1 S (\theta_1 + \theta_2)$$

$$\sum M_B = 0$$

$$2K\phi_1 - K(1+r)\theta_1 - K(1+r)\theta_2 = 0 \quad (D.3)$$

$$\sum V_B = 0$$

$$[2K(1+r) - PS + k_1 S^2]\theta_1 + [-2K(1+r) + PS]\theta_2 = 0 \quad (D.4)$$

Fig. D.2 Deflected Shape in Case of Three Lateral Springs

$$\sum V_B = 0$$

$$-2K(1+r)\phi_1 + k_1 S^2 \theta_1 + [4K(1+r) - 2PS + k_1 S^2] \theta_2 = 0 \quad (D.5)$$

Multiplying equation D.4 by two and adding it to D.5 and rearranging the above equations in a matrix form results in:

$$\begin{bmatrix} 4K & -2K(1+r) & -2K(1+r) \\ -2K(1+r) & 4K(1+r) - 2PS + 3k_1 S^2 & k_1 S^2 \\ -2K(1+r) & k_1 S^2 & 4K(1+r) - 2PS + k_1 S^2 \end{bmatrix} \begin{Bmatrix} \phi_1 \\ \theta_1 \\ \theta_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$$

Different trial values for P and the corresponding determinant of the above matrix are shown in Table D.2.

Table D.2 Trial Loads and Corresponding Matrix Determinant

$\frac{PS^2}{EI}$	Determinant
0	1.07×10^8
π	6.4×10^7
4	3.0×10^7
4.49	68429
4.493	28123
4.4934	-39
4.4935	-7289

The critical load corresponding to the least value of determinant is calculated as follows:

$$P_{cr} = (4.4934)^2 \frac{EI}{S^2} = 20.19 \frac{EI}{S^2} = 2.046 \frac{\pi^2 EI}{S^2} = \frac{\pi^2 EI}{(0.70 S)^2}$$

$$\sigma = \frac{2.046\pi^2 \times 29500 \times 0.000066708}{(2)^2 \times 0.02895}$$

$$= 228 \text{ ksi}$$

Using the tangent modulus

$$P_{cr} = 5.07 \text{ kips}$$