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RICE UNIVERSITY

LASER FLASH THERMAL DIFFUSIVITY DETERMINATION
PROCEDURE FOR HIGH TEMPERATURE LIQUID METALS

by

JENNIFER ANN MURPHY

A THESIS SUBMITTED
IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS FOR THE DEGREE

MASTER OF SCIENCE

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ABSTRACT

A method is proposed for determining the thermal diffusivity of high temperature liquid metals that involves levitating a small sample under microgravity conditions, melting it, and predicting its thermal response to a laser pulse from either side at time $t = 0$. The solution is obtained by determining the initial temperature distribution in the thermal penetration depth created by the laser pulses and then solving the two dimensional conduction equation in spherical coordinates for the cooling history of the droplet. The correct diffusivity is obtained by minimizing the difference between predicted temperatures and experimentally obtained temperatures. The method is demonstrated for nickel, iron, and copper, and is shown to have good accuracy. An error analysis reveals that the result is highly sensitive to bias error in the experimental temperature data, and an example is given in which order of magnitude estimates for diffusivity can be obtained from the curves provided.

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NOMENCLATURE

a	Width of the laser beam, m
b	Radius of droplet, m
Bi	Biot number, $Bi = \frac{\epsilon \bar{\sigma} T_i^3 b}{k}$
c_p	Specific heat of specimen, kJ/(kg-K)
dk	Nondimensional time step size
F	Shape factor of droplet; taken to be 1.0 for a sphere
k	Thermal Conductivity, W/(m-k)
M	Lumped property parameter, $\epsilon F/(\rho c_p)$
Ma	Marangoni number, $Ma = - \frac{\frac{d\gamma}{dT} \Delta T b}{\rho \nu k}$, where $\frac{d\gamma}{dT}$ is assumed constant
NI	Number of nodes in the r direction
NJ	Number of nodes in the phi direction
NK	Number of time steps
NL	Number of temperature measurement site locations
Nr	Conduction to radiation parameter, $Nr = \frac{k}{\epsilon \bar{\sigma} (T_i - T_\infty)^3 b}$
q	Heat flux of laser normal to sphere surface, $Q \cos \phi$
Q	Heat flux of the laser pulse, W/m ²
$\bar{Q}$	Nondimensional heat flux of laser pulse, $\frac{Q}{k(T_i - T_\infty)/b}$
r	Radial coordinate
$\bar{r}$	Nondimensional radial coordinate r/b
T	Temperature, K

T_i	Initial temperature, K
t	Dimensional time, s
x	Dimensional coordinate in penetration depth problem, m
$\bar{x}$	Nondimensional coordinate in penetration depth problem, x/b
α	Thermal diffusivity, m^2/s
ϵ	Hemispherical emissivity
δ	Dimensional penetration depth, m
Δ	Nondimensional penetration depth, δ/b
ϕ	Polar coordinate
ϕ_o	Angle subtended by the laser beam on the droplet, $\sin^{-1}(a/b)$
γ	Surface tension, N/m
ν	Kinematic viscosity, m^2/s
θ	Nondimensional temperature, $\left(\frac{T - T_\infty}{T_i - T_\infty} + \frac{T_\infty}{T_i - T_\infty} \right)^4$
ρ	Density, kg/m^3
σ	Stefan-Boltzmann constant
τ	Nondimensional time, $\alpha t / b^2$
τ_L	Nondimensional duration of laser pulse
subscripts:	
i	Radial direction
j	Phi direction
k	Time step number
l	Measurement site
∞	Ambient

superscripts:

n	Current time step
$n+1$	Next time step
$\sim$	Experimental
$'$	Estimated

INTRODUCTION

There is little data available in the literature on the thermal diffusivity of high temperature liquid metals. In nuclear reactors and metal foundries or almost any application in which high temperatures are involved, such data can be used to determine the thermal response time of the metals involved. Touloukian et. al. [1] is the most comprehensive collection of data on thermal diffusivity of metals, yet the data is limited to materials in the solid phase, and the uncertainty in the data given at or above the materials' melting points is often too large for good accuracy in heat transfer calculations.

There are several reasons for the difficulty in determining the diffusivity of high temperature liquid metals. Because a containment vessel is needed for the liquid, analysis becomes difficult because the sample's geometry is complicated, and due to the corrosive nature of high temperature liquid metals, the sample is also contaminated by the container. Other problems that arise are that the usual assumption of one dimensional conduction in the material being tested may not be valid due to other modes of heat transfer within the sample, and evaporation and vapor bubble formation at the high temperatures involved bias measurements.

Most methods available to determine a material's thermal diffusivity were designed for solid phase specimens--hence the difficulties that arise when dealing with liquids. One method developed for liquids is the transient hot wire method. It involves measuring the thermal response of a thin wire immersed in liquid following the stepwise application of a voltage across its ends [2,3]. To overcome the problems associated with studying conducting liquids, a thin coating is applied to the wire, but at high temperatures, this coating deteriorates. The stepwise heating method determines diffusivity by measuring the response of a slab to the application of a stepwise heating function at its surface [4]. Flash methods are identical except the heating is in the form of an intense pulse at time $t = 0$ rather

than a step function [5-9]. All of the above methods were developed for solids and/or low temperature liquids, and they do not overcome the problem of contact with a containment vessel.

