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OF OPTIMAL CAPITAL ACCUMULATION AND
CORPORATE FINANCIAL THEORIES.

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AN INTEGRATION OF NEOCLASSICAL THEORY
OF OPTIMAL CAPITAL ACCUMULATION AND
CORPORATE FINANCIAL THEORIES

by

Yutaka Imai

A THESIS SUBMITTED
IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY

Thesis Director's signature:

A handwritten signature in cursive script, reading "Irvin K. Levy". The signature is written in dark ink and is positioned below the text "Thesis Director's signature:". The signature is written in a fluid, cursive style with a large initial 'I' and a long, sweeping underline.

Houston, Texas
March 15, 1971

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CHAPTER I

INTRODUCTION

1. Toward a More Comprehensive Theory of the Firm

Post-war periods witnessed serious attempts to integrate the theory of capital or investment and the theory of production and cost which has traditionally been identified as the theory of the firm. This integration of two theories culminated in Vernon Smith's monograph, Investment and Production.¹ Smith's major contribution was the refinement of the relationship of short-run and long-run cost curves to replacement investment.

Peripheral to economics, there has developed a separate discipline, corporate finance, which has treated the productive decisions as given and has been concerned solely with the financial decisions.²

Recently, Douglas Vickers, relying upon Smith's results, attempted to establish a more comprehensive theory of the firm by incorporating financial decisions into the traditional theory.³ Put slightly differently, Vickers added the balance-sheet considerations to the theory of

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1. Vernon L. Smith, Investment and Production: A Study in the Theory of the Capital-Using Enterprise, Cambridge, Mass.: Harvard Press, 1961.
 2. For a summary of corporate financial theories, see J. Fred Weston and E. F. Brigham, Managerial Finance, New York: Holt, Rinehart and Winston, 1969.
 3. Douglas Vickers, The Theory of the Firm: Production, Capital, and Finance, New York: McGraw-Hill, Inc., 1968.

the firm, the only concern of which had been the profit-loss statement.¹ While Vickers made a significant contribution, his theory has the limitations inherent to a static analysis. In particular, investment for expansion and dividend policy cannot be explained by a static theory.²

In another strand of economic thought, Irving Fisher established the theoretical foundation for the theory of investment and finance, later extended by M. J. Bailey and Jack Hirshleifer.³ Fisher formulated a theory in which an owner-entrepreneur maximizes his (intertemporal) utility. Such a theory is often referred to as a neoclassical theory of optimal capital accumulation and is inherently dynamic. Recently, Dale Jorgenson rehabilitated a part of Fisher's theory into a theory of the firm's investment behavior.⁴ In his theory, however, the financial decisions exert no influence upon the investment decision.

To my knowledge, no attempts have been made to establish a dynamic theory of the firm which systematically encompasses both productive and financial decisions. It is the goal of this thesis to establish such a theory by integrating the neoclassical theory of optimal capital accumulation and the theories of corporate finance. The resulting theory should be able to take fuller account of various aspects of the firm's

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1. The importance of the balance sheet considerations has been emphasized by W. W. Cooper, "Theory of the Firm: Some Suggestions for Revision," American Economic Review, Vol. 39, December, 1949, pp. 1205-12.
 2. Vickers' model implies 100% dividend payout since all the money-capital requirements are satisfied by initial equity and borrowing.
 3. Irving Fisher, The Theory of Interest, New York: Macmillan Co., 1930. M. J. Bailey, "Formal Criteria for Investment Decisions," Journal of Political Economy, Vol. 67, October, 1959, pp. 476-488. Jack Hirshleifer, "On the Theory of Optimal Investment Decision," Journal of Political Economy, Vol. 66, August, 1958, pp. 329-352.

decisions which are left unexplained or inadequately explained by static theories.

2. Plan of the Thesis

In Chapter II we shall present some conceptual preliminaries for our theory construction. The first two sections of Chapter II point out the limitations of the original Fisherian analysis and suggest the ways to overcome these limitations. First, the original version of Fisher's theory is briefly presented and its relationship to Jorgenson's investment theory is discussed. Then, a Fisherian owner-entrepreneur firm is interpreted as a publicly owned enterprise, and the objective of maximizing the wealth of the owner-entrepreneur is replaced by the maximization of the wealth of a unit of ownership, i.e., the price of the common stock per share. Next, uncertainty is introduced and the relationships between various risks and the returns to capital are discussed. The last section of Chapter II briefly reviews some important issues in the corporate finance literature.

In Chapter III various models of the firm are constructed for different specifications of the cost of capital function. The relationships among the costs of capital, productive decisions, financial decisions, dividend policy, and the valuation of equity are examined. It will be shown that our theory not only integrates the neoclassical theory and corporate financial theories, but also connects the two most important and controversial issues in corporate finance. The thesis concludes with a summary of the salient results and with suggestions for further research including the implications of our theory for investment theories.

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4. Dale W. Jorgenson, "The Theory of Investment Behavior," in Macroeconomic Theory: Selected Readings, H. R. Williams and J. D. Hufnagle, eds., New York: Appleton-Century-Crofts, 1969.

CHAPTER II

CONCEPTUAL PRELIMINARIES

1. A Neoclassical Theory of Optimal Capital Accumulation

In this section we present Irving Fisher's theory of optimal capital accumulation in its simplest form.¹ Our presentation will be restricted to the two period analysis so that a graphical exposition can be employed. In Fig. 2-1-1 the horizontal and the vertical axes measure the incomes in period 0 (today) and in period 1 (tomorrow), respectively. The straight line A'A represents the financial opportunity and has the slope $-(1+r)$, where r is the given market rate of interest for both borrowing and lending. The concave curve B'B represents the investment opportunity, its concavity reflecting diminishing returns to investment. Therefore, if one invests OB dollars today, he receives OB' dollars tomorrow. Finally, the convex curve C'C is an indifference curve of a risk-averse, utility-maximizing owner-entrepreneur. He maximizes his utility in two steps: first, the optimal size of the capital budget is determined, and second, the optimal income stream is attained by either borrowing or lending. More specifically, the entrepreneur, with initial income OB, first maximizes his wealth (the present value of the income stream) obtainable from his investment opportunity by investing DB dollars (which is the optimal size of the capital budget) and attains the point S. He, then, borrows DE today and promises to

1. Irving Fisher, The Theory of Interest, New York: Macmillan Co., 1930.

Fig. 2-1-1

pay back D'E' tomorrow, and thus attains his optimum optimorum position S*, i.e., the optimal income stream is OE today and OE' tomorrow.¹

It should be noted that in the above analysis the first step of maximizing wealth is independent of the subsequent utility maximization through the financial decision. Jorgenson, taking advantage of this fact, separated the first step of optimization and derived his theory of investment.² In his theory, therefore, the financial decisions are not considered. We should remember that the only financial decisions in Fisherian analysis are borrowing and lending. But, from the viewpoint of corporate finance one important financial decision is implicit in the above discussion, namely, financing by equity capital. The entrepreneur essentially financed the investment by his own money. Thus, Jorgenson's firm seems to be purely equity-financed. But, doesn't the Fisherian firm ever borrow?

