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**Importance sampling for analysis of direct detection optical
communication systems**

Mandayam, Narayan Bhimarayan Thiru, M.S.

Rice University, 1991

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**300 N. Zeeb Rd.
Ann Arbor, MI 48106**

RICE UNIVERSITY

**Importance Sampling for Analysis of Direct
Detection Optical Communication Systems**

by

Narayan B. T. Mandayam

A THESIS SUBMITTED
IN PARTIAL FULFILLMENT OF THE
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APPROVED, THESIS COMMITTEE:

Dr. Behnaam Aazhang, Director
Associate Professor
Electrical and Computer Engineering

Dr. Naomi J. Halas
Assistant Professor
Electrical and Computer Engineering

Dr. Don H. Johnson
Professor
Electrical and Computer Engineering

Dr. James F. Young
Associate Professor
Electrical and Computer Engineering

Houston, Texas

April, 1991

Importance Sampling for Analysis of Direct Detection Optical Communication Systems

Narayan B. T. Mandayam

Abstract

Analytical solutions of the performance of optical communication systems are difficult to obtain and often, Monte Carlo simulations are used to achieve realistic estimates of the performance of such systems. However, for high performance systems, this technique requires a large number of simulation trials for the estimates to be in a reasonable interval of confidence, with the number of trials increasing linearly with the performance of the system. Various variance reduction schemes for Monte Carlo simulations, known as Importance Sampling have been employed to significantly reduce the number of simulation trials for radio communication systems, but have found little application in the optical domain. We develop an Importance Sampling technique to estimate the performance of direct detection optical systems, where the “gain” of Importance Sampling over Monte Carlo simulations is shown to increase linearly with the system performance.

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$$\dagger \Gamma \approx \frac{1}{P_e} \forall P_e \leq 10^{-8}$$

To all ye who dare to simulate!

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Chapter 1

Introduction

In most optical communication systems in existence today, the statistics of the underlying random processes and the structure of the detection systems are quite complicated, making it very difficult to achieve analytical performance estimates. Therefore, efforts have been concentrated on obtaining asymptotic approximations and bounds for estimating system error rates. The *Central Limit Theorem* has been the most powerful tool for approximating and simplifying the performance analysis of optical communication systems. This approach requires infinitely large sample sizes for analytically exact solutions. This rather unrealistic approach yields estimates of the system performance that are misleading for certain distributions and recently in a parallel field in the radio domain, the Central Limit Theorem has been shown to deviate exponentially in percent error from the true system performance for a wide class of signal detection systems [1].

A widely used method for achieving more realistic estimates of optical detection systems performance is the Chernoff-bound [2, 3]. Often this bound is not very tight and may result in erroneous estimates of the system performance. Specifically, the Chernoff bound has been shown to be loose for direct-detection optical systems with avalanche photodiodes [4]. Other bounds, utilizing a modified Gaussian quadrature rule based on approximate moments have also been derived [5]. The performance of optical receivers with avalanche diode detectors has also been studied through saddlepoint approximation techniques [6]. Due to the inherent complexity of such

systems, these techniques are often highly intractable, however, there is still a need to cater to such complicated systems since sensitive optical receivers operate at minimum detectable power levels of the order of a nanowatt and at typical fiber optic system wavelengths these weak currents are susceptible to being corrupted by the noise from the electronic amplifier stages following the detector. Avalanche photodiodes are used to increase the detector output before electronic amplification in order to obtain acceptable receiver performance and the complex statistical nature of these photodiodes makes performance analysis quite cumbersome.

Another, often more tractable and versatile method for obtaining performance estimates is Monte Carlo simulation. This method can produce very accurate estimates of system performance if the number of simulation trials is sufficiently large. However, the drawback of this approach is that today optical communication systems operate with a very small probability of error, P_e . This puts a tremendous computational burden on the simulation, in that a large number of simulation trials are required to achieve a meaningful estimate of the system performance. As an example, for a 95% confidence interval of $[2P_e/5, 8P_e/5]$ we require approximately $10/P_e$ simulation trials [7, 8]. This can be quite large for $P_e \leq 10^{-7}$, thus making conventional Monte Carlo methods impractical for simulating optical detection systems.

A technique known as *Importance Sampling* has been employed to greatly reduce the number of simulation trials [9, 10, 11, 12, 13, 14]. This technique is based on biasing the noise distribution such that more samples are taken from the *important* regions in the observation space (regions that cause errors) and then unbiasing the estimate of system performance with a weighting function. Within certain restrictions, it can be shown that this biasing technique will reduce the variance of the estimate, thereby reducing the number of simulation trials for the same level of accuracy. Shanmugam and Balaban [15] demonstrated the effectiveness of Importance Sampling applied to

a communication system problem, but offered no general technique for utilizing this powerful method. In the radio domain, Importance Sampling simulations have been applied for the analysis of detection systems [11, 12, 13, 14, 16, 17, 18]. Certain quick simulation techniques have been proposed for analysis of APD based digital light-wave communication links [19]. An Importance Sampling scheme has been proposed by Foschini [20] for efficient simulation of filtered light waves with phase noise, but in general, Importance Sampling has been rarely applied to optical systems with direct detection and more so for fiber based optical networks [21].

Optical media, namely fiber-optic links, are ideally suited for networking because of the ample available bandwidth. A semiclassical analysis of the error rate for optical CDMA systems has been proposed [22] and bounds have been developed. Code division multiplexing techniques have been proposed for multifiber networks [23], but have used electronic parts for encoding and decoding. In fiber-optic based systems, the limited speed of electronic signal processors curtails the fiber bandwidth that can be used. Optical components, when fully developed and integrated, will offer much higher speeds than their electronic counterparts [24, 25]. The technique of fiber-optic code division multiple access (FO-CDMA) takes advantage of the excess bandwidth in single mode fibers to map low information rate electrical or optical signals into high rate optical pulse sequences to provide random, asynchronous communications access, free of network control among many users. A class of optical codes called optical orthogonal codes (OOC's) have been developed [26] and applied to FO-CDMA systems [27], but the Poisson characteristics of the photodetectors have not been taken into consideration. Since these analyses are highly intractable, there is a great need for developing Importance Sampling techniques for the analysis of such FO-CDMA systems.

In this thesis, we analyze three different optical communications systems and arrive at estimates of the performance of these systems through both Monte Carlo and Importance Sampling simulation techniques. Moreover, it will be shown that the Importance Sampling technique allows for very easy implementation in computer simulations. We then proceed to analyze the “gain” using Importance Sampling simulations over standard Monte Carlo simulations by comparing the number of simulation trials required by each method to obtain a fixed estimator accuracy. In particular, we will consider a single user binary communication system employing an ideal avalanche photodiode, where the avalanche gain is unity. We will also analyze a single user system employing a random gain avalanche photodiode. For the multi user case, we will consider a fiber-optic code division multiple access (FO-CDMA) network employing OOC’s and develop an Importance Sampling technique of Monte Carlo simulations for the network, taking into consideration the Poisson statistics of the system and present our analysis for the case of chip synchronous codes.

Chapter 2

Optical Communication Systems with Ideal Photodiodes

A received optical signal can be modeled as a *photon arrival* process. In a simple setting, this process has a Poisson density with arrival rate proportional to the received optical power plus a dark current component dependent on the photodetector physical structure. The photodetector in an optical communication system converts the incident optical power, in terms of the number of photons seen, to electrical current and this electrical signal is processed to extract the information bit. In this chapter, we will consider systems with ideal photodiodes which are endowed with a constant avalanche gain of unity and develop an Importance Sampling scheme for efficiently estimating the performance of the system.

2.1 System Description

Consider a binary communications system where the information of the n^{th} bit in the bit interval $[(n-1)T, nT)$ is modulated onto the intensity of the incident light, giving rise to the following hypotheses testing problem at the receiver:

$$\begin{aligned} H_0 &: \text{photon intensity of } \lambda_0(t), \\ H_1 &: \text{photon intensity of } \lambda_1(t). \end{aligned} \tag{2.1}$$