A method developed by Bayazitoglu et. al. [10], overcomes the problem of contact with a containment vessel by levitating a small sample under microgravity conditions and melting it with an external heating device. The surface temperature history is recorded as a function of time so that it can be approximated with a curve fit and used as an extra boundary condition in the one dimensional solution of Fourier's law of conduction. An inverse conduction problem is formulated with this extra boundary condition, and the thermal diffusivity determined by minimizing a functional that requires a heat balance at the surface of the sphere. However, this method does not account for the two dimensional temperature profile that would result from an external heating device, is limited to the accuracy of the curve fit, and is highly sensitive the choice of the input data.

A new method is proposed here that accommodates a one or two dimensional cooling profile, has a high degree of accuracy, comparable sensitivity to variations in input data, and requires no curve fit or evaluation of derivatives in the inverse problem (as does the previous method). Criteria for choosing the experimental data are also suggested. This method consists of levitating a small sample and heating it to equilibrium with an external heating device such as a laser. Then, the sample is irradiated by a laser pulse from either side creating a two dimensional temperature profile (See Fig. 1). An advantage to using the lasers is that greater control over the temperature range of the experiment is obtained so that the value of diffusivity obtained more accurately reflects the diffusivity of the material at the desired temperature. The density, specific heat, and emissivity of the material are assumed to be known. The data that must be acquired are the pulse width, heat flux, and duration, droplet radius, and cooling history of the droplet at one or more locations on the surface of the sphere. The droplet must be large enough so that Fourier's law of conduction is valid

within the droplet, and small enough to be accommodated by a typical levitation device. Then, the cooling profile is predicted by a finite difference scheme and an inverse conduction problem is formulated. The correct diffusivity is obtained by minimizing the least squared error norm between the measured and predicted temperatures.

Experimental Equipment: For the experiment intended, it is very important to insure stability of the sample. Depending on the type of levitation technique used, different levels of stability can be obtained. Two well known levitation techniques are discussed here: electromagnetic levitation and acoustic levitation. Under a sufficiently strong time varying electromagnetic field created by a high frequency alternating current in oppositely wound coils, a sample can be levitated due to Lorentz forces and with sufficient current melted by ohmic heating from the induced currents within the sample. The degree to which a sample can be heated by the induced currents is directly related to the force necessary to levitate the sample. The greater the force and therefore frequency requirements, the greater the ohmic heating. On earth, the induced currents are of sufficient magnitude to melt the sample. However, in a reduced gravity environment, the force requirements are considerably less, and consequently an external heating device is necessary to melt the sample. From numerically predicted results it is observed that the power requirements necessary to melt the sample can easily be obtained by a CO₂ laser. Initially, the droplet is prone to spinning and shape oscillations due to fluctuations in the forces created by the electromagnetic levitator. It is assumed that these forces are counterbalanced and the droplet is stabilized. The entire apparatus, illustrated in Fig. 2, is contained in a chamber filled with an inert gas such as helium so that in a reduced gravity environment, the levitated liquid droplet is allowed to cool radiatively to space in the absence of other modes of heat transfer and free of container contamination. It is already the most widely used

levitation technique used in the determination of density, specific heat, and emissivity, and is standard equipment in space based experimentation.

Acoustically a sample can also be levitated with an ultrasonic standing wave. Therefore an additional heating device is required to melt the sample. However, for the experiment intended, the high temperature of the sample causes interference problems with the standing wave resulting in ejection of the sample from the levitator. Combinations of the electromagnetic and acoustic levitation methods have also been recommended incorporating the advantages of both methods.

FORMULATION

In the proposed experiment, a sample is levitated and melted with an alternate heating source so that the droplet's response to a laser pulse from either side can be predicted by the solution of Fourier's law of conduction in spherical coordinates with azimuthal symmetry. The solution is obtained by solving two problems in two separate time domains. A penetration depth problem is solved to approximate the temperature distribution in the thermal penetration depth created by the laser pulse. Then, the solution from the first time domain is used as the initial condition in the solution to the second time domain. The two laser pulses that irradiate the surface of the sphere from either side create a symmetric temperature profile about the origin of the sphere, and therefore only a quarter of the sphere need be analyzed in the solution.

First Time Domain: In the solution of the penetration depth problem, the sphere is initially at uniform temperature. Then it is irradiated by the laser pulses from either side creating a two dimensional temperature profile. Since the laser pulses are relatively short, the extent to which their effects are felt in the radial direction of the sphere are small. Thus, a thin layer on the surface of the sphere, as illustrated in Fig. 3, can be treated as a slab neglecting curvature effects and the resulting penetration depth problem solved to determine the nondimensional penetration depth, $\Delta(\tau)$. $\Delta(\tau) = \sqrt{6\tau}$ for $\tau > 0$, and the resulting quadratic approximation to the temperature profile for a given nondimensional laser pulse heat flux, $\bar{Q}$, in terms of the droplet coordinate system is:

$$\theta = 1 + \frac{\bar{Q} \cos \phi (\Delta(\tau_L) - \bar{r} - 1)^2}{2\Delta(\tau_L)}, \quad 1 - \Delta(\tau_L) \leq \bar{r} \leq 1, \quad 0 \leq \phi \leq \phi_0 \quad (1)$$

where $\Delta(\tau_L)$ is a fixed value of the nondimensional penetration depth of the laser pulse.