To answer the question posed above, we must return to Fisher's original definition of investment opportunity line:

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1. The financial opportunity line is also an iso-present-value line of the income stream with the discount rate r . To see this point more clearly, we first write down the expression for the present value (P.V.): $P.V. = \$ (0) + \$ (1)/(1+r)$. Solving this for $\$ (1)$, we obtain $\$ (1) = P.V.(1+r) - (1+r)\$ (0)$ with the slope $-(1+r)$. Thus, it is the financial opportunity line. The intercept of this line with the axis measuring $\$ (1)$ becomes higher as P.V. increases. Therefore, the point on the investment opportunity line which gives the highest line with the slope $-(1+r)$ maximizes the present value of the income stream accruing from the investment. For more details, see Jack Hirshleifer, "On the Theory of Optimal Investment Decision," Journal of Political Economy, Vol. 66, August, 1958, pp. 329-352.
 2. Dale W. Jorgenson, "The Theory of Investment Behavior," in Macroeconomic Theory: Selected Readings, H. R. Williams and J. D. Hufnagle, eds., New York: Appleton-Century-Crofts, 1969.

"It may be defined as the limiting line of a group of points which represent all the optional income situations available to an individual who neither borrows nor lends."¹

Thus, for example in Fig. 2-1-1, if the entrepreneur's initial income is DB, the investment opportunity line becomes SB according to the above definition. However, Fisher also writes: "In fact," the best point on the investment opportunity line "may be pushed even to the left of the vertical axis,....."² But, since the definition does not allow borrowing or lending for productive purposes, how can the best point be pushed to the left of the vertical axis where the investment opportunity line is not defined? Clearly, there is a contradiction. The correct definition, in my opinion, should delete the last part, "who neither borrows nor lends."³

We can now construct Fig. 2-1-2, where the investment opportunity line (OB) exists regardless of the initial income. Fig. 2-1-2 is almost identical to Fig. 2-1-1 except that investment is measured to the left and the investment opportunity line is extended. Let us consider an entrepreneur whose initial income is OH which is equal to OD in Fig. 2-1-2. First, he maximizes the present value of the income stream obtainable

1. Irving Fisher, op. cit., pp. 277-8.

2. Ibid., p. 274.

3. Jack Hirshleifer, "Investment Decision Under Uncertainty: Choice-Theoretic Approaches," Quarterly Journal of Economics, Vol. 79, November, 1965, pp. 509-536, took the same view as mine without pointing out this contradiction.

from his investment opportunity by investing OD' which is the optimal size of his capital budget. Of OD' , OD is equity-financed and DD' is debt-financed. His income stream so far is zero today and OA' tomorrow, since he must repay $SF = (1+r)DD'$ dollars out of SD' dollars for his productive borrowing. Next, he maximizes his utility by borrowing OE today and promising to pay back $A'G$ tomorrow. It should be noted that while the loan OE is not dependent upon his investment decision, the loan DD' is. Thus, in this case the firm does borrow.

Should Jorgenson's theory of investment include financial variables to allow for corporate borrowing? The answer is no, since in Fisherian analysis under certainty the optimal size of the capital budget is uniquely determined and independent of how it is financed, and the initial income level simply alters the magnitude of the income stream accruing from the investment. For example, in Fig. 2-1-2, had the initial income been OD' , the entrepreneur would have obtained SD' dollars in period 1. Thus, for those only interested in the time rate of change in capital under certainty Jorgenson's theory suffices.¹

2. Choosing the Firm's Objective Function

There are two distinctive characteristics in the neoclassical theory of optimal capital accumulation. One is that the firm is controlled by a single owner-entrepreneur, and the other is the assumption

1. For the case of uncertainty, Jorgenson relies on the cost of capital theory advanced by F. Modigliani and M. H. Miller, "The Cost of Capital, Corporation Finance, and the Theory of Investment," American Economic Review, Vol. 48, June, 1958, pp. 261-297. This issue is discussed in detail later in this thesis.

Fig. 2-1-2

of perfect certainty. On the other hand, the firm portrayed in corporate finance theories is a large publicly owned corporation which operates under uncertainty. Any attempt to synthesize these two separate bodies of literature must somehow relax these restrictive assumptions without impairing the theory.

Since the work of Berle and Means, the separation of ownership and control has become a widely recognized fact.¹ The modern corporation is not owned by a single entrepreneur, but by many stockholders.² Some theorists assume that the firm's goal is to maximize the stockholders' aggregate utility function.³ It is possible to extend the neoclassical theory with this approach. However, this involves the same conceptual difficulties associated with a social welfare function. For example, how can the management of the firm evaluate and compare the different tastes of its stockholders? Moreover, both the number and composition of the shareholders change over time. The alternative approach, which avoids the use of the aggregate utility function by separating the decisions of the firm from those of the stockholders, is to assume that the firm's goal is to maximize the market value per share of the firm's

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1. A. A. Berle and G. C. Means, The Modern Corporation and Private Property, New York: The Macmillan Company, 1944.
 2. Robin Marris, The Economic Theory of "Managerial" Capitalism, New York: Free Press of Glencoe, Inc., 1964, pp. 11-20, succinctly discusses this issue and its implications to economic theories.
 3. For example, John Lintner, "The Cost of Capital and Optimal Financing of Corporate Growth," Journal of Finance, Vol. 18, May, 1963, pp. 292-310, and R. K. Sau, "The Optimal Rate of Investment in a Firm," Journal of Finance, Vol. 24, March, 1969, pp. 1-12.

common stock.¹ This goal is consistent with maximizing the owner's wealth. The firm is not concerned with the subsequent decisions of the owners who will maximize their utilities in any case. Thus, the first step of maximization (the firm's decision) is effectively separated from the second step (the owners' decision).

Among the theories of share price determination, the discounted dividend theory is not only the most widely accepted but also the most appropriate for our purposes because it is consistent with the first step in the Fisherian analysis. It postulates that the market value of a share is equal to the present value of its dividend stream, or

$$e(0) = \sum_{t=0}^{\infty} d(t)/(1+q)^t$$

where

$e(0)$ = stock price per share today,

$d(t)$ = dividend per share at time t ,

and q = market rate of discount.

In this formulation, the share price is the wealth from owning a share since the dividend stream is the income stream accruing to a share.²

The market rate of discount is also called the required rate of return on, or the cost of, equity capital. Under the conditions of

1. For example, Myron J. Gordon, "Optimal Investment and Financing Policy," Journal of Finance, Vol. 18, May, 1963, pp. 264-272, and F. Modigliani and M. H. Miller, op. cit.

2. More precisely, $e(0)$ is the pre-dividend price per share. Usually in the discounted dividend theory, the ex-dividend price is employed. The use of the pre-dividend price is simply for convenience in our discussion. In the continuous time formulation the difference disappears.

perfect certainty and a perfect capital market, the cost of equity capital, q , is equal to the single market rate of interest, r , for every firm. Thus, the share price maximization based on the discounted dividend theory is identical to the first step of Fisherian solution, i.e., the maximization of the present value of the income stream, or wealth. This places the Fisherian analysis in a new light. Share price maximization based upon the discounted dividend theory enables us to remove the restrictive assumption of a single owner-entrepreneur in the neoclassical theory.

In Fig. 2-2-1 the horizontal axis measures dividend today, $D(0)$, to the right and investment today, $I(0)$, to the left, and the vertical axis dividend tomorrow, $D(1)$. The line $A'A$ now is not only a financial opportunity line but also an iso-share-price line which is the ultimate maximand in the present situation. Therefore, the optimal size of the capital budget is OC because at the point S , the highest iso-share-price line is attained. The final solution, however, must be determined in the first quadrant because the initial level of equity funds alters the magnitude of the dividend stream and thus the share price. For example, if the initial level is zero, the dividend stream is zero today and OF tomorrow because the firm has to pay back EF for borrowing OC today; and if OH is the initial level, the dividend stream is OK today and OE tomorrow because $CH = OK$ by assumption. The firm can, of course, change the shape of the dividend stream by borrowing and lending. For example, any point along the line FA may be obtained by borrowing. But, this does not change the share price because the financial opportunity line is identical to the iso-share-price line. The economic reason for this is

that the stockholders can always borrow or lend at the same single interest rate to obtain whatever shape of the dividend stream they desire.

