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AN INTEGRATED DECISION MODEL OF THE FIRM

by

Stephen P. Mezger

A Thesis Submitted
in Partial Fulfillment of the
Requirements for the Degree of
Doctor of Philosophy

Thesis Director's signature:

A handwritten signature in cursive script, reading "Ferdinand K. Levy", is written over a horizontal line.

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CHAPTER I

INTRODUCTION

1. Theory of the Firm: Its Diverging Evolution

Theories of the firm have, in the past thirty years or so, evolved roughly within two inherently conflicting channels.

The first of these channels, in its most pronounced form, views the firm as a set of interacting group and individual utility functions where interaction occurs so long as marginal benefits of cooperation outweigh the marginal disutility of sacrifice of more specific goal attainment. Less radical behavioral modeling of the firm emphasizes managerial objectives. The susceptibility of some of these latter models to mathematical exposition offers immediate and revealing comparisons with the neoclassical model with respect to resource allocating decision rules.

The second evolutionary process, rather than questioning neoclassical premises, has sought to buttress their relevance by extending the theoretical content. In the field of economics the major effort has been the conceptualization of the value of current capital services. In the field of finance recent work has attempted to incorporate the cost of financial capital and financial structure into neoclassical theory.

These two currents of thought are "inherently contradictory" in the sense that the first leads to a fragmentation of theories concerning the activities of the economic agents we call firms, whereas the second, at least in spirit, attempts by enrichment to enhance the scope and applicability of neoclassical theory. Ultimately, these richer models lay the basis for empirically testable hypotheses of business behavior. It is in this integrative spirit that this thesis is undertaken.

The possibilities of limiting the impact of managerial models appear to be quite good once the role of capital in the firm is delineated. The investment decision and the associated problems of capital budgeting comprise a nexus of decisions which are at once the most important and the most risky a firm undertakes. To the degree that firms and managers have committed their economic future to review by outside financial resources, the room for discretionary, non-profit maximizing objectives is reduced. The reduction of discretionary activities depends, of course, on the efficiency of the money capital market. On the other hand discretionary activities are most often associated with disequilibria in product markets. Only recently has much concern been shown for the impact of financial markets on firm behavior. This is surprising given the often voiced view, particularly in the field of corporate finance, that capital markets are relatively efficient in allocating investment resources.

2. Plan of the Thesis

The further and immediate aim of this introductory chapter is to develop more fully the integrative evolution in the theory of the firm. There is some historical interest in this since it serves to highlight the area of conflict between modern day neoclassicists and the managerial-behavioralist school. The primary purpose of the exposition is, however, to motivate the mathematical analysis of the following chapters.

A simple static equilibrium model of the firm, encompassing productive and financial decisions, is presented in Chapter II. Several permutations of the model are undertaken with the aim of adding realism.

The model developed in Chapter II is then used in Chapter III, in the context of a constrained sales maximizing objective function. Of particular interest is the effect of this non-profit maximizing behavior on the firm's ability to raise money capital and the implications for its vulnerability to takeover by a "raiding" firm.

Finally, in Chapter IV, the dividend decision is incorporated into the basic model (in a dynamic framework) and conclusions are drawn concerning the relevancy of dividend policy. Chapter V summarizes the integrative contribution of the analytical models presented in previous chapters and offers suggestions for theoretical extensions.

3.1. Theory of the Firm: Two Major Integrative Efforts by Economists

Perhaps the seminal contribution to an integrated view of the firm was that of Vernon Smith.^{1,2} The key aspect of a capital good, according to Smith,³ is that its presence in the form of capital stock is needed in order for production to take place. On the other hand, the decision concerning the number of units of capital to employ along with varying combinations of variable inputs requires some notion of the per unit current account cost of capital.

Using Smith's example and notations, suppose L is the fixed life of the capital input, W is the per unit price of the capital good, X , and r is the instantaneous discount rate. The present value of an outlay of WX every L years is

$$WX(1 + e^{-rL} + e^{-2rL} + e^{-3rL} + \dots) = \frac{WX}{1 - e^{-rL}}.$$

As a current cost this becomes $\frac{rWX}{1 - e^{-rL}}$. In the case of an indestructible capital good current cost is simply rWX . Note that if the capital acquisition were financed by debt, D , the latter term is rD . This equivalence is exploited in the model presented in Chapter

-
1. Vernon L. Smith, Investment and Production: A Study in the Theory of the Capital Using Enterprise (Cambridge, Mass.: Harvard University Press, 1961).
 2. Vernon L. Smith, "The Theory of Investment and Production," The Quarterly Journal of Economics (February, 1959), p. 61.
 3. Ibid., p. 65.

II. Smith goes on to provide a stimulating synthesis of the theories of investment, replacement and production.

In a later paper Smith discusses the relationship between market valuation of capital goods, replacement policy and investment theory.⁴ Market valuation in the context of this paper implies the economic valuation of capital goods at any point in time in terms of discounted benefits and secondary uses for the goods. Thus rather than an integration of securities market valuation and the theory of investment the result is a further amplification of investment and replacement theory and the development of a theory of second hand markets. If we revise some of the components of these integrated theories, we have an integration of an entirely different nature although conceptually closely related. For example suppose the capital good is instead a firm and the discounted stream of net returns is evaluated by shareholders rather than management. The result is an equity value for the firm. In place of the second hand market for real capital goods we substitute the securities market which provides in parallel manner an opportunity cost of capital investment. The integration conceived here encompasses the theory of production, investment and corporate finance and becomes the main topic of the thesis.

4. Vernon L. Smith, "Depreciation, Market Valuations, and Investment Theory," Management Science, Vol. 9, No. 4 (July, 1963), pp. 690-696.

In a paper, the theoretical content of which closely resembles that of Smith's earlier work, Dale Jorgenson systematically derives the rental value of capital services supplied by the firm to itself.⁵ The rental value of capital, c is shown to be

$$c = q(r+\delta) - \dot{q},^6$$

where q is the unit of cost of capital goods, r is the riskless discount rate, δ , the rate of depreciation and $\dot{q}$ the time change of the price of capital. The only difference in this formulation and that of Smith's is that the former is a dynamic, the latter static and that in the case of an infinitely durable capital good no term is needed for depreciation. Thus for $\dot{q} = \delta = 0$, the above expression reduces to $c = qr$ and is equivalent to rW .

From a historical standpoint it is interesting to note that the impetus for Jorgenson's article was provided by controversy over the adequacy of neoclassical theory to predict investment behavior. Jorgenson proceeds to show that the demand function for investment goods can be derived from neoclassical considerations. In the course of the exposition he shows that the neoclassical model of optimal capital accumulation can be derived by maximizing profit at each point

5. D.W. Jorgenson, "The Theory of Investment Behavior," in Macro-Economics Theory: Selected Readings, edited by H.R. Williams and D. Haffnagle (New York: Appleton-Century-Crafts, 1969), p. 207.

6. Ibid., p. 219.

in time, providing current capital services are valued by c , the rental value of capital.

3.2. Integrative Efforts in the Field of Corporate Finance

Integrative efforts by economists have, understandably, emphasized the uses side of the balance sheet. In the field of corporate finance it has been the sources side which has borne the focus of analysis.

Three efforts in the field of corporate finance are of particular interest. Douglas Vickers introduces financial variables explicitly into his model of the firm.⁷ His approach although thorough and insightful, proceeds on the basis of evaluating the shadow prices on given levels of debt and equity. In this manner the model ties in well to a logical sequential decision-making process.

For our purposes, however, a similar model presented by Stephen Turnovsky⁸ is more useful--primarily because it offers a simultaneous solution to the financial, investment and production decisions of the firm and is thus more closely aligned to the model presented in Chapter II--but also because a brief discussion of the Turnovsky model at this point raises a key question with respect to the formulation of the firm's objective function and the relationship of the objective function to

7. D. Vickers, The Theory of the Firm: Production, Capital, and Finance (New York: McGraw-Hill, Inc., 1968).

8. Stephen J. Turnovsky, "Financial Structure and Theory of the Firm," Journal of Finance (December, 1970), pp. 1061-1080.

the firm's discounted present value. The issue involves the use of book values in integrative models and is discussed more fully in Chapter II.2.

Turnovsky, although he does not make it clear in his article, assumes different profit expectations on the part of the firm's decision makers and holders of its issued securities. Thus the firm's objective becomes one of maximizing its net discounted present value, N :

$$N = \left(\frac{\bar{\pi} - rD}{i} \right) - K ,$$

where $\bar{\pi}$ is expected profits net of all current costs, including rental costs on capital, r and i the required returns on debt and equity, respectively, D , debt, and K , the book value of equity. The details of the model are not important at the moment. Three points should be raised concerning the formulation of the objective function.

The first point concerns the differentiation between book value of equity and market value, S , of equity defined as,

$$S = \frac{\bar{\pi} - rD}{i} .$$

This is not market value in the sense of being determined in the equity capital market. The value, S , is what is often called in investments textbooks the justified price of the firm's stock. The immediate market value of the firm's stock is in fact K , the book value, which along with the debt issue determines the size of the capital budget and the level of the real capital stock (assuming a

constant unit price for capital goods).

What is obfuscated in this formulation is the interaction of the firm's investment plans and the evaluation of those plans by financial investors. It is inherently more clear for purposes of integrating financial and productive decisions to assume an identity of expectations between the firm and its potential shareholders.

A further problem arises in the formulation of the term $(\bar{\pi} - rD)/i$. Expected profits, $\bar{\pi}$, are net of both variable factor costs and current account costs of capital, where current capital services are reckoned at a rental value. Previously we saw that Jorgenson's derived rental value in a static (or steady state) model with an infinitely durable capital stock was equivalent to Smith's rWX and this in turn was equivalent to rD . Therefore, the net expected profit expression which Turnovsky proposes contains a redundant term.

The last objection involves the steady-state discount term, i , the required rate of return on equity. If i is given (or, as in the Jorgenson model, is assumed to be the given riskless rate of interest), formulation of the firm's objective function in terms of a present value concept is quite legitimate. If the discount rate is to be determined as, say, a function of the firm's financial mix, the firm's objective function can no longer be expressed in terms of maximizing the present value of equity. The reason for this is that the concept of rental value of capital should now contain a term denoting the equity, as well as debt, rate cost of capital. Then if current account capital costs are expressed in terms of required flows to

holders of debt and equity, and if the expectational viewpoints of the firm and potential shareholders is identical, the solution of the productive and financial mix decision will simultaneously determine the market value of the firm's equity. In effect, the equity value of the firm is determined within a profit function which implicitly includes the firm's potential for generating revenue and the opportunity cost of using diverse forms of financial capital. These arguments will be stressed in Chapter II.

Predating and anticipating some of the results of the two previous works cited is the paper by Eli Schwartz.⁹ Schwartz brought together the profit potential of the firm in terms of the marginal rate of earnings, the argument of this function being the level of assets employed by the firm, and investors' preferences with respect to financial risk. The result of the synthesis is an optimum financial structure and a maximum share price for the firm's equity.

It is interesting to note that Schwartz does not confuse the analysis by attempting to draw a distinction between book value and market value. He does, however, make a careful distinction between the rate of return on reproducible assets and the rate of return on the value of the firm's equity. The contrast between industry and financial market equilibrium plays an important role in the discussion of managerial objectives in the context of financial markets in Chapter III.

9. Eli Schwartz, "Theory of the Capital Structure of the Firm," The Journal of Finance, Vol. XIV, No. 1 (March, 1959), pp. 18-39.

Schwartz's effort has always been given short-shrift by the "integrationists" which followed him. The reasons for this can probably be traced to the extensive use of graphical analysis, where in this case mathematical exposition would have been more convincing, and secondly, to his rather vague appeal to the "market indifference pattern between rate of return, capital stock and risk." As will be seen, this complexly stated tradeoff can be more easily handled by reference to the weighted average cost of capital.

