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AN EXTENDED APPROACH.

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DOPPLER LINE PROFILES IN A
PLANETARY CORONA:
AN EXTENDED APPROACH

by

Robert Anthony Prisco

A THESIS SUBMITTED
IN PARTIAL FULFILLMENT OF THE
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HOUSTON, TEXAS

October, 1978

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ABSTRACT

Measurements of the width of the line profile of the geocoronal Lyman alpha emission by the hydrogen cell aboard OGO-5 show a marked day-night asymmetry and have led Bertaux (1974, 1978) to conclude that the population of satellite particles in the exosphere cannot be described by any single value of the satellite critical radius (Chamberlain, 1963). Rather, he concludes (on the basis of a computer simulation of the effect) that Lyman alpha radiation pressure from the sun destroys satellites in a complicated manner which depends upon the initial orbital parameters of each particle and that the satellite critical radius formalism may no longer be a viable way to distinguish a satellite population. He does not, however, consider the alteration of the profile due to proton-neutral charge exchange when the ionic temperature differs from that of the neutrals (Chamberlain, 1977; Prisco and Chamberlain, 1978), nor does he take into account the fact that the satellite temperature may differ from that of the remaining neutral population.

This investigation takes the point of view that Bertaux's interpretation of his data may not be unique. In particular, the influence of these last two effects

on the inferred profile (characterized by a "reduction factor") is examined, and several conclusions are reached. First, we find that the perturbation to the reduction factor by charge exchange among the ballistic and escaping populations is negligible. Secondly, we are able to reproduce the OGO-5 data satisfactorily by introducing a satellite population characterized by $\lambda_{cs} = 1.5$ and a temperature which equals the neutral temperature beyond about $6 R_e$ but increases inward to about 1.2 times the neutral temperature at $2 R_e$. We also argue that radiation pressure effects are symmetrical and hence are unlikely to be the cause of the observed asymmetry. Thus the question as to how to best describe a satellite population appears to still be open.

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It is a privilege to be able to acknowledge the contributions of others to this work; they have been considerable.

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My family and particularly my parents have been a continual inspiration and an invaluable aid to me and

I thank them most sincerely for their constant prayers and support.

Finally I dedicate this paper to my wife Georgette. Goethe once said; "The sum which two married people owe to one another defies calculation. It is an infinite debt which can only be discharged through all eternity." My debt to her is already more than can ever be repaid.

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CHAPTER 1

INTRODUCTION

1.1 Hydrogen in the upper atmosphere.

Understanding the behavior of hydrogen in a planetary atmosphere is fundamental to our comprehension of atmospheric physics and chemistry both past and present. Consider for example, the evolution of atmospheres and on earth at least, its relation to the formation of life. Planetary atmospheres are generally believed to have accumulated gradually through out-gassing of planetary interiors and to have originally been in a highly reduced state with gases such as water vapor, methane, and ammonia predominating (Johnson, 1969). However, the present condition of the terrestrial planets is markedly unreduced. An obvious way to account for the free oxygen, nitrogen, and carbon dioxide found in these atmospheres today is to photodissociate the reduced compounds and demand the escape of the resulting free hydrogen. Knowing the loss rate of hydrogen from the primitive earth would establish the rate of accumulation of oxygen in the terrestrial atmosphere. Hence, one could presumably establish the point in geologic time when the environment became suitable to support respiratory forms of life.

Developing a theoretical basis for this intuitive picture of the escape of light gases from a planet has been of interest from the mid-nineteenth century until

the present. Interesting historical reviews of the work have been given by Tinsely (1974) and Chamberlain (1963). The concepts involved are relatively simple. Substantial numbers of light atoms on upward trajectories in the high velocity tail of a Maxwellian distribution will be energetic enough to escape the gravitational pull of a terrestrial planet provided those atoms were at the "top" of the atmosphere; that is, provided the atmosphere in the escaping regime is tenuous enough that the probability such particles would suffer collisions returning them to lower regions was small.

Jeans (1925) named this top of the atmosphere the critical level. Today, the term exobase is more frequently used and is precisely defined to be that height at which the mean free path is equal to a scale height, and the probability that a particle traveling straight up will experience no more collisions is e^{-1} . Thus

$$n_c' H_c' Q = 1 \quad (1)$$

where Q is the gas kinetic collision cross section, $H_c' = kT_c/M'g$ is the scale height, the subscript c refers to the critical level, the primes denote the major constituent, M is the mass of the atom, and g is the local acceleration due to gravity. The collisionless region above the exobase is called the exosphere while the designation corona is used for the hydrogen component of the exosphere.

Application of (1) to the earth with $n_c' \approx 2 \times 10^7 \text{ cm}^{-3}$ and $T_c = 1000^\circ\text{K}$ assigns a height of approximately 500 km above the surface for the exobase. In this particular case, atomic oxygen is the major constituent at the exobase with atomic hydrogen beginning to predominate at about 1500 km.

Jeans assumed that collisions maintained a complete Maxwellian distribution of velocities for hydrogen atoms up to the exobase and derived an expression for the thermal, or Jeans, flux of escaping hydrogen (see Appendix A):

$$\phi_J = n_c w_c = n_c \left[\frac{U(1+\lambda_c) e^{-\lambda_c}}{2 \pi^{1/2}} \right] \quad (2)$$

Here w_c is the so-called effusion velocity which is defined by the second equality while

$$\lambda \equiv \frac{GMM}{kT_c r} = \frac{V_{\text{esc}}^2}{U^2} = \frac{r}{H}$$

where G is the gravitational constant, M is the mass of the planet and r is the geocentric distance, $U^2 = 2kT_c/M$ is the square of the most probable Maxwellian velocity and $V_{\text{esc}}^2 = 2GM/r$ is the square of the escape velocity from height r . Also $\lambda_c = \lambda(r_c)$ and $n_c = n(r_c)$.

Of course, the source of this escaping hydrogen must lie somewhere in the lower atmosphere. In analyzing the chemistry of hydrogen-oxygen compounds, Bates and

Nicolet (1950) pointed out that atomic and molecular hydrogen would be produced by the photodissociation of water vapor and methane in the mesosphere and lower thermosphere. In the region above the turbopause (the height at which bulk or turbulent mixing of gases gives way to molecular diffusion), diffusion would transport a flow of hydrogen of approximately $10^8 \text{ cm}^{-2}\text{sec}^{-1}$ to the critical level where it would presumably be lost by Jeans escape. Furthermore, they noted that the amount of oxygen liberated, assuming a constant hydrogen escape flux of $10^8 \text{ cm}^{-2}\text{sec}^{-1}$ over geologic time, consistent with the present amount of oxygen in the earth's atmosphere.

So the picture of hydrogen streaming up towards the exobase and being injected into the collisionless region above emerged, and even before the advent of high altitude space probes, it was possible to conceive of an extensive geocorona of atomic hydrogen surrounding the earth. Furthermore the problem of describing the distribution of hydrogen in that corona was seen to be intimately related to the manner in which hydrogen escapes.

The actual discovery of the geocorona by rockets which mapped the Lyman alpha airglow above 120 km in the late 1950's stimulated a great deal of theoretical activity. The first attempts to derive the density distribution in the exosphere were made by Johnson and

Fish (1959) and by Öpik and Singer (1959, 1960) who pointed out that the usual exponential or barometric density law

$$n(\lambda) = n(\lambda_c) \exp[-(\lambda_c - \lambda)] \quad (3)$$

for an atmosphere in hydrostatic and thermodynamic equilibrium could not be valid in the exosphere. This had first been noted by Milne around 1910. This follows for several reasons. First, (3) predicts a constant density at large distances from the planet when in fact, the density there should vanish. Also, at very large distances, atoms should be streaming out from the planet with velocities becoming more and more radial, while incoming high speed particles would not be present at all. This is in opposition to the isotropic distribution of velocities in a Maxwellian distribution, and the concepts of pressure and even temperature must be used with care.

These theoretical efforts reached their fruition with the elegant and comprehensive paper of Chamberlain (1963). Using the Liouville theorem, he derived the actual phase space distribution function for a corona assuming spherical symmetry, a sharply defined critical level and neglecting rotational effects. The types of particles to be found in the exosphere may be classified as either ballistic (bound particles whose trajectories

cross the exobase), satellite (captive atoms which orbit above the exobase), or escaping. Those particles with velocity exceeding the escape velocity and whose orbits arise from the exobase in the outward direction are the only high energy particles assumed to occur in appreciable quantities. The contribution to the total density from each class of particles is then calculated by integration of the distribution function over velocities subject to the dynamical constraints imposed on the orbits. The resulting distribution is given by

$$n(\lambda) = n_c \exp(\lambda_c - \lambda) \zeta_{\text{total}}(\lambda)$$

with

$$\zeta_{\text{total}}(\lambda) = \zeta_{\text{ballistic}}(\lambda) + \zeta_{\text{satellite}}(\lambda) + \zeta_{\text{escaping}}(\lambda) \quad (4)$$

This type of approach will be carried over to the calculations of Chapter 2. This expression is seen to be equivalent to (3) except for the factor $\zeta(\lambda)$, the partition function. This factor is unity for a Maxwellian distribution of velocities when all possible velocities are allowed and decreases as greater dynamical restrictions apply, providing a means, for example, for the density to vanish at infinity.

Of special interest here is the role of satellite particles. Since satellites do not connect to the exobase, the Liouville theorem does not apply, and some ad hoc assumption has to be made concerning the population

of these orbits. For example, one does not expect to find satellites at infinity. As relatively long lived particles insofar as the amount of time spent above the exobase is concerned, satellites are subject to such destructive processes as occasional collisions with magnetospheric protons, photoionization and radiation pressure. Actually such collisions would be the means by which satellites attain their peculiar orientation in the first place since the Liouville theorem would predict that atoms on trajectories intersecting the exobase would remain on such trajectories in the strict absence of collisions. Chamberlain introduced the concept of satellite critical level (r_{cs}) to deal with this problem. According to this description, a full complement of satellite orbits exists below r_{cs} , but no satellite perigees exist above. Thus all captive particles are "ballistic" with respect to r_{cs} (see Chamberlain, 1963, Figure 3). In the steady state, an equilibrium between creation and destruction processes would be established and the value of r_{cs} determined. The satellite critical level description has the advantage of being analytically simple. Its disadvantage is that it may not be correct. One could for example, imagine satellite destruction mechanisms which behave in a more complicated manner. Chamberlain (1976) called for observations to help clarify the problem and showed that careful measurements of Lyman alpha or Balmer alpha line profiles would be capable of

identifying the location of r_{cs} . At any rate, the treatment of orbiting particles remains a controversial aspect of the theory to which we shall return later.

