70-23,477

BAKKEN, George Stewart, 1943-
OPTICAL DESIGN FOR BEAM-FOIL EXPERIMENTS.

Rice University, Ph.D., 1970
Physics, spectroscopy

University Microfilms, A XEROX Company, Ann Arbor, Michigan

RICE UNIVERSITY

OPTICAL DESIGN FOR BEAM-FOIL EXPERIMENTS

by

George Stewart Bakken

A DISSERTATION
SUBMITTED IN PARTIAL FULFILLMENT
OF THE REQUIREMENTS FOR THE DEGREE OF

Doctor of Philosophy

Thesis Director's signature:

Houston, Texas

September, 1969

PLEASE NOTE:

Some pages have small
and indistinct type.
Filmed as received.

University Microfilms

TABLE OF CONTENTS

	Page
INTRODUCTION	1
I. Historical Review	1
II. Organization of the Dissertation	5
SECTION ONE: GENERAL CONSIDERATIONS FOR OPTICAL DESIGN	7
I. Introduction	7
II. Irradiance and Doppler Profile	18
III. Convolutions and Lineshape	22
IV. Signal-to-Noise	31
V. Important Properties of the Real Beam	39
VI. Special Considerations	43
VII. Summary	45
SECTION TWO: THE SIDE-ON LENS	46
I. Introduction	46
II. Irradiance, Doppler Profile, and Line Slant	46
A. Analyzing Instrument	48
B. Entrance Slit Irradiance	50
C. Line Slant	51
D. Doppler Distribution of the Effective Irradiance	57
III. Convolutions and Lineshape	57
IV. Signal-to-Noise	61
A. General	61
B. (s/n) for the Side-On System	63
C. Numerical Evaluation	66
V. Special Considerations	67
A. Residual Gas	67
B. Wavelength Measurement	67
C. Lifetime Measurements	72
D. Relative Intensity Measurements	73
E. Other Measurements	75

	Page
VI. Modified Systems	75
A. Cylindrical Mirror	75
B. Segmented Side-On Lens	78
C. Interference-Type Analyzers	83
D. Ultraviolet Experiments	83
SECTION THREE: THE END-ON LENS	84
I. Introduction	84
II. Irradiance and Doppler Profile	86
A. Analyzing Instrument	86
B. Irradiance and Doppler Profile Calculation	89
III. Convolutions and Lineshape	98
IV. Signal-to-Noise	106
A. General	106
B. (s/n) for the End-On System	108
C. Numerical Evaluation	111
V. Special Considerations	115
A. Residual Gas	115
B. Continuum	115
C. Wavelength Measurement	118
D. Lifetime and Similar Measurements	122
E. Relative Intensity Measurements	122
F. Interference-Type Analyzers	123
G. Ultraviolet Systems	125
SECTION FOUR: THE AXICON	127
I. Introduction	127
II. Irradiance and Doppler Profile	129
A. Analyzing Instrument	130
B. Entrance Slit Irradiance and Doppler Distribution	131
III. Convolutions and Lineshape	137

	Page
IV. Signal-to-Noise	139
A. General	139
B. (s/n) for the Axicon System	142
C. Numerical Evaluation	143
V. Modifications	147
A. Interference-Type Analyzers	147
B. Vacuum Ultraviolet	147
SECTION FIVE: A STUDY OF THE CARBON SPECTRUM USING AN END-ON SYSTEM	149
I. Introduction	149
II. Experimental Details	150
III. Data Analysis	152
IV. Results	157
SUMMARY AND CONCLUSIONS	166
REFERENCES	173

INTRODUCTION

I. Historical Review of Beam-Foil Spectroscopy

For many years the field of classical atomic spectroscopy stagnated because of a lack of new source techniques. The ionic states accessible in conventional arc and spark sources were studied in detail, while the entire range of ionization states above IV or V were almost unknown, except in the brilliant work of Edlen¹, who was able to examine lines in states up to Cu XIX.

Some recent progress has been made with novel sources. Three such sources are the θ pinch technique, used primarily by the Swedish group at Lund^{2, 3}, the laser ionization technique reported by Fawcett and Irons⁴, and the beam-foil source which is the subject of this dissertation.

Kay⁵ and Bashkin⁶ employed similar techniques, using a beam of ions with an energy of a few MeV incident on a thin (10 gm/cm^2) carbon foil. The ions were excited by their passage through the foil and emitted optical radiation as they continued downstream of the foil. This procedure has many advantages. First, the atomic composition of the beam can be established with isotopic purity using the magnetic analyzer associated with the accelerator. Second, the foil has been observed to produce very few (often only 2 or 3) charge states, and all possible charge states at least as far as completely ionized neon.⁷ These factors facilitate

the assignment of lines to the proper charge state and atom. The nature of the beam further permits the use of specialized techniques to further simplify this assignment. Malmberg⁸ has successfully employed electrostatic separation of the charge states before examining the beam spectroscopically. A similar technique has been proposed by Trächslin and applied successfully.⁹ The beam is observed in the direction of deflection, so that lines from different charge states will have different Doppler-induced slants (cf., Section 2, II-C). This method allows identification of the charge state much closer to the foil, since physical separation is not required, and thus the method may be used with short-lived states.

Another advantage of the beam-foil is that the point of excitation is well defined in both time and space. This facilitates lifetime determinations by time of flight techniques.^{10, 11, 12} A similar technique has been employed using gas targets.¹³ The solid foil has the advantage of a more sharply defined point of excitation and is also more effective in generating highly ionized states.¹⁴ It is interesting to note that a similar time of flight technique was used successfully by Wien as early as 1919.¹⁵ The sharply defined excitation point produced by the foil has also allowed observation of Stark-induced mixing of fine structure levels, which gives rise to dramatic intensity fluctuations.¹⁶

The literature of beam-foil spectroscopy has expanded

rapidly. The reader is referred to the early review paper by Bickel¹⁷, the Proceedings of the Conference on Beam-Foil Spectroscopy¹⁸, and recent review papers by Bashkin¹⁹, Bickel²⁰, and Fink²¹, and the references contained therein.

Beam-foil spectroscopy experiments at Rice began with studies of oxygen using the HVEC Model EN Tandem accelerator.²² The spectral light emitted was analysed with an f/2 Meinel spectrograph. The results were very unsatisfactory. The light was very weak, necessitating exposures of up to 12 hours. Further, the resolution was very poor because of the Doppler shift in wavelength caused by the high velocity ($\approx 0.04 c$, c the velocity of light) of the emitting ions. The lines were 10 to 12 Å wide. The exposure times and poor resolution are strongly interrelated; the lines would have been much more intense if the total flux in the 12 Å wide lines had been concentrated into the 1 Å linewidth capability of the Meinel.

Accordingly, the author set out to examine the coupled problems of intensity and resolution presented by the beam-foil source. Because the Doppler shift of the observed light is sharply segregated according to the angle of emission θ relative to the beam velocity v , it is possible to design specialized optics which will optimize resolution and intensity. The general approaches were established very quickly and reduced to practice. First, in the conventional "side-on" configuration ($\bar{\theta} = \pi/2$, see Fig. 1) the lens and spectrograph

Figure 1

The conventional or side-on collecting system is compared schematically with the end-on collecting system. Here the angle between the ion beam axis and the optic axis of the collecting lens is denoted by $\bar{\theta}$

should be moved back from the beam so as to limit the range of emission angles collected by the system. Second, significant gains can be made by using the "end-on" system ($\bar{\theta} = 0$, see Fig. 1) to look lengthwise down the beam. Both approaches were tested successfully.^{23, 24, 25}

It became evident during the course of this work that the optical problems presented by the beam-foil source were unusual. Many accepted, proven theorems were invalidated by the source's unusual properties. While the basic changes in optical theory required were relatively clear to the author, the consequences were in apparent "obvious" violation of conventional optics and even common sense.[†] The results were received with considerable skepticism. Accordingly, the author set out to examine the effects of the Doppler shift on optical design in great detail, developing the rationale for the required modifications to conventional optical theory. The results of this analysis were then used to analyze the design of specific systems for beam-foil spectroscopy. New approaches for optical design resulted, as well as a more complete understanding of the earlier work. Several important conclusions were reached concerning the proper approach to the analysis of beam-foil optics:

1. Microscopic analysis, tracing photons from the point of origin to the detector should be used. The accepted,

[†]Witness the optimization of the side-on collecting system by moving it away from the source!

macroscopic, optical theorems and algorithms contain implicit assumptions which are not valid for the beam-foil source when a dispersing analyzer is used.

2. The analyzing instrument must be considered in detail. The dispersing element and the relation between the size of the entrance aperture (slit) and the resolution are critical to the analysis. It is in no way sufficient to calculate the irradiance distribution as a function of wavelength on the entrance slit.
3. Results should not be presupposed. The properties of the beam-foil source are highly unusual and "common sense" has proven to be a poor guide in the absence of rigorous analysis.

This dissertation presents the results of this detailed analysis, including modifications of fundamental theory, evaluation of specific systems, and discussion of related problems encountered in actual experiments.

II. Organization of the Dissertation

This dissertation has been broken into five principal sections to ease the burden of the reader in following the lengthy and somewhat unusual arguments involved. The first section discusses the general principles of optical design of beam-foil spectroscopic experiments, examining the consequences of the strong Doppler effect. Fundamental theorems

and definitions needed for later analyses are developed.

The three subsequent sections evaluate the properties and optimum design of three specific systems proposed, and used, for beam-foil work. Some of the most important results of the first section are repeated to clarify the discussion. Specific problems and experiments are considered for each system:

1. signal to noise
2. residual gas discrimination
3. effects of continuum radiation
4. accuracy of wavelength measurements
5. relative intensity measurements
6. lifetime determinations
7. specialized modifications of the basic design

The last section discusses the experimental use of the "end-on" lens system for the study of the beam-foil spectrum of carbon at 0.90 and 1.05 MeV.

A brief summary of the results obtained and of general recommendations for the design of given beam-foil spectroscopy experiments concludes the dissertation.

SECTION ONE: GENERAL CONSIDERATIONS FOR OPTICAL DESIGN

I. Introduction

The technique called beam-foil spectroscopy uses a very high velocity beam of ions excited by passage through a thin foil as a light source.^{1, 2} This source has been used successfully³ to measure the spectra and lifetimes of ionized species. The technique shows much promise, but some serious difficulties have been encountered. The most important is that the light emitted by the beam is very weak. Foil wear and accelerator design impose fundamental upper limits on the beam current. Thus the most promising approach for overcoming the weak light problem is careful design of the optics which collect light from the beam and transfer it into the analyzing instrument.

The beam-foil source presents very unusual problems in optical design because of the Doppler effect associated with the large unidirectional velocity of the emitting ions. This Doppler effect places a coupled limit on signal strength and resolution, and gives rise to unusual line shapes. Many of the theorems of optics, developed for Lambert's law emitters with an isotropic spectra, are subject to important restrictions or even break down completely when applied to the beam-foil source. Other considerations normally neglected in analysis assume much greater importance with a beam-foil source.

The purpose of this section of the thesis is to discuss the general considerations in analyzing the optical performance of a spectroscopic system when a beam-foil source is used. Necessary modifications of conventional optical theory will be developed. Other aspects of optical theory which are not modified, but which require special emphasis, will be reviewed. Special considerations required by certain measurements will be discussed, and the more important secondary features of the beam-foil source reviewed. Subsequent sections will apply the principles developed here to the design of some specific optical systems for beam-foil experiments.

It may assist the reader to understand the unusual nature of beam-foil optics if we review some of our early experience.

The low intensity problem presented by the beam-foil source appears, at first glance, to be a simple problem with a well-known solution. The usual optimization procedure is as follows:

- A. Chose the magnification of the transfer lens such that the image of the source will completely fill the entrance aperture (slit) of the analyzing instrument.
- B. Obtain a transfer lens which is fast enough to match or overfill the acceptance solid angle of the spectrograph.
- C. Obtain the fastest spectrograph available with the desired resolution.

In the usual case, the irradiance at the entrance slit is transmitted to the detector at the image plane with an efficiency dependent only on the spectrograph design and natural line width of the source. Since these factors are independent of the light collecting optics, they may be ignored.

We proceeded to assemble a system for our initial beam-foil experiments several years ago⁴ using this procedure. This system consisted of an f/2 Meinel spectrograph and a quartz lens which satisfied points B and C above. We examined the beam-foil spectrum of 12 MeV O^+ with this system, and obtained the highly unsatisfactory upper line shown in Fig. 1.

Because we neglected the large Doppler effect inherent in the high velocity v of the emitting ions ($v/c \approx .04$, c = velocity of light), our "optimized" system gave a decidedly less than optimum performance. The line was broadened from the instrumental width of $1 \text{ \AA}$ to nearly $16 \text{ \AA}$, with a consequent tenfold decrease in plate darkening. It has a very strange slant, and although the line is known to be single from earlier work, it appears double or self-reversed.

The general features of this line shape were explained some time ago⁵, and Marquet⁶ has done a very detailed analysis. The general outline of the analysis follows from Fig. 2 and the well-known relativistic equation for the observed wavelength λ of a fast moving source emitting light of wavelength λ_o :

$$\lambda = \lambda_o \gamma (1 - \beta \cos \theta) \quad (1-1)$$

Figure 1a

Oxygen line produced by early system satisfying the classical optimization on page 8. Poor performance results from Doppler vignetting of the system.

Figure 1b

Oxygen line produced by improved system which violates "A" of the classical optimization on page 8 to avoid Doppler vignetting. The improvement is obvious

Figure 2

Geometry of the side-on collecting system illustrating the cause of the Doppler line slant and broadening.

where $\beta = v/c$, $\gamma = (1-\beta^2)^{-\frac{1}{2}}$, and θ is the angle of emission relation to $\underline{v}$. Fig. 2 indicates that light is collected over a large range of θ values. Because β is large ($\beta \approx 0.04$ for 12 MeV 0^+) this gives the observed broadening. Note also that light falling on the bottom of the slit is emitted at a smaller average θ than light falling on the top. Hence the mean wavelength shifts from shorter to longer wavelengths along the slit, giving rise to the observed slant. The Cassegrain optics in the Meinel spectrograph block the central θ values at any point along the slit, giving rise to the doubled line. Fig. 3 shows the calculated line shape given by Marquet for the aperture in Fig. 4.

The combined effect of the unidirectional Doppler effect and the dispersing analyzer invalidates some of the assumptions used to derive the optimization algorithm. Let us review the rationale for the algorithm:

- A. For a given slit irradiance[†], the total flux entering the instrument is at a maximum when the slit is everywhere irradiated; therefore the image of the source should fill the slit.
- B.,C. The useful entrance slit irradiance is equal to the product of the photometric brightness of the source and the solid angle subtended by the exit pupil (the

[†]We will use photometric units as defined in U.S.A. Standard-Nomenclature and Definitions for Illuminating Engineering, Illuminating Engineering Society, New York, (1968). In discussing photon counting systems, watts will be replaced by counts/sec.

Figure 3

Plots of irradiance vs. plate wavelength for a Meinel spectrograph and the collector geometry of Fig. 2. Fig. 4 shows the vignetting and distribution of wavelengths on the entrance aperture of the system giving the results shown.

Figure 4

Two vignetting and Doppler wave-length distribution diagrams for different collector designs. One is placed close to the beam and has a total spread of $8\Delta\lambda$. The other is twice as far from the beam and collects $4\Delta\lambda$.

collecting lens aperture, corrected for any prior or subsequent vignetting) at the image. Therefore the solid angle should be maximized, i.e. a fast spectrograph used, and external vignetting of any sort must be avoided. This assumes that the light entering any one element of the pupil is as useful as that entering any other element. This assumption breaks down for beam-foil spectroscopy because the large Doppler effect produces effective vignetting in any spectroscopic experiment requiring better resolution than results naturally after classically optimizing the system!

- D. Because the Doppler effect couples $\Delta\lambda$ to $\Delta\theta$, the spectral line shape can vary when the collecting geometry changes. The spectral line shape couples with the dispersive element in the spectrograph to alter the transmission efficiency. This interaction between collector geometry and spectrograph transmission efficiency cannot be neglected.

The breakdown of the assumptions leading to (B) and (C) is shown clearly in Fig. 4. Here the light collecting aperture is superposed over the wavelength distribution of light emitted in a single line from a single point in a collimated monoenergetic ion beam. As Marquet⁶ noted, the light from a single point in the ion beam with $\lambda_1 \leq \lambda \leq \lambda_1 + \Delta\lambda_1$ falls between approximately parallel lines as shown. By Eq. (1-1), each of

the strips in Fig. 4 has the same range of wavelengths $\Delta\lambda$, but a different average wavelength. If a given experiment requires a resolution of $4\Delta\lambda$ rather than the $8\Delta\lambda$ given by the aperture of a classically optimized system, shown in Fig. 4a, all but four strips must be masked off as shown, vignetting the system and reducing the irradiance at the entrance slit to roughly half.[†] We will refer to this effective vignetting by the Doppler effect as "Doppler vignetting."

Because of the dispersing element, a spectrograph is sensitive to flux or irradiance^{††} per unit wavelength rather than the gross values at the entrance slit at the image plane. The effective wavelength spread is found by convoluting the Doppler spread with the various other spread functions resulting from the entrance slit and optics of the spectrograph, and the intrinsic line width of the source. These convolutions, which will be discussed later, usually give results which resemble the widest distribution in the convolution unless there are one or more others nearly as wide. Therefore, since masking selectively removes the longest and shortest wavelengths in the Doppler spread, as shown in Figs. 3a and 4a, the flux or irradiance per unit wavelength will not

[†]This is strictly true only for a more conventional spectrograph where an unvignetted square grating forms the system stop, and the irradiance distribution is rectangular. For the camel's back distribution of the Meinel, the masking should be done asymmetrically to remove one of the humps, rather than symmetrically, leaving only the weak center.

^{††}Irradiance for the photographic detector, flux through the exit slit for a photomultiplier.

be reduced until the Doppler spread function approaches the next widest spread function.[†] The image plane irradiance thus behaves as though the system is vignetted, independent of any masking to limit line width. This is due entirely to the Doppler effect, as implied by the term "Doppler vignetting."

An alternative approach to better resolution exists which avoids Doppler vignetting by violating (A), the requirement that the image of the source fill the slit. The lens may be moved back twice as far from the beam, reducing the range of θ values collected so that the wavelength spread is again reduced to $4\Delta\lambda$ as shown in Fig. 4b. This system is not affected by Doppler vignetting, the gross irradiance is unchanged, and the irradiance per unit wavelength is doubled as shown in Fig. 3b. The magnification is halved, so that the flux per unit wavelength for a given length of beam is unchanged as long as the beam image is wider than the slit and the Doppler spread is the widest distribution in the convolution. A greater length of beam is imaged which gives some gain in flux, dependent on the decay length $1/\alpha = v\tau$, where τ is the mean lifetime of the upper state. Therefore this technique has a slight advantage over masking for a flux sensitive detector, as long as the beam image is as wide or wider than the slit. The doubling of irradiance per unit wavelength is an obvious advantage for an irradiance sensitive

[†]A detailed treatment is given in Section III. The effect was noted in a simplified form by Bickel.⁷

detector such as a photographic plate. The advantage is a factor of $\sqrt{2}$ rather than 2 since the square root of the area enters the signal-to-noise calculations which must be used to properly evaluate the performance of a spectroscopic system.

Accordingly we chose to violate (A) of the optimization algorithm and moved our spectrograph farther from the ion beam and refocused the lens. We obtained the lower result shown in Fig. 1, namely a smaller, much darker image showing $2 \text{ \AA}$ resolution. The improvement in performance was qualitatively obvious; several lines invisible with the earlier configuration were clearly present in identical exposures (time, beam current, energy, and foil thickness) using the new system.⁸

This analysis has been necessarily rather qualitative, and serves mainly to indicate the areas needing detailed study. Ultimately this must be done for specific systems, as the author will do in following sections. There are, however, numerous considerations which can be discussed in very general terms. These form the remainder of section one and will be used as a basis for the subsequent analysis of specific systems.

II. Irradiance and Doppler Profile

The general properties of the ion beam have been discussed by Bickel.⁷ For our immediate purposes we will assume an "ideal" beam. This beam is assumed to be perfectly parallel

and monoenergetic, so that β and θ in Eq. (1-1) are well defined. We will further assume that the beam is excited only at the foil and decays exponentially with a single mean lifetime. Since the beam is optically thin, it behaves as a volume emitter rather than a Lambertian source. Then the photometric flux dF into an element of solid angle $d\Omega$ from a volume element dV in the beam at a distance z_0 from the foil is

$$dF = \frac{d\Omega}{4\pi} C(\theta) \exp\left\{-\frac{\alpha}{v} z_0\right\} dV \quad (1-2)$$

where

$$C(\theta) = \frac{hc}{\lambda} A_{ij} \frac{J \epsilon_i}{va} P(\theta)$$

Here h is Plank's constant, A_{ij} the Einstein transition probability, J the beam current in ions/sec, ϵ_i is the probability that an ion will be in the i^{th} state, α is the reciprocal mean lifetime of the upper state, and a is the cross sectional area of the beam. $P(\theta)$ is a factor which accounts for possible anisotropies in the radiation from the ions.

Eq. 1-1 implies that all the light flux emitted from a point in the beam, a distance z_0 from the foil (see Fig. 5), with a wavelength between λ and $\lambda + \Delta\lambda$ is emitted into the solid angle $(\Delta\Omega)_{\text{max}}$ from θ to $\theta + \Delta\theta$ and over the range $0 \leq \phi < 2\pi$. $(\Delta\Omega)_{\text{max}}$ as a function of $\Delta\lambda$ is

$$(\Delta\Omega)_{\text{max}} = 2\pi \frac{\Delta\lambda}{\lambda_0 \gamma \beta} \quad (1-3)$$

Figure 5.

Two elements of solid angle defined by the light emitted from a point in the beam within a spread $\Delta\lambda$. The elements are equal as given by Eq. (1-3) and denoted by $(\Delta\Omega)_{\max}$.

Eqs. (1-2) and (1-3) indicate that the total flux within a wavelength interval $\Delta\lambda$ at a particular θ angle varies only to the extent that $P(\theta)$ is anisotropic.

The irradiance at the slit E_s is just the flux per unit area on the entrance slit, $dF(z_o)/d\sigma_o$, integrated over the volume $V(d\sigma_o)$ of the ion beam imaged onto the element of area $d\sigma_o$. $dF(z_o)$ is given by Eq. (1-2) so that

$$E_s(d\sigma_o) = \int d\theta \int d\phi \frac{\sin\theta C(\theta)}{4\pi d\sigma_o} \tau_b \int \int \int_{V(d\sigma_o)} e^{-\frac{\alpha}{V} z_o} dV \quad (1-4)$$

Here τ_b is the transmission coefficient of the lens. The relation between dV and $d\sigma_o$ may be rather complex since nonimage-forming devices and out-of-focus images are common in real systems. Also the geometric relationships between the positions and values of $d\Omega$, dV , and $d\sigma_o$ may be complex and frequently appear in the limits. Consequently the precise method of evaluating Eq. (1-4) depends on the specific system being analyzed. Also the spectroscopic instrument itself often modifies the initial wavelength distribution by selectively blocking rays contributing to part of the distribution after they enter the instrument, as was discussed for the Meinel spectrograph. This effect must be included in any calculation.

The existence of Doppler vignetting, out-of-focus lens images and unusual optical components complicates the use of brightness theorems which are normally very useful in optics. These theorems usually must be used in differential

formulations, which differ little from the direct calculation of irradiance. Since irradiance integrated over the detector area is what is recorded in most experiments, we have chosen to use direct calculations of the form of Eq. (1-4). This choice is admittedly a personal bias, but one which has served us well. We will normally evaluate $E_s(d\sigma_o, \lambda)d\lambda$, the irradiance per unit wavelength interval at a wavelength λ on a slit plane element $d\sigma_o$, of the general form

$$E_s(d\sigma_o, \lambda, d\lambda) = \int dz_o \int d\phi \tau_b \frac{C(\theta)}{4\pi} \cdot \left(\frac{da}{d\sigma_o} \right) e^{-\frac{\alpha}{V} z_o} \frac{d}{\lambda_o^{\gamma\beta}} \quad (1-5)$$

Here we have set $dV = da dz$, where da is the element of cross sectional area imaged onto $d\sigma_o$, eliminating two of the volume integrals. We have also used Eq. (1-1) to relate $d\lambda$ and $d\theta$. Integration proceeds as indicated, ϕ and then z . The discussions of the "end-on" lens and axicon systems in subsequent papers illustrate the application of Eq. (1-4) to develop equations of the form of Eq. (1-5).

III. Convolutions and Lineshape

The calculation of the Doppler wavelength distribution of the entrance slit irradiance does not complete the analysis. The spectral analyzing instrument does not see the flux or irradiance in a given spectral line, or even the flux or irradiance per unit wavelength in a given spectral line. The output is rather some detector signal as a function of some apparatus parameter such as linear displacement on a photographic plate (spectrograph), grating rotation (monochrometer),

or even gas pressure (pressure scanned Fabry-Perot) which bears a known relation to the wavelength of the incident light. This function, the observed lineshape, is the product of numerous effects besides the distribution of flux per unit wavelength on the entrance slit. These effects are usually termed instrument lineshapes or spread functions. They result from such factors as finite entrance slit widths, optical aberrations, and the resolving power of the dispersing element. Since several of these factors may be adjusted in designing a beam-foil experiment, unlike experiments utilizing more conventional sources, it is useful to examine their interactions. A brief review of the technique for such calculations is helpful at this point.