The early standard work on capital costs is by David Durand.¹⁰ Some of the points raised by Durand were brought into question by the Miller-Modigliani paper¹¹ a few years later. Recently, the Miller-Modigliani view has been more firmly grounded in portfolio theory.¹² The M-M paper engendered an intense theoretical controversy and a large body of empirical work all directed to the question of whether or not the firm's financial mix decision affected share price. The controversy is far from being resolved, but it is interesting to

10. David Durand, "Cost of Debt and Equity Funds for Business: Trends and Problems of Measurement," Conference on Research on Business Finance, New York: National Bureau of Economic Research, 1952, pp. 215-247.

11. M.H. Miller and F. Modigliani, "The Cost of Capital, Corporation Finance, and the Theory of Investment," American Economic Review, Vol. 48 (June, 1958), pp. 261-297.

12. See for example, R. Hamada, "Portfolio Analysis, Market Equilibrium and Corporation Finance," Journal of Finance, 24 (March, 1969).

note that it proceeded for some time without a precisely specified relationship between leverage and financial market equilibrium. Empirical studies have tended to confirm, albeit imperfectly, a U-shaped cost of capital function. The concept of required rates of return to equity and debt and their weighted average, or the overall cost of capital, as used in all of the models presented in this thesis, is based on the empirical assumptions that underlie the "traditional" approach as articulated by Durand and reproduced in Chapter II.

3.3. Further Integrative Modeling: Shareholders' Utility, the Problem of Dividends and the Relationship of Optimal Capital Accumulation to Corporate Financial Decisions

Ultimately, the decisions of the firm must satisfy its shareholders. Jorgenson's model of optimal capital accumulation, wherein the objective of the firm is maximize present value, is firmly anchored in the ultimate objective of maximizing the utility of a consumption stream. As Jorgenson observes, "Maximization of the present value of the firm is the only criterion consistent with utility maximization..."¹³ For this conclusion he is indebted to Irving Fisher, and the latter's modern proponents, Bailey and Hirschleifer.¹⁴

13. Op.cit., p. 214.

14. Irving Fisher, The Theory of Interest (New York: Macmillan Co., 1930). M.J. Bailey, "Formal Criteria for Investment Decisions," Journal of Political Economy, LXVII (October, 1959), pp. 476-88. J. Hirschleifer, "On the Theory of the Optimal Investment Decision," The Journal of Political Economy, Vol. LXXVI, No. 4 (August, 1958), pp. 329-52.

The Fisherian model which included financing decisions under conditions of certainty was further extended by Yutaka Imai in his recent dissertation.¹⁵ Imai's contribution consists of extending the Fisherian framework to include corporate financing; that is, the firm's decisions with respect to borrowing and equity issue. He effectively demonstrates the dichotomization of the financing and productive decisions on the one hand, and the current period consumption decision on the other.

Imai then replaces the objective of maximization of present value with that of maximization shareholders wealth--that is, maximization of the price per share--and in parallel procedure to that of Jorgenson optimizes a set of productive and financial variables in the context of a dynamic model. Since the dividend decision is independent of financial and productive decisions the decision rules in these areas are in no way affected by subsequent consumption choices of shareholders. It is not clear, however, whether these choices will affect share price.

The analysis in the succeeding chapters is founded on many of the theoretical and empirical propositions mentioned in this brief introductory summary. In general the analysis is intended to advance the insights provided by the models discussed heretofore and follows the integrative approach of explicitly introducing financial variables

15. Yutaka Imai, An Integration of Neoclassical Theory of Optimal Capital Accumulation and Corporate Financial Theories, unpublished thesis dissertation, Rice University, 1971.

into the objective function. Somewhat more emphasis is placed on two critical concepts: First, the use of market value as opposed to book value of capital in building the analytical model; second, the dual role of the shareholder as a) a wealth maximizer and b) as owner of productive real capital. The former consideration leads to decision rules consistent with logical planning objectives of the firm, while the latter provides a basis for considering the investors' time preferences with respect to current and future income.

CHAPTER II

AN INTEGRATED DECISION MODEL OF THE FIRM

Introduction

The first part of this chapter presents a basic static equilibrium model of the firm, integrating production, financial and investment decisions. The original aspect of the model lies in the treatment and interpretation of capital costs. In particular, it is seen that if the firm equates the marginal rate of return on real capital to the required return on money capital, the firm simultaneously maximizes share price. This follows from the fact that the required rate of return on equity capital discounts an optimal net income stream (over an infinite horizon). The optimality of the net income stream is assured since its arguments, factor inputs, real capital and labor, debt and debt cost, are all optimal values, determined from the set of first order conditions of the objective function.

The simplicity of the model allows several realistic adaptations and improvements. First, it is argued that capital costs ought to be a function of the size of the capital fund as well as the debt-equity ratio, and in general many diverse aspects of the firm could, reasonably, affect the stability of the firm's earning stream and thus affect capital costs. Anyone who has read a stockbroker's analysis of a firm will

realize the diversity of topics which are of interest to the investor.

With these considerations in mind Section II.2 is devoted to adapting the model to the specific effects of operational leverage and advertising, where the latter is representative of a class of diverse firm decision variables which affect stock price not only through expected profits, but through their effect on required capital costs.

1. The Objective Function of the Firm

It has been suggested by several authors (Gordon [11]; Schwartz [27]; Imai [15]) that the firm should attempt to maximize (in each time period) the per share value of common stock. For Gordon and Imai this is equivalent to discounting the firm's stream of dividends. Schwartz, on the other hand, is more concerned with the effect of the firm's financing decisions on share price.

Schwartz's 1959 article preceded the more widely known work of Vickers and the recent paper of Turnovsky and in some ways is definitely superior. One advantage is that Schwartz's model arrives at a determinate solution, given the state of the financial, product and factor markets, rather than Vicker's sequential approach, and in addition avoids the obscure and confusing notion of net discounted value used by Turnovsky. The model presented here is closely aligned to the Schwartz model, but builds on the contributions of Smith, Vickers, and Turnovsky.¹

1. See general references to these authors in Chapter I.

To begin with the simplest approach is to conceive of the firm as an administrative entry, attempting to operate at a point on its minimum cost curve which maximizes profits. It will be shown that this concept of the firm's objective is consistent with

1. Minimization of overall capital costs; and
2. Maximization of current share price.

Further it will be seen that book value is irrelevant to the firm's decisions and use of it is somewhat contradictory.

In developing the analytical model it is beneficial to proceed by two or three approximations, grafting concepts from corporate finance literature on a step by step basis. Suppose we have a profit maximizing firm whose objective function may be expressed as

$$pf(K, L) - w_1 L - w_2 K \quad (\text{II.1.1})$$

where p is the constant price of output, K and L are the factors of production; i.e., real capital goods, assumed to be infinitely durable, and labor and w_1 , w_2 are their respective per unit current costs. The production function $f(K, L)$ is twice differentiable and $f_K, f_L > 0$, $f_{KK}, f_{LL} < 0$, where the subscripts denote first and second order partial derivatives. Following Jorgenson,² w_2 is the per unit current cost of capital services; that is,

2. Dale Jorgenson, "The Theory of Investment Behavior," in Williams and Hufnagle, eds., p. 219.

$$w_2 = kp_K ,$$

where k is the required rate of return on real capital investment and p_K is price per unit of capital item.³ This formulation is precisely equivalent to that of Smith.⁴ The term $p_K K$ is in dollar terms and can be viewed as the money capital fund needed to maintain the capital stock. Then $kp_K K$ is the required stream of returns to financial backers of the firm. Since the interests of financial backers of the firm are explicitly represented on the sources side of the balance sheet in terms of debt, D , and equity, E , let

$$kp_K K = k(D+E) .$$

Substituting the right hand side of the above equation into (II.1.1) puts the objective function in terms of variables of both sides, sources and uses, of the balance sheet:

$$pf(K, L) - wL - k(D+E) . \quad (II.1.2)$$

Those familiar with the literature of corporate finance will recognize

3. In Jorgenson's model, $w_2 = (r+\delta)p_K - \dot{p}_K$, where δ is the depreciation rate and $\dot{p}_K$ is the time change of the price of capital. Since the capital stock is indestructible and the model is static, $\delta = \dot{p}_K = 0$.

4. Vernon L. Smith, "The theory of Investment and Production,"....

k as the average cost of capital. In a world of certainty the concept has little meaning, however, since the required rates of return to debt and equity would be the same.

Under conditions of uncertainty a distinction must be made between required rates of return to debt and equity. The reason for this is that the two types of securities have different risk characteristics. Suppose income in each period is characterized as a random variable. Following common practice, let security holders' subjective expectations with respect to an unknown stream of income for a given period be represented by the mean of the probability distribution of the income random variable and let risk be associated with the coefficient of variation (ratio of the standard deviation of the distribution to its mean).

If the firm's financial structure has no debt, the risk associated with holding equity shares is the same as that associated with the income stream. Once the firm finances a portion of its productive assets with debt, the risk factor associated with equity holding increases because the mean of income stream available to shareholders is decreased by the amount of the current debt payment. The variance of the income stream, of course, remains constant. The firm's creditors, on the other hand, are only concerned with the probability of the income stream being insufficient to cover their senior claims.

In summary, the introduction of uncertainty has three results:

- a) Required returns on both debt and equity rise by the amount of a risk premium;
- b) The risk premium associated with the required rate of returns to equity is associated with the risk characteristics of the

firm's income stream, or what is commonly referred to as business risk. Risk is increased, however, if the firm is leveraged and this increased risk is denoted as financial risk. Financial risk (as well as total risk) is an increasing function of leverage;

- c) The risk premium associated with the required rate of return to debt is also an increasing function of leverage. The risk premium here is less than in the case of equity due to creditors' senior claims.

The divergence of required rates of return on debt and equity as functions of leverage is portrayed graphically in Figure II.1, where r is the required rate of return on debt, and i , the required rate on equity.

FIGURE II.1

The shape of these functions indicates the firm can economize on its overall cost rate of financing, k , by a judicious mixture of debt and equity. Note that

$$k(D+E) = iE + rD,$$

so that

$$k = k\left(\frac{D}{E}\right) = \frac{i\left(\frac{D}{E}\right) + r\left(\frac{D}{E}\right)\frac{D}{E}}{1 + \frac{D}{E}} \quad .5 \quad (\text{II.1.3})$$

If money capital is required only for the procurement of real capital goods, then

$$P_K K \leq D+E . \quad (\text{II.1.4})$$

If the firm solves its financial and productive decisions optimally, the equality must hold. This follows from the characteristic of this model that the value of equity is simultaneously derived as will be seen below.

The above expression is the simplest form of the money capital constraint. It indicates that all the firm need do to obtain one more capital item is to raise sufficient funds through debt and equity issues to cover the purchase price, P_K . The more fundamental question is, of course, whether or not the additional capital item should be purchased and this question is resolved by reference to the decision rules derived from the model's first order conditions.

The transformation of money capital into real capital as given

-
5. It should be clear that the overall cost of capital is a U-shaped function of leverage. If $D = 0$, $k = i$. For small increments of debt and equal decrements of equity, $k < i$, but over larger values of leverage the combined changes of Δi and $\Delta(r \cdot D/E)$ are greater than $1 + \Delta(D/E)$.

in the above expression can be depicted in the following graph (the heavy line) as a constant function of real capital, the slope of which

FIGURE II.2

is the unit price of capital, p_K . Typically, the transformation is not this efficient if we mean by efficiency the ability of the firm to convert capital funds into real capital against whose generated income stream shareholders have a claim. The dotted line indicates a less efficient transformation; its slope $O_K > p_K$. For the same level of the capital budget fewer units of real capital are obtained. This implies that the amount of funds which are left over, in the less efficient case, have been diverted to uses other than the acquisition of real capital.