For simplicity, we assume that the photodetector is also the decision making device, i.e., it is responsible for choosing between the hypotheses H_0 and H_1 . The received

electromagnetic signal at the photodetector can be characterized classically by the scalar field under each hypothesis as [28]

$$s_i(t) = \sqrt{2} \operatorname{Re}[S_i(t)\exp(j2\pi\nu t)] \quad ; \quad i = 0, 1,$$

at time t , where ν is the optical frequency of the carrier, $S_i(t)$ is a complex time envelope that is slowly varying in comparison to ν , and $\operatorname{Re}[\cdot]$ denotes the real part of the argument. The rate (or intensity) at which the photoelectrons emanate from the photodetector is related to the intensity of the incident light on its sensitive surface according to [28]

$$\lambda_i(t) + \lambda_d(t) = \frac{\eta}{h\nu} \int_{\wp} |S_i(t)|^2 d\wp + \lambda_d(t) \quad ; \quad i = 0, 1,$$

where η is the quantum efficiency of the detector (the probability that a quantum of light energy is converted into an electron), h is the Planck's constant, the integral is over the sensitive surface $\wp$ of the detector, and $\lambda_d(t)$ is the rate of generation of extraneous photoelectrons in the detector and accounts for the dark current. The arrival rate $\lambda_d(t)$ can be calculated by measuring the current produced when the incident optical power is zero, and it is assumed to be constant in time. The parameter T is the sampling period at which the output of the photodetector is sampled and $[(n-1)T, nT)$ denotes the n^{th} sampling interval. If $R(nT)$ denotes the number of photons received at the photodetector in time $[(n-1)T, nT)$ then the system can be described statistically as

$$p_{R(nT)|H_i}(r) = \frac{\exp(-\int_{(n-1)T}^{nT} (\lambda_i(t) + \lambda_d) dt) (\int_{(n-1)T}^{nT} (\lambda_i(t) + \lambda_d) dt)^r}{r!} \quad ; \quad i = 0, 1, \quad (2.2)$$

where $r \in \mathcal{Z} \triangleq \{0, 1, 2, \dots\}$ denotes the number of photons incident on the photodetector during the n^{th} interval namely $[(n-1)T, nT)$. The above expression represents the probability of receiving exactly r number of photons at the photodetector in

$[(n-1)T, nT)$. It can be seen that this expression is not that of an additive noise system, both the mean and the variance of the received signal being dependent on the arrival rates under the respective hypotheses unlike that of additive noise which has fixed mean and variance. The above equation will be the basis for the following development.

2.2 Problem Formulation

The two most popular binary signal sets for optical communication systems, that are easily realizable with today's technology, are on-off keying (OOK) and Manchester Coding. We will present all of our numerical results using the OOK signal set, however these results can be easily duplicated for Manchester codes.

We consider a homogeneous optical communication system employing $\lambda_0(t) = \lambda_0$ and $\lambda_1(t) = \lambda_1$. Without loss of generality, we consider the demodulation of the first bit which is in the interval $[0, T)$. The photodetector, therefore, faces the following two hypotheses

$$\begin{aligned} H_0 : p_{R|H_0}(r | H_0) &= \frac{\exp(-(\lambda_0 + \lambda_d)T)((\lambda_0 + \lambda_d)T)^r}{r!}, \\ H_1 : p_{R|H_1}(r | H_1) &= \frac{\exp(-(\lambda_1 + \lambda_d)T)((\lambda_1 + \lambda_d)T)^r}{r!}, \end{aligned} \quad (2.3)$$

where R is the photon count in $[0, T)$. The above set of expressions are the probability mass functions that will govern decision making under each hypothesis. Given equally likely hypotheses, (i.e., the aprior probabilities $\pi_i = \frac{1}{2}$ for $i = 0, 1$) we have the likelihood ratio as

$$\Lambda(r) = \exp((-\lambda_1 + \lambda_0)T) \left[\frac{\lambda_1 + \lambda_d}{\lambda_0 + \lambda_d} \right]^r, \quad (2.4)$$

which is compared to 1. Taking the logarithm and simplifying yields the following threshold

$$\gamma = \left\lceil \frac{(\lambda_1 - \lambda_0)T}{\log[(\lambda_1 + \lambda_d)/(\lambda_0 + \lambda_d)]} \right\rceil. \quad (2.5)$$

Thus the optimal detector counts the number of photoelectrons r in $[0, T)$ and compares it to the threshold γ and decides as follows

$$\begin{aligned} \text{bit } 0 \text{ was sent if } r < \gamma, \\ \text{bit } 1 \text{ was sent if } r \geq \gamma. \end{aligned} \quad (2.6)$$

The probability of error for the optimal detector can thus be computed as

$$P_e = \pi_0 \sum_{\Delta_1} p_{R|H_0}(r) + \pi_1 \sum_{\Delta_0} p_{R|H_1}(r), \quad (2.7)$$

where Δ_i is the decision region for H_i , and which in our case is given as

$$P_e = \frac{1}{2} \left[\sum_{r=0}^{\gamma-1} \frac{\exp(-(\lambda_1 + \lambda_d)T)((\lambda_1 + \lambda_d)T)^r}{r!} + \sum_{r=\gamma}^{\infty} \frac{\exp(-(\lambda_0 + \lambda_d)T)((\lambda_0 + \lambda_d)T)^r}{r!} \right] \quad (2.8)$$

It can be seen that the probability of error given H_0 is not equal to the probability of error given H_1 . This is a direct consequence of the probability density's dependence on the arrival rates, i.e., λ_i ; $i = 0, 1$.

A Monte Carlo (MC) simulation consists of generating a large number of samples and passing them through a mathematical model of the system at hand. The output sequence of this mathematical model is then compared to the input sequence and the errors are thus counted. The probability of error is then estimated as the ratio of the number of errors seen to the number of samples considered, that is

$$\hat{P}_{MC} = \frac{1}{M_{MC}} \sum_{j=1}^{M_{MC}} I(r_j), \quad (2.9)$$

where $I(r_j) = 1$ if the j^{th} bit is in error and $I(r_j) = 0$ if the j^{th} bit is correct, and M_{MC} represents the total sample size. It is easily shown that this estimator is unbiased, i.e., $E[\hat{P}_{MC}] = P_e$.

After minor algebraic manipulation, the variance of the Monte Carlo estimator can be shown to be

$$\text{var}(\hat{P}_{MC}) = \frac{P_e(1 - P_e)}{M_{MC}} \quad (2.10)$$

For small values of P_e the number of required samples for a given variance is very large, i.e., for values of $P_e \leq 10^{-7}$, the sample size required will be $M_{MC} \geq 10/P_e$, for an estimator accuracy of $\hat{P}_e \in [2P_e/5, 8P_e/5]$.

2.3 Importance Sampling Estimation

The Importance Sampling technique reduces the variance of the estimate of P_e by generating data from the biased probability mass functions $p_{R^*|H_i}$'s under the respective hypothesis, in place of $p_{R|H_i}$'s. This biasing mass function is chosen such that the random samples generated are more likely to come from the important regions (those that cause more errors) in the sample space. Since we will generate more errors than expected, we must weight each error such that an unbiased estimate is obtained. This biasing and weighting of events in the simulation will enable us to more accurately estimate P_e , i.e., obtain a reduced variance estimate of the system performance.

We obtain the Importance Sampling estimator by rewriting the error probability in (2.8) as

$$P_e = \frac{1}{2} \left[\sum_{\Delta_1} p_{R^*|H_0}(r | H_0) w(r | H_0) + \sum_{\Delta_0} p_{R^*|H_1}(r | H_1) w(r | H_1) \right], \quad (2.11)$$

where Δ_0 and Δ_1 are the decision regions defined in (2.7). The $p_{R^*|H_i}$ are chosen such that the above expression is summable. The resulting sample estimate of the above expectation is given by

$$P_e^* = \frac{1}{M_{IS}} \left[\sum_{j=1}^{M_{IS}} w(r_j^*) I(r_j^*) \right], \quad (2.12)$$

where $I(r_j^*)$ is the indicator function defined as before. However, the random samples r_j^* 's are now generated from the biasing density $p_{R^*|H_i}$, $i = 0, 1$. In the above

Importance Sampling estimator the weights associated with the biasing densities are given as

$$w(r | H_i) = \frac{p_{R|H_i}(r)}{p_{R^*|H_i}(r)}, \quad \forall r \in \mathcal{Z}, \quad (2.13)$$

such that $p_{R^*|H_i}(r) > 0$. Note that this estimator still counts errors based upon the original decision rule of choosing H_0 if $r \in \Delta_0$, and choosing H_1 if $r \in \Delta_1$, but each error is now weighted by $w(r_j)$ to compensate for this biasing effect.

The variance of the Importance Sampling estimate of the probability of a wrong decision can be shown to be

$$\text{var}[P_e^*] = \frac{\bar{W} - P_e^2}{M_{IS}}, \quad (2.14)$$

where $\bar{W}$ is defined as

$$\bar{W} = \frac{1}{2} \left[\sum_{\Delta_1} \frac{p_{R|H_0}^2(r) I_{\Delta_1}(r)}{p_{R^*|H_0}(r)} + \sum_{\Delta_0} \frac{p_{R|H_1}^2(r) I_{\Delta_0}(r)}{p_{R^*|H_1}(r)} \right], \quad (2.15)$$

or alternatively defined as

$$\bar{W} = E_{H_0}[w(r | H_0) I_{\Delta_1}(r)] + E_{H_1}[w(r | H_1) I_{\Delta_0}(r)], \quad (2.16)$$

where $I_{\Delta_i}(r)$ is equal to 1, $\forall r \in \Delta_i$, and is equal to 0, elsewhere.

