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DIAGNOSED BY UHF RADIO STAR SCINTILLATIONS

Rice University

PH.D. 1982

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HF PRODUCED IONOSPHERIC ELECTRON DENSITY IRREGULARITIES
DIAGNOSED BY UHF RADIO STAR SCINTILLATIONS

BY

ALFRED FREY

A THESIS SUBMITTED
IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS FOR THE DEGREE
DOCTOR OF PHILOSOPHY

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ABSTRACT

HF-waves incident on an overdense (HF-frequency $<$ penetration frequency) ionosphere are known to produce large scale electron density irregularities. It is predicted that similar irregularities are formed during underdense HF-modification. The propagation of UHF radio waves originating from radio stars will be affected by such irregularities in the ionosphere. The interest in a scintillation experiment is twofold. One may obtain information on the electron density irregularities and one may learn about the propagation of radio waves through such a perturbed medium. A thin screen (diffractive) theory is derived which allows to draw conclusions on the electron density irregularities from the intensity fluctuations measured on the ground if the phase perturbations are much less than one radian. Since radio stars suitable for scintillation measurements at UHF are very faint an antenna with a large collection area is required. The observations reported in this dissertation were performed with the 300m diameter spherical reflector of the Arecibo Observatory. Successful observations were performed at 430 MHz and at 1400 MHz. Intensity fluctuations at such high frequencies measured with a large antenna suffer

severe filtering in the thin phase screen regime. The theory presented in this dissertation includes these filtering effects. Many observations agree with the predictions of that theory. Some observations indicate that refraction effects have to be included to explain the data. HF-induced electron density irregularities were only observed during overdense heating. Several attempts to detect irregularities at an observing frequency of 430 MHz during underdense heating (HF-frequencies 1.1 to 2 times penetration frequency) failed. All the measurements reported in this dissertation are therefore during overdense heating except for the measurement on the 22 September 1980 which shows a transition from underdense to overdense HF-modification. Attempts were made to measure electron density irregularities at an observing frequency of 2380 MHz due to overdense heating but the fluctuations were too weak to be discriminated against instrumental uncertainties. The formation time for the irregularities after the HF-power was turned on was ~ 25 seconds. Their lifetime after the HF-power was turned off was on the order of hours. The electron density perturbations appear to exceed 1% at times. One observation on 11/12 September 1980 reveals that the density perturbation actually corresponds to a depletion.

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Table of symbols

<u>Symbol</u>	<u>Meaning</u>	<u>defined on page</u>
$\tilde{A}(\tilde{k})$	Fourier transform of $\tilde{E}(\tilde{R})$	8
$\mathcal{A}(\kappa)$	antenna filtering function	37
α	angular diameter of radio source	45
b	predetection frequency bandwidth	48
Γ	degree of coherence between the electric fields at two locations at two instants in time	35
Γ_s	degree of spatial coherence	35
Γ_T	degree of temporal coherence	35
$\tilde{E}(\tilde{R})$	electric field vector in 3-dimensional real space at position $\tilde{R}$	8
$\tilde{E}_o$	electric field vector of the waves unaffected by the thin screen	11
$\tilde{E}_d$	electric field perturbation due to the field diffracted by the irregularities in the thin screen	12
$\tilde{E}_R$	real part of the diffracted field $\tilde{E}_d$	12
$\tilde{E}_I$	imaginary part of the diffracted field $\tilde{E}_d$	12
$\tilde{E}_a$	electric field measured by the antenna	37
$\tilde{\mathcal{E}}_S(\tilde{\kappa})$	Fourier transform of 2-dimensional electric field structure $\tilde{E}(\tilde{r}_S, z=z_S)$ in the x-y plane representing the screen	9
$\tilde{\mathcal{E}}_G(\tilde{\kappa})$	Fourier transform of 2-dimensional electric field structure $\tilde{E}(\tilde{r}_G, z=z_G)$ in the x-y plane representing the ground	9
f_{HF}	HF-frequency	52
f_p	plasma frequency	18
f	observing frequency	17
F	introduced for convenience $(= \kappa^2 / \kappa_F^2)$	24
$\mathcal{F}(\tau_c)$	time response of filter, Fourier transform of $\mathcal{F}(f)$	48
f_o	center frequency of predetection BW	48

$\mathcal{F}(f)$	frequency response of filter (system)	48
ϕ	phase perturbation experienced by the wavefront passing through the irregular medium ($= \phi_i - \phi_h$)	16
ϕ_h	phase of propagating wave in a medium which is homogeneous in the x and y directions	18
ϕ_i	phase of propagating wave in a medium which is inhomogeneous in the x and y directions	18
$\tilde{\phi}(\tilde{\mathbf{k}})$	Fourier transform of $\phi(\tilde{\mathbf{r}})$	22
G	gain of HF-antenna	52
h	height above ground	43
I	measured intensity of radio source	14
I_0	measured intensity of radio source if there were no irregularities in the screen	14
ΔI	intensity fluctuations ($= I - I_0$)	25
$\tilde{\mathbf{k}}$	wavevector of propagating UHF wave (Fig.2.1.)	5
$\tilde{k}_z$	component of k perpendicular to the phase screen (Fig.2.1.)	8
$\tilde{\mathbf{K}}$	wavevector of irregularities in the screen (Fig.2.1.)	8
$\tilde{\mathbf{K}}_F$	wavevector characteristic to the Fresnel filtering	10
$\tilde{\mathbf{K}}_1$	wavevector corresponding to the first zero in the Fresnel filter	83
$\tilde{\mathbf{K}}_2$	wavevector corresponding to the second zero in the Fresnel filter	83
$\tilde{\mathbf{K}}_3$	wavevector corresponding to the third zero in the Fresnel filter	83
$\tilde{\mathbf{K}}_A$	wavevector corresponding to the first zero in the antenna filter $\mathcal{A}(\mathbf{K})$	83
L	spatial extent of density perturbations along the line of sight to the radio source	18

Λ	separation of a point in the thin screen from the line of sight (Fig.2.6.)	34
λ	wavelength of UHF waves	7
λ_o	wavelength of UHF waves in free space	17
λ_{ir}	wavelength of irregularities in the screen	7
λ_{HF}	HF-wavelength	52
n	index of refraction	17
N_e	electron density	18
ΔN_e	electron density perturbation	19
ν	frequency of intensity fluctuations	44
$\Delta\nu$	frequency resolution of power spectra	44
P	HF power fed into HF-antenna	52
ρ	power density of HF waves	52
$\tilde{r}$	component of position vector $\tilde{R}$ parallel to the screen (Fig.2.1.)	8
$\tilde{R}$	3-dimensional position vector (Fig.2.1.)	8
$\tilde{r}_s$	2-dimensional position vector in the x-y plane representing the thin screen	9
$\tilde{r}_G$	2-dimensional position vector in the x-y plane representing the ground	9
r_e	classical electron radius ($= 2.82 \times 10^{-13}$ cm)	18
ρ_o	radius of circular antenna	37
$\hat{U}(\tilde{k})$	Fourier transform of $\hat{E}_d(\tilde{r})$	13
T	total temperature measured by the receiver (including receiver noise)	55
ΔT	statistical error in the measurement of T	55
T_{sys}	system temperature	55
T_F	temperature equivalent to measured energy flux from the radio source	55
T_s	sampling period	44

τ_c	time delay due to finite frequency BW	48
τ_d	post detection integration time	55
τ	time delay due to the additional path-length χ	34
v_{RS}	apparent velocity of a fixed point on the line of sight to the radio source at a distance h from the antenna	43
v_{dr}	drift velocity of the irregularities	43
v_{sc}	velocity at which the line of sight to the radio source scans the irregularities which themselves are drifting with v_{dr}	43
χ	additional pathlength of the diffracted field E_d with respect to the unperturbed field E_o (Fig.2.6.)	34
$z=z_s$	corresponds to x-y plane that represents the screen	8
$z=z_G$	corresponds to x-y plane that represents the ground or plane of observation	8
$z=z_c$	corresponds to any arbitrary x-y plane	8
z_o	separation of the thin screen from the ground	10

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1. Introduction.

It is known that intense HF-waves incident on an overdense ionosphere (i.e.; HF-frequency $<$ penetration frequency) produce electron density irregularities with scale sizes on the order of several hundred meters. Such irregularities have previously been observed at Platteville, Colorado by an HF phased array [Allen et. al., 1974a,b; Thome & Perkins, 1974] and at Arecibo, Puerto Rico with the 430 MHz incoherent backscatter radar [Duncan & Behnke, 1978]. If radio waves originating from some distant radio source (i.e.; radio star or satellite beacon) propagate through such a perturbed ionosphere the intensity as measured on the ground fluctuates. Such fluctuations were previously observed at Platteville using frequencies below 400 MHz [Rufenach, 1973; Pope & Fritz, 1974; Bowhill, 1974; Basu et. al., 1980]. The objective of the experiments reported in this thesis is to investigate the effects of the artificially produced electron density irregularities on the propagation of frequencies in the UHF range. The question arises whether the intensity scintillations can be explained by the thin phase screen, diffractive approach or whether refractive effects, as recently proposed [Crain et. al., 1979] for SHF scintillations due to natural ionospheric irregularities, are at work. In addition one could imagine still other mechanisms like convection, turbulence, etc. Obtaining information on the mechanism producing the scintillations will in turn tell us something about the electron density irregularities which are produced by

HF-modification. The starting hypothesis is the correctness of the thin phase screen theory and that any disagreeing observation will hint at some different mechanism involved. For this purpose the thin phase screen theory is derived in chapter 2. The present experiment is different from other scintillation experiments in that we are using a large antenna (the 300 m dish of the Arecibo Observatory) and a high observing frequency. The combination of these two parameters introduces interesting filtering effects which are derived and illustrated in section 2.3. as an extension of the usual thin phase screen theory. Other experimental considerations are discussed in that same section. The experimental procedure is outlined in chapter 3. A short introduction to the theories attempting to explain the production of ionospheric electron density irregularities during HF-modification is presented in chapter 4. The observational results which were obtained at the observing frequencies of 430 MHz and 1400 MHz are shown in chapter 5. Observations at 2380 MHz do not show any discernible effects at the present HF power level. An interpretation of the results is given in chapter 6. And finally some conclusions are drawn in chapter 7.

2. Scintillation theory (diffraction-approach).

In the theory presented it is assumed that the intensity fluctuations (scintillations) on the ground are produced by electron density perturbations (diffractive theory) rather than electron density gradients (refractive theory). The latter theory [Crain, Booker & Ferguson, 1979] requires that the electron density gradients are nearly perpendicular to the line of sight with the radio source. It is also assumed that the phase fluctuations produced within the irregular medium are less than one radian which corresponds to the assumption that the wave-field undergoes single scattering. The relationship between the electron density irregularities and the observed intensity irregularities is very simple if the above approximations (conventionally called the thin phase screen approximation) hold. In practice this approximation may be achieved by increasing the observing frequency. Most theories following above assumptions proceed with statistical arguments (i.e.; random electron density fluctuations, gaussian variables) [Mercier, 1962; Salpeter, 1967]. The present theory assumes that the lifetime of the electron density irregularities is fairly long compared with the period of observation. The concept of an angular spectrum of plane waves [Booker, Ratcliffe & Shinn, 1950] is used and helps getting around the assumptions of [Mercier, 1961; Salpeter, 1967]. [Bowhill, 1961] used the same approach but obtained what the present theory would call (see section 2.1.b.) a 2nd order effect (i.e.; interference of perturbations with each

other) rather than the stronger 1st order effect (i.e.; interference of the perturbed field with the unperturbed field). The final result, namely eq.(2-68), is identical with the result of [Salpeter,1967]. It is shown in section 2.2.d. that the Fresnel-filtering term is a purely geometrical factor. An extension of the theory to include antenna effects (another filtering term) is straightforward and is shown in section 2.3.a.

2.1. The 2-dimensional screen approximation.

The spatial electric field structure within some homogeneous medium (i.e.; a nonrefracting medium for which the propagation vector $\tilde{\mathbf{k}}$ is independent of the spatial location) may always be represented in terms of an angular spectrum of plane waves which form a complete orthonormal set of eigenfunctions of the electromagnetic wave-equation if a rectangular x,y,z coordinate system (see Fig.2.1.) is used. This physical picture is equivalent to the mathematical statement that the electric field structure in 3-dimensional real space has a Fourier-transform into 3-dimensional $\tilde{\mathbf{k}}$ -space. The electric field satisfies Maxwell's equations by satisfying the electromagnetic wave-equation. If the field structure in the 3-dimensional medium is known, either by the real space representation or by specifying the angular spectrum, then we can determine the field structure at any 2-dimensional boundary or surface within the medium. The transition from 3-dimensional to 2-dimensional field-specification is trivial if we assume a geometry in which the planes of interest are perpendicular to the propagation path (line of sight to the radio-source) such that $|\tilde{\mathbf{z}}|$ can be fixed at any value of interest. The matching of the 2-dimensional field structure across a plane with the 3-dimensional angular spectrum is the basis of the concept formulated by [Booker, Ratcliffe & Shinn, 1950] and discussed further by [Ratcliffe, 1956].

Fig.2.1.

This concept allows the radiation pattern to be determined from the field distribution over any given aperture by a simple Fourier transform if the parameters are expressed in proper units (in essence identical to eq.(2-5)).

To relate the field structure in one plane to the field structure in another plane the theory presented in this chapter uses the above concept. It is written in terms of 3-dimensional vectors but in essence the vector quantities of interest are in the $\tilde{x}, \tilde{y}$ - plane and in that sense the analysis is 2-dimensional. The 3-dimensional analysis is chosen for convenience but is only a good approximation so long as the angular spectrum is restricted to small diffraction angles $\theta = \lambda / \lambda_{ir}$, a condition which is well satisfied for $\lambda_{ir} \gtrsim 10 \cdot \lambda$ (i.e.; the wavelength of the irregularities has to be larger than about 10 wavelengths of the radiowaves which propagate through the irregular medium). This condition restricts the angular spectrum to a cone which has an apex-angle of $\sim 11^\circ$ and is centered with its axis perpendicular to the plane of interest.

It is also assumed that the medium is isotropic at the frequencies of interest so that we may ignore Faraday-rotation of the electric field vector.

a.) The electric field structure.

The electric field structure at any given location $\tilde{\mathbf{R}}$ between the screen containing the irregularities ($\tilde{z}=\tilde{z}_s$) and the plane of observation on the ground ($\tilde{z}=\tilde{z}_g$) can be represented by an angular spectrum of plane waves;

$$\tilde{\mathbf{E}}(\tilde{\mathbf{R}}) = \int \tilde{\mathbf{A}}(\tilde{\mathbf{k}}) \cdot e^{-i \cdot \tilde{\mathbf{k}} \cdot \tilde{\mathbf{R}}} \cdot d\tilde{\mathbf{k}} \quad (2-1)$$

where $\tilde{\mathbf{A}}(\tilde{\mathbf{k}})$ is the representation of the electric field structure in $\tilde{\mathbf{k}}$ -space given by the inverse Fourier-transform;

$$\tilde{\mathbf{A}}(\tilde{\mathbf{k}}) = \int \tilde{\mathbf{E}}(\tilde{\mathbf{R}}) \cdot e^{i \cdot \tilde{\mathbf{k}} \cdot \tilde{\mathbf{R}}} \cdot d\tilde{\mathbf{R}} \quad (2-2)$$

We adopt a rectangular coordinate system $\tilde{x}, \tilde{y}, \tilde{z}$ (see Fig.2.1.) where $\tilde{z}$ is perpendicular to the screen of irregularities and $\tilde{\mathbf{r}}$ is restricted to a plane parallel to the screen containing the irregularities at a separation of $\tilde{z}=\tilde{z}_c$. Similarly $\tilde{\mathbf{k}}_z$ is perpendicular to the screen containing the irregularities and $\tilde{\mathbf{k}}$ is in the plane of the screen containing the irregularities. Hence;

$$\tilde{\mathbf{R}} = \tilde{\mathbf{r}} + \tilde{z} \quad (2-3)$$

$$\tilde{\mathbf{k}} = \tilde{\mathbf{k}} + \tilde{k}_z \quad (2-4)$$

With these assumptions the angular spectrum can be related to the electric field structure in any given plane ($z=\text{constant}$) which is parallel to the screen containing the irregularities.

In particular we have immediately below the screen containing the irregularities, where $\tilde{z}=\tilde{z}_s = \text{constant}$, for the angular spectrum (using eqs.(2-2) to (2-4));

$$\tilde{A}(\tilde{k}) = e^{i \cdot (\tilde{k}_z \cdot \tilde{z}_s)} \cdot \int \tilde{E}(\tilde{r}_s, \tilde{z}=\tilde{z}_s) \cdot e^{i \cdot (\tilde{\mathcal{K}} \cdot \tilde{r}_s)} d\tilde{r}_s \quad (2-5)$$

This last integral also corresponds to a Fourier-representation in 2-dimensional $\tilde{\mathcal{K}}$ -space of the electric field structure in the plane ($\tilde{z}=\tilde{z}_s$) forming a 2-dimensional real $\tilde{r}$ -space;

$$\tilde{\mathcal{E}}_s(\tilde{\mathcal{K}}) = \int \tilde{E}(\tilde{r}_s, \tilde{z}=\tilde{z}_s) \cdot e^{i \cdot (\tilde{\mathcal{K}} \cdot \tilde{r}_s)} \cdot d\tilde{r}_s \quad (2-6)$$

which has an inverse transform;

$$\tilde{E}(\tilde{r}_s, \tilde{z}=\tilde{z}_s) = \int \tilde{\mathcal{E}}_s(\tilde{\mathcal{K}}) \cdot e^{-i \cdot (\tilde{\mathcal{K}} \cdot \tilde{r}_s)} \cdot d\tilde{\mathcal{K}} \quad (2-7)$$

The angular spectrum can thus be written as;

$$\tilde{A}(\tilde{k}) = e^{i \cdot (\tilde{k}_z \cdot \tilde{z}_s)} \cdot \tilde{\mathcal{E}}_s(\tilde{\mathcal{K}}) \quad (2-8)$$

We may repeat the above arguments for a different plane, say the plane of observation at $\tilde{z}=\tilde{z}_G$. We then obtain;

$$\tilde{A}(\tilde{k}) = e^{i \cdot (\tilde{k}_z \cdot \tilde{z}_G)} \cdot \int \tilde{E}(\tilde{r}_G, \tilde{z}=\tilde{z}_G) \cdot e^{i \cdot (\tilde{\mathcal{K}} \cdot \tilde{r}_G)} \cdot d\tilde{r}_G \quad (2-9)$$

or;

$$\tilde{A}(\tilde{k}) = e^{i \cdot (\tilde{k}_z \cdot \tilde{z}_G)} \cdot \tilde{\mathcal{E}}_G(\tilde{\mathcal{K}}) \quad (2-10)$$

We obtain by comparison of eqs. (2-8) and (2-10);

$$\tilde{\epsilon}_G(\tilde{\kappa}) = \tilde{\epsilon}_S(\tilde{\kappa}) \cdot e^{i \cdot \tilde{\kappa}_z \cdot (\tilde{z}_S - \tilde{z}_G)} \quad (2-11)$$

This is equivalent to an application of the shift-theorem [Bracewell, 1978] to eq.(2-2).

