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**Optimal trajectories for aeroassisted, noncoplanar orbital
transfer**

Lee, Woon Yung, M.S.

Rice University, 1987

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OPTIMAL TRAJECTORIES FOR AEROASSISTED,
NONCOPLANAR ORBITAL TRANSFER

by

WOON YUNG LEE

A THESIS SUBMITTED
IN PARTIAL FULFILLMENT OF THE
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1987

Abstract**Optimal Trajectories for Aeroassisted,
Noncoplanar Orbital Transfer**

by

Woon Yung Lee

This thesis considers both classical and minimax problems of optimal control arising in the study of noncoplanar, aeroassisted orbital transfer. The maneuver considered involves the transfer from a high planetary orbit to a low planetary orbit with a prescribed atmospheric plane change.

With reference to the atmospheric part of the maneuver, trajectory control is achieved by modulating the lift coefficient and the angle of bank. The presence of upper and lower bounds on the lift coefficient is considered.

Within the framework of classical optimal control, the performance indexes studied are the energy required for orbital transfer and the time integral of the square of the path inclination. Within the framework of minimax optimal control, the performance index studied is the peak heating rate.

Numerical solutions are obtained by means of the sequential gradient-restoration algorithm for optimal control problems. Numerical examples are presented, and their engineering implications are discussed.

Key Words. Orbital transfer, noncoplanar orbital transfer, aeroassisted orbital transfer, plane change, optimal trajectories, optimal control, Bolza problems, Chebychev problems, numerical methods, computing methods, sequential gradient-restoration algorithm.

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Dedication

The author dedicates this work to his beloved wife and his son.

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1. Introduction

This paper considers both classical and minimax problems of optimal control arising in the study of noncoplanar, aeroassisted orbital transfer. The maneuver considered involves the transfer from a high planetary orbit to a low planetary orbit with a prescribed atmospheric plane change. An example is the HEO-to-LEO transfer of a spacecraft with a prescribed plane change, where HEO denotes high Earth orbit and LEO denotes low Earth orbit.

The basic idea is to employ the hybrid combination of propulsive maneuvers in space and aerodynamic maneuvers in the sensible atmosphere. Hence, this type of flight is also called synergetic space flight. With reference to the atmospheric part of the maneuver, trajectory control is achieved by modulating the lift coefficient (hence, the angle of attack) and the angle of bank. The presence of upper and lower bounds on the lift coefficient is considered.

Previous Research. Previous research on the topics covered in this paper can be found in Refs. 1-44. For an overview of theoretical calculus of variations and optimal control, see Refs. 1-3. For an analysis of the performance of hypervelocity vehicles, see Refs. 4-5. For a discussion of optimal atmospheric trajectories, see Ref. 6. For a discussion of optimal space trajectories, see Ref. 7.

Concerning the algorithms useful for solving optimal control problems of the Bolza type, see Refs. 8-19. In particular, first-order methods (gradient methods) are presented in Refs. 8-15 and second-order methods (quasilinearization methods) are presented in Ref. 16. Among the first-order methods, Refs. 8-13 deal with the primal formulation and Refs. 14-15 deal with the dual formulation. Both first-order methods and second-order methods

require the solution of a linear, two-point boundary-value problem at each iteration. In this connection, one of the most effective techniques is the method of particular solutions, discussed in Refs. 17-19.

Concerning optimal control problem of the Chebyshev type (minimax problems), see Refs. 20-33. While the Chebyshev problem is not covered by the Bolza formulation, it can be brought into the Bolza format by various transformations. These transformations are discussed in Refs 20-28.

Aerospace applications of minimax optimal control are presented in Ref. 22 and Refs. 29-33.

Finally, concerning aeroassisted orbital transfer, see Refs. 30-44. In particular, the topics of optimization and guidance of the trajectories of AOT vehicles are discussed in Refs. 38-44 as well as in Refs. 30-33.