We next introduce uncertainty into the modified version of the neoclassical theory shown above.¹ When uncertainty is introduced, the first thing to notice is that the market rate of discount will be higher, reflecting the premium for risk-bearing.² How much higher depends upon the investors' attitudes toward risk. Moreover, the market rate of discount for debt and equity will diverge since each is associated with a different degree of risk. This is because debt has a senior claim on the firm's income. Suppose that the income (before debt obligations) is conceived by investors as having a (subjective) probability distribution with a mean and a variance. The coefficient of variation (ratio of the standard deviation to the mean) of this income is often used as a measure of business risk which is defined as the risk inherent to the income of the firm. If the firm has no debt, the returns to the stockholders have the same probability distribution as the firm's income. Therefore, the risk associated with the stockholders' returns, which is measured by the coefficient of variation, equals the business risk. If the firm

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1. The original Fisherian theory has been extended to the domain of uncertainty by J. Hirshleifer, "Investment Decision Under Uncertainty: Choice-Theoretic Approaches," op. cit., and its sequel, "Investment Decision Under Uncertainty: Applications of the State-Preference Approach," Quarterly Journal of Economics, Vol. 80, May, 1966.
 2. An excellent discussion of risk and the discount rate can be found in J. Hirshleifer, "Risk, the Discount Rate, and Investment Decisions," American Economic Review, Vol. 51, May 1961, pp. 112-120; and V. L. Smith, "Risk, the Discount Rate, and Investment Decisions: Discussion," ibid., pp. 124-127.

has some debt, the stockholders' risk increases since the mean of the returns decreases by the payment to the bondholders while the standard deviation remains constant. This additional risk caused by an increase in debt is called financial risk.¹ Thus, as the proportion of debt in the firm's financial structure (leverage) is increased, the market discount rate for equity (the cost of equity) will increase to compensate for the financial risk.

Theoretically, as the leverage increases, the market rate of discount for debt (the interest rate) should also increase to compensate for the increasing risk of bankruptcy. Given a probability distribution of the firm's income, the probability of failing to meet its debt obligations increases with leverage. Often, in spite of the presence of uncertainty, a constant interest rate is assumed. This is a justifiable approximation if the bond market is in equilibrium and the amount of debt raised by the firm is too small to affect the interest rate significantly. The interest rate gives the lower bound for the cost of equity since the stockholders can always purchase a bond and realize the rate of return equal to the interest rate.

Introducing uncertainty into the modified version of the neoclassical theory causes a divergence between the interest rate (the cost of debt) and the cost of equity and makes these costs increasing functions of leverage. The firm's objective is still the maximization of the stock price except that the discount rate depends upon the firm's financial decisions. Exactly how the firm achieves this goal is the main theme of this thesis and the topic of Chapter III.

1. For more rigorous discussion of these risk concepts, see my paper "Financial Structure and the Theory of Production: Comment," forthcoming in Journal of Finance.

3. Issues in Corporate Finance

In corporate finance the two most important theoretical controversies have been: (1) the cost of capital and the financial structure of the firm, and (2) dividend policy and share price valuation. The first controversy is whether leverage influences the weighted average cost of capital. One school argues that in the absence of corporate income tax, the cost of capital for a class of firms with the same business risk is identical and independent of leverage, while the other maintains that the cost-of-capital function is U-shaped with respect to leverage. Modigliani and Miller (hereafter M-M), the proponents of the former position, argue that the low cost of debt-financing will be exactly offset by the increasing cost of equity capital through an arbitrage process in the capital market.¹ Their opponents, while agreeing that M-M's propositions are valid given their assumptions, have attacked the realism of their assumptions by pointing out the institutional factors and other imperfections in the capital market that would hinder the arbitrage process.² Empirical studies tend to favor the U-shaped cost-of-capital function although they are by no means conclusive.³

1. F. Modigliani and M. H. Miller, op. cit.

2. Some of the major proponents of this position include David Durand, "The Cost of Capital, Corporation Finance, and the Theory of Investment: Comment", American Economic Review, Vol. 49, September, 1959, pp. 639-655, and Ezra Solomon, "Leverage and the Cost of Capital", Journal of Finance, Vol. 18, May, 1963.

3. These studies include A. Barges, The Effect of Capital Structure on the Cost of Capital, Englewood Cliffs, N. J.: Prentice-Hall, 1963; and E. Brigham and M. J. Gordon, "Leverage, Dividend Policy, and the Cost of Capital", Journal of Finance, Vol. 23, March, 1968, pp. 85-104.

The second controversy is whether the firm's dividend policy affects its share price. In the absence of taxes, M-M argue that, given the investment policy of the firm, if investors are rational and expect that all other investors are rational, and if they have no non-pecuniary reasons to prefer either dividends or capital gains, then dividend policy does not matter.¹ Gordon, on the other hand, argues that since dividends are more certain than capital gains, the investors' rate of discount increases for future time periods; thus dividend policy changes the share price.² The empirical studies favoring either side have been sparse.³

These controversies have been debated in isolation of each other. In other words, in the cost of capital controversy dividend policy is assumed to be fixed, and in the discussion of dividend policy the firm is assumed to be purely equity financed. But, the issue of dividend policy is closely related to the cost of capital function. For example, in the previous section we showed that dividend policy does not matter under certainty since every investor's rate of time preference for dividends was the cost of capital. Once uncertainty is introduced, the relationship between the cost of capital and dividend policy is no longer obvious. To analyze this relationship is another important topic of this thesis.

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1. M. H. Miller and F. Modigliani, "Dividend Policy, Growth, and the Valuation of Shares", Journal of Business, Vol. 34, October, 1961, pp. 411-433.
 2. Myron Gordon, op. cit.
 3. E. Brigham and M. J. Gordon, op. cit. and M. H. Miller and F. Modigliani, "Some Estimates of the Cost of Capital to the Electric Utility Industry, 1954-1957", American Economic Review, Vol. 56, June, 1966, pp. 333-391.

CHAPTER III

A THEORY OF THE FIRM

1. Introduction

In this chapter, using the concepts in Chapter II, we shall construct the models of the firm based on various specifications of the equity cost and debt cost functions. These models assume the following:

- (a) The firm's objective is to maximize its share price at the beginning of the planning period.¹
- (b) The share price is assumed to be the discounted sum of the future dividend stream, and the discount rate is the cost of equity capital.
- (c) The shareholders' dividend expectation is identical to the firm's planned dividend stream.
- (d) The means of financing are retained earnings and permanent debt.
- (e) The investment undertaken by the firm does not change its business risk.²

The objective of this chapter is to examine the relationships among productive decisions, financial decisions, the costs of capital, dividend policy, and the valuation of the firm's shares, under different specifications of the equity cost and debt cost functions. In particular, we shall prove the following propositions: (1) Regardless of the

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- 1. This objective is consistent with Fisherian wealth maximization.
 - 2. This assumption restricts our analyses to the case of widening in risk.

specifications of the equity cost and debt cost functions, the book value weighted average cost of capital (denoted by ρ) is the appropriate cutoff rate of investment; (2) Productive decisions are independent of financial decisions only if ρ is constant; (3) Productive decisions cannot, in general, be independent of financial decisions if both the cost of equity and cost of debt are increasing functions of the book value debt-equity ratio (denoted by B/E); (4) If the equity cost function is linear in B/E of a specific form, then the book value of equity is equal to its market value; (5) If ρ has a minimum with respect to B/E , then the book value of equity is equal to its market value only if ρ is minimized; (6) If ρ is (is not) constant, then dividend policy does not (does) affect the share price.