Calculation of the combined result of several spread functions is done by representing each factor by a mathematical distribution $G_i(T-T_i)$, where T is the running variable and T_i a reference origin, usually the center of the distribution. These distributions are then combined by convoluting them:

$$G(T-T_O) = \int dT_{n-1} \cdots \int dT_1 G_1(T-T_1) G_2(T_1-T_2) \cdots G_n(T_{n-1}-T_O) \\ \equiv G_1 * G_2 * \cdots * G_n \quad (1-6)$$

The convolution operator $*$ is associative and defined by Eq. (1-6). The independent variable T must represent the identical physical quantity in each distribution in the convolution. For example take G_1 as the Doppler distribution

with the independent variable τ , in $\text{\AA}$, and G_2 as the slit distribution of a monochromator with the independent variable x in cm. Before convoluting them, the reciprocal dispersion D ($\text{\AA}/\text{cm}$) and the internal magnification M_s must be used in the relation

$$\lambda_1 - \lambda_0 = (x_1 - x_0) D M_s \quad (1-7)$$

to convert the slit distribution, measured in centimeters, to the spectral slit $\Delta\lambda_s$, measured in $\text{\AA}$.

In general a number of such distributions G_i , representing each of the contributing spread functions, must be convoluted to calculate the observed lineshape. The result, G_o , must be suitably normalized to the total incoming flux. The exact procedure will depend on the analyzing instrument.

For example let us consider a grating spectrograph,* and define the common independent variable for the distributions to be the focal plane (plate) coordinate in the direction of dispersion, ξ . Further let ζ be the Cartesian coordinate perpendicular to ξ . The normalization of the distribution G_o to obtain the irradiance at the image plane $E_p(\xi, \zeta)$ is found by conservation of energy: the flux falling on a line element of height $d\zeta$ in the focal plane,

$$F = [N \int_{-\infty}^{\infty} G_o(\xi - \xi_0, \zeta) d\xi] d\zeta$$

where N is the normalization constant, must equal the flux entering the corresponding entrance slit element, multiplied

* Assumed to be stigmatic; the general case is more complex.

by the transmission factor τ_s for the line and diffraction order $F = E_s(\zeta/M_s) \tau_s W(d\zeta/M_s)$. $E_s(\zeta/M_s)$ is the irradiance at the slit and may be a function of position along the slit, ζ/M_s , and W is the slit width. Then

$$E_p(\xi - \xi_0) = \frac{E_s(\zeta/M_s) W \tau_s (d\zeta/M_s)}{\int_{-\infty}^{\infty} G(\xi - \xi_0, \zeta) d\xi d\zeta} G(\xi - \xi_0, \zeta) \quad (1-8)$$

The use of M_s in equating the entrance slit and image plane coordinates is required because the dependent variables ξ and ζ must be identically the same physical quantity, not merely have the same units, for the convolution and normalization to be correct. Conservation of energy would be violated if M_s was not used on the grounds that both slit and image plane coordinates are measured in centimeters.

For the purposes of illustrating the properties of convolutions, let us convolute the wavelength distribution in Fig. 3 with various slit widths, and assume all other contributions to be negligible. The slit is described by

$$G_s(\xi - \xi_0) = \frac{1}{M_s W} \cdot \begin{cases} \frac{M_s W}{2} & |\xi - \xi_0| \leq \frac{M_s W}{2} \\ 0 & \text{otherwise} \end{cases} \quad (1-9)$$

The results are shown for a fixed E_s and Doppler distribution G_D and varying choices of W in Fig. 6. Several effects are obvious:

1. As long as G_s is wider than G_D , $E_p = E_s$ at the center of the observed line.

Figure 6

Comparison of true convolution of G_s given by Eq. (1-9) and G_D of Fig. 3. The Central Limit Theorem Gaussian is shown as a broken curve. Note the rapid convergence of the C.L.T. for $\text{FWHM}(G_s) = \text{FWHM}(G_D)$.

True Convolution —
 Central Limit Approximation ----

2. E_p decreases rapidly when G_s is narrower than G_D .
3. The full width at half maximum of $G_s * G_D$, denoted by $\text{FWHM}(G_s * G_D)$, is approximately equal to the FWHM of the widest distribution in the convolution.
4. The shape of $G_s * G_D$ is similar to the widest distribution in the convolution, if the difference is large.
5. When the distributions in the convolution are nearly equal in width, $G_s * G_D$ has a roughly Gaussian appearance.

Two conclusions can be drawn from these effects:

- I. AN OPTIMUM IRRADIANCE-RESOLUTION CONDITION exists when $\text{FWHM}(G_s)$ equals $\text{FWHM}(G_D)$. Points 1-3 above indicate that this gives a good compromise between maximizing irradiance E_p and minimizing the FWHM of $G_s * G_D$, and represents the best operating condition for most experiments. Some experiments may require that irradiance be maximized at the expense of resolution or vice versa, of course.
- II. The optimum irradiance-resolution condition corresponds to point 5 above, where $G_s * G_D$ has a Gaussian appearance. This implies that the CENTRAL LIMIT THEOREM⁹ IS A GOOD APPROXIMATION TO THE LINESHAPE GIVEN BY THE OPTIMUM IRRADIANCE-RESOLUTION CONDITION.

These results are based rather qualitatively on a conventional grating or prism spectrograph, but actually are much more general.

Let us review the central limit theorem. This theorem, which is fundamental to statistical theory, shows that the detailed choice of the spread functions G does not significantly change the result of a series of convolutions. Essentially the theorem states: If the densities[†] $G(\xi)$ are subject to certain conditions, the convolution integral

$$G = G_1 * G_2 * \dots * G_n$$

tends to the limit

$$G(T) = \frac{1}{\mu \sqrt{2\pi}} e^{-(T-T_0)^2/2\mu^2}$$

where the line center is

(1-10)

$$T_0 = \sum_i T_i = \sum_i \int_{-\infty}^{\infty} T G_i(T) dT$$

and

$$\mu^2 = \sum_i \mu_i^2 = \sum_i \int_{-\infty}^{\infty} (T-T_i)^2 G_i(T) dT$$

The convergence of this theorem is extremely rapid if the densities G are roughly Gaussian to begin with. Even if the distributions are very un-Gaussian, such as the one in Fig. 3, convergence is rapid when the widths are nearly equal.¹⁰ We have observed this in Fig. 6, where the central limit theorem

[†]The term "density" denotes a distribution or spread function normalized to unit area.

Gaussian is shown by the superposed broken curve. These features make the central limit theorem very useful in discussing lineshapes since the optimum irradiance-resolution conditions will usually be assumed. It will be useful to have the FWHM of the central limit Gaussian,

$$2|T_{\frac{1}{2}} - T_0| = (8\mu^2 \ln 2)^{\frac{1}{2}} \quad (1-11)$$

where $T_{\frac{1}{2}}$ is either coordinate where $G(T)$ drops to half its maximum value.

We may now restate the OPTIMUM IRRADIANCE-RESOLUTION CONDITION more precisely:

- I. For a slit type spectrograph or monochrometer, the best resolution which is obtainable which allows the image plane irradiance to reach the maximum value $E_p = E_s \tau_s / M_s^2$ occurs approximately when the second moment μ_s^2 of the entrance slit function G_s ,

$$\mu_s^2 = \frac{M_s^2 W^2}{12} \quad (1-12)$$

equals the second moment μ_λ^2 of the convolution G_λ of ALL OTHER CONTRIBUTING DISTRIBUTIONS,

$$G_\lambda = G_D * G_B * G_O * \dots$$

REGARDLESS OF THEIR EXACT SHAPE.[†] Here G_D is the

[†]Certainly regardless of any exact shape likely to be found in practice. Fig. 6 should be sufficient to convince the most skeptical reader of the wide latitude allowed.

Doppler distribution, G_B the distribution due to ion beam properties, G_D due to optical aberrations, etc. JUSTIFICATION: Since many distributions are involved, Eq. (1-10) may be used to describe G_λ . The relation between the FWHM of G_λ and G_S is given by Eqs. (1-11) and (1-12):

$$\text{FWHM}(G_\lambda) = \left(8 \frac{M_S^2 W^2}{12} \ln 2\right)^{1/2} = .68 M_S W$$

Therefore the entrance slit should be set about 1.47 times as wide as the FWHM of the convolution of all other distributions. Because FWHM (G_S) and FWHM (G_λ) are roughly equal we may definitely use Eq. (1-10), and find the observed lineshape to have a full width at half maximum

$$\text{FWHM}(G) = \left(\frac{4}{3} M_S^2 W^2 \ln 2\right)^{1/2} = .97 M_S W$$

essentially equal to the projected entrance slit width. The peak irradiance follows from Eqs. (1-8) and (1-10):

$$\begin{aligned} E_p(\xi - \xi_o) &= \frac{E_S W_\tau S}{M_S [1]} \frac{\sqrt{6}}{M_S W \sqrt{2\pi}} \\ &\cong \frac{E_S \tau S}{M_S^2} \end{aligned}$$

as desired. The only important assumption is that

G_λ is Gaussian. Typically it will be more nearly rectangular and FWHM (G_s) will be more nearly equal to FWHM (G_λ).

IV. Signal-to-Noise

In the preceding two sections we have outlined the calculations of flux or irradiance falling on the detector of the analyzing instrument. The most useful measure of the information incident on the detector is the signal to noise ratio, (S/N) .[†] This is usually formulated as a dimensionless digital quantity, the ratio of the number of incident signal photons N_s (in our case, photons resulting from an atomic transition in the beam) to the noise signal, taken to be the standard deviation of the total detector output. This definition of noise means that the (S/N) will reflect the uncertainty in the true value of N_s . Since photons obey Poisson statistics, the noise signal is given by $(N_s + N_c)^{1/2}$, where N_c is the number of incident background photons (for example, continuum due to the beam striking the foil). Then defining the signal rates n_s and n_c such that $N_s = n_s t$ and $N_c = n_c t$

$$(S/N)_{in} = \frac{n_s t^{1/2}}{(n_s + n_c)^{1/2}} \quad (1-13)$$

[†]We will follow J. C. Marchant^{13, 14}, with modifications for non-photographic detectors, since photographic detection is the most commonly used in beam-foil work.

The signal to noise of the detector output, $(S/N)_{out}$ is related to $(S/N)_{in}$ by the Detective Quantum Efficiency $\hat{Q}$

$$\hat{Q} = \frac{(S/N)_{in}^2}{(S/N)_{out}^2} \quad (1-14)$$

where $(S/N)_{out}$ is defined for a noiseless detector. Detector noise must be included in $(S/N)_{in}$ as a noise equivalent signal N_d , which may or may not be time dependent ($N_d = n_d t$). This concept was developed by Rose¹¹ and Jones.¹² Marchant^{13, 14} has applied this work to photographic detection of threshold signals. Then

$$(S/N)_{out} = \hat{Q}^{\frac{1}{2}} \frac{n_s t^{\frac{1}{2}}}{(n_s + n_c + n_d)^{\frac{1}{2}}} \quad (1-15)$$

A more useful definition of $(S/N)_{out}$ for beam-foil spectroscopy is the threshold detection case $(s/n)_{out}$ formed by dropping n_s from the denominator of Eq. (1-15).

$$(s/n)_{out} = \hat{Q}^{\frac{1}{2}} \frac{n_s t^{\frac{1}{2}}}{(n_c + n_d)^{\frac{1}{2}}} \quad (1-16)$$

Eq. (1-16) for $(s/n)_{out}$ is essentially the same as Eq. (1-15) for $(S/N)_{out}$ when $N_s < N_c + N_d$. If this is not the case, then it reflects the probability that N_s is a random fluctuation in the no-signal background. In fact, $(s/n)_{out}$ is then just the signal measured in standard deviations of the background with no signal present. Therefore $(s/n)_{out}$ represents the probability that a line is or is not present,

whereas $(S/N)_{\text{out}}$ gives the uncertainty in the absolute intensity of the line. This threshold definition, $(s/n)_{\text{out}}$, was used by Marchant^{13, 14} to discuss the detection of weak photographic signals and appears to be satisfactory. Also $(s/n)_{\text{out}}$ gives a good indication of the minimum time required to detect a line. As indicated in the introduction, the difficulty presented by the beam-foil source is that n_s is small, requiring long integrating times. Photographic exposures with t up to 12 hours are required, with monochromator or Fabry-Perot scanning rates correspondingly slow. As a result the threshold case frequently holds in practice because beam-foil experiments are often time limited either by other demands on accelerator time or reciprocity failure¹⁵ of the spectroscopic plates. Comparing Eqs. (1-15) and (1-16) we see that $(S/N)_{\text{out}}$ varies as $n_s^{1/2}$ for the usual case and $(s/n)_{\text{out}}$ as n_s for the threshold case, so that the distinction is important. This n_s dependence indicates that the use of Eq. (1-15) underestimates the exposure time advantage of a given gain in n_s , so that the threshold case, Eq. (1-16), should be used in analyzing various light collecting systems.

Two specific formulations for $(s/n)_{\text{out}}$ are usually sufficient for discussing most spectroscopic systems. For a photographic plate the signal may be expressed in terms of irradiance:

$$N_s = \lambda/hc \quad E_p \quad t \quad A$$

$$N_c = \lambda/hc \quad E_c \quad t \quad A$$

$$N_d = \lambda/hc \quad E_d \quad t \quad A$$

Here h is Plank's constant, E_p the signal irradiance, and E_c the continuum irradiance, and A the area of the plate exposed. The noise equivalent irradiance of the detector E_d is an inverse function of exposure time for a purely photographic instrument since the gross fog E_f , $E_f = E_d t$, is independent of t . For a system including an image intensifier tube^{16, 17}, E_d has two components--image tube noise E_i and gross fog E_f/t , so that $E_d = E_i + E_f/t$. Then

$$(s/n)_{out} = \left(\frac{\hat{Q}\lambda}{hc}\right)^{\frac{1}{2}} \frac{E_p A^{\frac{1}{2}} t^{\frac{1}{2}}}{(E_c + E_d)^{\frac{1}{2}}} \quad (1-17)$$

Now in general E_p may vary with the image plane coordinates ξ and ζ . Since Eq. (1-17) assumes a uniform irradiance over A , it is not directly applicable. There is no accepted definition for $(S/N)_{out}$ of a nonuniformly irradiated signal, because in general the nonuniform irradiance contains information which does not fit this simple theory. In most cases we will be dealing with a slit spectrograph where the variation in irradiance contains no useful information[†] and we may use the form

$$d(s/n)_{out}^2 = \frac{\hat{Q}\lambda}{hc} \frac{E_p^2(\xi, \zeta) t d\xi d\zeta}{[E_c(\xi, \zeta) + E_d(\xi, \zeta)]} \quad (1-18)$$

with $(s/n)_{out} = \sqrt{\int d(s/n)_{out}^2}$. The actual evaluation of this

[†]This is not strictly true, since in some experiments the variation of irradiance with ζ may contain information about the lifetime of the upper level in the decay.¹⁶

expression depends very strongly on the light collecting system and spectrograph used, and may be very complex, especially in a prism spectrograph crossed with a Fabry-Perot étalon.

For photoelectric detection one may write a similar expression for $(s/n)_{\text{out}}$. The detector noise depends on the photocathode area A_p which is generally not the same as the area receiving signal. Then

$$(s/n)_{\text{out}} = \frac{\hat{Q}\lambda}{hc}^{\frac{1}{2}} \frac{E_p A t^{\frac{1}{2}}}{(E_c A + E_d A_p)^{\frac{1}{2}}} \quad (1-19)$$

with the form for nonuniform irradiance

$$(s/n)_{\text{out}} = \frac{\hat{Q}\lambda}{hc}^{\frac{1}{2}} \frac{\int_{\xi_1}^{\xi_1+M} d\xi^S \int d\zeta E_p(\xi, \zeta) t^{\frac{1}{2}}}{[E_d A_p + \int_{\xi_1}^{\xi_1+M} d\xi^S \int d\zeta E_c(\xi, \zeta)]^{\frac{1}{2}}} \quad (1-20)$$

Generally an instrument using a photoelectric detector is scanned continuously as a function of time, so that Eq. (1-20) must be modified. Because of the weak signal in beam foil studies, scans must be so slow that instruments which scan by a series of finite steps[†] nearly equal to the FWHM of the observed lineshape are used. In this case t is well defined in Eq. (1-20), equal to the integrating time at each step. For continuous scanning,

[†]A stepping monochrometer system is nearing completion in the author's lab, and a similar system has been built by Bridwell.¹⁸

$$(s/n)_{out} = \left(\frac{\hat{Q}\lambda}{hc} \right)^{1/2} \frac{\int dt \int_{\xi_1(t)}^{\xi_1(t)+M_s W} d\xi \int d\zeta E_p(\xi, \zeta)}{[\int dt \{ E_d A_p + \int_{\xi_1(t)}^{\xi_1(t)+M_s W} d\xi \int d\zeta E_c(\xi, \zeta, t) \}]^{1/2}} \quad (1-21)$$

In evaluating photoelectric scanning the independent variable in the convolution of spread functions, τ , is time t , with t related to wavelength λ and image plane coordinate ζ by the scanning rate $d\lambda/dt$ and the reciprocal dispersion D

$$t - t_o = \frac{1}{(d\lambda/dt)} (\lambda_1 - \lambda_o) = \frac{D}{(d\lambda/dt)} (\zeta_1 - \zeta_o) \quad (1-22)$$

where the subscripted variables refer to the arbitrary origin for the convolution integral.

Several properties of signal to noise should be remembered. First, a substantial increase in E_c , the background continuum, may be tolerated in increasing E_s . Second, the detector noise equivalent signal E_d must be included since it is frequently the dominant noise term in beam-foil work. To illustrate, consider two different collecting arrangements for the same spectrograph and detector. Assume that the detector is a photon counting photomultiplier with $n_d = 8$ counts/sec. For identical ion beams the arrangements have following hypothetical properties:

System #1: $n_s = 25$ $n_c = 1$, so that from Eq. (1-16)

$$(s/n)_{in} = \frac{25}{1} t^{1/2}; \quad (s/n)_{out} = \hat{Q}^{1/2} \frac{25}{3} t^{1/2}$$

and from Eq. (1-15)

$$(S/N)_{in} = \frac{25}{5} t^{1/2}; \quad (S/N)_{out} = \hat{Q}^{1/2} \frac{25}{6} t^{1/2}$$

System #2: $n_s = 125$, $n_c = 100$, so that Eq. (1-16) gives

$$(s/n)_{in} = \frac{25}{2} t^{\frac{1}{2}}; (s/n)_{out} = \hat{Q}^{\frac{1}{2}} \frac{25}{2} t^{\frac{1}{2}}$$

and from Eq. (1-15)

$$(S/N)_{in} = \frac{25}{3} t^{\frac{1}{2}}; (S/N)_{out} = \hat{Q}^{\frac{1}{2}} \frac{25}{3} t^{\frac{1}{2}}$$

comparing these two systems we note

1. System 1 has a negligible continuum, as contrasted with system 2, which has a continuum nearly as strong as the signal. The signal in system 2 is five times as strong as in system 1.
2. System 1 has twice the $(s/n)_{in}$ of system 2. However the detector noise n_d affects system 1 more severely than system 2. As a result, system 2 has better $(s/n)_{out}$ than system 1.
3. System 2, despite the very strong continuum background, shows better $(S/N)_{in}$ and $(S/N)_{out}$ than system 1.

Photographic systems introduce additional properties because of non-linear response and a limited dynamic range, effects which make $\hat{Q}$ a function of $n_s t$. Marchant^{13, 14} has studied these effects in great detail, and the reader is referred to those papers. To summarize his results briefly, note that $\hat{Q}^{\frac{1}{2}}$ shows a fairly sharp maximum located as shown on Fig. 7.

1. If exposure time is limited so that $\log(N_s + N_c + N_D)$ lies below the maximum, $(s/n)_{out}$ may be increased by the well-known technique of prefogging the plate

Figure 7

Detective quantum efficiency (Q) of Kodak Tri-X Film. This figure is taken from Marchant, ref. 13, Figure 1.

to increase the total exposure to a value where $\hat{Q}^{\frac{1}{2}}$ is near its maximum. Although prefogging decreases $N_p/(N_c+N_d)^{\frac{1}{2}}$, the increase in $\hat{Q}^{\frac{1}{2}}$ more than compensates.

2. If the exposure time is large so that $\log(N_s+N_c+N_d)$ lies past the point where $\hat{Q}^{\frac{1}{2}}$ has a slope of $-\frac{1}{2}$, $(S/N)_{out}$ may be increased by using a longer exposure with a slower film, provided the film has the same maximum $\hat{Q}$. For example, this may be done by using a Kodak IIIa-J emulsion rather than 103a-0.

These results indicate qualitatively that the earlier comments about the effects of increasing both background continuum and signal still hold. If the resulting total exposure exceeds the optimum value for a fast plate, one may simply use a suitable slower emulsion.

V. Important Properties of the Real Beam

So far we have assumed a rather idealized ion beam. The real beam has some properties which must be considered when designing optics for beam-foil spectroscopy.

First, as intimated by the polarization factor $P(\theta)$ in Eq. (1-2), the excitation process may polarize the emitted light to produce a preferred angle of emission. This effect has been observed in the light emitted by electron impact or gas targets.¹⁹ A few polarization measurements have been made on the beam-foil source, and have not shown a significant effect.²⁰ Comparisons of spectra taken at $\theta = 0^\circ$ and $\theta = 90^\circ$ ^{8, 21}

provide indirect evidence for preferred angles of emission, since lines appearing at $\theta = 90^\circ$ occasionally are absent at $\theta = 0^\circ$ and vice versa. Not enough is known about the beam-foil excitation process to allow generalization, and references 8 and 21 indicate that observation at both $\theta = 0^\circ$ and $\theta = 90^\circ$ are required to obtain complete spectra.

The assumption that the ion beam is monoenergetic and perfectly collimated is justified when calculating the Doppler distribution due to the collector. In practice, both β and θ in Eq. (1-1) are not well defined, but have a distribution of values G_β which must be included in the convolutions used to find the observed lineshape. The result is an ultimate lower limit on the observed linewidth. From Eq. (1-1) it is clear that θ spread is the dominant effect for a mean θ near 90° ¹ and β spread is dominant for a mean θ near 0° . Little is known as to which case gives the best ultimate resolution, and both β and θ spreads are a strong function of the ion, foil, and ion energy. Stoner²² has found a θ spread equivalent to $2 \text{ \AA}$ at $5000 \text{ \AA}$ for $1.03 \text{ MeV } O_2^+$ viewed at 90° , less than predicted by multiple scattering theory. No detailed study of β spread has been done, but $0.75 \text{ \AA}$ total linewidths have been obtained for C^+ at 0.9 MeV ²³ viewed at $\theta = 0^\circ$. The effects appear comparable when Stoner's data are extrapolated to higher energy per nucleon, but again not enough is known to allow generalization.

Four sources of noise exist in the beam-foil source:

Residual gas due to poor vacuum is excited by the beam, producing line and band spectra which may be confused with ion spectra. Techniques utilizing the Doppler shift of the ion lines have been used successfully to eliminate these lines, and they generally disappear if the vacuum is of the order 10^{-6} Torr or less. Two continuum sources exist, one due to incandescence of the foil where the beam strikes it, and the other fluorescence of the quartz windows produced by ion, electron, γ ray and ultraviolet bombardment. The light from the foil has been examined by Bashkin⁵ and found to be a weak continuum of importance only at very long wavelengths. A trace of the quartz fluorescence band is shown in Fig. 8. This trace was taken with about $2\mu\text{A}$ of He^+ at 1 MeV impinging directly on a fused silica plate. The resultant bright blue spot visually overwhelmed the ion emission, but the $4686\text{ \AA}$ line in He II shows clearly above the continuum since the irradiance per unit wavelength in the line greatly exceeded the irradiance per unit wavelength of the $1000\text{ \AA}$ wide fluorescence band. This emphasizes the point made earlier that the important parameter in a spectrograph is the flux or irradiance per unit wavelength interval, not the gross values. The intense fluorescing spot may be prevented by placing a small beam stop in front of it. The remaining area fluoresces too weakly to be seen at all except in very long exposures at $4500\text{ \AA}$ peak.* Note however, that certain spectrographs, such as the Fabry-Perot, concentrate all the continuum

* For example, see Fig. 7 in ref. 8 or Fig. 2 in ref. 23.

radiation in a few Angstrom free spectral range, so that the effective flux per unit wavelength of the continuum may be very large indeed. Much of the continuum may be eliminated by suitable predispersion however. The final source of noise applies only to scanning instruments, and is due to the ion beam current fluctuating with time. This may be eliminated by normalizing the spectrum to the ion beam current or gross light output as discussed by Bickel⁷ and others.²⁴

VI. Special Considerations

So far we have considered only the calculation of the fundamental optical parameters--resolution and signal-to-noise--of spectroscopic experiments with the beam-foil source. Particular classes of experiments impose additional design criteria which must be satisfied.