Quite recently, a number of experimental and theoretical results changed our outlook on the behavior of hydrogen in the exosphere. A prevalent but fallacious belief concerning the escape flux was that somehow atmospheric structure determined the value of n_c . According to this view, if the exospheric temperature increased, as it might in periods of high solar activity, the product $n_c w_c$ and hence the Jeans escape flux would increase enormously because of the exponential dependence of w_c on T_c . Others assumed that the escape rate was set by the flux of hydrogen diffusing upward from the thermosphere. They calculated thermospheric density distributions and the value of n_c in the presence of a given amount of flow by solving the diffusion equation and imposing the boundary condition at the exobase that the Jeans flux must equal the flux from the source region. Thus T_c would again set w_c and n_c could be calculated via (1) since ϕ_J was assumed to be known. However, the correct way to attack the problem was demonstrated by Hunten (1973a,b, 1974). He did not impose the Jeans flux as an upper boundary condition but instead concentrated on the theoretical evaluation of the upward flowing flux in the lower atmosphere. Very briefly, his idea was that the flux should be relatively insensitive to the

details of the photochemistry, or even the exospheric temperature and that the flow through the critical level should be determined by the total mixing ratio of hydrogen in all forms in the stratosphere. This is the so-called limiting flux principle of Hunten (see Appendix B). Model calculations by Hunten and Strobel (1974) and Liu and Donahue (1974a,b,c) produced complimentary results and predicted the value of the upward flux to be between 1.6×10^8 and $3.1 \times 10^8 \text{ cm}^{-2} \text{ sec}^{-1}$. In the same period, four different spacecraft experiments (OSO-5, Vidal-Madjar, et al. 1974, OGO-5, Bertaux 1974, 1977; OGO-6, Thomas and Anderson 1976, Thomas and Vidal-Madjar 1976; and D2-A, Emerich et al. 1976) made measurements of the exospheric hydrogen content and temperature. Application of (1) gave a consistently lower value of only $7 \times 10^7 \text{ cm}^{-2} \text{ sec}^{-1}$ for the Jeans flux. Later, Brieg et al. (1976) presented in situ measurements of the hydrogen flow in the thermosphere which verified the model calculations. By this time, several investigators realized that the Jeans flux was from two to four times too small to remove all of the hydrogen arriving at the exobase and the search for other escape mechanisms began.

1.2 Charge exchange escape and the present work.

Cole (1966) had observed that the reaction

might be a significant non-thermal source of escaping hydrogen because of the relatively large reaction cross section. In this scenario, the high energy protons of the magnetosphere would be effectively confined by the geomagnetic field. However, upon exchanging an electron with a cold neutral, the resulting fast neutral could escape provided it was traveling outward. Tinsley (1973) made a rough but quantitative estimate of the magnitude of charge exchange contribution to the total escape flux and obtained a non-thermal flux of approximately four times the Jeans value. For completeness, it should be mentioned that the polar wind constitutes another form of hydrogen loss for the earth. Here H^+ , a minor constituent compared with O^+ , is increasing with altitude and is further accelerated by the O^+ -e electric field to supersonic velocities so that protons may escape from the open field lines of the polar region rather than settle into diffusive equilibrium. These polar fluxes are probably comparable to the charge exchange flux in magnitude (Banks and Holzer, 1969). However averaged over the entire globe, this contribution is probably only about one third that of the Jeans flux for moderate exospheric temperatures (Liu and Donahue, 1974b). Recently though, Tinsley (1978) has suggested that polar ion loss may occur at lower latitudes than was originally believed.

With the need to understand the implications of charge exchange becoming more and more obvious, Chamberlain (1977) presented a detailed collisional model with the Boltzmann equation. He provided calculations for the earth and Venus which showed the same enhancement of the escape flux that was required by the previous investigators and also gave new density profiles which incorporated the effect. Incidentally, the fact that the hydrogen escape rate is several times greater than was previously believed eases the requirements on the earth's oxygen budget if the primitive atmosphere was sufficiently rich in hydrogenous molecules. For example, Hunten and Donahue (1976) have calculated that a mixing ratio $f_t \approx 0.4\%$ would result in an escape flux sufficient to produce all of the oxygen in the atmosphere as well as in CO_2 and Fe_2O_3 in about 10^9 years. Thus several authors were obtaining convergent results insofar as the escape flux problem was concerned. However, the problem of satellite orbits remained troublesome because Bertaux (1974, 1978) in his exhaustive analysis of the Lyman alpha line profiles measured by OGO-5 offered a new interpretation.

Bertaux found that no single value of r_{cs} fit his data well, but that satellite particles were absent below about $2 R_e$ and gradually increased in number until they constituted about half of the total number of hydrogen atoms at $7 R_3$. The satellite population then

decreased outward from $7 R_e$ on the dayside. The destruction mechanism preferred by Bertaux to explain this behavior is solar Lyman alpha radiation pressure which had also been suggested by Thomas and Bohlin (1972) to elucidate their OGO-5 results. To assess the consequences of this effect, Bertaux and Blamont (1973) performed a computer "experiment". They placed atoms in orbits of various eccentricities and inclinations, "turned on" the sun, and monitored the evolution of the orbits. The conclusion they reached was that the results of the simulation predict qualitatively the observed features of the geocorona. For example, particles with perigees below $2 R_e$ were rapidly pushed below the exobase. This explanation is apparently considered satisfactory by a number of authors (Donahue, 1976). There may, however, be problems here too. Apart from any difficulties with the computer simulation (which may be considerable), the radiation pressure solution may not be unique. Other effects not considered by Bertaux and Blamont are the enhancement of the line profile due to the presence of charge exchange induced density (Prisco and Chamberlain, 1978) and the charge exchange induced temperature variation of the satellite population. Of course, in basic coronal theory, ballistic, escaping and satellite particles are considered to have the same temperature. However, as was noted previously, satellites can only exist because some atom traveling in a ballistic

orbit from the critical level experienced a collision. Since such a collision is most likely to involve a hot proton in the manner of (5), the satellite population should more realistically be characterized by a temperature somewhat hotter than that of the non-orbiting population. Furthermore, since the ion temperature in the magnetosphere varies with geocentric distance (Serbu and Maier, 1970), it is expected that the satellite temperature may do likewise.

The purpose of this paper then, is to consider how these other effects will modify the reduction factor (i.e., line profile) which we proceed to do in the following section. We will also argue that radiation pressure effects cannot account for the observed day-night asymmetry in the hydrogen distribution.

CHAPTER 2

THEORETICAL CONSIDERATIONS

Background information and details of most of the theoretical approach have been given by Chamberlain (1963, 1976, 1977) and by Bertaux (1974, 1978) and were recently summarized by Prisco and Chamberlain (1978). Therefore many of those results will be stated without explicit reference.

2.1 The reduction factor of the hydrogen cell.

The hydrogen cell is used in this work because the Doppler width of the geocoronal Lyman alpha emission is too small ($\sim 10^{-2}$ Å) to be observed directly. However, as we shall see, the so-called reduction factor of the H cell provides an indirect means of determining line width. The observing apparatus consists of an H cell placed in front of a photomultiplier tube. The cell itself contains a filament along with a supply of molecular hydrogen. When heated the filament dissociates a portion of the H_2 into atomic hydrogen with an optical thickness which is known by prior testing for a given line of sight, a portion of the corona is observed with the cell off and then with the cell turned on. Observations are conducted so that there is no Doppler shift between spacecraft and geocorona. The experimental design and working principles of the cell are discussed in greater detail by Blamont et al. (1975). The reduction factor of the cell is then defined to be the ratio of the total Lyman alpha intensity measured

with the cell on to the same intensity measured with the cell off. That is

$$R \equiv \frac{I(\tau_{\text{cell}})}{I(\tau=0)} \quad (6)$$

where I is the total emission rate in rayleighs (1 rayleigh = 10^6 photon $\text{cm}^{-2}\text{sec}^{-1}$). The observations are made from far outside the geocorona looking in (Figure 1). The width of the core will be characteristic of the temperature of the cell which for OGO-5 at about $24 R_e$ was 300°K . Hence the "cell on" configuration represents a relatively narrow, constant absorption core superimposed on the wider geocoronal emission. Figure 2 shows a typical unattenuated emission line profile. Observation with the cell off gives the total emission of the line; that is, the profile integrated over frequency. This, of course, is just the area under the second solid curve. The dotted line shows the same profile following absorption. The reduction factor will then be the ratio of the area under the dotted curve to the area under the solid line.

To gain a more intuitive feeling for the reduction factor, suppose that the H cell is monitoring the corona quite far from the critical level where the temperature might be close to that of the spacecraft. The emission profile of the corona will then be narrow and will be almost totally blotted out when the cell is on. Hence the reduction factor will be close to zero. On the other

hand, when observing a point close to the exobase where the temperature can be several times that of the cell, the emission profile will be much wider and only a relatively small portion will be absorbed. In this case, R will be closer to unity. In essence, the wider the emission line, the less it will be affected by the cell, and the larger will be R . One can see then that the reduction factor gives an indirect measure of line width. Sample reduction factors are shown in Figure 6.

Of course, other factors besides temperature influence line width. One is multiple scattering which we along with Bertaux neglect in this study. The second is the fact that the velocity distribution in the exosphere is not Maxwellian. This is discussed at length in the background papers, and one need only examine Chamberlain (1976, Figures 1 and 2), Prisco and Chamberlain (1978, Figure 4), or Bertaux (1973, Figure 7) to see the dramatic departure of the profiles from the usual Gaussian shape. Thus calculating the theoretical line shape for a non-Maxwellian distribution is our next concern.