For our purposes the following simple model will suffice:

Model G

$$\max. \int_0^{\infty} e^{-u} [pf(k) - qI + Z - rB] dt$$

subject to

$$\dot{k} = I$$

$$\dot{B} = Z$$

$$\dot{u} = i$$

where

$$u = \text{discount factor} = \int_0^t i(x) dx$$

i = cost of equity capital

p = price of output

k = capital stock

I = purchase of capital goods

q = price of capital goods

B = debt outstanding

Z = borrowing

r = cost of debt capital

f = production function, $f_k > 0$ and $f_{kk} < 0$.

The objective functional is the market value of equity. The maximization of this value is equivalent to that of the share price because the number of the shares does not change in our model. The time derivatives are indicated by the dots on the state variables. For example, $\dot{k}$ is a rate of change in capital stock and is equal to the purchase of capital goods, I . Derivatives are denoted by the primes on the functions. In some cases the partial derivatives are denoted by the subscripts corresponding to the appropriate arguments of the functions. Furthermore, to simplify our presentation we shall assume that capital goods are infinitely durable and are the sole inputs to the production process. Dropping these assumptions does not alter the qualitative conclusions of this chapter.¹

1. If we assume that the firm produces a variety of goods using various inputs, our model may be regarded as a mathematical programming model for the allocation of funds among various capital projects, which has been developed by H. M. Weingartner, Mathematical Programming and the Analysis of Capital Budgeting Problems, Englewood Cliffs, N. J.: Prentice-Hall, Inc., 1963.

2. The Costs of Capital, Productive Decisions, and Financial Structure

The first model assumes that the cost of equity capital is an increasing function of the market value debt-equity ratio and that the cost of debt is constant.

Model 1

$$\max. \int_0^{\infty} e^{-u} [pf(k) - qI + Z - rB] dt$$

subject to

$$\dot{k} = I$$

$$\dot{B} = Z$$

$$\dot{u} = i(B/S)$$

$$\dot{S} = -[pf(k) - qI + Z - rB] + i(B/S)S$$

where

S = market value of equity.

This is a problem in optimal control theory with the controls I and Z and the state variables k , B , u and S . The derivation of the fourth constraint requires some explanation. S at time τ has the form,

$$S(\tau) = \int_{\tau}^{\infty} e^{-\int_{\tau}^t i(x) dx} [pf(k) - qI + Z - rB] dt.$$

Therefore, differentiating $S(\tau)$ with respect to τ yields the fourth¹

1. We have made use of the formula,

$$\frac{\partial}{\partial a} \int_{v(a)}^{w(a)} g(x, a) dx = \int_{v(a)}^{w(a)} \frac{\partial}{\partial a} g(x, a) dx + g(w, a) \frac{\partial}{\partial a} w - g(v, a) \frac{\partial}{\partial a} v.$$

constraint: $\dot{S} = -D + iS$, where D is the dividend which equals the terms in the brackets. This may also be written as $\dot{S} + D = iS$, which has the following economic interpretation: capital gains $\dot{S}$ plus dividend (D) must be equal to the returns to equity capital (iS).

The Hamiltonian of this problem is expressed as

$$H = (e^{-u} - \lambda_4)[pf(k) - qI + Z - rB] + \lambda_1 I + \lambda_2 Z + \lambda_3 i + \lambda_4 iS$$

where λ_i , $i = 1$ to 4 , are the Lagrangian multipliers associated with the differential constraints. The "maximum principle" thus yields the following first order conditions:¹

$$\frac{\partial H}{\partial I} = 0 \rightarrow \lambda_1 = q(e^{-u} - \lambda_4) \quad (1)$$

$$\frac{\partial H}{\partial Z} = 0 \rightarrow \lambda_2 = -(e^{-u} - \lambda_4) \quad (2)$$

$$\frac{\partial H}{\partial k} = -\dot{\lambda}_1 \rightarrow -\dot{\lambda}_1 = (e^{-u} - \lambda_4)pf_k \quad (3)$$

$$\frac{\partial H}{\partial B} = -\dot{\lambda}_2 \rightarrow -\dot{\lambda}_2 = -r(e^{-u} - \lambda_4) + \lambda_3 i' / S + \lambda_4 i' \quad (4)$$

$$\frac{\partial H}{\partial u} = -\dot{\lambda}_3 \rightarrow -\dot{\lambda}_3 = -e^{-u}[pf(k) - qI + Z - rB] \quad (5)$$

$$\frac{\partial H}{\partial S} = -\dot{\lambda}_4 \rightarrow -\dot{\lambda}_4 = -\lambda_3 i' B / S^2 - \lambda_4 (i' B / S - i) \quad (6).$$

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1. For the "maximum principle", see K. J. Arrow, "Applications of Control Theory to Economic Growth", in American Mathematical Society, Mathematics of the Decision Sciences: Part 2, Providence, 1968, pp. 85-119; R. Dorfman, "An Economic Interpretation of Optimal Control Theory", American Economic Review, Vol. 59, December, 1969, pp. 817-831; L. S. Pontryagin, et al., The Mathematical Theory of Optimal Processes, (tr. by K. N. Trirogoff), New York: Interscience, 1962.

Unfortunately, the problem as formulated above does not have a meaningful solution. To see this, note first that integrating (5) over time gives¹

$$\lambda_3 = -e^{-u}S. \quad (7)$$

Next, from (2) we obtain

$$\dot{\lambda}_2 = ie^{-u} + \dot{\lambda}_4. \quad (8)$$

Substituting (8) into (4) yields

$$-ie^{-u} - \dot{\lambda}_4 = -r(e^{-u} - \lambda_4) + \lambda_3 i' / S + \lambda_4 i'. \quad (9)$$

Subtracting (6) from (9) and substituting (7) into the resulting difference, we obtain

$$\lambda_4(i - i' - i'B/S - r) = e^{-u}(i - i' - i'B/S - r).$$

Therefore,

$$\lambda_4 = e^{-u}. \quad (10)$$

Because of equation (10), it is clear that equations (1), (2), and (3) yield $\lambda_1 = \lambda_2 = \dot{\lambda}_1 = 0$. Furthermore, the relations in (7) and (10) force equation (4) to give $\dot{\lambda}_2 = 0$ and (6) to give an identity, $\dot{\lambda}_4 = -ie^{-u}$. Equation (5) by itself yields no information about the solution to our problem.

1. We have assumed the transversality condition that λ_3 at infinity is equal to zero.

The economic interpretation of these results suggests trivial decision rules. Since Lagrangian multipliers are interpreted as the marginal increment in the maximand due to the marginal increment in the respective state variables, $\lambda_1 = 0$ would imply that the capital stock, k , must be increased until its marginal contribution to the objective functional becomes zero.¹ Similarly, $\lambda_2 = 0$ implies that the total debt outstanding, B , must be increased to the point where its marginal contribution to the market value of equity today is zero. These results are essentially telling us to do the best we can without telling us how.