From eq.(2-4) we have for $|\tilde{\kappa}_z|$;

$$|\tilde{\kappa}_z| = |\tilde{\kappa} - \kappa| = \sqrt{k^2 - \kappa^2} = k \cdot \sqrt{1 - \frac{\kappa^2}{k^2}} \quad (2-12)$$

Since the wavelength of the irregularities $\lambda_{ir} = 2 \cdot \pi / |\tilde{\kappa}|$ is assumed larger than the wavelength λ of the incident radiation field we may approximate;

$$\frac{\lambda^2}{\lambda_{ir}^2} = \frac{\kappa^2}{k^2} < 1 \quad (2-13)$$

Therefore;

$$|\tilde{\kappa}_z| \cong |\tilde{\kappa}| \left(1 - \frac{1}{2} \cdot \frac{\kappa^2}{k^2} + \dots \right) \quad (2-14)$$

Now we can rewrite eq.(2-11) as;

$$\tilde{\epsilon}_G(\tilde{\kappa}) = \tilde{\epsilon}_S(\tilde{\kappa}) \cdot e^{-i \cdot |\tilde{\kappa}| \cdot |\tilde{z}_O|} \cdot e^{i \cdot \frac{\kappa^2}{\kappa_F^2}} \quad (2-15)$$

where we define;

$$\kappa_F^2 = \frac{4 \cdot \pi}{\lambda \cdot |\tilde{z}_O|} \quad (2-16)$$

and;

$$\tilde{z}_O = \tilde{z}_G - \tilde{z}_S \quad (2-17)$$

Equation (2-15) is an important result since it shows that the $\tilde{\mathbf{K}}$ -space representation of the electric field structure on the ground is equal to the $\tilde{\mathbf{K}}$ -space representation of the electric field structure immediately below the screen containing the irregularities but multiplied by a filtering factor. The term $e^{-i \cdot \mathbf{k} \cdot \mathbf{z}_0}$ corresponds to the ordinary propagation of the undisturbed incident wave, and the term;

$$e^{i \frac{\mathbf{K}^2}{\mathbf{K}_F^2}} \quad (2-18)$$

is a term which selectively filters certain $\mathbf{K}$ -values,

The electric field structure in the observation plane may then be written as;

$$\tilde{\mathbf{E}}(\tilde{\mathbf{r}}_G, z=z_G) = e^{-i \cdot \mathbf{k} \cdot \mathbf{z}_0} \cdot \int \tilde{\mathbf{E}}_s(\tilde{\mathbf{K}}) \cdot e^{i \frac{\mathbf{K}^2}{\mathbf{K}_F^2}} \cdot e^{-i \tilde{\mathbf{K}} \cdot \tilde{\mathbf{r}}_G} \cdot d\tilde{\mathbf{K}} \quad (2-19)$$

This field can then be separated into;

$$\tilde{\mathbf{E}}(\tilde{\mathbf{r}}_G, z=z_G) = \left\{ \tilde{\mathbf{E}}_0 + \int_{\mathbf{K} \neq 0} \tilde{\mathbf{E}}_s(\tilde{\mathbf{K}}) \cdot e^{i \frac{\mathbf{K}^2}{\mathbf{K}_F^2}} \cdot e^{-i \tilde{\mathbf{K}} \cdot \tilde{\mathbf{r}}_G} \cdot d\tilde{\mathbf{K}} \right\} e^{-i \mathbf{k} \cdot \mathbf{z}_0} \quad (2-20)$$

where $\tilde{\mathbf{E}}_0$ corresponds to the component $\tilde{\mathbf{E}}_s(\tilde{\mathbf{K}} = 0)$ and is that part of the incident field which is unaffected by the irregularities. The integral then has a lower limit for $|\tilde{\mathbf{K}}|$ corresponding to the longest wavelength of the irregularities in the screen and an upper limit according to the approximation (2-13) made earlier.

b.) The intensity distribution.

The intensity pattern in a given plane ($z=z_c$) parallel to the irregularities screen is given as;

$$I(\tilde{r}_c) = \tilde{E}(\tilde{r}_c) \cdot \tilde{E}^*(\tilde{r}_c) \quad (2-25)$$

where;

$$\tilde{E}(\tilde{r}_c) = \tilde{E}_O + \tilde{E}_d(\tilde{r}_c) \quad (2-26)$$

$\tilde{E}_O$ corresponds to the incident field and is assumed real and constant.

$\tilde{E}_d(\tilde{r}_c)$ is the electric field perturbation at the plane of interest ($z=z_c$) produced by the irregularities in the screen ($z=z_s$) and is in general complex. i.e.;

$$\begin{aligned} \tilde{E}_d(\tilde{r}_c) &= \tilde{E}_R(\tilde{r}_c) + i \cdot \tilde{E}_I(\tilde{r}_c) \\ &= \text{Re } \tilde{E}_d(\tilde{r}_c) + i \cdot \text{Im } \tilde{E}_d(\tilde{r}_c) \end{aligned} \quad (2-27)$$

The intensity is then given as;

$$I(\tilde{r}_c) = (\tilde{E}_O + \tilde{E}_R + i \cdot \tilde{E}_I) \cdot (\tilde{E}_O + \tilde{E}_R - i \cdot \tilde{E}_I) \quad (2-28)$$

$$I(\tilde{r}_c) = E_O^2 + 2 \cdot \tilde{E}_O \cdot \tilde{E}_R + E_R^2 + E_I^2 \quad (2-29)$$

The first term is the intensity which would be measured in the absence of irregularities. Under the assumption of weak perturbations (thin screen) we have;

$$|\tilde{E}_d(\tilde{r}_c)| < |\tilde{E}_0| \quad (2-30)$$

The second term then dominates the third and fourth terms in eq.(2-29). It is this second term which is of primary interest at present and which is responsible for the first order scintillation effects. To determine the intensity pattern in the plane $z=z_c$ is therefore equivalent to the problem of determining $\tilde{E}_R(\tilde{r}_c)$. The complex electric field $\tilde{E}_d(\tilde{r}_c)$ in any given plane $z=z_c$ has a corresponding representation in $\tilde{\mathcal{K}}$ -space;

$$\tilde{U}(\tilde{\mathcal{K}}) = \int \tilde{E}_d(\tilde{r}_c) \cdot e^{i\tilde{\mathcal{K}} \cdot \tilde{r}_c} d\tilde{r}_c \quad (2-31)$$

with the corresponding inverse transformation;

$$\tilde{E}_d(\tilde{r}_c) = \int \tilde{U}(\tilde{\mathcal{K}}) \cdot e^{-i\tilde{\mathcal{K}} \cdot \tilde{r}_c} d\tilde{\mathcal{K}} \quad (2-32)$$

$\tilde{U}(\tilde{\mathcal{K}})$ can then be written as a sum of even and odd functions $\tilde{U}^e(\tilde{\mathcal{K}})$ and $\tilde{U}^o(\tilde{\mathcal{K}})$ respectively, each of which is in general complex and can therefore be separated into a real and an imaginary part. We thus write [Bracewell, 1978];

$$\tilde{U}(\tilde{\mathcal{K}}) = \text{Re} \tilde{U}^e(\tilde{\mathcal{K}}) + i \cdot \text{Im} \tilde{U}^e(\tilde{\mathcal{K}}) + \text{Re} \tilde{U}^o(\tilde{\mathcal{K}}) + i \cdot \text{Im} \tilde{U}^o(\tilde{\mathcal{K}}) \quad (2-33)$$

The same may be done with $\tilde{E}_d(\tilde{r}_c)$;

$$\tilde{E}_d(\tilde{r}_c) = \text{Re} \tilde{E}_d^e(\tilde{r}_c) + i \cdot \text{Im} \tilde{E}_d^e(\tilde{r}_c) + \text{Re} \tilde{E}_d^o(\tilde{r}_c) + i \cdot \text{Im} \tilde{E}_d^o(\tilde{r}_c) \quad (2-34)$$

Each term in one representation corresponds to a term in the other representation. The transformation rules are as follows [Bracewell, 1978];

$$\left. \begin{array}{l} \text{real even} \\ \text{real odd} \\ \text{imag even} \\ \text{imag odd} \end{array} \right\} \text{ transforms to } \left\{ \begin{array}{l} \text{real even} \\ \text{imag odd} \\ \text{imag even} \\ \text{real odd} \end{array} \right. \quad (2-35)$$

We then have for the real part of the electric field perturbations due to the irregularities $\tilde{E}_R(\tilde{r}_c)$ using eqs. (2-27, 2-33, 2-34) and the transformation rules (2-35);

$$\begin{aligned} \tilde{E}_R(\tilde{r}_c) &= \text{Re} \tilde{E}_d(\tilde{r}_c) = \text{Re} \tilde{E}_d^o(\tilde{r}_c) + \text{Re} \tilde{E}_d^e(\tilde{r}_c) \\ &= \int [i \cdot \text{Im} \tilde{\Gamma}^o(\tilde{\mathbf{r}}) + \text{Re} \tilde{\Gamma}^e(\tilde{\mathbf{r}})] \cdot e^{i \tilde{\mathbf{r}}_c \cdot d \tilde{\mathbf{r}}} \end{aligned} \quad (2-36)$$

The general expression for the intensity pattern then becomes;

$$I(\tilde{r}_c) = I_0 + 2 \cdot \tilde{E}_0 \cdot \int [i \cdot \text{Im} \tilde{\Gamma}^o(\tilde{\mathbf{r}}) + \text{Re} \tilde{\Gamma}^e(\tilde{\mathbf{r}})] \cdot e^{i \tilde{\mathbf{r}}_c \cdot d \tilde{\mathbf{r}}} \quad (2-37)$$

The quantities of interest are therefore $\text{Im } \tilde{\sigma}^0(\hat{k})$
and $\text{Re } \tilde{\sigma}^e(\hat{k})$. These will be derived in section 2.2.c.
for the special case of a thin phase screen.

2.2. The thin phase screen approximation.

As a plane wave coming from a distant radio source enters a medium with spatial changes in index of refraction across the wavefront (i.e.; irregular electron density in the ionosphere), different parts of the wavefront experience different phase delays. In the thin phase screen approximation it is assumed that the relative phase delays across the wavefront are small (< 1 radian). It is further assumed that the diffraction angle $\theta = \lambda/\lambda_{ir}$ is much smaller than the angle given by λ_{ir}/L (where L is the thickness of the irregularity layer) so that we can integrate meaningfully the phase perturbation along a given ray-path (perpendicular to the wavefront) through the perturbed medium. The 3-dimensional irregular medium can then be thought of as being projected onto a 2-dimensional plane containing the integrated effects. This plane is commonly called the thin phase screen and may be specified by the phase-perturbation $\phi(\tilde{r}_s)$ which it introduces to the incident wavefront. In section a.) it is shown how the electron density perturbations in the ionosphere are related to these integrated phase perturbations and under which conditions $\phi(\tilde{r}_s) < 1$ radian holds up. In section b.) it is shown that the intensity immediately below the thin phase screen is constant. In propagating to the ground the electric field from different parts of the screen will interfere and the resulting intensity pattern on the ground as a function of the phase perturbations in the screen is derived in section c.).

a.) Relation of the phase to the electron density perturbations.

As a plane wave propagates through a slab of irregular medium extending from $z_s - L$ to z_s (note in Fig.2.1. that $\tilde{z}$ points downwards), the infinitesimal change of phase at some location on the wavefront $d\phi(\tilde{r})$ for an infinitesimal amount of propagation dz within the medium is;

$$d\phi(\tilde{r}, z) = 2 \cdot \pi \cdot \frac{dz}{\lambda_0} \cdot n(\tilde{r}, z) \quad (2-40)$$

where; λ_0 = wavelength in free space

$\tilde{z}$ and $\tilde{r}$ are defined in Fig.2.1.

If the medium is a plasma we have for the index of refraction $n(\tilde{r}, z)$;

$$n(\tilde{r}, z) = \sqrt{1 - \frac{f_p^2(\tilde{r}, z)}{f^2}} \cong 1 - \frac{1}{2} \frac{f_p^2(\tilde{r}, z)}{f^2} \quad (2-41)$$

where we used the fact that;

$$f_p \ll f$$

(for the present experiment it is always true that

$$f_p < 12 \text{ MHz and } f > 430 \text{ MHz})$$

The plasma frequency $f_p(\tilde{r}, z)$ is related to the electron density $N_e(\tilde{r}, z)$ as;

$$f_p^2(\tilde{r}, z) = \left(\frac{e^2}{\pi \cdot m_e} \right) \cdot N_e(\tilde{r}, z) \quad (2-42)$$

numerically;

$$f_p [\text{Hz}] = 9000 \cdot \sqrt{N_e [\text{cm}^{-3}]}$$

where; e = charge of electron

m_e = mass of electron

Combining eqs. (2-40) to (2-42) and integrating along z from $z_s - L$ across the slab to z_s , the total change of phase experienced by a plane wave propagating through a plasma which is homogeneous across the wavefront but changes in the propagation direction is;

$$\phi_h(z_s) = \lambda_o \cdot r_e \cdot \int_{z_s - L}^{z_s} N_e(z) \cdot dz \quad (2-43)$$

$$\text{where; } r_e = \left(\frac{e^2}{m_e \cdot c^2} \right) = 2.82 \times 10^{-13} \text{ cm} \quad (2-44)$$

= classical electron radius

The total change of phase experienced by a plane wave propagating through a plasma which is slightly inhomogeneous (i.e.; having electron density irregularities) is given as;

$$\phi_i(\tilde{r}_s, z=z_s) = \lambda_o \cdot r_e \cdot \int_{z_s - L}^{z_s} N_e(\tilde{r}, z) \cdot dz \quad (2-45)$$

We then obtain for the difference in phase below the phase screen ($z=z_s$) due to the electron density irregularities;

$$\begin{aligned}\phi(\tilde{r}_s, z=z_s) &= \phi_i(\tilde{r}_s, z=z_s) - \phi_H(z=z_s) \\ &= \lambda_o \cdot r_e \cdot \int_{z_s-L}^{z_s} \Delta N_e(\tilde{r}, z) \cdot dz\end{aligned}\quad (2-46)$$

where;

$\Delta N_e(\tilde{r}, z)$ is the electron density perturbation superimposed on a plasma which is homogeneous across the wavefront.

For the thin phase screen approximation to hold we have the condition that $\phi < 1$ radian . Therefore we have from eq.(2-46) that;

$$\lambda_o \cdot r_e \cdot \int_{z_s-L}^{z_s} \Delta N_e(\tilde{r}, z) \cdot dz < 1 \quad (2-47)$$

We therefore can apply the thin phase screen approximation only if the perturbation in electron content per column satisfies;

$$\int_{z_s-L}^{z_s} \Delta N_e(\tilde{r}, z) \cdot dz < 5 \times 10^{10} [\text{electrons/cm}^2] \quad \text{for; 430MHz}$$

$$1 \quad < 25 \times 10^{10} [\text{electrons/cm}^2] \quad \text{for; 2380MHz}$$

Assuming that the irregularities extend over ~ 50 km we have for the upper limit on the average electron density perturbation;

$$\begin{aligned} \langle \Delta N_e \rangle &< 10^4 \text{ [electrons/cm}^3\text{]} \quad \text{for; 430 MHz} \\ & \quad \quad \quad (2-48) \\ \text{" } &< 5 \times 10^4 \text{ [electrons/cm}^3\text{]} \quad \text{for; 2380 MHz} \end{aligned}$$

Assuming further an electron density of $N_e \sim 10^6 \text{ [elec/cm}^{-3}\text{]}$ for the background plasma near the interaction height we obtain for the upper limit on the percentage electron density fluctuations;

$$\begin{aligned} \frac{\langle \Delta N_e \rangle}{N_e} &< 1 \% \quad \text{for; 430 MHz} \\ & \quad \quad \quad (2-49) \\ \frac{\langle \Delta N_e \rangle}{N_e} &< 5 \% \quad \text{for; 2380 MHz} \end{aligned}$$

If these limits are exceeded then the thin phase screen approximation is invalid.

b.) Intensity pattern at the lower boundary of the screen.

By the definition of a thin phase screen we have that the total phase variations experienced by the radio waves in transversing the irregular medium have to be less than one radian.i.e.;

$$\phi(\tilde{\mathbf{r}}_s, z_s) < 1 \text{ radian}$$

$\phi(\tilde{\mathbf{r}}_s, z_s)$ is the phase variation (2-46) of the electric field at the lower boundary of the phase screen. The electric field structure at the lower boundary can then be written as;

$$\tilde{\mathbf{E}}(\tilde{\mathbf{r}}_s, z_s) = \tilde{\mathbf{E}}_0 \cdot e^{i \cdot \phi(\tilde{\mathbf{r}}_s, z_s)} \quad (2-50)$$

$$= \tilde{\mathbf{E}}_0 \cdot (1 + i \cdot \phi(\tilde{\mathbf{r}}_s, z_s) + \dots)$$

$$= \tilde{\mathbf{E}}_0 + i \cdot \tilde{\mathbf{E}}_0 \cdot \phi(\tilde{\mathbf{r}}_s, z_s) \quad (2-51)$$

where; $\phi < 1$ radian and; $\mathbf{E}_0$ and ϕ are real.

comparing with eqs. (2-26, 2-27) for $z_c = z_s$ we have;

$$\tilde{\mathbf{E}}_R(\tilde{\mathbf{r}}_s, z_s) = 0 \quad (2-52)$$

Using eq. (2-29) this result implies that the intensity is constant and therefore there are no 1-st order scintillations immediately below a thin phase screen.

c.) The intensity pattern on the ground for an isotropic antenna.