2.2 The spectrum.

Expression (6) may be written explicitly in terms of the spectrum. The expression we will use is the distribution of velocities along a particular line of sight in space, summing over particles between the observer and infinity. This is proportional to the

spectral profile only in the optically thin regions where multiple scattering and extinction are negligible. With this assumption (6) becomes

$$R = \int_0^{\infty} \frac{I(\nu) \exp\{-\exp[-\frac{\lambda_0}{U}(\nu-\nu_0)]^2\}}{\int_0^{\infty} I(\nu) d\nu} d\nu \quad (7)$$

where the units of $I(\nu)$ are photon $\text{cm}^{-2}\text{sec}^{-1}\text{Hz}^{-1}$ and

$$I(\nu_s, r_1) d\nu_s = \frac{g}{4\pi} \left\{ \int_{r_1}^{\infty} \int_{\nu_\alpha}^{\infty} \int_{\nu_\beta}^{\infty} M^3 f d\nu_\alpha d\nu_\beta \frac{dr}{\mu} \right\} d\nu_s \quad (8)$$

(Chamberlain, 1978, Equation 7.1.68) which represents the total number of particles along a line of sight s , lying above a reference height r_1 , with velocity between ν_s and $\nu_s + d\nu_s$ (see Figure 1). Here ν_α and ν_β are the velocity components perpendicular to the line of sight, f is the collisional phase space distribution function and g is the number of solar photons scattered per second per hydrogen atom (Chamberlain, 1961). Thus the reduction factor is also a function of r_1 . We now proceed to evaluate (8).

In the theory of Chamberlain, the spherical symmetry reduces the number of independent phase space variables for describing particles undergoing central force motion in the exosphere to three: the radial velocity, angular

velocity and the height (or gravitational potential).

In dimensionless units these are

$$\xi \equiv \frac{p_r}{(2MkT_c)^{1/2}} = \frac{v_r}{U} \quad \nu \equiv \frac{p_\chi^2}{2MkT_c r^2} = \frac{v_\chi^2}{U^2} \quad (9)$$

and λ . (The symbol ν in (9) should not be confused with the symbol for frequency in (7). It will always be clear from the context which is meant.) An equivalent but more convenient set of variables involves the polar angle of the velocity vector θ , the total kinetic energy of the particle ψ , and the height λ . Thus

$$\psi = \xi^2 + \nu = \frac{v^2}{U^2} \quad (10)$$

The transformation between the two sets of variables is given by

$$\xi = \psi^{1/2} \cos \theta \quad (11)$$

$$\nu = \psi \sin^2 \theta$$

The collisional distribution function f is obtained by a linear perturbation to the collisionless situation by $H-H^+$ collisions. The collisionless phase space density f_0 is the known distribution function for the exosphere

obtained from Liouville's theorem with no satellite orbits (Chamberlain, 1963, Equation 13). That is

$$f(x_i, v_i) = f_0(x_i, v_i) [1 + \phi(x_i, v_i)] \quad (12)$$

where $f_0 \phi$ represents the accumulation of density along a dynamical trajectory (Chamberlain, 1977). Chamberlain also shows that $\phi(x_i, v_i)$ satisfies the differential equation

$$\frac{d\phi}{ds} = \frac{Ke^{\lambda/2\alpha}}{\lambda^{3/2}} \left\{ \frac{\zeta(\lambda)}{\alpha^{3/2}} \exp[(\xi^2 + v)(\frac{\alpha-1}{\alpha})] - 1 \right\}$$

The protons which participate in the charge exchange collisions are assumed to have a temperature T^* which is at a constant ratio α to the neutral temperature. Their velocity distribution is taken to be Maxwellian up to a plasmopause where the spatial distribution terminates. Using Chamberlain's result and the ψ, θ, λ set of variables we obtain

$$M^3 f = \frac{n_c}{\pi^{3/2} \bar{U}^3} e^{-(\lambda - \lambda_c)} e^{-\psi} [1 + \phi(\psi, \theta, \lambda)] \quad (13)$$

Then (8) takes the form

$$\frac{4\pi}{g} I(v_s, r_1) dv_s = \frac{n_c e^{-\lambda} c_{r_1} \lambda_1}{\pi^{1/2} U} \int_0^\lambda \frac{e^\lambda}{\lambda^2 \mu'(\lambda, \lambda_1)} v(v_s, \lambda) d\lambda dv_s$$

(14)

where

$$v(v_s, \lambda) = \frac{1}{\pi U^2} \int_{v_\alpha} \int_{v_\beta} e^{-\psi} [1 + \phi(\psi, \theta, \lambda)] dv_\alpha dv_\beta$$

(15)

is the velocity distribution along the line of sight.

2.3 The velocity distribution for an arbitrary line of sight.

Equation (14) has been evaluated by Prisco and Chamberlain (1978) for the relatively simple case when the line of sight coincides with a radius vector. For the observations of OGO-5, however, this rather special geometry no longer applies (Figure 1), and the calculation must be performed for an arbitrary line of sight. Bertaux (1974, 1978) has suggested an elegant approach to the problem, and our strategy here is similar.

The geometry of the situation is depicted in Figure 3. First define

$$\alpha \equiv \frac{v_\alpha}{U} \qquad \beta \equiv \frac{v_\beta}{U} \qquad \sigma \equiv \frac{v_s}{U}$$

so that

$$\psi = \alpha^2 + \beta^2 + \sigma^2$$

Then (α, β, σ) can be considered to be a set of cartesian axes in velocity space with σ oriented along the line of sight s . Notice that this figure contains a cone of half angle θ_0 . Because this cone plays a fundamental role in determining the shape of the non-Maxwellian distribution, we digress a moment to discuss its physical significance. A more comprehensive view of the cone is given in Figure 4. Since ballistic and escaping particles lie on trajectories which touch the exobase, they must have velocity vectors which lie inside the cone. The upper cone contains only ascending particles while the lower cone holds only descending ones since particles travel in both directions on each trajectory. For a given height and energy, particles whose velocity vectors lie outside of the cone cannot have originated their flight at the exobase. These assertions are easily proved. Supposed an atom of mass M has velocity vector of magnitude v which makes an angle θ with the local vertical of height r . Defining similar quantities at the critical level conservation of angular momentum requires

$$Mvr\sin\theta = Mv_c r_c \sin\theta_c$$

or

$$\frac{\sin^2 \theta}{\sin^2 \theta_c} = \frac{\psi_c \lambda^2}{\psi \lambda_c^2} \quad (16)$$

Now we wish to differentiate particles which touch the exobase from those which don't. The maximum angle a velocity vector may have with the radius at the exobase and still touch the exobase is $\theta_c = \pi/2$ when the two vectors are tangent. This condition by virtue of (16) defines the maximum angle which a particle originating at the exobase can make with the radius at height λ . This angle is denoted θ_o and can be considered to be the half angle of a cone with axis along r and apex at height r . Thus (16) becomes

$$\sin^2 \theta_o = \frac{\psi_c \lambda^2}{\psi \lambda_c^2} = \frac{\lambda^2}{\lambda_c^2} \left(1 + \frac{\lambda_c^{-\lambda}}{\psi}\right)$$

since conservation of energy requires that

$$\psi - \lambda = \psi_c - \lambda_c \quad .$$

So

$$\cos \theta_o = \left[1 - \frac{\lambda^2}{\lambda_c^2} \left(1 + \frac{\lambda_c^{-\lambda}}{\psi}\right)\right]^{\frac{1}{2}}$$

or

$$\cos \theta_0 = \frac{(\lambda_c^2 - \lambda^2)^{1/2}}{\lambda_c} \left(1 - \frac{\psi_1}{\psi}\right)^{1/2} \quad \psi \geq \psi_1$$

$$(17)$$

$$= 0 \quad \psi < \psi_1$$

where

$$\psi_1 \equiv \frac{\lambda^2}{\lambda + \lambda_c}$$

Note that except for very large energies or for $\psi < \psi_1$, the size of cone depends on the energy. This may be understood in the following way. If the total energy is less than a certain minimum amount, the particle must have come from the exobase because it doesn't have enough angular momentum to orbit outside the exobase. What is this minimum amount? Since $\psi = \xi^2 + v$ and $\xi^2 \geq 0$, the minimum energy necessary to support satellites of height λ is

$$\psi_{\min} = v$$

(i.e., invest all of the energy in angular momentum).

Then

$$\psi_{\min} - \lambda = \psi_c - \lambda_c$$

or

$$v - \lambda = v_c - \lambda_c$$

Since $\xi_c = 0$ for satellites just tangent to the exobase. But conservation of angular momentum requires

$$\frac{v_c}{\lambda_c^2} = \frac{v}{\lambda^2}$$

so that

$$v = \frac{\lambda^2}{\lambda + \lambda_c} = \psi_{\min}$$

Therefore $\psi_{\min} = \psi_1$. So that any particle with kinetic energy less than ψ_1 must be a ballistic since it lacks sufficient angular momentum to orbit as a satellite, regardless of the orientation of its velocity vector. For such particles then, the cone of acceptance covers all space and there is an isotropic distribution of ballistics for $\psi < \psi_1$ (Figure 4B).

Now suppose $\psi = \psi_1 + \Delta\psi$ where $\Delta\psi$ is small. Most particles will still be ballistics as discussed previously. However, some will now have angular momentum great enough to be satellites with some small remanent radial velocity. Hence the cone will have a half angle somewhat less than 90° to exclude these from the ballistic population (Figure 4C). As ψ increases, the cone continues to decrease so that satellites with even larger radial velocities at λ can be accommodated. There is a limit, however, for a given λ beyond which the cone can decrease no further from purely geometrical considerations (see Figure 4A).