Thus we have a method for estimating the error rate which counts errors based on the original decision rule, however, we generate data using the probability mass function p_{R^*} rather than p_R . The estimate is then unbiased by scaling the errors with the weighting function $w(r)$. We must note that the variance of P_e^* is dependent upon $\bar{W}$, P_e , and M_{IS} , whereas the variance of $\hat{P}_e$ is dependent upon P_e and M_{MC} . The parameter $\bar{W}$ will allow for the reduction of the variance of the estimate for a fixed number of trials and for a given system as compared to the standard Monte Carlo simulations.

The fundamental problem in Importance Sampling is to determine the biasing density p_{R^*} such that $\text{var}[P_e^*]$ is minimized for all values of P_e . From (2.15), we see

that $\bar{W}$ must be minimized with respect to p_{R^*} under each hypothesis, since Δ_0 and Δ_1 are determined by the form of the detector. Using the Calculus of Variations [12], it has been shown that the optimum probability mass function is degenerative and is given as

$$p_{R^*|H_i}^{(opt)}(r) = \frac{I_{\Delta_i}(r)p_{R|H_i}(r)}{P_e}, \quad i = 0, 1, \quad \forall r \in \mathcal{Z}. \quad (2.17)$$

Therefore, we must restrict ourselves to determining a suboptimal biasing density which yields a reduced variance estimate compared to conventional Monte Carlo techniques. Since variance reduction is not an easily quantifiable parameter, the expression used to define the usefulness of the Importance Sampling technique is the ratio of the number of samples needed for Monte Carlo simulation to the number of samples needed for Importance Sampling to achieve a fixed variance of the performance estimate. This term is referred to as the “gain” and is given by

$$\Gamma = \frac{M_{MC}}{M_{IS}} = \frac{P_e(1 - P_e)}{\bar{W} - P_e^2}. \quad (2.18)$$

From this expression, an idea of the time savings achieved through Importance Sampling is obtained. Thus maximizing this ratio Γ would mean saving time through Importance Sampling as compared to the standard Monte Carlo simulations. Clearly from the expression for Γ we see that the following condition would yield a realistic gain

$$P_e^2 < \bar{W} \leq P_e, \quad (2.19)$$

which for our binary system, via equations (2.11) and (2.15), it is sufficient to require

$$p_{R^*|H_1}(r | H_1) \geq p_{R|H_1}(r | H_1); \quad \forall r \in \Delta_0, \quad (2.20)$$

and

$$p_{R^*|H_0}(r | H_0) \geq p_{R|H_0}(r | H_0); \quad \forall r \in \Delta_1. \quad (2.21)$$

The above set of conditions definitely reduces the variance of the estimate of the system performance in the sense described above. There are several methods of arriving at a suboptimal density for any Importance Sampling situation. The commonly used methods are the *linear shift*, *increased variance*, and *exponential tilt* [12, 29, 30, 31, 32].

We will use the *exponential tilt* method for our problem as it has certain characteristics which will minimize the variance of our Importance Sampling estimator.

Definition : An exponential change of probability measure from $P_{R^*}(z)$ to $P_R(z)$ is defined at z as

$$dP_{R^*}(z) = \frac{\exp(sz)dP_R(z)}{M_R(s)}, \quad (2.22)$$

where $s \in \Re$ is a free parameter and $M_R(s)$ is the characteristic function of the measure P_R . We now present a modified version of the theorem due to Cottrell *et al.* [33]

Theorem 1(Cottrell *et al.*) [33] : For a Poisson process, if P_R (with parameter λ) is absolutely continuous with respect to P_{R^*} (with parameter λ^*), then among all the exponential changes of measure, the transformation from $P_R(z)$ to $P_{R^*}(z)$ has the asymptotic property that as the intensity approaches its extreme value,

$$\lim_{\lambda \rightarrow \lambda_e} \int \left[\frac{dP_R}{dP_{R^*}} \right]^2 dP_{R^*}, \quad (2.23)$$

is minimum. The proof of the above theorem can be found in [33].

As we are looking to minimize $\bar{W}$ in (2.15), the above theorem provides an exponential change of measure that can asymptotically (i.e., as the intensities approach their extreme values) minimize the variance of the Importance Sampling estimator of the binary optical communication system. If we make our sampling interval such that $T = 1$, then under hypothesis H_i we have the probability density function

$$p_{R|H_i}(r) = \frac{\exp[-(\lambda_i + \lambda_d)](\lambda_i + \lambda_d)^r}{r!}, \quad \forall r \in \mathcal{Z}. \quad (2.24)$$

Applying the exponential change of measure, the biasing density can be found as

$$p_{R^*|H_i}(r) = p_{R|H_i}(r) \frac{\exp(sr)}{M_{R|H_i}(s)}, \quad \forall r \in \mathcal{Z}, \quad (2.25)$$

where $M_{R|H_i}(s) = \exp[(\lambda_i + \lambda_d)(\exp(s) - 1)]$ is the characteristic function for a Poisson distribution. The above transformation results in the biasing density of the form

$$p_{R^*|H_i}(r) = \frac{\exp[-(\lambda_i + \lambda_d)\exp(s)][(\lambda_i + \lambda_d)\exp(s)]^r}{r!}, \quad \forall r \in \mathcal{Z}, \quad (2.26)$$

which is again Poisson distributed with parameter $(\lambda_i + \lambda_d)\exp(s)$. Thus we see that the transformation $p_{R|H_i}$ to $p_{R^*|H_i}$ results in an equivalent transformation of arrival rates from $\lambda_i + \lambda_d$ to $(\lambda_i + \lambda_d)^* \triangleq (\lambda_i + \lambda_d)\exp(s)$. Therefore, this exponential change of measure results in a new parametric class of probability measures with the intensity as the parameter. Considering hypothesis H_0 , we see that the weights for compensating the biased density $p_{R^*|H_0}$ are given as

$$w(r | H_0) = \frac{\exp(-(\lambda_0 + \lambda_d))(\lambda_0 + \lambda_d)^r}{\exp(-(\lambda_0 + \lambda_d)^*)((\lambda_0 + \lambda_d)^*)^r}, \quad \forall r \in \mathcal{Z}. \quad (2.27)$$

Recall from (2.21) that tilted biased density will reduce the estimator variance if

$$\exp(-(\lambda_0 + \lambda_d)^*)((\lambda_0 + \lambda_d)^*)^r > \exp(-(\lambda_0 + \lambda_d))(\lambda_0 + \lambda_d)^r, \quad \forall r \in [\gamma, \infty). \quad (2.28)$$

Simplifying and taking the logarithm yields,

$$-[(\lambda_0 + \lambda_d)^* - (\lambda_0 + \lambda_d)] + r \log\left[\frac{(\lambda_0 + \lambda_d)^*}{\lambda_0 + \lambda_d}\right] > 0, \quad \forall r \in [\gamma, \infty). \quad (2.29)$$

We need to maximize the left side of the above inequality with respect to $(\lambda_0 + \lambda_d)^*$, since this will yield a value of $\bar{W}$ which is closest to its lower bound, P_e^2 . It is easily seen that for any given $r \in [\gamma, \infty)$,

$$(\lambda_0 + \lambda_d)^* = r \quad \forall r \in [\gamma, \infty) \quad (2.30)$$

achieves the maximum value of the left side of the inequality in (2.29). Since the maximization requires $(\lambda_0 + \lambda_d)^* \in [\gamma, \infty)$, and since $\gamma > \lambda_0 + \lambda_d$, the left hand side of (2.29) is an increasing function of r . So it is sufficient to find a $(\lambda_0 + \lambda_d)^*$ that maximizes the left hand side of (2.29) for $r = \gamma$. Therefore the value of $(\lambda_0 + \lambda_d)^*$ at $r = \gamma$ is found to satisfy the inequality (2.28), that is

$$(\lambda_0 + \lambda_d)^* = \gamma. \quad (2.31)$$

This means that the “biased” equivalent arrival rate $(\lambda_0 + \lambda_d)^*$ should be set to the arrival rate corresponding to the threshold number of arrivals. Thus the resulting biasing probability mass function would be given as

$$p_{R^*|H_0}(r) = \frac{\exp(-\gamma)\gamma^r}{r!}, \quad (2.32)$$

where γ is the threshold number of arrivals and is given in (2.5).

A similar derivation can be performed for the case of hypothesis H_1 , and using the conditions set forth by (2.13) and (2.20), it can be seen that the equivalent arrival rate for the biased probability mass function, denoted by $(\lambda_1 + \lambda_d)^*$, is required to be fixed to the arrival rate corresponding to the threshold number of arrivals, i.e., $(\lambda_1 + \lambda_d)^* = \gamma$. Thus the biasing density $p_{R^*|H_1}(r)$ is identical to the density in (2.32), i.e., $p_{R^*|H_1}(r) = p_{R^*|H_0}(r)$, and these constitute the suboptimal probability mass functions which satisfy the equations (2.20) and (2.21) and are to be used for the Importance Sampling estimator. The numerical results demonstrating the merits of such a biasing scheme are presented in Chapter 5 along with a detailed comparison of both Monte Carlo and Importance Sampling simulation techniques for the above system. For small values of the error rate P_e , the analytical evaluation of the gain Γ is intractable, and therefore upper and lower bounds for Γ are obtained in the Appendix A. In Chapter 5 we also examine the tightness of these bounds for various examples.