The spatial phase variation across the phase screen $\phi(\tilde{\mathbf{r}}_s, z=z_s)$ is given by eq. (2-46). Its representation in $\tilde{\mathbf{K}}$ -space is obtained by the corresponding Fourier transform and we define;

$$\Psi(\tilde{\mathbf{K}}, z=z_s) = \int \phi(\tilde{\mathbf{r}}_s, z=z_s) \cdot e^{i\tilde{\mathbf{K}} \cdot \tilde{\mathbf{r}}_s} \cdot d\tilde{\mathbf{r}}_s \quad (2-53)$$

where $\phi(\tilde{\mathbf{r}}_s, z=z_s)$ is a real function and can be separated into an even and an odd contribution;

$$\phi(\tilde{\mathbf{r}}_s, z=z_s) = \phi^e(\tilde{\mathbf{r}}_s, z=z_s) + \phi^o(\tilde{\mathbf{r}}_s, z=z_s)$$

According to the transformation rules (2-35) the corresponding two terms of $\Psi(\tilde{\mathbf{K}}, z=z_s)$ are;

$$\Psi(\tilde{\mathbf{K}}, z=z_s) = \text{Re } \Psi^e(\tilde{\mathbf{K}}, z=z_s) + i \cdot \text{Im } \Psi^o(\tilde{\mathbf{K}}, z=z_s) \quad (2-54)$$

We also have from eqs. (2-51, 2-26) for $z_c = z_s$;

$$\tilde{\mathbf{E}}_d(\tilde{\mathbf{r}}_s, z=z_s) = i \cdot \tilde{\mathbf{E}}_o \cdot \phi(\tilde{\mathbf{r}}_s, z=z_s) \quad (2-55)$$

Separating the electric field given by eq. (2-7) into the incident field and the field perturbations produced by the irregularities at $z=z_s$;

$$\tilde{E}(\tilde{r}_s, z=z_s) = \tilde{E}_0 + \int_{\kappa \neq 0} \tilde{E}_s(\tilde{\kappa}) \cdot e^{-i\tilde{\kappa} \cdot \tilde{r}_s} \cdot d\tilde{\kappa} \quad (2-56)$$

comparing with eq. (2-26) for $z_c = z_s$;

$$\tilde{E}_d(\tilde{r}_s, z=z_s) = \int_{\kappa \neq 0} \tilde{E}_s(\tilde{\kappa}) \cdot e^{-i\tilde{\kappa} \cdot \tilde{r}_s} \cdot d\tilde{\kappa} \quad (2-57)$$

Using the inverse transform of (2-57) combined with eqs. (2-53, 2-55) ;

$$\tilde{E}_s(\tilde{\kappa}, z=z_s) = i \cdot \tilde{E}_0 \cdot \mathcal{Y}(\tilde{\kappa}, z=z_s) \quad \text{for; } \kappa \neq 0 \quad (2-58)$$

The electric field structure on the ground ($z=z_G$) due to the irregularities in the screen ($z=z_s$) may be written as (compare eqs. (2-20, 2-26)) ;

$$\tilde{E}_d(\tilde{r}_G, z=z_G) = \int_{\kappa \neq 0} \left[\tilde{E}_s(\tilde{\kappa}, z=z_s) \cdot e^{i \frac{\kappa^2}{\kappa_F^2}} \right] \cdot e^{-i\tilde{\kappa} \cdot \tilde{r}_G} \cdot d\tilde{\kappa} \quad (2-59)$$

Substituting from eq. (2-58) ;

$$\tilde{E}_d(\tilde{r}_G, z=z_G) = i \cdot \tilde{E}_0 \int_{\kappa \neq 0} \left[\mathcal{Y}(\tilde{\kappa}, z=z_s) \cdot e^{i \frac{\kappa^2}{\kappa_F^2}} \right] \cdot e^{-i\tilde{\kappa} \cdot \tilde{r}_G} \cdot d\tilde{\kappa} \quad (2-60)$$

Comparing eq. (2-60) with the general expression eq. (2-32) ;

$$\tilde{V}(\tilde{\kappa}) = i \cdot \tilde{E}_0 \cdot \mathcal{Y}(\tilde{\kappa}, z=z_s) \cdot e^{i \frac{\kappa^2}{\kappa_F^2}} \quad (2-61)$$

We then define for convenience;

$$F = \frac{\kappa^2}{\kappa_F^2}$$

Eq.(2-61) is then evaluated with the use of eq.(2-54) to be;

$$\begin{aligned} \tilde{\sigma}(\tilde{\kappa}) &= i \cdot \tilde{E}_0 \cdot [\operatorname{Re} \psi^e(\tilde{\kappa}) + i \cdot \operatorname{Im} \psi^o(\tilde{\kappa})] \cdot \\ &\quad [\cos(F) + i \sin(F)] \\ &= \tilde{E}_0 \{ i \operatorname{Re} \psi^e \cdot \cos(F) - \operatorname{Im} \psi^o \cdot \cos(F) \\ &\quad - \operatorname{Re} \psi^e \cdot \sin(F) - i \cdot \operatorname{Im} \psi^o \cdot \sin(F) \} \end{aligned}$$

where $\cos(F)$ and $\sin(F)$ are even functions of κ .

Comparing with eq.(2-33) matching terms according to their being even or odd, real or imaginary we obtain for the terms needed in eqs.(2-36,2-37);

$$\begin{aligned} \operatorname{Im} \tilde{\sigma}^o(\tilde{\kappa}) &= -\tilde{E}_0 \cdot \operatorname{Im} \psi^o(\kappa) \cdot \sin(F) \\ \operatorname{Re} \tilde{\sigma}^e(\tilde{\kappa}) &= -\tilde{E}_0 \cdot \operatorname{Re} \psi^e(\kappa) \cdot \sin(F) \end{aligned} \tag{2-62}$$

We then obtain for the intensity pattern on the ground after substituting eqs.(2-62) into eq.(2-37) and simplifying with the use of eq.(2-54);

$$I(\tilde{r}_G, z=z_G) = I_0 - 2 \cdot \tilde{E}_0 \cdot \tilde{E}_0 \int \psi(\tilde{\kappa}, z=z_s) \cdot \sin(F) \cdot e^{-i\tilde{\kappa} \cdot \tilde{r}_G d\tilde{\kappa}} \tag{2-63}$$

Normalizing the intensity-fluctuations;

$$\Delta I(\tilde{\mathbf{r}}_G, z=z_G) = I(\tilde{\mathbf{r}}_G, z=z_G) - I_0$$

by setting; $I_0 = |\tilde{\mathbf{E}}_0|^2 = 1$

we have for the intensity fluctuations;

$$\Delta I(\tilde{\mathbf{r}}_G, z=z_G) = -2 \cdot \int \Psi(\tilde{\mathbf{K}}, z=z_s) \cdot \sin(F) \cdot e^{-i\tilde{\mathbf{K}} \cdot \tilde{\mathbf{r}}_G} d\tilde{\mathbf{K}} \quad (2-64)$$

If we Fourier transform the spatial intensity fluctuations $\Delta I(\tilde{\mathbf{r}}_G, z=z_G)$ we obtain its representation in $\tilde{\mathbf{K}}$ -space;

$$\Delta I(\tilde{\mathbf{K}}, z=z_G) = \int \Delta I(\tilde{\mathbf{r}}_G, z=z_G) \cdot e^{i\tilde{\mathbf{K}} \cdot \tilde{\mathbf{r}}_G} d\tilde{\mathbf{r}}_G \quad (2-65)$$

with its inverse transform;

$$\Delta I(\tilde{\mathbf{r}}_G, z=z_G) = \int \Delta I(\tilde{\mathbf{K}}, z=z_G) \cdot e^{-i\tilde{\mathbf{K}} \cdot \tilde{\mathbf{r}}_G} d\tilde{\mathbf{K}} \quad (2-66)$$

Comparing eq.(2-66) with eq.(2-64) we obtain for the intensity fluctuations of any particular irregularity scale size;

$$\Delta I(\tilde{\mathbf{K}}) = -2 \cdot \Psi_s(\tilde{\mathbf{K}}) \cdot \sin(F) \quad (2-67)$$

where;

$$\mathbf{K} = \frac{2 \pi}{\lambda_{ir}}$$

The power spectrum of the spatial intensity fluctuations $\Delta I(\tilde{\mathbf{r}}_G)$ is then;

$$\Delta I^2(\kappa) = 4 \cdot \mathcal{F}_s^2(\kappa) \cdot \sin^2\left(\frac{\kappa^2}{\kappa_F^2}\right) \quad (2-68)$$

which is the same result as obtained by [Salpeter, 1967; equation 15]. The last factor is called the Fresnel filtering term and determines what scale sizes of irregularities any scintillation experiment is most sensitive to. This Fresnel filter as a function of κ is plotted in Fig. 2.2 for reference.

If we neglect the Fresnel filter we obtain by substituting eq. (2-53) into eq. (2-64) and interchanging the order of integration;

$$\Delta I(\tilde{r}_G, z=z_G) = -2 \cdot \int \phi(\tilde{r}_s, z=z_s) \int e^{i\tilde{\mathbf{r}} \cdot (\tilde{r}_s - \tilde{r}_G)} \cdot d\tilde{\mathbf{r}} \cdot d\tilde{r}_s$$

The integral over $\tilde{\mathbf{r}}$ represents the Dirac delta function and requires that;

$$\begin{aligned} \int e^{i\tilde{\mathbf{r}} \cdot (\tilde{r}_s - \tilde{r}_G)} \cdot d\tilde{\mathbf{r}} &= \delta(\tilde{r}_s - \tilde{r}_G) = 0 \quad \text{for } \tilde{r}_s \neq \tilde{r}_G \\ &= 1 \quad \text{for } \tilde{r}_s = \tilde{r}_G \end{aligned}$$

We therefore obtain for the intensity fluctuations on the ground as a function of the phase variations within the screen;

$$\Delta I(\tilde{r}_G, z=z_G) = -2 \cdot \phi(\tilde{r}_G, z=z_s) \quad (2-69)$$

where ΔI is expressed in fractions of $I_0 = 1$ and ϕ is in radian.

This expression allows us to deduce the actual electron density perturbations (using eq.(2-46)) from the intensity fluctuations on the ground. The intensity fluctuations used in such calculations however have to be corrected for the filtering effects present in eqs.(2-67,2-88) and Fig.2.5.

Example: Intensity fluctuations due to irregularities with $\lambda_{irr} \sim 680m$ are diminished by a factor of $\sqrt{.59}$ as seen from Fig.2.5 and eq.(2-88). Therefore the actual electron density perturbations are larger than estimated by eq.(2-69) by a factor of;

$$\frac{1}{\sqrt{.59}} = 1.3$$

similarly irregularities with $\lambda_{irr} \sim 380m$ are underestimated by eq.(2-69) by a factor of $\sqrt{.16}$ as seen from Fig.2.5. Therefore the actual electron density perturbations are larger by a factor of;

$$\frac{1}{\sqrt{.16}} = 2.5$$

The Fresnel filtering function for
irregularity spectra measured at the
Arecibo Observatory

Figure 2.2.

d.) The effect of the Fresnel-filter on the intensity pattern.

This section serves to illustrate in a more intuitive way the Fresnel filter function derived in the previous section. Some important effects as to the type of intensity fluctuations on the ground will become apparent. Let us assume a plane wave is incident on a thin phase screen with irregularities of only one wavelength $\lambda_{ir} = 2\pi/\kappa$. Immediately below the phase screen the electric field vector may be represented by the phase diagram shown in Fig.2.3.a. We have the unperturbed wave-field represented by $\tilde{E}_0$ and we have the electric field $\tilde{E}_d = i \cdot \tilde{E}_0 \cdot \phi$ due to the irregularities in the screen (see eq.(2-55)). It is the fact that the phase ϕ is much less than one radian (i.e.; single scattering) and the fact that $\tilde{E}_d$ is purely imaginary ($\tilde{E}_0 \cdot \phi$ is real) which accounts for the often used term "thin phase screen". As was demonstrated in section 2.2.b this would not by itself produce first order intensity fluctuations. As the radiation field propagates to the ground however the field perturbation $\tilde{E}_d$ due to the irregularities at B (see Fig.2.3.c) experience a phase shift relative to the unperturbed wave field $\tilde{E}_0$ when observed at point C due to the longer path length. This phase shift is essential to produce intensity fluctuations from phase irregularities. The phase shift for a given wavelength of the irregularity is determined by the factor $\exp[i \cdot (\kappa^2 / \kappa_F^2)]$ as derived in section 2.1.a. The effect of this phase shift is demonstrated in Fig.2.3.b. As the phase angle

determined by $\exp[i \cdot (\kappa^2 / \kappa_F^2)]$ increases (i.e.; the irregularity scale size decreases), the vector of the electric field due to the irregularities $\tilde{E}_d = i \cdot \tilde{E}_0 \cdot \phi$ rotates and $\tilde{E}_d$ has an increasing real component pointing in the opposite direction to $\tilde{E}_0$. This makes the intensity fluctuations $\Delta I = \tilde{E}_0 \cdot \tilde{E}_d$ negative. In other words, for irregularities with a scale size $\lambda_{ir} = 2 \cdot \pi / \kappa$ such that $\kappa^2 / \kappa_F^2 \cong \pi / 2$ an increase in ϕ (or in the electron density N_e) will result in a decrease of the intensity measured on the ground. A decrease in electron density in turn will produce positive intensity fluctuations for $\kappa^2 / \kappa_F^2 \cong \pi / 2$. For arbitrary κ^2 / κ_F^2 the efficiency of conversion between phase fluctuations in the screen and the intensity fluctuations on the ground is given by the real part of the phase factor $i \cdot \exp[i \cdot (\kappa^2 / \kappa_F^2)]$. This is the mathematical origin of the Fresnel filter function $\sin(\kappa^2 / \kappa_F^2)$. It becomes now also apparent that the zeroes of the Fresnel filter function correspond to the condition that $\tilde{E}_d = i \cdot \tilde{E}_0 \cdot \phi(\kappa) \cdot \exp[i \cdot (\kappa^2 / \kappa_F^2)]$ is purely imaginary (i.e.; $\kappa^2 / \kappa_F^2 = n \cdot \pi$ (where; $n=1,2,\dots$)) and no intensity scintillations are produced.

In other words, no intensity scintillations are produced on the ground for phase irregularities in the screen which satisfy the condition $\lambda_{ir} = \ell_{Fn} / n$ where $\ell_{Fn} = \sqrt{n \cdot \lambda \cdot z_0}$ is the radius for the n -th Fresnel zone. We may now in general conclude that for;

$$(n-1) \cdot \pi < \kappa^2 / \kappa_F^2 < n \cdot \pi \quad \text{where; } n=1,2,3,\dots$$

- a) if n is odd (corresponding to the odd Fresnel zones) the real part of $\tilde{E}_d$ is opposite to $\tilde{E}_0$ in Fig.2.3.b and electron density depletions (re-

presented by negative phase in the phase screen) will be measured as positive intensity fluctuations.

- b) if n is even (corresponding to the even Fresnel zones) the real part of $\tilde{E}_d$ is in the same direction as $\tilde{E}_0$ in Fig.2.3.b and electron density depletions (represented by negative phase in the phase screen) will be measured as negative intensity fluctuations.

All the features discussed above are conveniently contained within the factor $\sin(\kappa^2/\kappa_F^2)$ and the eq.(2-67). It is also apparent that the Fresnel filter is a purely geometrical effect depending only on spatial parameters and not on any given statistics of the irregularities.

2.3. Experimental considerations.

In a radio-star scintillation experiment one measures the intensity fluctuations as a function of time with the radio-star moving at a rate of 15° per hour from east to west and with the ionospheric irregularities drifting themselves with respect to the observer. To understand how these intensity fluctuations are related to the spatial electron density irregularities in the ionosphere several experimental factors have to be considered. For highly directive antennas which have an angular beamwidth ($\sim \lambda / 2 \cdot S_0$) comparable to or smaller than the angular size of the first Fresnel-zone ($\sim 2\sqrt{\lambda / z_0}$) the effect of the antenna on the observed irregularity power spectra is very important. As will be shown in section a.) the measured irregularity power spectra contain an antenna filtering term which makes the measurements very insensitive to irregularities with scale sizes comparable to or smaller than the diameter (see Fig.2.4.) of the observing antenna. Using the Arecibo 300-m dish for the measurements we therefore cannot observe efficiently electron density irregularities with wavelengths smaller than 300m.

The necessity for a relative drift between the electron density irregularities and the line of sight to the radio source (to be called the scanning velocity) is pointed out in section b.). The effect of this relative drift on the quality of the power spectra (i.e.; spectral resolution) is also discussed there.