In this view then, ballistic and escaping atoms fill the cone of acceptance while satellites fill the remaining space. As has been discussed before, a more realistic situation would admit only a partial population of satellites. The satellite critical level concept of Chamberlain (1963) was designed to deal with this problem. One defines a level $r_{cs} > r_c$ which assumes a role for satellite particles analogous to the critical level for ballistic and escaping particles. Specifically we suppose that below r_{cs} , occasional collisions maintain a complete isotropic distribution of satellites while above r_{cs} , collisions are assumed negligible, and only those satellite particles will be present which pass through perigee between r_{cs} and r_c . Assuming satellite destruction took place by the same charge exchange collisions and photoionization, Chamberlain (1963) showed how one might calculate the value of r_{cs} for the earth by requiring an equilibrium between the creative and destructive processes. Thus the concept of satellite critical level was basically designed to accommodate interactions which occur near perigee. This seems reasonable since the regions of greatest ion and neutral densities are found closest to the critical level. If however, destructive processes which act primarily at apogee are found to predominate, then we might expect that the satellite critical level description would no longer be completely valid. We will return to this point later.

Assuming a sharply defined satellite critical level, computations are carried out by simply replacing λ_c with λ_{cs} in (17). The cone will now be wider to accommodate all particles which are "ballistic" with respect to r_{cs} , and the calculation is carried through for the total captive population. The satellite contribution may then be isolated from that of the true ballistic population by subtracting the contribution of the ballistics. Of course, the ballistic and escaping populations are not affected by this modification. It is just that the ballistic formula may now be used to describe all captive (ballistic plus satellite) particles.

Returning to Figure 3A, we ask what particles contribute to the distribution an observer views at a particular line of sight velocity σ . (Henceforth, this discussion will concentrate only on particles which touch the exobase since satellites are easily included via the satellite critical level artifact just mentioned above.) Consider all velocity vectors with a particular magnitude $\psi^{\frac{1}{2}}$ which have a projection σ along the line of sight. As is evident from the figure, all such vectors lie on a ring of radius R in the $\sigma = \text{constant}$ plane. However, only those lying on the circle which are simultaneously inside the θ_0 cone have orbits which touch the exobase, and hence, count in the distribution. This portion of the ring is shown dotted in the figure.

Looking at Figure 3B, which is the observers view of the $\sigma = \text{constant}$ plane and taking advantage of the symmetry about the local vertical we see that $2\phi_1$ is the portion of the ring inside of the upper cone. If the cone is wide enough and the ring is large enough, the ring will also intersect the lower cone. In this case $2\phi_2$ is the portion of the ring within the lower cone. This circular geometry suggests the use of "polar" velocity coordinates rather than cartesian velocity coordinates to span the $\sigma = \text{const}$ plane. That is

$$\int_{\alpha} \int_{\beta} d\alpha d\beta = 2 \int_{\sigma^2}^{\infty} R \int_{\phi} d\phi dR \quad (18)$$

where

$$R = (\psi - \sigma^2)^{\frac{1}{2}}$$

The limits on the ϕ integral will be zero and ϕ_1 or ϕ_2 and π if only one cone is intersected. If both cones are ϕ cut, the integral becomes the sum of two integrals. Transforming the integration variable from R to ψ we obtain

$$V(\sigma, \lambda) = V_{\text{ballistic}}(\sigma, \lambda) + V_{\text{escaping}}(\sigma, \lambda) \quad (19)$$

where

$$V_{\text{ballistic}}(\sigma, \lambda) = \frac{1}{\pi} \int_{\psi=\sigma^2}^{\psi=\lambda} e^{\psi} \int_{\phi} [1 + \phi(\psi, \theta, \lambda)] d\phi d\psi$$

and

$$V_{\text{escaping}}(\sigma, \lambda) = \frac{1}{\pi} \int_{\psi=\lambda}^{\psi=\infty} e^{\psi} \int_{\phi} [1 + \phi(\psi, \theta, \lambda)] d\phi d\psi$$

where $\psi = \lambda$ by virtue of the definition of λ is the maximum amount of kinetic energy a bound particle may possess.

Two further matters complicate the evaluation of the velocity distribution. The first is the determination of the ϕ integral limits ϕ_1 and ϕ_2 . The other is the fact that the perturbation $\phi(\psi, \theta, \lambda)$ is a function of θ instead of ϕ . This requires the establishment of a relationship between ϕ and θ . Both of these problems are geometrical in nature and their solutions (which are quite straightforward) are constructed in Appendix C. We will henceforth use

$$\phi = \phi(\psi, \phi, \lambda)$$

Now (8) can be written in the form

$$\frac{4\pi}{g} I(v_s, r_1) dv_s = \frac{n_c e^{-\lambda} c r_1 \lambda_1 e^{-\sigma^2}}{\pi^{\frac{1}{2}}} S(\sigma, \lambda_1) d\sigma$$

(20)

where

$$S(\sigma, \lambda_1) = e^{\sigma^2} \int_0^{\lambda_1} \frac{e^{\lambda}}{\lambda^2 \mu(\lambda, \lambda_1)} V(\sigma, \lambda) d\lambda$$

is called the spectrum function in analogy to Chamberlain (1976, Equations 1 and 3). As discussed in that paper and elsewhere (Prisco and Chamberlain, 1978), knowledge of the spectrum function affords an excellent means of determining which classifications of particles are present in a certain distribution. One such function illustrating the effect of adding satellites to the distribution is plotted in Figure 5.

Finally (6) can be transformed to integrals over σ . Using the Doppler formula

$$v - v_0 = \frac{v_s}{c} v_0$$

we find

$$R = \frac{\int_{-\infty}^{\infty} I(\sigma, r_1) \exp[-\tau \exp(-\sigma^2)] d\sigma}{\int_{-\infty}^{\infty} I(\sigma, r_1) d\sigma} \quad (21)$$

One more refinement remains. Because the temperature of the satellite population may differ from that of the remainder of the population, the normalization factors that involve temperature will also be different.

Let

$$D \equiv \frac{1}{\pi^{\frac{1}{2}}} \int_{\sigma=-\infty}^{\sigma=\infty} e^{\sigma^2} S(\sigma, \lambda_1) d\sigma$$

and

$$D' \equiv \frac{1}{\pi^{\frac{1}{2}}} \int_{\sigma=-\infty}^{\sigma=\infty} e^{\sigma^2} S(\sigma, \lambda_1) \exp[-\tau \exp(-\sigma^2)] d\sigma$$

Then using (19)

$$R = \frac{\lambda_1 e^{-\lambda_c} D'_{\text{bal.}+\text{esc.}} + \lambda_1^* e^{-\lambda_c^*} D'_{\text{sat}}}{\lambda_1 e^{-\lambda_c} D_{\text{bal.}+\text{esc.}} + \lambda_1^* e^{-\lambda_c^*} D_{\text{sat}}}$$

where λ_1^* and λ_c^* involve the temperature of the satellites T^* . Thus

$$R = \frac{D'_{\text{bal.}+\text{esc.}} + e^{-\lambda_c \left(\frac{\alpha-1}{\alpha}\right)} \frac{D'_{\text{sat}}}{\alpha}}{D_{\text{bal.}+\text{esc.}} + e^{-\lambda_c \left(\frac{\alpha-1}{\alpha}\right)} \frac{D_{\text{sat}}}{\alpha}} \quad (22)$$

CHAPTER 3

DETAILS OF THE COMPUTATIONS

3.1 Discontinuities in the integrals.

As a first step, expression (19) was evaluated numerically using Gauss-Legendre or Gauss-Laguerre quadrature for the ballistic and escaping populations, the value of $\phi(\psi, \phi, \lambda)$ was calculated for each Gaussian point by integrating the differential equation for ϕ along a dynamical trajectory from the exobase to the required value of λ . This particular aspect of the calculation has been described in great detail by Prisco and Chamberlain. In determining the satellite contribution to the velocity distribution, however, we set $\phi(\psi, \phi, \lambda) = 0$. This is because subjecting satellites, which are products of $H-H^+$ collisions in the first place, to further collisions would be a second order effect. Even if one should desire to assess this effect, he would be presented with an infinite path of integration since satellites do not touch the exobase.

In calculating (19) without satellites, it was anticipated that discontinuities similar to those discussed by Chamberlain (1977) due to the sudden cut off of an entire class of particles from the distribution would manifest themselves. Another way of thinking of this is in terms of the cone of acceptance of ballistic and escaping orbits. The cone itself represents a physical discontinuity in that in crossing

the surface of the cone, one goes from a region full of particles into a region suddenly devoid of them. The point $\psi = \psi_1$ represents another point of discontinuity in the ψ integral since the cone itself abruptly changes from covering all space to covering a finite domain. For a given range of energy, the ring for each ψ may be completely within or completely outside of the cone. At the particular energy where the ring breaks the surface of the cone, there will be a discontinuity in the integrand. Mathematically, this occurs when the right hand side of (C-1) equals unity. In order that we do not integrate through points of discontinuity, we solve

$$\frac{\psi^{\frac{1}{2}} \cos \theta_0(\psi) + \sigma \cos \theta'}{R \sin \theta'} = 1$$

to establish the critical values of ψ and incorporate them into the integration limits. By breaking the ψ integrals up in this fashion, accurate results were achieved for each interval with a relatively coarse mesh size. It is certainly desirable to keep the number of Gaussian points as low as possible since otherwise, the computer time required to run the code rapidly becomes prohibitively large.

Another horizon effect is present in the λ integral of (20), since only escaping particles occur for $\lambda < \sigma^2$. This is treated by breaking the integral into

two portions at $\lambda = \sigma^2$.