Second Time Domain: With the droplet radius and pulse heat flux, width, and duration known, the initial condition for the second time domain is given by Eq. (1). The droplet is now exposed to the radiatively cooling environment of space and the formulation of the problem in nondimensional coordinates is:

$$\frac{\partial \theta}{\partial \tau} = \frac{2}{\bar{r}^2} \frac{\partial \theta}{\partial \bar{r}} + \frac{\partial \theta^2}{\partial \bar{r}^2} + \frac{\cot \phi}{\bar{r}^2} \frac{\partial \theta}{\partial \phi} + \frac{1}{\bar{r}^2} \frac{\partial \theta^2}{\partial \phi^2} \quad (2)$$

subject to:

$$\frac{\partial \theta(\bar{r} = 0, \phi, \tau > 0)}{\partial \bar{r}} = 0, \quad \frac{\partial \theta(\bar{r} = 1, \phi, \tau > 0)}{\partial \bar{r}} = -\frac{1}{Nr} \theta^4 \quad (3a)$$

$$\frac{\partial \theta(\bar{r}, \phi = 0, \tau > 0)}{\partial \phi} = 0, \quad \frac{\partial \theta(\bar{r}, \phi = \pi/2, \tau > 0)}{\partial \phi} = 0 \quad (3b)$$

where T_∞ is taken to be zero and outside the penetration depth layer, $\theta = 1$. The solution of this set of equations, will predict the unsteady temperature profile provided that: the assumption of two dimensional conduction with azimuthal symmetry applies, the droplet is spherical in shape, the material is isotropic and homogeneous, no modes of heat transfer other than conduction within the sample and surface radiation occur, Fourier's law of conduction is valid within the droplet, and the diffusivity can be assumed constant over the temperature range of the experiment.

Discretization: Due to the nonlinearity of the radiation boundary condition and the sample geometry, the problem must be solved numerically. The finite difference Alternating Direction Implicit (ADI) method [11] is chosen to solve the governing system of equations in the second time domain because it has been shown to be stable for all time

steps, unlike fully explicit methods [12], and it is computationally more efficient than fully implicit algorithms [13,14]. The equations are discretized with forward differencing in time and central differencing in the spatial derivatives $\bar{r}$ and ϕ . The ADI method consists of defining the spatial derivatives in one direction implicitly so that a tridiagonal system of equations in the direction chosen can be solved. In the subsequent time step, the spatial derivatives in the other direction are defined implicitly and the resulting system of equations solved. Therefore, at each time step, an $N \times N$ matrix is solved P times, where N is the number of nodes in the implicitly defined direction, and P is the number of nodes in the explicitly defined direction. See Appendix A for a brief description of ADI as it applies to this problem. Using ADI, the governing equation is solved subject to the boundary conditions and initial condition. Special cases for the tridiagonal system of equations arise at the boundaries, and a detailed listing of the different discretized forms of the governing equation encountered can be found in Appendix B. The only departure from straightforward application of ADI occurs at the surface of the sphere where the nonlinear radiation boundary condition must be incorporated into the equations. To get the radiation boundary condition in suitable linear form for a matrix solution, it is assumed that the temperatures at subsequent time steps are relatively equal. Then the temperature of the unknown temperature at the $n+1$ time-step can be linearized as follows:

$$\theta_{i,j}^{n+1} \approx 4\theta_{i,j}^{n+1}(\theta')^3 - 3(\theta')^4 \quad (4)$$

where θ' can be approximated with $\theta_{i,j}^n$. This equation is an excellent approximation when θ' is approximately equal to $\theta_{i,j}^{n+1}$ as is the case in subsequent time steps. The following equations are a few forms of the discretized governing equation for the case in

which the $\bar{r}$ - direction derivatives are defined implicitly. Again, refer to Appendix B for a more detailed listing.

For an interior node:

$$\begin{aligned} \frac{1}{\alpha} \frac{\theta_{ij}^{n+1} - \theta_{ij}^n}{\Delta\tau} = & \frac{1}{(\Delta\bar{r})^2} \left[\left(1 - \frac{1}{i}\right) \theta_{i+1,j}^{n+1} - 2\theta_{ij}^{n+1} + \left(1 + \frac{1}{i}\right) \theta_{i-1,j}^{n+1} \right] + \\ & + \frac{1}{2i^2(\Delta\bar{r}\Delta\phi)} \left[(2 - \cot\phi\Delta\phi) \theta_{i,j-1}^n - 4\theta_{ij}^n + (2 + \cot\phi\Delta\phi) \theta_{i,j+1}^n \right] \end{aligned} \quad (5)$$