In the derivation of the expressions in section 2.1. & 2.2. it was assumed that the incident plane waves were monochromatic. The radiation field coming from a radio star is only quasimonochromatic however with limited coherence in time and space. To obtain information on the phase screen the unperturbed field $\tilde{E}_0$ and the field $\tilde{E}_d$ diffracted by the phase irregularities in the screen need to have some coherence. Only the coherent part of the radiation field can interact to produce the interference effects between the unperturbed wavefronts coming from the source and the wavefronts diffracted by the phase irregularities. Without this coherence there would be no interference effects and therefore no scintillations due to the diffracted waves. If the field incident at point A (Fig.2.6.) is $\tilde{E}_0(\tilde{r}_s, t')$ then the field incident at point B which is responsible for the scintillation effects at the antenna is given by $\tilde{E}_0(\tilde{r}_s + \tilde{\mathcal{K}}, t' - \tilde{\tau})$. The displacement $\tilde{\mathcal{K}}$ (Fig.2.6.) represents the separation within the screen between a phase irregularity of a given wave-vector $\mathcal{K}$ (located at point B) and the line of sight to the radio source (point A) and is given by;

$$\tilde{\mathcal{K}} = \frac{\mathcal{K}}{k} \cdot z_0 \quad \text{for small } \frac{\mathcal{K}}{z_0}$$

The time delay $\tilde{\tau}$ represents the additional pathlength $\mathcal{X}$ (Fig.2.6.) that the diffracted field has to travel when propagating from point D to the antenna. $\tilde{\tau}$ may also be written as a function of $\mathcal{K}$ (see section d.);

$$\tilde{\tau} \approx \frac{z_0}{2 \cdot c} \left(\frac{\mathcal{K}}{k} \right)^2$$

The complex degree of coherence between the electric fields $\tilde{E}_0(\tilde{r}_s, t')$ and $\tilde{E}_0(\tilde{r}_s + \tilde{\lambda}, t' - \tau)$ measured at the two different locations (separated by a distance $\tilde{\lambda}$) and at two different instants in time (separated by τ) can be defined as [Born & Wolf, 1965; Perina, 1971];

$$\Gamma(\tilde{\lambda}, \tau) = \Gamma(\tilde{\lambda}) \cdot \Gamma(\tau) \quad (2-71)$$

where we have for the spatial degree of coherence;

$$\Gamma(\tilde{\lambda}) = \frac{\int \tilde{E}_0(\tilde{r}_s) \cdot \tilde{E}_0^*(\tilde{r}_s + \tilde{\lambda}) \cdot d\tilde{r}_s}{\int |\tilde{E}_0(\tilde{r}_s)|^2 \cdot d\tilde{r}_s} \quad (2-72)$$

and for the temporal degree of coherence;

$$\Gamma(\tau) = \frac{\int \tilde{E}_0(t') \cdot \tilde{E}_0^*(t' - \tau) \cdot dt'}{\int |\tilde{E}_0(t')|^2 \cdot dt'} \quad (2-73)$$

Since both coherence functions can be expressed as functions of the irregularity wavevector $\tilde{\mathcal{K}}$ and since the coherence is given by the modulus of the complex degree of coherence we can rewrite eq. 2-63) as;

$$I(\tilde{r}_G, z=z_G) = I_0 - 2\tilde{E}_0 \cdot \tilde{E}_0 \cdot \int |\Gamma_S(\tilde{\mathcal{K}})| \cdot |\Gamma_T(\tilde{\mathcal{K}})| \cdot \mathcal{V}(\tilde{\mathcal{K}}, z=z_S) \cdot \sin(F) \cdot e^{-i\tilde{\mathcal{K}} \cdot \tilde{r}_G} \cdot d\tilde{\mathcal{K}}$$

where;

$$\begin{aligned} \Gamma_S(\tilde{\mathcal{K}}) &= \Gamma(\tilde{\lambda}) \\ \Gamma_T(\tilde{\mathcal{K}}) &= \Gamma(\tau) \end{aligned} \quad (2-74)$$

Similarly we can rewrite eq.2-87 and using the steps that led from eq.(2-63) to (2-68) we obtain for the irregularity wavevector power spectrum instead of eq.2-89 ;

$$|\Delta I(\hat{\mathbf{K}})|^2 = 4 \cdot |g(\hat{\mathbf{K}})|^2 \cdot \sin^2\left(\frac{\kappa^2}{\kappa_F^2}\right) \cdot \mathcal{A}^2(\hat{\mathbf{K}}) \cdot |\Gamma_S(\hat{\mathbf{K}})|^2 \cdot |\Gamma_T(\hat{\mathbf{K}})|^2$$

(2-75)

The spatial coherence $\Gamma_S(\hat{\mathbf{K}})$ will be determined by the angular size of the radio source and is further discussed in section c.

The temporal coherence $\Gamma_T(\hat{\mathbf{K}})$ will be determined by the bandwidth of the measured radiation field and is further discussed in section d.

a.) The effects of an antenna with a circular aperture.

The electric field structure $\tilde{E}_a(\tilde{r}, z=z_G)$ measured by a circular antenna of radius ρ_0 is obtained as the average of the electric field $\tilde{E}(\tilde{r}_G, z=z_G)$ given by eq. (2-20) over the whole receiving area A (refer to Fig.2.1.) weighted by the aperture distribution function of the antenna. In the following derivation the aperture distribution function is assumed to be uniform. Hence;

$$\tilde{E}_a(\tilde{r}_G, z=z_G) = \frac{1}{A} \cdot \int_A \tilde{E}(\tilde{r}_G + \tilde{\mathcal{G}}, z=z_G) \cdot d\tilde{\mathcal{G}} \quad (2-80)$$

$$\tilde{E}_a(\tilde{r}_G) = e^{ikz_0} \left\{ \tilde{E}_0 + \frac{1}{A} \cdot \int_A \int_{\kappa \neq 0} \tilde{E}_s(\tilde{\kappa}) \cdot e^{iF} \cdot e^{-i\tilde{\kappa} \cdot (\tilde{r}_G + \tilde{\mathcal{G}})} d\tilde{\kappa} d\tilde{\mathcal{G}} \right\} \quad (2-81)$$

where; $F = \frac{\kappa^2}{\kappa_F^2}$

Interchanging the order of integration;

$$\tilde{E}_a(\tilde{r}_G, z=z_G) = e^{ikz_0} \left\{ \tilde{E}_0 + \frac{1}{A} \int_{\kappa \neq 0} \tilde{E}_s(\tilde{\kappa}) \cdot e^{iF} \cdot e^{-i\tilde{\kappa} \cdot \tilde{r}_G} \int_A e^{-i\tilde{\kappa} \cdot \tilde{\mathcal{G}}} d\tilde{\mathcal{G}} d\tilde{\kappa} \right\} \quad (2-82)$$

It can be shown (Appendix A) that for a disk of radius ρ_0 and area A ;

$$\int_A e^{i\tilde{\kappa} \cdot \tilde{\mathcal{G}}} d\tilde{\mathcal{G}} = A \cdot \mathcal{A}(\kappa) \quad (2-83)$$

where we define the antenna filtering term $\mathcal{A}(\kappa)$ as;

$$\mathcal{A}(\kappa) = 2 \cdot \frac{J_1(\kappa \cdot \rho_0)}{(\kappa \cdot \rho_0)} \quad (2-84)$$

The electric field structure measured by the antenna may then be written as;

$$\tilde{E}_a(\tilde{r}_G, z=z_G) = e^{ikz_0} \left\{ \tilde{E}_0 + \int_{\kappa \neq 0} \tilde{E}_s(\tilde{\kappa}) \cdot e^{iF} \mathcal{A}(\kappa) \cdot e^{-i\tilde{\kappa} \cdot \tilde{r}_G} d\tilde{\kappa} \right\} \quad (2-85)$$

comparing the curly bracket with eqs. (2-26, 2-32) we obtain;

$$\tilde{U}(\tilde{\kappa}) = \tilde{E}_s(\tilde{\kappa}) \cdot e^{iF} \mathcal{A}(\kappa) \quad (2-86)$$

substituting from eq. (2-58);

$$\tilde{U}(\tilde{\kappa}) = i \cdot \tilde{E}_0 \cdot \mathcal{Y}(\tilde{\kappa}, z=z_s) \cdot e^{iF} \mathcal{A}(\kappa)$$

This is the same as eq. (2-61) except for $\mathcal{A}(\kappa)$. Since $\mathcal{A}(\kappa)$ is real and even the results of eq. (2-62) are unaffected and $\mathcal{A}(\kappa)$ is just a multiplication factor in all the terms on the righthand side. We therefore have for the intensity instead of eq. (2-64);

$$I(\tilde{r}_G, z=z_G) = I_0 - 2 \cdot \tilde{E}_0 \cdot \tilde{E}_0 \cdot \int [\mathcal{Y}(\tilde{\kappa}, z=z_s) \cdot \sin(F) \cdot \mathcal{A}(\kappa)] e^{i\tilde{\kappa} \cdot \tilde{r}_G} d\tilde{\kappa} \quad (2-87)$$

Starting with eq. (2-87) and using the steps that led from eq. (2-63) to eq. (2-67) we obtain for the intensity fluctuations of any particular irregularity scale size;

$$\Delta I(\tilde{\kappa}) = -2 \cdot \mathcal{Y}(\tilde{\kappa}) \sin(F) \cdot \mathcal{A}(\kappa) \quad (2-88)$$

And finally we have for the power spectrum $|\Delta I(\tilde{\kappa})|^2$ of the spatial intensity fluctuations $\Delta I(\tilde{r}_G)$;

$$|\Delta I(\tilde{\kappa})|^2 = 4 \cdot |\Psi(\tilde{\kappa})|^2 \cdot \sin^2\left(\frac{\kappa^2}{\kappa_F^2}\right) \cdot \mathcal{A}^2(\kappa) \quad (2-89)$$

This last factor (the square of eq.(2-84)) representing the antenna filtering effect on the power spectrum of the intensity fluctuations for any given power spectrum of the phase fluctuations in the screen is plotted as a function of κ in Fig.2.4. for two different radii S_0 . At the Arecibo Observatory $S_0 = 150\text{m}$ corresponds to the effective area of the spherical reflector which is illuminated by the 430 MHz feed and $S_0 = 110\text{m}$ corresponds to the effective area illuminated by the 1400 MHz and the 2380 MHz feeds. It is seen that this filter greatly reduces the sensitivity to observe phase irregularities with scale sizes less than two times the antenna radius S_0 (i.e.; less than the aperture diameter). The combined effects of the Fresnel filtering and the antenna filtering on the wavevector power spectra of the intensity fluctuations (measured on the ground) for any given wavevector power spectra of the phase fluctuations (in the screen) is illustrated in Fig.2.5. It represents the product of Fig. 2.2. (the Fresnel filter) and Fig.2.4. (the antenna filter) for different values of the height of the screen and different observing frequencies.

The antenna filtering function for
irregularity spectra measured at the
Arecibo Observatory

Figure 2.4.

The filtering (sensitivity) function for
the irregularity spectra measured at the
Arecibo Observatory

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Fig.2.5.

The filtering (sensitivity) function for the
irregularity spectra measured at the Arecibo

Observatory

Figure 2.5.

b.) The effect of relative motion between the radio source and the irregularities.

The spatial intensity pattern calculated in the previous sections would not be very useful if it was stationary with respect to the antenna. We need the intensity pattern to drift past the antenna so that we can record the intensity fluctuations as a function of time. Fortunately the earth rotates and therefore the radio source appears to move from east to west at a rate of $15^\circ/\text{hr}$. If the irregularities are assumed stationary with respect to the earth and if they are located at a height h , they will be scanned by the line of sight to the radio source at an approximate rate of;

$$v_{\text{RS}} [\text{m/s}] = 7.3 \times \left(\frac{h [\text{km}]}{100 \text{ km}} \right)$$

For irregularities at a height of 300 km we then have $v_{\text{RS}} = 22 \text{ m/s}$. In addition the irregularities themselves drift (with the background plasma) at speeds up to $v_{\text{dr}} \sim 50 \text{ m/s}$ [Richmond et.al., 1980]. The relative velocity between the drifting irregularities and the line of sight to the radio source will from now on be called the scanning velocity v_{sc} . From the measured temporal intensity fluctuations we will calculate frequency power spectra as discussed in section 3.d. A high scanning velocity will improve the spectral resolution of features in the frequency power spectra, since a given irregularity wavelength will translate into a higher frequency (i.e.; contain more elementary frequency resolution intervals).

The frequency resolution of the spectra $\Delta\nu$ is given as;

$$\Delta\nu = \frac{1}{T_s}$$

where; T_s = sampling period

In the present experiment T_s is at the most on the order of 1/2 hour, hence $\Delta\nu$ will always be larger than 1/2 mHz. In the previous sections it was shown that the longest wavelengths of the irregularities observable in the present experiment are on the order of $\lambda_{ir} \sim 1000$ m. With a scanning velocity of 10 m/s we have for the frequency of the intensity fluctuations ν ;

$$\nu = \frac{v_{sc}}{\lambda_{ir}} = \frac{10}{1000} = 10 \text{ mHz (i.e.; } \sim 20 \cdot \Delta\nu)$$

The drifts reported by [Richmond et.al.,1980] are westward (in the same direction as the motion of the radio source) between 0200 LST and 1400 LST, but eastward (opposite to the motion of the radio source) the rest of the time. From this point of view it is best to do the experiment between late afternoons and midnight to obtain a high relative velocity v_{sc} between the drifting irregularities and the line of sight to the radio source.

c.) Source size ; transverse spatial coherence.

As was shown at the beginning of this chapter the intensity fluctuations measured at the antenna are dependent on the existence of partial transverse spatial coherence defined as $\Gamma(\Lambda)$. If the angular diameter of the source is small and Λ is also small compared to the distance of the radio source (a condition well satisfied by celestial radio sources selected for this experiment) then spatial coherence can be calculated from the normalized Fourier transform of the intensity distribution over the source. This is known in optics as the Van Cittert-Zernike theorem [Perina, 1971; eq. 3.57.; Born & Wolf, 1965]. The intensity distribution of the radio source is usually not known in detail. If we assume a 2-dimensional Gaussian distribution with an approximate radius r_0 for the source, we obtain;

$$\begin{aligned}\Gamma(\Lambda) &= \frac{\int e^{-\pi \cdot (\frac{r}{r_0})^2} \cdot e^{i \cdot k \cdot \frac{\Lambda}{s} \cdot r} dr}{\int e^{-\pi \cdot (\frac{r}{r_0})^2} \cdot dr} \\ &= e^{-\pi \cdot r_0^2 \cdot k^2 \cdot (\frac{\Lambda}{s})^2} = e^{-\pi \frac{\alpha^2}{4} \cdot k^2 \cdot \Lambda^2}\end{aligned}$$

where; s is the distance of the radio source

$$\alpha = \frac{2 \cdot r_0}{s} \quad \text{is the angular diameter of the radio source.}$$

From Fig. 2.6. we have for small Θ ;

Figure 2.6.

$$\Delta \cong \theta \cdot z_0 \cong \frac{\kappa}{k} \cdot z_0$$

We therefore have for $|\Gamma_S(\kappa)|$ in eq.2-85 ;

$$|\Gamma_S(\kappa)| = e^{-\frac{\pi}{4} \cdot \alpha^2 \cdot \kappa^2 \cdot z_0^2}$$

To have a transverse spatial coherence close to unity we require;

$$\alpha < \frac{1}{2\pi} \cdot \frac{\lambda_{ir}}{z_0}$$

Choosing as the shortest irregularity wavelength which we are able to observe with the antenna used (see Fig.2.4.);

$$\lambda_{ir} \cong 2 \cdot s_0 \cong 300\text{m}$$

and for the height of the irregularities;

$$z_0 = 300 \text{ km}$$

we have for the maximum allowable angular diameter of the radio source;

$$\alpha \leq 0.6 \text{ arcmin.}$$

d.) Bandwidth ; temporal coherence.

If we wish to measure diffraction effects (such as used to explain scintillations) we have to make sure that there is some temporal coherence in the radiation field which is longer than the time delays involved in the diffraction problem (due to different propagation paths). For a more detailed discussion see the beginning of this chapter. The temporal coherence in the radiation field measured at some location is determined by the accepted frequency bandwidth of the measurement. Most often this bandwidth is chosen by an appropriate filter and in what follows we choose a Gaussian filter (as was done in the experiment) with the following frequency response;

$$\mathcal{F}(f) = e^{-\left(\frac{f - f_o}{b}\right)^2} \quad (2-90)$$

where;

$$f_o = \frac{c}{\lambda_o} \quad \text{is the center frequency}$$

$$b = \quad \text{frequency bandwidth}$$

The corresponding time response is obtained by a Fourier transform as;

$$f(\tau_c) = e^{-\pi^2 \cdot b^2 \cdot \tau_c^2} \quad (2-91)$$

The electric fields measured at two instants in time separated by a time τ_c are therefore related as;

$$\hat{E}_o(t' - \tau_c) = \tilde{E}_o(t') \cdot f(\tau_c)$$

substituting into eq.(2-73) we obtain for the coherence in the measured radiation field;

$$\Gamma_T(\tau_c) = f(\tau_c) \quad (2-93)$$

The coherence between radiation from point C (see Fig.2.6.) and radiation from point D is therefore determined by the frequency bandwidth and by the time delay $\tau_c = \chi/c$ (see Fig.2.6.) and can be written as;

$$\Gamma_T(\tau_c) = e^{-\pi^2 \cdot b^2 \cdot \tau_c^2} \quad (2-94)$$

We also have from Fig.2.6. that;

$$z_o^2 + \Lambda^2 = (z_o + \chi)^2$$

for small θ (i.e.; small χ);

$$\Lambda^2 \cong 2 \cdot z_o \cdot \chi$$

$$\theta \cong \frac{\Lambda}{z_o}$$

and; $\theta \cong \sin \theta = \frac{\kappa}{k}$

therefore;

$$\tau_c = \frac{\chi}{c} \cong \frac{\Lambda^2}{2 \cdot z_o \cdot c} \cong \frac{z_o \cdot \theta^2}{2 \cdot c} = \frac{z_o}{2 \cdot z} \left(\frac{\kappa}{k} \right)^2$$

using eq.(2-16) we may write;

$$\tau_c = \frac{1}{2 \cdot \pi \cdot f_o} \cdot \left(\frac{\kappa^2}{\kappa_F^2} \right) \quad (2-96)$$

The temporal coherence between the radiation fields received from point C and the radiation fields received from point D can then be reexpressed in terms of the irregularity wavevector κ by the use of eq. (2-96);

$$\Gamma_T(\tau_c) = \Gamma_T(\kappa) = e^{-\left(\frac{b^2}{4 \cdot f_o^2}\right) \cdot \left(\frac{\kappa}{\kappa_F}\right)^4}$$

Assuming that the shortest irregularity wavelength that can be detected by the antenna (see Fig. 2.4.) is;

$$\lambda_{ir} \cong 2 \cdot \lambda_o \cong 300 \text{ m}$$

and that the irregularities are at a height of 300 km we can calculate the coherence to be expected for the different observing frequencies. If we select the largest bandwidth possible we obtain the lowest coherence for a given κ . According to the Arecibo Observatory users manual;

$$\text{maximum bandwidth} \left\{ \begin{array}{ll} b = 10 \text{ MHz} & \text{for } f_o = 430 \text{ MHz} \\ b = 50 \text{ MHz} & \text{for } f_o = 1400 \text{ MHz} \\ b = 40 \text{ MHz} & \text{for } f_o = 2380 \text{ MHz} \end{array} \right.$$

We then obtain for the coherence Γ_T ;

$$\begin{array}{ll} \Gamma_T = 0.9928 & \text{for } f_o = 430 \text{ MHz} \\ \Gamma_T = 0.9985 & \text{for } f_o = 1400 \text{ MHz} \\ \Gamma_T = 0.9999 & \text{for } f_o = 2380 \text{ MHz} \end{array}$$

The coherences are close to perfect. We should therefore choose the largest bandwidth possible so that we can improve the signal to noise as discussed in section 3.b.