These trivial results are caused by our use of the market value of equity in the argument of the equity cost function. Because of this specification of the equity cost function, we have defined a new state variable S and specified its time path. However, it must be recalled that to obtain the time path of S we have taken a part of the objective functional and differentiated it with respect to time. Therefore, no new information was contained in the resulting differential equation, which was nevertheless required to complete the model. If we do not specify the time path of S , the problem is not defined. If we do, triviality results. Thus, we have to abandon Model 1.

The next model employs the book value debt-equity ratio as the argument in the equity cost function, and assumes constant cost of debt. From the accounting identity, the book value of equity capital can be expressed as the difference between the book value of total assets and

1. See Dorfman, op. cit. for the economic interpretation of λ 's.

the book value of total debt outstanding. Therefore, the book value of equity (denoted by E) is

$$E = qk - B ,$$

since the book value of total assets in our present formulation is equal to qk (the resale value of capital stock).

Model 2

$$\max. \int_0^{\infty} e^{-u} [pf(k) - qI + Z - rB] dt$$

subject to

$$\dot{k} = I$$

$$\dot{B} = Z$$

$$\dot{u} = i(B/(qk-B))$$

The Hamiltonian for this problem is

$$H = e^{-u} [pf(k) - qI + Z - rB] + \lambda_1 I + \lambda_2 Z + \lambda_3 i(B/(qk-B)).$$

The first order conditions are the following:

$$\frac{\partial H}{\partial I} = 0 \rightarrow \lambda_1 = e^{-u} q \quad (2.1)$$

$$\frac{\partial H}{\partial Z} = 0 \rightarrow \lambda_2 = -e^{-u} \quad (2.2)$$

$$\frac{\partial H}{\partial k} = -\dot{\lambda}_1 \rightarrow -\dot{\lambda}_1 = e^{-u} pf_k - \lambda_3 i' qB/(qk-B)^2 \quad (2.3)$$

$$\frac{\partial H}{\partial B} = -\dot{\lambda}_2 \rightarrow -\dot{\lambda}_2 = -e^{-u} r + \lambda_3 i' qk/(qk-B)^2 \quad (2.4)$$

$$\frac{\partial H}{\partial u} = -\dot{\lambda}_3 \rightarrow -\dot{\lambda}_3 = -e^{-u} [pf(k) - qI + Z - rB] \quad (2.5).$$

The time derivative of equation (2.1) gives

$$\dot{\lambda}_1 = -ie^{-u}q + e^{-u}\dot{q} . \quad (2.6)$$

Similarly, equation (2.2) yields

$$\dot{\lambda}_2 = ie^{-u} . \quad (2.7)$$

Next, substituting (2.6) into (2.3), we obtain

$$e^{-u}(iq - \dot{q}) = e^{-u}pf_k - \lambda_3 i'qB/(qk-B)^2 . \quad (2.8)$$

Also, from (2.7) and (2.4),

$$-ie^{-u} = -e^{-u}r + \lambda_3 i'qB/(qk-B)^2 . \quad (2.9)$$

Solving (2.9) for λ_3 and substituting the resulting expression for λ_3 into (2.8), we obtain

$$pf_k = iq - \dot{q} - (i - r)B/k .$$

Therefore,

$$\begin{aligned} pf_k/q &= i - \dot{q}/q + (r - i)B/qk \\ &= (i(qk-B) + rB)/qk - \dot{q}/q . \end{aligned} \quad (2.10)$$

We should note that the first term on the left side of equation (2.10) is the weighted average cost of capital in terms of book values, since qk is the book value of total assets and $qk-B$ is the book value of equity, E . Therefore, equation (2.10) may be rewritten as

$$pf_k/q = \rho - \dot{q}/q \quad (2.11)$$

where $\rho \equiv (iE + rB)/(E + B)$.

Since $(pf_k + \dot{q})/q$ is the rate of return on investment, equation (2.11) indicates that the weighted average cost of capital in book values should be employed as the appropriate cutoff rate of (or, equivalently, the required rate of return on) the firm's investment.¹ It follows, therefore, that if ρ is constant, productive decisions are independent of financial decisions, and vice versa.² Since these results are important, we shall state them in the form of propositions.

Proposition 1

If the cost of equity is an increasing function of the book value debt-equity ratio and the cost of debt is constant, then the appropriate cutoff rate of investment is the book value weighted average cost of capital.

Proposition 2

If the book value weighted average cost of capital is constant, then productive decisions are independent of financial decisions.

-
1. The investment decision rule given by (2.11) may seem myopic since it seems that the firm takes account only of the immediate gains, pf_k and $\dot{q}$. But, it should be recalled that because of the recursive optimization, all the future decisions have already been made when the firm makes investment decisions today. Therefore, the decision rule is not as myopic as it appears at first glance.
 2. If the cost of capital is constant, it is possible for a large firm to decentralize its investment decisions. A division manager may make his own investment decisions as long as each project's rate of return is at least equal to the company's cost of capital.

To explore some further implications of this model, integrating equation (2.5) over time, assuming $\lambda_3(\infty) = 0$, gives

$$\lambda_3 = -e^{-u} S \quad (2.12)$$

where S is the market value of equity. Substituting (2.12) into (2.9) yields

$$i - r = Si'(E + B)/E^2,$$

which, using the definition of ρ , may be written as

$$E(\rho - r) = Si'. \quad (2.13)$$

Equation (2.13) reveals the relationship between the form of the capital cost function and the book value and market value of equity.

Let us first analyze the case in which the book value weighted average cost of capital is constant so that the cost of equity capital is a linear function of the book value debt-equity ratio, $i = \rho + (\rho - r)B/E$. The term $(\rho - r)B/E$ is the premium for financial risk and increases with leverage.¹ These capital cost functions are depicted in Fig. 3-2-1.

If the book values are replaced by the market values, this linear equity cost function is identical to that derived by Modigliani and Miller (hereafter M-M).² It is important to note, however, that M-M's specification of equity cost function holds only in equilibrium as they clearly state.³

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1. See Section 2 in Chapter II of this thesis for more detailed discussion of the concepts of risks and its relation to the costs of capital.
 2. F. Modigliani and M. H. Miller, "The Cost of Capital, Corporation Finance, and the Theory of Investment," American Economic Review, Vol. 48, June, 1958, pp. 261-297.
 3. F. Modigliani and M. H. Miller, ibid., p. 268.

The derivative of this linear equity cost function with respect to B/E is

$$i' = \rho - r .$$

Therefore, equation (2.13) implies $E = S$, i.e., the book value of equity is equal to its market value in equilibrium. Thus, we establish

Proposition 3

If the equity cost function is $i = \rho + (\rho - r)B/E$, then, as a result of the firm's maximizing its market value of equity, the book value of equity will equal its market value. Thus, the market value weighted average cost of capital becomes congruent to the book value weighted average cost of capital and is constant.

We have thus shown that Model 2 with the linear equity cost function $i = \rho + (\rho - r)B/E$ is consistent with the Modigliani-Miller cost of capital theorems. Moreover, since our model of the firm is based on the neoclassical theory of optimal capital accumulation, Proposition 3 builds a bridge between the cost of capital theorems and the neoclassical theory. In so doing, Proposition 3 offers a theoretical justification for Jorgenson's use of the market value weighted average cost of capital in his neoclassical theory of investment. Further discussion of this point and of the implications of the other propositions for investment theories is deferred until Chapter IV.

Let us now assume that ρ has a minimum with respect to B/E . This case corresponds to a so-called traditional position in the cost of

capital controversy and is shown in Fig. 3-2-2.¹ We shall show below that in this case the book value of equity is equal to its market value only if ρ is minimized.