For a node on $\bar{r} = 1, \phi = 0$ boundary:

$$\begin{aligned} \frac{1}{\alpha} \frac{\theta_{ij}^{n+1} - \theta_{ij}^n}{\Delta\tau} = & \frac{2}{(\Delta\bar{r})^2} (\theta_{i+1,j}^{n+1} - \theta_{ij}^{n+1}) - \frac{2}{i N_{\bar{r}}} (i+1) (\theta_{ij}^{n+1})^4 + \\ & + \frac{2}{i^2(\Delta\bar{r}\Delta\phi)} [\theta_{i,j+1}^n - \theta_{ij}^n] \end{aligned} \quad (6)$$

For a node at the origin:

$$\frac{1}{\alpha} \frac{\theta_{ij}^{n+1} - \theta_{ij}^n}{\Delta\tau} = \frac{6}{(\Delta\bar{r})^2} (\theta_{i+1,j}^{n+1} - \theta_{ij}^{n+1}) \quad (7)$$

The discretized form of the equations can be implemented in an ADI FORTRAN computer program to predict the nondimensional temperature profile of the droplet as a function of diffusivity at discrete time steps. The inverse conduction problem is then

solved by minimizing the error between the experimentally obtained surface temperature history and the numerically predicted temperatures at one or more locations on the surface of the sphere. The minimization is accomplished by using a modified quasilinearization algorithm (MQA). The functional to be minimized is:

$$\sum_{k=0}^{NK} \sum_{l=1}^{NL} (\tilde{\theta} - \theta)^2 = 0 \quad (8)$$

where θ is the numerically predicted temperature at the same location and time where $\tilde{\theta}$ is measured.

RESULTS AND DISCUSSION

Demonstration of procedure: To demonstrate the method, both one and two dimensional temperature profiles of nickel, iron, and copper were studied with conduction to radiation parameters of ten, twenty, and thirty respectively. The analysis given consists of two portions. The first study consists of determining actual values of diffusivity by substituting a theoretically predicted one dimensional cooling profile for the experimental data and comparing it to the profiles generated by the computer program with a zero heat flux condition on the lasers. This portion of the demonstration also provides the mesh size and timestep requirements necessary for convergence of the solution. The second portion of the analysis involves studying variations in the cooling profile for different nondimensional input parameters and then studying the error propagation that results from perturbations in the input data. In the error analysis, the cooling profile calculated for a known diffusivity and error-free input parameters is compared to the cooling profile generated with variations in input data. This analysis provides insight to the sensitivity of the final result on the accuracy of the input data. Then, an example is given that shows how order of magnitude estimates for any material's diffusivity can be obtained from the data given.

Summary of Experimental Procedure:

- 1) In the experimental portion of the method, a sample is levitated, and melted with an external heating device such as a laser. The droplet size is measured, and the surface temperature history of one or more points on the surface of the sphere is recorded. It is assumed that the pulse width, duration, and heat flux are known in addition to the density, specific heat, and emissivity of the droplet. The

experiment is expected to last approximately 30 seconds during which temperature readings will be taken at approximately each second.

- 2) The data acquired during the experimental portion is then used in a computer program that converts the data to nondimensional parameters and compares the result with a numerically predicted nondimensional profile. The value of α that minimizes the functional (8) is the correct α .

Algorithm:

- 1) Input the pulse radius, duration and heat flux; the material's specific heat, emissivity, density; the nondimensional time-step size and mesh size; and the droplet initial temperature and surface temperature history.
- 2) Guess α and compute the conduction to radiation parameter.
- 3) Compute the calculated nondimensional temperature profile using a suitable differencing technique.
- 4) Compute the norm of the least squared error and adjust the value of α so that the norm between the measured and calculated temperatures is decreased.
- 5) Repeat steps 2 through 4 until the error is within acceptable limits. The value of α that minimizes the functional is the correct α .

Droplet Size Requirements: For Fourier's law of conduction to be valid as the droplet cools, the size of the droplet must be such that its Biot number is greater than 0.1. This requirement sets the lower limit on the droplet size. The upper limit on droplet size is dictated by the levitator-- the maximum force it can generate, and the size of its

levitation coils. For this experiment, droplet sizes of approximately three to five centimeters are expected.

Marangoni Convection: For space based application, the absence of gravity removes the possibility of forced convection brought on by buoyant instabilities. However, with the temperature gradients in the droplet caused by the laser pulse and subsequent cooling of the sphere, it must be shown that the onset of convective instability, predicted by the Marangoni number, can be avoided. An order of magnitude estimate for the Marangoni number, Ma , in terms of droplet radius is $10^{-8}b$. Pratzner and Homsy [15] show that the lowest critical Marangoni number for a cooling sphere is approximately ten. It is observed that the Marangoni numbers encountered in this experiment are several orders of magnitude smaller than the critical Marangoni number. Therefore Marangoni convection can be neglected safely.