3. The experiment.

The experiment was conducted at the Arecibo Observatory in Puerto Rico during three observation periods. The first observations were between the 2 and 13 September 1980, the second set of observations was between 15 and 23 December 1980 and the final observations were between the 2 and 7 April 1981. Ionospheric irregularities were produced by an HF-facility located 17 km to the north-northeast of the Arecibo Observatory. The HF-facility is described in section 3.a. Information on ionospheric irregularities was obtained by measuring fluctuations in the intensity of discrete radio sources using the 1000 ft.-dish of the Arecibo Observatory. The measurement of these intensity fluctuations is described in section 3.b. Since the volume modified by the HF-waves has some finite dimensions (on the order of tens of km) and since the HF-facility is located 17 km from the 300 m dish we have to select sources with declinations on the sky such that the line of sight will intersect the modified volume at some time. This is discussed in section 3.c. where the observing geometry is considered in detail. And finally in section 3.d. the procedure for data acquisition and analysis is described. On the basis of different considerations a choice of suitable sources had to be made. These considerations are summarized in section 3.e. and a table of the most suitable sources is given.

a.) The HF-facility.

The HF-facility is located at a distance of 17 km from the 300 m dish of the Arecibo Observatory which was used for the scintillation measurements. For more details on the observing geometry see section 3.c. The HF-antenna consists of an array of 32 inverted pyramid log-periodic antennas. The 4x8 array is made of two halves aligned in an east west direction. Each half of the array consists of 4 by 4 inverted pyramids with a separation of 85 m between the centers of the pyramids. The transmitters can feed the full array or only one half of the array. When the full antenna was fed there were 8 antenna elements in the east west direction and if only half the antenna was fed there were 4 elements in the east west direction. In either case there were 4 antenna elements in the north south direction. The HF-frequencies f_{HF} used in the present experiment are given in Table 3.1. with the corresponding halfpower beamwidth for 4 antenna elements and for 8 antenna elements. The transmitters are rated at 200 kW each but due to technical problems the total power input into the antenna never exceeded 300 kW during the observations reported in this dissertation. The gain of the full HF-antenna at the different HF-frequencies was measured on a scaled model (A.Veldhuis, private communication) and is also given in Table 3.1. The power density at any given height (neglecting absorption effects) is then calculated by;

$$\mathcal{P} = \frac{P \cdot G}{4 \cdot \pi \cdot h^2}$$

where; P = total power fed into the antenna

G = gain of antenna

h = height of interest

For a height of 300 km and during operation of 4 transmitters at 75 kW each feeding the full antenna, the power density has been calculated and is also listed in Table 3.1.

f_{HF} [MHz]	λ_{HF} [m]	HPBW(4 elements) [rad]	HPBW(8 elements) [rad]	Gain* [dB]	Power-density [$\mu W / m^2$]
3.2	95	.28	.14	28.5	95
5.1	59	.19	.090	25.0	42
7.3	41	.14	.064	24.0	33
8.2	37	.13	.059	24.5	37

Table 3.1. HF-beam parameters

- *) full antenna (4 x 8 elements) - measured on a scale model.
- §) for an ionospheric height of $h=300$ km, assuming 4 transmitters (75 kw each) are feeding the full antenna array.
neglecting absorption and other ionospheric effects on the HF-waves,
assuming an overall efficiency of 0.5 .

b.) Measuring scintillations with the Arecibo
radiotelescope.

The scintillation measurements were done at the observing frequencies of 430 MHz, 1400 MHz and 2380 MHz. At these frequencies the Arecibo radiotelescope has the following system temperatures and sensitivities (users manual for the Arecibo Observatory) ;

<u>f_o</u>	<u>T_{sys}</u>	<u>sensitivity</u>
430 MHz	120°K	19.7°K/f.u.
1400 MHz	80°K	8.0°K/f.u.
2380 MHz	45°K	6.5°K/f.u.

where; 1 f.u. (flux unit) = $10^{-26} \text{ W/m}^2 \text{ Hz}$

To measure intensity scintillations due to the irregularities in the ionosphere we have to be able to measure the intensity with an accuracy that is better than the intensity fluctuations which we wish to measure. The statistical error in measuring the intensity (which is commonly expressed as an equivalent temperature) is given by the following equation [Kraus ,1966];

$$\Delta T = \frac{T_{\text{sys}}}{\sqrt{b \cdot \tau_d}}$$

where;

- T_{sys} = system temperature
- b = bandwidth of predetection stage
- τ_d = postdetection integration time
- ΔT = error in the measurement of T

$$T = T_{\text{sys}} + T_{\text{F}} = \begin{array}{c} \text{system plus source} \\ \text{temperature} \end{array}$$

Ionospheric scintillations which are measured with the 300 m dish of the Arecibo Observatory are produced by irregularities that have a wavelength longer than 300 m, the shortest wavelength that can be detected by the antenna according to the thin phase screen approximation (see Fig.2.4.). Assuming that the line of sight to the radio source scans the irregularities with a maximum scanning velocity (at the height of the irregularities) of 100 m/s we have as the shortest possible intensity fluctuation period about 3 seconds. It should therefore be sufficient to select for the integration time constant about 1 second. The choice of the frequency bandwidth b is a compromise between two effects, namely;

- a) Decreasing the error in the intensity measurement according to the above equation by increasing the bandwidth.
- b) Increasing the coherence time by decreasing the bandwidth b and therefore increasing the coherence over a volume of the ionosphere from where contributions to the intensity fluctuations are expected. This coherence is essential for a diffraction phenomena such as scintillations and is further discussed in section 2.3.d.

It was shown in section 2.3.d. that as far as the temporal coherence T_T is concerned there is no restriction on the bandwidth b other than the systems limitations (usually the feed bandwidth). If we use a filter with the maximum

bandwidth allowed by the system for any observing frequency
we have for the error in the intensity measurement ΔT ;

$\underline{f_o}$	$\underline{T_{sys}^*)}$	$\underline{b^*)}$	$\underline{\Delta T}$
430 MHz	120°K	10 MHz	0.038°K
1400 MHz	80°K	50 MHz	0.011°K
2380 MHz	45°K	40 MHz	0.007°K

*) quoted from the users manual for the Arecibo Observatory

If the flux of the discrete radio sources used for the
scintillation measurements is on the order of 5 [f.u.]
and with the sensitivities of the radiotelescope quoted on
the previous page we have for the source temperatures T_F
and the percentage errors in the intensity measurements
 $\Delta T/T_F$;

$\underline{f_o}$	$\underline{T_F}$	$\underline{\Delta T/T_F}$
430 MHz	99°K	0.04 %
1400 MHz	40°K	0.03 %
2380 MHz	33°K	0.02 %

c.) Observing geometry.

The HF-facility (geodesic coordinates: $18^{\circ}28.5'$ N / $66^{\circ}40.0'$ W) is located at a distance of 17 km from the Arecibo Observatory (geodesic coordinates: $18^{\circ}20.8'$ N / $66^{\circ}45.2'$ W). If we wish to observe with the 300 m - dish of the Arecibo Observatory a point at any height on the perpendicular passing through the center of the HF-antenna (i.e.; on the axis of the HF-beam if refraction effects are neglected) we have to point in a uniquely defined direction. This direction corresponds to an azimuth of 34° east of north and to a zenith angle depending on the height of interest. Neglecting the curvature of the earth surface and the height difference between the Arecibo Observatory and the HF-facility one calculates following values for the zenith angle as a function of the height z_0 ;

<u>height z_0 [km]</u>	<u>zenith angle [deg]</u>
200	4.9
250	3.9
300	3.2
350	2.8
400	2.4

The line of sight to the radio source intersects (for zenith angles attainable by the AO beam) a plane at any given height within the ionosphere which is parallel to a plane tangential to the earth surface near the Arecibo Observatory. As the discrete radio source drifts from

east to west at a rate of 15° /hour this intersection point traces out a line on the plane at the height chosen. The height of interest in the experiment is naturally the height where we expect the formation of the HF-induced irregularities. The chosen declination of the radio source then uniquely defines a line at that height. We thus select a radio source with a declination which will allow us to observe that part of the ionosphere where we expect the formation or existence of irregularities (i.e.; within the HF-beam). In Fig.3.1. a set of 3 such lines is shown for a source at a declination of 21° N. The 3 lines correspond to the choice of 3 different heights for the planes, namely; 100 km , 200 km & 300 km. Also shown is the HF-beamwidth at the corresponding heights for an HF-frequency of 7.3 MHz. (For HF-beam parameters see Table 3.1.). In Fig.3.2. another set of lines is shown for different choices of the radio source declinations and for a fixed height of 300 km. In Fig.3.3 yet another set of lines is shown for different declinations of the radio sources and for a fixed height of 200 km. The tic-marks on the lines correspond to 4 min. time intervalls and allow a determination of the location of the intersection point in terms of minutes before meridian transit of the source. In Fig.3.2 it is apparent that the line of sight to the radio source with a declination of 21° passes near the center of the HF-beam at a height of 300 km about 8 minutes before the source transits the meridian of the Arecibo Observatory. In Fig.3.3 it is apparent that the line of

sight to a radio source with a declination of 22.5° N passes near the center of the HF-beam at a height of 200 km about 10 minutes before the source transits the meridian of the Arecibo Observatory. Sources with different declinations are also used to select particular heights within the HF-beam (i.e.; the HF-wave interaction height). Some of the most suitable sources are listed in Table 3.2 (p.68).

Fig.3.1

Fig. 3.2

Fig.3.3

d.) Data acquisition and analysis.

Several different feeds, each designed for a specific observing frequency are available for reception of radiation collected by the 300m spherical reflector of the Arecibo Observatory. These feeds are installed on the bottomside of either of two carriage houses. The carriage houses can move along the bottomside of the azimuth arm and thereby control the zenith angle of the feed. The azimuth arm itself can rotate with respect to the fixed triangle which is suspended by cables from 3 big towers. This rotation allows for azimuth control of the feed in use. For a more illustrative description see [Gordon, 1964]. During the experiments reported in this thesis observations were made at 430, 1420 and 2380 MHz. Local oscillators within the carriage houses beat the received signal down to an intermediate frequency of 30 MHz. This signal is then amplified and carried to the control-room. In the control-room a frequency filter of 1 or 2 MHz bandwidth was usually used (for reasons discussed in section 2.3.d.) during observations at 430 MHz. For observations at 1420 and 2380 MHz a bandwidth of 20 MHz was used. The signal was then fed into a square-law detector to obtain the intensity of the measured field. This signal was then fed into the Sanborn amplifier which had an integration time constant of 1 sec. The output of the amplifier was recorded analog on paper and digitized and recorded on magnetic tape. The sampling rate was 1 (in some cases 2) sample(s) per second which is fast enough for the expected ionospheric scintillations.

During analysis the data was normalized so that the mean intensity of the radio source (after subtracting the sky noise in the vicinity of the source) from it) was unity. These intensity recordings are shown in chapter 5. Then a polynomial of degree 2 was fitted to the data and then subtracted such as to detrend the data (i.e.; remove any dc-component and slow variations due to antenna-gain and -temperature changes with varying zenith angle. As a next step a Fast Fourier Transform was performed on the temporal intensity fluctuations $\Delta I(t)$ to obtain $\Delta I(\nu)$ which is complex, but after squaring the real and imaginary parts and adding them together the frequency power spectra $|\Delta I(\nu)|^2$ shown in chapter 5 were obtained.

e.) Selection of suitable radio-sources.

Radio-sources to be useful for scintillation measurements have to be selected according to several criteria. First of all we wish to use sources which have a relatively steady flux over the time scale of the measurement (usually on the order of one hour). Since we wish to study ionospheric irregularities produced within the HF-beam, we have to choose radio-sources that have a line of sight through the HF-beam at the height of interest. From the considerations in section c) it becomes apparent that sources with declinations in the range of $19.0^{\circ} \leq \delta \leq 24.0^{\circ}$ are the most suitable for the study of F-region ($z_o \sim 200\text{km}$ to 400km) irregularities produced within the HF-beam. This criteria was used in selecting the most suitable radio-sources for scintillation measurements which are listed in Table 3.2. Another criteria that sources in Table 3.2. fulfill is that their angular diameter be less than $1/4$ minute of arc. This guarantees that the transverse spatial coherence is long enough that irregularities with scale sizes of $\lambda_{\text{irr}} \approx \lambda_o \sim 150\text{m}$ may be detected (section 2.3.c.). Radio sources with such small angular diameters are likely to produce interplanetary scintillations (IPS) if their line of sight passes near the sun. To avoid contributions from IPS on the spectra of the ionospheric irregularities all observations were performed at night. Finally from Table 3.2. one should select the sources with the strongest flux to increase the sensitivity of the scintillation measurements (section b.) and the right ascension is naturally dictated by the time of the night and the time of the year.

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Radio sources selected for scintillation measurements

Name of source	Right ¹⁾ Ascension h : m : s	Declination ¹⁾ ° : ' : "	Flux [10^{-26} W/m ² Hz]	
			1400 MHz ¹⁾	178 MHz ²⁾
3C43.0	01:27:15.1	23:22:53	2.8	11.6
3C132	04:53:42.4	22:44:42	3.3	12.4
3C166	06:42:24.7	21:24:54	2.4	14.7
PK0820+22	08:20:28.3	22:32:46	2.3	4.5
3C263.1	11:40:49.2	22:23:37	3.0	18.3
PK1354+19	13:54:42.0	19:33:44	2.1	7.6
PK1422+20	14:22:37.3	20:14:01	1.7	7.4
3C318.0	15:17:50.7	20:26:54	2.5	13.2
3C324.0	15:47:37.3	21:34:44	2.3	13.6
3C333	16:15:05.1	21:14:51	2.0	6.9
3C336.0	16:22:32.5	23:52:06	2.7	12.6
3C409	20:12:18.2	23:25:42	13	79
3C466	23:37:51.9	22:04:14	2.1	6.1

Table 3.2.

1) [Bridle et.al.,1972]

2) [Pilkington,Scott,1964] & [Gower et.al.,1967]

Note: The flux at 430 MHz and at 2380 MHz can be estimated from the above data by assuming a linear relationship between the flux and the frequency.

4. Theories on the production of artificial ionospheric irregularities.

It can be shown that a radio beam becomes unstable in an isotropic nonlinear focusing medium. This was demonstrated in the geometrical optics approximation by [Gurevich, 1978; section 5.4.1.]. The plasma in the ionosphere is a self-focusing medium if the local electron density decreases with an increase in the local electric field intensity (true for altitudes above ~ 200 km). Such a decrease in the local electron density results in an increase in the index of refraction. Radio waves propagating in such a medium will refract towards the electron density depletion further strengthening the electric field intensity which in turn further decreases the electron density and so on. This instability is usually called a self-focusing or modulation instability. The possibility of producing this instability with HF-waves incident on the ionosphere was suggested by [Litvak, 1968]. Since in the F-region of the ionosphere diffusion along the geomagnetic field is much faster than the diffusion across the geomagnetic field, we expect the electron density depletions to be aligned with the geomagnetic field. We also expect the time scale for the formation of such depletions to be determined by the diffusion process along the geomagnetic field. This is indeed observed and will be discussed in section 6.d. The effects are also stronger near the HF-wave reflection height due to the

swelling of the HF-wave electric field. As the electron density irregularities are growing the HF-waves experience increasing refraction which eventually goes over into the diffraction regime due to the increasing phase variations across the wavefront. This may reduce the focusing effects and occurs first for the irregularities of shorter wavelength λ_{ir} . Such effects are included in the theoretical treatments by following authors [Perkins & Valeo, 1974; Vaskov and Gurevich, 1975, 1976]. The former authors suggest that for overdense heating in the ionosphere the diffraction is dominant. In this case neighboring irregularities support each other by diffracting energy of the HF-field into the adjacent irregularities. Some experimental evidence supports this idea and will be discussed in section 6.e. An alternative explanation for the formation of the large scale irregularities is given by [Cragin et.al., 1977]. According to their theory the incident HF-field excites a parametric instability whereby the incident high frequency electromagnetic wave decays into a stationary ion-acoustic wave and another high frequency electromagnetic wave. The stationary wave is maintained by the differential heating of the electrons in the interference field of the two high frequency electromagnetic waves.

Such larger scale irregularities can affect in two ways the production of short scale irregularities which are aligned with the geomagnetic field and which have been observed in Boulder [Fialer, 1974; Minkoff et.al., 1974] and at Arecibo [Frey, 1980]. The large scale electron density stratification can generate short scale irregularities due to a drift instability [Borisov et.al., 1976]. The focusing

also locally intensifies the electric field strength and therefore increases the strength of short scale irregularities produced by other mechanisms as suggested by [Perkins, 1974; Lee & Fejer, 1978; Grach et.al., 1978; Vaskov & Gurevich, 1977].

None of the proposed theories on the production of large scale irregularities give more than a good qualitative picture of the mechanisms involved. This is understandable in view of the complexity of the problem as posed by the natural environment of the ionosphere. It is therefore not attempted to do any detailed comparison of the theoretical predictions with the observations presented in this dissertation. The theories that predict growth rates give the right order of magnitude and the theories that predict an HF-power threshold show it to be low enough for the irregularities to occur as is observed in practice. It appears to the author that the theories which were suggested have to be supplemented by more realistic models (preferably based on measurements during a specific observation) about the initial state of the natural ionospheric environment (drift velocity, electron density profiles, local magnetic field, etc.). Dynamic computer modeling as the electron densities grow during HF-modification based on the ideas given by the theories will be an important tool enabling comparison between the theories and the observations. As an input to the theoretical considerations may serve some of the conclusions that were reached in chapter 7.

5. Observational results.

The data reported in this thesis were obtained during three separate observing periods at the Arecibo Observatory. The first observing period was during the first two weeks of September 1980 and the second observation period was during the third week of December 1980. The final set of observations was concluded during the first week of April 1981. The measurement of intensity-fluctuations caused by ionospheric irregularities was attempted at three different frequencies, namely 430 MHz, 1400 MHz and 2380 MHz. Observations at 430 MHz and 1400 MHz have shown effects that were the result of overdense HF-modification of the ionosphere. Several observations at 2380 MHz under very similar experimental conditions showed no effects other than intensity fluctuations which were intrinsic to the instrument. Several attempts at 430 MHz and 2380 MHz were made during underdense HF-modification but no evidence of ionospheric irregularities caused by HF-modification was seen. The HF-power level and the HF-polarization during underdense heating were very similar to the ones which produced strong effects during overdense heating. The ratio of the HF-frequencies used to the penetration frequency spans the range from 1 (a transition from underdense to overdense was observed on the 22.December 1980; see page 90 and Fig.5.5.) to about 2.