First, recall that the measured wavelength λ is not the same as the emitted wavelength, but is related to it by Eq. (1-1). Any uncertainty in the mean value of β or θ immediately results in a corresponding uncertainty in λ_0 . For example, the slanted lines in Fig. 1, resulting from the change in the mean value of θ along the slit, present the obvious problem: which point on the line corresponds to λ_0 ? Other designs pose similar problems.²³ The simplest method of correction is to identify prominent lines in the spectrum with lines observed for the same element in another source. Techniques²⁵ also exist for particular designs which allow direct correction

for the Doppler shift. Other problems with wavelength measurement result from blending of unresolved multiplets or accidental coincidences, and asymmetric Doppler spread functions.

The localized excitation and exponential decay of the beam-foil source complicate relative intensity measurements. Two definitions of relative intensity may be used. The first, corresponding most closely to the conventional source intensity, is the value at $z_0 = 0$, $A_{ij}N_ihc/\lambda$. The second definition is the relative total intensity, $A_{ij}N_ihc/d\lambda$, measured by integrating the decay curve. Both require detailed resolution of the decay curve, usually with the techniques used for radiative lifetimes described in the next paragraph. However good relative total intensity measurements may be made more rapidly with systems which view the beam "end-on" to integrate the decay curve automatically. With any experimental approach, any anisotropic behaviour of $P(\theta)$ will result in inaccurate relative intensities.

The beam-foil technique allows the measurement of radiative lifetimes by time-of-flight techniques.^{24, 26, 27} These measurements require that the optics examine only the radiation from a short segment Δz_0 of the beam at a known but variable distance z_0 from the foil. Also it is often desirable to separate the lines originating in different charge states in the beam (N II, N III, etc.) by deflecting the beam with electrostatic or magnetic fields and looking at physical

separation²⁸ or differential Doppler shifts¹⁶ of each charge state. For both types of experiments it is necessary to examine the beam roughly perpendicular to its axis. A third class of experiments, examining time dependent Stark shift mixing of excited levels²⁹, also requires perpendicular viewing.

VII. Summary

We have reviewed the steps in the optical design of beam-foil spectroscopy experiments, examining the consequences of the Doppler effect. It was found that the accepted techniques for the optimization of signal-to-noise and resolution in the spectroscopy of faint sources are partially invalidated by a "Doppler vignetting" of the system stop. The dispersing properties, resolution and entrance aperture of the analyzing instrument must be included in the design of the light gathering optics in some detail, although they usually may be neglected. Other considerations in the optical design of specialized beam-foil experiments are also required.

Subsequent sections will apply these principles and considerations to the optimization of specific optical systems for beam-foil spectroscopy.

SECTION TWO: THE SIDE-ON LENS

I. Introduction

The light collecting system for beam-foil spectroscopy using a lens with its optic axis perpendicular to the beam axis is diagrammed in Fig. 1. This system was used in the first beam-foil experiments^{1, 2} and will be called the "side-on" system. Many of the properties of this system have been discussed qualitatively^{3, 4} and Marquet⁵ has made a detailed analysis of the wavelength distribution of the flux collected. Bickel⁴ noted that the Doppler distribution could be narrowed by masking the collecting lens. The author^{6, 7} noted that masking reduces the photographic signal-to-noise (s/n) whereas moving the lens and spectrograph away from the beam accomplishes the same narrowing with no loss of (s/n). These results were summarized qualitatively in the first section of this thesis. The purpose of this section is to develop these results quantitatively. Some modifications suggested to improve the performance of this system will be analysed quantitatively as well. The reader is assumed to be familiar with the results and notation of section one.

II. Irradiance, Doppler Profile, and Line Slant

Marquet⁵ has analysed the Doppler profile of the system in Fig. 1. We will use the case for small β ($\beta = v/c$, v beam velocity and c the speed of light) and θ . Then light emitted

Figure 1.

Geometry of the side-on lens system for beam-foil spectroscopy. Vignetting is shown by projecting the grating aperture on the collecting lens. The spectrograph collimating lens is omitted for clarity.

from a point located a distance z_0 from the foil with a wavelength between λ and $\lambda + \delta\lambda$ falls between two parallel lines as shown on Fig. 2a.

A. Analysing Instrument

We will assume that a stigmatic spectrograph or monochromator is used to analyse the light emitted by the beam, and that the instrument has an effectively square grating or prism which is not vignetted by the auxiliary internal optics. This is a more common case[†] than the Meinel used in the discussion in section one. Other instruments, such as the Fabry-Perot are not well suited for use with the side-on lens system, as will be discussed later. The focal ratio of the entrance optics, $(f/)$, will be taken to correspond to the circular cone of light which will completely fill the grating, and is thus numerically less than the effective focal ratio. The reciprocal linear dispersion $D(\lambda)$ may be assumed constant over a narrow range of wavelengths.^{††} The slit

[†]This holds for most Fastie-Ebert, Czerny-Turner, and curved grating designs. It is also true of almost all prism instruments. However, the effective aperture is usually rectangular rather than square, and actual shape may be a function of wavelength in instruments that scan by grating or prism rotation. In this case, the square grating assumed corresponds to the larger dimension of the effective grating.

^{††}We have chosen to separate the linear reciprocal dispersion $D(\lambda)$ (eg in Å/cm, measured in the image plane) from the internal magnification M_i and introduce the λ dependence so that the calculations S will hold for prism instruments such as the one used in reference 1.

Figure 2a.

Entrance aperture of Fig. 1 with the assumed and actual grating stop projections.

Figure 2b.

Wavelength distributions of the light flux through the lens, assumed, and actual grating stops.

dimensions will be denoted by W wide and L long, and the internal magnification by M_s .

B. Entrance Slit Irradiance

The Doppler wavelength distribution in Fig. 2b then results from the vignetting shown in Fig. 2a. The total slit irradiance $E_s(d\sigma_o)$ is then calculated using Eq. (1-4)

$$E_s(d\sigma_o) = \int_0^\theta \sin\theta \, d\theta \int_0^\Phi d\phi \frac{1}{4\pi} C(\theta) \iiint_{V(\sigma_o)} \frac{dV(\sigma_o)}{d\sigma_o} e^{-\frac{\alpha}{V} z_o} \quad (1-4)$$

The integrals over θ and ϕ give $\sim \theta \phi$ and the integrals over dV give T/M_b^2 . Here T is the beam thickness along the optic axis and M_b is the magnification of the collecting lens. For the assumed vignetting and definition of $(f/)$,

$$\phi \doteq M_b / \sqrt{2} (f/) \quad (2-1)$$

From the well-known Doppler shift equation

$$\lambda = \lambda_o \gamma (1 - \beta \cos\theta) \quad (1-1)$$

we find

$$\theta \doteq \frac{\Delta\lambda}{\lambda_o \gamma \beta} \quad (2-3)$$

where $\Delta\lambda$ is the width of the Doppler distribution shown in Fig. 2. It may be narrower than the linewidths defined by the assumed square effective aperture due to masking, rotation of the prism or grating, or the grating being rectangular

rather than square. Then defining τ_b as the transmission of the collecting lens

$$E_s(d\sigma_o) = \frac{C(\theta) \tau_b T}{4\pi\sqrt{2} M_b(f/)} \left(\frac{\Delta\lambda}{\lambda_o \gamma \beta}\right) e^{-\frac{\alpha}{v} z_o} \quad (2-4)$$

C. Line Slant

Before writing down the exact expression for $E_s(d\sigma_o, \lambda, d\lambda)$ we must recall that the line is slanted by the Doppler effect since the mean value of θ , $\bar{\theta}$, varies as z_o changes. Bickel⁴ has discussed this effect qualitatively, but neglected the effect of the internal spectrograph optics. Their effects can be quite significant, affecting the observed slant, the width of the Doppler distribution and the effective slit and image plane irradiances, E_s and E_p .

The basis for the effect is shown in Fig. 3. The grating is assumed to be the effective stop. The effective value of $\bar{\theta}$ has usually been assumed to be measured from the optic axis so that

$$\bar{\theta} = \tan^{-1} \left(\frac{z_o - z_c}{f_b} \right) + \frac{\pi}{2} \quad (2-5)$$

In fact, $\bar{\theta}$ must be measured relative to the center of the projected stop imposed by the grating. This coincides with the optic axis only for the beam at z_c imaged onto the slit at L_c . For all other cases, such as z_o shown in Fig. 3, it

Figure 3.

Geometry of a typical side-on lens and spectrograph for the line slant analysis leading to Eq. (2-6).

will be displaced a distance $(Z_3 + Z_4)$ from the axis. From the geometric consideration of similar triangles in Fig. 3,

$$\begin{aligned}
 Z_3 &= L_O - L_C \\
 Z_1 &= \frac{f_g}{f} Z_3 \\
 Z_2 &= Z_3 \left(1 - \frac{f_g}{f}\right) = Z_3 - Z_1 \\
 Z_4 &= \frac{f_s}{f} Z_2 = Z_3 \frac{f_s}{f} \left(1 - \frac{f_g}{f}\right) \\
 \bar{\theta} &= \tan^{-1} [(Z_3 + Z_4 + Z_O - Z_C)/f_B] + \frac{\pi}{2} \\
 &= \tan^{-1} \left\{ \left[\frac{L_O - L_C}{f_B} \right] \left[1 + \frac{f_B}{f_s} + \frac{f_s}{f} \left(1 - \frac{f_g}{f}\right) \right] \right\} + \frac{\pi}{2}
 \end{aligned} \tag{2-6}$$

This result differs from Eq. (2-5) in all but a few special cases. It is interesting to note that this has a set of solutions giving $\bar{\theta}(d\sigma_O) = \frac{\pi}{2}$ for all values of $(L_O - L_C)$ - i.e. the line will not be slanted! Fig. 4 shows one such solution, $3f_g = f = f_s = f_B$. Unfortunately, a larger value of $f_B/f_s (>>1)$ corresponding to a fast collecting lens and good resolution, implies a large f_g/f ratio ($>>1$). However a large f_g/f ratio implies that the projection of the grating stop will rapidly vignette the collecting lens. For off-axis points on the slit the effective speed of this system is therefore always slow and closely coupled to the resolution. It is unlikely that the elimination of the slant is worth the inherent loss of speed, large size and inflexibility of this system. The important point is to emphasize again the point made in the

Figure 4.

Geometry of a side-on lens and spectrograph design for beam-foil spectroscopy which does not give a slanted line.

first section: The Doppler effects of the beam-foil source require that the entire optical system, from ion beam to detector, be included in any analysis.

For most systems in the laboratory, the values of f_B/f_s , f , and f_g will be fixed, and only the absolute values of f_B or f_s may be varied. Large values of $f_B \propto f_s$ give the minimum slant unless $|1-f_g/f| \gg 1$, which is not usually the case.

In the early part of this section, we indicated that the internal optics could have effects other than changing the expected line slant. If the collecting lens in Fig. 3 is too small, the light falling at either end of the slit will pass through a smaller aperture than light falling on the center. This is particularly the case if the lens has been masked to reduce the Doppler distribution width. In this event, Eq. (2-6) will no longer hold, since the effective aperture will be determined by the mask on one side and the grating on the other. As a result the line slant will decrease towards the ends of the slit, giving the line a slight "S" curve. Fig. 5 compares the line given by Bickel's calculation⁴, by Eq. (2-6), and the "S" shaped line resulting from Eq. (2-6) and masking for a relatively extreme case.

Now the exact slant of the line or slight "S" shape is not usually significant. However, the variation in collecting efficiency resulting from masking has three detrimental effects which may be significant:

1. The total flux and signal-to-noise is reduced.

λ $\rightarrow$

ξ $\rightarrow$

Figure 5.

Line slants given by different
calculations for the system of
Fig. 4.

A. Bickel, ref. 4

B. Eq. (2-6)

C. Eq. (2-6) showing effect of
masking the lens.

2. Lifetime measurements from photographic plates⁸ become inaccurate.
3. Relative intensity measurements become inaccurate.

The actual effect is usually small but significant, especially for lifetime measurements. Therefore, one of two precautions should be observed:

1. The collecting lens must not form the part of the system stop. Therefore, any resolution masking must be done at the grating.
2. A field lens must be placed on the slit.

Approach 1 is generally preferable since the field lens introduces additional transmission losses.

D. Doppler Distribution of the Effective Irradiance

Returning to the calculation of $E_s(d\sigma_o, \lambda, d\lambda)$ we see

$$E_s(d\sigma_o, \lambda, d\lambda) = \frac{C(\bar{\theta}) \tau_b T}{4\pi\sqrt{2} M_b(f/)} \left(\frac{d\lambda}{\lambda_o \gamma \beta} \right) e^{-\frac{\alpha}{v} z_o}, \quad (2-7)$$

$$|\lambda - \lambda_o \gamma [1 - \beta \cos \bar{\theta} (d\sigma_o)]| \leq \frac{\lambda_o \gamma \beta \Theta}{2}$$

$$= 0, \text{ otherwise}$$

where $\bar{\theta}(d\sigma_o)$ is given by Eq. (2-6). Fig. 6a shows an example of this distribution.

III. Convolutions and Lineshape

The observed lineshape in the detector plane must be

True Convolution ———

CLT Approximation - - - -

Figure 6.

Convolutions of G_S and G_D for the case $\Delta\lambda/D = M_S W$. The slit distribution G_S is given by Eq. (2-9) and the Doppler distribution G_D by Eq. (2-10). Note the fast convergence of the central limit theorem.

computed by convoluting the Doppler distribution G_D with the other distributions such as the entrance slit function G_S , variation in $\bar{\theta}$ in the beam due to scattering, G_B , optical aberrations in the spectrograph G_O , etc., to find the observed lineshape G :

$$G = G_D * G_S * G_B * G_O * \dots \quad (2-8)$$

where the convolution operator $*$ is defined in the usual manner.[†]

A problem is presented because G_B is a function of ion and foil thickness, and G_O a function of the particular spectrograph used, so that a general result which can be used for comparing the side-on lens with other light collecting designs is difficult since G_D may vary. The central limit theorem^{††} may be used to good advantage here. The central theorem indicated that the result of several convolutions approaches a Gaussian, so that the second moment of the contributing distributions, rather than their exact shape, is the important parameter. The convergence is rapid if the curves are roughly Gaussian or if the second moments are nearly equal. Therefore, for the purposes of comparison, we may neglect all but G_S and G_D , since for a given $G_S * G_D$,

[†]See Eq. (1-6) in section one or reference 9.

^{††}See Eq. (1-10) in section one and the accompanying discussion or reference 9.

the remaining spread functions will affect all systems approximately equally. To ensure rapid convergence, we will use an approximation to the optimum irradiance-resolution condition of section one - i.e. set the second moments of G_S and G_D equal, rather than set the second moment of G_S equal to that of all other convolutions.

Defining the slit function

$$G_S = \frac{1}{M_S W} \quad , \quad |\xi - \xi_0| \leq \frac{M_S W}{2} \quad (2-9)$$

where as in section one we are choosing the common variable for the convolution ξ to be the image plane coordinate in the direction of dispersion and ζ the perpendicular coordinate.

Normalizing Eq. (2-7) to unit area

$$G_D = \frac{D}{\Delta\lambda} \quad , \quad |\xi - \xi_0| \leq \frac{\Delta\lambda}{2D} \quad (2-10)$$

The normalized convolutions of $G_S * G_D$ and $G_S * G_S * G_D$ for $\Delta\lambda/D = M_S W$ are shown in Fig. 6b and 6c. The general analytic solution, normalized to irradiance by Eq. (1-8), is

$$E_p(\xi - \xi_0, \zeta) = \frac{DE_S(d\sigma_0)}{M_S^2 \Delta\lambda} \left[\mathcal{L} \left(\xi + \frac{M_S W}{2}, \xi_0 + \frac{\Delta\lambda}{2D} \right) - \mathcal{L} \left(\xi - \frac{M_S W}{2}, \xi_0 - \frac{\Delta\lambda}{2D} \right) \right] \tau_s \quad (2-11)$$

when $2|\xi - \xi_0| \leq M_S W + (\Delta\lambda/D)$, and is otherwise equal to zero.

$\mathcal{L}$ denotes the lesser of the quantities in parentheses,

[†]The convolutions are well-known (ref. 9) and normalization is discussed in section one.

and $\mathcal{G}$ denotes the greater. Quantities of interest in our analysis are the peak irradiance in the image plane, $E_p(0, \zeta)$, and the full width at half maximum (FWHM) of $E_p(\xi - \xi_0, \zeta)$. The peak irradiance is given by setting $\xi = \xi_0$ in Eq. (2-11)

$$E_p(0, \zeta) = E_s(\zeta/M_s) \left[\frac{\tau_s^{DW}}{M_s \Delta\lambda} \right], \quad W < \frac{\Delta\lambda}{M_s D} \quad (2-12a)$$

$$E_p(0, \zeta) = E_s(\zeta/M_s) \left[\frac{\tau_s}{M_s} \right], \quad W > \frac{\Delta\lambda}{M_s D} \quad (2-12b)$$

In Eqs. (2-11) and (2-12), $d\sigma_0$ on the entrance slit corresponds to the image plane point $(\xi - \xi_0, \zeta)$ and dimensions and coordinates are scaled from one to the other by the spectrograph magnification M_s as in Eqs. (2-12), or onto the ion beam with M_b , the collecting lens magnification, as will be done later.

IV. Signal-to-Noise

A. General

We will use the threshold equations for signal-to-noise out of the detector (Eqs. 1-18 and 1-21). There is generally no significant continuum, so that Eq. (1-18) has the form

$$(s/n)_p = \left(\frac{\hat{Q}\lambda}{hc} \right)^{1/2} \left[\iint \frac{E_p^2(\xi - \xi_0, \zeta)}{E_d(\xi - \xi_0, \zeta)} d\xi d\zeta \right]^{1/2} \quad (2-13)$$

Since E_p is separable in the two coordinates, we may approximate the lineshape by a pulse function with the same FWHM and peak height. For the case $W \leq \Delta\lambda/M_s D$ we replace

$$\int_{-\infty}^{\infty} E_p^2(\xi - \xi_0, \zeta) d\xi \quad \text{by} \quad k_p E_p^2(0, \zeta) \frac{\Delta\lambda}{D}$$

where

$$k_p = (D/\Delta\lambda) \int_{-\infty}^{\infty} E_p^2(\xi - \xi_0, \zeta) / E_p^2(0, \zeta) d\xi$$

This $k_p^{1/2}$ factor, which ranges typically from 0.5 to 0.8, contains all the relevant information about line shape. This definition is based on the central limit theorem and allows us to eliminate the detailed properties of G_p in comparisons between various collecting systems.

Then for the photographic case, noting that E_d is independent of ξ and $\zeta^\dagger$,

$$(s/n)_p = \left(\frac{\hat{Q} k_p \lambda}{E_d h c} \right)^{1/2} \left[\frac{\Delta\lambda}{D} \right]^{1/2} \left[t \int_0^{M_s L} d\zeta E_p^2(0, \zeta) \right]^{1/2} \quad (2-14)$$

Similarly Eq. (1-20) for incremental photoelectric scanning the (s/n) for each step may be written

$$(s/n)_{pe} = \left(\frac{\hat{Q} k_{pe} \lambda}{E_d A_p h c} \right)^{1/2} M_s W \quad t^{1/2} \int_0^{M_s L} d\zeta E_p(0, \zeta) \quad (2-15)$$

where

[†]This may not be true for some systems using an image intensifier tube (see ref. 10).

$$k_{pe}^{\frac{1}{2}} = \frac{1}{M_s W} \int_{\xi_1}^{\xi_1 + M_s W} E_p(\xi - \xi_0, \zeta) / E_p(0, \zeta) d\xi$$

and Eq. (1-21), for continuous scanning, at a rate $(d\lambda/dt)$

$$(s/n)_{pes} = \left(\frac{\hat{Q}k_{pes} \lambda}{E_d A_p h c} \right)^{\frac{1}{2}} M_s W \left[\frac{\Delta \lambda}{(d\lambda/dt)} \right]^{\frac{1}{2}} \int_0^{M_s L} d\zeta E_p(0, \zeta) \quad (2-16)$$

where we assume $w = \lambda / M_s D$ and

$$k_{pes}^{\frac{1}{2}} = \frac{(d\lambda/dt)}{(\Delta \lambda)} \frac{1}{M_s W} \int_{-\infty}^{\infty} dt \int_{\xi_1(t)}^{\xi_1(t) + M_s W} d\xi \frac{E_p(\xi - \xi_0, \zeta)}{E_p(0, \zeta)}$$

B. (s/n) for the Side-On System

We may now proceed to combine the preceeding calculations to find the comparison (s/n) of the side-on lens. Let us consider the case where masking is used to achieve a given resolution $\Delta \lambda$, and assume $W < \Delta \lambda / M_s D$. This will allow us to see the behavior of (s/n) as $\Delta \lambda$ is adjusted by masking the optics. Eqs. (2-4), (2-12a) and (2-14) give

$$(s/n)_p = C_p \left[\frac{D}{2M_s M_b} \right]^{\frac{1}{2}} \frac{W T}{4\pi (f/)} \frac{(\Delta \lambda)^{\frac{1}{2}}}{\lambda_o \gamma \beta} \left(t \int_0^{L/M_b} e^{-\frac{2\alpha}{v} z_o} dz_o \right)^{\frac{1}{2}} \quad (2-17)$$

for $W < \Delta \lambda / M_s D$, and where

$$C_p = \left[\frac{\lambda}{h c} \frac{\hat{Q}k_p}{E_d} \right]^{\frac{1}{2}} C(90^\circ) \tau_s \tau_b \quad (2-18)$$

So that $(s/n)_p$ decreases as $(\Delta\lambda)^{1/2}$ when masking is used to reduce $\Delta\lambda$ to the optimum irradiance-resolution condition

$$\Delta\lambda/D = M_s W.$$

Now we noted that $\Delta\lambda$ may be reduced by moving the lens and spectrograph away from the beam without affecting (s/n) , as long as the image of the beam was wider than the slit.

For this case $\theta = \phi$ and $M_b = \sqrt{2}(f/) \Delta\lambda/\lambda_o \gamma\beta$ (again assuming that the grating is effectively square so that

$$(s/n) = C_p \left[\frac{D}{2\sqrt{2} M_s (f/)} \right]^{1/2} \frac{TW}{4\pi (f/)} \left[\frac{1}{\lambda_o \gamma\beta} \right]^{1/2} \left(t \int_0^{L/M_b} e^{-\frac{2\alpha}{v} z_o} dz_o \right)^{1/2} \quad (2-19)$$

for $W < \Delta\lambda/M_s D$ and $W < M_b H$, where H is the height of the beam. In this case (s/n) is independent of $\Delta\lambda$ - that is the resolution may be improved by moving the lens and spectrograph away from the beam without affecting (s/n) .[†]

For the purposes of later comparisons we will write the optimum irradiance-resolution condition form of Eq. (2-19) by substituting $W = \Delta\lambda/M_s D$. For the beam image wider than the slit,

$$(s/n)_p = C_p \left[\frac{1}{2\sqrt{2} M_s^3 D (f/)} \right]^{1/2} \frac{T}{4\pi} \left[\frac{\Delta\lambda^2}{\lambda_o \gamma\beta} \right]^{1/2} \left(t \int_0^{L/M_b} e^{-\frac{2\alpha}{v} z_o} dz_o \right)^{1/2}.$$

[†]This effect of reducing $\Delta\lambda$ may also be accomplished by using a lens of smaller diameter and focal length. As noted in the line slant discussion (II-C), this will give a more slanted line than moving the spectrograph.

if

$$1 \leq \sqrt{2} H (f/) M_S D / \lambda_O \gamma \beta \quad (2-20)$$

The condition for the validity of Eq. (2-20) and hence for the limit of the moving back process is independent of the Doppler wavelength spread $\Delta\lambda$ and dependent only on the apparatus parameters and beam height. This is because both W and the image size decrease in the same way as $\Delta\lambda$ is reduced. The ultimate limit is only imposed by a rapid drop in k_p as $\Delta\lambda$ approaches the ultimate resolution of the plate or spectrograph. If the condition is not satisfied, $M_b = \Delta\lambda / M_S D H$ is the smallest allowed value and Eq (2-17) becomes

$$(s/n)_p = C_p \left[\frac{H}{2} \right]^{\frac{1}{2}} \frac{T}{4\pi M_S (f/)} \left[\frac{\Delta\lambda}{\lambda_O \gamma \beta} \right] \left(t \int_0^{L/M_b} e^{-\frac{2\alpha}{v} z_O} dz_O \right)^{\frac{1}{2}} \quad (2-21)$$

$$\Delta\lambda / M_S D = W > \sqrt{2} H (f/) / \lambda_O \gamma \beta$$

Similarly, for stepped scanning, Eqs. (2-4). (2-12a) and (2-15) give for $W = \Delta\lambda / M_S D$

$$(s/n)_{pe} = C_{pe} \frac{T}{4\pi\sqrt{2}} \left[\frac{\Delta\lambda}{\lambda_O \gamma \beta} \right]^2 t^{\frac{1}{2}} \int_0^{L/M_b} e^{-\frac{\alpha}{v} z_O} dz_O \quad (2-22)$$

where

$$C_{pe} = \left(\frac{\hat{Q} k_{pe} \lambda}{E_d A_p h c} \right)^{\frac{1}{2}} \frac{\tau_s \tau_b C(90^\circ)}{M_S D (f/)}$$

As noted in the first section, $(s/n)_{pe}$ does not depend on M_b , and hence shows no preference for masking vs. moving back,

except for the upper limit on the integral. Since this dependence can be significant for long decay lengths v/α , the same procedure should be used as for the photographic case:

$$M_b = \sqrt{2} \left(f/ \right) \frac{\Delta \lambda}{\lambda_o \gamma \beta}$$

or

$$M_b = \Delta \lambda / M_s D H$$

whichever is larger.