One Dimensional Results: In studying the one dimensional cooling profile of the droplet, the experimental cooling profile is approximated numerically by a method developed by Ayers [16], and the thermal property parameters are obtained from [1]. With the input parameters specified, criteria on finite difference mesh size requirements, and timestep requirements were established. As can be seen in Fig. 4, a mesh size of one hundred by one hundred nodes for the quarter sphere was established as sufficient. Further refinement of the mesh beyond 100×100 nodes leads to unnecessary round off error. Referring to Fig. 5, it is also shown that a nondimensional timestep size of $1.0e-04$ is sufficient for a stable solution over nondimensional time periods greater than 0.001. For nondimensional time periods less than 0.001, Fig. 6 shows that a nondimensional timestep of $1.0e-05$ provides a stable solution. With the stated mesh criteria, Table 1 shows that this method yields results accurate to within approximately a tenth of a percent of the values

given by [1] for approximately one hundred data points over a nondimensional time period of 0.01. This translates to about thirty to sixty seconds for the experiment corresponding to approximately a twenty to thirty degree drop in temperature, and the results obtained are well within the uncertainty range given by the tables in [1].

Two Dimensional Results: By studying the two dimensional cooling profile of the droplet for a desired maximum rise in temperature, it is possible to obtain estimations for the laser power requirements and dimensions. For a ten percent rise in temperature above the initial temperature, rough estimations require the laser to generate several hundred watts per square centimeter over the duration of approximately a second. From Eqs. (1) and (3) it can be seen that the nondimensional parameters that affect the two dimensional cooling profile of the droplet are $\bar{Q}$, ϕ_o (defined by the ratio a/b), $\Delta(\tau_L)$, and Nr . With each of these nondimensional parameters specified, nondimensional cooling curves can be generated so that with the necessary experimental input data all that is required to obtain an estimate for a specific material's diffusivity is a comparison of the experimentally obtained and numerically predicted curves.

Figs. 7 and 8 show the dependence of the cooling curve on the conduction to radiation parameter, Nr , at $\phi = 0$ and $\phi = \pi/2$. As expected, the predicted temperatures increase with increasing Nr because the relative importance of the radiative heat transfer at the surface of the sphere is decreased. Fig. 9 shows the effect of varying the penetration depth of the laser pulse on the cooling profile. In effect, the magnitude of the penetration depth, $\Delta(\tau_L)$, is a measure of the pulse duration or magnitude so that for a penetration depth of zero, the droplet has a one dimensional cooling profile. The greater the penetration depth, the greater the temperature rise at the surface of the sphere. The effect of varying the ratio a/b on the cooling profile of the droplet is displayed in Figs. 10 and 11 for $\phi = 0$ and

$\phi = \pi/2$ respectively. For a pulse width of zero, the droplet has a one dimensional cooling profile. In Fig. 11 at $\phi = \pi/2$, note that for $a/b = 1$ the droplet experiences a slight rise in temperature before it cools. This rise shows that initially, the heat from the laser pulse diffuses to this point more quickly than it is dissipated at the surface. Figs. 12 and 13 show how the cooling profile of the droplet changes with different nondimensional pulse heat fluxes, $\bar{Q}$. At $\phi = 0$, the temperature profiles increase in magnitude with increased pulse heat flux. Again, a heat flux of zero from the laser pulse corresponds to a one dimensional cooling profile. At $\phi = \pi/4$ just outside the range of the pulse width, it is shown that although variations in $\bar{Q}$ are noticeable, they are much less significant because as the pulse heat flux ranges from approximately 0.0 to 1000 w/cm², the temperature range changes only a half a percent corresponding to about 10K.

Error Analysis: In an overall comparison with the results obtained by [10], this method yields results that are less sensitive to error in the lumped property parameter M , and more sensitive to bias error in temperature (error in which the entire surface temperature history is shifted a specified percentage) and error in droplet radius.

Studying the effect of bias error on the result for diffusivity revealed that a 1% error in temperature yields approximately a 10% error in diffusivity depending on the material. This result shows the strong dependence of α on the accuracy of the temperature measuring device. It is assumed that this error is eliminated.

The effect of a normal error distribution in the experimental temperature profile is also studied and shows that a 10% error in temperature results in approximately a 15% error in diffusivity (see Fig. 14). The difference in error for calculated diffusivity also varied significantly from material to material. This error can be attributed to the widely varying thermal properties, or propagation of error in the data used from [1]. The results

obtained for nickel are assumed to be the most reliable because the data obtained from [1] for nickel at 1400K had the least uncertainty compared to the other materials studied.

The effect of varying the lumped property parameter, M , resulted in a one to one relation in percent error in diffusivity to percent error in M . The method developed by [10] gives a slightly more sensitive two to one relation in percent error in diffusivity to percent error in M .

A 10% error in droplet radius resulted in about a 25% error in diffusivity. In comparison, this method gives almost a ten percent greater error in diffusivity for the same error in droplet radius than the method presented by [10]. In the actual experiment, however, the error in droplet size measurement is not expected to be large.

The effect of error in the pulse width, duration, and heat flux measurements, and the error in the determination of the temperature measurement site location was also studied. Variations in the determination of pulse width resulted in approximately a 7% error in diffusivity for about a 10% error in pulse width. Error in the measurement of the duration of the laser pulse resulted in errors of less than 10% in diffusivity for a 10% error in $\Delta(\tau_L)$. A 10% error in the heat flux of the beam created approximately a 3 - 8% error in diffusivity for the materials tested. In studying the effects of measurement site location error, it is shown in Fig. 15 that a 2° error in location determination results in about a 1-2% error in diffusivity.