Typically, money capital is associated with nearly all the firm's inputs. Examples include the short term financing of payrolls, of raw materials purchases and of inventory production--in effect the multitude of transactions which relate to the timing of receipts and expenditures and the firm's credit arrangements. Further, since short term

debt obligations give rise to the need for liquid assets, the money capital transformation function provides an integrating mechanism for many firm decisions which have, so far, been ignored in neoclassical theory. These suggestions go beyond the scope of this thesis, however.⁶

For purposes of the model presented the money capital constraint provides the necessary link between the sources and uses side of the balance sheet. As a result, marginal conditions on production and investment may be equated to the required rate of return which in turn reflects the judgments of investors.

Formally, the assumptions of the model are:

1. A world of uncertainty when the firm's net earning stream is expectational but constant over time for any level of production. The external risk associated with the stability of the firm's earnings is constant over all levels of output. Subjective risk-return estimations by potential investors are the same as those of the firm.

2. Investors' preferences with respect to financial risk and compensating return are reflected in "cost of capital curves" extending over a range of debt-equity ratios.

Thus if i is the required return on equity, E , and r is the required return on debt, D , then:

6. For more extensive discussions of the money capital constraint see Stephen J. Turnovsky, "Financial Structure and the Theory of Production," Journal of Finance (December, 1970), pp. 1074-1076 and Douglas Vickers, The Theory of the Firm:..., Chs. 7 and 11.

$$\frac{di}{dD/E} > 0 ; \quad \frac{d^2i}{dD/E^2} > 0$$

$$\frac{dr}{dD/E} > 0 ; \quad \frac{d^2r}{dD/E^2} > 0 ,$$

but r is everywhere less than i . It should be understood that both i and r , in addition, are defined with respect to the external risk of the firm's income stream. As external risk increases (from firm to firm) all levels of i and r over the range of debt-equity ratios are higher. The fact that r is less than i over the range of debt-equity values implies that varying combinations of debt and equity will lower (over some of the D/E range) the overall cost rate of the capital fund. This overall cost rate (denoted by k) will reach a minimum and then rise.

3. Financial risk is a function of the debt-equity ratio alone and is independent of the size of the money capital fund. This is not an unrealistic assumption given the model we are building. The firm's capital needs are small relative to the total size of the capital market.

4. There is full market comprehension of the investment opportunity afforded by the firm. In addition we assume no firm entry into the industry in which the firm in question operates.

5. All earnings are paid out as dividends. This is not an assumption of course, but an outcome of the model. In equilibrium shareholders will receive all the net earnings of the firm as dividends, since, in effect, the firm has exhausted all investments returning more than the market rate.

6. The real capital stock is considered indestructible so that problems of depreciation and capital stock renewal decisions are by-passed.

The firm wishes to maximize (II.1.2) subject to (II.1.4), expressed as an equality. In Lagrangian form the firm maximizes

$$H = pf(K, L) - wL - iE - rD + \lambda[D + E - p_K K] .^7 \quad (\text{II.1.5})$$

Assuming second order conditions for a maximum prevail, first order conditions with respect to K , L , D and E are:

$$\frac{\partial H}{\partial K} = pf_K - \lambda p_K = 0 \quad (\text{II.1.6})$$

$$\frac{\partial H}{\partial L} = pf_L - w = 0 \quad (\text{II.1.7})$$

$$\frac{\partial H}{\partial D} = -r' \frac{D}{E} - r - i' + \lambda = 0 \quad (\text{II.1.8})$$

$$\frac{\partial H}{\partial E} = i' \frac{D}{E} - i + r' \left(\frac{D}{E} \right)^2 + \lambda = 0 \quad (\text{II.1.9})$$

$$\frac{\partial H}{\partial \lambda} = D + E - p_K K = 0 . \quad (\text{II.1.10})$$

The first question which arises from the set of first order conditions is whether the firm has financed its real capital acquisitions such that its overall cost of capital is minimized. Equating (II.1.8) and (II.1.9), the following condition is obtained:

$$\frac{i-r}{1 + \frac{D}{E}} = i' + r' \frac{D}{E} . \quad (\text{II.1.11})$$

7. Note that $iE + rD$ has been substituted for $k(E+D)$.

From (II.1.3), taking the derivative of k with respect to (D/E) and setting it equal to zero:

$$\frac{\partial k}{\partial \left(\frac{D}{E}\right)} = 0 = i' \left(1 + \frac{D}{E}\right) + r' \frac{D}{E} \left(1 + \frac{D}{E}\right) + r - i ,$$

which implies,

$$\frac{i-r}{\left(1 + \frac{D}{E}\right)} = i' + r' \frac{D}{E} .$$

Overall capital costs have thus been minimized.

Using (II.1.11), λ is found to be equal to k . Substituting $(i-r)/\left(1 + \frac{D}{E}\right)$ for the expression $i' + r' \frac{D}{E}$ in (II.1.8),

$$\frac{i-r}{1 + \frac{D}{E}} + r = \lambda ,$$

and

$$\frac{iE + rD}{E+D} = k = \lambda . \quad (\text{II.1.12})$$

If this result is substituted into (II.1.6), then

$$\frac{pf_K}{p_K} = k \quad (\text{II.1.13})$$

and k becomes the cutoff point for investment. Further we note that at the margin the discounted value of capital, pf_K/k , is equal to the unit price. The relationships in (II.1.12) and (II.1.13) indicate

that λ is equal to the marginal rate of return on the money capital fund and in equilibrium this is equated to the cost of capital. On the other hand the results conform to neoclassical conditions on factor usage. The ratio of (II.1.6) to (II.1.7) is

$$\frac{f_K}{f_L} = \frac{kp_K}{w},$$

where, again, kp_K is the current account cost of capital goods' services.

One can view the firm's decision process as sequential, commencing with finding the optimal debt-equity ratio and solving for the total capital fund from (II.1.13). The proportion of equity in the capital fund is determined directly from the capital fund and the debt-equity ratio.

The interpretation of the objective function should leave little doubt that for a given distribution of shares maximization of the function will result in a maximum share price. Given the firm's external or business risk, an optimal financial mix decision will insure a maximum return per share. Finding the optimal debt-equity ratio implies finding a share price (and a corresponding debt value) which optimizes (II.1.5). It may not be clear that this price is a maximum, or to put it another way, the earnings price ratio, i , is a minimum.

From the first order condition (II.1.6) and the definition of k ,

$$i = \frac{pf_K}{p_K} + \left[\frac{pf_K}{p_K} - r \right] \frac{D}{E} . \quad (\text{II.1.14})$$

That is, the required rate of return on equity capital consists of a "pure" equity return and an additional return due to leverage. Of course, all values in the expression are known, having been solved through first order conditions. In fact, i implies E , given the firm's relative profitability. That is, for example, if the firm is more profitable than competitors of identical risk characteristics, then share price is correspondingly bid upward (and the capital budget expanded), reducing i to the value required by external and financial risk.

The importance of the financial mix decision may be seen by establishing E , from the maximization process above, as a "target" equity value and determining whether any financial mix other than the optimal one will achieve an equal or a greater share price. Holding E constant at the target level and deriving first order conditions from (II.1.6)⁸ with respect to K , L and D only,

$$pf_K - \lambda p_K = 0 \quad (\text{II.1.15})$$

$$pf_L - w = 0 \quad (\text{II.1.16})$$

$$-k'1/E(D+E) - k + \lambda = 0 . \quad (\text{II.1.17})$$

Substituting for λ from (II.1.17),

8. Having established overall capital cost minimization, $k(D+E)$ will be returned to the objective function as it appears in (II.1.2). This is done for purposes of expediting the exposition.

$$pf_K = [k'1/E(D+E) + k]p_K \quad (II.1.18)$$

and

$$i = \left[\frac{pf_K}{p_K} - \frac{k'}{E(D+E)} \right] + \left[\frac{pf_K}{p_K} - \frac{k'}{E(D+E)} - r \right] \frac{D}{E} . \quad (II.1.19)$$

Comparing any two points on the negatively-sloped portion of the cost of capital curve, it is clear that as the debt-equity ratio is increased k falls more rapidly than $k'1/E(D+E)$ rises. Therefore, the marginal conditions in (II.1.18) are at a lower level the greater the debt-equity ratio. Since, in addition, the capital fund has been increased (by moving to a higher debt-equity ratio) total returns to capital have been increased.

On the other hand if the debt-equity ratio is increased beyond the capital cost minimization point, the marginal conditions of (II.1.18) are higher and the total fund is greater. This in turn implies increasing returns to scale which precludes the optimal solution derived from equations (II.1.6)-(II.1.10).

Inspection of (II.1.19) will affirm the latter point. Compare the debt-equity value that minimizes capital costs to any other higher debt-equity ratio. At the cost minimization point, (II.1.19) is equivalent to (II.1.14), but as debt is increased (II.1.18) indicates marginal returns to capital must increase at a rate equal to the sum of the increases in k and $k'(D+E)/E$. Moving to a higher than optimal debt-equity ratio implies the first term on the right-hand side of (II.1.19) will increase. Further, since k must rise more quickly than r over higher debt-equity ratios, and it is hypothesized that

D/E increases, the second term on the right in (II.1.19) increases.

Thus i must rise, but this contradicts a target value of E .

This analysis emphasizes the importance of the financial mix decision: Share price is maximized only if capital costs are minimized.

An additional check of the correctness of the model and its decision rules is provided by comparing the marginal conditions expressed in (II.1.14) to the general relationship of the required rate of return to equity to the average cost of capital, the required rate on debt and the debt-equity ratio.⁹

Irrespective of the method of valuation of the firm,¹⁰ the relationship, $i = k + (k-r) \frac{D}{E}$, will hold. Note that for any set of values for K , L , D and E (and where V is the total market value of the firm)

$$i = \frac{pf(K, L) - wL - rD}{E}, \text{ and}$$

$$kV = pf(K, L) - wL = k(D+E).$$

Therefore by substitution,

$$i = \frac{k(D+E) - rD}{E} = k + (k-r) \frac{D}{E}.$$

9. This analysis follows that of Ezra Solomon, "Leverage and the Cost of Capital," The Journal of Finance, Vol. XVIII, No. 2 (May, 1963), p. 274.

10. See David Durand, "Cost of Debt and Equity Funds for Business..." for an extensive exposition of the net income and net operating income approaches to firm valuation.

It is clear that the decision rule in (II.1.14) conforms to the general proposition outlined above. Since the first order conditions of the model allow a simultaneous solution of optimal values for the variables involved, equity value in particular is maximized.

2. Book Values vs. Market Values

Two of the contributions toward integrating traditional neoclassical and corporate financial models have included book values of financial variables rather than market values.¹¹ The issue of the choice between book and market values is important from at least two viewpoints,¹² which will be developed sequentially.

First, in the context of the models developed how can we effectively differentiate between book values and market values? Second, supposing the differentiation can be made, what are the implications for the interaction between the firm's managers and financial investors in general?

A generally accepted definition of book value is the net worth of the corporation (assuming no preferred stock) plus the total of debt liabilities. Since we are concerned with a firm whose assets are

11. Yutaka Imai [15], Chapter III and Stephen J. Turnovsky [29].

12. Whether or not decision makers consider book value or market value is, of course, an empirical question, but the models ought to, through the derived decision rules, indicate the implications of decision-makers' actions.

productive rather than financial

$$D + S = p_K K ,$$

where D is debt (and it is assumed that book = market value), S is the firm's net worth which consists of common stock at par value, capital surplus and retained earnings. As of some point in time, the firm's book value is given, reflects past financial and productive decisions and is essentially not (in common practice) subject to change. This does not mean, of course, that the firm's managers and financial investors are not influenced by book value. Book value incorporates a history of the firm's activities and these past activities are indicators of the firm's future prospects. The important point about book values is that they are not variable, and as such present obvious difficulties for incorporation as variables in an integrative model. Differences between book values and market values, which are commonly observed, if not the rule in the real world, occur because the economic prospects of firms vary over time, where these changes in prospects were unforeseen by shareholders at the time of the original equity issue (or at the time of any other past issue).