Chapter 3

Optical Communication Systems with Avalanche Photodiodes

Having developed a general approach for estimating the performance in a single-user system employing an ideal photodetector, we look now to extend these results to a more general setting where instead of an ideal photodetector, the system employs a random gain Avalanche photodiode/detector (APD). Sensitive optical receivers operate at minimum detectable power levels of the order of a nanowatt. At typical fiber optic system wavelengths, even a high-quantum-efficiency photodiode would produce a response to this power level on the order of a nanoampere. Such weak currents are severely corrupted by noise from electronic amplifier stages following the detector. In order to obtain acceptable receiver performance, means are needed to increase the detector output before electronic amplification. This is accomplished by using an avalanche photodiode. In such a device, each primary electron-hole pair created by optical absorption generates tens or hundreds of secondary electron-hole pairs through the process of collision ionization [34]. Thus each primary electron-hole pair is replaced by a bunch of secondary pairs. This multiplication process is statistical in nature, i.e., each primary electron-hole pair generates a random number of secondary pairs, depending upon the type of doping of the semiconductor, the electric field level in the high-field region, and the width of the high-field region. Thus the average gain of the device is dependent on the device geometry and the reverse bias voltage applied to it. Due to their inherent complexity, the analysis of these systems is typi-

cally intractable. The existing results on the performance of such a system are due to analysis by means of asymptotic bounds and approximations [3, 4, 5] which appear to be loose. Here we will develop the concept of Importance Sampling for such a system, with the accuracy growing linearly with computational cost. The underlying principle is to model the detection process, based on avalanche detection, as a marked point process [28]. In such a scheme, the photon arrivals at the photodetector correspond to the underlying point process and the random avalanche gains associated with the avalanche photodetector are viewed as the random marks of the marked point process. This model enables us to modify the simulation of avalanche photodetector based systems, in order to improve the performance analysis as outlined in the following sections.

3.1 System Description

In this section, we will consider the underlying point process on a time interval and present the following definitions and results based on the above underlying point process. Without loss of generality, we will consider the time interval $[0, t]$ and N_t will denote the number of points/arrivals in $[0, t]$. The photoelectron count at the output of the avalanche photodetector can be represented as a marked point process given by

$$R_t = \sum_{j=1}^{N_t} G_j \quad (3.1)$$

where N_t is the underlying Poisson process with parameter $\lambda(\sigma)$, $0 \leq \sigma < t$, and G_j , $\forall 1 \leq j \leq N_t$, are the random avalanche gains by which the primary photons incident on the avalanche photodetector are multiplied. The above process can be viewed as a particular case of a marked point process, namely the compound Poisson process defined as :

Definition : A Compound Poisson process comprises of a counting process N_t with intensity function $\lambda(\sigma)$ for $\sigma \geq 0$ and a sequence of mutually independent and identically distributed random variables G_j 's which are also independent of N_t .

The statistics of the above compound Poisson process can be described by means of its sample function density as [28],

$$\mathcal{P}_{N_t(g)} = \begin{cases} \exp(-\int_0^t \lambda(\sigma) d\sigma) & N_t = 0 \\ (\prod_{j=1}^{N_t} \lambda(w_j) p_{G_j}(g_j)) \exp(-\int_0^t \lambda(\sigma) d\sigma), & N_t = n \geq 1, \end{cases}$$

where $p_{G_j}(g_j)$ is the probability that the mark G_j takes on the value g_j for a particular realization of the point process on $[0, t)$ with $N_t = n$ points located for $n \geq 1$ at times $W_1 = w_1, W_2 = w_2, \dots, W_n = w_n$, or alternately the sample function density may be written as

$$\mathcal{P}_{N_t(g)} = \exp[-\int_0^t \lambda(\sigma) d\sigma + \int_0^t \log(\lambda(\sigma)) dN_\sigma + \sum_{j=1}^{N_t} \log(p_{G_j}(g_j))], \quad (3.2)$$

where the Riemann-Stieltjes integral has the evaluation

$$\int_0^t \log(\lambda(\sigma)) dN_\sigma = \begin{cases} 0 & N_t = 0 \\ \sum_{j=1}^{N_t} \log(\lambda(W_j)) & N_t \geq 1. \end{cases}$$

In general in an avalanche detector, both holes and electrons can produce secondary electron-hole pairs. Such detectors are characterized by the carrier ionization ratio α which is defined as the ratio of the probabilities per unit length (in the high field region) of the holes and electrons. For a detector with a mean avalanche gain $\bar{G}$ and ionization ratio α , the probability density function $p_G(g)$ of the multiplication has been determined to be [35]:

$$p_G(g) = \left\{ (1 - \alpha)^{g-1} \Gamma\left(\frac{g}{1 - \alpha}\right) \left[\frac{1 + \alpha(\bar{G} - 1)}{\bar{G}} \right]^{[1 + \alpha(g-1)]/(1-\alpha)} \left(\frac{\bar{G} - 1}{\bar{G}} \right)^{g-1} [(1 + \alpha(g-1))(g-1)! \Gamma\left(\frac{1 + \alpha(g-1)}{1 - \alpha}\right)]^{-1} \right\},$$

where $\Gamma(\cdot)$ is the Gamma function. Various approximations have been derived for use in performance calculations [4, 35, 36]. The analysis of these avalanche photodetector based systems by the marked point process approach is often highly intractable and have yielded bounds on the performance of the system, rather than the actual error probabilities [5, 3, 4]. For such systems employing direct detection schemes, an “exact” method has been proposed [37], where the total probability of the photoelectron count at the output of the avalanche photodetector in the time interval $[0, t)$ can be described as

$$p_{R_t}(r) = \sum_{m=0}^{\infty} p_{N_t}(m) p_{R_t|N_t}(r | m), \quad (3.3)$$

where r is the photoelectron count at the avalanche photodetector output, m is the number of photons incident on the avalanche photodetector at the input end, $p_{N_t}(m)$ is the Poisson probability mass function with parameter $\lambda(\sigma)$, $0 \leq \sigma < t$ and $p_{R_t|N_t}(r | m)$ is the conditional probability that m incident photons give rise to r photoelectrons at the avalanche photodetector output. The complete expression for the above conditional probability has been found to be [35]

$$p_{R_t|N_t}(r | m) = \left\{ m (1 - \alpha)^{r-m} \Gamma\left(\frac{r}{1 - \alpha}\right) \left[\frac{1 + \alpha(\bar{G} - 1)}{\bar{G}} \right]^{[1 + \alpha(r-m)]/(1-\alpha)} \left(\frac{\bar{G} - 1}{\bar{G}} \right)^{r-m} [(m + \alpha(r - m))(r - m)! \Gamma\left(\frac{1 + \alpha(r - m)}{1 - \alpha}\right)]^{-1} \right\},$$

where the conditional probability must be normalized such that $\sum_{m=r}^{\infty} p_{R_t|N_t}(r | m) = 1$. The expression in (3.3) forms the basis for the developments in the analysis of the performance of such avalanche photodetector based systems.

3.2 Problem Formulation

We will consider a single-user binary hypothesis system where the intensity is modulated based on the information to be transmitted. The system is assumed to be

homogeneous in the sense that the intensities under each hypothesis are kept constant, i.e., the intensity under H_i is assumed to be λ_i . There is a constant dark current factor λ_d under each hypothesis, to account for the extraneous generation of photoelectrons in the photodetector. The probability mass function governing the statistics of the system under hypothesis H_i is given as

$$p_{R_T|H_i}(r) = \sum_{m=0}^{\infty} \exp((- \lambda_i - \lambda_d)T) \frac{[(\lambda_i + \lambda_d)T]^m}{m!} p_{R_T|N_i}(r | m), \quad i = 0, 1; \quad (3.4)$$

where $p_{R_T|N_i}(r | m)$ is given in (3.4) and is assumed to be identical under both hypotheses and r is the photoelectron count in the time interval $[0, T)$. The photoelectron count at the output of the avalanche photodetector is compared to a threshold and gives rise to the decision making process. Using the maximum likelihood decision rule, the threshold for detection is given as

$$\gamma_g = \left\lceil \arg_r \left[\frac{p_{R_T|H_1}(r)}{p_{R_T|H_0}(r)} = 1 \right] \right\rceil. \quad (3.5)$$

The probability of error can be computed as

$$P_e = \pi_0 \sum_{\Delta_1} p_{R_T|H_0}(r) + \pi_1 \sum_{\Delta_0} p_{R_T|H_1}(r), \quad (3.6)$$

where Δ_i is the decision region for hypothesis H_i . The problem at hand is the evaluation of the above quantity through the Importance Sampling technique of Monte Carlo simulations in order to alleviate the burden on the computer system, and in general is highly intractable due to the complex nature of the statistics describing the above system.