3.2 Transformation of the lambda integral.

The factor $\mu'(\lambda, \lambda_1)$ in the denominator of (20) arises from the spherical geometry of the problem. The law of sines can be applied to show that

$$\mu'(\lambda, \lambda_1) \equiv \cos\theta' = \left[1 - \frac{\lambda^2}{\lambda_1^2} (1 - \mu_1^2)\right]^{\frac{1}{2}}$$

where θ_1 is the inclination of the line of sight with the local vertical (Figure 1). In using Gaussian quadrature on (20) it was discovered that μ' rapidly approaches zero as λ approaches λ_1 causing a singularity in the integrand. It was found that instead of resorting to a different integrating scheme the problem could be circumvented by changing the variable of integration from λ to θ . From Figure 1

$$b = r_1 \sin\theta_1 = r \sin\theta = \text{constant}$$

and

$$s = b \cot\theta$$

Thus

$$ds = -r_1 \sin\theta_1 \csc^2\theta d\theta \quad \theta_1 \neq 0 .$$

The relation for b also specifies the relationship between θ and λ . That is

$$\lambda = \frac{\lambda_1}{\sin\theta_1} \sin\theta$$

In actuality then, (8) was replaced with

$$\frac{4\pi}{g} I(v_s, r_1) dv_s = \frac{n_c e^{-\lambda} c_{r_1, \lambda_1}}{\pi^{\frac{1}{2}}} \left[\frac{\sin \theta_1}{\lambda_1} \int_0^{\theta_1} e^{\lambda(\theta)} V(\sigma, \lambda(\theta)) \times \right. \\ \left. \csc^2 \theta d\theta \right] d\sigma$$

for

$$\theta_1 \neq 0.$$

Of course $\csc \theta$ diverges at $\theta = 0$. However this occurs at $\lambda = 0$ ($r = \infty$) where the density and hence, the rest of the integrand vanishes causing the integral itself to remain finite.

3.3 Checking the results.

From the definition we have used to construct the spectrum (page 16), it is easy to see that

$$N(\lambda_1) = \frac{4\pi}{g} \int_{\sigma=-\infty}^{\sigma=\infty} I(\sigma, \lambda_1) d\sigma$$

where $N(\lambda_1)$ is the integrated density or total number of atoms along the line of sight. Using a different approach, Chamberlain had calculated and tabulated values of $N(\lambda_1)$ for the special cases $\theta_1 = 0$ and $\theta_1 = \pi/2$ (Chamberlain, 1963, Tables 3 and 4). Since the reduction factor is basically a ratio of integrated densities it was straightforward to reproduce Chamberlain's tables using the present code (of course $\phi(\psi, \phi, \lambda)$ was set equal to zero for this check). Gauss-Hermite quadrature

was used to perform the σ integrals. An error was discovered in Table 4 and those using it to calculate integrated densities are advised to proceed with caution. The mistake is in the numerical evaluation of equation (78) of that paper which is the correction factor to the integrated density for the ballistic plus the satellite component when the lower limit of the integral is replaced by λ_{cs} . Those familiar with quadrature formulae will find that the correct expression for $\Delta K(\mu_1, \lambda_1, \lambda_{cs})$ may be quickly and easily obtained by performing a double integration where the integral form of the relevant partition function is given by equation (33).

There is another error in Table 3 of the same paper which was first pointed out by D. L. Anderson, Jr. (private communication, 1976). In the case $\lambda_1 > \lambda_{cs}$, Table 3 cannot be used to find $K(\mu_1, \lambda_{cs})$ in the interval $\lambda_{cs} > \lambda > 0$ if $\mu_1 = 0$. This is because at the point $\lambda = \lambda_{cs}$ the angle between the radius vector and line of sight is no longer a right angle. The table can still be used however for $\mu_1 = 1$.

3.4 Comparison with the OGO-5 data.

The theoretical reduction factors computed by Bertaux to fit the OGO-5 data allowed for the absorption of the emitted radiation between the emitting region being observed and the detector. This can be accomplished quite simply by inserting a factor $\exp[-\tau(s)]$ in (8). Here $\tau_v(s)$ is the optical thickness along the line of

sight and

$$d \tau_v(s) = k_v \rho ds \quad (23)$$

where k_v is the mass absorption coefficient in $\text{cm}^2\text{gm}^{-1}$ and ρ is the mass density. Now

$$k_v \rho = \alpha_v n(r)$$

where α_v is the molecular cross section and $n(r)$ is the number density at r .

Integrating both sides over v and using $\alpha_v = \alpha_o \exp[-(\frac{\lambda_o}{U} (v-v_o)^2)]$ we obtain

$$\int_v k_v \rho dv = n(r) \alpha_o \pi^{\frac{1}{2}} \frac{U}{\lambda_o} \quad (24)$$

But (13) and (15) imply that

$$n(r) = n_c e^{-(\lambda_c - \lambda)} \frac{1}{\pi^{\frac{1}{2}}} \int_{\sigma=-\infty}^{\sigma=\infty} V(\sigma, \lambda) d\sigma \quad (25)$$

Substituting (25) into (24) and comparing yields

$$k_v \rho = \frac{\alpha_o n_c e^{-(\lambda_c - \lambda)}}{\pi^{\frac{1}{2}}} V(\sigma, \lambda)$$

Putting this result into (23) and integrating we obtain

$$\tau_v(s) = \frac{\alpha_0 n_c e^{-\lambda_c r_1 \lambda_1} e^{-\sigma^2}}{\pi^{\frac{1}{2}}} S(\sigma, \lambda_1) \quad (26)$$

It was decided, however, to ignore this effect in our computations for the following reasons. First, to calculate the collisional spectrum function contained in (26) for each Gaussian point involved in the determination of (20) would increase an already time consuming calculation by an incredible factor. Secondly, using several simple functions for the optical depth, it was verified that the effect of this absorption is to increase the reduction factor by a more or less constant amount for all values of r_1 which we will use in this study. Since we are concerned with how charge exchange and satellite particle temperature effect the shape of the curve, the absolute scale of the ordinate (see Fig. 6) really does not matter.

The OGO-5 observations were made along a tangent looking in and the reduction factor was plotted as a function of the impact parameter of this line of sight (Bertaux, 1978, Figure 6). The investigators had also deduced that $T_c = 1100^\circ\text{K}$ and determined the optical depth of the H cell to be $\tau = 4$. With this information we set $\theta_1 = \pi/2$, $\lambda_c = 6.395$ and used the "earth" set of parameters for the plasma (Chamberlain, 1977). The results are plotted in Figure 6 and are discussed in the next chapter.

CHAPTER 4

DISCUSSION AND CONCLUSIONS

4.1 The day-night asymmetry and radiation pressure.

First let us review the interpretation given by Bertaux of the OGO-5 data. Initial indications of the possible inadequacy of the satellite critical level description were reported by Thomas and Bohlin (1972) and by Bertaux and Blamont (1973) who had instruments aboard OGO-5, and by Wallace et al. (1970) for Mariner 5. The most thorough analysis was given by Bertaux and Blamont for their Lyman alpha photometer data. They reported a sizeable day-night asymmetry in the hydrogen density distribution. Specifically, they found that the night-side data was compatible with a Chamberlain model density distribution with $r_{cs} = 2.5 R_e$. However, they could obtain a fit for the dayside with the same r_{cs} only if they removed all satellite particles above $15 R_e$. The same conclusion was reached by Wallace et al. Bertaux and Blamont went on to assess the importance of various loss processes which might destroy particles at their apogee preferentially on the day side. Charge exchange collisions with solar wind protons were examined first. This would be a high altitude process which would be less effective on the night side where the solar wind cannot penetrate the magnetosphere. They also computed the lifetime against photoionization. Finally they attempted to assess the effect of radiation pressure on the orbits

and found this mechanism to be at least as important as the other two. They accomplished this latter investigation by simulating the earth-sun-satellite particle system in a computer. One family of satellite particles with perigee at $2 R_e$ and apogee at $20 R_e$ but with otherwise differing orbital parameters was subjected to a constant force, and the change in the orbital parameters was monitored as each particle was followed through several orbits. Their interpretation was that the behavior of the orbits under this influence could qualitatively explain the asymmetries in the data. For example, the simulation showed that for a constant value of the line of apsides, the stationary configuration was one in which the highest density of apogees was found at radial distances smaller than the initial value of apogee on the dayside, whereas on the night side, the highest density is found at radial distances greater than the initial value. ..

Bertaux (1974, 1978) presented the final analysis of OGO-5 measurements. This data was from the hydrogen cell experiment and was concentrated on the dayside. The observed reduction factor vs. radial distance from the earth was plotted and various model reduction factors were computed in an attempt to fit the dayside data. Bertaux found that his best fitting Chamberlain model was one in which no satellites at all were present. However, even this model, though passing through all of

the error bars, deviated in a systematic way from the data points themselves above about $4 R_e$. He then went back and sketched another curve through the data which, in his opinion, removed this systematic error. At each height he computed the number of satellite particles necessary to push the Chamberlain-model curve with no satellites up to touch his new modified curve. Finally he computed the value of r_{cs} necessary to yield the required number of satellite particles. His conclusion was that no single value of r_{cs} would suffice to describe the requisite satellite population. Rather, the number of satellites was a maximum at about $5 R_e$ and was nil below $3 R_e$ and above $10-12 R_e$.

4.2 Results of this study.

The first result that was noticed was that the modification of the reduction factor by including charge exchange collisions among the ballistic and escaping populations was negligible. This conclusion was not too surprising for several reasons. First, even though the addition of such collisions greatly adds to the earth's escape flux, the addition to the density is quite small because the relative effect is greater on the high velocity tail. (Chamberlain, 1977, Table 2.) Looking at the line profiles (Prisco and Chamberlain, 1978, Figure 4) provides another important clue. These profiles are all markedly distorted from the Maxwellian shape, but the bulk of the distortion is in the wings.

Since the H- cell absorbs roughly just the line center, the area under the wings appears virtually unaffected both in the numerator and denominator of (6) and the distorting effect just cancels out.

As was mentioned before, charge exchange collisions affect the temperature of the satellite population. As a first approximation the satellite temperature would be that of the ions. But this sounds misleadingly simple. For one thing, the exospheric temperature varies diurnally by as much as several hundred degrees. This effect can lead to a maximum to minimum daily temperature ratio of from 1.3 to 1.6 depending upon the level of solar activity (Tinsley, 1974). This time variation is short compared to the satellite production and loss process. Thus particles are continually being created at different temperatures, and which portion of the satellite population is characterized by which temperature may be very difficult if not impossible to determine. For another, occasional collisions with neutrals might tend to bring the temperature somewhat lower. At any rate, let us examine the effect varying satellite temperatures has on the reduction factor (Figure 6). The seemingly peculiar behavior which makes the curves with satellites cross the curve with no satellites can be understood by examining Chamberlain (1976, Figure 1). These are profiles normalized to the line center, as is, essentially the reduction factor. Note particularly the $\lambda_1 = 3.0$

portion of the graph. The addition of satellites at large heights occurs very close to the line center, so that after normalization, the wings in the curves with satellites are actually depressed relative to the satellite curves because the extra particles have been added at small radial velocities.