Therefore, from the error analysis performed, it is shown that the method described is most sensitive to bias error in temperature measurement. It is also sensitive to error in measurement site location and droplet radius, but it is assumed that these errors can be minimized so that the final result for diffusivity is within acceptable error limits. The results from the above error analysis are summarized in Table 2 along with order of magnitude estimations for expected experimental parameters.

Example: To demonstrate how diffusivity estimations can be obtained from the curves provided, an example is given for nickel at 1400K. Typical experimental parameters were chosen as described in Fig. 16, and the experimental cooling profile simulated with the expected value of α from [1]. The experimental cooling profile also reflects a 1% normal error distribution in temperature, a 1% error in M , and a 1° error in site location, the parameters in which error is most likely to occur. Since varying the diffusivity in predicted the cooling profile enters the problem through the conduction to radiation parameter in the thermal conductivity, the problem is solved by finding the conduction to radiation parameter that makes the predicted cooling curve coincide with the experimental cooling curve. Fig. 16 shows a band of predicted cooling profiles for nickel with diffusivities ranging from $1.52\text{e-}05$ - $1.64\text{e-}05$ m^2/s so that after smoothing the experimental temperature data, the curves can be examined to determine the correct value of diffusivity. In Fig. 17, it can be seen that the cooling curve corresponding to a diffusivity of $1.60\text{e-}05$ m^2/s matches the expected curve best for the given parameters. This predicted value differs from the expected value of $1.58\text{e-}05$ m^2/s (given by [1]) by approximately 1%. In like manner, any experimental cooling profile can be compared to the curves provided in Figs. 7-13 to obtain rough estimates for a specific material's diffusivity.

SUMMARY AND CONCLUSIONS

A method is proposed by which the thermal diffusivity of high temperature liquid metals can be determined. The method consists of levitating a droplet and melting it with laser beams from either side so that a two dimensional cooling profile is created that is symmetric about the origin. The cooling profile of the droplet is predicted by an ADI finite difference method and the inverse conduction problem solved for the correct diffusivity by minimizing the difference in the calculated and experimentally measured temperatures at one or more locations on the surface of the sphere.

It is found that the method yields results accurate to within a tenth of a percent of expected values. Since the problem discussed requires highly sophisticated instrumentation, the effects of error propagation on the final determination of diffusivity was studied extensively so that recommendations can be made for the actual experimental procedure (see Table 2). It was found that the method is most sensitive to bias error in temperature measurement. Therefore the accuracy of the temperature measuring device is one of the most important factors in the final determination of α in addition to accurate determination of the location of the measurement site, and droplet radius. Variations in the rest of the input parameters result in much lower values of error for α .

Although sensitive to variations in input data, performing the experiment within acceptable error limitations yields accurate results as has been shown in the example provided. In studying the one dimensional cooling profile of the droplet, the error in values obtained for the diffusivity of the materials studied was only on the order of a tenth of a percent. To verify the method however, and further study the sensitivity of the method to error in input parameters, experimentation with solids on earth is recommended before implementation in space.

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Fig. 1 Droplet Geometry

Fig. 2 Experimental Setup

Fig. 3 Computational Domain and Penetration Depth

Fig. 4. Cooling Profile for Different Mesh Sizes
 $N=N_I=N_J$, $N_r=10$, $dk=1.0e-05$

Fig. 5. Cooling Profile for Large Time-Steps, dk
 $Nr=20$, $Nl=Nj=100$

Fig. 6. Cooling Profile for Small Time-Steps, dk
 $Nr=20$, $Nl=NJ=100$

Fig. 7. Variation of Cooling Profile at $\phi = \pi/2$
for different Nr : $Nl=NJ=100$, $dk=1.0e-04$
 $\bar{Q}=5.0$, $a/b=0.3$, $\Delta_L=10\%$

Fig. 8. Variation of Cooling Profile at $\phi = 0$
for different Nr : $Nl=NJ=100$, $dk=1.0e-04$
 $\bar{Q}=5.0$, $a/b=0.3$, $\Delta_L=10\%$

Fig. 9. Variation of Cooling Profile at $\phi = 0$
for Different Penetration Depths: $Nl = Nj = 100$,
 $dk = 1.0e-04$, $\bar{Q} = 5.0$, $Nr = 10$, $a/b = 0.3$

Fig. 10. Variation of Cooling Profile at $\phi = 0$ for Different a/b Ratios: $Nl=Nj=100$, $dk=1.0e-04$, $\bar{Q}=5.0$, $Nr=10$, $\Delta_L=10\%$

Fig. 11. Variation of Cooling Profile at $\phi = \pi/2$ for Different a/b Ratios: $N_I = N_J = 100$, $dk = 1.0e-04$, $\bar{Q} = 5.0$, $Nr = 10$, $\Delta_L = 10\%$

Fig. 12. Variation of Cooling Profile at $\phi = 0$
for Different $\bar{Q}$: $Nl=Nj=100$, $dk=1.0e-04$
 $a/b=0.3$, $Nr=10$, $\Delta_L=10\%$