3.3 Importance Sampling Estimation

As before, we will formulate the probability of error in terms of the biased sample function density, and then weight the samples to account for the biasing. The

probability of error is now rewritten as

$$P_e = \pi_0 \sum_{\Delta_1} p_{R_T|H_0}^*(r) w(r | H_0) + \pi_1 \sum_{\Delta_0} p_{R_T|H_1}^*(r) w(r | H_1), \quad (3.7)$$

where the weights are given by

$$w(r | H_i) = \frac{p_{R_T|H_i}(r)}{p_{R_T|H_i}^*(r)}, \quad \forall r \in \mathcal{Z},$$

and $p_{R_T|H_i}^*(r)$ denotes the biased sample function density under H_i . As the optimum probability mass function yields a degenerative solution to the problem, we need to find a suboptimal probability mass function belonging to the parametric family $\{p_{R_T|H_i}^*(r); i = 0, 1\}$ with the parameters being the photon intensity and the mean avalanche gain. From the analysis in Chapter 2, the following conditions

$$p_{R_T|H_1}^*(r) > p_{R_T|H_1}(r) \quad \forall r \in \Delta_0, \quad (3.8)$$

and

$$p_{R_T|H_0}^*(r) > p_{R_T|H_0}(r) \quad \forall r \in \Delta_1. \quad (3.9)$$

are sufficient to achieve a gain of Importance Sampling over Monte Carlo simulations.

We will now present a lemma which is similar to the one due to Verdu and Brady [22]

Lemma : Let $N_t(\lambda)$ be a Poisson random variable with mean λ , and let $R_t(\lambda) = \sum_{j=1}^{N_t} G_j$ where $\{G_j\}$ is a collection of nonnegative independent and identically distributed random variables that is independent of $N_t(\lambda)$. Let $0 < \lambda' \leq \lambda$. Then $Pr[R_t(\lambda) = r] \leq Pr[R_t(\lambda') = r]$, $\forall r \in \Delta_0$.

Proof :

$$\begin{aligned} Pr[R_t(\lambda) = r] &= \sum_{m=0}^{\infty} P[N_t(\lambda) = m] P[G_1 + \dots + G_m = r] \\ &= \sum_{m=0}^{\infty} \{P[N_t(\lambda) \leq m] - P[N_t(\lambda) \leq m-1]\} \cdot P[G_1 + \dots + G_m = r] \\ &= \sum_{m=0}^{\infty} P[N_t(\lambda) \leq m] \{P[G_1 + \dots + G_m = r] - P[G_1 + \dots + G_{m+1} = r]\} \end{aligned}$$

$$\begin{aligned} &\leq \sum_{m=0}^{\infty} P[N_t(\lambda') \leq m] \{P[G_1 + \dots + G_m = r] - P[G_1 + \dots + G_{m+1} = r]\} \\ &= Pr[R_t(\lambda') = r]. \end{aligned}$$

The inequality follows from the fact that the Poisson cumulative distribution function is a decreasing function of the mean. Considering hypothesis H_1 , if we can find $p_{R_T|H_1}^*(r)$ with parameter λ_1^* , such that $\lambda_1^* < \lambda_1$, then we have from the lemma that $p_{R_T|H_1}(r) < p_{R_T|H_1}^*(r) \forall r \in \Delta_0$, and this is a sufficient condition for achieving a gain of Importance Sampling over Monte Carlo simulations. Therefore, it is sufficient to look for biasing probability mass functions $\{p_{R_T|H_1}^*(r)\}$ with only the photon intensity as the parameter and thus leave the conditional probabilities $p_{R_T|N_i}(r | m)$ unchanged. The analysis reduces to that considered in Chapter 2 and we can obtain the biased probability mass function as

$$p_{R_T|H_1}^*(r) = \sum_{m=0}^{\infty} \exp(-\gamma) \frac{\gamma^m}{m!} p_{R_T|N_i}(r | m), \quad (3.10)$$

where γ is given as

$$\gamma = \frac{(\lambda_1 - \lambda_0)T}{\log[(\lambda_1 + \lambda_d)/(\lambda_0 + \lambda_d)]}. \quad (3.11)$$

i.e., the intensity under hypothesis H_1 is set to γ , the optimum threshold for the ideal photodetection case. A similar analysis can be shown for the case of hypothesis H_0 , i.e., it can be shown that for $0 < \lambda_0 < \lambda_0^*$, we have $p_{R_T|H_0}(r) < p_{R_T|H_0}^*(r) \forall r \in \Delta_1$. Thus the biased probability mass functions under each hypothesis are set identical and given by

$$p_{R_T|H_i}^*(r) = \sum_{m=0}^{\infty} \exp(-\gamma) \frac{\gamma^m}{m!} p_{R_T|N_i}(r | m), \quad i = 0, 1; \quad (3.12)$$

The above probability mass function represents the suboptimal biased densities to be used in the Importance Sampling simulations for estimating the performance of the system. Empirical analysis also supports the conjecture that biasing the gain distribution of the avalanche photodetector does not improve the "gain" of Importance

Sampling over standard Monte Carlo simulations. The results of such a biasing scheme are discussed in Chapter 5 and a comparison of Monte Carlo and Importance Sampling techniques for the above system are also presented.

Chapter 4

Optical Code Division Multiple Access Systems

The technique of fiber-optic code division multiple access (FO-CDMA) takes advantage of the excess bandwidth in single mode fibers to map low information rate electrical or optical signals into high rate optical pulse sequences to provide random, asynchronous communications access which is free of network control to many users. We will develop an Importance Sampling scheme of Monte Carlo simulations for a FO-CDMA system where an optical encoder maps each bit of information into a very high rate optical sequence, that is then coupled into a single-mode fiber channel. At the receiver end of the FO-CDMA, the optical pulse sequence is compared to a stored replica of itself (correlation process) and compared to a threshold at the comparator for data recovery. However, the photodetectors used at the receiving end are endowed with Poisson counting statistics and in direct detection schemes the photoelectron count at the output of the photodetector is compared to a threshold for the purpose of recovering the transmitted data. A class of optical codes called optical orthogonal codes (OOC's) have been developed [26] and applied to FO-CDMA systems [27], but the Poisson statistics of the photodetectors have not been taken into consideration. We will consider a FO-CDMA system where the intensity of the transmitted light is in the form of the above OOC's and employ a direct detection scheme where the Poisson nature of the photodetector is incorporated in the analysis. The Importance Sampling technique of Monte Carlo simulations is developed for

the purpose of estimating the performance of the system in an efficient manner that alleviates the computational burden imposed on the computer system.

4.1 System Description

We will consider a FO-CDMA system where there are K users, and hence K transmitter and receiver pairs. The set of optical pulse sequences essentially is a set of signature sequences for the network. The essence of such a system is to extract data with the desired optical pulse sequence in the presence of interfering optical pulse sequences of the other users. So there is a need to design sequences such that, each sequence is distinguishable from a shifted version of itself and also from other sequences [38, 39, 40]. A new class of high rate sequences called the optical orthogonal codes (OOC's) have been developed by Salehi and Brackett [41]. These OOC's are endowed with some interesting correlation properties and have been shown to be very well suited for FO-CDMA systems [27, 41]. These OOC's are a set of signature sequences that conform to the autocorrelation and crosscorrelation properties prescribed below

- For any sequence $\{x_n\}$ in the set

$$|R_{x,x}(l)| = \left| \sum_{n=0}^{N-1} x_n x_{n+l} \right| = \begin{cases} J & \text{for } l = 0 \\ \leq \rho_a & \text{for } 1 \leq l \leq N-1 \end{cases} \quad (4.1)$$

- For each pair of sequences $\{x_n\}$ and $\{y_n\}$

$$|R_{x,y}(l)| = \left| \sum_{n=0}^{N-1} x_n y_{n+l} \right| \leq \rho_c \quad \text{for } 0 \leq l \leq N-1 \quad (4.2)$$

where $\{x_n\}$ and $\{y_n\}$ are periodic sequences with period N , and where J , ρ_a , and ρ_c are constants. A sequence is considered to be orthogonal with respect to its shifted

version if ρ_a takes on its minimal value and two sequences are considered to be orthogonal if ρ_c takes on its minimal value. In general, an (N, J, ρ_a, ρ_c) optical orthogonal code C , is a family of $\{0, 1\}$ sequences of length N and weight J with autocorrelation and crosscorrelation constraints ρ_a and ρ_c . In this case J is the number of 1's in the sequence. We will present our analysis for the case where ρ_a and ρ_c are equal to one. However, these results may be extended to any general setting. For a given integer number N (code length) and weight J where $J(J-1) \leq N-1$ and $\rho_a = \rho_c = 1$, the number of users K is upper bounded by [27]

$$K \leq \lfloor \frac{N-1}{J(J-1)} \rfloor \quad (4.3)$$

where $\lfloor z \rfloor$ denotes the integer portion of the real value z . Several techniques like projective geometry, the greedy algorithm, algebraic coding theory, etc. have been studied [26] for the design of families of OOC's and these can be constructed easily on a computer for the purpose of simulations.