Models using book values can (and indeed, must) be salvaged by either one of two arguments. One might propose that the decision rules involve the book values of incremental investments or that the decisions are the initial ones of a firm previously non-existent as an administrative entity. In this case, however, the difference between book values and market values completely disappears.

On the other hand it could be argued that the expectations of the firm and those its financial backers differ.¹³ In the two models which incorporate book values relationships between market values and book values are derived although what is actually meant by market value is the managers' view of the "justified" price of the stock and what is conceived of as book value is also market value in the generally accepted sense of being the value of the firm as reflected by the collective judgment of financial investors.

The assumption of preexisting equity ownership presents no particular problems for the model introduced in Section II.1.

If owners' equity is reflected in the book value of the firm, and a new issue is to be floated then current owners will exchange their stock for some amount of the shares to be issued. The market will then value these shares along with the rest of the issue.

Let βE be the value of new equity funds raised for investment. Total equity value (including shares distributed to owners) is E and $0 < \beta < 1$. The overall capital cost may be considered a function of the "total" debt-equity ratio, but

$$P_K K = D + \beta E$$

13. It is interesting to note that Imai [15] precludes this possibility by assumption. See Chapter III, p. 1. Turnovsky [29] only implicitly makes the differing expectations assumption by constructing his model such that the difference between "market value" and book value is maximized. See pp. 1067, 1068.

and current account capital costs are,

$$k(D + \beta E) .$$

These expressions may be substituted into (II.1.5) and first order conditions derived. Only (II.1.8), (II.1.9) and the closing condition are changed:

$$-k'1/E(D + \beta E) - k + \lambda = 0 \quad (\text{II.1.8})'$$

$$k'D/E^2(D + \beta E) - \beta k + \beta \lambda = 0 \quad (\text{II.1.9})'$$

$$D + \beta E - p_K K = 0 . \quad (\text{II.1.10})'$$

Equating (II.1.8)' and (II.1.9)', we again find $k' = 0$. Again $\lambda = k$. In neither case was book value a factor.

What are the implications of this analysis for management-shareholder interaction? If the integrative model is predicated on a divergence of stockholder and management expectations, the way has been opened toward a more formal theory of the separation of ownership and management. The model developed in this chapter, however, is based on the assumption of identical expectations between shareholders and management. Actually, the assumption need not be so restrictive. As long as management is more optimistic than shareholders, the money capital constraint will hold as an equality. Even though the managers may feel their prospects are superior to the evaluations of the capital market, they can do no worse, in an ex ante sense, than shareholders' expectations. The formulation of the integrated model, remains unchanged.

Since the integrative model presented in this chapter is based on an interaction of shareholders and management, it should be adaptable to situations where non-optimal (from the viewpoint of shareholders) or managerial objectives are hypothesized to exist. This topic becomes the subject of Chapter III.

Given the simultaneous interaction of expected profits from production and stock market evaluation which characterizes this model, it is useful to reinterpret the objective function in a manner which emphasizes this aspect. Thus the firm may be said to maximize above normal profits where normal profits are given by the opportunity costs imposed on the firm by the capital market; i.e., required returns, $k(D+E)$.

The reinterpretation allows us to draw a sharp distinction between financial market equilibrium and product market equilibrium. Clearly, the decision rules involve financial market equilibrium.¹⁴ The model is thus short-run from the standpoint of neoclassical theory, but it is the length of the short-run which has been the main concern of managerialists. Without the threat of significant market entry, the latter contend, non-profit maximizing objectives of managers become feasible. Little formal attention has been given, however, to the constraining affects of financial markets.

14. This may have been clear to the reader from the start since we have specifically assumed no firm entry; i.e., a given industry structure.

3. Other Variables Affecting Capital Costs

3.1. Size of the Capital Fund and Capital Costs

Although in terms of mathematics modification of the cost of capital function to include the size of the firm's capital budget is straight forward, the reasoning behind such a modification is more complex.

Seemingly, there are two possible sources of concern with respect to the capital budget size. First, as individual investors increase their holdings of the firm's stock in greater and greater proportions to their entire portfolio, diversification will be lost and returns must be proportionately larger. On the other hand since the firm's money capital needs are small in relation to total asset holdings of potential investors (we are still operating under the assumption of full market knowledge of the firm's investment opportunity), this sort of additional required return is avoided. In the case of restricted market knowledge, however, this source of additional capital costs may be of considerable importance.

A second source of concern lies in changes in risk associated with the firm's net income stream associated with various levels of asset accumulation. Thus, for example, an output of 20,000 units may well have different risk characteristics than an output of ten times that amount. Capital budgets may be considered too expensive on the one hand, or too modest to achieve market penetration, on the other.

Risk characteristics associated with capital budget size inherently involve complex considerations. Such aspects of firm enterprise as the allocation to the advertising budget, management quality

as well as quantity,¹⁵ and the relative efficiency of the firm in transforming money capital into productive assets, are just some of the obvious areas of concern to investors. Over a cross section of publicly-owned firms, it is in the above mentioned areas that the greatest differences in risk occur and render any concept of broad, encompassing risk classes suspect.

Perhaps the most satisfactory way of handling the cost of capital as a function of the capital fund size is in terms of operational leverage. As it is usually defined, operational leverage arises because of fixed operating costs present in the enterprise structure. Since these costs do not fluctuate with output, but must be met in order to sustain production, they tend to magnify the fluctuations in net operating income and induce higher levels of external risk.

As the firm reaches higher levels of real capital stock, these fixed costs will increase. Therefore, the firm cannot finance larger capital funds without incurring greater required capital costs.

To proceed formally in adapting the basic model to considerations of operational leverage we must first relax that part of the first assumption relating to the constancy of external risk over all levels of output. Clearly, this assumption is precluded by the above discussion.

15. See Edith Penrose, The Theory of the Growth of the Firm (New York: John Wiley and Sons, Inc., 1959), pp. 43-55, for a discussion of managerial capacity in terms of a receding managerial limit. Also see T.Y. Shen, "Economics of Scale, Penrose Effect, Growth of Plants and Their Size Distribution," Journal of Political Economy, Vol. 78, No. 4 (July/August 1970), pp. 702-716.

Further, since fixed costs are primarily associated with the depreciable assets of the firm and may be considered as those costs on current account necessary to maintain the capital stock, it is necessary to modify the sixth assumption. The capital stock may be considered indestructible in the sense that although it physically depreciates, it is maintainable at a constant, "like new" efficiency.

3.2. Capital Costs as a Function of Other Firm Decisions and Characteristics

Before explicitly adding the effects of capital fund size to our basic model, it is worthwhile to briefly explore other variables which may well affect capital costs.

As has been suggested, capital cost as a function of the size of the capital budget is related to many underlying considerations. The basic model is flexible enough to handle most of these aspects. For example, an advertising input may be added to the revenue function, its cost accounted for, and its influence on capital costs recorded by including it as an argument in the cost of capital function. Similarly, the effects of management and, in general, the administrative and organizational aspects of the firm may be demonstrated.

By way of emphasis, by incorporating advertising or management variables into the objective function we are able to gauge their contribution to a maximum share price. This is not unimportant from a conceptual standpoint since the model now ties in another loose strand of firm activity--which has its own voluminous body of literature--

that of administrative behavior. We need only investigate the model in terms of one of these variables--advertising, since the analysis would be quite similar for other variables.

In addition to real capital, advertising effort (in dollar terms) will be added as an argument in the money capital requirements function. Given the complex strategies associated with advertising campaigns and their obvious lagged effects over more than one operating period, the need for current and continued financing of such efforts is a realistic assumption.

The combined effects of capital budget size and advertising may now be added to the cost of capital function. Let

$$g = g(K, A) = D + E$$

be a more general form of the money capital transformation function which now includes A , an advertising variable.¹⁶ As before

$$\frac{\partial g}{\partial K} = p_K,$$

but let

$$\frac{\partial g}{\partial A} > 0; \quad \frac{\partial^2 g}{\partial A^2} < 0.$$

16. Advertising effort is extremely diverse in form. The variable A could be considered a vector $(A_1 \dots A_n)$ where A_1 was newspaper space per period, A_2 , television time per period, etc. (See Lester G. Telser, "Advertising and Competition," Journal of Political Economy (December, 1964), pp. 537-562, for a comprehensive analysis of advertising variables.) For our purposes it is sufficient to treat advertising efforts as homogeneous.

The marginal conditions on advertising indicate that the greater the advertising effort the greater are the financing needs, but the requirements increase at a decreasing rate. The second order conditions indicate economies of scale in advertising insofar as financing requirements are concerned. These conditions are exemplified by lower rates per minute or per square inch per period of time for large media advertisers.

Then

$$k = k(D/E, g(K, A)) ,$$

or,

$$\hat{k} = \hat{k}(D/E, K, A) ,$$

where $\hat{k}$ is a U-shaped function of leverage, and,

$$\frac{\partial \hat{k}}{\partial K} > 0 ; \quad \frac{\partial^2 \hat{k}}{\partial K^2} > 0$$

$$\frac{\partial \hat{k}}{\partial A} < 0 ; \quad \frac{\partial^2 \hat{k}}{\partial A^2} < 0 .$$

The effects of operational leverage are reflected on overall capital costs as a monotonically increasing function of the capital stock and the marginal increase is hypothesized to rise at an increasing rate.

The marginal conditions with respect to A simply describe the inverse effects of changes in advertising on the risk to investors. While the effects of advertising on expected return have been duly

noted in the literature, corresponding effects on net income stability and thus on the cost of capital have been given short shrift. Yet the effects of advertising in creating a sustained level of product demand over time should be--conceptually, at least--clear. Second order marginal conditions with respect to advertising seem reasonable although there may well be some range of advertising effort over which capital costs decline at an increasing rate. The objective function now becomes:

$$\text{Max } \hat{H} = p(Q, A)f(K, L) - \hat{k}(D, E, K, A)[D+E] - wL - A + \lambda[D + E - g(K, A)] . \quad (\text{II.3.1})$$

First order conditions with respect to L , D and E are basically unchanged from previously derived first order conditions--once again $k' = 0$ and $\lambda = \hat{k}$. Additional elements must be added to capital costs, however, as may be seen through the marginal effects of K and A on $\hat{k}$. First order conditions with respect to K and A are:

$$\frac{\partial \hat{H}}{\partial K} = \left[p + \frac{\partial p}{\partial Q} \right] f_K - \hat{k}_K[D+E] - \lambda p_K = 0 \quad (\text{II.3.2})$$

$$\frac{\partial \hat{H}}{\partial A} = \frac{\partial p}{\partial A} f(K, L) - \hat{k}_A[D+E] - 1 - \lambda g_A = 0 . \quad (\text{II.3.3})$$

Since $\hat{k}_A < 0$, it is obvious from (II.3.3) that the marginal benefits of advertising are not limited to those derived through its effects on net operating income. The marginal effects of advertising on price, $(\partial p / \partial A)$, will be less given the additional effects on capital costs than if these latter effects were not present. Since additional units of advertising expenditure will be applied beyond the

range of increasing returns, the salutary effects on risk imply larger advertising budgets than would otherwise be the case. Thus the usual marginal conditions carried out in terms of net operating income, do not apply. Eliminating the second term of (II.3.3)

$$\frac{\left[p + \frac{\partial p}{\partial Q} \right] f_K}{\frac{\partial p}{\partial A} F(K, L)} > \frac{\hat{k}_K [D+E] + \hat{k}_K p_K}{1 + \hat{k}_K g_A}, \quad (\text{II.3.4})$$

indicating that in terms of net operating income (marginal profits in the usual sense), the firm is over-advertising.