Fig. 13. Variation of Cooling Profile at $\phi = \pi/4$ for Different $\bar{Q}$: $Nl=Nj=100$, $dk=1.0e-04$, $a/b=0.3$, $Nr=10$, $\Delta_L=10\%$

Fig. 14. Influence of Normal Error in Temperature on α : $N_I=N_J=100$, $dk=1.0e-04$

Fig. 15. Influence of error in Measurement Site Location on α : $N_I=N_J=100$, $dk=1.0e-04$

Fig. 16. Variation of Predicted Cooling Profile for Nickel at 1400K and $\phi = \pi/2$ for Different α :
 $N_I = N_J = 100$, $dk = 1.0e-04$, $\bar{Q} = 5.0$, $a/b = 0.3$,
 $\Delta_L = 10\%$

Fig. 17. Variation of Predicted Cooling Profile at $\phi = \pi/2$
for Different α : $N_I = N_J = 100$, $dk = 1.0 \times 10^{-4}$
 $\bar{Q} = 5.0$, $a/b = 0.3$, $\Delta_L = 10\%$

Table 1 Results

Material	Nr	ε^*	ρ kg/m ³	c_p J/(kg-K)	calculated α m ² /s	percent error
Nickel	10	0.25	8324.61	611.3	1.5863e-05	0.3984
Iron	20	0.30	7368.71	648.9	6.6626e-06	0.1897
Copper	30	0.32	785.52	5275.4	8.0697e-05	0.1197

*Values for ε , ρ , and c_p are taken from [1].

Table 2 Experimental Parameters**Order of magnitude estimates for Experimental Parameters:**

Droplet size, b	~ 3 - 5 cm
Pulse heat flux, Q	~ 200 w/cm ²
Pulse duration	~ 0.01 - 1.0 sec
Duration of experiment	~ 30 - 60 sec

Dependence of α on Experimental parameters:

10.0%	Bias temperature error	}	1% error in α
4.5 %	Normal temperature error		
2.0°	Measurement site location error		
1.0%	Error in M		
1.0%	Error in Droplet radius		
1.0%	Error in Pulse heat flux		
1.0%	Error in Pulse width		
1.0%	Error in Pulse duration		

Appendix A

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Alternating Direction Implicit Method

$\bar{r}$ direction derivatives defined implicitly:

$$f(\theta_{i-1,j}^{n+1}, \theta_{i,j}^{n+1}, \theta_{i+1,j}^{n+1}) = f(\theta_{i,j-1}^n, \theta_{i,j}^n, \theta_{i,j+1}^n)$$

$$\begin{array}{ccc} \text{NI} \times \text{NI} & \text{NI} \times 1 & \text{NI} \times 1 \\ \left[\begin{array}{cccc} - & - & 0 & \\ - & - & - & 0 \\ 0 & - & - & - & 0 \\ \cdot & & \text{.....} & & \cdot \\ 0 & & & 0 & - & - & - & 0 \\ 0 & & & & 0 & - & - & - \\ 0 & & & & & 0 & - & - \end{array} \right] \begin{bmatrix} \theta_{1,j}^{n+1} \\ \vdots \\ \theta_{\text{NL},j}^{n+1} \end{bmatrix} & = & \begin{bmatrix} f(\theta_{1,j-1}^n, \theta_{1,j}^n, \theta_{1,j+1}^n) \\ \vdots \\ f(\theta_{i,j-1}^n, \theta_{i,j}^n, \theta_{i,j+1}^n) \\ \vdots \\ f(\theta_{\text{NL},j-1}^n, \theta_{\text{NL},j}^n, \theta_{\text{NL},j+1}^n) \end{bmatrix} \end{array}$$

ϕ direction derivatives defined implicitly:

$$f(\theta_{i,j-1}^{n+1}, \theta_{i,j}^{n+1}, \theta_{i,j+1}^{n+1}) = f(\theta_{i-1,j}^n, \theta_{i,j}^n, \theta_{i+1,j}^n)$$

$$\begin{array}{ccc} \text{NJ} \times \text{NJ} & \text{NJ} \times 1 & \text{NJ} \times 1 \\ \left[\begin{array}{cccc} - & - & 0 & \\ - & - & - & 0 \\ 0 & - & - & - & 0 \\ \cdot & & \text{.....} & & \cdot \\ 0 & & & 0 & - & - & - & 0 \\ 0 & & & & 0 & - & - & - \\ 0 & & & & & 0 & - & - \end{array} \right] \begin{bmatrix} \theta_{i,1}^{n+1} \\ \vdots \\ \theta_{i,\text{NJ}}^{n+1} \end{bmatrix} & = & \begin{bmatrix} f(\theta_{i-1,1}^n, \theta_{i,1}^n, \theta_{i+1,1}^n) \\ \vdots \\ f(\theta_{i-1,j}^n, \theta_{i,j}^n, \theta_{i+1,j}^n) \\ \vdots \\ f(\theta_{i-1,\text{NJ}}^n, \theta_{i,\text{NJ}}^n, \theta_{i+1,\text{NJ}}^n) \end{bmatrix} \end{array}$$