In this chapter the data will be represented in chronological order and from a phenomenological point of view. The interpretation of the data will be deferred to the next

chapter. Most measurements were performed with radio sources that are listed in table 3.2. and therefore satisfy the criteria outlined in section 3.e. At times less suitable sources were used if available transit time of source demanded so. The data for each observation is presented in 3 parts. First the recording of the intensity versus time for the particular source is shown. Then the observing geometry which was discussed in general in section 3.c. is shown for the particular source(s) and the corresponding estimated HF-wave interaction height. Finally the frequency power spectra which were obtained by the procedure outlined in section 3.d. are shown. Features on the spectra which are predicted by the thin screen theory (see section 2.3.a. and Fig.2.5.) are indicated where identification is possible. The labels κ_1 , κ_2 , and κ_3 indicate the features which can be identified with the 1st, 2nd and 3rd zero of the Fresnel filtering (see eq.(2-68) and Fig.2.2.) and κ_A indicates the features which can be identified with the first zero of the antenna filtering (see eqs.(2-89 & 2-84) and Fig. 2-4). If such features which were predicted by the thin phase screen theory for the irregularity wavevector power spectrum can be identified with features on the measured frequency power spectrum, we obtain the scaling factor between κ and ν and therefore the scanning velocity v_{sc} . And according to the discussion in section 2.3.b. we may estimate the drift velocity of the electron density irregularities.

6. September 1980; 0:51 to 1:30 AST.

Starting at 0:51 AST the radio source 3C466 was tracked until 1:30 AST. The intensity was recorded in 4 segments and is displayed in Fig.5.1.a. The HF-frequency was $f_{\text{HF}} = 5.1$ MHz and the critical frequency was $f_{\text{OF2}} = 5.6$ MHz resulting in overdense HF-modification. The HF-wave reflection height is assumed to be about 250 km. The HF-power density of $\rho = 15 \mu\text{W}/\text{m}^2$ at 250 km (neglecting absorption) was rather low compared to the other observations reported in this thesis, since only 2 transmitters were feeding 75 kW each into half the antenna array. The modified volume was rather large and its projection on a horizontal plane at 250 km is shown in Fig. 5.1.b. Also shown in that figure is the curve which is followed (as the radio source drifts in its daily motion) by the point where the line of sight to the radio source intersects the horizontal at 250 km. Note that the time flow in Fig.5.1.a. is in the opposite direction to the spatial east-west motion in Fig.5.1.b. On that curve are also shown the segments during which data was recorded. The labels of the segments are identical to the labels in Fig.5.1.a. This observation was one of the first observations and was designed to look for higher frequency intensity fluctuations. The integration time constant was therefore set to 0.1 sec which accounts for the noisy intensity recording. 8 samples were read per second for a sampling period of 512 seconds corresponding to 4096 data points. 4096 is a power of 2 and therefore suitable for a Fast Fourier Transform. Segment 1 however was truncated

and therefore not used to produce a power spectrum. Frequency power spectra of the intensity fluctuations were calculated for the segments 2,3, and 4 according to the procedure outlined in section 3.d. and are shown in Fig. 5.1.c. Notice the change in the vertical scale when comparing the three spectra. The magnitudes are not normalized and these spectra should therefore not be compared with spectra obtained on other days. The label λ_1 points to a feature which is believed to correspond to the first zero of the Fresnel filter and the corresponding irregularity wavelength is deduced from Fig.2.5.d. for the parameters under consideration.

The HF-power was off since 0:35 AST giving the ionosphere ~16 minutes to relax from previous heating. The HF-power was not turned back on until 1:05:39 AST when the line of sight to the radio source was passed the HF-beam center at a height of 250 km yet well within the volume modified by the HF-beam (segment 2 in Fig.5.1.). No immediate effects are seen until time advances to segment 3 where strong intensity fluctuations become apparent in the intensity record (Fig.5.1.a.) and in the power spectrum of the intensity fluctuations (Fig.5.1.c.). It appears that the response time for the intensity fluctuations after HF-power turn on is about 4 minutes. As the radio source drifts on and the line of sight leaves the HF-beam (segment 4) the intensity fluctuations diminish again.

Projection of volume
modified by HF-beam.

6. September ; 0:51 to 1:30 AST

$f_{HF} = 5.1 \text{ MHz}$ $f_{OF2} = \text{unknown}$

height of reflection (assumed): 250km

2 transmitters 75kW each; half antenna

radio source 3C466; observing frequency 430MHz

Horizontal plane
of projection
located at a
height of 250km.

Figure 5.1.

6. September 1980 ; 0:51 to 1:30 AST

Figure 5.1.c.

9. September 1980

Starting at 0:21 AST the radio source 3C466 was tracked until 2:09 AST. The intensity was recorded in 3 segments of 2048 seconds each which are displayed in Figure 5.2.a. If the radio sources are observed more than 1/2 hour away from the meridian transit time the antenna gain and the antenna temperature changes as the effective area seen by the feed becomes smaller. These changes are apparent in Fig. 5.2.a.

The HF-frequency was $f_{\text{HF}} = 8.175 \text{ MHz}$ and the critical frequency was $f_{\text{OF2}} \sim 9.5\text{--}10.5 \text{ MHz}$ resulting in overdense HF-modification. The HF-wave reflection height is assumed to be $h_{\text{refl}} = 300 \text{ km}$. 4 transmitters were feeding 75kW each into the full antenna array. If absorption is neglected this corresponds to a power density at 300km of $P = 37 \mu\text{W/m}^2$ (see table 3.1.). The projection of the modified volume onto a horizontal plane at a height of 300 km is shown in Figure 5.2.b. Also shown in that figure is the curve which is followed (as the radio source drifts in its daily motion) by the point where the line of sight to the radio source intersects the horizontal plane at 300km. On that curve are also indicated the segments during which the 'on source' intensity was recorded. The labels on the segments are identical to the labels in Fig. 5.2.a. The power spectra of the intensity fluctuations for the three segments are shown in Fig. 5.2.c. Notice the change in the vertical scale when comparing the three spectra. The HF-power had been on since 23:30 AST and one would

expect steady state conditions to be established. Intensity fluctuations as large as 15 to 18 % (excursion from mean intensity) with periods longer than 3 minutes are seen during the whole observation period, i.e.; well outside the HF-beam. Intensity fluctuations of shorter time-scale (tens of seconds) however are only present within the HF-beam. This is evident within the interval corresponding to the halfpower-beamwidth= HPBW shown in segment 1 of Fig.5.2.a.

9. September ; 0:21 to 2:09 AST

$f_{HF} = 8.175 \text{ MHz}$ $f_{OF2} \sim 9.5 - 10.5 \text{ MHz}$

height of reflection (assumed): 300km

4 transmitters 75kW each; full antenna
HF-on since 23:20 AST (8.Sept.80)

radio source ; 3C466

b.)

Figure 5.2.

9. September 1980; 0:21 to 2:09 AST

Figure 5.2.c.

10. September 1980 ; 1:10 to 2:43 AST.

During this observation the radio source 3C14.0 was tracked for an extended period and the intensity of the source was measured at 430 MHz. The two intensity recordings obtained are shown in the top portion of Fig.5.3.a. The ionosphere penetration frequency for the ordinary mode was 9.5MHz at the beginning of the observation period and gradually dropped to 8.0 MHz at the end of the observation period. The HF-frequency was chosen 8.2 MHz which resulted in overdense HF-modification. The height of the HF-wave reflection is assumed to be 300 km. A projection of the HF-modified volume onto a horizontal plane at 300 km height is shown in the lower portion of Fig.5.3.a. Also shown in that figure is the curve which is followed (as the radio source drifts in its daily motion) by the point where the line of sight to the radio source intersects the horizontal plane at 300 km. Indicated are two segments A and B which correspond to the intensity recordings shown in the upper portion and labeled correspondingly. Note that the direction of time on the intensity recordings is opposite to the spatial east-west motion in the plane of projection. At the beginning of period A (1:10:15 to 1:44:21 AST) no HF-power has been transmitted for an extended period. The intensity fluctuations present ($\sim 6\%$) correspond to the natural environment. There is also increased system noise since a bandwidth of 100 kHz or less was used for this measurement.

At 1:15 AST the HF-power was turned on and about 3 minutes later the fluctuations increased even though the line of sight to the radio source is about 10-15 km to the east of the modified volume. The fluctuations increased to a level of 13% (excursion from mean) as the line of sight to the radio source approached and entered the modified volume. 4 HF-transmitters were feeding 75 kW each into the full antenna array. Neglecting absorption this corresponds to a power density of $37 \mu\text{W}/\text{m}^2$ at a height of 300 km (see table 3.1.). The fluctuations started to decrease before the line of sight was moving out of the modified volume (as defined by the HF halfpower beam-width). The second intensity recording B was obtained about 20 minutes later as the line of sight was about 35 km to the west of the modified volume. The intensity fluctuations during this period were about 6% (excursion from mean). The HF-power stayed up for the remainder of this observation.

10. September; 1:10 to 2:43 AST

$f_{HF} = 8.2 \text{ MHz}$

height of reflection (assumed): 300 km

Horizontal plane of projection
of projection
located at a
height of 300km.

4 transmitters 75kw each;
full antenna

radiosource: 3C14.0

observing-frequency: 430MHz

Figure 5.3.a.

10. September 1980 ; 1:10 to 2:43 AST.

vertical scale = $\left| \Delta I(\nu) \right|^2$

Figure 5.3.b.

11./12. September 1980 ; 22:11 to 01:32 AST.

This observation period extends over more than three hours during which 4 different radio sources were tracked in sequence. The corresponding intensity recordings are shown in Fig.5.4.a. The motion of the point where the line of sight to the radio source intersects a horizontal plane at a height of 310 km (the HF-wave interaction height) is shown in Fig.5.4.b. for all four radio sources. Note that the time flow in Fig.5.4.a. is in the opposite direction to the spatial east-west motion in Fig.5.4.b. Also shown is the halfpower beamwidth of the HF-beam at the HF-wave interaction height. It can be seen that all four radio sources have a line of sight which passes through the modified volume at some time. The segments A1,A2,B1,B2,B3,B4,C1,C2,C3,D1 represent the actual recording of the source intensity, each segment being 1024 seconds long and suitable for a fast Fourier transform. The corresponding intervals are also labeled on the intensity recordings of Fig.5.4.a. The power spectra obtained from these intervals by the procedure outlined in section 3.d. are shown with the same label in Fig.5.4.c. The features on these frequency power spectra which are labeled with κ_A are identified with the first zero in the antenna filtering of the theoretically predicted wavevector power spectrum (see eqs.(2-88,2-84)). With such an identification a scanning velocity of $v_{sc} = 3$ m/s is obtained in the manner discussed in section 6.a. Considering the arguments of section 2.3.b. we obtain for the east-west component of the irregularity drift velocity

$v_{dr} = v_{RS} + v_{sc} = 7.3 \times (310/100) + 3 = 19 \text{ or } 25 \text{ m/s}$
westwards.

The first interval AL on radio source 3C433 was recorded before any HF-power was transmitted and therefore represents the natural (unperturbed) effect of the ionosphere on the measured source intensity. The baseline (zero intensity) in all intensity recordings corresponds to the off-source intensity or the sky-noise 2° away from the radio source. During the second interval A2 the HF-power was turned on at 22:35 AST but the line of sight to the radio source was outside the HF-halfpower beamwidth (compare Fig.5.4.b.) and still no effects are seen. As the line of sight to the next source, namely PK2223+21 approaches the volume modified by the HF-waves first irregularities appear. The HF-power has now been on for ~ 20 to 30 minutes. 4 transmitters have been feeding 75 kW each into the full antenna array at an HF-frequency of 7.3 MHz. If we neglect absorption, we obtain at a height of 310 km a power density of $30 \mu\text{W/m}^2$ (see table 3.1.). At first only intensity fluctuations with periods longer than 2 minutes are seen. As the line of sight to the radio source approaches the center of the HF-beam these fluctuations become stronger and additional fluctuations with periods as short as 20 seconds appear. See segments B1 and B2. The extent of the geometric HF-halfpower beamwidth is indicated and labeled as HF-HPBW. As the line of sight to the radio source leaves the HF-beam some residuals of intensity fluctuations with periods longer than 2 minutes remain even well outside the HF-beam. As we

start to track the next source 3C466 intensity fluctuations with periods longer than 2 minutes are seen well outside the HF-beam. The HF-power is then turned off for the remainder of the observation at 00:21 AST just before the line of sight to the radio source enters the modified volume. Up to 12 minutes later we still observe strong intensity fluctuations within the modified volume with periods as short as 20 seconds. As the line of sight to the radio source leaves the modified volume only fluctuations with periods longer than one minute remain. About one hour later as the next source 3C14.0 is tracked and has a line of sight passing through the volume which was previously modified by HF waves only fluctuations with periods longer than one minute remain.

11 September / 12 September 1980 ; 430 MHz - Intensity fluctuations.

Figure 5.4.a.

b.)

c.)

Figure 5.4.

11.September 22:20 AST to 12.September 1:33 AST

1980.

$v_{sc} = 3 \text{ m/sec.}$

$\Delta\nu = 1 \text{ mHz}$

Figure 5.4.c.

22. December 1980 ; 6:30 to 7:04 AST

This observation was performed during sunrise in the F-region of the ionosphere using the radio source 3C274.1 for the intensity fluctuation measurements. The intensity of the source versus time is shown in Figure 5.5.a. The two large downwards spikes are due to interference from the ionosonde as ionograms were taken. The increase of the critical frequency f_{oF2} with the rising sun is shown in Fig.5.5.c. for two different days which appear to behave very similar. Only three data points (the crosses) refer to the day of this observation but they fit well within the empirical model formed from the data measured 5 days earlier on the 17. December (the circles).

The HF-frequency was 5.08 MHz and it appears from Fig. 5.5.c. that we start with underdense heating and around 6:30 AST the o-mode goes from overdense to underdense. The HF-reflection height was deduced from ionograms to be near 250 km. The HF-power had been on since 5:30 AST. 4 transmitters were feeding 75 kW each into the full antenna. Neglecting absorption this corresponds to a power density of $60 \mu\text{W}/\text{m}^2$ at a height of 250 km (compare with table 3.2.).

The path of the point (as the radio source drifts) where the line of sight to the radio source intersects a horizontal plane at a height of 250 km is shown in Fig.5.5.b. Indicated is the segment during which the intensity recording shown in Fig.5.5.a. was obtained. Also shown is the projection of the volume (defined by the HF-halfpower

beamwidth) which is modified at a height of 250 km. It appears from the intensity recording of Fig.5.5.a. that the intensity fluctuations are very weak, namely less than 1.5% (excursion from mean) for the time before 6:40 AST even though the line of sight to the radio source is well within the volume modified by the HF-waves. As seen from Fig.5.5.c. the HF-frequency is very close (within $\pm .2\text{MHz}$) to the o-mode critical frequency for that period. After 6:40 AST the intensity fluctuations increase within a period of ~ 20 seconds by a factor of ~ 6 and reach $\sim 18\%$ after a few minutes. At 6:55 AST the HF-power is turned off and the fluctuations die away within 10 minutes as the line of sight to the radio source leaves the modified volume.

22. December 1980; 6:30 to 7:04 AST

a.)

$f_{HF} = 5.08 \text{ MHz}$

Height of HF-reflection as estimated from ionograms; 250 km
4 transmitters 75 kW each; full antenna.

radiosource: 3C274.1 - 430MHz

Horizontal plane of projection located at a height of 250 km.

Figure 5.5.

2.April 1981; 22:50 to 23:25 AST.

Starting at 22:50:41 AST the radio source 3C263.1 was tracked and its intensity recorded with a bandwidth of 2 MHz centered at 430 MHz until 23:24:48 AST. This intensity recording is shown in Fig.5.6.a. The HF-power had been on (2x150kW) for over one hour and was turned off at 21:45 AST, about 65 minutes before this intensity recording starts. The small intensity fluctuations during the first 5 minutes of the recording could be residues from this previous heating or they could be naturally caused. At 22:55:35 AST the HF-power was turned on transmitting at a frequency of 8.175 MHz. The o-mode penetration frequency was above 12 MHz which resulted in overdense HF-modification. The HF-wave reflection height was deduced from ionograms to be near 300 km. 2 transmitters were feeding 150kW each into the full antenna array. If absorption is neglected this corresponds to a power density of $\mathcal{P} = 37 \mu\text{W}/\text{m}^2$ at a height of 300 km. The projection of the modified volume onto a horizontal plane at a height of 300 km is shown in Fig.5.6.c. Also shown in that figure is the curve which is followed (as the radio star drifts in its daily motion) by the point where the line of sight to the radio source intersects the horizontal plane at 300 km. On that curve is indicated the segment during which the intensity recording shown in Fig.5.6.a. was obtained. Note that the time flow in Fig.5.6.a. is in the opposite direction to the spatial east-west motion in Fig.5.6.c.

The intensity fluctuations start to increase 4-5 min. after the HF-power is turned on. This corresponds to a line of sight to the radio source which passes about 8 km to the east of the volume modified by the HF-beam (as defined by the HPBW at 300 km). As the line of sight to the radio source enters the volume modified by the HF-beam the intensity fluctuations (excursion from mean) increase to about 14% (on two individual spikes the excursion reaches 25% and 22%). As the line of sight to the radio source leaves the modified volume the intensity fluctuations promptly die away. A power spectrum was obtained for the most active segment of the intensity recording. The segment is 1024 sec. long (extending from 23:02:21 to 23:19:25 AST) and is indicated in Fig.5.6.a. The calculated power spectrum is shown in Fig.5.6.b.

2. April 1981 ; 22:50 to 23:25 AST

$f_{\text{HF}} = 8.175 \text{ MHz}$

radiosource; 3C263.1

2 transmitters 150 kW each ; full antenna

Figure 5.6.