Stockholders of the firm benefit both from returns derived from real capital accumulation and the firm's advertising effort. Adding (II.3.2) and (II.3.3), these gains are indicated by the level of total marginal returns:

$$\left[p + \frac{\partial p}{\partial Q} \right] f_K + \frac{\partial p}{\partial A} F(K, L) - 1 = [D+E] (\hat{k}_K + \hat{k}_A) + k(p_K + g_A) \quad (\text{II.3.5})$$

Since $\hat{k}_A$ is negative, the level of marginal returns on the left-hand side is less than if capital costs were not affected by advertising. Thus the total return to shareholders will rise and current share price will be higher. On the other hand, the effect of operational leverage is to reduce total returns and decrease share price.

Summary

By including current account capital costs and the sources of real capital financing in the profit function of the firm it was

possible to derive decision rules for production, investment--in terms of the level of capital stock acquisition, and the proportion of debt and equity in the firm's financial structure.

It is possible for the firm to maximize share price only if overall financial costs are minimized. The decision rules resulting from the model are compatible with the general relationship between the required rate of return on equity, the overall cost of capital, required rate of return on debt and the debt-equity ratio, a relationship which must hold regardless of the method used for calculating the value of the firm.

Although the integrated model presented in this chapter was developed directly from the neoclassical theory of the profit maximizing firm, it is useful once financial variables have been incorporated to reinterpret the objective function as one in which the firm attempts to maximize above normal profits. This interpretation emphasizes the capital market as imposing an opportunity cost on the profit potential of the firm, and further, stresses the difference between financial market equilibrium, on the one hand, and product market equilibrium, on the other.

CHAPTER III

THE CAPITAL MARKET AND MANAGERIAL OBJECTIVES

Introduction

This chapter is concerned with the effect of the explicit use of capital costs in objective functions which are "managerial" by nature rather than stockholder oriented. At least three authors who have developed hypotheses relating to firm behavior in the context of managerial utility functions have been greatly concerned with constraints on these utility functions generated by a fully or at least reasonably competitive capital market. Each finds it necessary to show (in one way or another) that there is sufficient market imperfection in the form of barriers to money capital accumulation for purposes of take-over, affording leeway for managerial objectives to be realized on a consistent basis.

Oliver Williamson, for example, suggests that the questions to be considered are: "Can the capital market, operating through external controls, either constrain or otherwise induce the management to operate the firm in what is effectively a profit maximizing manner?"¹ Williamson

1. Oliver E. Williamson, Corporate Control and Business Behavior (Englewood Cliffs, N.J.: Prentice Hall, Inc., 1970), p. 90.

cites several categories of control over management including incentive constraints in the form of stock options and inside trading, and displacement constraints in the form of mergers, or proxy battles at the instigation of stockholders.² It is in relation to this latter category that Robin Marris constructs his "theory of take-over."³ Marris is careful to link the theory to the discrepancy between current market valuation of the firm and its expected potential valuation as seen by a prospective "raider"; i.e., the take-over raid or merger; or for that matter internal displacement is ultimately linked to a general market reevaluation under differing management policies.

Therefore, it would seem most important to consider the effects of what Williamson calls direct capital market control, but to which he apparently devotes little analysis. Williamson is content to cite Baumol in this regard, where the latter points out that since relatively little use is made of equity financing by the large majority of the nation's "leading" firms, the capital market has little opportunity to discipline corporate management directly.⁴

In another well-known work Baumol finds leeway for the existence of constrained sales maximization in the non-symmetric investment op-

2. Ibid., pp. 91-103.

3. Robin Marris, The Economic Theory of 'Managerial' Capitalism (New York: The Free Press of Glencoe, 1964), pp. 29-45.

4. William J. Baumol, The Stock Market and Economic Efficiency (New York: Fordham University Press, 1965), p. 76.

portunities of large and small firms.⁵ In addition "...the businessman's desire to increase his profits lends itself to translation into a desire to expand his firm....This, in turn, means that large size can increase the magnitude of the funds he can accumulate to finance further expansion. The businessman must have amassed wealth to be an efficient wealth amasser."⁶

It is not Baumol's point that large firms may have greater profit opportunities than smaller ones that is of direct interest to us. Rather, we are interested in what the large firm does with the opportunity --specifically, what is the firm's objective function and how is the opportunity correspondingly evaluated in the capital market? Baumol's reference to the efficient wealth amasser in this regard is confusing.

If the businessman has been successful in the past in accumulating wealth, then the capital market may extrapolate from the historical record an estimated stream of returns for the future. The mere level of wealth at a point in time, however, is relatively unimportant. Given some potential for greater than normal profits, it is the firm's or businessman's policies, as Marris emphasizes, with respect to what he intends to do with the raised money capital that determines the size and availability of financing. If money capital is raised on the basis of a set of expectations which is not fulfilled, then the value of that

5. William J. Baumol, Business Behavior, Value and Growth (New York: Harcourt, Brace and World, Inc., 1959), Ch. 5.

6. Ibid., pp. 43-44.

fund in the secondary equity market will decline.

The analysis of Chapter III looks, in particular, at the effect of constrained sales maximization on the firm's ability to raise money capital as compared with a profit maximizing objective in the same firm. In this manner we get an idea of the precise effect of the capital market constraint. Once the effect of this constraint is clear, then and only then can we draw inferences concerning the vulnerability of the "managerial" firm to take-over.

1. Stock Price and Constrained Sales Maximization

This section expands the models represented in equations (II.1.5) and (II.3.1), although the advertising variable will be dropped. Our purpose here is to analyze the feasibility of the constrained sales maximization hypothesis within the context of the capital fund market. We are particularly interested in specifying the effect of the profit constraint (capital costs treated explicitly) on the required return on equity capital and the level of the capital fund. The objective function now becomes:

$$H_s = p(Q)f(K, L) + \lambda_1 [p(Q)f(K, L) - k(D/E, K)[D+E] - wL - \bar{W}] + \lambda_2 [(D+E) - p_K K],$$

(III.1.1)

where the constraints in Lagrangian form are, respectively, those associated with a specified profit level, $\bar{W}$, and money capital availability.

First order conditions are:

$$\frac{\partial H_s}{\partial K} = \left[p + \frac{dp}{dq} f(K, L) \right] f_K (1 + \lambda_1) - \lambda_1 k_K (D+E) - \lambda_2 p_K = 0 \quad (\text{III.1.2})$$

$$\frac{\partial H_s}{\partial L} = \left[p + \frac{dp}{dL} f(K, L) \right] f_L (1 + \lambda_1) - \lambda_1 w = 0 \quad (\text{III.1.3})$$

$$\frac{\partial H_s}{\partial D} = -\lambda_1 [k' l / E (D+E) + k] + \lambda_2 = 0 \quad (\text{III.1.4})$$

$$\frac{\partial H_s}{\partial E} = -\lambda_1 [-k' l / E^2 (D+E) + k] + \lambda_2 = 0 \quad (\text{III.1.5})$$

$$p(Q)f(K, L) - k[D+E] - wL - \bar{W} = 0 \quad (\text{III.1.6})$$

$$D + E - p_K K = 0 . \quad (\text{III.1.7})$$

Note first that the money capital requirements constraint relates not to profit maximization (and thus share price maximization) as in (II.1.5), but to revenue maximization. Thus the raising of money capital is related entirely to a managerial objective against which stockholders have only a constraining voice. Secondly, we see from equating (III.1.4) and (III.1.5) that the financial mix decision is unaffected by the change of objective function. That is, the debt-equity ratio is still chosen at a point of minimum capital costs. There is the possibility, however, that the size of the total budget will affect the optimum debt-equity ratio through the risk associated with operational leverage.

Substituting $k' = 0$ into (III.1.4) or (III.1.5),

$$\lambda_2 = \lambda_1 k . \quad (\text{III.1.8})$$

Using (III.1.8) in (III.1.2),

$$\left[p + \frac{dp}{dQ} f(K, L) \right] f_K \left(\frac{1 + \lambda_1}{\lambda_1} \right) = k_K (D+E) + kp_K, \quad (\text{III.1.9})$$

and we note that investor reaction to constrained profits is a higher effective cost of capital. ($\lambda_1 > 0$ and implies $(1 + \lambda_1)/\lambda_1 > 1$.)

The key point in the marginal conditions on capital, expressed in (III.1.9) is that the actual capital fund becoming available to the firm is not independent of these conditions. The usual interpretation of the marginal conditions on factor inputs under constrained sales maximization is that the firm employs factors beyond the point where the marginal revenue product of the factor is equal to its price. In the above model where the cost of capital funds as well as the size of the fund is simultaneously determined with the production decision (the size of the capital fund then determining the amount of real capital accumulation by the firm), such an interpretation is no longer possible.

To simplify the analysis we will assume the overall cost of capital is a function of the debt-equity ratio alone. Thus the first term on the right-hand side of (III.1.9) is eliminated. Suppose in the following diagram we represent the locus of points of equilibrium conditions with respect to capital, given optimum labor usage, as a downward sloping schedule consistent with decreasing marginal revenues over output. This schedule is labeled PP. Required capital cost is given by kp_K .

FIGURE III.1

Let $kp_K \equiv MC'_K$ and $\left[p + \frac{\partial p}{\partial Q} f(K, L) \right] f_K \equiv MRP'_K$, where MC'_K is the marginal cost of capital and MRP'_K is the marginal revenue product of capital. Thus

$$MRP'_K \left(\frac{1 + \lambda_1}{\lambda_1} \right) = MRP'_K + Z = MC'_K .$$

The first and last terms are equal from first order conditions and Z is simply the difference between marginal return on capital and its marginal cost and is so noted in Figure 1. Using the first two terms,

$$Z = MRP'_K \cdot 1/\lambda_1 .$$

λ_1 may be interpreted as the marginal change in revenue given a small change in the profit constraint, or dR/dW . In terms of managerial advantage, the Lagrangian may be viewed as the marginal gain in revenue due to an easing of the profit constraint.⁷

To the investor, however, λ_1 must be interpreted differently, though symmetrically. The investor must view the marginal managerial gain as an additional cost, since profits, the variable upon which he has a claim, are being marginally reduced.

The reciprocal of λ_1 ($= dW/dR$) may be viewed in terms of investor advantage as the incremental gain in profits due to a marginal restriction of revenues. The terms, $MRP'_K \cdot (dW/dR)$, is the additional marginal return to real capital (under managerial control of the firm) required by investors to achieve equality between the marginal returns of their transformed money capital and the marginal return to the legal claims on the firm's income stream which they hold as stocks and bonds. It is precisely the dual role of the investor as the ultimate owner of real assets as well as the supplier of money capital which is reflected in our model. Since

$$MRP'_K + MRP'_K(dW/dR) = MRP^*_K,$$

where the term on the right-hand side is the marginal return on capital

7. The constraint on sales may be seen to be the dual of a constraint on profits where the latter constraint is in terms of a specified market share. See Appendix at the end of this chapter.

consistent with profit maximization (both terms are equal to the required cost of capital), investors will restrict money capital availability to that amount consistent with the profit maximizing level of real capital stock.

If the efficient combination of labor and capital is used at a level consistent with profit (and share price) maximization, then constrained sales maximization would simply cease to exist as a management objective. Thus in order to attain a greater than profit maximizing output management must employ variable factors inefficiently. As our model is in ex ante terms, however, inefficient usage of variable inputs will have simultaneous effects on the firm's ability to raise money capital. (See Section III.3 for a discussion of ex post sales maximization.)

With reference to Figure III.1, these simultaneous effects may be seen by a leftward shift of the PP schedule to TT. The full reduction of capital in the production function is $(\hat{K} - K')$. This reduction has two components, $(\hat{K} - K^*)$, the reduction caused by the extra cost to investors of the firm's profit constraint, and $(K^* - K')$, the additional profit loss due to inefficient labor usage.