Let us first express ρ in terms only of B/E . Thus, we get

$$\rho = (i + rB/E)/(1 + B/E) .$$

Next, this expression is differentiated with respect to B/E and the derivative is set equal to zero:

$$\frac{\partial \rho}{\partial (B/E)} = \frac{(i' + r)(1 + B/E) - (i + rB/E)}{(1 + B/E)^2} = 0 .$$

Therefore,

$$\begin{aligned} i' &= (i - r)/(1 + B/E) \\ &= (iE + rB - rB - rE)/(E + B) \\ &= \rho - r . \end{aligned}$$

Thus, from equation (2.13) we establish

Proposition 4

If the book value weighted average cost of capital has a minimum, then the book value of equity is equal to its market value only if ρ is minimized.

1. For a brief review of this controversy, see Section 3 in Chapter III of this thesis. This case is not exactly a traditional position since the traditional position postulates that the cost of debt eventually goes up; whereas, in our case, the cost of debt is assumed to be constant. However, it seems permissible to classify the cases in which ρ has a minimum as a traditional position as opposed to the M-M position in which ρ is constant.

Fig. 3-2-1

Fig. 3-2-2

Fig. 3-2-3

On the other hand, $\rho' \geq 0$ implies $i' \geq \rho - r$. Therefore, again from equation (2.13) we can derive the following.

Proposition 5

If the book value weighted average cost of capital is increasing (decreasing), the book value of equity is greater (smaller) than its market value.

Thus far, the cost of debt capital has been assumed to be constant. In reality, the cost of debt may be an increasing function of leverage. Sometimes, firms may find it impossible to borrow. In this case, the cost of debt may be regarded as infinite. The cost of debt may increase as the firm is more levered since the risk of bankruptcy increases. Given the (subjective) probability distribution of the firm's profit, the probability of the firm's becoming unable to repay its debt increases as the firm is more heavily levered. Therefore, the lenders will require higher returns for bearing additional risk. Let us now formulate a model in which both the equity cost and the debt cost are the increasing functions of the book value debt-equity ratio.

Model 3

$$\max. \int_0^{\infty} e^{-u} [pf(k) - qI + Z - r(\frac{B}{qk-B})B] dt$$

subject to

$$\dot{k} = I$$

$$\dot{B} = Z$$

$$\dot{u} = i(\frac{B}{qk-B})$$

The Hamiltonian is

$$H = e^{-u}[pf(k) - qI + Z - rB] + \lambda_1 I + \lambda_2 Z + \lambda_3 i .$$

The first order conditions are identical to those in Model 2 except for those involving k and B since Model 3 differs from Model 2 only in the specification of the debt cost function. Thus, in this problem equations (2.3) and (2.4) are replaced by the following:

$$-\dot{\lambda}_1 = e^{-u}(pf_k + r'qB^2/(qk-B)^2) - \lambda_3 i'qB/(qk-B)^2 \quad (2.14)$$

$$-\dot{\lambda}_2 = -e^{-u}(r + r'qkB/(qk-B)^2) + \lambda_3 i'qk/(qk-B)^2 . \quad (2.15)$$

Following the procedures similar to those employed in Model 2, we obtain the following expression:

$$pf_k/q = \rho - \dot{q}/q$$

which is identical to equation (2.11). It is clear that the book value weighted average cost of capital must be the appropriate cutoff rate of investment even when the cost of debt is an increasing function of leverage. Furthermore, since Model 3 is the most general formulation of the firm's decision process of our concern, we may generalize Proposition 1.

Proposition 6

Regardless of the specifications of the equity cost and debt cost functions, the book value weighted average cost of capital is the appropriate cutoff rate of the firm's investment.

This proposition seems to imply that even though the cost of debt is an increasing function of leverage, productive decisions can be independent of financial decisions when ρ is constant. However, for ρ to be constant while the cost of debt is an increasing function of leverage, it is absolutely necessary that the cost of equity must, at some point, decline enough to offset the increasing cost of debt, contrary to a generally accepted behavior of risk-averse investors. Thus, the book value weighted average cost of capital, when both the cost of equity and the cost of debt are increasing functions of leverage, will not generally be constant.¹ Therefore, we obtain

Proposition 7

When the cost of debt as well as the cost of equity is an increasing function of the book value debt equity ratio, productive decisions cannot, in general, be independent of financial decisions.

Next, the relation between the book value of equity and its market value, derived by operations similar to Model 2, is

$$\rho - r = i'S/E + r'B/E .$$

-
1. It is generally agreed in the literature on the cost of capital that the absence of the risk of bankruptcy, i.e., a constant cost of debt, is by far the most crucial assumption in the Modigliani-Miller cost of capital theorems. The concept of equivalent return class is not a necessary assumption since the M-M theorems have been proved in a general equilibrium framework by J. E. Stiglitz, "A Re-examination of the Modigliani-Miller Theorem," American Economic Review, Vol. 59, December, 1969, pp. 784-793; J. Hirshleifer, "Investment Decision Under Uncertainty: Applications of the State-Preference Approach," Quarterly Journal of Economics, Vol. 80, May, 1966; and R. S. Hamada, "Portfolio Analysis, Market Equilibrium and

In this case, since the first order condition for the minimum ρ is

$$\rho - r = i' + r'B/E,$$

the book value of equity is equal to its market value only if ρ is minimized, which is Proposition 4. Proposition 5 follows similarly.¹

Finally, it should be pointed out that the analyses in this section were "short-run" in that we did not consider what might happen after the valuation process is completed. What would happen if $S \neq E$? If all the markets are competitive, in long-run equilibrium we expect S will equal E through various adjustment mechanisms. First, let us consider the case where $S < E$. The stockholders will liquidate the firm because their wealth will increase by $E-S$. As the number of liquidations increases, i.e., more firms leave the industry, various forces work to equate S and E : E will be lowered because the increased supply of the capital goods will depress their prices; S will increase because of (a) the decreased supply of shares and (b) the increased future profitability stemming from a higher output price and lower prices of capital goods. On the other hand, if $S > E$, the stockholders will

Corporation Finance," *Journal of Finance*, Vol. 24, March, 1969, p. 13. The assumption of no risk of bankruptcy was nevertheless retained in their proofs. J. E. Stiglitz, *ibid.*, has shown that only under very restrictive conditions the M-M theorems hold even though risk of bankruptcy is present.

1. The capital cost curves corresponding to this case are shown in Fig. 3-2-3.

sell their shares and acquire the capital goods. If, for example, S is twice as high as E , the stockholders can double their wealth through this venture, since they can double their capital stock, while the income stream accruing to a unit of capital stock remains constant. As a result, the increased demand for the capital goods will raise E , and the increased supply of shares and the decreased profitability in the future will lower S . Thus, in long-run equilibrium, $E = S$, and, therefore, ρ will be minimized.¹

3. Graphical Exposition and Some Further Implications to Dividend Policy

In this section we shall elucidate our mathematical analyses with Fisherian diagrams. Also, these diagrams are used to analyze whether the firm's dividend policy affects the share price.

We shall first assume that the equity cost function is $i = \rho + (\rho - r)B/E$ and the cost of debt, r , is constant. This case corresponds to M-M's view and is a special case of Model 2. It has already been shown that productive decisions are independent of financial decisions because ρ is constant. Since equation (2.11) involves only one unknown, k , the optimum level of capital stock can be determined. With this value of k , equation (2.13) can be solved for B since (2.13) reduces to $E = S$ where $E = qk - B$ and S is already (and theoretically) obtained from the recursive relationships.