3. April 1981 ; 2:36 to 4:24 AST.

The intensity fluctuations of two different radio sources were measured at two different frequencies. From 2:36:41 to 3:10:48 AST the intensity fluctuations of radio source 3C318 was measured with a bandwidth of 2 MHz centered at 430 MHz. The intensity recording obtained is shown in the top portion of Fig.5.7.a. From 3:49:38 AST to 4:23:45 AST the intensity of radio source 3C336 was measured with a bandwidth of 40 MHz centered at 1400 MHz. The intensity recording obtained is shown in the lower portion of Fig.5.7.a. The HF-power had been off since 2:05 AST and the small intensity fluctuations ($\sim 3\%$ excursion from mean) during the first ten minutes of observation at 430 MHz are most likely caused by natural irregularities in the ionosphere. At 2:48:00 AST, as the line of sight to the radio source 3C318 was intersecting the modified volume close to the center of the HF-beam, the HF-power was turned on. The penetration frequency for the o-mode was near 9.5 MHz and the HF-frequency was 5.08 MHz which corresponds to overdense HF-modification. The HF-wave reflection height for $f_{HF}=5.08$ MHz was deduced from ionograms to be near 260 km. A projection of the volume, modified by the HF-waves, onto a plane at a height of 260 km is shown in Fig.5.7.b. Also shown in that figure are the two curves which are followed (as the radio stars drift in their daily motion) by the points where the line of sight to the corresponding radio source intersects the horizontal plane at 260 km. The segments indicated on each curve represent the actual intensity recording periods which are shown

in Fig.5.7.a. Note that the time flow in Fig.5.7.a. is in the opposite direction to the spatial east-west motion in Fig.5.7.b. At $\sim 2:48:20$ AST, 20 seconds after the HF-power was turned on, strong initial fluctuations ($\sim 16\%$ excursion from mean) appear which after about 1 minute reduce to moderate ($\sim 8\%$) intensity fluctuations. 2 HF-transmitters were feeding 100 kW each into the full antenna array. Neglecting absorption this corresponds to a power density of $P = 37 \mu\text{W}/\text{m}^2$ at a height of 260 km. As the line of sight to the radio source moves out of the HF-beam at $\sim 3:00$ AST the fluctuations reduce to the natural level observed before the HF-power was turned on. With the HF-power still on since 2:48:00 AST, observation of the radio source 3C336 at a frequency of 1400 MHz was started at 3:49:38 AST. At this time the line of sight to the radio source was intersecting the modified volume well within the halfpower beamwidth of the HF-beam and intensity fluctuations of 1.2% (excursion from mean) with periods of ~ 15 seconds were observed at 1400 MHz. At 3:56:00 AST the HF-power was increased to 120 kW each transmitter and a slight increase in the fluctuation amplitude to 1.5% is apparent in Fig.5.7.a. As the line of sight to the radio source moves out of the modified volume these fluctuations promptly disappear. The slow intensity fluctuations (periods of several minutes) are instrumental. Power spectra were obtained for three time segments which are indicated in Fig.5.7.a. Segment A covers the most active part of the 430 MHz observation. Segment B covers the most active

part of the 1400 MHz observation and segment C represents the 1400 MHz intensity in the absence of irregularities produced by the HF-waves. The resulting spectra are shown in Fig.5.7.c. and are normalized such as to be comparable with any other spectra obtained during April 1981. The bump in the spectra B and C near 0.4 Hz corresponds to a resonance in the azimuth arm and is therefore instrumental.

3. April 1981 ; 2:36 to 4:24 AST.

$f_{HF} = 5.08 \text{ MHz}$ HF-reflection height as deduced from ionograms; 260 km
2 transmitters 100 kw each ; full antenna

Figure 5.7.a.

3.April 1981 ; 2:36 to 4:24 AST.

Horizontal plane of projection
located at a height of 260 km

Figure 5.7.b.

3.April 1981 ; 2:36 to 4:24 AST.

Figure 5.7.c.

4. April 1981 ; 1:23 to 1:51 AST.

During this period the intensity of the radio source PK 1422+20 was measured with a bandwidth of 40 MHz centered at 1400 MHz. The first intensity recording which extends over a period of 1024 seconds (from 1:23:31 to 1:40:35 AST) is shown in Fig.5.8.c. together with the frequency power spectrum calculated from it. The HF-power was off since 1:05 AST and was not turned back on until 1:44:40 AST. The first intensity recording was therefore obtained during a period where no HF-power was applied and should represent the natural environment. The slow (~ 1 minute period) intensity fluctuations are probably instrumental. As seen in the second intensity recording (Fig.5.8.a.) stronger intensity fluctuations ($\sim 3\%$ excursion from mean) with periods of ~ 5 to 10 seconds appear about 25 seconds after the HF-power was turned on and last through the whole recording. The HF-frequency was 5.08 MHz and the o-mode penetration frequency was about 11 MHz resulting in overdense modification. The HF-wave reflection height was estimated from ionograms as ~ 250 km. 2 HF-transmitters were feeding 130 kW and 145 kW respectively into the full antenna array. If absorption is neglected this corresponds to a power density of $\rho = 51 \text{ uW/m}^2$ at a height of 250 km. The projection of the modified volume (as defined by the HF halfpower beamwidth) onto a horizontal plane at a height of 250 km is shown in Fig.5.8.b.

Also shown in that figure is the curve which is followed (as the radio star drifts in its daily motion) by the point where the line of sight to the radio source intersects the horizontal plane at 250 km. Note that the direction of time for the intensity recordings in Fig.5.8.a. & c. is opposite to the direction of the spatial east-west motion in Fig.5.8.b. From Fig.5.8.b. it is seen that the whole intensity recording shown in Fig.5.8.a. was obtained well within the modified volume. The frequency power spectra are normalized and can be compared quantitatively with other spectra obtained during April 1981.

4. April 1981 ; 1:23 to 1:51 AST.

Figure 5.8.

c.)

$$f_{\text{HF}} = 5.08 \text{ MHz}$$

$$f_{\text{OF2}} \sim 11 \text{ MHz}$$

Height of HF-wave reflection as estimated from Ionograms;
250 km

Observing frequency ; 1420 MHz

Radio source ; PK 1422 + 20

2 HF-transmitters (130 kW & 145 kW)

Figure 5.8.

4. April 1981 ; 2:09 to 4:16 AST.

During this observation the distribution of irregularities within the modified volume was partially mapped by observing sequentially 3 radio sources with different declinations. The intensities were recorded with a bandwidth of 2 MHz centered at 430 MHz. The intensity recordings for all 3 sources are shown in Fig.5.9.a. The frequency power spectra calculated from these recordings are shown in Fig.5.9.c. with the corresponding labels A,B,C and D. The F-region was relatively stable over the whole observation period. The penetration frequency was between 8 and 9 MHz. The HF-frequency was selected as 5.08 MHz which resulted in overdense HF-modification. The HF-wave reflection height was estimated from ionograms to be near 250 km. A projection of the modified volume (as defined by the HF halfpower beamwidth) onto a horizontal plane located at a height of 250 km is shown in Fig.5.9.b. Also shown in that figure are the three curves which are followed (as the radio sources drift in their daily motion) by the points where the line of sight to the corresponding radio source intersects the horizontal plane at 250 km. Note that the direction of time for the intensity recordings in Fig.5.9.a. is opposite to the direction of the spatial east-west motion in Fig.5.9.b. Indicated in Fig.5.9.b. are the segments labeled A,B,C,D and E which correspond to the periods during which the intensity recordings shown in Fig.5.9.a. were obtained. The HF-power had been off since 1:56:30 AST. The small intensity fluctuations ($\sim 3\%$) measured during period A

(2:09:24 to 2:26:27 AST on radio source 3C318) may be residues from this previous heating or they may represent the natural environment. During period B (2:27:54 to 3:02:01 AST on radio source 3C318) the HF-power was turned on (at 2:33:53 AST) slightly before the line of sight to the radio source entered the modified volume (defined by the HF halfpower beamwidth). After about 2 - 3 minutes (see Fig.5.9.a.) the intensity fluctuations increased and reached 7.3 % (excursion from mean) over the central part of the modified volume. 2 HF-transmitters were feeding 145 kW each into the full antenna array. Neglecting absorption this corresponds to a power density of $\rho = 58 \mu\text{W}/\text{m}^2$ at a height of 250 km. As the line of sight to the radio source was moving out of the modified volume the intensity fluctuations died away to the level measured during period A. The intensity fluctuations measured during period C (3:08:56 to 3:25:59 AST on source 3C324) reached a level of 5.8 % (excursion from mean). During most of this recording the line of sight to the radio source was intersecting the modified volume. The intensity fluctuations during period D (3:32:32 to 4:06:39 AST on radio source 3C336) started before the line of sight to the radio source entered the modified volume and reached a level of 5.8 % (excursion from mean) through most of the time when the line of sight to the radio source intersected the modified volume. As the line of sight moved away from the modified volume the fluctuations

died away. During period E the fluctuation level dropped to 1 % , the lowest value over the whole observation period. The HF-power stayed on for the rest of this observing period.

4. April 1981 ; 2:09 to 4:16 AST.

observing frequency ; 430 MHz

heating frequency; $f_{HF} = 5.08$ MHz

2 HF-transmitters 145kW each;
full antenna

Figure 5.9.a.

4. April 1981 ; 2:09 to 4:16 AST.

Horizontal plane of projection
located at a height of 250 km

Figure 5.9.b.

4. April 1981 ; 2:09 to 4:16 AST.

Figure 5.9.c.

6. Interpretations

a.) The power spectra of the spatial intensity fluctuations.

Most of the temporal intensity recordings reported in chapter 5 were used to obtain frequency power spectra $|\Delta I(\nu)|^2$ by the procedure outlined in section 3.d. These frequency power spectra are also shown in chapter 5. Using the thin screen theory expressions were derived in chapter 2 to calculate the wavevector power spectrum of the intensity fluctuations $|\Delta I(k)|^2$ expected for any given wavevector power spectrum of the phase fluctuations $|\varphi_s(k)|^2$ in the phase screen. The expression obtained (see eq.(2-89)) contains two filtering terms which produce very characteristic features on the wavevector power spectrum of the intensity fluctuations for any given set of experimental parameters (i.e.; height of phase screen, observing frequency, antenna radius). These features can be seen in Fig.2.5. where the combined effect of the two filters on the wavevector power spectrum of the intensity fluctuations are plotted for different experimental parameters. The calculated wavevector power spectrum $|\Delta I(k)|^2$ and the measured frequency power spectrum $|\Delta I(\nu)|^2$ of the intensity fluctuations are related to each other since;

$$k = \frac{2\pi}{\lambda_{ir}} = \frac{2\pi}{v_{sc}} \cdot \nu$$

The scanning velocity v_{sc} of the line of sight to the radio source across the phase irregularity screen was discussed in section 2.3.b.

In what follows the theoretically calculated features will be identified (where possible) on the measured frequency power spectra and the scanning velocity deduced. From the scanning velocity the drift velocity of the phase irregularities in the screen can be estimated. Within the accuracy of this identification the measured frequency power spectra of the intensity fluctuations are now measured wavevector power spectra of the intensity fluctuations.

A spectrum which shows all the expected features was obtained on the 10 September 1980. Refer to the spectrum A2 in Fig.5.3.b. The labels κ_1 , κ_2 , and κ_3 correspond to the 1st, 2nd and 3rd zero (i.e.; $\kappa_1 = \sqrt{\pi} \kappa_F$, $\kappa_2 = \sqrt{2\pi} \kappa_F$ and $\kappa_3 = \sqrt{3\pi} \kappa_F$ respectively) of the Fresnel filter function

$$\sin^2\left(\frac{\kappa^2}{\kappa_F^2}\right) \quad \text{contained in eqs. (2-68 and 2-89).}$$

The label κ_A corresponds to the zero of the antenna filtering term given by eq. (2-74) which occurs at $\kappa_A \cdot \xi_0 \cong 3.8$. From this last expression $\kappa_A = \frac{3.8}{150} = 0.025 \text{ [m}^{-1}\text{]}$ and from Fig.5.3.b κ_A is 0.095 Hz.

We can now convert the horizontal scale of Fig.5.3.b to $\kappa \text{ [m}^{-1}\text{]}$ by multiplying $\nu \text{ [Hz]}$ by 0.26. Since now κ_1 , κ_2 and κ_3 are determined by that scale we can calculate κ_F to be 0.0071 and since λ is known to be 0.7 m for an observing frequency of 430 MHz we can calculate the height of the screen from eq. (2-16) to be 350 km. This procedure can only be used if we can identify κ_A and one of κ_1 , κ_2 or κ_3 . Unfortunately such an identification is not very often possible in practice and the height z_0 has to be estimated from ionograms.

The frequency power spectrum (Fig.5.6.b.) calculated from the intensity recorded on the 2.April 1981 (Fig.5.6.a.) may be compared with Figure 2.5.a. after deducing from ionograms an HF-wave reflection height of ~ 300 km. The measured spectrum shows the features expected from the thin phase screen theory with a slight indication of the first zero of the Fresnel filter at a frequency of 0.12 Hz. For a height of 300 km the first zero of the Fresnel-filter $\kappa_1 = \sqrt{2} \kappa_F$ corresponds to an irregularity wavelength of $\lambda_{ir} \approx 460$ m. The scanning velocity is therefore

$$v_{sc} = \lambda_{ir} \nu = 55 \text{ m/s.}$$

which corresponds to an eastward drift (opposite to the motion of the radio source) of about 30 m/s. As seen in the above example one obtains drift velocities as a byproduct when matching the calculated wavevector power spectra to the measured frequency power spectra.

The frequency power spectra obtained on the 4.April (Fig. 5.9.c.) when the HF-wave reflection height was ~ 250 km may be compared with Fig.2.5.d. and it appears that spectra C indicates a feature near 80 MHz corresponding to $\lambda_{ir} \approx 420$ m which results in a scanning velocity of $v_{sc} = 34$ m/s or an eastward drift of the irregularities of about 15 m/s. The spectra obtained for the 6.September 1980 (Fig.5.1.c.), 9.September 1980 (Fig.5.2.c.) and 11./12. September 1980 (Fig.5.4.c.) make the identification of features more difficult since the frequency resolution is comparable to the bandwidth of the features in the spectra. This is a result of the slow scanning velocity during these observations, an effect that was discussed in section 2.3.b.

In the frequency power spectra obtained for the 9. September 1980 (Fig.5.2.c.) and for the night from the 11. to the 12. September 1980 (Fig 5.4.c.) some features appear in the spectra at frequencies up to 5 times higher than the highest frequency expected by the diffractive scintillation theory. For the above observations the antenna filtering starts to suppress scintillations at a frequency of about 10 mHz (the scanning velocity is $\sim 3\text{m/s}$ since the irregularities drift westward, in the same direction as the line of sight to the radio source moves, and at comparable speeds) in the context of the diffractive scintillation theory. Spectra obtained when the line of sight to the radio source was passing through the volume modified by the HF-waves (spectrum 1 in Fig.5.2.c.; spectra B2, C1, and C2 in Fig.5.4.c.) show intensity fluctuations with frequencies 5 times higher (namely up to 50 mHz). These are the result of the differential refraction between the "unperturbed" radiation field $\tilde{E}_0$ coming from point C (see Fig.2.6.) and the perturbed radiation field $\tilde{E}_g$ diffracted by an irregularity of wavelength $\lambda_{ir} = \frac{2\pi}{k}$ and located at point D. This effect will be further discussed in section b.

b.) The electron density perturbations.

In chapter 2 it was shown that only a narrow band of wavevectors out of the total wavevector power spectrum of the phase irregularities present in the phase screen can be observed on the ground in any observation (see eq. (2-79)). This is the result of the combination of Fresnel filtering and antenna filtering. The effect of this combined filtering on the wavevector power spectrum of the intensity fluctuations measured on the ground is illustrated in Fig.2.5. for different experimental parameters. According to the correspondence between the measured frequency power spectrum and the calculated wavevector power spectrum, discussed in section a, we expect a narrow bandwidth of the measured temporal intensity fluctuations. Such quasiperiodic fluctuations are indeed observed quite regularly (see for example Figs.5.7.a. & 5.9.a.). Furthermore such quasimonochromatic intensity fluctuations (most likely associated with an irregularity wavelength λ_{ir} such that the wavevector $K = 2\pi/\lambda_{ir}$ is near the maximum of the filtering function (Fig.2.5.)) can be used directly to estimate the magnitude of the electron density perturbations using eqs.2-78, 2-46 and 2-53 with K constant. The intensity fluctuations (due to diffraction) at 430 MHz reported in this dissertation span the range from 6% to 40%. The phase fluctuations $\psi(K)$ in the screen can then be calculated from the intensity fluctuations $\Delta I(K)$ (using eq.2-78) to be in the range 0.04 to 0.26 radians. According to eq.2-46 this corresponds to a total electron

content perturbation integrated along the line of sight to the radio source of;

$$\int_L \Delta N_e(z) \cdot dz = \frac{\phi}{\lambda_e \cdot r_e} = 2 \text{ to } 13 \times 10^9 \left[\frac{\text{el.}}{\text{cm}^2} \right]$$

This later case with the stronger irregularities was observed on the night from 11 to 12 September 1980.

Assuming the irregularities have cylindrical shape with a diameter of ~ 1 km (estimated from the temporal extent of the individual perturbations in the intensity recording of Fig.5.4.a.) and are aligned along the magnetic field (having a dip angle of $\sim 50^\circ$ at Arecibo) we may set $L \cong 2$ km. This results in an electron density perturbation of;

$$\Delta N_e = 6 \times 10^4 \left[\frac{\text{electrons}}{\text{cm}^3} \right]$$

During that observation the HF-frequency was 7.3 MHz which corresponds to an electron density at the HF-wave reflection height of (using eq.2-42);

$$N_e = 6.6 \times 10^5 \left[\frac{\text{electrons}}{\text{cm}^3} \right]$$

The fractional electron density perturbation is then;

$$\frac{\Delta N_e}{N_e} = 10\%$$

If the irregularities have the shape of a slab with a thickness of ~ 1 km (estimated from the temporal extent of the individual perturbations in the intensity recording

of Fig.5.4.a) aligned with the magnetic meridian plane. we may set $L \cong 10$ to 20 km and the fractional electron density, perturbation is then;

$$\frac{\Delta N_e}{N_e} \cong 1 \text{ to } 2 \%$$

Since the slow intensity fluctuations (period ~ 2 minutes) correspond to positive intensity excursions we can conclude according to the arguments given in section 2.2.d (p.32) that the electron density perturbations are actually depletions. The faster (period ~ 20 seconds) intensity fluctuations would correspond to ~ 60 m irregularities, a wavelength of the irregularities at which the filtering inherent in the thin screen theory as applied to the experimental conditions at the Arecibo Observatory suppresses the intensity fluctuations to less than 2%. The measured intensity fluctuations are ~ 30 to 40 % (Fig.5.4.a) and contradict the prediction. It is for that reason that an alternative explanation based on refraction arguments is suggested in section 6.f.

c.) Comparison of observations at 430 MHz and 1400 MHz.