The reduced real capital is directly attributable to a reduced profit stream net of debt costs discounted at the required rate of return. This implies both a lower share price, and as the optimal debt equity ratio is unchanged, a decreased amount of debt. Thus the total capital fund is reduced leading to the reduction of real capital acquisition. It is helpful to review these results by reflecting the

reduced real capital in terms of increased costs of money capital. This is equivalent to raising kp_K in Figure III.1 to a level where the intersection with PP is consistent with K' . (See dotted line.)

The change in relative prices plus the money capital requirements constraint results in an equilibrium shown in Figure III.2, the usual two-dimensional portrayal of the production function, where the isoquant Q^S is the optimal output for the sales maximizer.

FIGURE III.2

The money capital constraints, M^S and M^P , are indicated as cutoff levels of real capital usage. M^P indicates the higher level of money capital available to the profit maximizer whereas M^S is the indicated level of real capital available to the constrained sales maximizer. Both of these constraints intersect the tangency of the price line and optimal isoquant.

If the overall cost of capital is a function of the size of the capital fund as well as the debt-equity ratio, the analysis is changed only in degree. That is, investor reaction in restricting the level of capital stock is even more severe.

2. Capital-Labor Ratios and Output Levels Under Constrained Sales Maximization

The discussion in the previous section is now expanded upon with the intent of analyzing both the feasibility of sales maximization in the context of a perfect, though uncertain, capital market and the relative use of factors compared to the profit maximizer.

With respect to the latter point, it has already been noted that the sales maximizer will always have a smaller money capital fund to work with than the profit maximizer. As a result, only if the sales maximizer produces less than the profit maximizer is there any possibility that the former will use a higher capital-labor ratio than the latter. Naturally, such a level of output would be inconsistent with the sales maximization hypothesis where in its usual form profits are less than maximum. The only alternative then is that the sales maximizer will use proportionately greater amounts of labor than the profit maximizer. Graphically, this result may be seen in Figure III.2 simply by noting the effect of the money capital constraint on real capital acquisition. The money capital constraint on the sales maximizer is M^S as compared to M^P of the profit maximizer. Clearly, where M^S intersects the profit maximizing level of output, Q^P , the proportion

of labor in the production function is greater than that used by the profit maximizer. It follows that any output level beyond Q^P involves even greater relative labor usage.

We can formerly derive marginal conditions on factor usage for the sales and profit maximizer for purposes of comparison. We proceed by deriving the marginal conditions for minimized costs where production is constrained first by Q^P , the profit maximizing output and then by Q^S , any given output attained by the constrained sales maximizer. We denote terms relating to the profit maximizer and constrained sales maximizer by the superscripts P and S, respectively.

Thus for

$$\text{Min } \theta^P = k^P(D/E, K)[D+E] + wL - \lambda_1[f(K, L) - Q^P] + \lambda_2[D + E - p_K K],$$

$$\frac{f_K^P}{f_L^P} = \frac{k_K^P(D+E) + k^P p_K}{w}$$

are the marginal conditions for factor employment for the profit maximizer. Similarly,

$$\frac{f_K^S}{f_L^S} = \frac{k_K^S(D+E) + k^S p_K}{w}$$

are the marginal conditions for the sales maximizer.

From (III.1.9),

$$k_K^S(D+E) + k^S p_K > \text{MRP}_K^*,$$

the marginal revenue product of capital of the profit maximizer, and therefore

$$k_K^S(D+E) + k_{p_K}^S > k_K^P(D+E) + k_{p_K}^P,$$

which implies, since w is constant for both expressions,

$$\frac{f_K^S}{f_L^S} > \frac{f_K^P}{f_L^P}.$$

This further implies from the total derivative of the total cost function, that,

$$\frac{dL^S}{dK^S} < \frac{dL^P}{dK^P}.$$

We have noted previously that the sales maximizer's higher capital costs implied a lower total capital fund, thus fewer real capital inputs could be produced. The less-steep slope of the constant total cost function (dL^S/dK^S) must, therefore, intersect the production function at a lower capital-labor marginal product ratio. The constant-cost plane, isoquant tangency is shown in Figure III.2 at the point where $C'B'$ touches the isoquant Q^S . This tangency is consistent with the money capital constraint, M^S , which intersects the tangency point.

The result of this analysis is that under most circumstances constrained sales maximization is a feasible ex ante objective even

in the context of a perfect capital market. In one case, however, it is not. Suppose the production function is such that fixed factor proportions must be employed. Then only profit maximization is a feasible objective. The reduced money capital would simply force the sales maximizer to produce a smaller output than the profit maximizer.

In general, the degree to which ex ante constrained sales maximization is possible depends largely on the elasticity of substitution of labor for capital. Therefore, complementing the external constraints of the capital market, the effects of stock options as inducements to managements and the threat of management displacement, the additional aspect of capital-labor substitutability must be considered.

The analysis thus allows us to make some inferences with respect to firms and industries in which sales maximization is likely to be practical. These will be summarized in the concluding section.

3. Ex Post Sales Maximization

The previous analysis demonstrated the feasibility of ex ante constrained sales maximization. This section notes the fact that within the context of the manager-stockholder dichotomy it is entirely possible for the firm to raise money capital on a profit maximizing pretext and proceed then to apply labor in sufficient quantities to achieve a sales level greater than that consistent with profit maximization.

Given the factor of capital-labor substitution, this course of action might be attractive to management. The firm, in effect, may

take its capital stock as given and proceed along its money capital constraint (M^S in Figure III.2) to some desired output level.

Given the amount of real capital acquired, the target sales level and the product demand curve, the amount of labor required and the level of revenue are immediately forthcoming. The inefficient labor usage must subsequently reflect on share price, however. At this point it is useful and correct to distinguish between the book value and market value of the firm. It is clear that ex-post sales maximization will result in the book value of the firm, that is, those funds raised previously on a profit maximizing pretext, being greater than the market value. Further, the greater the disparity between output consistent with profit maximization and the target output (implied by the sales goal), the greater the disparity between book and market value. Given reasonably widespread knowledge and homogeneity of production methods, such a course of action will leave the firm open for takeover, the disparity in book and market value being a clear signal of "mismanagement."

It is unlikely, therefore, that ex-post sales maximization would be a widespread phenomenon except where barriers to industry entry were prevalent.⁸

8. It may be argued that the growth of the conglomerate firm has significantly reduced, or is reducing barriers to entry. If so, the above analysis may be viewed as an indicator of the proper anti-trust policy toward these firms.

Managerial Objectives and Capital Market Constraints: Conclusions

The managerial objective of constrained sales maximization results in higher effective capital costs. In the ex-ante situation this is reflected in a lower level of obtainable money capital. Thus to achieve outputs larger than those consistent with profit maximization labor must be sufficiently substitutable for capital in the production function.

To escape the problems of low factor substitutability in the production function, the sales maximizer may be tempted to apply labor inputs, ex-post, to the raising of money capital. This course of action will lead to a subsequent decline in the market price of equity so that it will be necessarily less than book value, thus leaving the firm vulnerable to takeover.

The ex-ante and ex-post models presented here indicate the sales maximizer's heavy reliance on variable factors to achieve sales goals. This implies that any management seeking to take over the sales maximizing firm need only reduce variable inputs to increase profits, reduce the cost of capital and pave the way to increased real capital accumulation. The ease with which the corporate raider may effect a gain in the firm's value makes the sales maximizing management particularly vulnerable to displacement.

With respect to the capital market alone both the ex-ante and in particular, the ex-post sales maximizer are vulnerable to takeover unless adequately protected. While the need for protection from entry or takeover is perhaps obvious, it is the fact that financial markets

constitute a signal of mismanagement that is important. If financial markets are relatively efficient and adjust rapidly, they provide an important link in assuring behavior which maximizes shareholders' welfare.

The results and conclusions of the sales maximization analysis can lead us to fruitful inferences regarding industries and firms in which this particular managerial objective may be most prevalent.

The results with respect to capital-labor ratios provide interesting inferences. Industries with greater substitutability of capital and labor would appear to be more susceptible to sales maximization.

While large firms in oligopolistic industries may well enjoy the greatest takeover protection, it is not at all clear that the protection is ironclad, particularly in view of the rise of conglomerate enterprise and what Oliver Williamson has called the multi-form organization.⁹ But further, the need for capital-labor substitutability would direct our attention away from industries whose manufacturing processes were highly automated, and in general we might suspect a prevalence of sales maximization in service rather than manufacturing industries.

The combined factors of the money capital constraint and the substitutability of capital and labor suggest a totally different milieu in which sales maximization--if not a conscious objective, at least is closely approximated--may be prevalent. This is the area of industries dominated by small firms.

9. Oliver Williamson, op.cit., Ch. 5, 6.

With respect to capital-labor substitutability the hypothesis is straightforward: small businesses use less sophisticated, less specialized capital equipment more freely substitutable for labor in terms of intensity of use or because of the existence of a large variety of capital goods within a general category of such goods which may be applied with varying amounts of labor to the firm's production function.

Insulation from capital market effects is accomplished simply because ownership equity in small firms is closely held and therefore not subject to widespread investor review. Since owner-entrepreneurship is the rule, management need not fear tender offers, although problems of this nature arise even in closely-held firms.

Market structures in industries dominated by small firms are sufficiently diverse to impose a noncompetitive solution. This is clearly brought out in W. Warren Haynes's study of small business pricing.¹⁰

Finally, strong ethical attitudes prevail in many small business industries regarding a "fair" level of profit,¹¹ which although possibly consistent with a long-run profit constraint, may just as well supply a motivation for constrained sales maximization among small owner-operated businesses.

10. W. Warren Haynes, Pricing Decisions in Small Business (Lexington, Ky.: University of Kentucky Press, 1962), Ch. 5.

11. Ibid., pp. 52, 80.

APPENDIX

CONSTRAINED PROFIT MAXIMIZATION

It has been suggested by at least two authors that firms are more likely to maximize profits subject to a sales constraint, this constraint being conveniently and realistically formulated in terms of a fixed market share.¹

Professor Osborne demonstrates the formal equivalence between the marginal conditions of the constrained sales maximizer and the constrained profit maximizer. Our purpose in reviewing this proof is simply to note the effects of these differing objective functions (though as Osborne points out the problems bear a dual relationship and we have already noted the duality from the standpoint of managerial and shareholder evaluations) on the size of the capital fund.

Let MR_L , MR_K , MP_L , and MP_K be the marginal revenue of labor and capital and the marginal profit of labor and capital, respec-

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1. See Franklin M. Fisher's review of Baumol, op.cit., Journal of Political Economy, Vol. LXVIII, (June, 1960), and D.K. Osborne, "On the Goals of the Firm," Quarterly Journal of Economics, Vol. LXXVIII (November 1964). Both of these authors suggest a constrained profit model, although their primary concern is the firm's attitude toward sacrificing profits in the short run to achieve greater sales. They are not concerned with the cost of capital and the money capital requirements constraint.

tively, such that

$$MR_L = \left[p + \frac{\partial p}{\partial Q} f(K, L) \right] f_L$$

$$MR_K = \left[p + \frac{\partial p}{\partial Q} f(K, L) \right] f_K$$

$$MP_L = MR_L - w$$

$$MP_K = MR_K - (k_K(D+E) + kp_K) .$$

The objective function in Lagrangian form for the constrained profit maximizer is

$$\text{Max } N = pf(K, L) - k(D/E, K)[D+E] - wL - \mu_1[pf(K, L) - \bar{R}] + \mu_2[D + E - p_K K] ,$$

(III.A.1)

where the first constraint is a level of sales consistent with a fixed market share; we assume the constraint to be binding.

For the sales maximizer in (III.1.1),

$$\lambda_1 = \frac{MR_K}{MP_K} = \frac{MR_L}{MP_L} ,$$

whereas in the above expression

$$\mu_1 = \frac{MP_K}{MR_K} = \frac{MP_L}{MR_L} .$$

Therefore, $\mu_1 = 1/\lambda_1$.