Finally, for continuous scanning, Eqs. (2-4), (2-12a) and (2-16) give for $W = \Delta \lambda / M_s D$

$$(s/n)_{\text{pes}} = C_{\text{pes}} \frac{T}{4\pi\sqrt{2}} \left[\frac{\Delta \lambda^2}{\lambda_o \gamma \beta} \right] \left[\frac{\Delta \lambda}{(d\lambda/dt)} \right]^{1/2} \int_0^{L/M_b} e^{-\frac{\alpha}{v} z_o} dz_o$$

where

$$C_{\text{pes}} = \left(\frac{\hat{Q} k_{\text{pes}} \lambda}{E_d A_p h c} \right)^{1/2} \frac{\tau_s \tau_b C(90^\circ)}{M_s D (f/)} \quad (2-23)$$

with the same behavior as Eq. (2-22). Note that $\Delta \lambda / (d\lambda/dt)$ has the dimensions of time.

C. Numerical Evaluation

Fig. 7a plots the $(s/n)_p$ given by Eq. (2-20) for a typical spectrograph and range of v/α and β . The scale factor is

$$\frac{v}{\alpha} \left(\frac{\lambda}{h c} \frac{\hat{Q} t}{E_d D M_s^3 (f/)^3} \right)^{1/2} C(90^\circ) \tau_s \tau_b \equiv B \quad (2-24)$$

where v/α is included to normalize the area under the decay curve to 1. Fig. 7b plots $(s/n)_{pe}$ for $M_b = \sqrt{2} f / (\Delta\lambda/\lambda_o \gamma\beta)$ with the scale factor

$$\frac{v}{\alpha} \left(\frac{\lambda}{hc} \frac{\hat{Q}}{E_d A_p} t \right)^{\frac{1}{2}} \frac{\tau_s \tau_b C(90^\circ)}{M_s D(f/)} = S \quad (2-25)$$

Since $(s/n)_{pes}$ exhibits the same behavior as $(s/n)_{pe}$ except for $[\Delta\lambda/(d\lambda/dt)]^{\frac{1}{2}}$ replacing $t^{\frac{1}{2}}$, it will not be plotted. In both curves we assume $T = H = 4\text{mm}$, $f/ = 10$, $L = 2\text{cm}$, and $\lambda_o = 5000 \text{ \AA}$.

V. Special Considerations

A. Residual Gas

Residual gas in the experimental chamber can be excited by the beam and give rise to line and band spectra which may be confused with emission from the ion beam. This is not a serious problem for pressures below 10^{-6} Torr. If a problem exists, residual gas lines may be distinguished by their lack of Doppler slant discussed in II-C. Thus the procedures for minimizing or eliminating slant should be used cautiously, leaving enough slant to distinguish ion emission from residual gas emission.

B. Wavelength Measurement

Accurate wavelength measurements in beam-foil spectroscopy require precise knowledge of the mean angle of emission $\bar{\theta}$, to allow λ_o to be determined from λ by Eq. (1-1). The

line slant complicates this problem since $\bar{\theta}$ varies over the slit unless the special system discussed in II-C is used. The point where $\bar{\theta} \equiv 90^\circ$ is generally used as a reference point, since only the second order shift, $\lambda = \lambda_0 \gamma$, is present. It would be desirable to locate this point at the foil to ensure that short lived states may be measured there. The foil holder generally produces asymmetric vignetting problems, so that the choice of Fig. 3 is normally used. The $\bar{\theta} = 90^\circ$ point must then be located by auxiliary means, such as an accurately placed opaque bar producing a break in the spectral line at $\bar{\theta} = 90^\circ$, which in turn allows measurement of $\bar{\theta}$ at the foil.

An accurate approach is similar to that used by Ives and Stillwell¹¹ and is shown in Fig. 8. A spherical mirror is arranged to image the beam and slit on themselves. The opaque bar may then be placed either on the slit or near the beam and carefully adjusted so that it is imaged onto itself by the spherical mirror. The bar then lies exactly on the optic axis. At this point, the light from the beam and the image produced by the spherical mirror will have equal and opposite $\bar{\theta}$ values. The result seen on the spectrograph image plane is a pair of crossed lines with a break in the middle, as shown in Fig. 9a. The light reflected by the mirror gives the more slanted line, the exact amount dependent on the mirror radius of curvature f_m and the distance to the collecting lens f_b . The mean value of the wavelengths at the break is

Figure 8.

Geometry of a side-on lens system for accurate wavelength measurements with a beam-foil source. The resulting lines are shown in Fig. 9.

Figure 9.

Line shapes resulting from the system of Fig. 8. The interpretation of the patterns for accurate wavelength measurement is described in the text.

just $\lambda_{O\gamma}$. The cross will be located relative to $\lambda_{O\gamma}$ and the bar at distances

$$\Delta\xi = \frac{\lambda_{O\gamma}}{D} - \beta \cos \left[\sin^{-1} \left(\frac{f_b \sin \omega}{(f_m^2 + f_b^2 + 2f_m f_b \cos \omega)^{1/2}} \right) \right] \quad (2-26)$$

$$\Delta\zeta = M_s M_b f_m f_b \left(\frac{\sin \omega}{(f_m^2 + f_b^2 + 2f_m f_b \cos \omega)^{1/2}} \right)$$

Here ω is the angle between the beam axis and perpendicular to the optic axis. By adjusting the apparatus so that $\omega = 0$ the x will be superimposed on the bar as shown in Fig. 9b. The bar may then be removed, and the point where the lines cross will indicate $\lambda_{O\gamma}$. This crossover point will have roughly twice the irradiance of the rest of the lines, as shown in Fig. 9c, aiding its recognition. It is desirable, for short lived states, to have the crossing point as close to the foil as possible while avoiding vignetting by the foil holder. Such a case is shown in Fig. 9d.

This and similar procedures for absolute alignment are tedious and difficult. A rather simpler procedure is to identify several lines in the beam-foil spectrum with lines previously measured in other sources, and determine a correction factor. This may not always be possible, and there is danger of error due to misidentification.[†]

Once the point where $\lambda = \lambda_{O\gamma}$ is determined, the accuracy

[†]For example, Brown et al.⁸ obtained sodium spectra where only the D lines could be identified with earlier work.

of wavelength measurement will depend on the full width at half maximum (FWHM) of the line, and the precision of determining the line center. The line center determination is affected by any asymmetry in the line shape, usually due to asymmetric vignetting and easily avoided, and the signal-to-noise ratio of the line. The ultimate limit on resolution is the irreducible minimum FWHM imposed by scattering in the foil. The effect is not well understood. Stoner¹² has examined scattering effects and found them to be significantly less than predicted by conventional multiple scattering theory. The limit may be as much as 2 or 3 Å FWHM for O_2^+ at 1.03 MeV and 5000 Å. This allows λ measurement to about .02 FWHM (.06 Å) for a strong single line. Weak or blended lines will be subject to much greater uncertainty. These values are based on very meager evidence and much more experimental work is needed in this area.

C. Lifetime Measurements

Radiative lifetimes are measured in the beam-foil source by time of flight methods and have been described in some detail in the literature.^{8, 13-17} Lifetime measurements require the ability to measure the light flux emitted from a given line as a function of distance z_0 from the foil. Only the side-on lens (or no lens at all, with the slit very close to the beam - which may be regarded as a lens with infinite focal length) configuration, of the various systems

proposed for beam-foil studies, is well suited to this work. Design of experiments is straightforward, and for isolated lines, the resolution requirements may be less severe, allowing larger s/n in the experiments. Care must be taken to ensure that detecting efficiency is uniform for all values of z_0 . Inequality may result from vignetting by the foil holder¹⁴ or by lens masking as discussed in II-C. Also detector efficiency may vary with wavelength, although the effect will be small unless line slant is extreme. Finally, the detector may be non-linear, as is the case with photographic plates, and special calibration is required.^{8, 17}

D. Relative Intensity Measurements

The localized excitation of the beam-foil source presents problems for relative intensity measurements because the brightness of the beam decays exponentially as the distance from the exciting foil z_0 increases, as given by Eq. (1-2). The relative intensity of two lines of different reciprocal mean lifetimes α will thus depend on the choice of z_0 . Two definitions of relative intensities have been used: The first, corresponding most closely to the relative intensity in a conventional source, is the value at $z_0 = 0$, $P(\bar{\theta})A_{ij}N_i hc/\lambda$. Since a point measurement at $z_0 = 0$ is essentially impossible, this measurement is made by measuring the spatially resolved decay curve and extrapolating to $z_0 = 0$. The necessary spatial resolution of the decay

requires the use of the side-on lens collecting system. The second definition is the relative total intensity, $P(\bar{\theta})$ $A_{ij} N_i hc/\alpha\lambda$. Total relative intensity measurements are made by integrating the area under the decay curve. This must be done explicitly for the side-on lens because of the spatial resolution of the decay. For example, a measurement of 10% absolute accuracy requires integrating the decay for a distance of about $2.3 v/\alpha$. Relative measurements may be done over an arbitrary distance (say $1 v/\alpha$) as long as it is the same for each decay (in units of v/α) to a precision which decreases as the length (in v/α) increases.

It is often desirable to make approximate estimates of relative total line strengths without resorting to detailed integration under the decay curve, usually using photographic detectors. This may be done by analysing the plate with the densitometer slit set to include the full ($2.3 v/\alpha$ or more) decay of the longest lifetime, and using this setting for all lines. Unfortunately this gives true results only as long as all parts of all lines ($z_0 = 0$ to $z_0 \geq 2.3 v/\alpha$) lie on the linear part of the sensitometric curve. For the most commonly used plates the linear range is not much more than a factor of 10 in exposure.* Since the irradiance in

* For example, Kodak Ia-0, Ia-E, fast plates frequently used by the author, have such sensitometric curves.¹⁸ However, special multilayer emulsions have been developed with much wider response, but requires a specialized densitometer for analysis.¹⁹ Note that we are making a distinction between the linear range and the useful range for most photographic purposes. The latter is much larger.

a single line will vary this much from $z_0 = 0$ to $z_0 = 2.3 v/\alpha$, lines with significantly different strengths cannot be compared accurately without complete decay curve analysis and corrections for non-linear response. We will describe a modified system which reduces this problem somewhat a bit later.

This problem of limited dynamic range does not exist if direct photoelectric scans are used, and the exit slit is slanted to match the dispersed line. Normalization of photoelectric data may present equally severe problems however.⁴ In summation then, the beam-foil source presents unusual problems for relative line intensity measurements.

E. Other Measurements

Certain other measurements, specifically the identification of the ionization state of a line by either the Malmberg²⁰ or Trächslin⁹ methods require the use of a side-on lens system.

VI. Modified Systems

Two modified side-on lens systems show promise for improving the s/n ratio and reducing the difficulty of making relative intensity measurements.

A. Cylindrical Mirror

Fig.10 shows that a cylindrical mirror may be placed

Figure 10.

System using a cylindrical lens to enhance the apparent brightness of the beam. The range of Doppler shift of the light is unchanged if the mirror is properly aligned, but the distribution of collected flux may be peaked toward shorter λ .

behind the beam to roughly double the light flux collected.*
 Because the mirror is afocal in the θ coordinate, the light emerging from any point in the beam due to direct emission at an angle θ will be augmented by reflected light emitted at an identical angle θ from another point $|2f_m \cot\theta|$ away from the first. If, for this line under study, $v/\alpha \gg |2f_m \cot\theta|$ the light contributed by the mirror will have identically the same wavelength distribution in θ and ϕ as the directly emitted light. This assumes the beam and cylindrical axis of the mirror are collinear. If they are not, two more or less identical parallel lines will result. For cases where v/α is only a few times greater than $|2f_m \cot\theta|$, the observed lines will be skewed to the blue since the beam radiates most strongly near the foil, where the blue - shifted reflected light originates. For good resolution, $|2f_m \cot\theta|$ can be made sufficiently small that the asymmetry of the observed line is not significant for most lifetimes.

It is important to remember that near the foil, vignetting by the foil holder will reduce the intensity of both direct and reflected light and produce line asymmetry. Also

* This approach was mentioned by Bashkin²¹ very briefly with the sentence "For example, a cylindrical or plane mirror behind the beam will contribute only 90° - shifted light so enhancement in a particular part of the image is achieved." This remark is somewhat misleading since we note in this text that the additional light contributed by the mirror has the same range of shifts as the direct light, rather than "only 90° - shifted."

the afocal property of the mirror will obscure the spatial decay characteristics (lifetimes, stark mixing resonances, etc.) of the beam.

B. Segmented and Anamorphic Side-On Lens Systems

Another approach for increasing s/n , but which is even more useful in making relative intensity measurements, is shown in Fig. 11. This system is essentially a modified form of masking which utilizes all parts of the lens and acceptance solid angle of the spectrograph. Normally masking will leave only the center strip of the lens, producing a normal sized image with reduced irradiance at the entrance slit. Any adjacent strip with the same angular width has exactly the same shape and FWHM. However, it is displaced in ξ by one FWHM because of the difference in the mean value of θ , $\bar{\theta}$, for each strip. The result of using two strips is thus a line which is twice as broad. However, prisms can be used, as shown in Fig. 11, to change the $\bar{\theta}$ for each segment of the lens so that they are all equal. The result is to displace the image of the beam produced by light from one segment an amount corresponding to the change in $\bar{\theta}$. For $v/\alpha \rightarrow \infty$, and n equal segments in the lens, all n images will overlap in the center for $1/n$ th the total distance, $n-1$ for the two adjacent $1/n$ ths, etc. The slit irradiance E_s will be n times that of the image from one strip, and equal to the E_s produced by the lens alone. This is illustrated in

Figure 11.

Modified form of masking using prisms to adjust θ for each section of the lens to superpose light with identical Doppler shift distributions on identical points of the spectrograph slit.

Fig. 12. The FWHM is reduced to $1/n$ th, so that the image plane sees E_p n times greater than the lens alone, over $1/n^2$ the area, for the central part, and areas of lesser irradiance on either side. The s/n of the central piece is thus just the same as for the wide line produced by the lens alone, showing the same improvement over simple masking as moving the lens back.

The width of the beam image is unchanged, so this process is not subject to the image size limit seen for the procedure of moving the lens back, and it is easier to align the larger image on the slit. ~~It is important~~ to construct and align the lens very carefully to ensure that $\bar{\theta}$ for any point on the slit is the same for each strip of the lens. This may present a serious problem at high resolution because λ varies rapidly with $\bar{\theta}$ at 90° , and θ must be the same for each segment within a small fraction of the FWHM for accurate wavelength measurements.

The actual gain of this system for values of v/α usually encountered will be less than n because of the variation of E_p along the ζ coordinate, and is further limited by the rapid decrease in k_p as $\Delta\lambda$ is reduced to approach the spectrograph resolution. The actual irradiance gain is thus less than when the lens is moved back for the central segment of the line, but the s/n may be less severely affected because of contributions from the other segments. The actual (s/n) of the central strip follows from Eq. (2-17)

Figure 12.

Line shape produced by the lens alone (left) and by the lens and prism system (right) shown in Fig. 11. The letters refer to the contribution of the corresponding segments of the lens.

by noting that $\Delta\lambda$ is replaced by $n\Delta\lambda$ in Eq. (2-4)

$$(s/n)_p = C_p \left[\frac{D}{2M_s M_b} \right] \frac{W}{4\pi(\bar{f})} \frac{\Delta\lambda^{\frac{1}{2}}}{\lambda_o \gamma \beta} \left(t \int_0^{L/nM_b} \sum_{i=0}^{n-1} e^{-\frac{\alpha}{v}(z_o + \frac{i}{n} \frac{L}{M_b})} \right)^2 dz_o \quad (2-27)$$

We expected (s/n) to be $n^{\frac{1}{2}}$ times the masked lens, and indeed Eq. (2-27) equals Eq. (2-17), with $\Delta\lambda \rightarrow n\Delta\lambda$, in the limit of $v/\alpha \rightarrow \infty$. Other parts of the line contribute signal as well, so the net result is increased s/n as well as improved resolution.

The most important feature of this collecting system is the reduction in the range of the integral in Eq. (2-27), and consequent reduction in the variation of intensity along the line of $v/\alpha > L/nM_b$. The effect is to equalize the apparent strength of lines with differing v/α in relative total intensity measurements. This reduces the problem presented by the limited linear range of the sensitometric curve of photographic plates discussed in V-D.

The segmented side-on lens reduces to an anamorphic lens in the limit of $n \rightarrow \infty$. This anamorphic lens has a focal length appropriate for normal imaging of the beam onto the entrance slit of the spectrograph in the direction perpendicular to the beam axis. The focal length in the direction parallel to the beam axis is just equal to the image distance. Consequently the light falling on any point along the slit will have been emitted at the same angle θ . The irradiance of the image at the center of the image slit will correspond

to the brightness of the beam averaged over the length of beam viewed by the lens, and will accurately reflect the relative total intensity of the line. Eq. (2-25) will not accurately reflect the (s/n) performance of this system, which actually depends in a complex way on α and the detailed design of the anamorphic system.

C. Interference-Type Analysers

The side-on lens system is not well suited to use with Fabry-Perot or Fourier Transform Spectrographs because of the change of wavelength with the angle of incidence. This means that parts of each interference order are produced by light of different wavelength which gives rise to serious errors in wavelength measurement. This problem has already been considered for much lower velocity beams.²² Since the axicon system does not show this effect to the same extent, and provides better (s/n), we will not consider the use of interference detectors with the side-on lens.

D. Ultraviolet Experiments

Equivalent designs using only reflecting elements, and hence suitable for use below $1200 \text{ \AA}$ exist for all variations of the side-on lens. Most of these, using conventional normal or grazing incidence mirrors are trivially obvious.

SECTION THREE: THE END-ON LENS

I. Introduction

The use of a collecting system with the optic axis collinear with the beam axis, called the end-on system, for beam-foil spectroscopy was first proposed by the author¹⁻³ and independently by a French group.⁴ One example is diagrammed in Fig. 1. The end-on lens makes use of the result in II of section one that the solid angle containing all the light emitted with wavelength from λ to $\lambda + \Delta\lambda$ from a point in the beam, given by

$$\Delta\Omega_{\max} = 2\pi \frac{\Delta\lambda}{\lambda_o \gamma \beta} \quad (1-3)$$

is independent of the choice of the mean angle of emission $\bar{\theta}$. As a result, we see the side-on lens system can only collect a fraction $\phi/2\pi$ of $\Delta\Omega_{\max}$, whereas the end-on lens collects all the available flux.* Because $\Delta\theta$ must change as $\bar{\theta}$ goes from 90° to 0° , changing the magnification, and because parts of the beam are out of focus, the gains in signal-to-noise (s/n) and irradiance are less than the total flux gain. Evaluating these quantities requires a much more detailed analysis. The purpose of this section of the thesis is to develop quantitative expression for these parameters. We will also discuss the design of such systems and modifications of the basic system for special experiments. The

* See section one, Fig. 5 and the introduction, Fig. 1.

Figure 1.

Geometry of the end-on lens system. Vignetting is shown by the shading on the grating. The spectrograph collimating lens has been omitted for clarity.

uses and limitations of the end-on system will be discussed.

II. Irradiance and Doppler Profile

The Doppler profile of the light emitted from a point in the beam is very simple. We will again take the usual case where both β ($\beta=v/c$, v beam velocity and c the speed of light) and the range of θ values collected is small. The light emitted from a point a distance z_0 from the foil ($z=0$) with wavelength between λ and $\lambda+\Delta\lambda$ will fall between two of the concentric rings in the "bulls-eye" pattern shown in Fig. 2a. The radius R of the ring corresponding to a wavelength λ is given by

$$R = (z_L - z_0) \tan\theta \quad (3-1)$$

where z_L is the position of the lens relative to the foil and θ is related to λ by Eq. (1-1)

$$\lambda = \lambda_0 \gamma (1 - \beta \cos\theta) \quad (1-1)$$

A. Analysing Instrument

We shall assume that a stigmatic spectrograph or monochromator is used to analyse the light emitted by the beam, and that the instrument has an effectively square grating or prism which is not vignetted by the auxiliary internal

Figure 2a.

View of collecting lens plane with the Doppler wavelength distribution of the light from one point in the beam. The square is the projection of the grating. Vignetting is indicated by shading.

Vignetted by Faraday Cup

p_G

From Corners
of Grating

λ

Figure 2b.

Doppler wavelength distribution
 G_D resulting from the geometry
of Figs. 1 and 2a.

optics.[†] The focal ratio of the entrance optics, $(f/)$, will be taken to correspond to the circular cone of light which will completely fill the grating, and is thus numerically less than the effective focal ratio. The reciprocal linear dispersion $D(\lambda)$ may be assumed constant over a narrow range of wavelengths. The slit dimensions will be denoted by W wide and L long, and the internal magnification by M_s .^{††} The vignetting is thus as shown in Fig. 2a, and the resulting line shape is shown in Fig. 2b. Normally a round Faraday cup is used to protect the lens from the direct beam, vignetting the part labeled FC. The "tail" on the red side of the distribution contributes nothing to s/n and distorts the symmetry of the line. It should thus be masked off, and for the same reasons discussed in II-C of section two, the masking should be done on the grating. The resulting Doppler distribution is the same as that obtained for the side-on lens, Eq. (2-10).

It is interesting to note that this vignetting is

[†]This holds for most Fastie-Ebert, Czerny-Turner, and curved grating designs. It is also true of almost all prism instruments. However, the effective aperture is usually rectangular rather than square, and actual shape may be a function of wavelength for instruments that scan by grating or prism rotation. In this case, the square grating assumed corresponds to the larger dimension of the effective grating.

^{††}We have chosen to separate the linear reciprocal dispersion $D(\lambda)$ (eg in $\text{\AA}/\text{cm}$, measured in the image plane) from the internal magnification M_s and introduce the λ dependence so that the calculations will hold for prism instruments.

essentially similar to that in the Meinel spectrograph. The central stop imposed by the secondary mirror of the Cassegrain collimator in the Meinel corresponds to the Faraday cup stop⁺, and both vignette the blue light in the distribution. This contrasts with the bifurcated line resulting when the side-on lens is used. The side-on Meinel is discussed in I of section one and was analysed by Marquet.⁵

The mean value of θ collected, $\bar{\theta}$, varies as z_0 changes -- the bull's-eye pattern expanding or contracting as given by Eq. (2-1). The light from each point z_0 is not separated along the slit, but superposed in varying degrees of focus. The result is a slightly asymmetric line shape, the extent of the asymmetry depending on the decay distance v/α (α the reciprocal mean lifetime) and the design of the collimator. The calculation of the spectral irradiance at the slit, $E_s(\lambda)$, is therefore rather complex.