We will consider a K user FO-CDMA system where the information bit of each user is modulated onto the intensity of the laser transmitted through a single-mode fiber channel. The intensities are in the form of optical pulse sequences, i.e., $(N, J, 1, 1)$ OOC's, and satisfy the auto and crosscorrelation properties set forth by (4.1) and (4.2). If user k is sending bit 1, under hypothesis H_1 , then the intensity of the modulated light will be $\lambda_1^{(k)}(t)$. If user k is sending the information bit 0, under hypothesis H_0 , then intensity of the modulated light will be $\lambda_0^{(k)}(t)$ where

$$\lambda_i^{(k)}(t) = \sum_{n=1}^N \lambda_i^{(k)}(n) \Pi_{T_c}(t - nT_c), \quad i = 0, 1; \text{ , for } t \in [0, T) \quad (4.4)$$

and $\Pi_{T_c}(t)$ is a unit rectangular pulse of duration T_c , and $\underline{\lambda}^{(k)} = (\lambda_1^{(k)}(1), \dots, \lambda_1^{(k)}(N))$ is a signature sequence of length $N = T/T_c$. At the receiving end, the receiver corresponding to user k has a replica of the signature sequence assigned to k , and

the light in the channel due to the user k and other interferers is correlated with this replicated signature sequence. Prior to correlation, this gives rise to the following two hypotheses at receiver of the desired user (taken to be user 1) in the time interval $[0, T)$ as

$$\begin{aligned} H_0 : \tilde{\lambda}_0^{(1)}(t) &= \lambda_0^{(1)}(t) + \sum_{k=2}^K \lambda_b^{(k)}(t), \\ H_1 : \tilde{\lambda}_1^{(1)}(t) &= \lambda_1^{(1)}(t) + \sum_{k=2}^K \lambda_b^{(k)}(t), \end{aligned} \quad (4.5)$$

where $\tilde{\lambda}_i^{(1)}(t)$ is the sum of the intensities in the channel under hypothesis H_i , due to the 1st user and $K - 1$ interferers, and in $\lambda_b^{(k)}(t)$ the symbol b denotes the information of the k^{th} user. The correlated intensities are incident on an ideal photodiode and the photoelectron count at the output of the photodiode is compared to a threshold for the purpose of data recovery, as shown in Figure 4.1. Each user is employing on-off keying, and hence at the 1st receiver, the intensities are correlated with $\underline{\lambda}_1^{(1)}$, since $\underline{\lambda}_0^{(1)} = \underline{0}$. The statistical nature of the photoelectron count at the output of the photodiode at the 1st receiver can be described, under H_i , by a doubly stochastic Poisson process as

$$p_{R(\tilde{\nu})|H_i}(r) = \frac{\exp[-\int_0^T (\tilde{\nu}_i^{(1)}(t) + \nu_d)dt][\int_0^T (\tilde{\nu}_i^{(1)}(t) + \nu_d)dt]^r}{r!}, \quad i = 0, 1; \quad (4.6)$$

where r is the photoelectron count in the time interval $[0, T)$, and $\tilde{\nu}_i^{(1)}(t)$ is the random valued intensity resulting from correlating the replicated signature sequence at the 1st receiver with the intensity of the light transmitted through the channel, and ν_d is a deterministic constant associated with the dark current of the photodiode. The above equation forms the basis for all the results presented in this section.

Figure 4.1 A K-user FO-CDMA system with the optical correlation receiver shown for the 1st receiver.

4.2 Problem Formulation

From the analysis presented in [41], it can be shown that, for the case of chip-synchronous codes, equation (4.6) can be written as

$$p_{R(\bar{\nu})|H_i}(r) = \frac{\exp[-(\nu_i^{(1)} + \nu_d T)][\nu_i^{(1)} + \nu_d T]^r}{r!}, \quad (4.7)$$

where $\nu_i^{(1)} = (b_i^{(1)}J + I^{(1)})T_c$, $i = 0, 1$; and for the case of on-off keying we further have $b_1^{(1)} = 1$ and $b_0^{(1)} = 0$. The symbol $I^{(1)}$ is the random valued interference term due to the presence of other users. Without loss of generality, we will consider user 1 with $T_c = 1$, and present our analysis for the data recovery, at the receiver end of user 1, in the presence of interference from the other $K - 1$ users. In the intensity of the correlated received optical signal, $b_i^{(1)}J$ is the desired intensity term and $I^{(1)} = \sum_{k=2}^K I_k^{(1)}$ is the undesired intensity term due to the interferers. Each interference term $I_k^{(1)}$, due to user k , is a random variable given by

$$I_k^{(1)} = \begin{cases} 0 & \text{with probability } (1 - J^2/2N) \\ 1 & \text{with probability } (J^2/2N) \end{cases} \quad (4.8)$$

For a direct detection optical system, it is essential to determine the counting statistics of the underlying process. The counting statistics for doubly stochastic Poisson process can be determined in principle by conditioning on the information process, using the fact that the conditioned process is a Poisson counting process, and then removing the conditioning by taking the expectation over the paths of the information process [42, 28]. Thus the probability of r photoelectrons occurring, under H_i , in the sampling interval T is given as

$$p_{R|H_i}(r) = E[p_{R(\bar{\nu})|H_i}(r \mid \bar{\nu}_i)], \quad i = 0, 1, \quad (4.9)$$

where the expectation is taken over the paths of the information process. Performing the above conditioning and later averaging and from (4.6), (4.7), (4.8) and the fact that $b_1^{(1)} = 1$ and $b_0^{(1)} = 0$, we obtain that the probability mass functions under the two hypothesis are given as

$$p_{R|H_1}(r) = \sum_{k=0}^{K-1} \binom{K-1}{k} \frac{\exp[-(J + \nu_d T + k)](J + \nu_d T + k)^r}{r!} [1 - J^2/2N]^{K-1-k} [J^2/2N]^k \quad (4.10)$$

$$p_{R|H_0}(r) = \sum_{k=0}^{K-1} \binom{K-1}{k} \frac{\exp[-(\nu_d T + k)](\nu_d T + k)^r}{r!} [1 - J^2/2N]^{K-1-k} [J^2/2N]^k \quad (4.11)$$

where $\binom{K}{k} = \frac{(K)!}{(K-k)!k!}$.

The number of photoelectrons r counted at the output of the photodetector in the interval $[0, T)$ is compared to a threshold for the purpose of deciding whether a 0 or 1 was transmitted. Using the maximum likelihood decision rule, the threshold γ

for the detection process is given as

$$\gamma = \left\lceil \arg_r \left[\frac{p_{R|H_1}(r)}{p_{R|H_0}(r)} = 1 \right] \right\rceil \quad (4.12)$$

The probability of bit error at the 1st receiver is now given by

$$P_e = \pi_1 \sum_{\Delta_0} p_{R|H_1}(r) + \pi_0 \sum_{\Delta_1} p_{R|H_0}(r) \quad (4.13)$$

where π_i is the aprior probability of H_i , and Δ_i is the decision region under H_i . We will develop an Importance sampling technique of Monte Carlo simulations for this system in order to estimate the probability of bit-error for the desired user described in the above equation.

4.3 Importance Sampling Estimation

For a K -user FO-CDMA network, given the length, N , of the OOC's, the number of weights J , can be varied as long as they satisfy the condition set forth by (4.3), i.e., the number of 1's in the sequence can be varied within certain limits. Since the probability mass functions $p_{R|H_i}(r)$ have been derived by averaging the Poisson probability mass functions $p_{R(\tilde{\nu})|H_i}(r | \tilde{\nu})$ we will apply the *exponential tilt* method [33] to arrive at the biased probability mass functions $p_{R^*|H_i}$, $i = 0, 1$. Therefore, we consider the parametric family of probability mass functions $\{p_{R^*|H_i}, i = 0, 1\}$, and since J and $\nu_d T$ are the only free parameters in the probability mass functions governing the statistics of the system, we will look for a biased probability mass function $p_{R^*|H_1}$ with parameter $(J + \nu_d T)^*$, rather than $(J + \nu_d T)$. Under hypothesis H_0 the probability mass function $p_{R|H_0}$ is shifted to its biased version $p_{R^*|H_0}$ belonging to above parametric family with $(\nu_d T)^*$ as the new parameter, in place of $\nu_d T$. The probability of error can now be rewritten as

$$P_e = \frac{1}{2} \left[\sum_{\Delta_0} p_{R^*|H_1}(r) w(r | H_1) + \sum_{\Delta_1} p_{R^*|H_0}(r) w(r | H_0) \right], \quad (4.14)$$

where we have assumed that the hypotheses are equiprobable, and $w(r \mid H_i)$ are weights associated with the biased probability mass function $p_{R^*|H_i}$ and has the evaluation given in (2.13). The conditions for achieving a realistic gain through Importance Sampling over Monte Carlo simulations are given by (2.20) and (2.21), which reduce to

$$\frac{\exp[-(J + \nu_d T + k)]}{\exp[-((J + \nu_d T)^* + k)]} \left[\frac{J + \nu_d T + k}{(J + \nu_d T)^* + k} \right]^r < 1, \quad \forall r \in \Delta_0, \quad (4.15)$$

$$\frac{\exp[-(\nu_d T + k)]}{\exp[-((\nu_d T)^* + k)]} \left[\frac{\nu_d T + k}{(\nu_d T)^* + k} \right]^r < 1, \quad \forall r \in \Delta_1, \quad (4.16)$$

for all $k \in [0, K - 1]$. Since k is nonnegative, the above conditions can further be reduced to

$$\exp[-(J + \nu_d T) + (J + \nu_d T)^*] \left[\frac{J + \nu_d T}{(J + \nu_d T)^*} \right]^r < 1, \quad \forall r \in \Delta_0 \quad (4.17)$$

$$\exp[-\nu_d T + (\nu_d T)^*] \left[\frac{\nu_d T}{(\nu_d T)^*} \right]^r < 1, \quad \forall r \in \Delta_1 \quad (4.18)$$