Taking the first order conditions with respect to capital in (III.A.1), having solved for financial mix,

$$\left[p + \frac{\partial p}{\partial Q} f(K, L) \right] f_K (1 + \mu_1) = k_K^N [D+E]^N + k_K^N p_K, \quad (\text{III.A.2})$$

where the superscript, N , refers to capital costs of the constrained profit maximizer.

Since $\mu_1 = 1/\lambda_1$, $1 + \mu_1 = (1 + \lambda_1)/\lambda_1$. If the market share constraint is such that the marginal revenue to capital is the same for both the constrained profit and sales maximizers (denoted by superscript S), then

$$k_K^N [D+E]^N + k_K^N p_K = k_K^S [D+E]^S + k_K^S p_K,$$

and the capital fund sizes would be the same.

In either model, the capital fund is reduced from that available to the unconstrained profit maximizer.

CHAPTER IV

A DYNAMIC MODEL OF THE DIVIDEND DECISION

Introduction

If dividend policy is to be relevant at all, it must be considered outside of the static framework. This chapter uses the tools of control theory to analyze the problem of dividends in the context of all the other decisions discussed heretofore and in an environment where the firm enjoys the prospect of profitable investment opportunities in the current and future periods.

In Section IV.1 the model is developed and decision rules derived. In the following section a brief comparison of these results is made with Hirshleifer's extension by the Fisherian investment model. In Sections IV.3 and IV.4 conclusions are drawn concerning the relevancy of dividend policy with respect to share price and the degree to which these conclusions depend on the effect of the financial mix decision on share price.

1. Incorporation of the Dividend Decision

Up to this point the conceptual framework of the analytical models has been a static one. The static framework was quite adequate to derive the decision rules we were seeking. Even, it may be recalled, the

dividend decision was obtained since given declining returns to scale and investment opportunities fixed, the marginal conditions on the size of the capital stock is

$$\frac{pf_K}{p_K} = k ;$$

that is, the marginal rate of return on capital is equal to the (minimized) overall cost of financial capital.

This result implies that any further increments to the capital stock in the form of investment will return less than the required return. Therefore, all net income is returned to shareholders. It may be concluded that a dividend policy, implying some portion of net return is reinvested, depends on the availability of new, current period investment opportunities.¹ Accounting for dividends in this manner, however, necessitates the use of a dynamic model.

Results for dividend distribution from the static model plus the necessary assumption concerning investment opportunities in a dynamic model of dividend policy suggest that the shareholder evaluates the firm's capital stock as of a point in time (the size of the capital stock being simultaneously determined by that evaluation) and attempts

1. Emphasis is placed on the distinction between dividend policy and the distribution of all net income as dividends which results from the decision rules of the static framework. Dividend policy must be analyzed in terms of shareholders' preference function for current consumption; the static model dividend distribution need not.

to forecast future investment opportunities.

In the static equilibrium case, dividends, c , are equal to required return on equity:

$$c = iE . \quad (IV.1)$$

If investment opportunities are present in the current period, then shareholders can invest in them by allowing the firm to reinvest some of its net income (i.e. some portion of $pf(K) - rD$). In this case $c(t) < iE(t)$ and in general,

$$c(t) \leq iE(t) .^2 \quad (IV.2)$$

The degree to which current shareholders increase the value of their wealth in light of the firm's current investment opportunities depends on how much they currently reinvest in the firm, and the extent to which current period external financing, borrowing and new equity issues, is available (and desirable).

Both $E(t)$ and $c(t)$ reflect investment opportunities but are affected in the opposite directions, equity value rising in proportion to the incremental additions to income over time provided by investment,

2. Equation (IV.2) may be rewritten as:

$$E(t) \geq c(t)/i , \quad (IV.2')$$

which means that current period equity valuation reflects some (unspecified) growth in dividends in succeeding periods. If the growth rate, g , is specified, then $E(t) = c(t)/(1-g)$. It is the set of decisions over time, reflected in g , which the model of this chapter portrays. (Of course if $c(t) = c(t+1) = \dots = c(t+n)$, and $n \rightarrow \infty$, then, again, the equality in (IV.2') must hold.)

and dividends falling as current shareholders reinvest.

Let w be the discrepancy between $c(t)$ and $iE(t)$:

$$w(t) = c(t) - iE(t) . \quad (\text{IV.3})$$

We assume shareholders wish, at each point in time, to maximize utility, U , where

$$U = U(w(t)) . \quad (\text{IV.4})$$

Stated in another fashion, for $E(t) \geq c(t)/i$ there exists an optimal time path of capital accumulation and an optimal time path of current period consumption--given expected succeeding period investment opportunities--such that $U(c(t) - iE(t))$ is maximized in each period.³

The arguments of the proposed utility functional and their relationship do not, in fact, differ greatly from other models in the literature which incorporate utility functions of consumption. There is here, however, an explicit beginning of the period comparison of future period dividends--summarized in E --with those forthcoming in the current period, c .

A further important aspect of the utility functional is the fact that it stresses the dual role of the shareholder as a wealth maximizer (he discounts the flow of income which he expects to come directly to him) and an owner of real capital (the discounted stream of dividends

3. The assumption is thus made that at those values of the variables where the marginal functions are zero we obtain a global maximum of the functional.

results in a capital fund which constitutes a claim against the benefits derived from acquired real capital).

As will be seen from the decision rules forthcoming, the values of i and E are determined without reference to the utility function so that the expression, iE , can be interpreted as entering the utility function as a constant. Informally, this result may be anticipated by recognizing that while E depends on succeeding period investment opportunities, it does not depend on the availability of retained earnings for future period financing. The degree to which shareholders withhold current net income in the form of dividends depends on their time preference for current consumption; but whatever further financing is required in the current period for investment purposes is supplied by the market in the form of borrowing and new equity issues.

The utility function in any given period, t , may be graphically portrayed in the following figure. (Note that $-iE \leq w \leq 0$ so that the graph is depicted in the negative quadrant.) The values $-iE'$ and $-iE''$ represent two possible equity valuations, the higher valuation iE'' rendering greater utility, U'' , than the lower valuation. If investors decide to reinvest all net income, U'' is the level of utility obtained. If some net income is paid out, say, $c^* = w^* + iE''$, then utility is increased to U^* . The amount of dividends paid out depends on shareholders' time preference for current consumption. It is assumed that $c(t) > 0$ for all t . Then

$$U'(w) > 0 ; \quad U''(w) < 0 .$$

In order to account for the firm's investment, production and financial mix decisions--production is a function of capital only--let,

$$c(t) = p(t)f(K(t)) - r(t)D(t) - Q(t) \quad (\text{IV.1.5})$$

where Q is retained earnings and the remaining symbols are the same as in previous chapters. Note that the price of output is now a function of time. The firm's investment policy may be viewed as a response to increasing demand for its product. The value of current period investment is

$$p_K(t)I(t) = B(t) + S(t) + Q(t) \quad (\text{VI.1.6})$$

where I is investment, p_K the price of capital goods and B and S are current period borrowing and new equity issues, respectively. Note that S cannot be interpreted as a financing of dividends.⁴

4. At least not entirely. See the discussion on dividend irrelevance in Section IV.3.

Substituting for Q in (IV.1.5) and dropping the time arguments

$$c = pf(K) - rD - p_K I + B + S ,$$

$$w = pf(K) - k(D+E) - p_K I + B + S . \quad (IV.1.7)$$

Investors wish to maximize the functional

$$\int_{t=0}^{\infty} e^{-\rho t} U(pf(K) - k(D+E) - p_K I + B + S) dt , \quad (IV.1.8)$$

where K , D , and E are the state variables and I , B , and S are the controls, subject to the transition constraints

$$\begin{aligned} \dot{K} &= I \\ \dot{D} &= B \\ \dot{E} &= S , \end{aligned} \quad (IV.1.9)$$

The Hamiltonian of this problem is:

$$H = e^{-\rho t} U(pf(K) - k(D+E) - p_K I + B + S) + \lambda_1 I + \lambda_2 B + \lambda_3 S , \quad (IV.1.10)$$

where λ_1 , λ_2 , λ_3 are the auxiliary variables associated with K , D , and E , respectively, and are all functions of time. Second order conditions are guaranteed by the strict concavity of $U(w)$. First order conditions are:

$$\frac{\partial H}{\partial K} = -\dot{\lambda}_1 = e^{-\rho t} U_w p_K \quad (IV.1.11)$$

$$\frac{\partial H}{\partial I} = 0 = -e^{-\rho t} U_w p_K + \lambda_1 \quad (IV.1.12)$$

$$\frac{\partial H}{\partial D} = -\dot{\lambda}_2 = -e^{-\rho t} U_w (k' 1/E(D+E) + k) \quad (\text{IV.1.13})$$

$$\frac{\partial H}{\partial B} = 0 = e^{-\rho t} U_w + \lambda_2 \quad (\text{IV.1.14})$$

$$\frac{\partial H}{\partial E} = -\dot{\lambda}_3 = e^{-\rho t} U_w (k' D/E^2 (D+E) - k) \quad (\text{IV.1.15})$$

$$\frac{\partial H}{\partial S} = 0 = e^{-\rho t} U_w + \lambda_3 . \quad (\text{IV.1.16})$$

It is clear from inspection of (IV.1.14) and (IV.1.16) that $\lambda_2 = \lambda_3$. Since this equation holds for all t , $\dot{\lambda}_2 = \dot{\lambda}_3$. It is thus possible to equate (IV.1.13) and (IV.1.15), eliminate $\dot{\lambda}_2$ and $\dot{\lambda}_3$ and determine whether or not the utility function enters the capital mix decision. The answer is no since the term $e^{-\rho t} U_w$ cancels out. Further, the firm continues to minimize capital costs, $k' = 0$. Substituting this result into (IV.1.13) and (IV.1.15),

$$\dot{\lambda}_2 = \dot{\lambda}_3 = e^{-\rho t} U_w k . \quad (\text{IV.1.17})$$

From (IV.1.14) and (IV.1.16)

$$\dot{\lambda}_2 = \dot{\lambda}_3 = \rho e^{-\rho t} U_w - e^{-\rho t} \dot{U}_w . \quad (\text{IV.1.18})$$

Equating the right-hand sides of (IV.1.18) and (IV.1.17), dividing through by $e^{-\rho t} U_w$ and transposing,

$$\rho = k + \frac{\dot{U}_w}{U_w} . \quad (\text{IV.1.19})$$

The market's time preference discount rate is thus equal to the overall cost of capital plus the percentage change in utility over time. Therefore, only if $\dot{U}_w = 0$ will the market's time preference discount rate be equal to the overall cost of capital.

The production decision and the level of capital stock can be determined from (IV.1.11) and (IV.1.12). Using the latter expression,

$$\dot{\lambda}_1 = -\rho e^{-\rho t} U_w p_K + e^{-\rho t} \dot{U}_w p_K + e^{-\rho t} U_w \dot{p}_K . \quad (\text{IV.1.20})$$

Equating (IV.1.20) to (IV.1.11) and cancelling,

$$\rho = \frac{p f_K + \dot{p}_K}{p_K} + \frac{\dot{U}_w}{U_w} . \quad (\text{IV.1.21})$$

In this case the market's time preference discount rate is equal to the marginal rate of return on capital, plus the percentage capital gain, plus the percentage change in utility. It may be seen that it is the market's time preference discount rate which links the production decision and the capital mix decision. Equating (IV.1.19) and (IV.1.21),

$$\frac{p f_K + \dot{p}_K}{p_K} = k . \quad (\text{IV.1.22})$$

The level of the capital stock is still determined by equating the marginal rate of return on capital (including capital gains) to the overall cost of capital. Having found the optimal level of the capital stock and the value of the debt-equity ratio which minimizes capital costs, the values of debt and equity may be determined.