To explain the reduction factor variation with the ion to neutral temperature ratio α , consult Figure 5. Since increasing the temperature increases the density, letting α become larger has the same effect as adding more satellites. At large α , the line center becomes so sharp and the wings so depressed that the tendency for the reduction factor with satellites to dip under the reduction factor without satellites pervades even down to low altitudes and the former curve lies everywhere under the latter.

We can now follow Bertaux's procedure exactly. Drawing the same modified curve which he did, we can try to boost the satellite population at each height from the Chamberlain model with no satellites to meet the modified curve. The results of this effort are shown in Figure 6. For a constant r_{CS} we are able to vary the reduction factor in the desired manner by postulating a smooth but small variation in α . Therefore, the interpretation of Bertaux is not unique. A temperature variation in the satellite population will produce the same result.

In addition, there may be a more fundamental difficulty with the computer simulation itself. Many analytic studies of the effect of radiation pressure on orbiting bodies have been published and the results do not support the findings of Bertaux and Blamont. For example, Shapiro (1963) and Chamberlain (1978) have examined the rate of decay of perigee for various trajectories by computing the time variation of the orbital elements which define a Keplerian orbit subject to a constant perturbing force. This type of analysis shows that radiation pressure displaces the particles in a direction perpendicular to the earth-sun line rather than parallel to it as the computer simulation showed. Let us briefly examine the process to understand why this is true.

At first it may not be obvious that radiation pressure can be considered to exert a constant force on the atoms. The mechanism involved, of course, is the absorption and reemission of a solar Lyman alpha photon. To a high degree of accuracy, the emission is isotropic. Then, since the photon initially possesses momentum $p_i = h\nu/c$, the net change in momentum of the atom will be also just p_i . The g-factor, which is the rate at which photons are absorbed at the top of the atmosphere, was introduced on page 16. Tables of g indicate a value for Lyman alpha of $g = 2.3 \times 10^{-3}$ photon sec^{-1} . The magnitude of the perturbing force is therefore

$$F_p = Ma_p = \frac{gh\nu}{c}$$

corresponding to an acceleration of .75 cm/sec². Since an absorption occurs every $g^{-1} = 435$ sec, and typical orbital periods range from 1.5×10^4 sec for a circular orbit at $2 R_e$ to 3.3×10^5 sec for an orbit with perigee at $2 R_e$ and apogee at $15 R_e$, the acceleration is essentially constant.

In assessing the effects of this acceleration on a particle in orbit, we note first of all that (neglecting the earth's shadow) the force is conservative. Therefore since it is well known from celestial mechanics that the total energy of an object in orbit is

$$E = -\frac{GMM}{2a}$$

where a is the semi-major axis, the energy and hence the size of the orbit remain constant. However, since the net force acting is no longer central, a torque will be exerted on all non-circular orbits and the angular momentum (and with it the orbital eccentricity) will change.

To picture the situation more intuitively, it is useful to construct an analogy to the motion of charged particles that are "ExB" drifting. Consider, for example, Figure 9a. Without the perturbation, an atom is executing circular motion in the equatorial plane of the earth.

In the perturbed situation, the atom gets a "boost" at point x from the radiation force component in the plane. This particular boost tends to distort the orbit in such a way as to produce apogee 180° away with the particle returning to perigee at x. In actuality, however, the particle is retarded in the vicinity of apogee so that it reaches perigee at a point lower than x. Hence the net displacement of the orbit is perpendicular to the earth-sun line. If the relative orientation of the axes remains approximately the same over several orbits, the height of perigee will monotonically decrease until it dips below the exobase. Actually, with regard to hydrogen atoms in the exosphere, Shapiro deems it unlikely that even one revolution could be completed.

The same type of analysis can be applied to the orbits with orientations given in Figure 9b and c. Since the second harmonic of the earth's gravitational field causes the position of perigee to precess, the orbit of Figure 9c would begin to distort in the usual way but would eventually end up being rotated by 180° with a large eccentricity. Then the torque would reverse and the eccentricity would diminish. Figure 9b illustrates an orbit which just precesses without distortion through approximately 90° until its orientation is that of Figure 9c. All this, of course, assumes that orbiting particles remain above the exobase long enough for these effects to manifest themselves.

An analytic expression for the decay rate of perigee has been worked out by Chamberlain (1978, Equation 7.2.15). Since $r_p = a(1-e)$ we have

$$\frac{dr_p}{dt} = - \frac{a}{\omega_0} \frac{de}{dt}$$

The change in e can be calculated via the change in angular momentum by finding the difference in the area swept out by the radius vector with the perturbation and without it. The result is

$$\left\langle \frac{dr_p}{dt} \right\rangle = -\frac{3}{2} \frac{a_p}{\omega_0} (1-e^2)^{\frac{1}{2}}$$

where $\omega_0 = \langle \dot{\theta} \rangle$. For illustrative purposes, the values relevant to a circular orbit at $2 R_e$ a lifetime of about 2×10^5 sec which is slightly more than the period for one revolution.

So we see that, although radiation pressure is important in destorying satellite particles, its effect is not such as to be the cause of the observed day-night asymmetry. With particles initially moving in all directions, the perturbation will be symmetrical. On the other hand, the varying temperature model for satellite can. For example, the night side data of Bertaux and Blamont fit a standard Chamberlin model with a reasonable value of r_{cs} , so we don't have to worry

about distorting a reduction factor in this case. We simply have to invoke a nightside temperature close to the dayside temperature at $2 R_e$. Actually since the error bars on the nightside data allowed for values of $R_c < r_{cs} \lesssim 3 R_e$ there could be a slight temperature gradient here too. One could certainly argue about the reasonableness of the required temperature variation. To do so, however, would be missing the point, which is essentially this: considering the ambiguity which has been demonstrated, surely the real situation with regard to the distribution of satellite particles in the geocorona is quite a bit more complicated than any current explanation.

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LEGEND

Figure 1. The spherical geometry for coronal observations.

In the top portion of the figure, the observer is at O and the inclination of his line of sight with respect to the local vertical is specified by θ_1 , and the element of length is $ds = dr/\cos\theta'$. Another useful expression for ds is discussed in the text. The OGO-5 observations were made from outside of the geocorona looking in. This situation is depicted in the lower portion of the figure. The impact parameter of the line of sight is r , and hence $\theta_1 = 90^\circ$. This line of sight traverses an integrated density of twice that of an observer at B. The center of the earth is at A.

Figure 2. Geometrical interpretation of the reduction factor. The curves are normalized so that $e^{-\sigma^2}$ $S(\sigma, \lambda_1)$ represents a full Maxwellian distribution of velocities. The outer envelope shows the profile for a complete Maxwellian. The solid curve is the profile for $\lambda_c = 7.5$, $\lambda_1 = 6.5$ and no satellite particles. The area underneath is just the denominator of (7). The profile after passing through the absorbing cell is depicted by the dashed line. The area beneath it is the numerator of 7.

Figure 3. Detail of the geometry for calculating the velocity distribution where the line of sight makes an arbitrary angle with the radius vector. This detail can be put into the proper perspective by examining Figure 1. The origin of the (α, β, σ) co-ordinate system lies at height r (at the intersection of the lines s and r) while the σ axis coincides with s . The local vertical is chosen to be in the α - σ plane. Thus the dashed part of the ring is symmetric about the diameter O-A. The bottom portion of the figure is the view that an observer at height r has of the $\sigma = \text{constant}$ plane. Only velocity vectors with magnitude $\psi^{\frac{1}{2}}$ that terminate on the portion of the ring within the cone (shown dashed here) have trajectories which touch r_c . That portion of the ring is $2\phi_1$ where $\cos\phi_1 = \ell_1/R$. Should the ring intersect the lower cone also, analogous quantities ϕ_2 and ℓ_2 are defined. Figure 8 shows the derivation of ℓ_1 and ℓ_2 .

Figure 4. Projection of the cone of acceptance for ballistic and escaping particles onto the plane of orbital motion for a particle at height r . The height of the exobase is denoted by r_c . Section A illustrates the cone for large energies. For a given ψ and λ this is the smallest θ_0 can be since (17) degenerates into the Pythagorean theorem. Orbit B represents an atom with velocity vector within the cone. It must therefore be ballistic. On the other hand, orbit A must belong to a satellite. The situation for extremely low energies is shown in Section B. Here the cone widens the maximum amount; that is, it covers all space. As explained in the text, particles of such low energy possess insufficient angular momentum to orbit as satellites. Therefore, both of the trajectories shown must be ballistic even though one has a relatively large amount of radial velocity and the other has practically none. Section C illustrates an intermediate case.

Figure 5. Normalized collisionless spectrum functions illustrating the effect of adding satellite particles. Here $\lambda_1 = 3.0$, $\theta_1 = 90^\circ$ and $\lambda_c = 7.5$.

Figure 6. Reduction factor vs. r_1 for various α .

It is easily seen that by taking $\alpha = 1.2$ below $r_1 = 3 R_e$, and then letting it vary smoothly so that $\alpha = 1.15$ at about $r_1 = 3.5 R_e$, $\alpha = 1.10$ at around $r_1 = 4.5 R_e$, $\alpha = 1.05$ just below $r_1 = 5.5 R_e$ and $\alpha = 1.0$ above $r_1 = 6.5 R_e$, we can duplicate Bertaux's modified reduction factor with only the single value $\lambda_{cs} = 1.5$.

Figure 7. Geometry rotated to the Jeans escape formula.

The lower portion shows the element of volume in momentum space.

Figure 8. Projection of the cone of acceptance onto the α - σ plane. In this instance the ring of Figure 3 intersects both the upper and lower portions of the cone. Determination of ℓ_1 and ℓ_2 of Figure 3 is now straightforward.