1. This conclusion is analogous to the Price Theory result that under perfect competition the average cost is minimized in long-run equilibrium.

Fig. 3-3-1 illustrates this case. Let us assume that the initial amount of internal funds is OA. The lines indicated by ρ , i , and r correspond respectively to the iso-total-value line, iso-market-value-of-equity line, and financial opportunity line. In the present case, since ρ is constant and is also the cutoff rate of investment, the optimal capital budget is determined where the iso-total-value line is tangent to the productive opportunity line (point R in Fig. 3-3-1). Thus, the optimal capital budget is OB of which AB is debt-financed. The total market value of the firm today must be OC. Now, since the total amount of debt is AB, subtracting the same amount (DC) from the point C, we can find the market value of equity, OD. On the other hand, since OH is the income accruing to equity capital, the slope of the line HD must be $-(1+i)$, where i is the equity cost.

Next, we illustrate the case where $i = i(B/E)$, $r = \text{constant}$, and ρ has a minimum. As was shown in Proposition 4, the market value of equity is equal to its book value only when ρ is at a minimum. Therefore, let us suppose, in Figs. 3-3-2 and 3-3-3, that the point R corresponds to the minimum ρ . No points beyond R will be chosen, because by definition the slope of the opportunity line beyond R is smaller than the minimum ρ which is the cutoff rate of investment. Up to R, however, there are two possibilities depending on whether ρ is decreasing or increasing. Fig. 3-3-2 illustrates the case where $\rho' > 0$. The arrow indicates the direction to which the equilibrium point, R^* , would be shifted if B/E were higher. Proposition 5 suggests that, in this case, the book value of equity is greater than its market value. Therefore, the total book value must also be greater than the market value. It is

Fig. 3-3-2

difficult to explain how all the decisions are made because, unlike the M-M case, they are made simultaneously. Therefore, Figs. 3-3-2 and 3-3-3 are snapshots of equilibria. The line GC, indicated by ρ , is for the total book value of the firm; whereas, the line GD, denoted by $\bar{\rho}$, gives the total market value. Therefore, subtracting the amount of debt $AB = FD = JC$ from the respective total values, we obtain the market value of equity, OF, and its book value, OJ. Fig. 3-3-3 is similarly constructed.

The case corresponding to our Model 3 shall not be illustrated because the graphs are identical to Figs. 3-3-2 and 3-3-3 except that the financial opportunity line becomes concave due to an increasing cost of debt.

Let us now discuss the issue of dividend policy in the M-M case. M-M asserted that, given the firm's investment policy, the market value of a share is not affected by changes in the expected dividend stream.¹ They essentially argued that an increase in the dividend which requires a new issue of the shares is exactly offset by a decrease in the ex-dividend price of the shares because of the dilution resulting from an increased number of shares. It is important to note that M-M may assume that the firm's investment policy is given only if ρ is constant.

Model 2 does not allow the issuing of new shares. Therefore, let us evaluate the consequences of financing the dividend by debt.²

1. M. H. Miller and F. Modigliani, "Dividend Policy, Growth, and the Valuation of Shares," Journal of Business, Vol. 34, October, 1961, pp. 411-433. For a brief discussion of this issue, see Section 3 in Chapter II of this thesis.

2. This case was not discussed by M-M.

Presently, the dividend stream is zero today and OH tomorrow in Fig. 3-3-1. Let us alter the dividend stream to the one indicated by the point J which requires the firm to borrow the additional amount OE . The equity cost must increase due to the increased leverage. The value of equity, however, is still OD since the investment decision is not affected. The additional borrowing, OE , increases the total market value by $CF(= OE)$ to OF . Thus, the new equity cost, $\bar{i}$, can be obtained from the slope of the line JD which is $-(1+\bar{i})$. Therefore, in the M-M case, a change in the dividend stream financed by debt does not change the market value of equity because an increase in debt raises the total market value exactly by the same amount.

What is the dividend policy if ρ is not constant? We cannot separate dividend policy from investment policy as was done by M-M because all the decisions are made simultaneously. Therefore, the dividend stream resulting from the firm's optimal decisions must be optimal. Although the cases depicted in Figs. 3-3-2 and 3-3-3 do not show borrowing to finance dividends, it does not mean that the firm never borrows for this purpose. However, even when the firm finances dividends by debt, the amount of borrowing will still be optimal.

Thus, we establish

Proposition 8

If ρ is (is not) constant, then dividend policy does not (does) affect the market value of equity.

This proposition unifies the two most important controversies in corporate finance. We now see that the protagonist's position in the cost of capital controversy inevitably leads to his conclusion on dividend policy. Thus, empirical evidence on the shape of the cost of capital function may now be employed to support a corresponding position on dividend policy.

CHAPTER IV

SUMMARY AND SUGGESTIONS FOR FURTHER RESEARCH

1. Summary of the Results

The primary concern of this thesis was to show how the neoclassical theory of optimal capital accumulation and the theories of corporate finance can be integrated into a comprehensive theory of a publicly owned corporation operating under uncertainty. We first extended the original Fisherian theory by allowing corporate borrowing. Then, this Fisherian firm was interpreted as a publicly owned enterprise, and the maximization of the wealth of the owner-entrepreneur was replaced by the share price maximization. Next, uncertainty was introduced by making the equity cost an increasing function of leverage.

Having laid the cornerstones, we proceeded to construct various models of the firm under different specifications of the cost-of-capital functions. We first attempted to build a model with the market value debt-equity ratio as the argument in the equity cost function. This model was abandoned because the resulting decision rules were trivial. We then built models which employed the debt-equity ratio in book values. From these models, we obtained important results which are summarized below.

First, the weighted average cost of capital in book values (denoted by ρ) was shown to be the appropriate cutoff rate of investment. Second, it was shown that productive decisions are independent of financial decisions only when ρ is constant. Third, when ρ is constant,

the market value of equity becomes equal to its book value in equilibrium so that ρ is congruent to the weighted average cost in market values. This result established a formal correspondence between the Modigliani-Miller cost of capital theorems and Jorgenson's investment theory. Fourth, if ρ has a minimum with respect to the book value debt-equity ratio, then the market value of equity is equal to its book value only when ρ is minimized. This proposition is analogous to the Price Theory result that under perfect competition the average cost is minimized in longrun equilibrium. Finally, we have shown that whether dividend policy affects the market value of equity depends upon the shape of ρ function. More specifically, if ρ is (is not) constant, then dividend policy does not (does) change the market value of equity. This proposition clarifies the relationship between the two longstanding controversies in the literature of corporate finance.

2. Implications for Investment Theories

There have been many attempts in the economics literature to explain the firm's investment behavior and thereby the behavior of aggregate investment. Most works endeavored to explain the firm's investment decision by studying what they felt to be the most important factor, e.g., the early versions of the acceleration principle emphasized the change in output, and the liquidity theories concentrated upon the

internally available funds.¹ It was soon recognized that the factors singled out by each theory are interdependent elements of the firm's decision process. Consequently, later theories considered combinations of these factors, e.g., the accelerator-residual funds theory of Meyer and Kuh.² These attempts are merely empirical hypotheses. The factors alleged to influence investment decisions are not systematically related in the firm's decision process, nor are the hypotheses logically deduced from the postulates of the firm's behavior.