B. Irradiance and Doppler Profile Calculation

We proceed by evaluating the equation for the spectral irradiance, Eq. (1-4) from section one:

⁺This is not strictly true, since the mirror must be supported. Marquet⁵ shows one example which includes the support in the vignetting. This support is not the only, or even the optimal, design. A three-legged "spider" support, such as used in small Cassegrain telescopes, would give a better line shape but is less convenient mechanically. Marquet's vignetting is reproduced in Fig. 4 of section one.

$$E_S(d\sigma_O) = \int_{\theta} \sin\theta \, d\theta \int_{\phi} d\phi \frac{\tau_b C(\theta)}{4\pi} \iiint_{V(\sigma_O)} \frac{dV(\sigma_O)}{d\sigma_O} e^{-\frac{\alpha}{V} z_O} \quad (1-4)$$

First we must determine $dV(\sigma_O)/d\sigma_O$ for any z_O . Consider Fig. 3, which shows the distribution of light from a point a distance z_O from the foil and a radial distance $\vec{r}_p + \vec{r}$ from the common beam and optic axis. If z_O lies exactly on the object focal point z_b , $r=0$ and the wavelength distribution is exactly as given in Fig. 2b. If it is not, the wavelength will be distributed over a bull's-eye pattern identical with the one on the lens given by Fig. 2a, except the radius r_D at the slit position z_s is

$$r_D = R \frac{|z_f - z_s|}{z_f - z_L} \quad (3-2)$$

where z_f is the true focal point for dV at z_O . The radius r_D is measured relative to the paraxial ray through dV which defines the center of the bull's eye and is the path of the light emitted at $\theta \approx 0$. Then $d\sigma_O$ is just $d\sigma_O = r_D dr_D d\phi$ or

$$d\sigma_O = \frac{(z_f - z_s)^2 (z_L - z_O)^2}{(z_f - z_L)^2} \tan\theta \sec^2\theta \, d\theta d\phi \quad (3-3)$$

The corresponding element of volume located at z_O is defined by the amount dr that the position of the paraxial ray through dV moves in the beam as it moves dr_D in the slit plane. The relation between r and r_D is independent of z_O since r is defined by a paraxial ray, and is given by

$$r = r_D/M \quad (3-4)$$

Where the magnification M_b is given by

$$M_b = \frac{z_s - z_L}{z_L - z_b} \quad (3-5)$$

Thus, the element of volume $dV = r dr d\phi dz$ becomes

$$dV(\sigma_o, z_o) = \frac{1}{M^2} d\sigma_o dz \quad (3-6)$$

Then using Eq. (3-6) to evaluate $dV/d\sigma_o$ in Eq. (1-4), we have

$$E_s(d\sigma_o) = \int d\theta \sin\theta \int d\phi \frac{C(\theta)\tau_b}{4\pi} \int_0^{z_L} \frac{dz_o}{M_b^2} e^{-\frac{\alpha}{V} z_o} \quad (3-7)$$

This result, that the irradiance at a point in the image plane due to an out-of-focus image is the same as for the focused image, results from the overlapping of adjacent out-of-focus images of adjacent volume elements dV . As soon as adjacent images cease to exist, the irradiance will decrease rapidly. The evaluation of Eq. (3-7) thus depends on determining the ϕ limits on the integral caused by the loss of the adjacent images.

Let us define $\phi=0$ to be the line from the common beam and optic axis to $d\sigma_o$, and r_o the radial distance of $d\sigma_o$ from the common axis. Then the radial distance from the paraxial ray through σ_o projected into the beam element dV to the common axis is $r_p=r_o/M$. Also r_b denotes the beam

radius. These definitions are shown in Fig. 3. Several points should be noted:

1. The rays emitted at an angle θ which fall on $d\sigma_o$, regardless of their origin, are collinear with the rays from the focused image point at z_b .
2. Therefore the volume elements emitting light at large θ values will begin to fall outside the beam sooner than light emitted with small θ . If the beam is emitting in the region where the large θ rays lie outside the beam, the long wavelength side of the line will be less intense than the rest of the line, and the observed line will be asymmetric.
3. The ray from dV falling on $d\sigma_o$ will always pass through the lens on a circle of radius $R+r$ centered on the paraxial ray through $d\sigma_o$.

The limits on ϕ in Eq. (3-7) may now be computed.

The most important limit on ϕ comes when the volume element dV lies outside the beam. Looking at the normal section of the beam shown in Fig. 4, and applying the cosine law to the triangle formed by $\vec{r}$, $\vec{r}_b$, and $\vec{r}_p$, the upper limit on ϕ , imposed by the beam, ϕ_b , is

$$\begin{aligned}
 \phi_b &= 0 \quad , \quad r_b > r + r_p \\
 \phi_b &= \pi \quad , \quad r_b < r - r_p \\
 \phi_b &= \cos^{-1} \left[\frac{r^2 + r_p^2 - r_b^2}{2 r r_p} \right] \quad , \quad \text{otherwise}
 \end{aligned}
 \tag{3-8}$$

Figure 4.

Geometry used to derive Eq. (3-8). This is a normal section of Fig. 3 at the point z_0 .

Next in importance, the point where the ray emitted at an angle θ passes through the lens may be obscured by the Faraday cup, as shown in Fig. 5. The Faraday cup radius is centered on the optic axis and denoted by r_F . From the triangle $R+r$, r_p and r_F , we have the lower limits on ϕ , ϕ_F

$$\begin{aligned}\phi_F &= 0 \quad , \quad r_F < R + r - r_p \\ \phi_F &= \pi \quad , \quad r_F > R + r + r_p \\ \phi_F &= \cos^{-1} \left[\frac{(R+r)^2 + r_p^2 - r_F^2}{2r_p(R+r)} \right] \quad , \quad \text{otherwise}\end{aligned}\tag{3-9}$$

The remaining two limits are imposed either by the lens edge or the edge of the projection of the effective spectrograph aperture. The upper limit on ϕ imposed by the lens, ϕ_L is given by trigonometric arguments similar to those preceeding:

$$\begin{aligned}\phi_L &= 0 \quad , \quad r_L > r + R + r_p \\ \phi_L &= \pi \quad , \quad r_L < r + R - r_p \\ \phi_L &= \cos^{-1} \left[\frac{(R+r)^2 + r_p^2 - r_L^2}{2r_p(R+r)} \right] \quad , \quad \text{otherwise}\end{aligned}\tag{3-10}$$

Here r_L denotes the lens radius and is centered on the optic axis. The upper limit on ϕ imposed by the projected spectrograph stop is

$$\phi_s = 0 \quad , \quad r_s > R + r + r_p + r_c$$

Figure 5.

Geometry used to derive Eq. (3-9). This figure is a superposition of normal sections of Fig. 3 at the points z_0 and z_L .

$$\begin{aligned}\phi_s &= \pi, \quad r_s < R + r - r_p - r_c \\ \phi_s &= \cos^{-1} \left[\frac{(r+R)^2 + (r_p+r_c)^2 - r_s^2}{2(r+R)(r_p+r_c)} \right], \quad \text{otherwise}\end{aligned}\quad (3-11)$$

The radius of the spectrograph stop, assumed circular and determined by masking as discussed earlier, has a projected radius at the lens of r_s centered on a point a distance r_c from the optic axis. The direction of $\vec{r}_c$ is diametrically opposite r_p and the magnitude follows from Eq. (2-6).

$$r_c = r_o \left[1 + \frac{z_s - z_L}{f} \left(1 - \frac{f_g}{f} \right) \right] \quad (3-12)$$

Here f is the slit to collimator distance in the spectrograph and f_g the collimator to grating distance.

Eq. (3-7) may then be written

$$E_s(d\sigma_o) = \int_{\phi} d\theta \frac{\sin\theta}{M_b} \frac{\tau_b C(\theta)}{4\pi} \int_0^{z_L} dz_o \int_{\phi_F}^{\mathcal{L}(\phi_b, \phi_L, \phi_s)} d\phi e^{-\frac{\alpha}{v} z_o} dz_o \quad (3-13)$$

where $\mathcal{L}$ denotes the least of the quantities in the parentheses. Eqs. (3-1), (3-2), (3-4), (3-5) and (3-8)-(3-12) must be used to evaluate Eq. (3-13). Because of the complexity of the calculation, computer analysis is required.

The wavelength distribution may be found by omitting the integral over θ and substituting the derivative of the Doppler shift equation

$$\lambda = \lambda_o \gamma (1 - \beta \cos \theta) \quad (1-1)$$

with respect to θ ,

$$d\theta = \frac{d\lambda}{\lambda_o \gamma \beta \sin \theta} \quad (3-14)$$

Then the irradiance per unit wavelength interval is

$$E_s(d\sigma_o, \lambda) d\lambda = \frac{d\lambda}{\lambda_o \gamma \beta} \frac{C(\theta) \tau_b}{M_b^2 4\pi} \int_0^{z_L} dz_o e^{-\frac{\alpha}{v} z_o} \int_{\phi_F}^{\mathfrak{L}(\phi_b, \phi_L, \phi_s)} d\phi \quad (3-15)$$

Some representative examples for various choices of v/α and $d\sigma_o$ are shown in Figs. 6 and 7 for a system used by the author. This system is shown in Fig. 13.

III. Convolutions and Lineshape

The observed lineshape in the detector plane must be computed by convoluting the Doppler distribution G_D with the other distributions such as the entrance slit function G_S , spread in values of β due to energy loss in the foil G_B , optical aberrations in the spectrograph G_O , etc. to find the observed lineshape G :

$$G = G_D * G_S * G_B * G_O * \dots \quad (1-6)$$

where the convolution operator $*$ is defined in the usual manner.[†] As we did in III of section two, the central

[†]See Eq. (1-6) in section one or some text such as Papoulis.⁸

Figure 6.

Oblique plot of $G_D \alpha E_s$ computed by computer evaluation of Eq. (3-15) for the system of Fig. 13. and various values of r_0 . We have used a value of $v/a = 3$ cm.

Figure 7.

Oblique plot of $G_D \propto E_s$ computed by computer evaluation of Eq. (3-15) for the system of Fig. 13, $r_0 = 0$, and various values of v/a .

limit theorem[†] may be used to eliminate explicit evaluation of the contributing distributions, so that we only need consider $G_D * G_S$. Let us define the slit function as before

$$G_S = \frac{1}{M_S W} (\xi - \xi_0) \leq \frac{M_S W}{2} \quad (2-9)$$

Fig. 8 shows $G_D * G_S$ for the same system used in Figs. 6 and 7.

Note that even though the lineshape of G_D may be very asymmetric for v/α large and $d\sigma_0$ some distance from the center of the image, the peak and centroid of $G_S * G_D$ is constant to within a small fraction of the FWHM. Table 1 gives numerical data for a wide range of v/α and resolutions $\Delta\lambda/\lambda_0\gamma\beta$. The basis for the independence of the location of the convoluted line center and the asymmetry is, of course, the rapid convergence of the central limit theorem, which is sensitive only to the centroid and second moment of the distribution. The centroid usually determines the observed value of λ . As v/α increases, the Doppler distribution G_D tends to broaden on the long wavelength side at the same time as it becomes asymmetrically depressed on the long wavelength side. The two effects nearly cancel, so that the centroid is not displaced. After multiple convolutions, the lineshape G is nearly Gaussian and centered on the wavelength of the centroid of G_D .

[†]See Eq. (1-6) and the accompanying discussion in section one or Papoulis.⁸

Figure 8.

True Convolution —
CLT Approximation ---

Comparison of G_s from Eq. (2-9), G_D from Eq. (2-10), and $G_{D'}$ from Eq. (3-15), and of the convolutions $G_s * G_D$ and $G_s * G_{D'}$. The Central Limit Theorem approximation is shown as the superposed broken line.

TABLE 1

Variation of Observed Wavelength with v/α

$$\beta = 0.01$$

	FWHM, $\Delta\lambda/\lambda_0$		
	1×10^{-6}	3×10^{-6}	1×10^{-5}
$v/\alpha = 1$	.99005001	.99005090	.9900539
$v/\alpha = 5$	.99005001	.99005090	.9900541
$v/\alpha = 10$	.99005001	.99005090	.9900539
% variation	$\sim 2\% \Delta\lambda$	$\sim 2\% \Delta\lambda$	$2\% \Delta\lambda$
			$3\% \Delta\lambda$

This table shows the position of the maximum of $G_S * G_D$, in terms of λ/λ_0 for $\beta = 0.01$. Comparing columns, each computed for a particular system, we see that the variation of measured wavelength over a typical range of v/α is on the order of the precision of the computer calculation, 2% FWHM, except for the lowest resolution. The increase of λ/λ_0 as the FWHM increases is a result of the short wavelength side of the line being fixed by $\theta = 0$, so that any increase of the FWHM is due to adding to the red side of the line. This shifts the center of the line to longer wavelengths.

Now a few precautions must be observed for the preceding discussion of asymmetry to hold. As noted earlier, the asymmetry results from the locus of dV contributing red-shifted light falling outside the beam before the volume contributing blue-shifted light does. Since the beam is emitting most strongly near the foil, particularly when v/α is small, it is important that the foil be close enough to the true focus point z_b that the volume contributing red light does not fall outside the beam. It is difficult to do this without focusing directly on the foil ($z_b = 0$) which increases the amount of asymmetry and decreases the slit irradiance by losing light farther away from the foil for larger v/α . The choice used in computing the curves in Figs. 6-8 is to have the locus of volume elements contributing the reddest light fall outside the beam exactly at the foil for $d\sigma_0$ at the exact center of the beam image, that is

$$z_b = z_p \equiv \left(\frac{r_b}{r_b + r_c} \right) z_L \quad (3-16)$$

where we assume that the circularly masked grating forms the system stop, as recommended earlier.

For the purpose of calculating (s/n) , the asymmetry of G_D is even less important. The $G_S * G_D$ profile in Fig. 8 corresponds to the result Eq. (2-11) for the side-on lens to a very good approximation, especially near the optimum irradiance-resolution condition $\Delta\lambda/D = M_S W$. Therefore, we

may use Eqs. (2-12) from section two in the (s/n) computations here:

$$E_p(0, \zeta) = E_s(\zeta/M_s) \left[\frac{\tau_s^{DW}}{M_s \Delta \lambda} \right] , \quad W < \frac{\Delta \lambda}{M_s D} \quad (2-12a)$$

$$E_p(0, \zeta) = E_s(\zeta/M_s) \left[\frac{\tau_s}{M_s} \right] , \quad W > \frac{\Delta \lambda}{M_s D} \quad (2-12b)$$

It is also useful to have a rough estimate of E_s which is less complex than Eq. (3-15). It is adequate to take the limits on the ϕ integral to be $\pm \pi$ and integrate from $z=0$ to $z=2z_p$, where Eq. (3-16) is used to determine z_p . The first assumption overestimates E_s and the second underestimates it, the effects roughly cancelling. This approximation is most accurate for v/α less than z_p , and is thus most useful for systems showing good resolution (r_c/z_L small). The adequacy of this approximation will be discussed later. With these assumptions then Eq. (3-15) becomes

$$E_s(d\sigma_o, \lambda, d\lambda) = \frac{d\lambda}{\lambda_o \gamma \beta} \frac{C(\theta) \tau_b}{2M_b^2} \int_0^{2z_L \left(\frac{r_b}{r_b + r_c} \right)} dz_o e^{-\frac{\alpha}{v} z_o} \quad (3-18)$$

and

$$E_s(d\sigma_o) = \frac{\Delta \lambda}{\lambda_o \gamma \beta} \frac{C(\theta) \tau_b}{2M_b^2} \int_0^{2z_L \left(\frac{r_b}{r_b + r_c} \right)} dz_o e^{-\frac{\alpha}{v} z_o} \quad (3-19)$$

IV. Signal-to-Noise

A. General

We will use the threshold Eqs. (1-18) and (1-21) as we did in discussing the side-on system. Since there may be observable continuum in some wavelength intervals we will use the full form of Eq. (1-18), but take E_c independent of ξ and ζ . Then

$$(s/n)_p = \left[\frac{\hat{Q}\lambda}{hc(E_c + E_d)} \right]^{1/2} \left(\int_{-\infty}^{\infty} E_p^2(\xi - \xi_0, \zeta) d\xi d\zeta \right)^{1/2}$$

Because of the complex variation of $E_p(\xi, \zeta)$, as in the example of Figs. 7 and 9, it may be desirable to evaluate Eq. (3-15) exactly by computer. Except in cases where extreme accuracy is required, the assumptions justified by the central limit theorem may again be used as in section two. Since E_p is separable in the two coordinates, we may approximate the lineshape by a pulse function with the same FWHM and peak height. We then replace

$$\int_{-\infty}^{\infty} E_p^2(\xi - \xi_0, \zeta) d\xi$$

by

$$k_p E_p^2(0, \zeta) \Delta\lambda/D$$

where

$$k_p = \frac{D}{\Delta\lambda} \int_{-\infty}^{\infty} \frac{E_p^2(\xi - \xi_0, \zeta)}{E_p^2(0, \zeta)} d\xi$$

This k_p factor, which ranges typically from 0.25 to 0.7, contains all the relevant information about lineshape. This definition and the central limit theorem has allowed us to eliminate most of the detailed properties of G_p to facilitate comparisons between various collecting systems.

Then for the photographic case,

$$(s/n)_p = \left(\frac{\hat{Q}k_p \lambda}{(E_c + E_d)hc} \right)^{\frac{1}{2}} \left[\frac{\Delta \lambda}{D} \right]^{\frac{1}{2}} \left(t \int_0^{\mathcal{L}(M_s L, 2M_s M_b r_b)} d\zeta E_p^2(0, \zeta) \right)^{\frac{1}{2}} \quad (3-20)$$

Similarly Eq. (1-20) for incremental photoelectric scanning may be written

$$(s/n)_{pe} = \left(\frac{\hat{Q}k_{pe} \lambda}{(E_c + E_d)A_p hc} \right)^{\frac{1}{2}} M_s W t^{\frac{1}{2}} \int_0^{\mathcal{L}(M_s L, 2M_s M_b r_b)} d\zeta E_p(0, \zeta) \quad (3-21)$$

where

$$k_{pe}^{\frac{1}{2}} = \frac{1}{M_s W} \int_{\xi_1}^{\xi_1 + M_s W} \frac{E_p(\xi - \xi_0, \zeta)}{E_p(0, \zeta)} d\xi$$

and Eq. (1-21), for continuous scanning, at a rate $(d\lambda/dt)$

$$(s/n)_{pes} = \left(\frac{\hat{Q}k_{pes} \lambda}{(E_c + E_d)A_p hc} \right)^{\frac{1}{2}} M_s W \left[\frac{\Delta \lambda}{d\lambda/dt} \right]^{\frac{1}{2}} \int_0^{\mathcal{L}(M_s L, 2M_s M_b r_b)} d\zeta E_p^2(0, \zeta) \quad (3-22)$$

where

$$k_{\text{pes}}^{\frac{1}{2}} = \frac{(d\lambda/dt)}{(\Delta\lambda) M_S W} \int_{-\infty}^{\infty} dt \int_{\xi_1(t)}^{\xi_1(t) + M_S W} d\xi \frac{E_p(\xi - \xi_0, \zeta)}{E_p(0, \zeta)}$$

B. (s/n) for the End-On System

As in the case of the side-on lens, any given resolution (full width at half maximum or FWHM) may be achieved either by masking or moving the lens back to reduce $\Delta\theta$. In this case, of course, the masking is done concentrically to leave a washer-like acceptance angle, similar to that encountered in conventional Cassegrain optics. Some masking of this sort is produced automatically if a Faraday cup is used.[†] An analysis essentially identical to that done for the side-on lens in section two gives the same result, namely that moving away from the foil gives better signal-to-noise. There is a difference in that the end-on lens has a larger magnification than the side-on for a given resolution, and thus will not encounter the problem of the beam image being smaller

[†]The Faraday cup may be eliminated by using electromagnetic beam deflection such as that used for charge state analysis.⁷ Deflection may be applied after the beam has left the region of observation (determined by the depth of field $2z_c$) to cause the beam to fall outside the window area. This is most easily done for small ranges of θ .

than the slit.[†] Also, the end-on lens is always collecting nearly all the available light in a wavelength interval λ to $\lambda + \Delta\lambda$, so the special techniques used with the side-on lens discussed in VI of section two have no analogs.

If we remove the Faraday cup and denote the FWHM of the resulting line by $\Delta\lambda$, the magnification of the system is related to $\Delta\lambda$ and the focal ratio of the spectrograph ($f/$) by

$$M_b = 2\sqrt{2}(f/) \tan\left(\cos^{-1}\left[1 - \frac{\Delta\lambda}{\lambda_o \gamma \beta}\right]\right) \quad (3-23)$$

$$\doteq 4(f/) \left(\frac{\Delta\lambda}{\lambda_o \gamma \beta}\right)^{\frac{1}{2}}, \quad \frac{\Delta\lambda}{\lambda_o \gamma \beta} \text{ small}$$

Substituting Eqs. (3-19), (2-12a) and (3-23) into (3-20) we have for $(s/n)_p$ when $\Delta\lambda/D = M_s W$

$$(s/n)_p = C_p \left[\frac{1}{M_s^3 D (f/)^3}\right]^{\frac{1}{2}} \frac{(2\text{tr}_b)^{\frac{1}{2}}}{16} \left(\frac{\Delta\lambda}{\lambda_o \gamma \beta}\right)^{\frac{3}{4}} \int_0^{2z_p} e^{-\frac{\alpha}{v} z_o} dz_o \quad (3-24)$$

where we assume $M_s L > 2M_s M_b r_D$, and

$$C_p = \left[\frac{\lambda}{hc} \frac{\hat{Q}k_p}{(E_c + E_d)}\right]^{\frac{1}{2}} C(0^\circ) \tau_s \tau_b$$

[†] Compare Eq. (3-23) with IV-B of section two which gave $M_b = \sqrt{2}(f/)\Delta\lambda/\lambda_o \gamma \beta$ for the unmasked side-on lens. Since $(\Delta\lambda/\lambda_o \gamma \beta) < 1$, both the coefficient and resolution term of the side-on magnification M_b are less than in the end-on case, Eq. (3-23).

Note that the resolution dependence is $(\Delta\lambda^3/\lambda_o\gamma\beta)^{1/4}$ compared with $(\Delta\lambda^2/\lambda_o\gamma\beta)^{1/2}$ for the side-on lens, Eq. (2-20). This results from the dependence of the Doppler effect on $\cos\theta$, which is slowly varying for small θ .

Similarly we have $(s/n)_{pe}$ by substituting Eqs. (3-12a), (3-19) and (3-23) into Eq. (3-21)

$$(s/n)_{pe} = \frac{C_{pe}}{8} \left(\frac{\Delta\lambda^3}{\lambda_o\gamma\beta} \right)^{1/2} 2r_b t^{1/2} \int_0^{2z_p} e^{-\frac{\alpha}{v} z_o} dz_o$$

where (3-25)

$$C_{pe} = \left[\frac{\hat{Q}k_{pe}\lambda}{(E_c + E_d)A_p hc} \right]^{1/2} \frac{\tau_s \tau_b C(0^\circ)}{M_s D(f/)}$$

Further we obtain $(s/n)_{pes}$ by substituting into Eq. (3-22):

$$(s/n)_{pes} = \frac{C_{pes}}{8} \left(\frac{\Delta\lambda^3}{\lambda_o\gamma\beta} \right)^{1/2} 2r_b \left(\frac{\Delta\lambda}{d\lambda/dt} \right)^{1/2} \int_0^{2z_p} dz_o e^{-\frac{\alpha}{v} z_o}$$

where (3-26)

$$C_{pes} = \left[\frac{\hat{Q}k_{pes}\lambda}{(E_c + E_d)A_p hc} \right]^{1/2} \frac{\tau_s \tau_b C(0^\circ)}{M_s D(f/)}$$

C. Numerical Evaluation

The comparison (s/n) - that is the (s/n) based on only the convolution $G_S * G_D$ is shown in Fig. 9 for the photographic case. The solid curves are based on Eq. (3-24) with the $k_p^{1/2}$ value was estimated to about 0.55 from lineshape plots similar to Figs. 6-8. The individual points represent complete computer evaluations of (s/n), including all effects. Some scatter results from the coarse integrating grid used in the computation. The points for $\Delta\lambda/\lambda_0 \gamma\beta$ of 3×10^{-3} or greater includes the vignetting from a Faraday cup since the required deflection angle is rather large. Fig. 9 uses units of B,

$$B = \frac{v}{\alpha} \left[\frac{\lambda}{hc} \frac{\hat{Q} t}{D(E_c + E_d) M_s^3 (f/)^3} \right]^{1/2} C(\theta) \tau_s \tau_b \quad (3-27)$$

The v/α term from the integration over z is included in the units so that in Fig. 9 the area under the total decay curve is normalized to 1. This facilitates the comparison of curves with differing v/α values. We are assuming $r_b = 2\text{mm}$, corresponding to the side-on curves computed earlier.

The results for the side-on case are shown in Fig. 9 as the **lower** curves, with the units as defined in section two. The only differences are in $C(\theta)$ and the possible continuum term E_c present in the end-on case. $C(\theta)$ may be assumed constant for the purpose of comparing the systems.

The continuum radiation E_c , although it may have a large integrated intensity, has such a low intensity per unit wavelength that it is rarely observed at all, and may usually be neglected. This point will be discussed in more detail in the next section.

In any event, the superiority of the end-on lens is clearly evident; (s/n) is at least a factor of 10 higher at any given resolution. Since $\hat{Q}$ is a function of irradiance, comparison of the curves for photographic detection must be done with some care, but the general trend is obvious.

The (s/n) results for photoelectric detection from Eqs. (3-25) and (3-26) are shown in Fig. 10 as the solid line. Again, the side-on case is shown as the broken lines. The units for the end-on case are,

$$S = \frac{v}{\alpha} \left[\frac{\lambda}{hc} \frac{\hat{Q} t}{(E_c + E_d) A_p} \right]^{\frac{1}{2}} \frac{\tau_s \tau_b C(\theta)}{M_s D(f/)} \quad (3-28)$$

and the same assumptions made earlier are used in plotting the side-on units except $k_p^{\frac{1}{2}}$ is estimated to be about 0.7 because of the different definition. For photoelectric detection, $\hat{Q}$ is nearly independent of irradiance and flux, especially for weak signals so that the comparison of the curves in Fig. 10 is straightforward.

Figure 10.

Plot of $(s/n)_p \times 1/S$ for the end-on lens system, with the side-on results repeated for comparison. Eq. (3-25) is used here, since the computer results for $(s/n)_p$ indicated the approximations used for Eqs. (3-24 to 26) to be adequate. The scale factor S is given by Eq. (3-28).