Note that neither determining minimum k nor finding the optimal level of capital involved information from the utility function. Thus the determination of i and E may be viewed as the result of decisions by the firm, simultaneously evaluated by investors in the financial markets, which are separable from the dividend decision.

At this point an economic interpretation by the auxiliary variables will be helpful. Let

$$V(K, E, D, t_0) = \max \int_0^{\infty} e^{-\rho t} U(w) dt ; \quad (\text{IV.1.23})$$

that is, (IV.1.23) represents the maximization of the utility function from time t_0 to infinity. History affects the utility function before t_0 only through the state of K , E , and D at t_0 . For example, the auxiliary variable associated with E , λ_3 , is defined such that

$$\lambda_3 = \partial V / \partial E . \quad (\text{IV.1.24})$$

The auxiliary variable measures the contribution of the incremental value of equity to the utility functional at time t_0 .

The expression $\lambda_3 S$ may be interpreted as the rate of increase of utility due to the current rate of increase in E . Similar interpretations hold for the auxiliary variables associated with the other state variables.

The Hamiltonian, rewritten as

$$H = e^{-\rho t} U(w) - \lambda_1 I + \lambda_2 B + \lambda_3 S , \quad (\text{IV.1.10})'$$

is the current flow of utility immediately forthcoming through U and anticipated in future periods through investment and the offsetting financing of that investment.

Taking the derivatives of the Hamiltonian with respect to the state variables set up an equilibrium condition for these variables so that for a point in time they may be held constant. For example, since λ_1 is the addition to utility from the incremental unit of capital stock, $\dot{\lambda}_1$ is the time change of that addition and may be interpreted as speculative gains expected to result from current investment. Then $\partial H / \partial K$, which is the gain in utility due to a change in the capital stock, plus speculative gain must equal zero. If not, then the firm (and investors) would have wanted either more or less capital stock. Having secured this condition on all the state variables, the control variables, I , B , and S may then be chosen so that utility is maximized.

It was shown that the equilibrium conditions on the state variables were such that the decision rules for the capital stock, the financial mix (and thus the values of D and E) were independent of the utility function. Therefore, we can surmise that at any point in time the state variables will enter the utility function as constants--albeit their values will differ over time thus changing utility over time--leaving only the controls to be optimized. Optimizing $pf(\bar{K}) - \bar{r}\bar{D} - p_K I + B + S$, where the bars represent variables which have been solved from (IV.1.11), (IV.1.13), and (IV.1.15), will result in an optimal dividend in a given period.

Note that since $e^{-\rho t} U_w > 0$, equations (IV.1.14) and (IV.1.16)

imply that $\lambda_2, \lambda_3 < 0$. On the other hand, (IV.1.12) shows that

$$\lambda_1/p_K = -\lambda_2 = -\lambda_3 .$$

This implies that the incremental addition to total utility, achieved through a change in the capital stock (and deflated by price) must just offset the total incremental losses of utility due to a change in debt or equity. Put another way, equilibrium depends on an incremental use of real capital of sufficient value to just offset the incremental losses in utility incurred by shifting other assets in the financial market into claims against the firm.

From equations (IV.1.3) and (IV.1.4) it may be seen that the term $e^{-\rho t} U_W$ represents discounted marginal utility of dividends, as well as, from (IV.1.12), (IV.1.14), (IV.1.16) the discounted marginal utility of investment, borrowing, and new equity issues. Therefore, the discounted marginal utility of dividends must, in turn, be equal to the negative marginal losses of total utility due to changes in the stock of debt and equity and to the incremental gain in total utility due to changes in the capital stock, divided by the price of capital goods. These rules cover all the specified current period decisions.

Several aspects of the model presented here and the derived results bear comparison to previous analyses in the literature. Specifically, the effort of Jack Hirshleifer⁵ in extending the Fisherian investment model and Modigliani's and Miller's⁶ paper on the irrelevancy of

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5. J. Hirshleifer, "On the Theory of Optimal Investment Decision," The Journal of Political Economy, Vol. LXVI, No. 4 (August, 1958), pp. 329-52.
 6. Merton H. Miller and Franco Modigliani, "Dividend Policy Growth, and the Valuation of Shares," The Journal of Business of the University of Chicago, Vol. XXXIV, No. 4 (October, 1961), pp. 411-33.

dividends are of particular interest.

2. Comparison of the Dividend Model with Hirshleifer's Extension of the Fisherian Model

In his two period model of investment and consumption decisions in a world of certainty, Hirshleifer extends the Fisherian framework to include differing borrowing and lending rates. A further possibility is a productive opportunity locus which has: a) a steeper slope than the lending rate over some range of current period consumption commencing at the intercept of the locus with the axis of current income; and b) a flatter slope than the borrowing rate over a range commencing at the intercept of the locus with the axis of next period's income.

The diagram below indicates the nature of the problem. A'RA represents the productive opportunity locus, the dotted line the borrowing rate and the flatter, solid line the lending rate. U is an iso-utility curve indicating combinations of this and next period's consumption to which investors would be indifferent.⁷

7. No precise analogy between Hirshleifer's model and our model is attempted. Hirshleifer makes no attempt to include a specific variable which constitutes an evaluation of the productive opportunity locus, although such a variable is implied by the tangency of U with A'RA. The translation of Hirshleifer's utility function and ours would be cumbersome: his is a measure of the sum of the total incremental utility of two adjacent time periods in our model--providing it is proper to assume an implicit evaluative variable in his function.

The point of this case is that neither the borrowing nor the lending rate is the appropriate discount rate for the present value computation. The correct rate here is what Hirshleifer calls the "marginal productive opportunity rate" and in equilibrium it will be equal to investors' time-preference rate for current consumption.⁸ This latter rate was solved for in the previous section in terms of the percentage change in utility, overall capital costs (IV.1.19), and the marginal rate of return to physical capital (equivalent to Hirshleifer's marginal production opportunity rate) (IV.1.21), where the first term was common to both referenced equations.

8. Ibid., p. 336.

The dividend model developed in this chapter thus specifically incorporates the evaluation of the productive opportunity locus via the securities market. For a model of the firm this latter element is an explicit consideration in its planning process. Hirshleifer states that the proper discount rate used for present value computation in the case under discussion is undiscoverable until the solution is attained and thus is of no assistance in finding the solution. This contention appears to disregard the evaluative processes of the securities market. The stock market directly evaluates productive opportunity and the mere absence of uncertainty would not preclude the existence of the equivalence of an equity market.

3. Do Dividends Affect the Value of the Firm?

The model under discussion provides a useful tool to investigate this controversy which arose upon the publication of the Miller-Modigliani article in 1961.⁹ The circumstances of this model go beyond the MM model since in the latter case investment was held constant under differing dividend policies. Here differing dividend payouts over time are stimulated by investment opportunities in the current and future periods.

We have already noted that investors' productive evaluation of the firm as well as the firm's decision rule with respect to the capital stock--the decision and the evaluation occur simultaneously--is unaffected by the utility function. The question remains whether there are feed-

9. Ibid.

back effects from the control variables which might alter the initial productive evaluation.

The decision rules governing investment, dividends, and current period financing, summarized at the end of Section IV.1, provide the guide to determining the effect on share price. It was noted that dividends would be desired to the point where the marginal gain in utility of dividends in the current period, $e^{-\rho t} U_w$, was just equal to the incremental gain in total utility due to an increase in the capital stock, deflated by the price of capital goods, $\lambda_1/p_K = (\partial V/\partial K)/p_K$.

Suppose the actual dividend pay out is too large; i.e., $e^{-\rho t} U_w < \lambda_1/p_K$. Shareholders may then purchase sufficient amounts of the new equity issues (the equivalent of the firm issuing a stock dividend) to bring about an equality in the marginal tradeoffs. Note that the above inequality also implies inequalities in the decision rules for borrowing and new shares issues. The decision rule with respect to new shares is brought into equilibrium by the voluntary movement of paid out dividends into the market for these shares. Borrowing is adjusted appropriately to maintain a constant debt-equity mix.

If, on the other hand, the dividend payout is too little, $e^{-\rho t} U_w > \lambda_1/p_K$. In this case either the firm will be required to use part of the proceeds from the new equity issue to increase its payout to current shareholders or shareholders will cash in some of their holdings to augment their current income.

Therefore, even though the current dividend decision on the part

of the firm is non-optimal, there is sufficient compensating interaction on the part of shareholders to insure that optimality is achieved. Equity valuation at the beginning of the period is unaffected. This result extends the MM hypothesis on the irrelevancy of dividends to the case where investment policy varies.

4. Dividend Policy and Leverage

It is probably evident by now that dividend policy is in no way affected by the attitudes of shareholders regarding financial mix. Specifically, and contrary to the conclusions of Modigliani and Miller, the irrelevance of dividend policy does not depend on shareholder indifference to financial mix. The model presented in this chapter takes as a basic premise the U-shaped cost of capital function.

The reason for this conflict of conclusions is also apparent. Under the MM formulation it was simply impossible to determine that the cost of capital could be determined independently of the dividend decision (and for that matter, independently of the investment decision).

Summary and Conclusions

The results for the dividend model do not affect the previous analysis with respect to managerial objectives. A managerial objective imposed on the dividend model must, in effect, constrain shareholder utility.

This constraint on utility imposes a higher capital cost on the firm compared to existing firms and compared to the firm operating under

profit maximization the money capital funds available are smaller.

One further result of the dividend model emerges. The model accounts for a systematic time preference in the market. This time preference does not affect the results for irrelevancy. Again, this is contrary to the conclusions of Modigliani and Miller. Assuming the market's desire to take advantage of the firm's investment opportunities and investors' indifference to the form in which they obtain current period funds for consumption, the market's time preference may be satisfied.

CHAPTER V

CONCLUSIONS, SUGGESTIONS FOR RESEARCH

Managerial economists have rested comfortably under the protective umbrella of the Berle-Means hypothesis of separation of corporate ownership and control.¹ The analysis of this thesis has related firm performance to the desires of its owners via the imputed cost of capital as it is forthcoming from securities markets.

The discussion has shown, particularly that of Chapter III, that managerial behavior is highly constrained by a perfect capital market. It therefore behooves managerialists to show either

1. Money capital markets are inefficient allocators of funds; or
2. Investors do not, as a rule, wish to maximize wealth.

If one is to demonstrate the former proposition, a clear-cut formulation of an adjustment process over time is needed, for it is precisely the speed, or lack of it, with which the market evaluates the firm's operations which is important. The latter proposition is not likely to be questioned.

Theoretical extensions of the models presented would serve to integrate several other areas of firm decision-making which heretofore have been treated in isolation. For example it should be possible to

1. A. A. Berle and G.C. Means, The Modern Corporation and Private Property, New York: The Macmillan Co., 1944.

construct a theory of financial asset holding from the money capital constraint. If D and E represent funds directed to the purchase of productive capital and $g(K)$ is a general expression of the money capital function, then the characteristics,

$$g'(K) > 0 ; \quad g''(K) < 0 ,$$

imply decreasing efficiency in the transformation of capital funds into productive capital goods. Thus $g(K)$ can be considered a partial function, the more general form of which is, $g(K, X_1, X_2, \dots)$, where the X 's indicate other non-productive financing needs such as payroll, inventories, financial assets, etc.

A second theoretical extension leads to the consideration of the firm's expansion decision. Vernon Smith in references previously cited has examined this decision in the framework of the Kuhn-Tucker conditions. The extension would allow an analysis of conditions in the financial markets which would be necessary for the contemplated expansion. This case would present an interesting variation since the capital stock representing an in place facility is fixed. Therefore, the situation affords the possibility of differentiating between book value and market value in the context of an expansion decision and a comparison of decision rules involving each.

Yet another extension of the model, although closely related to the expansion decision, is the replacement decision. Perhaps the most promising avenue for research would be the development of models considering the effects of differing depreciation measures in accounting on investment and share price and the results for money capital allocation.

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