Considering hypothesis H_1 , the left hand side of (4.17) needs to be minimized with respect to $(J + \nu_d T)^*$, since this will make the quantity $\bar{W}$ closest to its lower bound P_e^2 , and hence maximize the gain of Importance Sampling over Monte Carlo simulations. Minimizing the left hand side of (4.17) with respect to $(J + \nu_d T)^*$ results in the following condition

$$(J + \nu_d T)^* = r, \quad (4.19)$$

where $r \in \Delta_0$. Since $(J + \nu_d T)$ is greater than γ which is the maximum value of r in the region Δ_0 , it is seen that the left hand side of (4.17) is an increasing function of r . Therefore if the condition in (4.17) holds for $r = \gamma$, it would hold for all values of $r \in \Delta_0$. Therefore at $r = \gamma$, the parameter $(J + \nu_d T)^* = \gamma$, and this guarantees

the gain of Importance Sampling over Monte Carlo simulations. Therefore the biased probability mass function $p_{R^*|H_1}$ is given as

$$p_{R^*|H_1}(r) = \sum_{k=0}^{K-1} \binom{K-1}{k} \frac{\exp[-(\gamma+k)](\gamma+k)^r}{r!} \frac{1}{[1 - J^2/2N]^{K-1-k} [J^2/2N]^k} \quad (4.20)$$

This implies that the parameter $(J + \nu_d T)^*$ is to be set to the threshold number of arrivals. A similar analysis of hypothesis H_0 results in the parameter $(\nu_d T)^*$ of the probability mass function $p_{R^*|H_0}$ being set equal to the threshold number of arrivals, i.e., $(\nu_d T)^* = \gamma$. Thus the biased probability mass functions are identical under both hypotheses and these constitute the suboptimal probability mass functions that are to be used for Importance Sampling simulations. The numerical results of the performance analysis of the above FO-CDMA system is presented in the next chapter along with a comparison of Importance Sampling and Monte Carlo simulations.

Chapter 5

Simulations and Numerical Results

5.1 Single User System

For the case of the single user system with an ideal photodiode, the analytical probability of error was computed for different values λ_1 (the intensity of modulation when the user is sending the bit 1). The dark current rate for the photodiode, λ_d , was kept fixed at a value of unity. The number of trials required for an estimate $\hat{P}_e$ within 0.1% of the actual probability of error P_e is computed for both Monte Carlo and Importance Sampling simulations and the resulting simulated “gain”, $\Gamma = M_{MC}/M_{IS}$ is shown in Figure 5.1 against the probability of bit-error on a logarithmic scale. The lower and upper bounds developed for the simulated “gain” Γ are plotted in Figure 5.2 along with the actual simulated “gain” for different values of λ_1 . The lower bound is seen to closely follow the actual value of Γ . In the case of a single user system employing a random gain APD, the analytical probability of error is computed for several values of the intensity of modulation, λ_1 with λ_d being held constant at unity. The mean avalanche gain, $\bar{G}$, of the APD was set to a value of 50.0, with the ionization ratio being held at a value of $\alpha = 0.5$. The simulated “gain” Γ is plotted for different values of the bit error probability on a logarithmic scale in Figure 5.4 where the number of trials for both Monte Carlo and Importance Sampling simulations are that required for an estimate $\hat{P}_e$ within 0.1% of the actual probability of error P_e . For the Importance Sampling estimator for the single user system with an ideal photodiode, the intensity of modulation was set equal to the threshold under both hypotheses,

i.e.,

$$p_{R^*|H_i}(r) = \frac{\exp(-\gamma)\gamma^r}{r!}, \quad i = 0, 1.$$

For the system employing an APD, only the intensities were biased to new values, leaving the conditional probability mass functions $p_{R_T|N_t}(r | m)$ unchanged, i.e.,

$$p_{R_T|H_i}^*(r) = \sum_{m=0}^{\infty} \exp(-\gamma) \frac{\gamma^m}{m!} p_{R_T|N_t}(r | m), \quad i = 0, 1;$$

For both the systems described above, the gain of Importance Sampling over Monte Carlo simulations is seen to increase with the performance of the system, i.e., as the probability of error for the system decreases, the value of the “gain” Γ increases almost linearly as $1/P_e$.

5.2 Multi User System

For the FO-CDMA system, the probability of bit-error of the desired user is computed for increasing number of weights J in Figure 5.6 for a 2-user FO-CDMA system. The OOC's were designed such that the total code length was $N = 2000$, i.e., a family of $\{2000, J, 1, 1\}$ sequences. weight J for any K corresponds to the maximum allowable weight for OOC's of length of $N = 1000$. The variation of the probability of bit-error with changing weights J shows that the performance of the system is enhanced as the weights increase for a fixed code length. The maximum allowable J is governed by the relation given in (4.3). For the Importance Sampling estimator the biasing densities used were of the form

$$p_{R^*|H_i}(r) = \sum_{k=0}^{K-1} \binom{K-1}{k} \frac{\exp[-(\gamma+k)](\gamma+k)^r}{r!} [1 - J^2/2N]^{K-1-k} [J^2/2N]^k, \quad i = 0, 1.$$

The simulated “gain” Γ is plotted on a logarithmic scale for a 2-user FO-CDMA system against the bit-error probability for the desired user in Figure 5.8. The OOC's

were chosen from a family of $\{1000, J, 1, 1\}$ sequences. In Figure 5.12, the logarithm of the quantity $C = \log(\Gamma \times P_e)$ is plotted against the number of users K for a family of $\{1000, J_{K,max}, 1, 1\}$ OOC's, where $J_{K,max}$ corresponds to the maximum allowable J for any given number of users K , i.e., the maximum value of J that satisfies (4.3) for $N = 1000$ and given value of K . For a given number of users K , i.e., a K -user system, the relationship between the simulated “gain” Γ and the performance of the system tends to become linear, i.e., as the probability of bit-error for the desired user decreases there is an almost linearly related increase in the “gain” Γ . Due to the added interference from other users, the gain of Importance Sampling over Monte Carlo simulations is seen to decrease as the number of users K increases, in the sense that $C = \Gamma \times P_e$ decreases as the number of users increases.

Figure 5.1 MC versus IS Simulations for Ideal Photodetection

The logarithm of the simulated "Gain" Γ of Importance Sampling over Monte Carlo simulations for a single user system with an ideal photodiode. The simulations were run till $\hat{P}_e$ was within 0.1% of P_e . The intermediate step type behaviour is due to the quantization error in fixing the detection threshold γ , i.e., γ is fixed to the next integer value even when it is real.

Figure 5.2 Lower and Upper bounds for Γ
 The simulated "Gain" Γ is plotted against the intensity λ_1 for a single user ideal photodiode system along with the lower and upper bounds on the "Gain". The dark current was set to a constant value of $\lambda_d = 1.0$.

Figure 5.3 MC versus IS Simulations for Ideal Photodetection

The number of trials for Importance Sampling and Monte Carlo simulations for an estimate $\hat{P}_e \in [2P_e/5, 8P_e/5]$ with 95% confidence for a single user system with an ideal photodiode.

Figure 5.4 MC versus IS Simulations for Avalanche Photodetection
 The simulated "Gain" Γ versus the probability of error P_e for a single user system employing an APD with mean avalanche gain of 50 is plotted on a logarithmic scale. The simulations were run until an estimate of $\hat{P}_e$ was within 0.1% of P_e .

Figure 5.5 MC versus IS Simulations for PD and APD Systems

A comparison of the simulated "Gain" Γ of Importance Sampling for single user system with APD of mean avalanche gain of 50 and an ideal photodiode of constant avalanche gain of unity. The simulations were run till $\hat{P}_e$ was within 0.1% of P_e .

Figure 5.6 Performance versus Weights for 2-user FO-CDMA
 Probability of bit-error for the desired user versus the number of weights J for a 2-user FO-CDMA system employing OOC's of length $N = 2000$.