V. Special Considerations

A. Residual Gas

Residual gas lines are not detected as easily with the end-on system as with the side-on system since the slanted line characteristic of beam emission is not present. Residual gas lines may be detected by taking two exposures at slightly (10%) different energies. The lines originating in the beam will change their observed wavelength by a small but observable amount ($1-5 \text{ \AA}$ in the visible region), whereas the residual gas lines will not. The change in energy required is not large enough to significantly change the observed spectra. This procedure has been employed with success by the authors.^{3, 8} An additional advantage is that any accidental blend of beam and residual gas emission will be revealed.

B. Continuum

Continuum emission is much more obviously present with the end-on system since the incandescent spot on the foil produced by the beam impact is viewed directly by the system. Bashkin⁹ has examined the spot and found the emission to be a continuum. The continuum source is probably thermal since this author has observed it to appear increasingly reddish as the beam current decreases, but some additional mechanisms were also suggested by Bashkin. Also, scattered ions

from the beam may strike the quartz window of the chamber, producing a fluorescence band about $1000 \text{ \AA}$ wide centered on $4600 \text{ \AA}$. Neither source, although visually overwhelming the light from the ion beam itself, has significant radiant power per unit wavelength interval. Computation of the brightness of either source is usually impossible. The authors have surveyed He, C, and N spectra^{3, 8, 10} with end-on systems, with beam currents from $3 \mu\text{A}$ (He^+) to $0.6 \mu\text{A}$ (C,N) and seen no evidence of continuum, except near the fluorescence peak at $4600 \text{ \AA}$, and never enough to affect (s/n). Others⁴ have used the system and have not reported any difficulty.

Even when detectable continuum is present, it will not strongly affect the (s/n) performance of the system. The discussion near the end of IV of section one is particularly relevant here.

The band emission from the quartz may be eliminated by using a 45° solid aluminum mirror to turn the beam emission through 90° , as shown in Fig. 11.[†] The scattered beam strikes only the mirror, and no fluorescence is produced. This system may also overcome the problem of radiation damage affecting the short wavelength transmission of the quartz window. This has been a minor problem in the author's work, necessitating the frequent replacement of quartz windows. It implies

[†]This is just the arrangement used by Denis et. al.⁴, although in their case the purpose was to "fold" the system to overcome space limitations.

Figure 11.

Diagram of modified end-on lens system using 45° mirror to avoid quartz fluorescence. The evaporator may be included to renew the mirror surface if it is degraded by radiation damage.

that the spectra below $3000 \text{ \AA}$ should be examined first with a new window before too much total exposure is received. No problem seems to be present above $3000 \text{ \AA}$ as long as all Faraday cup surfaces exposed to the beam are of tantalum. Sputtering of other materials such as brass produces a broad band absorbing coating on the window. If the reflecting properties of the aluminum mirror are degraded by the beam, it is relatively simple to arrange an evaporator in the chamber as shown to renew the surface as required.

C. Wavelength Measurement

The side-on lens presents a different set of problems for wavelength measurement than the end-on lens. As can be seen by looking at Eq. (1-1), the end-on lens is insensitive to slight errors in the mean value of the θ collected, $\bar{\theta}$, but is quite sensitive to variation in β , whereas the opposite is true in the side-on case. The primary problem for the end-on lens is to determine the exact value of β so that λ_0 may be computed from λ . The best approach again is essentially that used by Ives and Stillwell¹¹ and is shown in Fig. 12. A plane mirror is used as the foil holder for a small foil or foils. The mirror produces a virtual image of the beam going away from the foil holder and emitting light with a red shift. To a good approximation, the centroid of the red-shifted and blue-shifted line is just $\lambda_0 \gamma$.

Figure 12.

Geometry of system for measurement of the true emitted wavelength λ_0 . This is similar to that used by Ives and Stillwell, ref. 11.

A few precautions must be observed. First, z_p should be zero, that is the lens focused on the foil, so that the real and virtual images occupy equal parts of the depth of field. Second, a detailed lineshape computation should be made for both the red- and blue-shifted lines. Because the apparent position of the virtual beam emitting red-shifted light is further from the lens, the red-shifted line will show better resolution than the blue. Recall that the narrower line will have its centroid closer to the limit, in this case $\theta=\pi$ reflected to $\theta=0$. The average wavelength (centroid) of the red and blue lines will therefore be somewhat to the red or long wavelength side of $\lambda_0\gamma$ and must be corrected in the analysis.

It is often possible⁸ to use the reflection from the foil itself rather than introducing a mirror foil holder. The foil will often reflect well enough that several of the stronger lines may be measured to make a β determination for all lines. Conversely, the foil should be inclined slightly to the beam axis to avoid accidental red-shifted lines when they are not wanted. If a mirror foil mount must be used, the presence of the less reflective foil must be included in the lineshape calculation.

This and similar procedures for absolute alignment are tedious and difficult. A simpler procedure is to identify several lines in the beam-foil spectrum with lines previously measured in other sources, using the approximate value of

β by the analysing magnet of the accelerator. The identifications may then be used to calculate the true value of β . This may not always be possible, and there is always the danger of error due to misidentification.[†]

Once the point where $\lambda = \lambda_0 \gamma$ is determined, the accuracy of wavelength measurement will depend on the full width at half maximum (FWHM) of the line, and the precision of determining the line center. The line center determination is affected by any asymmetry in the lineshape, but as indicated in Table 1, this is not a serious problem. The much greater (s/n) of the end-on system will usually more than compensate, either directly since strong lines are more easily measured than weak ones, or by allowing the use of higher resolutions, so that the absolute value of the percentage error is reduced. The ultimate limit on resolution is the irreducible minimum FWHM imposed by scattering in the foil. The effect is not well understood.

As discussed in V of section one, little is known as to the relative importance of β and θ spread produced by the foil. Also the energy regulation of the accelerator contributes to the β distribution. The combined β spread does not seem to be any more serious a limit than θ scattering,

[†]For example, L. Brown et. al.¹² obtained sodium spectra where only the D lines could be identified with earlier work.

and good resolution (better than $0.75 \text{ \AA}^{\circ}$ FWHM) has been achieved for 1 MeV carbon.⁸ Much more experimental work is required before any general conclusions may be made.

In practice, the resolution in an experiment is more often determined by (s/n) limitations due to reciprocity failure of plates or limitations on operating time. The higher (s/n) attainable with the end-on system, when compared with the side-on system, is thus a significant advantage for high resolution. This is clearly evident when the resolution of the side-on and end-on curves in Figs. 9 and 10 are compared at equal (s/n) values.

D. Lifetime and Similar Measurements

Lifetime and similar measurements which require resolution of light emitted from the beam at different distances from the foil are not possible with the end-on system.

E. Relative Intensity Measurements

Relative intensity measurements fall into two classes, as discussed in section two. The first type, relative intensity at $z_0=0$, cannot be done with the end-on system. The second, relative total intensity, may be done fairly easily. The end-on system has much less severe problems with this measurement than the side-on system because of the automatic integration, but errors are less easily corrected than in

the side-on system when problems exist. Table 2 shows the irradiance at the center of the beam image for a range of resolutions and decay lengths. Note that for v/α below a certain value which is dependent on $\Delta\lambda/\lambda_0\gamma\beta$, there is no error in relative total intensity, and the error increases rather slowly for larger v/α . The range of v/α values which give undistorted total intensities increase as $\Delta\lambda/\lambda_0\gamma\beta$ decreases.[†] Thus relative total intensity measurements should be made with systems having the narrowest Doppler distribution consistent with adequate (s/n), even if this resolution is better than that of the other contributing distributions. This high resolution will also assist in resolving blends which frequently cause serious problems in relative intensity measurements.

F. Interference-Type Analysers

The end-on system is not well suited to use with Fabry-Perot or Fourier transform spectrographs because of the change of wavelength with the angle of incidence. This means that each interference order is produced by light of a different wavelength, which gives rise to serious errors in wavelength measurement. This problem has already

[†]As long as this is done by moving the lens back. Masking does not improve relative intensity measurements and, as discussed earlier, loses (s/n), and thus should be doubly avoided with the end-on system.

TABLE 2

Variation of Entrance Slit Irradiance with v/α

	FWHM, $\Delta\lambda/\lambda_O \gamma\beta$			
	1×10^{-6}	3×10^{-6}	1×10^{-5}	3×10^{-5}
$v/\alpha = 1$	5.14	5.17	5.22	4.46
$v/\alpha = 5$	5.33	5.47	5.24	3.33
$v/\alpha = 10$	5.51	5.14	4.14	2.16
				1×10^{-4}
				5.09
				2.76
				1.63

Entrance slit irradiance for different systems showing different resolution as a function of v/α . Note the desirability of using high (Doppler) resolution systems for relative intensity measurements. The variations in irradiance result from limited depth of field and the choice of the focus given by Eq. (3-16).

been considered for much lower velocity beams and the side-on configuration.¹⁴ The axicon system discussed in the next paper in this series does not show this effect, and, when used with a Fabry-Perot, has a (s/n) comparable to the end-on lens used with a slit type instrument. We will not consider the use of the end-on lens with an interference analyser. However, the effective gain of 10 in throughput¹⁵ may be sufficient to justify the use of a Fabry-Perot or Fourier transform instrument in certain circumstances despite the increased complexity of data analysis. Predispersion must be used to avoid foil and quartz continua.

G. Ultraviolet Systems

The extension of the end-on lens system into the vacuum-ultraviolet by the use of normal or grazing incidence reflection optics is fairly obvious. As an illustration, the system nearing completion in the author's lab is shown in Fig. 13. This system was used for the lineshape calculations presented in Figs. 6-8.

Figure 13.

Diagram of the vacuum ultraviolet end-on collecting system designed by the author. This system was used in the calculation of Figs. 6-8.

SECTION FOUR: THE AXICON

I. Introduction

The term axicon refers to a family of optical devices described by McLeod.¹ These devices have the general property that a ray passing through the optic axis at an angle ω and incident upon the axicon will be transformed to some other angle ω' and focused back through the axis.

The version described by Bickel² and for beam-foil spectroscopy attributed to Meinel³, shown in Fig. 1, has $\omega=90^\circ$ and $\omega'=0^\circ$. A lens is placed after the axicon so that their optic axes are collinear. The lens focuses rays that are parallel to the common axis ($\omega'=0^\circ$) to the secondary focal point. Light which leaves the axicon at a small angle relative to the axis will fall on a ring of radius $\rho=f\delta$ centered on the secondary focal point. If the system is now placed with the optic axis coincident with the ion beam axis, light emitted by the beam with $\theta=90^\circ$ and wavelength $\lambda_{O\gamma}$ will fall on the secondary focus, and light emitted at some angle $\theta=90^\circ\pm\delta$ and wavelength $\lambda_{O\gamma}(1\pm\beta\delta)$ will fall on the ring of radius $\rho=f\delta$. In principle, the wavelength spread of the light may be restricted by placing a suitable aperture at the secondary focus. Since it collects over all θ , one would expect that all light emitted into the region λ to $\lambda+\Delta\lambda$ is collected. That is, the system collects $\Delta\Omega_{\max}$

Figure 1

Geometry of the axicon based collecting system as described by Bickel in ref. 2.

$$\Delta\Omega_{\max} = 2\pi \frac{\Delta\lambda}{\lambda_0 \gamma \beta} \quad (1-3)$$

as did the end-on lens discussed in section three of this thesis. In practice, the beam has finite size, and not all the light emitted in $\Delta\theta=2\delta$, and thus in $\Delta\Omega_{\max}$, is collected. Light emitted into a cone of angle δ about a perpendicular to the axicon axis will fall on a ring of radius $f\delta$ about the focus. In particular, light emitted at an angle $\theta=90^\circ$ relative to $\vec{v}$ but a distance $r=R\delta$ from the axis will be treated the same as light emitted from the axis at $\theta=90^\circ \pm \delta$ relative to $\vec{v}$. R is the radius of the axicon at the perpendicular. The collection efficiency may be improved by increasing R for a given beam diameter, but for a fixed $(f/)$ for the analysing instrument, the focal length of the lens, f , must also be increased. The light from the desired wavelength interval will then be spread over a larger area and the irradiance at the slit will be decreased. A balance between collection efficiency and spot size must be made to optimize the (s/n) available with the axicon.

II. Irradiance and Doppler Profile

One of the more interesting and useful properties of axicon based systems is that the Doppler distribution profile is independent of vignetting and the angle of incidence on the entrance aperture of the analysing instrument. Because of this, axicon systems are particularly well suited

to use with interference-type analysers such as Fabry-Perot or Fourier transform spectrographs. We will perform the analysis for a conventional slit instrument, since these are generally more convenient for survey work, and for comparison with the other systems. Interference-type analysers will be discussed later.

A. Analysing Instrument

We shall assume that a stigmatic spectrograph or monochromator is used to analyse the light emitted by the beam, and that the instrument has an effectively square grating or prism which is not vignetted by the auxiliary internal optics. The focal ratio of the entrance optics, $(f/)$, will be taken to correspond to the circular cone of light which will completely fill the grating, and is thus numerically less than the effective focal ratio. The reciprocal linear dispersion $D(\lambda)$ may be assumed constant over a narrow range of wavelengths.[†] The slit dimensions will be denoted by W wide and L long, and the internal magnification by M_s .

[†]We have chosen to separate the linear reciprocal dispersion $D(\lambda)$ (eg in $\text{\AA}/\text{cm}$, measured in the image plane) from the internal magnification M_s and introduce λ dependence so that the calculations will hold for prism instruments.

B. Entrance Slit Irradiance and Doppler Distribution

We proceed by analysing Eq. (1-4)

$$E_s(d\sigma_o) = \frac{C(90^\circ)}{4\pi} \tau_b \int_0^\theta \sin\theta \, d\theta \int_\phi d\phi \iiint \frac{dV(\sigma_o)}{d\sigma_o} e^{-\frac{\alpha}{V} z_o} \quad (1-4)$$

The volume element may be found by considering Fig. 2. The figure shows a single perpendicular to the axis, with a cone of angle δ surrounding it. The light falling on the point of the axicon defined by the perpendicular, with an angular deviation δ to $\delta+d\delta$, originates from the indicated element of volume. We approximate this volume by a cylindrical shell of radius $R\delta$, thickness $R \, d\delta$, and height $2(r_b^2 - R^2 \delta^2 \sin^2 \psi)^{1/2}$, where r_b is the beam radius and ψ is defined in Figs. 2 and 4.

$$dV = 2R\delta d\delta \int_0^{2\pi} (r_b^2 - R^2 \delta^2 \sin^2 \psi)^{1/2} d\psi \quad (4-1)$$

The corresponding element of solid angle is $d\Omega \approx \frac{dz_o d\phi}{R}$ and the area of the image of this element is $d\sigma_o = 2\pi f^2 \delta d\delta$. Substituting this result into Eq. (1-4) and noting that the θ and ϕ integrals give $d\Omega$

$$dE_s = \frac{R\tau_b C(90^\circ)}{4\pi^2 f^2} e^{-\frac{\alpha}{V} z_o} dz_o d\phi \int_0^{2\pi} (r_b^2 - R^2 \delta^2 \sin^2 \psi)^{1/2} d\psi \quad (4-2)$$

Figure 2.

Geometry of the axicon system defining parameters used in the analysis leading to Eq. (4-3).

Figure 3.

Geometry of the modified axicon system. The axicon collects most efficiently at the maximum radius R , and thus the maximum radius should be placed at $z_0 = 0$ where the beam radiates most strongly, as shown here.

This differential contribution to the irradiance is proportional to the local radius R , which implies that the system collects most efficiently at the maximum radius R_m . Since the beam radiates most strongly near the foil, the axicon arrangement shown in Fig. 3 is more efficient for beam-foil spectroscopy than that shown in Fig. 1. We will analyse this modified system since it sets an upper limit to the performance of any axicon system. For the modified system, $R=R(z_o)=z_m+r_f-z_o$, where the Faraday cup radius is denoted by r_f . The total irradiance is given by Eq. (4-2) integrated over all z_o and ϕ . For $R\delta < .5 r_b$, we may replace the integrand of Eq. (4-2) by r_b and introduce less than a 10% error. Thus

$$E_s = \frac{\tau_b C(90^\circ) r_b}{f^2} \int_0^{z_m} (z_m+r_f-z_o) e^{-\frac{\alpha}{v} z_o} dz_o \quad (4-3)$$

$$= \frac{C(90^\circ) r_b \tau_b}{8 [(z_m+r_f)f]^2} \frac{v}{\alpha} [z_m + (r_f - \frac{v}{\alpha}) (1 - e^{-\frac{\alpha}{v} z_m})]$$

$$r_b (f/) > \rho$$

E_s will drop rapidly for larger values of ρ .

To find the wavelength distribution of E_s we must find the relation between ψ and λ and evaluate Eq. (4-2) in terms of λ . Defining $\eta = \pi/2 - \theta$ in the plane formed by the optic

[†]J. O. Stoner, Jr.³, has done a somewhat simpler lineshape analysis using a similar procedure. My result, Eq. (4-7), reduces to his result for his special assumptions.

axis and the perpendicular of Fig. 2, Fig. 4 shows

$$\psi \approx \cos^{-1} \frac{\eta}{\delta} \quad (4-4)$$

From Eq. (1-1), η is related to λ by

$$\eta \approx \frac{1}{\beta} \left(1 - \frac{\lambda}{\lambda_{OY}}\right) \quad (4-5)$$

Now note that both $\pm\psi$ contribute at the same λ and

$$\sin\psi = \left[1 - \frac{1}{\beta^2 \delta^2} \left(1 - \frac{\lambda}{\lambda_{OY}}\right)^2\right]^{\frac{1}{2}} \quad (4-6)$$

$$d\psi = [\lambda_{OY}^2 \beta^2 \delta^2 - (\lambda_{OY} - \lambda)^2]^{-\frac{1}{2}} d\lambda$$

Then substituting into Eq. (4-2) and integrating over z_O and ϕ rather than ψ , we have

$$E_S(\rho, \lambda) d\lambda = \frac{\tau_b C(90^\circ)}{\pi f^2} \int_0^{z_m} dz_O e^{-\frac{\alpha}{V} z_O} \frac{fR\{r_b^2 - (\frac{R}{f})^2[\rho^2 - \frac{f^2}{\beta^2} (1 - \frac{\lambda}{\lambda_{OY}})^2]\}}{[\lambda_{OY}^2 \beta^2 \rho^2 - f^2(\lambda_{OY} - \lambda)^2]^{\frac{1}{2}}},$$

for

$$r_b > \frac{R}{f} [\rho^2 - (\frac{f}{\beta})^2 (1 - \frac{\lambda}{\lambda_{OY}})^2]^{\frac{1}{2}}$$

$$= 0, \text{ otherwise} \quad (4-7)$$

This expression must be evaluated numerically because of the dependence on $R=(z_m+r_f-z_O)$ of the validity condition on Eq.

(4-7).

Figure 4.

Geometry used in the derivation
of Eq. (4-7).

III. Convolutions and Lineshape

The observed lineshape in the detector plane must be computed by convoluting the Doppler irradiance distribution G_D with the other distributions contributing to the final lineshape. As in the preceeding sections of this thesis, we will consider only the Doppler and entrance slit distribution, given respectively by Eqs. (4-7) and (2-9)

$$G_S = \frac{1}{M_S W} \quad , \quad \xi - \xi_0 \leq \frac{M_S W}{2} \quad (2-9)$$

As before, ξ is the coordinate in the direction of dispersion and ζ is the perpendicular coordinate. Fig. 5 plots G_D from Eq. (4-7) and the convolution $G_S * G_D$.

Now in evaluating (s/n) for a general axicon system it is desirable to avoid evaluating $G_S * G_D$ in detail by computer. Looking at Fig. 5, we can make a couple of useful observations.

1. Since G_D is much narrower than G_S near $\rho=0$, the convoluted lineshape $G_S * G_D$ strongly resembles G_S , a general property of convolutions noted in section one. We may approximate E_p by Eq. (2-12b)

$$E_p(0, \zeta) = E_S(\zeta/M_S) \left[\frac{\tau_S}{M_S} \right] \quad (2-12b)$$

2. As the full width at half maximum of G_D , $\text{FWHM}(G_D)$, approaches $\text{FWHM}(G_S)$ $G_S * G_D$ converges rapidly to the central limit theorem Gaussian, Eq. (1-10). As

Figure 5.

Oblique plot of the entrance slit irradiance per unit wavelength interval $E_s(\rho, \lambda)$ is shown on the left, and the spectrograph image plane irradiance $E_p(\xi, \zeta)$ is shown on the right. Since the total entrance slit irradiance is fixed and $\Delta\lambda \rightarrow 0$ as $\rho \rightarrow 0$, $E_s \rightarrow \infty$ but $E_p \rightarrow$ constant.

FWHM(G_D) exceeds FWHM(G_S), $E_p(\xi, \zeta)$ drops rapidly, again a general property of convolutions and the normalization discussed in III of section one.³

The point where FWHM(G_D)=FWHM(G_S) for $\lambda/D = M_S W$ is

$$\rho_m = \frac{f \Delta \lambda}{2 \lambda_o \gamma \beta} \quad (4-8)$$

Since $G_S * G_D$ in Fig. 5 starts to fall rapidly at this point, one is led to suspect that if Eq.(4-8) is used for ρ , Eq. (4-3) still holds. This is generally the case since substituting Eq.(4-8) in the condition on Eq.(4-3) gives

$$\frac{r_b}{R} \geq \frac{\Delta \lambda}{\lambda_o \gamma \beta} \quad (4-9)$$

For example, if $R=10$ cm, $r_b=2$ mm, $\Delta \lambda / \lambda_o \gamma = 2 \times 10^{-4}$ (1 Å at 5000 Å), the relation of Eq. (4-9) holds for $\beta > 0.01$.

For approximate (s/n) calculations we will assume that E_p is given by Eq. (2-12b), E_s by Eq. (4-3), and the appropriate range of ζ is defined as $2 M_S \rho_m$, ρ_m given by Eq. (4-8). This will underestimate the (s/n) if Eq. (4-9) is satisfied and overestimate (s/n) if it is not. This seems to be a reasonable compromise and should give (s/n) within $\pm 30\%$ of the value given by a detailed computer evaluation.