Figure 9. All orbits are in the equatorial plane of the earth. Radiation pressure on the circular orbit accelerates the atom at X. If there were no retarding force, the particle would reach apogee as shown and return to perigee at X. Similarly a retarding force acts at Y and, in the absence of an acceleration would produce apogee there and perigee at Z. Therefore the combined effect produces a cycbidal motion to the left with the eccentricity increasing with each revolution. Eventually, the perigee point will reach the exobase and the particle will be lost. Orbits oriented as in B will not be initially deformed, but will precess until the major axis becomes perpendicular to the earth-sun line. The trajectory will then decay according to the previous description. In reality, there is a secondary precession caused by the second harmonic of the earth's gravitational field. Assuming that the particle survives long enough, C illustrates the fact that the torque on the orbit will eventually reverse itself and trajectory will switch back and forth as shown.

APPENDIX A

THE JEANS FLUX

To calculate the thermal evaporation rate of a planetary atmosphere, Jeans (1925) adopted the expression

$$\phi_J = \int v_z f_m(r_c, v) d^3v = \langle n_c v_z \rangle$$

Here (Figure 7A) v_z is the velocity component normal to the sphere and $f_m(r_c, v)$ is the Maxwellian velocity distribution which was assumed to persist at the critical level. Above the critical level, it was supposed that hydrogen experienced such infrequent collisions that the atoms would be traveling on essentially Keplerian orbits. The atoms contributing to the escape flux will be those on trajectories directed into the upward hemisphere whose velocities exceed the escape velocity at r_c . Since $U = (2kT_c/M)^{1/2}$

$$f_m(r_c, v) = \frac{n_c}{U^3 \pi^{3/2}} e^{-v^2/U^2}$$

With $v_z = v \cos \theta$ we obtain

$$\begin{aligned} \phi_J(r_c) &= \frac{n_c}{U^3 \pi^{3/2}} \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi/2} \int_{v=v_{esc}(r_c)}^{\infty} e^{-v^2/U^2} v^3 \cos \theta \sin \theta d\theta dv d\phi \\ &= \frac{n_c}{U^{3/2} \pi^{1/2}} \int_{v=v_{esc}(r_c)}^{\infty} e^{-v^2/U^2} v^3 dv \end{aligned} \quad (A.1)$$

After a change of variable and integration by parts one finds

$$\phi_J(r_c) = n_c U e^{-v_{\text{esc}}^2(r_c)/U^2} \left(\frac{v_{\text{esc}}^2(r_c)}{U^2} + 1 \right) \quad (\text{A.2})$$

or

$$\phi_J(\lambda_c) = \frac{U n_c e^{-\lambda_c}}{2\pi^{1/2}} (1 + \lambda_c)$$

At first it seems rather curious that this flux is dependent on the choice of reference level r_c . This was explained by Jeans in the following way. Because of the assumed Maxwellian distribution of the atmosphere, there will be some atoms on hyperbolic trajectories which come from infinity, pass within a distance b of the planet and travel back out to infinity. Now if $b < r_c$, such particles will be counted in the escaping population while if $b > r_c$ they will not be counted. Thus as r_c increases, more of this type of particles will be included and the escape flux will increase.

Still the flux is quite insensitive to the choice of r_c since in actuality, the product $n_c e^{-\lambda_c}$ is a constant independent of λ_c . To see this use (3) to eliminate n_c . Then the flux is

$$\phi_J(\lambda_c) = \frac{U n e^{-\lambda}}{2\pi^{1/2}} (1 + \lambda_c)$$

To get some feeling for the factor $(1 + \lambda_c)$, recall that

$$\lambda_c = \frac{GMM}{kT_c r_c}$$

Taking $T_c = 940^\circ\text{K}$ and $r_c = 6.87 \times 10^8$ cm corresponding to a critical level 500 km above the surface, we find $\lambda_c = 7.5$. Shifting the critical level by a large amount to say 600 km above the surface gives $\lambda_c = 7.4$ and the resulting change in the Jeans flux is only about one percent. Thus the thermal flux can be evaluated at practically any height as long as the temperature excursion is not too great.

It is interesting to note that, because Jeans thought the temperature of the upper atmosphere to be 219°K , his conclusion was that the escape rate of hydrogen from the earth was negligible. He did not, though, that the flux would be appreciable should the temperature reach 550°K .

It was mentioned in the text that in exospheric theory, a typical calculation involves starting with the appropriate distribution function and integrating over the desired coordinates subject to the dynamical constraints imposed on the orbits. The preceding was an example of just such a calculation. The dynamical restrictions were: (1) Only those orbits directed into the upward hemisphere would be included, (2) Only those particles with $v > v_{\text{esc}}$ counted. In the absence of

restrictions, all velocities and all directions would be allowed.

To gain some familiarity with the dimensionless coordinates of Chamberlain, it would be instructive to repeat the calculation using f_0 (see (13) with $\phi = 0$) and (9), and the fact that (Figure 7B)

$$d^3p = \frac{2\pi}{r^2} dp_r p_\chi dp_\chi$$

we find

$$\phi_J = \frac{U n_c e^{-(\lambda_c - \lambda)}}{\pi^{\frac{1}{2}}} \int_{\xi} \int_{v} \xi e^{-\xi^2 - v} d\xi dv \quad (A.3)$$

Now the restrictions require ξ to be positive and the total velocity of the particle to be greater than the escape velocity. That is

$$v^2 = v_r^2 + v_\chi^2 > v_{esc}^2$$

Divide everything by U . Then

$$\psi^2 = \xi^2 + v > \lambda \quad (A.4)$$

These are two unique combinations of velocities which will satisfy (A.4). If $\xi^2 > \lambda$ then $v = \text{anything}$. That

is, if the radial component of the velocity already exceeds the escape velocity, then the angular component can take on any value. But if $\xi^2 < \lambda$, then $\xi^2 + v > \lambda$ implies $v > \lambda - \xi^2$. Then A.3 becomes

$$\phi_J = \frac{n_c e^{-(\lambda_c - \lambda)}}{\pi^{\frac{1}{2}}} U \left\{ \int_{\xi=\lambda^{\frac{1}{2}}}^{\xi=\infty} \int_{v=0}^{v=\infty} \xi e^{-\xi^2 - v} d\xi dv + \int_{\xi=0}^{\xi=\lambda} \int_{v=\lambda - \xi^2}^{v=\infty} \xi e^{-\xi^2 - v} d\xi dv \right\}$$

Performing the elementary integrations and setting $\lambda = \lambda_c$ at the exobase we obtain (A.2).

The flux at any level also can be computed. Let's do this and illustrate the case of the ψ, θ, λ set of variables. Using (11) we have

$$\begin{aligned} \phi_J &= \frac{n_c e^{-(\lambda_c - \lambda)}}{\pi^{3/2} U^2} \int_v \int_{\theta} \int_{\phi} (\psi^{\frac{1}{2}} U \cos \theta) e^{-\psi} v^2 \sin \theta d\theta d\phi dv \\ &= \frac{n_c e^{-(\lambda_c - \lambda)}}{\pi^{\frac{1}{2}}} U \int_{\psi} \int_{\theta} \psi e^{-\psi} \cos \theta d(\cos \theta) d\psi \end{aligned}$$

The cone of acceptance for ballistic and escaping orbits is discussed at length in the text. It is shown there that all particles whose trajectories touch the exobase must lie within a cone of half angle θ_0 where the

expression for θ_0 is given by (17). The dynamical restrictions then require $\lambda < \psi < \infty$ and $0 < \theta < \theta_0$. Performing the θ

$$\phi_J(\lambda) = \frac{n_c e^{-(\lambda_c - \lambda)} U}{2 \pi^{\frac{1}{2}}} \int_{\psi=\lambda}^{\infty} \psi e^{-\psi} [1 - \cos^2 \theta(\psi)] d\psi$$

Substituting for $\cos^2 \theta$ we find

$$\phi_J(\lambda) = \frac{n_c e^{-\lambda_c} U}{2 \pi^{\frac{1}{2}}} \left(\frac{\lambda}{\lambda_c} \right)^2 \left\{ \int_{\lambda}^{\infty} \psi e^{-\psi} d\psi + (\lambda_c - \lambda) \int_{\lambda}^{\infty} e^{-\psi} d\psi \right\}$$

Finally

$$\phi_J(\lambda) = \frac{U n_c e^{-\lambda_c}}{2 \pi^{\frac{1}{2}}} (1 + \lambda_c) \frac{\lambda^2}{\lambda_c^2} \quad (A.5)$$

which, as we might expect, is identical to (A.2) except for the r^{-2} factor.

One refinement to the thermal escape flux calculation was completed recently. In reality the velocity distribution at the exobase cannot be truly Maxwellian since escaping atoms are bound to deplete the high velocity tail. Brinkmann (1971) and Chamberlain and Smith (1971) evaluated the corresponding reduction in the escape flux using a Monte Carlo technique and found the effect to be dependent on the ratio of masses of the escaping and background gases. Typical values for the

reduction of the escape flux are 28% for H in O and 50% for H in CO₂.

The Jeans flux then, represents a lower limit to the amount of light gas escaping from a planetary atmosphere. Other non-thermal mechanisms such as proton-neutral charge exchange, polar ion escape, the sweeping of ions from an unmagnetized planet by the solar wind, or highly exothermic chemical reactions, in short, any process which can create a fast atom must be considered when one is attempting to assess the total rate of escape of a light gas from a planet.

APPENDIX B

LIMITING FLOW

The limiting flow principle was rediscovered by Hunten (1973a,b) in the course of his work on Titan. In addition to these original papers, excellent discussions of the subject are given by Chamberlain (1978, Chapter 7), Hunten and Donahue (1976) and Donahue (1976). Briefly stated the principle reads (Hunten and Donahue, 1976):

The mixing ratio f_t of total hydrogen is the same at the homopause and in the lower stratosphere. The escape flux is then closely approximated by $\phi = bf_t/H_a$.