On the 3 April 1981 the intensity fluctuations were first measured at a frequency of 430 MHz and about 50 minutes later at a frequency of 1400 MHz (Fig.5.7.a.). With intensity fluctuations (excursion from mean) of $\sim 8\%$ (corresponding to phase fluctuations of < 0.04 radian according to eq.2-78) at an observing frequency of 430 MHz the thin screen theory should be a good approximation. The HF-power level was kept constant between 2:48:00 AST and 3:56:00 AST and the ionosphere was relatively stable. Under these conditions we may assume that the electron density irregularities were about the same for the 430 MHz observations and the 1400 MHz observations. With this assumption we can estimate the expected intensity fluctuations at 1400 MHz from the fluctuations at 430 MHz if the thin screen theory is indeed a good approximation. According to eqs. (2-46 & 2-53) the phase perturbation ϕ for any given irregularity scale size $\lambda_{ir} = \frac{2\pi}{k} = \text{constant}$ is proportional to the wavelength λ_o . If we further assume a relatively flat irregularity spectrum for the electron density over the interval $\lambda_{ir} \sim 700\text{m}$ to $\lambda_{ir} \sim 400\text{m}$ where the measurements for the two different frequencies have the sensitivity as shown in Fig.2.5.a.&b. we may estimate the intensity fluctuations at 1400 MHz by the use of eq.2-78. For intensity fluctuations of 8% as observed at 430 MHz we would expect the intensity fluctuations at 1400 MHz to be about 2%. The actually observed intensity fluctuations of 1.2% at 1400 MHz prior to 3:56:00 AST (before the HF-power level was raised by

about 20%) are in agreement with this expectation within the errors inherent in all the assumptions made.

d.) Response times.

There are two observations which allow us to deduce the time required for the formation of the irregularities after the HF-power is turned on. In both observations (3.April 1981; 2:48 AST and 4.April 1981; 1:45 AST) the HF-power was turned on when the line of sight to the radio source passed close to the center of the volume modified by the HF-waves. Another observation on the 4.April 1981; 2:35 AST (Fig.5.9.a.) is not considered here since the HF-power was turned on while the line of sight to the radio source was outside of the HF halfpower beamwidth at the HF-wave interaction height. We therefore do not know whether the increase in intensity fluctuations is determined by temporal or spatial changes. In the observations on the 3.April 1981; 2:48 AST (Fig.5.7.a) and on the 4.April 1981; 1:45 AST (Fig.5.8.) the irregularities form within 20 to 30 seconds after the HF-power is turned on. At Platteville (near Boulder, Colorado) the appearance of spread-F on ionograms, which is believed to be an effect caused by electron density irregularities, begins within seconds after the HF-power is turned on [Utlaut & Violette, 1974]. The irregularities observed with an HF phased array [Allen et.al., 1974] also show a formation time of 15-30 seconds after the HF-power is turned on. The observed response time of about 25 seconds corresponds to the time τ_N required for an average electron to diffuse over a distance L_N of about 3.5 km along the magnetic field by ambipolar diffusion. The thermal velocity of the ions v_{th} is assumed to be 700 m/s and the ion collision frequency ν_i is

about $1 \text{ [sec}^{-1}]$ which corresponds to typical F-region conditions. Hence;

$$\tau_N \simeq \frac{L_N^2}{v_{th}} = \left(\frac{3500}{700} \right)^2 \cdot 1 = 25 \text{ sec}$$

The time necessary for the decay of the irregularities is much longer (nighttime observations) and may only be observed if the irregularities drift in the same direction (east-west) as the line of sight to the radio source moves. During the night from the 11 to the 12 September 1980 strong irregularities were observed over an hour after the HF-power was turned off. At Platteville spread-F has been observed to last for several hours [Utlaut & Violette, 1974]. Using above equation again to obtain an estimate of the distance L_N over which the ionization diffuses along the magnetic field to fill the density depletion within a time $\tau_N \sim 1 \text{ hr}$;

$$L_N = v_{th} \sqrt{\frac{\tau_N}{\gamma_i}} = 700 \cdot \sqrt{\frac{3600}{1}} = 40 \text{ km}$$

This corresponds to a length of the irregularities along the geomagnetic field of $\sim 80 \text{ km}$. Similar scale sizes of the irregularities along the geomagnetic field were observed at Platteville with the HF phased array [Allen et.al., 1974]. In the above arguments the diffusion across the field is negligible for irregularities with scale sizes larger than $\sim 100 \text{ m}$ perpendicular to the geomagnetic field and for F-region conditions. It is also interesting that all three observations [Utlaut & Violette, 1974; Allen et.

al.1974; this dissertation] have shown comparable times for the formation of the irregularities even though they are sensitive to different scale sizes of the irregularities perpendicular to the geomagnetic field. This is another indication that a principal ingredient in the production mechanism of the irregularities is the ambipolar diffusion along the geomagnetic field.

e.) Spatial distribution of the electron density
irregularities.

From the data presented in chapter 5 it appears that the horizontal extent over which irregularities are produced near a height of 250 km (usually corresponding to an HF-frequency of 5.08 MHz) are well defined by the HF halfpower beamwidth (see for example; 4 April 1981 (Fig. 5.9.a.), 3 April 1981 (Fig.5.7.a.)). If irregularities are seen well outside the HF-beam they have drifted out of the HF-beam with a drift velocity that may be estimated from the spectra as explained in section 6.a. If the irregularities are produced at a height of 300 km or higher (corresponding to an HF-frequency of 7.3 or 8.2 MHz) some irregularities appear well outside the HF-beam (see for example; 11/12 September 1980, Fig.5.4.a. or 9 September 1980, Fig.5.2.). Some of these cannot be explained as having drifted out of the HF-beam because the drift velocity deduced from the spectrum indicates a drift in the opposite direction. In two examples (segment 1 of Fig.5.2. and segment B1 & C1 of Fig.5.4.a.) irregularities are observed well before the line of sight to the radio source enters the modified volume but the corresponding spectra indicate a drift in the other direction (same direction as the radio source moves). It therefore appears that these irregularities were produced by a sidelobe of the HF-antenna radiation pattern. The power level in the first sidelobe is approximately 5% of the power level in the main lobe. It further appears that in most observations the irregularities fill out the whole perturbed volume evenly except

for the observations on the 11/12 September (Fig.5.4.). During these latter observations one can distinguish individual isolated electron density perturbations. It appears that these observations show a more pure focusing (with a corresponding increase in the perturbations) into individual selfcontained filaments. In most other observations diffraction of the HF-waves on the density irregularities which they initially produced may have reduced the focusing effects thereby distributing the irregularities more evenly. This may be an important saturation mechanism in the formation of large scale irregularities.

f.) Scintillations due to refraction.

In the derivation of the thin screen diffraction theory in chapter 2 it was assumed that $\tilde{E}_0$, corresponding to the unperturbed wavefronts coming from the radio source, was unaffected by the irregularities in the screen. This approximation is valid as long as $\Delta N_e/N_e$ is very small. The calculations in section 6.b. showed that $\Delta N_e/N_e$ may exceed 1%. Under these circumstances the so far unperturbed wavefronts will be refracted by an angle [Appendix B] ;

$$\theta_r = \frac{f_p}{f} \cdot \sqrt{\frac{\Delta N_e}{N_e}} \approx \frac{7.3}{430} \cdot \sqrt{10^{-2}} \approx \frac{1}{10}^\circ$$

corresponding to a change in phase on the ground equal to;

$$\phi_r = \frac{2 \cdot \pi \cdot \theta_r^2 \cdot z_0}{\lambda_0} \approx 8 \text{ radians}$$

The first order intensity fluctuations according to the thin screen theory in chapter 2 are produced by the second term on the RHS of eq. (2-29), namely;

$$\Delta I = 2 \cdot \tilde{E}_0 \cdot \operatorname{Re} \tilde{E}_d$$

The refraction of the wavefronts unaffected by the diffraction at the phase screen can therefore modulate the intensity fluctuations which were due to the diffraction field by changing the phase between $\tilde{E}_0$ and $\tilde{E}_d$. This can be achieved by electron density perturbations $\Delta N_e/N_e$ of 1% or larger having horizontal dimensions which are shorter than

the separation of the points where $\tilde{E}_0$ and $\tilde{E}_d$ leave the thin screen. Since this modulation is proportional to $\Delta N_e/N_e$ a large scale irregularity as observed by the diffraction effects may produce intensity fluctuations due to refraction effects which appear to correspond to irregularities of shorter wavelength which do not actually exist. This effect has been observed on the 11/12 September 1980 (Figs. 5.4.a & c) and on the 9 September 1980 (Figs. 5.2.a & c ; segment 1).

This explanation differs from the refractive theory of [Crain et.al., 1979] which also assumes a refraction of $\tilde{E}_0$ but considers the interference between the refracted $\tilde{E}_0$ and the unaffected $\tilde{E}_0$ as the cause of the intensity fluctuations.

7. Summary and Conclusions.

In chapter 2 a theory was developed by the author which relates the intensity fluctuations on the ground to the electron density irregularities in the ionosphere. The theory is conceptually founded on the ideas of [Booker et.al.,1950] but is mathematically more general such as to explain more of the observable details in a scintillation measurement. It was assumed that the irregularities could be represented by a thin phase screen. It was shown that the intensity fluctuations measured on the ground are affected by a geometrical filtering term which is important in any intensity scintillation experiment and is usually called the Fresnel filtering term (discussed in sections 2.2.c. &d.). In the thin phase screen theory the intensity fluctuations on the ground result from the interference between the unperturbed field and the field diffracted from the irregularities in the ionosphere. The diffraction angle ($\theta = \lambda / \lambda_{ir}$) increases as the wavelength of the irregularities decreases. Information on the ionospheric irregularities with shorter scale sizes originates at larger angles from the line of sight to the radio source. If the radiation pattern of the observing antenna is very directive there is additional filtering due to the antenna which is less sensitive to larger angles and therefore less sensitive to irregularities of shorter scale sizes. The observations reported in this dissertation were affected by a combination of both filtering terms. In most of my observations reported the diffraction theory appeared to be a reasonably good description of the observed phenomena (compare Fig.2.5. with Figs.5.3.b., 5.6.,

5.7.c., and 5.9.c.). On two occasions however (see section 6.f.) differential refraction between the unperturbed wavefronts and the diffracted wavefronts had to be invoked to explain the 'fictitious' shorter scale intensity fluctuations which were strongly suppressed according to the thin screen theory. The origin of this differential refraction is a horizontal density gradient between the location where the line of sight to the radio source intersects the layer of irregularities and the location where the electron density irregularities of a given horizontal wavelength diffract the incident field. For one case where the thin screen theory was a good description at 430 MHz, the frequency scaling appeared to hold true in a comparison (section 6.c.) between observations at 430 MHz and 1400 MHz. The strongest electron density depletions produced by the HF-waves (section 6.b.) reached at least 1% of the ambient density for horizontal scale sizes between $\sim 1/2$ and 1 km (where the observations were most sensitive). The formation time of the irregularities after the HF-power was turned on was on the order of 25 seconds comparable with the measurements at Boulder, Colorado. Since the different experiments do not see the same range of horizontal scale sizes but appear to have the same formation time, one can conclude (section 6.d.) that diffusion along the geomagnetic field is an important factor in the formation of the electron density depletions. It further appears that the formation of the irregularities is rarely the result of focusing of the HF-wave by refraction alone (section 6.e.). Instead diffraction of the HF-waves on the electron density irregulari-

ties which are forming may spread the HF-wave energy into adjacent irregularities. That diffraction is important in overdense modification was explicitly stated in the theoretical treatment of [Perkins and Valeo, 1974]. Diffraction effects become stronger as the electron density perturbations grow and may actually form a saturation mechanism in the formation of electron density irregularities which initially grow by focusing of the HF-waves due to refraction. Some additional evidence of such an effect is observed on the 3 April 2:48 AST (Fig.5.7.a.) where the intensity fluctuations show an overshoot before the saturation sets in.

It is planned to observe in the near future the development of the irregularities as the HF-power is slowly raised by $\sim 20\%/min$ rather than abruptly turned on. This should give additional clues as to the production mechanism of the irregularities. A further important question to be resolved is the relative importance of the polarization of the HF-waves on the formation of the irregularities. This type of observation is planned to be done in the near future as the HF-polarimeter under construction nears completion. Since intensity scintillations have shown strong filtering effects it is suggestive to do phase scintillations which are less susceptible to such effects and which may therefore reveal new information not obtainable by intensity scintillations. Such an experiment is planned for the very near future and uses the 300 m dish of the Arecibo Observatory and the 30 m dish at Los Cannos which is at a distance of 10.7 km from the 300 m dish. It also has been observed (11/12 September 1980;

Fig.5.4.a.) that the irregularities have a lifetime exceeding one hour at times during nighttime. If the irregularities drift at a typical ionospheric drift velocity of ~ 30 m/s they should be observable at distances of ~ 100 km or more from the HF-facility. Such irregularities could be observed with a scintillation experiment using an antenna located 50 to 100 km to the east or west of Arecibo. With a favorable observing geometry and the right drift direction of the irregularities one could actually obtain field aligned observation even from Puerto Rico. If the drift instability proposed by [Borisov et.al.,1976] is an effective mechanism in producing short scale irregularities from the large scale irregularities then we should also be able to see radar backscatter from (field aligned) short scale irregularities outside the HF-beam.

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Appendix A

The objective is to show, that;

$$\int_A e^{i \cdot \tilde{\mathbf{K}} \cdot \tilde{\mathbf{S}}} \cdot d\tilde{\mathbf{S}} = 2 \cdot A \cdot \left[\frac{J_1(\kappa \cdot \mathcal{S}_0)}{(\kappa \cdot \mathcal{S}_0)} \right]$$

using circular coordinates;

$$\int_A e^{i \tilde{\mathbf{K}} \cdot \tilde{\mathbf{S}}} d\tilde{\mathbf{S}} = \int_0^{\mathcal{S}_0} \int_0^{2\pi} e^{i \cdot \kappa \cdot \mathcal{S} \cdot \cos \varnothing} \cdot \mathcal{S} \cdot d\varnothing d\mathcal{S}$$

It is known [Abramowitz and Stegun, 1970; eq. (9.1.21.)]

that;

$$J_0(\kappa \mathcal{S}) = \frac{1}{\pi} \int_0^{\pi} e^{i \cdot \kappa \mathcal{S} \cdot \cos \varnothing} d\varnothing$$

Since the integrand is even;

$$\int_0^{2\pi} e^{i \kappa \mathcal{S} \cos \varnothing} \cdot d\varnothing = 2\pi \cdot J_0(\kappa \mathcal{S})$$

Therefore;

$$\begin{aligned} \int_A e^{i \tilde{\mathbf{K}} \cdot \tilde{\mathbf{S}}} d\tilde{\mathbf{S}} &= 2 \cdot \pi \cdot \int_0^{\mathcal{S}_0} J_0(\kappa \mathcal{S}) \mathcal{S} d\mathcal{S} \\ &= \frac{2 \cdot \pi}{\kappa^2} \int_0^{\mathcal{S}_0} J_0(\kappa \mathcal{S}) \kappa \mathcal{S} d(\kappa \mathcal{S}) \end{aligned}$$

From the recurrence formula for Bessel functions

[Abramowitz & Stegun, 1970; eq (9.1.30.) setting $k=1, \nu=1, z=\kappa \mathcal{S}$];

$$\frac{d}{d(\kappa \mathcal{S})} \left[(\kappa \mathcal{S}) \cdot J_1(\kappa \mathcal{S}) \right] = (\kappa \mathcal{S}) J_0(\kappa \mathcal{S})$$

Hence;

$$\begin{aligned}
 \int_A e^{i \vec{\mu} \cdot \vec{S}} d\vec{S} &= \frac{2 \cdot \pi}{\mu^2} \left[(\mu S) J_1(\mu S) \right]_0^{\mu S_0} \\
 &= \frac{2 \cdot \pi}{\mu^2} \cdot (\mu S_0) \cdot J_1(\mu S_0) \\
 &= 2 \cdot A \cdot \left[\frac{J_1(\mu S_0)}{(\mu S_0)} \right]
 \end{aligned}$$

where; $A = \pi \cdot S_0^2$

Appendix B

The objective is to show that the angle of refraction θ_r for an incident wave with a propagation vector nearly perpendicular to the electron density gradient may be written in the limit of small angles as;

$$\theta_r \approx \frac{f_p}{f} \sqrt{\frac{\Delta N_e}{N_e}}$$

We start with Snell's law;

$$n \cdot \sin \mu = \text{constant}$$

where for a plasma the index of refraction is given by eq. (2-41). It is assumed that the gradient in the index of refraction is almost perpendicular to the propagation vector of the incident plane waves. It is convenient to express Snell's law in terms of a grazing angle $\theta_g = 90^\circ - \mu$ rather than the angle of incidence μ (see following Figure);

$$\sin \mu = \cos \theta_g$$

As seen from this Figure we may write for the angle of refraction θ_r ($\ll \frac{\pi}{2}$) for a given grazing angle θ_g ($\ll \frac{\pi}{2}$) and for a change in the index of refraction Δn across the slab of thickness Δx ;

$$(n - \Delta n) \cdot \cos \theta_g = n \cdot \cos (\theta_g + \theta_r)$$

$$\cong n \cdot \cos \theta_g \cdot \cos \theta_r$$

we also approximate;

$$\cos \theta_r = 1 - \frac{\theta_r^2}{2} + \dots$$

and obtain;

$$\frac{\Delta n}{n} = \frac{\theta_r^2}{2} \quad (B-1)$$

Differentiating eq.2-41 and dividing the result by eq.2-41 we obtain;

$$\frac{dn}{n} = \frac{\frac{f_p^2}{f^2} \left(\frac{df_p}{f_p} \right)}{1 - \frac{f_p^2}{f^2}}$$

for the refraction angle to be small we also require that;

$$\frac{f_p^2}{f^2} \ll 1$$

therefore;

$$\frac{\Delta n}{n} = \frac{f_p^2}{f^2} \left(\frac{\Delta f_p}{f_p} \right) \quad (B-2)$$

Differentiating eq.2-42 and dividing the result by eq.2-42 we obtain;

$$\frac{\Delta N_e}{N_e} = 2 \frac{\Delta f_p}{f_p} \quad (B-3)$$

Combining eqs. (B-1 to B-3) we may now write for the angle of refraction;

$$\theta_r = \frac{f_p}{f} \cdot \sqrt{\frac{\Delta N_e}{N_e}}$$

This expression corresponds to eq.(4) of [Crain et.al., 1979] and will be used in section 6.b.