IV. Signal-to-Noise

A. General

We will use the threshold Eqs. (1-18) and (1-21) to

calculate (s/n) . There is generally no significant continuum, so that Eq. (1-18) has the same form as for the side-on lens:

$$(s/n)_p = \left(\frac{\hat{Q}\lambda}{hc} \right)^{1/2} \left[\iint \frac{E_p^2(\xi - \xi_o, \zeta) t}{E_d(\xi - \xi_o, \zeta)} \right]^{1/2} \quad (2-13)$$

Since E_p is separable in the two coordinates, we will again approximate the lineshape by a pulse function with the same FWHM and peak height. We then replace

$$\int_{-\infty}^{\infty} E_p^2(\xi - \xi_o, \zeta) d\xi$$

by

$$k_p E_p^2(0, \zeta) \frac{\Delta\lambda}{D}$$

where

$$k_p = \left(\frac{D}{\Delta\lambda} \right) \int_{-\infty}^{\infty} \frac{E_p^2(\xi - \xi_o, \zeta)}{E_p^2(0, \zeta)} d\xi$$

Now when we made this definition earlier, k_p was independent of ζ which made it a much more useful quantity. For the axicon based system, $k_p^{1/2}$ varies from about .6 at $\zeta = M_s \rho_m$ to 1 at $\zeta = \rho = 0$. This definition of k_p , justified by the central limit theorem, has allowed us to eliminate most of the detailed properties of G_D to facilitate comparisons between various collecting systems. We will retain it by

taking an average over ζ .

$$\bar{k}_p = \frac{D}{\Delta\lambda} \frac{1}{2M_{s\rho m}} \int_{-M_{s\rho m}}^{M_{s\rho m}} d\zeta \int_{-\infty}^{\infty} d\xi \frac{E_p^2(\xi - \xi_0, \zeta)}{E_p^2(0, \zeta)}$$

Then for the photographic case, noting that E_d is independent of ξ and ζ ,

$$(s/n)_p = \left[\frac{\hat{Q} \bar{k}_p \lambda}{E_d h c} \right]^{1/2} \left[\frac{\Delta\lambda}{D} \right]^{1/2} \left[t \int_{-M_{s\rho m}}^{M_{s\rho m}} d\zeta E_p^2(0, \zeta) \right]^{1/2} \quad (2-14)$$

Similarly Eq. (1-20) for incremental photoelectric scanning may be written

$$(s/n)_{pe} = \left[\frac{\hat{Q} \bar{k}_{pe} \lambda}{E_d A_p h c} \right]^{1/2} M_{sW} t^{1/2} \int_{-M_{s\rho m}}^{M_{s\rho m}} d\zeta E_p(0, \zeta) \quad (2-15)$$

where

$$k_{pe}^{1/2} = \frac{1}{2M_{s\rho m}} \frac{1}{M_{sW}} \int_{-M_{s\rho m}}^{M_{s\rho m}} d\zeta \int_{\xi_1}^{\xi_1 + M_{sW}} d\xi \frac{E_p(\xi - \xi_0, \zeta)}{E_p(0, \zeta)}$$

and Eq. (1-21), for continuous scanning, at a rate $(d\lambda/dt)$

$$(s/n)_{pes} = \left[\frac{\hat{Q} \bar{k}_{pes} \lambda}{E_d A_p h c} \right]^{1/2} M_{sW} \left[\frac{\Delta\lambda}{d\lambda/dt} \right] \int_{-M_{s\rho m}}^{M_{s\rho m}} d\zeta E_p(0, \zeta) \quad (2-16)$$

where

$$\bar{k}_{pes}^{1/2} = \frac{(d\lambda/dt)}{2M_{s\rho m}^2 (\Delta\lambda) W} \int_{-M_{s\rho m}}^{M_{s\rho m}} d\zeta \int_{-\infty}^{\infty} dt \int_{\xi_1(t)}^{\xi_1(t) + M_{sW}} d\xi \frac{E_p(\xi - \xi_0, \zeta)}{E_p(0, \zeta)}$$

B. (s/n) for the Axicon System

We may proceed to calculate the (s/n) relations for the axicon based systems, substituting Eqs. (4-3), (2-12b), and (4-8) into Eq. (2-14), we have

$$(s/n)_p = C_p \left[\frac{t \sqrt{2}}{32 M_s^3 D(f/)^3 (z_m + r_f)^3} \right]^{\frac{1}{2}} r_b \left(\frac{\Delta \lambda^2}{\lambda_o \gamma \beta} \right)^{\frac{1}{2}} \frac{v}{\alpha} \\ [z_m + (r_f - \frac{v}{\alpha}) (1 - e^{-\frac{\alpha}{v} z_m})] \quad (4-10)$$

$$C_p = \left[\frac{\lambda \hat{Q} \bar{k}_p}{hc E_d} \right]^{\frac{1}{2}} C(90^\circ) \tau_s \tau_b$$

The $(s/n)_p$ may be maximized with respect to z_m in a numerical calculation to give the balance between image size and collection efficiency discussed earlier. For the photoelectric case, Eqs. (4-3), (2-12b) and (4-8) substituted in Eq. (2-15) give

$$(s/n)_{pe} = C_{pe} \left[\frac{r_b t^{\frac{1}{2}} \sqrt{2}}{4 (z_m + r_f)} \right] \left[\frac{\Delta \lambda^2}{\lambda_o \gamma \beta} \right] \frac{v}{\alpha} \\ [z_m + (r_f - \frac{v}{\alpha}) (1 - e^{-\frac{\alpha}{v} z_m})] \quad (4-11)$$

$$C_{pe} = \left[\frac{\hat{Q} \bar{k}_{pe} \lambda}{E_d A_p hc} \right]^{\frac{1}{2}} \frac{\tau_s \tau_b C(90^\circ)}{M_s D(f/)}$$

and from Eq. (2-16)

$$\begin{aligned}
 (s/n)_{\text{pes}} &= C_{\text{pes}} \left[\frac{r_b \sqrt{2}}{4(z_m + r_f)} \right] \left[\frac{\Delta \lambda}{d\lambda/dt} \right]^{\frac{1}{2}} \left[\frac{\Delta \lambda^2}{\lambda_o \gamma \beta} \right] \\
 &\quad \frac{v}{\alpha} [z_m + (r_f - \frac{v}{\alpha}) (1 - e^{-\frac{\alpha}{v} z_m})] \\
 C_{\text{pes}} &= \left[\frac{\hat{Q}_{\text{pes}}}{E_d A_p h c} \right]^{\frac{1}{2}} \frac{\lambda^{\frac{1}{2}} \tau_s \tau_b C(90^\circ)}{M_s D(f/)}
 \end{aligned} \tag{4-12}$$

The maximum with respect to z_m usually occurs for impractically large values (> 1 m) of z_m and correspondingly long slits.

C. Numerical Evaluation

Numerical evaluation of (s/n) for the axicon is difficult even for the simplified expressions Eqs. (4-10) and (4-12) because of the optimization with respect to z_m , which is strongly dependent on (v/α) of the transitions, since this will be different for different lines.

To illustrate the general characteristics of axicon systems, we present curves showing the (s/n) available as a function of resolution. In this example, the source beam is assumed to be radiating at $\lambda_o = 5000 \text{ \AA}$ with $\alpha = 3 \times 10^8$. For the lens cases, $T = H = 0.30$ cm, and for the axicon system, $r_b = 0.17$ cm. Calculations are made for $\beta = 0.1$ and $\beta = 0.01$. The (s/n) units for the

photographic case (Fig. 6) are

$$B = \frac{v}{\alpha} \left[\frac{\lambda}{hc} \frac{\hat{Q} t^{\frac{1}{2}}}{E_d^{DM} S^3 (f/)^3} \right] C(90^\circ) \tau_s \tau_b \quad (4-13)$$

where v/α is included to normalize the area under the decay curve to 1. Eq. (4-10) is used in Fig. 7 to plot $(s/n)_{pe}$ with the units

$$S = \frac{v}{\alpha} \left[\frac{\lambda}{hc} \frac{\hat{Q} t^{\frac{1}{2}}}{E_d^A p} \right] \frac{\tau_s \tau_b C(90^\circ)}{M_s D(f/)} \quad (4-14)$$

Since $(s/n)_{pes}$ exhibits the same behavior as $(s/n)_{pe}$ except for $[\Delta\lambda/(d\lambda/dt)]^{\frac{1}{2}}$ replacing $t^{\frac{1}{2}}$, it will not be plotted.

The length of the beam z_m examined by the axicon system in the photographic case (curves C of Fig. 6) is 1.98 cm for $\beta = 0.01$ and 20.59 cm for $\beta = 0.1$. These are the optimum values. For the photoelectric case, z_m is taken to be 10 cm for both values of β , since the optimum values are impractically large. In all axicon cases, $r_f = 1.0$ cm. Comparison curves for the end-on and side-on lens systems are also plotted in Figs. 6 and 7.

Because the (s/n) is little improved over the side-on lens, and because the axicon shares all the problems of limited usefulness for special experiments with the end-on lens, we conclude that the axicon is of little value for use with a slit spectrograph.

V. Modifications

A. Interference-Type Analysers

The axicon system has the singular characteristic that the wavelength of the light falling on the entrance aperture of the analysing instrument is independent of the angle of incidence. The axicon is thus the only system that may be used conveniently with Fabry-Perot or similar interference-type analysing instruments. Since Fabry-Perot type instruments have a throughput about 10 x as great as grating instruments⁴, the (s/n) will become comparable with the end-on lens. Stoner³ has described the use of the axicon with the Fabry-Perot.

B. Vacuum Ultraviolet

The extension of the axicon and Fabry-Perot system into the vacuum ultraviolet may be done by using a modified axicon design first described by the author⁵ and discussed by Jordan⁶, together with a thin-film Fabry-Perot described by Stoner.⁷ This system is diagrammed in Fig. 8.

Figure 8.

Geometry of a collecting system with the function of the axicon and lens combined. The system uses a segment of the surface generated by rotating a parabola around its latus rectum,

SECTION FIVE: A STUDY OF CARBON SPECTRA USING AN END-ON LENS SYSTEM

I. Introduction

The objective of the theoretical study contained in the preceeding sections was to develop optical systems which would overcome the (s/n) and resolution problems encountered in the early experiments done at Rice with the side-on system. This study led to the development of the end-on lens system described in section three of this thesis, which showed significant theoretical gains in (s/n) and resolution while retaining experimental simplicity. Preliminary experiments with the first end-on lens system indicated that the theoretical gains could be achieved in practice.^{1, 2} A co-investigator at Rice³ employed the same apparatus very successfully in a study of the beam-foil excited spectrum of nitrogen.

The author then undertook the first study of the beam-foil excited spectrum of carbon. In this work a 0.4 μ A beam of C^+ at 0.90 and 1.05 MeV incident upon a thin carbon foil was used as a light source. The spectra from the excited carbon beam showed 111 distinct emission lines, 102 of which have been identified with known lines in C II⁴, C III⁵, and Carbon IV.⁶ Nine of the observed lines from the carbon ion beam have not previously been reported.

II. Experimental Details

A conventional radio-frequency arc in CO gas provided a 6-8 μA beam of C^+ to the Rice University 5.5 MV Van de Graaff accelerator. (attempts to obtain suitable ion beams using CH_4 in the arc proved unsatisfactory.) The energy, and the energy stability, of the accelerated beam was limited by the 90° turning magnet. Beams with a nominal energy of 0.90 and 1.05 MeV were used, although the energy had to be determined for each spectroscopic plate, as is described below.

After magnetic analysis, a collimated beam of 0.4-0.5 μA of C^+ entered the experimental chamber. (see Fig. 1) The beam was ionized and electronically excited as it passed through the thin (nominal area density $\approx 10 \mu\text{g}/\text{cm}^2$) carbon foil. As the excited ions continued to the Faraday cup, they decayed, emitting the observed spectra.

The observed emission was collected by a fused silica lens in the end-on lens configuration, and focused upon the spectrograph entrance slit. The spectrograph used in the experiment was a Jarrell-Ash 1 m Czerny-Turner instrument with a 590 1/mm grating, blazed at $10,000 \text{ \AA}$ in first order. The spectrograph was modified for use with an RCA C33034 two-stage-cascade image-intensifier tube.⁷ The image tube has an S-20 photocathode and a quartz face-plate so that its spectral response is in the region

Figure 1.

Schematic diagram of the apparatus.

8000 Å-2000 Å. The tube has a P-11 phosphor, which is peaked at 4600 Å, and Kodak Ia0 plates were used to record the intensified images. The focus of the image tube exhibited drift because unregulated power supplies were used.

The large Doppler effects inherent in the high-velocity ($v \approx 0.01 c$) ion beam source are described by Eq. (1-1):

$$\lambda = \lambda_0 \gamma (1 - \beta \cos \theta) \quad (1-1)$$

where λ is the observed wavelength, λ_0 is the emitted wavelength, $\beta = v/c$, $\gamma = (1 - \beta^2)^{-1/2}$, and θ is the angle of emission with respect to the ion beam direction. Because the end-on system was used, the data obtained in this apparatus had to be corrected for both the first and second order Doppler shifts. The spread in collected values of $\cos \theta$ defines a Doppler linewidth. The optical system used was designed to give a Doppler linewidth (FWHM) of ~ 0.6 Å at 5000 Å. The best spectra exhibited a linewidth of 0.75 Å, while spectra obtained during periods of severe turning-magnet or image-tube-focus drift exhibited linewidths as large as 2 Å.

III. Data Analysis

Spectral data were obtained on 36 plates, 18 at each incident ion energy. Each plate was exposed for 20 minutes using a 0.4-0.5 $\mu A C^+$ beam current. The 18 plates covered

the region from $7000 \text{ \AA}$ to $14,000 \text{ \AA}$, in first order, in 18 steps of $400 \text{ \AA}$. This coverage permitted examination of the entire response region of the image tube in one or more of the first five diffraction orders at each energy. An Fe-Ne hollow cathode discharge lamp provided a wavelength reference spectrum for all the plates.

The plates were reduced using a W. G. Pye Model 6147 two-coordinate measuring microscope. Both the carbon ion and reference spectra were measured, and the measurements punched on computer data cards. For each plate, a computer routine identified lines in the reference spectrum and fitted a fifth-order polynomial to that spectrum. The program then computed the observed wavelengths of the unknown lines.

The blue Doppler shift inherent in the end-on collecting system was determined by the foil reflection technique of section three. An approximate energy calibration indicated that a very strong observed triplet was probably the $3s^3S-3p^3P^o$ transition in C III at 4647.42 , 4650.25 , and $4651.47 \text{ \AA}$. This very strong triplet was also observed as a weak triplet in red-shifted light reflected from the foil (see Fig. 2). The average wavelengths, corrected for the second-order effect, gave the expected wavelengths to within $0.15 \text{ \AA}$, and photoelectric measurements (see Fig. 3) gave the expected 5:3:1 intensity ratio. This identification allowed us to compute the actual velocity of the carbon ions emerging from the carbon foil. Because of

Figure 2.

Plate showing the C III triplet at 4647.2, 50.25, 51.47 Å. The lines labeled blue result from light emitted toward the lens and the lines labeled red from light emitted away from the lens and reflected by the foil. In each case the Doppler blue or red shift is about 60 Å.

Figure 3.

Photoelectric scan of the C III triplet made using the Czerny-Turner spectrograph-spectrometer in the scanning mode, an EMI 6256S photomultiplier, a Kiethly 150A electrometer, and a TI Servewriter strip chart recorder

instabilities in the turning magnet and in the image-intensifier tube power supplies, a final correction was made for each plate by identifying three to five lines with known carbon ion lines and computing the beam velocity from the observed Doppler shift for those selected lines.

Several hundred lines were measured in the unknown spectrum. Some of the lines on the plates resulted from excitation of residual gas in the chamber by the ion beam. Others were very strong lines in the Fe-Ne reference spectrum which continued to phosphoresce on the image tube during the carbon exposures. The Doppler effect associated with the fast ion beam was used to identify the contaminating lines. Lines originating in the beam at 1.05 MeV were shifted a small but obvious amount ($\sim 5 \text{ \AA}$) farther blue than in the 0.90 MeV exposures, but both gave the same corrected wavelength. Residual gas or phosphorescence lines showed no such shift, and hence gave different corrected wavelengths and could thereby be identified and eliminated.

The remaining lines were assigned to the appropriate diffraction order by two methods. For the stronger lines, filters placed over the ends of the slits provided a positive determination of order. Weaker lines were assigned by examining the entire spectrum to verify that the lines appeared in all orders appropriate to the assumed wavelength. About 20% of the observed lines could not be assigned to a diffraction order and were neglected in the

further analysis.

A total of 111 lines were positively determined to have the excited carbon ion beam as their source.

IV. Results

Of the 111 carbon beam lines, 62 coincided with lines in C II (Glad 1952), 33 with lines in C III (Bockasten 1955), and 7 with lines in C IV (Bockasten 1956). No lines were observed which could be attributed to C I, C V, or C VI. The observed lines are listed in Tables 1-3.

Nine observed lines could not be identified with any previously classified carbon lines. These lines are shown in Table 4. The uncertainties assigned to the wavelength determinations were caused primarily by the inability to assign a well-defined energy because of the magnet drift, and because of drift in the image intensifier magnification. Intensities given are on a 0 to 5 scale.

TABLE 1
SINGLY IONIZED CARBON - C II

AVERAGE	S. GLAD	TRANSITION	J
7120.01	7119.90	$3p \ ^4D - 3d \ ^4F^\circ$ $3p \ ^4D - 3d \ ^4F^\circ$	$7/2-9/2$ $3/2-3/2$
7116.12	7115.63	$3p \ ^4D - 3d \ ^4F^\circ$	$5/2-7/2$
7112.90	7112.48 7113.04	$3p \ ^4D - 3d \ ^4F^\circ$ $3p \ ^4D - 3d \ ^4F^\circ$	$1/2-3/2$ $3/2-5/2$
6783.98	6783.90	$3s \ ^4P^\circ - 3p \ ^4D$	$5/2-7/2$
6780.21	6780.61 6779.93	$3s \ ^4P^\circ - 3p \ ^4D$ $3s \ ^4P^\circ - 3p \ ^4D$	$1/2-3/2$ $3/2-5/2$
6582.54	6582.88	$3s \ ^2S - 3p \ ^2P^\circ$	$1/2-1/2$
6577.66	6578.05	$3s \ ^2S - 3p \ ^2P^\circ$	$1/2-3/2$
5889.36	5889.27 5889.77	$3d \ ^2D - 4p \ ^2P^\circ$ $3d \ ^2D - 4p \ ^2P^\circ$	$3/2-3/2$ $5/2-3/2$
5662.52	5662.47	$3s \ ^4P^\circ - 3p \ ^4S$	$5/2-3/2$
5647.80	5648.07	$3s \ ^4P^\circ - 3p \ ^4S$	$3/2-3/2$
5640.64	5640.55	$3s \ ^4P^\circ - 3p \ ^4S$	$1/2-3/2$
5537.67	5537.61	$4s \ ^2S - 5p \ ^2P^\circ$	$1/2-1/2$
5534.47	5535.35	$4s \ ^2S - 5p \ ^2P^\circ$	$1/2-3/2$
5477.96	5478.59	$3d \ ^4D^\circ - 4p \ ^4D$	$7/2-7/2$
5258.83	5259.71 5259.06	$3d \ ^4F^\circ - 4p \ ^4D$ $3d \ ^4F^\circ - 4p \ ^4D$ $3d \ ^4F^\circ - 4p \ ^4D$	$5/2-3/2$ $3/2-1/2$ $7/2-5/2$
5256.67	5257.24 5256.09	$3d \ ^4F^\circ - 4p \ ^4D$ $3d \ ^4F^\circ - 4p \ ^4D$	$9/2-7/2$ $3/2-3/2$
5150.94	5151.09	$3s \ ^4P^\circ - 3p \ ^4P$	$5/2-3/2$
5145.14	5145.16	$3s \ ^4P^\circ - 3p \ ^4P$	$5/2-5/2$

C II - cont.

AVERAGE	S. GLAD	TRANSITION	J
5143.37	5143.49	$3s\ 4P^{\circ} - 3p\ 4P$	$3/2-1/2$
5138.92	5139.17	$3s\ 4P^{\circ} - 3p\ 4P$	$3/2-3/2$
5136.79	5137.26	$3s\ 4P^{\circ} - 3p\ 4P$	$1/2-1/2$
5132.95	5133.28	$3s\ 4P^{\circ} - 3p\ 4P$	$3/2-5/2$
	5132.94	$3s\ 4P^{\circ} - 3p\ 4P$	$1/2-3/2$
5035.85	5035.91	$2p^3\ 2P^{\circ} - 3p\ 2D$	$1/2-3/2$
5032.10	5032.07	$2p^3\ 2P^{\circ} - 3p\ 2D$	$3/2-5/2$
4619.34	4619.23	$3d\ 2F^{\circ} - 4f\ 2G$	$7/2-9/2$
	4618.40	$3d\ 2F^{\circ} - 4f\ 2G$	$5/2-7/2$
4411.50	4411.506	$3d\ 2D^{\circ} - 4f\ 2F$	$5/2-7/2$
	4411.163	$3d\ 2D^{\circ} - 4f\ 2F$	$3/2-5/2$
4409.96	4409.979	$3d\ 2D^{\circ} - 4f\ 4F$	$5/2-7/2$
	4409.161	$3d\ 2D^{\circ} - 4f\ 4F$	$3/2-5/2$
4374.15	4374.272	$3d\ 4P^{\circ} - 4f\ 4D$	$5/2-7/2$
4372.47	4372.487 4372.350	$3d\ 4P^{\circ} - 4f\ 2D$	$5/2-5/2$
		$3d\ 4P^{\circ} - 4f\ 4D$	$1/2-1/2$
		$3d\ 4P^{\circ} - 4f\ 4D$	$3/2-3/2$
		$3d\ 4P^{\circ} - 4f\ 4D$	$3/2-5/2$
4370.17	4370.661	$3d\ 4P^{\circ} - 4f\ 2D$	$1/2-3/2$
	4369.857	$3d\ 4P^{\circ} - 4f\ 4D$	$3/2-1/2$
4367.33	4368.047	$3d\ 4P^{\circ} - 4f\ 4D$	$3/2-3/2$
	4368.263	$3d\ 4P^{\circ} - 4f\ 4D$	$5/2-5/2$ $5/2-3/2$
4317.20	4317.260	$3p\ 4P - 4s\ 4P^{\circ}$	$5/2-5/2$
4313.08	4313.100	$3p\ 4P - 4s\ 4P^{\circ}$	$3/2-5/2$
4267.01	4267.258	$3d\ 2D - 4f\ 2F^{\circ}$	$5/2-7/2$
	4267.003	$3d\ 2D - 4f\ 2F^{\circ}$	$3/2-5/2$
4075.84	4075.851	$3d\ 4D^{\circ} - 4f\ 4F$	$5/2-5/2$
		$3d\ 4D^{\circ} - 4f\ 4F$	$7/2-9/2$

C II - cont.

AVERAGE	S. GLAD	TRANSITION	J
	4074.518	3d $4D^{\circ}$ - 4f $4F$	3/2-5/2
		3d $4D^{\circ}$ - 4f $4F$	1/2-3/2
4074.72	4074.845	3d $4D^{\circ}$ - 4f $4F$	5/2-7/2
3980.30	3980.323	3d $4D^{\circ}$ - 4f $4D$	7/2-7/2
3952.01	3952.058	3d $4F^{\circ}$ - 4f $4F$	9/2-9/2
3920.53	3920.693	3p $2P^{\circ}$ - 4s $2S$	3/2-1/2
3918.91	3918.978	3p $2P^{\circ}$ - 4s $2S$	1/2-1/2
	3880.588	3d $4F^{\circ}$ - 4f $4G$	9/2-9/2
3880.32	3879.640	3d $4F^{\circ}$ - 4f $4G$	7/2-7/2
3877.92	3878.028	3d $4F^{\circ}$ - 4f $4G$	5/2-5/2
	3876.664	3d $4F^{\circ}$ - 4f $4G$	5/2-7/2
	3879.408	3d $4F^{\circ}$ - 4f $4G$	7/2-9/2
	3876.187	3d $4F^{\circ}$ - 4f $4G$	9/2-11/2
3876.26	3876.055	3d $4F^{\circ}$ - 4f $4G$	3/2-5/2
3871.86	3871.669	3d $4F^{\circ}$ - 4f $2G$	5/2-7/2
3835.76	3835.730	4p $2P^{\circ}$ - 3p $2D$	1/2-3/2
3831.78	3831.743	4p $2P^{\circ}$ - 3p $2D$	3/2-5/2
	3590.862	3p $4D$ - 4s $4P^{\circ}$	5/2-3/2
		3p $4D$ - 4s $4P^{\circ}$	3/2-1/2
3589.88	3589.657	3p $4D$ - 4s $4P^{\circ}$	7/2-5/2
	3585.809	3p $4D$ - 4s $4P^{\circ}$	1/2-3/2
3585.45	3584.977	3p $4D$ - 4s $4P^{\circ}$	5/2-5/2
	3361.721	3d $2D$ - 5p $2P^{\circ}$	3/2-1/2
3361.18	3361.051	3d $2D$ - 5p $2P^{\circ}$	5/2-3/2
	3360.891	3d $2D$ - 5p $2P^{\circ}$	3/2-3/2
3167.97	3167.931	2p ³ $2D^{\circ}$ - 3p $2P$	3/2-1/2
	3165.974	2p ³ $2D^{\circ}$ - 3p $2P$	3/2-3/2
3165.54	3165.467	2p ³ $2D^{\circ}$ - 3p $2P$	5/2-3/2
2992.58	2992.618	3d $2D$ - 5f $2F^{\circ}$	3/2-5/2
			5/2-7/2

C II - cont.

AVERAGE	S. GLAD	TRANSITION	J
		$3d \ ^4D^\circ - 5f \ ^2F$	5/2-7/2
		$3d \ ^4D^\circ - 5f \ ^2F$	3/2-5/2
2885.33	2885.496	$3d \ ^4D^\circ - 5f \ ^2F$	7/2-9/2
2837.55	2837.603	$2p^2 \ ^2S - 3p \ ^2P^\circ$	1/2-1/2
2836.60	2836.710	$2p^2 \ ^2S - 3p \ ^2P^\circ$	1/2-3/2
	2801.43	$3d \ ^4F^\circ - 5f \ ^4G$	3/2-5/2
		$3d \ ^4F^\circ - 5f \ ^4G$	5/2-7/2
	2801.21	$3d \ ^4F^\circ - 5f \ ^4G$	7/2-9/2
2801.40		$3d \ ^4F^\circ - 5f \ ^4G$	9/2-11/2
	2641.425	$3p \ ^4D - 4d \ ^4F^\circ$	3/2-3/2
		$3p \ ^4D - 4d \ ^4F^\circ$	7/2-9/2
2641.10	2640.894	$3p \ ^4D - 4d \ ^4F^\circ$	5/2-7/2
		$3p \ ^4D - 4d \ ^4F^\circ$	3/2-5/2
2640.41	2640.560	$3p \ ^4D - 4d \ ^4F^\circ$	1/2-3/2
	2512.065	$2p^2 \ ^2P - 2p^3 \ ^2D^\circ$	3/2-5/2
2511.93	2511.734	$2p^2 \ ^2P - 2p^3 \ ^2D^\circ$	3/2-3/2
2509.28	2509.121	$2p^2 \ ^2P - 2p^3 \ ^2D^\circ$	1/2-3/2
	2401.761	$3p \ ^2P^\circ - 5s \ ^2S$	1/2-1/2
2402.10	2402.402	$3p \ ^2P^\circ - 5s \ ^2S$	3/2-1/2
	2269.70	$3p \ ^4D - 5s \ ^4P^\circ$	7/2-5/2
	2270.20	$3p \ ^4D - 5s \ ^4P^\circ$	5/2-3/2
2269.76		$3p \ ^4D - 5s \ ^4P^\circ$	3/2-1/2

TABLE 2
DOUBLY IONIZED CARBON - C III

AVERAGE	BOCKASTEN	TRANSITION	J
5304.28	5305.10	5f $^3F^\circ$ - 7g 3G	-
5271.14	5272.53	3s $^3P^\circ$ - 3p 3S	2-1
4859.96	4859.6	5d 3D - 7f $^3F^\circ$	3-4
4673.91	4673.95	3s $^3P^\circ$ - 3p 3P	2-1
4666.01	4665.86	3s $^3P^\circ$ - 3p 3P	2-2
4662.93	4663.64	3s $^3P^\circ$ - 3p 3P	1-0
4658.71	4659.06	3s $^3P^\circ$ - 3p 3P	1-1
	4652.06	3s $^3P^\circ$ - 3p 3P	0-1
	4651.47	3s 3S - 3p $^3P^\circ$	1-0
4651.40	4651.01	3s $^3P^\circ$ - 3p 3P	1-2
4650.21	4650.25	3s 3S - 3p $^3P^\circ$	1-1
4647.42	4647.42	3s 3S - 3p $^3P^\circ$	1-2
4516.84	4516.77	4p $^3P^\circ$ - 5s 3S	2-1
	4515.78	4p $^3P^\circ$ - 5s 3S	1-1
4515.86	4515.33	4p $^3P^\circ$ - 5s 3S	0-1
4361.89	4361.87	4f $^3F^\circ$ - 5g 3G	?-?
4325.47	4325.560	3s $^1P^\circ$ - 3p 1D	1-2
4186.99	4186.900	4f $^1F^\circ$ - 5g 1G	3-4
4166.49	4166.95	3p 1D - 6p $^1P^\circ$	2-1
	4163.26	3p 3D - 5f $^3F^\circ$	3-3
4162.99	4162.86	3p 3D - 5f $^3F^\circ$	3-4
	4156.76	3p 3D - 5f $^3F^\circ$	2-2
4156.68	4156.49	3p 3D - 5f $^3F^\circ$	2-3
4152.60	4152.512	3p 3D - 5f $^3F^\circ$	1-2

C III - cont.