Figure 5.7 Performance versus Weights for K-user FO-CDMA
 Probability of bit-error for the desired user versus the number of weights J for $K = 3$ and $K = 4$ FO-CDMA system employing OOC's of length $N = 2000$.

Figure 5.8 MC versus IS Simulations for 2-user FO-CDMA
 The simulated "Gain" Γ is plotted against the Probability of bit-error of the desired user for a 2-user FO-CDMA system employing OOC's of length $N = 1000$.

Figure 5.9 MC versus IS Simulations for 3-user FO-CDMA
 The simulated "Gain" Γ is plotted against the Probability of bit-error of the desired user for a 3-user FO-CDMA system employing OOC's of length $N = 2000$.

Figure 5.10 MC versus IS Simulations for 4-user FO-CDMA
 The simulated "Gain" Γ is plotted against the Probability of bit-error of the desired user for a 4-user FO-CDMA system employing OOC's of length $N = 2000$.

Figure 5.11 MC versus IS Simulations for 2-user FO-CDMA
 The number of trials for Importance Sampling and Monte Carlo simulations for an estimate $\hat{P}_e \in [2P_e/5, 8P_e/5]$ with 95% confidence for a 2-user FO-CDMA system.

Figure 5.12 Relative Variation of Γ and P_e for FO-CDMA
 $\log[C = \Gamma \times P_e]$ versus Number of users K for a FO-CDMA system employing
 OOC's of length $N = 1000$, the number of weights J for each user corresponds to the
 maximum allowable J .

Chapter 6

Conclusions

We have considered in this thesis the problem of performance evaluation of different classes of optical communication systems employing direct detection schemes. It is well known that in most realistic optical communication systems analytic results are not readily obtained, and therefore numerical techniques must be utilized. Furthermore, the complex statistical nature of these systems makes analytical evaluation of the performance of these systems highly intractable and often unrealistic estimates of the performance of these systems is obtained by means of approximations and bounds. Due to the high performance of these systems, the Monte Carlo simulations, required to estimate the error probabilities of the order of 10^{-7} and below, imposed a great computational burden on the computer system. Clearly there was a need to employ some variance reduction technique to reduce the number of trials required for estimates in a reasonable interval of confidence.

As such, Importance Sampling techniques of Monte Carlo simulations were employed in this thesis to reduce the number of trials required for estimating the performance analysis of different classes of direct-detection optical communication systems. The fundamental problem in using this technique is to determine the appropriate biasing density such that the number of simulation trials required for estimating the performance of the system are reduced. In Chapter 2 we considered the case of the single user system with an ideal photodiode and employed the exponential tilt method of biasing densities as the strategy for arriving at the new tilted density

which was again Poisson distributed with the intensity of the Poisson process as the biasing parameter. One immediate benefit of this biasing scheme is the relative ease in implementation. Moreover in the examples investigated, the “gain” of Importance Sampling over Monte Carlo simulations was found to vary approximately in an inverse linear manner with the probability of error of the system for Importance Sampling and Monte Carlo estimates in identical confidence intervals. For the case of the system with an avalanche photodiode, the biasing strategy with the intensity of the Poisson process as the lone parameter was shown to be sufficient for achieving a “gain” of Importance Sampling over Monte Carlo simulations. In the examples investigated, results demonstrated that the reduction of simulation trials required to obtain the equivalent confidence interval in using standard Monte Carlo simulations is almost inversely proportional to the true value of the error rate.

The above parametric class of biasing densities was extended in Chapter 4 for estimating the performance of a fiber-optic code division multiple access in terms of the bit-error probability of the desired user. For a given K-user system the “gain” of Importance Sampling over Monte Carlo simulations was again found to vary in an almost inverse linear manner with the probability of bit-error, i.e., for high performance systems the “gain” was found to be linearly increasing as the performance of the system.

In all cases of direct-detection optical communication systems considered, the “gain” was shown to be considerable for simulation of the performance of these systems, thereby alleviating the computational burden imposed on the computer system. The “gain” was found to be linearly increasing as the performance of the system, i.e., $\Gamma \approx 1/P_e$ for small values of P_e . However, the “gain” does not reflect the hidden computation required for computing the weights for Importance Sampling estimator. It was also shown that Importance Sampling simulation required as low as 50 – 100

trials for estimating error probabilities of the order of 10^{-10} and below. In summary, it is believed that similar results will be obtained in the extension of these methods to more complex FO-CDMA systems.

Appendix A

Lower and Upper Bounds on Γ

For a single user system employing an ideal photodiode, we have from (2.18) that the gain Γ is given by,

$$\Gamma \approx \frac{P_e}{\bar{W}}, \quad (\text{A.1})$$

where P_e and $\bar{W}$ are given in (2.8) and (2.15) respectively. For small error rates we can assume (A.1) holds with equality and write

$$\Gamma = \frac{\frac{1}{2}[\sum_{r=0}^{\gamma-1} \frac{\exp(-(\lambda_1+\lambda_d))(\lambda_1+\lambda_d)^r}{r!} + \sum_{r=\gamma}^{\infty} \frac{\exp(-(\lambda_0+\lambda_d))(\lambda_0+\lambda_d)^r}{r!}]}{\frac{1}{2}\{\sum_{r=0}^{\gamma-1} [\frac{\exp(-(\lambda_1+\lambda_d))(\lambda_1+\lambda_d)^r}{r!}]^2 \frac{1}{\exp(-\gamma)(\gamma)^r} r! + \sum_{r=\gamma}^{\infty} [\frac{\exp(-(\lambda_0+\lambda_d))(\lambda_0+\lambda_d)^r}{r!}]^2 \frac{1}{\exp(-\gamma)(\gamma)^r} r!\}}, \quad (\text{A.2})$$

where γ is given by (2.5). Since we have $(\lambda_0 + \lambda_d) \leq \gamma \leq (\lambda_1 + \lambda_d)$, the lower and upper bounds for Γ are given as

$$\Gamma \geq \frac{1}{2} \left[\sum_{r=0}^{\gamma-1} \frac{\exp(-(\lambda_1 + \lambda_d))(\lambda_1 + \lambda_d)^r}{r!} + \sum_{r=\gamma}^{\infty} \frac{\exp(-(\lambda_0 + \lambda_d))(\lambda_0 + \lambda_d)^r}{r!} \right] \times (\mu)^{-1} \quad (\text{A.3})$$

where

$$\mu = \frac{1}{2} \left\{ \sum_{r=0}^{\gamma-1} \left[\frac{\exp(-(\lambda_1 + \lambda_d))(\lambda_1 + \lambda_d)^r}{r!} \right] \left[\frac{\lambda_1 + \lambda_d}{\gamma} \right]^{\gamma-1} e^{(\gamma-(\lambda_1+\lambda_d))} + \sum_{r=\gamma}^{\infty} \left[\frac{\exp(-(\lambda_0 + \lambda_d))(\lambda_0 + \lambda_d)^r}{r!} \right] \left[\frac{\lambda_0 + \lambda_d}{\gamma} \right]^{\gamma} e^{(\gamma-(\lambda_0+\lambda_d))} \right\},$$

and

$$\Gamma \leq \frac{\frac{1}{2}[\sum_{r=0}^{\gamma-1} \frac{\exp(-(\lambda_1+\lambda_d))(\lambda_1+\lambda_d)^r}{r!} + \sum_{r=\gamma}^{\infty} \frac{\exp(-(\lambda_0+\lambda_d))(\lambda_0+\lambda_d)^r}{r!}]}{\frac{1}{2}\exp(\gamma - (\lambda_1 + \lambda_d))[\sum_{r=0}^{\gamma-1} \frac{\exp(-(\lambda_1+\lambda_d))(\lambda_1+\lambda_d)^r}{r!}]}. \quad (\text{A.4})$$

The above lower and upper bounds for Γ can be put in a concise form as

$$\frac{1}{\max\{[\frac{\lambda_1+\lambda_d}{\gamma}]^{\gamma-1} \exp(\gamma - \lambda_1 - \lambda_d), [\frac{\lambda_0+\lambda_d}{\gamma}]^{\gamma} \exp(\gamma - \lambda_0 - \lambda_d)\}} \leq$$

$$\Gamma \leq 2 \exp(\lambda_1 + \lambda_d - \gamma) \left[\frac{P_e}{Q(2(\lambda_1 + \lambda_d) | 2\gamma)} \right], \quad (\text{A.5})$$

where the function $Q(X^2 | \nu)$ is associated with a *Chi-Square* distribution and has the following description,

$$Q(X^2 | \nu) = \sqrt{2\pi} Z(X) \left\{ 1 + \sum_{i=1}^{\frac{\nu-2}{2}} \frac{X^{2i}}{2 \cdot 4 \cdots (2i)} \right\}, \quad (\text{A.6})$$

where $Z(X) = \frac{1}{\sqrt{2\pi}} \exp(-X^2/2)$, and ν is even.

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