Appendix B

Finite Difference Equations for Implicitly Defined $\bar{r}$ - Direction Derivatives

Case 1 - Interior Node:

$$\begin{aligned} \frac{1}{\alpha} \frac{\theta_{ij}^{n+1} - \theta_{ij}^n}{\Delta\tau} = & \frac{1}{(\Delta\bar{r})^2} \left[\left(1 - \frac{1}{i}\right) \theta_{i-1,j}^{n+1} - 2\theta_{ij}^{n+1} + \left(1 + \frac{1}{i}\right) \theta_{i+1,j}^{n+1} \right] + \\ & + \frac{1}{2i^2(\Delta\bar{r}\Delta\phi)} \left[(2 - \cot\phi\Delta\phi) \theta_{i,j-1}^n - 4\theta_{ij}^n + (2 + \cot\phi\Delta\phi) \theta_{i,j+1}^n \right] \end{aligned}$$

Case 2 - Node on the $\phi = 0$ boundary:

$$\begin{aligned} \frac{1}{\alpha} \frac{\theta_{i,j}^{n+1} - \theta_{i,j}^n}{\Delta\tau} = & \frac{1}{(\Delta\bar{r})^2} \left[\left(1 - \frac{1}{i}\right) \theta_{i-1,j}^{n+1} - 2\theta_{i,j}^{n+1} + \left(1 + \frac{1}{i}\right) \theta_{i+1,j}^{n+1} \right] + \\ & + \frac{2}{i^2(\Delta\bar{r}\Delta\phi)} [\theta_{i,j+1}^n - \theta_{i,j}^n] \end{aligned}$$

Case 3 - Node on the $\phi = \pi/2$ boundary:

$$\begin{aligned} \frac{1}{\alpha} \frac{\theta_{i,j}^{n+1} - \theta_{i,j}^n}{\Delta\tau} = & \frac{1}{(\Delta\bar{r})^2} \left[\left(1 - \frac{1}{i}\right) \theta_{i-1,j}^{n+1} - 2\theta_{i,j}^{n+1} + \left(1 + \frac{1}{i}\right) \theta_{i+1,j}^{n+1} \right] + \\ & + \frac{2}{i^2(\Delta\bar{r}\Delta\phi)} [\theta_{i,j-1}^n - \theta_{i,j}^n] \end{aligned}$$

Case 4 - Node on the $\bar{r} = 1$ boundary:

$$\begin{aligned} \frac{1}{\alpha} \frac{\theta_{ij}^{n+1} - \theta_{ij}^n}{\Delta\tau} = & \frac{2}{(\Delta\bar{r})^2} (\theta_{i-1,j}^{n+1} - \theta_{ij}^{n+1}) - \frac{2}{i N_r} (i+1) (\theta_{ij}^{n+1})^4 + \\ & + \frac{2}{i^2 (\Delta\bar{r} \Delta\phi)} [(2 - \cot\phi \Delta\phi) \theta_{ij-1}^n - 4\theta_{ij}^n + (2 + \cot\phi \Delta\phi) \theta_{ij+1}^n] \end{aligned}$$

Case 5 - Node on the $\bar{r} = 1, \phi = 0$ boundary:

$$\begin{aligned} \frac{1}{\alpha} \frac{\theta_{i,j}^{n+1} - \theta_{i,j}^n}{\Delta\tau} = & \frac{2}{(\Delta\bar{r})^2} (\theta_{i-1,j}^{n+1} - \theta_{i,j}^{n+1}) - \frac{2}{i N_r} (i+1) (\theta_{i,j}^{n+1})^4 + \\ & + \frac{2}{i^2 (\Delta\bar{r} \Delta\phi)} [\theta_{i,j+1}^n - \theta_{i,j}^n] \end{aligned}$$

Case 6 - Node on the $\bar{r} = 1, \phi = \pi/2$ boundary:

$$\begin{aligned} \frac{1}{\alpha} \frac{\theta_{i,j}^{n+1} - \theta_{i,j}^n}{\Delta\tau} = & \frac{2}{(\Delta\bar{r})^2} (\theta_{i-1,j}^{n+1} - \theta_{i,j}^{n+1}) - \frac{2}{i N_r} (i+1) (\theta_{i,j}^{n+1})^4 + \\ & + \frac{2}{i^2 (\Delta\bar{r} \Delta\phi)} [\theta_{i,j-1}^n - \theta_{i,j}^n] \end{aligned}$$

Case 7 - Node at the origin:

$$\frac{1}{\alpha} \frac{\theta_{i,j}^{n+1} - \theta_{i,j}^n}{\Delta\tau} = \frac{6}{(\Delta\bar{r})^2} (\theta_{i+1,j}^{n+1} - \theta_{i,j}^{n+1})$$

Finite Difference Equations for Implicitly Defined ϕ Direction Derivatives

The finite difference equations for the implicitly defined ϕ derivatives are the same as those for the implicitly defined $\bar{r}$ derivatives except the "n+1" superscripts are interchanged with the "n" superscripts.