AVERAGE	BOCKASTEN	TRANSITION	J
4121.76	4121.843	4p $^1P^\circ$ - 5d 1D	1-2
4070.33	4070.261	4f $^3F^\circ$ - 5g 3G	4-5
4069.00	4068.912	4f $^3F^\circ$ - 5g 3G	3-4
4068.00	4067.940	4f $^3F^\circ$ - 5g 3G	2-3
3889.18	3889.475	4d 3D - 5f $^3F^\circ$	3-3
	3889.144	4d 3D - 5f $^3F^\circ$	3-4
3886.12	3885.941	4d 3D - 5f $^3F^\circ$	2-3
3883.74	3883.816	4d 3D - 5f $^3F^\circ$	1-2
3703.53	3703.71	3p 1P - 3d $^1P^\circ$	1-1
3609.36	3609.625	4p $^3P^\circ$ - 5d 3D	2-2,3
	3609.063	4p $^3P^\circ$ - 5d 3D	1-1,2
	3608.81	4p $^3P^\circ$ - 5d 3D	0-1
2799.60	2799.47	3p 3D - 6f $^3F^\circ$	3-4
2725.77	2725.90	4f $^3F^\circ$ - 6g 3G	4-5
	2725.30	4f $^3F^\circ$ - 6g 3G	3-4
	2724.85	4f $^3F^\circ$ - 6g 3G	2-3
2697.78	2697.75	4p $^3P^\circ$ - 6s 3S	2-1
	2697.42	4p $^3P^\circ$ - 6s 3S	1-1
2671.26	2671.318	4d 3D - 6f $^3F^\circ$	2-3
	2670.240	4d 3D - 6f $^3F^\circ$	1-2
2296.99	2296.870	2p $^1P^\circ$ - 2p ² 1D	1-2

TABLE 3
TRIPLY IONIZED CARBON - C IV

AVERAGE	BOCKASTEN	TRANSITION	J
4685.47	4685.4	6f $^2F^\circ$ - 8g 2G etc.	-
4658.06	4658.30	5f $^2F^\circ$ - 6g 2G etc.	-
4441.15	4441.49	5p $^2P^\circ$ - 6d 2D	3/2-5/2
	4440.34	5p $^2P^\circ$ - 6d 2D	1/2-5/2
3689.61	3689.6	6f $^2F^\circ$ - 9g 2G etc.	-
2697.77	2698.67	4p $^2P^\circ$ - 5s 2S	3/2-1/2
	2697.75	4p $^2P^\circ$ - 5s 2S	1/2-1/2
2530.03	2530.6	4f $^2F^\circ$ - 5f 2F	7/2-5/2, 7/2
	2529.98	4f $^2F^\circ$ - 5g 2G	7/2-7/2, 9/2
2335.53	2335.90	5f $^2F^\circ$ - 8g 2G etc.	-

TABLE 4
NEW LINES

WAVELENGTH	STRENGTH
$5130.8 \pm 0.3 \overset{\circ}{\text{\AA}}$	2
$4943.4 \pm 0.2 \overset{\circ}{\text{\AA}}$	3
$4944.3 \pm 0.6 \overset{\circ}{\text{\AA}}$	1
$4943.2 \pm 0.6 \overset{\circ}{\text{\AA}}$	1
$4640.1 \pm 0.2 \overset{\circ}{\text{\AA}}$	2
$4635.0 \pm 0.2 \overset{\circ}{\text{\AA}}$	2
$4428.4 \pm 0.3 \overset{\circ}{\text{\AA}}$	2
$4131.9 \pm 0.2 \overset{\circ}{\text{\AA}}$	2
$4128.6 \pm 0.2 \overset{\circ}{\text{\AA}}$	2

SUMMARY AND CONCLUSIONS

The work reported in this dissertation has been quite successful in improving the quality of spectral data taken using the beam-foil source. The technique of moving the side-on lens away from the beam was key to the success of the early work on oxygen¹, resulting in the observation of several lines which had previously been undetected.

The performance of the end-on lens system was even more dramatic. It permitted the acquisition of good data with the 1 meter Czerny-Turner spectrograph in the experiment reported in section five, as well as in other experiments^{2, 3, 4}, whereas we were unable to obtain satisfactory spectra in the side-on system, in spite of using the faster (f/2) Meinel spectrograph.

The general principles of analysis of beam-foil optical systems is well established in this dissertation, and the most useful specific designs have been considered in detail. However, one important area has not been covered. This dissertation has been almost exclusively concerned with conventional slit spectrograph analysing instruments. Fabry-Perot and Fourier transform analysers are of considerable value for the spectroscopy of weak sources because of their high throughput, which is typically ten times that of a slit spectrograph.⁵ The sensitivity of these instruments to variation in wavelength as a function of

the angle of the incident light makes detailed analysis imperative.⁶ The possible pitfalls in the analysis of even the relatively simple slit spectrographs should be evident to the reader, and analogous problems are much more severe for interference analysers.

The range of specific systems has not been exhausted, although the ones considered are the most commonly found. Several other designs have occurred to the author in the course of this work, and undoubtedly many other are possible.

To assist the reader in the design of optical systems for beam-foil spectroscopy, Table 1 summarizes the properties of the systems considered in this dissertation.

Table 1

Comparison of the Optical Systems

I. Side-on Systems

A. Advantages

1. simplicity
2. spatial resolution of the decay, allowing
 - a. lifetime measurements
 - b. Stark mixing experiments
 - c. charge state assignments
3. direct identification of residual gas emission lines
4. energy regulation of the ion beam not critical

5. with special effort, very accurate relative intensity measurements possible.

B. Disadvantages

1. very low signal, and signal-to-noise, at a given resolution
2. small beam image requires accurate spectrograph alignment
3. strong dependence of λ on θ requires accurate alignment of the optic axis
4. line slant complicates wavelength measurements
5. line slant severely complicates use of scanning monochromators
6. variation of λ on θ makes Fabry-Perot use difficult
7. simple relative^{total} intensity measurements are difficult
8. linear beam image requires use of straight slits.

C. Resolution limit set by angular scattering

1. collimated beam required
2. minimum scattering at the foil required.

II. End-on Systems

A. Advantages

1. simplicity
2. alignment relatively non-critical

3. excellent signal, and signal-to-noise, at a given resolution
4. simple relative^{total} _{λ} intensity measurements usually possible
5. identification of residual gas emission lines possible by use of two energies
6. no line slant, facilitating use of scanning monochromators
7. round beam image, allowing use of straight or curved spectrograph slits.

B. Disadvantages

1. sensitivity to variations in β requires careful energy regulation
2. no spatial resolution of the decay, prohibiting lifetime and similar measurements
3. large Doppler shift correction required
4. radiation damage a more serious problem than in other systems
5. continuum difficulties if precautions are not observed, probably precluding use with Fabry-Perot analysers
6. variation of λ on θ makes use of interference analysers difficult.

C. Resolution limit set by velocity spread

1. careful energy regulation required

2. minimum straggling at foil required.

III. Axicon Systems

A. Advantages

1. no variation of λ on θ , permitting more convenient use of interference analysers
2. signal, and signal-to-noise, slightly better than the side-on lens, for slit spectrographs
3. signal, and signal-to-noise, much better than than the side-on lens, for Fabry-Perot analysers; possibly comparable with the end-on lens
4. energy regulation not critical.

B. Disadvantages

1. complex
2. poor signal, signal-to-noise, and strange line-shape, when used with a slit spectrograph
3. collection efficiency varies along beam, prohibiting relative intensity measurements
4. optimum design is a function of lifetime
5. no spatial resolution of decay, making lifetime and similar measurements impossible
6. no way of identifying residual gas lines
7. alignment critical
8. no obvious grazing incidence analogs, thus probably restricting use to $\lambda \gtrsim 500 \text{ \AA}$.

C. Resolution limit set by angular scattering

1. careful collimation required
2. minimum foil-induced scattering required.

Keeping the comparisons shown in Table 1 in mind, the various systems should be used to compliment each other. Certain measurements are best made using the end-on system, while others demand the use of a side-on system. The recommended applications of the systems are:

General spectral survey

A general survey of the spectrum of beam-foil excited ions should be done first with an end-on system. The high signal-to-noise is important in allowing the highest possible resolution and detection of weak lines. This should be complimented with a low resolution survey of the same spectrum with a side-on system to detect any lines with strongly anisotropic emission.

Accurate wavelength measurements

The strong anisotropic Doppler effect makes absolute wavelength determinations extremely difficult regardless of the system used. The best approach is to use lines measured in other sources for calibration. Where this is impossible, a few lines should be measured with a very carefully aligned side-on or axicon system, and these lines should be used as standards. This leaves only the second-order

correction, which is quite small. Errors here are instrumental in nature and may be eliminated by calibrating with an ion beam producing a known line or lines.

Lifetime, Stark mixing, and charge state analysis

A side-on system is required for these measurements.

Relative intensity measurements

The most accurate relative intensity measurements require a side-on system and careful integration under the decay curve. For rapid estimates (10-20% accuracy) or where the low signal-to-noise of the side-on system precludes accurate determination or even detection of the line, an end-on system with good depth of field will give good total intensity measurements.

REFERENCES - INTRODUCTION

1. B. Edlen, Zeitschrift fur Physik, 100, 621 (1936).
2. K. Bockasten, R. Hallin, K. B. Johannson, P. Tsui, Phys. Letters 8, 181 (1964).
3. K. Bockasten, R. Hallin, and T. P. Hughes, Proc. Phys. Soc. (London), 81, 522 (1963).
4. B. C. Fawcett and T. E. Irons, Proc. Phys. Soc. (London) 89, 1063 (1967).
5. L. Kay, Phys. Letters 5, 36 (1963).
6. S. Bashkin, A. B. Meinel, P. R. Malmberg, and S. G. Tilford, Phys. Letters 10, 63 (1964).
7. H. H. Heckman, E. L. Hubbard, and W. G. Simon, Phys. Rev. 129, 1240 (1963).
8. P. R. Malmberg, S. Bashkin, and S. G. Tilford, Phys. Rev. Letters 15, 98 (1965).
9. L. Brown, W. K. Ford, V. Rubin, W. Trächslin, and W. Brandt, in S. Bashkin, ed., Beam-Foil Spectroscopy, Gordon and Breach, New York, (1968).
10. A. S. Goodman and D. J. Donahue, Phys. Rev. 141, 1 (1966).
11. W. S. Bickel and A. S. Goodman, Phys. Rev. 148, 1 (1966).
12. J. A. Jordan, Jr., G. S. Bakken, and R. E. Yager, J. Opt. Soc. Am. 57, 530 (1967).
13. R. H. Hughes, H. R. Dawson, and B. M. Doughry, J. Opt. Soc. Am. 56, 830 (1966).
14. E. Alqvist, C. Broude, M. A. Clark, J. A. Kuchner, and A. E. Litherland, Can. J. Phys. 40, 954 (1964).
15. W. Wien, Ann. der Physik 60, 597 (1919).
16. S. Bashkin, W. S. Bickel, D. Fink, and R. K. Wangsness, Phys. Rev. Letters 15, 284 (1965).
17. W. S. Bickel, Appl. Optics 6, 1309 (1967).

18. S. Bashkin, ed., Beam-Foil Spectroscopy, Gordon and Breach, New York, (1968).
19. S. Bashkin, Appl. Optics 7, 2341 (1968).
20. W. S. Bickel, Appl. Optics 7, 2367 (1968).
21. D. Fink, Appl. Optics 7, 2373 (1968).
22. J. A. Jordan, Jr., G. S. Bakken, A. C. L. Barnard, and K. E. Kissell, Bull. Am. Phys. Soc. 11, 70 (1968).
23. G. S. Bakken and J. A. Jordan, Jr., Bull. Am. Phys. Soc. 12, 186 (1967).
24. G. S. Bakken, M. A. Thesis, Rice University, 1967.
25. J. A. Jordan, Jr., in S. Bashkin, ed., Beam-Foil Spectroscopy, Gordon and Breach, New York, (1968).

REFERENCES - SECTION ONE

1. L. Kay, Phys. Letters 5, 36 (1963).
2. S. Bashkin, A. B. Meinel, P. R. Malmberg, and S. G. Tilford, Phys. Letters 10, 63 (1964).
3. S. Bashkin, ed., Beam-Foil Spectroscopy, Gordon and Breach, New York, (1968).
4. J. A. Jordan, Jr., G. S. Bakken, A. C. L. Barnard, and K. E. Kissell, Bull. Am. Phys. Soc. 11, 70 (1966).
5. S. Bashkin, D. Fink, P. R. Malmberg, A. B. Meinel, and S. G. Tilford, J. Opt. Soc. Am. 56, 1064 (1966).
6. L. C. Marquet, J. Opt. Soc. Am. 57, 878 (1967).
7. W. S. Bickel, Appl. Opt. 6, 1309 (1967).
8. J. A. Jordan, Jr., in S. Bashkin, ed., Beam-Foil Spectroscopy, Gordon and Breach, New York, (1968).
9. A. Papoulis, The Fourier Integral and Its Applications, McGraw-Hill, New York, (1965) or any similar text.
10. E. W. Marchand, J. Opt. Soc. Am. 54, 263 (1964).
11. A. Rose, J. Soc. Motion Pict. Television-Engrs. 47, 273 (1946).
12. R. C. Jones, Phot. Sci. Eng. 2, 57 (1958).
13. J. C. Marchant, J. Opt. Soc. Am. 54, 798 (1964).
14. J. C. Marchant and A. G. Millikan, J. Opt. Soc. Am. 55, 907 (1965).
15. C. E. K. Mees and T. H. James, ed., The Theory of the Photographic Process, 3rd Ed., Macmillan, New York, (1966).
16. L. Brown, W. K. Ford, V. Rubin, W. Trächslin, and W. Brandt, in S. Bashkin, ed., Beam-Foil Spectroscopy, Gordon and Breach, New York, (1968).
17. J. A. Jordan, Jr., G. S. Bakken, and A. C. Conrad, Jr., Advances in Electronics and Electron Physics, 28, 907 (1969).

18. L. Bridwell, W. E. Maddox, L. M. Beyer, and R. C. Ether-ton, Bull. Am. Phys. Soc. 14, 618 (1969).
19. B. L. Moiseiwitsch and S. J. Smith, Rev. Mod. Phys. 40, 238 (1968).
20. L. A. Spitzberg, private communication.
21. A. Denis, J. Desquelles, M. Dufay, and M. C. Poulizac, in S. Bashkin, ed., Beam-Foil Spectroscopy, Gordon and Breach, New York, (1968).
22. J. O. Stoner, Jr., Appl. Opt., 9, 53, (1970).
23. G. S. Bakken, A. C. Conrad, Jr., and J. A. Jordan, Jr., J. Phys. (B), 2, 1378, (1969).
24. J. A. Jordan, Jr., G. S. Bakken, and R. E. Yager, J. Opt. Soc. Am. 57, 530 (1967).
25. H. E. Ives and G. R. Stillwell, J. Opt. Soc. Am. 28, 215 (1938).
26. K. Berkner, W. S. Cooper III, S. N. Kaplan, R. V. Pyle, Phys. Letters 16, 35 (1965).
27. W. S. Bickel, Appl. Opt. 7, 2367 (1968).
28. P. R. Malmberg, S. Bashkin, and S. G. Tilford, Phys. Rev. Letters 15, 98 (1965).
29. S. Bashkin, W. S. Bickel, D. Fink, R. W. Wangsness, Phys. Rev. Letters 15, 284 (1965).

REFERENCES - SECTION TWO

1. L. Kay, Phys. Letters 5, 36 (1963).
2. S. Bashkin, A. B. Meinel, P. R. Malmberg, and S. G. Tilford, Phys. Letters 10, 63 (1964).
3. S. Bashkin, D. Fink, P. R. Malmberg, A. B. Meinel, and S. G. Tilford, J. Opt. Soc. Am. 56, 1064 (1966).
4. W. S. Bickel, Appl. Opt. 6, 1309 (1967).
5. L. C. Marquet, J. Opt. Soc. Am. 57, 878 (1967).
6. G. S. Bakken, M. A. Thesis, Rice University, Houston, Texas, (1967).
7. J. A. Jordan, Jr., in S. Bashkin, ed., Beam-Foil Spectroscopy, Gordon and Breach, New York, (1968).
8. L. Brown, W. K. Ford, Jr., V. Rubin, W. Trächslin, and W. Brandt, in S. Bashkin, ed., Beam-Foil Spectroscopy, Gordon and Breach, New York, (1968).
9. A. Papoulis, The Fourier Integral and Its Applications, McGraw-Hill, New York, (1968) or any similar text.
10. J. A. Jordan, Jr., G. S. Bakken, and A. C. Conrad, Jr., Adv. Electronics and Electron Physics, 28, 907, (1969).
11. H. E. Ives and G. R. Stillwell, J. Opt. Soc. Am. 28, 215 (1938).
12. J. O. Stoner, Jr., Appl. Opt., 9, 53, (1970).
13. K. Berkner, W. S. Cooper III, N. S. Kaplan, and R. V. Pyle, Phys. Letters 16, 35 (1965).
14. J. A. Jordan, Jr., G. S. Bakken, and R. E. Yager, J. Opt. Soc. Am. 57, 530 (1967).
15. W. S. Bickel, Appl. Opt. 7, 2367 (1968).
16. L. Heroux, in S. Bashkin, ed., Beam-Foil Spectroscopy, Gordon and Breach, New York, (1968).
17. W. S. Bickel, in S. Bashkin, ed., Beam-Foil Spectroscopy, Gordon and Breach, New York, (1968).

18. Kodak Publication P-14, Rev. 12-66, Eastman Kodak Co., Rochester, New York, (1966).
19. W. P. Boquist, Sky and Telescope, January, 12, (1964).
20. P. R. Malmberg, S. Bashkin, S. G. Tilford, Phys. Rev. Letters 15, 98 (1965).
21. S. Bashkin, Appl. Opt. 7, 2341 (1968).
22. R. L. Barger and K. G. Kessler, J. Opt. Soc. Am. 50, 352 (1960).

REFERENCES - SECTION THREE

1. G. S. Bakken, M. A. Thesis, Rice University, Houston, Texas, (1967).
2. G. S. Bakken and J. A. Jordan, Jr., Bull. Am. Phys. Soc. 12, 186 (1967).
3. J. A. Jordan, Jr., in S. Bashkin, ed., Beam-Foil Spectroscopy, Gordon and Breach, New York, (1968).
4. A. Denis, J. Desquelles, M. Dufay, and M. C. Poulizac, in S. Bashkin, ed., Beam-Foil Spectroscopy, Gordon and Breach, New York, (1968).
5. L. C. Marquet, J. Opt. Soc. Am. 57, 878 (1967).
6. A. Papoulis, The Fourier Integral and Its Applications, McGraw-Hill, New York, (1965).
7. P. R. Malmberg, S. Bashkin, S. G. Tilford, Phys. Rev. Letters 15, 98 (1965).
8. G. S. Bakken, A. C. Conrad, Jr., and J. A. Jordan, Jr., J. Phys. (B), 2, 1378, (1969).
9. S. Bashkin, D. Fink, P. R. Malmberg, A. B. Meinel, and S. G. Tilford, J. Opt. Soc. Am. 56, 1064 (1966).
10. J. A. Jordan, Jr., A. C. Conrad, Jr., and G. S. Bakken, Abstracts of the International Conference on Atomic Physics, N.Y.U., p. 66, (1968).
11. H. E. Ives and G. R. Stillwell, J. Opt. Soc. Am. 28, 215 (1938).
12. L. Brown, W. K. Ford, V. Rubin, W. Trächslin, and W. Brandt, in S. Bashkin, ed., Beam-Foil Spectroscopy, Gordon and Breach, New York, (1968).
13. S. Bashkin, in S. Bashkin, ed., Beam-Foil Spectroscopy, Gordon and Breach, New York, (1968).
14. R. L. Barger and K. G. Kessler, J. Opt. Soc. Am. 50, 352 (1960).
15. D. M. Hunter, Space Science Reviews 6, 493 (1967).

REFERENCES - SECTION FOUR

1. J. H. McLeod, J. Opt. Soc. Am. 44, 592 (1954).
2. W. S. Bickel, Appl. Opt. 6, 1309 (1967).
3. J. O. Stoner, Jr., Appl. Opt., 9, 53, (1970).
4. D. M. Hunter, Space Science Reviews 6, 493 (1967).
5. G. S. Bakken, M. A. Thesis, Rice University, Houston, Texas, (1967).
6. J. A. Jordan, Jr., in S. Bashkin, ed., Beam-Foil Spectroscopy, Gordon and Breach, New York, (1968).
7. J. O. Stoner, Jr. and H. L. Supp, Appl. Opt. 8, 831 (1969).

REFERENCES - SECTION FIVE

1. G. S. Bakken, M. A. Thesis, Rice University, (1967).
2. J. A. Jordan, Jr., in S. Bashkin, ed., Beam-Foil Spectroscopy, Gordon and Breach, New York, (1968).
3. A. C. Conrad, Jr., M. A. Thesis, Rice University, (1969).
4. S. Glad, Arkiv För Fysik 7, (1952).
5. K. Bockasten, Arkiv För Fysik 9, (1955).
6. K. Bockasten, Arkiv För Fysik 10, (1956).
7. J. A. Jordan, Jr., et. al., in Proceedings of the Fourth Symposium on Photo-Electronic Image Devices, ed., J. S. McGee, Academic Press, Inc., London, p. 907, (1969). Volume 28 of the Advances in Electronics and Electron Physics series.

REFERENCES - SUMMARY AND CONCLUSIONS

1. J. A. Jordan, Jr., G. S. Bakken, A. C. L. Barnard, and K. E. Kissell, Bull. Am. Phys. Soc. 11, 70 (1966).
2. G. S. Bakken, A. C. Conrad, Jr., and J. A. Jordan, Jr., J. Phys. (B), 2, 1378, (1969).
3. A. C. Conrad, Jr., M. A. Thesis, Rice University, (1969).
4. J. A. Jordan, Jr., in S. Bashkin, ed., Beam-Foil Spectroscopy, Gordon and Breach, New York, (1968).
5. D. M. Hunter, Space Science Reviews 6, 493 (1967).
6. Some of this analysis has been done by R. L. Barger and K. G. Kessler, J. Opt. Soc. Am. 50, 352 (1960).

Acknowledgments

I would like to express my appreciation to my research advisor, Dr. James A. Jordan, Jr., for his guidance and helpful suggestions.

I also wish to thank Mr. J. F. Van der Henst, now retired, and the staff of the Rice University Physics Shop for the construction of the apparatus used in the experimental work. Special thanks are due to Mr. A.C. Conrad for his work on the image intensifier modification to the Czerny-Turner spectrograph used in the experiment, and for his assistance in the execution of the experiment.

I gratefully acknowledge my financial support by the National Aeronautics and Space Administration, as well as the support of the experimental and computer work by Rice University, the Department of Defense (project THEMIS), and the Atomic Energy Commission.

And last, but far from least, to my parents: my father, who taught me many of the engineering and shop skills needed for the experimental work; and my mother, who was for many years the only one who really thought I would get this far.