Credit must be given to Jorgenson for proposing the first rigorous theory of the firm's investment behavior. His theory is based on the neoclassical theory of optimal capital accumulation which was employed to derive the demand for capital services. In his theory, however, the firm's financial decisions do not bear upon its investment decisions. Jorgenson justifies this by resorting to the M-M cost of capital theorems:

"The neoclassical theory of investment behavior is based on an optimal time path for capital accumulation. It also implies a theory of the cost of capital. This theory has been developed by Modigliani and Miller.... In the Modigliani and Miller theory the cost of capital is shown to be independent of the financial structure of the firm or of dividend policy;...."³

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1. For the original version of acceleration principle, see J. M. Clark, "Business Acceleration and the Law of Demand: A Technical Factor in Business Cycles," Journal of Political Economy, Vol. 25, March, 1917, pp. 217-235, and a representative liquidity theory would be J. Meyer and E. Kuh, The Investment Decision, Cambridge, Mass.: Harvard University Press, 1957.
 2. J. Meyer and E. Kuh, ibid.
 3. D. W. Jorgenson and C. D. Siebert, "A Comparison of Alternative Theories of Corporate Investment Behavior," American Economic Review, Vol. 58, September, 1968, p. 685.

As can be seen from the quotation, Jorgenson asserts that the neo-classical theory of optimal capital accumulation implies the M-M cost of capital theory. This assertion is incorrect. These two theories are separate entities and one does not necessarily imply the other. Since, as was shown in Chapter III, the neoclassical theory of optimal capital accumulation is compatible with any reasonable specifications of the cost of capital function, Jorgenson can obtain his investment theory only when these two theories are combined.

In its simplest form, Jorgenson's model may be expressed as the following calculus of variation problem:

$$\max. \int_0^{\infty} e^{-\rho t} [pf(L,k) - wL - q\dot{k}] dt$$

where

ρ = constant weighted average cost of capital in

market value

w = wage rate

L = labor

and the rest of the symbols are the same as those in Chapter III.¹

1. We have assumed no depreciation since it is not essential to discussing the nature of his model in a tax-free world. It should be noted that financial decisions are absent from the model and ρ is assumed to be constant. It is also interesting to note that the objective functional in Jorgenson's model is the total value of the firm since no financial charge is subtracted from the integrand. This fact suggests that if ρ is constant, equity value maximization and total value maximization are identical.

The Euler first order condition gives¹

$$pf_L = W$$

$$pf_k = q\phi - \dot{q} \quad (4.1.1)$$

Next, assuming a Cobb-Douglas production function, $Q \equiv f(L, k) = k^a L^{1-a}$, Jorgenson derives the demand for capital stock. Since $f_k = ak^{a-1}L^{1-a} = aQ/k$, equation (4.1.1) may be solved for the optimal capital stock, yielding

$$k^* = aqQ/c$$

where $c = q\phi - \dot{q}$ is the user cost of capital. He then empirically tested his theory with a distributed lag model.

Jorgenson's investment theory seems to have been highly successful in empirically explaining the firm's investment behavior as well as the behavior of the aggregates.² On the other hand, other empirical studies found that the investment decision was strongly influenced by the amount of internally available funds.³ Also, the good results obtained by Jorgenson may be attributable more to the distributed lag function than

1. The Euler condition for the problem $\max. \int_a^b g(x, \dot{x}, t) dt$ is $g_x - d(g_{\dot{x}})/dt = 0$.

2. For example, D. W. Jorgenson, "Anticipations and Investment Behavior" in J. S. Duesenberry, et al., eds., The Brookings Quarterly Econometric Model of the United States, Chicago: 1965, pp. 35-92; D. W. Jorgenson and J. A. Stephenson, "Investment Behavior in U.S. Manufacturing, 1947-60," Econometrica, Vol. 35, April, 1967, pp. 169-220; and D. W. Jorgenson and C. D. Siebert, op. cit. The first two are concerned with aggregates, and the last with individual firms.

3. Many such studies are summarized in J. Meyer and E. Kuh, op. cit.

to the neoclassical theory per se.¹ Furthermore, since the publication of the M-M cost of capital paper, there have been numerous attempts to estimate the cost of capital functions.² Although still not conclusive, the results seem to favor a U-shaped weighted average cost of capital curve.³ These results suggest that the productive and financial decisions are interdependent, contrary to Jorgenson's assumption. In summary, the theory, although it rejects the importance of financial variables in investment decision, is nevertheless supported empirically, yet many other empirical studies found these variables significant. Therefore, a significant contribution would be to construct a rigorous

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1. G. Fromm and L. R. Klein, "The Brookings Model Volume: A Review Article : A Comment," Review of Economics and Statistics, Vol. 50, May, 1968, pp. 235-240. On p. 236, they write: "The good fits of the equations are to a large extent due to serial correlation and the inclusion of lagged investment as explanatory variables. In fact, after introducing two lags of the dependent variable, there is very little left to explain by user cost or anything else; the R^2 for investment in quarter t related only to investment in $t-1$ and $t-2$ for the period 1954-1962 is 0.91."
 2. Alexander Barges, The Effect of Capital Structure on the Cost of Capital, Englewood Cliffs, N. J.: Prentice-Hall, 1963. J. Fred Weston, "A Test of Cost of Capital Propositions," Southern Economic Journal, Vol. 30, October, 1963, pp. 105-112. M. H. Miller and F. Modigliani, "Some Estimates of the Cost of Capital to the Electric Utility Industry, 1954-1957," American Economic Review, Vol. 56, June, 1966, pp. 333-391. E. Brigham and M. J. Gordon, "Leverage, Dividend Policy, and the Cost of Capital," Journal of Finance, Vol. 23, March, 1968, pp. 85-104.
 3. A. Barges, ibid., and E. Brigham and M. J. Gordon, ibid., have shown evidence consistent with the U-shaped cost of capital function. J. Fred Weston has obtained the results against the constant cost of capital. Also the comment by M. J. Gordon, "Some Estimates of the Cost of Capital to the Electric Utility Industry, 1954-1957: Comment," American Economic Review, Vol. 57, December, 1967, pp. 1267-1278 argues persuasively against the results obtained by M. H. Miller and F. Modigliani, ibid. which reports statistical tests supporting the constant cost of capital.

theory which reflects the influences of financial variables on the investment decision. To achieve this task it is prerequisite to construct a model of the firm which systematically integrates both the productive and financial decisions. Our Chapter III may in fact be regarded as one such attempt. It was shown that ρ , whether it is constant or not, has to be the appropriate cutoff rate of investment. Therefore, if the weighted average cost of capital in book values (rather than in market values) is employed to derive the demand for capital stock, it will reflect the influence of financial variables whenever it is present. The construction of a full-fledged investment theory based on our theory of the firm and the subsequent empirical tests are beyond the scope of this study.

3. Other Suggestions for Further Research

Throughout this study it has been assumed that the investment decision does not alter the basic risk characteristics of the firm. In the language of capital theory, we have considered the case of widening in time and risk. A theory of the firm which is capable of analyzing the cases of both deepening and widening in time and risk has been absent in the literature. On the other hand, investment decisions made by modern corporations are far more complex than the simple case of widening in time and risk, e.g., research and development, merger decisions, etc. The development of the theory of the firm which is able to analyze such investment decisions is certainly a subject for future work. This thesis hopefully offers a point of departure for such an attempt.

Only two types of financial instruments, consols and retained earnings, were considered. This is because the theoretical discussions of optimal financial decisions in the literature have included only these means of finance. We must wait for the development of more sophisticated financial theories which include the issuing of common and preferred stocks and convertible debentures. Then, it will be possible to modify our theory to incorporate these financial instruments.

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