Here subscripts a refer to the background atmosphere and b is the binary collision parameter (to be defined precisely below) for H_2 in air. Then $f_t = n_{1t}/n_a$ where n_{1t} refers to hydrogen in all forms. For purposes of this discussion, the atmosphere is conveniently divided into two portions. In the lower portion or homosphere, bulk vertical mixing dominates the distribution of the atmospheric constituents while in the heterosphere, stable components tend to separate out into diffusive equilibrium. The homopause is the height which separates the two regions. An unstable or escaping component will not quite settle into diffusive equilibrium if escape from the top of the atmosphere is easy, but will flow to the top.

The flow of a minor constituent with density distribution $n_1(z)$ in a heavy background atmosphere when bulk vertical mixing can be neglected is governed by the molecular diffusion equation (Chamberlain, 1970, Equation 2.3.2)

$$\phi_1 = n_1 w_1 = -n_1 D \left(\frac{1}{n_1} \frac{\partial n_1}{\partial z} + \frac{M_1 g}{kT} + \frac{1}{T} \frac{\partial T}{\partial z} \right) \quad (\text{B.1})$$

The subscript 1 refers to the minor constituent and D is the molecular diffusion coefficient in $\text{cm}^2 \text{sec}^{-1}$. Since D is inversely proportional to the background density, b is defined such that $D = b/n_a$. The last two items on the right hand side above are immediately recognizable as the hydrostatic equilibrium value of the density scale height (cf. Chamberlain, 1978, Equation 1.1.5). Although we will neglect it here, if thermal diffusion is important, the last term becomes $(1 + \alpha_T)/T$ times the temperature gradient, where α_T is the thermal diffusion coefficient. Thus (B.1) could be rewritten in the form

$$\phi_1 = -b f_1 \left(\frac{1}{H_{1E}} - \frac{1}{H_1} \right) \quad (\text{B.2})$$

where

$$\frac{1}{n_1} \frac{\partial n_1}{\partial z} = -\frac{1}{H_1}$$

and H_1 is the actual density scale height of species 1. One can therefore see that departure of the real density gradient from the hydrostatic equilibrium value of the gradient will set up a flow with velocity given by the product of D and the difference in the reciprocal density scale heights. The negative sign assures the proper sense of the flow. If the real density distribution decreases more rapidly than the hydrostatic distribution, the flow will be upward; otherwise, it will be downward.

An analytic solution of B.2 is discussed by Tinsley (1974). He finds that

$$n_1(z) = \frac{\phi_1 H_a n_a(z_0) e^{-(z-z_0)/H_a}}{b} + (\text{const.}) e^{-(z-z_0)/H_1}$$

(B.3)

Thus if ϕ_1 is large, the light component's density is proportional to the ambient density and its scale height is the same as that of the major constituent, which is nearly the same as in the mixing region below the homopause. On the other hand, if $\phi_1 \approx 0$, the second term, which is just the hydrostatic equilibrium solution, dominates.

A diffusion equation appropriate to all regions of the atmosphere would include the effects of vertical mixing in the lower atmosphere.

Such an equation is

$$\phi_1 = bf_1 \left(\frac{1}{H_1} - \frac{1}{H_{1E}} \right) - Kna \frac{\partial f_1}{\partial z} \quad (B.4)$$

K is the so-called eddy diffusion coefficient. It is treated as a parameter to be determined empirically by fitting a solution of the above to observations of some tracer. It is large in the lower atmosphere and decreases with height as mixing becomes less important. D, on the other hand, varying as n_a^{-1} grows rapidly with height. So usually, in any given region of the atmosphere, either one term or the other in (B.4) is negligible. At approximately 100 km on the earth however, the two coefficients become comparable. The homopause is then precisely defined to be the height at which $D = K$.

We now inquire into the conditions under which the flow (B.4) is a maximum. Since everything in (B.4) is positive, ϕ_1 is greatest when $\partial f_1 / \partial z = 0$. (The mixing ratio gradient cannot be negative when $M_1 < M$.) This implies that M_1 is thoroughly mixed and is consistent with our discussion of the solution (B.3). Under those circumstances we saw that $H_1 = H_a$ and the flow is said to have its limiting value. Then (B.4) implies

$$\phi_1 = bf_1 \left(\frac{1}{H_a} - \frac{1}{H_{1E}} \right) \quad (B.5)$$

or

$$\phi_1 \approx \frac{bf_1}{H_a}$$

if $M_1 \ll M_a$. With this definition (B.4) can be rearranged to give

$$\phi_1 = \phi_\ell - (D + K)n_a \frac{\partial \phi_1}{\partial z} \quad (B.6)$$

These ideas are supplied at the homopause, so that if escape from the top of the atmosphere is easy, the maximum amount of flow that diffusion will allow through the homopause on its way to the top is given by (B.5). At great heights, the concentration gradient starts to increase, and by (B.6), the value of the flow starts to fall below the limiting value until at very great heights, hydrogen reverts to diffusive equilibrium with an extremely large scale height.

To make this idea more intuitive, one could define an effusion velocity at the homopause in direct analogy to the Jeans effusion velocity at the critical level (2). Using (B.5)

$$\phi_e = n_{1h} w_h = n_{1h} \frac{D}{H_a} \quad (B.7)$$

Thus for hydrogen at the homopause, w_h is usually a few centimeters per second. Using (2), the Jeans velocity is about 500 cm sec^{-1} for $T_c = 1000^\circ\text{K}$, while with $T_c = 500^\circ\text{K}$, $w_J = 1.5 \text{ cm sec}^{-1}$. These velocities describe the ease with which the hydrogen can flow through a certain portion of the atmosphere. For example, at

$T_c = 1000^\circ\text{K}$, the "bottleneck" to the flow is the homopause since $w_h < w_J$. That is, the flux there is already at the maximum value diffusion will allow and no amount of temperature increase in the exosphere will increase the flow. Although the flow velocity will increase, n_1 will decrease so that ϕ_ℓ remains constant. This called the limiting flow situation. For low exospheric temperatures though, $w_J < w_h$ and the bottleneck is at the top. Then hydrogen would pile up in the thermosphere and sizeable backflows down toward the homopause would result. The net upward flux would then be very small and (B.6) tells us that the concentration gradient must be large. That is, the distribution of the light constituent is breaking away from that of the background atmosphere and the situation approximates one of diffusive equilibrium. This is the so-called classical or Spitzer situation and describes the state of affairs for helium on earth.

Thus far the discussion has implicitly assumed that all hydrogenous molecules have been dissociated into H_2 at the homopause and that all of the H_2 converts to atomic hydrogen in the thermosphere. Part of the limiting flow idea, however, is that the value of the limiting flux can be determined much lower in the atmosphere. At heights where $D \ll K$, (B.6) reduces to

$$\phi_1 \approx \text{Kn}_a \frac{\partial f_1}{\partial z} \quad (\text{B.8})$$

Since hydrogen occurs in many forms at these heights (H_2O , CH_4 , H_2), it is necessary to formulate an equation like (B.8) for each form weighted by the number of hydrogen atoms in that molecule. These equations are then added to recover an equation for total hydrogen, namely

$$\phi_1 = -\text{Kn}_a \frac{\partial f_t}{\partial z} = \text{constant}.$$

The constancy of ϕ_1 occurs because there can be no sources or sinks of total hydrogen even if various processes shuffle about the relative amounts of the various hydrogenous forms. Now Kn_a is so large that f_t is essentially constant throughout the homosphere provided it is evaluated above the troposphere where H_2O rains out.

So the picture that emerges is the following one. The mixing ratio of total hydrogen can be measured at say, 30 km on earth. Then if the chemical form at the homopause is known, (B.5) furnishes a number for the limiting flux. The density distribution in the thermosphere will be such as to maintain this flux and the density at the critical level will adjust itself so that

$$\phi_1 = \phi_\infty = n_c w_c$$

With this type of thorough analysis, it was immediately recognized that $w_c \neq w_J$ for hydrogen on the earth and the search for additional escape processes began in earnest.

APPENDIX C

GEOMETRICAL DETAILS OF THE CALCULATIONS

In order to perform the ϕ integration in (19), the integration limits ϕ_1 and ϕ_2 must be determined. These limits represent the portions of the ring of Figure 3 which lie within the cone of acceptance. All atoms with velocity vector $\psi^{\frac{1}{2}}$ and line-of-sight velocity σ will have trajectories lying within the upper cone if a particular trajectory is oriented such that $0 < \phi < \phi_1$. If the geometry is such that the ring intersects the lower cone, the same situation requires $\phi_2 < \phi < \pi$. The coordinates which are normally used to describe an orbit, however, are ψ , θ , and λ where θ is the angle between the velocity vector and the local vertical. Thus, we also need to establish a relation between θ and ϕ .

Figure 8 shows a detailed view of the cone in the α - σ plane. The angle θ' which specifies the inclination of the line of sight with respect to the local vertical of the cone is given in Section 3.2. A velocity vector lying on the ring and just touching the surface of the upper cone will have $\theta = \theta_0$ and $\phi = \phi_1$. Such a velocity vector has a projection $\psi^{\frac{1}{2}} \cos \theta_0$ on the local vertical. Then from Figure 8

$$\cos\phi_1 = \frac{\ell_1}{R} = \frac{\psi^{\frac{1}{2}} \cos\theta_0 - \sigma \cos\theta'}{R \sin\theta'} \quad (C.1)$$

$$\cos\phi_2 = \frac{\ell_2}{R} = \frac{\psi^{\frac{1}{2}} \cos\theta_0 + \sigma \cos\theta'}{R \sin\theta'}$$

If the right-hand side of either of the above expressions is greater than unity, the ring is completely outside of the cone, while a value less than negative one indicates that the ring is entirely within the cone.

These equations also provide the desired relation between θ and ϕ . For fixed ψ and θ' , and regarding θ and ϕ as variables we find

$$\theta = \cos^{-1} \left[\frac{R \sin\theta'}{\psi^{\frac{1}{2}}} \left(\cos\phi \pm \frac{\sigma \cot\theta'}{R} \right) \right]$$

where the positive sign is used if $\phi > \pi/2$.