Present Research. This paper deals with noncoplanar, aeroassisted orbital transfer, under the assumptions that: (i) the initial and final orbits are circular; (ii) three impulses are employed, one at HEO, one at LEO, and one at the exit from the atmosphere; (iii) the prescribed plane change is performed entirely in the atmospheric part of the trajectory; and (iv) the gravitational field is central and is governed by inverse square law. The maneuver is performed partly in outer space and partly in the sensible atmosphere. The following terminology is employed: r_e is the radius of the Earth; r_a is the radius of the outer edge of the sensible atmosphere; r_{00} is the HEO radius; and r_{11} is the LEO radius. The four key points of the maneuver are these: point 00 (exit from HEO); point 0 (entrance into the atmosphere); point 1 (exit from the atmosphere); and point 11 (entrance into LEO).

The maneuver starts with a tangential propulsive burn, having characteristic velocity ΔV_{00} , at point 00; here, the spacecraft exits from the

high Earth orbit and enters into an elliptical transfer orbit, connecting the points 00 and 0. At point 0, the spacecraft enters into the atmosphere; after traversing the upper layers of the atmosphere, it exits from the atmosphere at point 1; during the atmospheric pass, the velocity of the spacecraft is reduced, due to the aerodynamic drag; in addition, the plane change is performed. At point 1, the spacecraft exits from the atmosphere; right at the exit, a tangential propulsive burn takes place, having characteristic velocity ΔV_1 ; then, the spacecraft enters into an elliptical transfer orbit connecting the points 1 and 11; this elliptical transfer orbit is such that its apogee occurs at r_{11} . The maneuver ends with a tangential propulsive burn, having characteristic velocity ΔV_{11} , at point 11; here, the spacecraft enters into the low Earth orbit, in that the magnitude of ΔV_{11} is such that the desired circularization into LEO is achieved.

For the AOT maneuver described above, optimal trajectories were investigated in Refs. 30-32, under the assumptions (i)-(iv) and under the additional assumption that (v) the flight time is given. Here, assumption (v) is removed, and the flight time is optimized simultaneously with the lift coefficient history and the bank angle history. Therefore, this paper extends to noncoplanar transfer the approach employed in Ref. 33 for coplanar transfer.

Specifically, within the framework of classical optimal control, the performance indexes studied are the energy required for orbital transfer and the time integral of the square of the path inclination; these yield Bolza problems of optimal control. Within the framework of minimax optimal control, the performance index studied is the peak heating rate; this yields a Chebyshev problem of optimal control, which can be converted into a Bolza problem by suitable transformations.

Numerical solutions are obtained by means of the sequential gradient-restoration algorithm for optimal control problems. Numerical examples are presented, and their engineering implications are discussed.

Outline. In Section 2, the basic equations to be solved for noncoplanar, aeroassisted orbital transfer using lift modulation and bank angle modulation are stated. In Section 3, the performance indexes of interest for AOT maneuvers are reviewed. In Sections 4-5, the numerical experiments performed are summarized and the results obtained using the sequential gradient-restoration algorithm for optimal control problems are presented. Finally, in Section 6, the conclusions are presented.

2. Basic Equations

In this section, the atmospheric portion of the trajectory of an AOT vehicle is considered, and the basic equations to be solved are stated. The following assumptions are employed: (a) the atmospheric pass is made with engine shut-off; hence, in this portion of the flight, the AOT vehicle behaves as a particle of constant mass; (b) Coriolis acceleration terms and transport acceleration terms are neglected; and (c) the aerodynamic forces are evaluated using the inertial velocity, rather than the relative velocity.

The symbols employed are as follows: r is the local radius; r_{00} is the HEO radius; r_{11} is the LEO radius; r_e is the radius of the Earth; r_a is the radius of the outer edge of the sensible atmosphere; and H is the height of the sensible atmosphere. Also, h is the altitude above sea level; V is the velocity; D is the drag, L is the lift; g is the local acceleration of gravity; m is the mass; and τ is the flight time.

In addition, γ is the path inclination, the angle between the local horizon and the velocity vector $\vec{V}$; γ is positive when $\vec{V}$ is directed upward with respect to the local horizon; θ is the longitude, measured along the Equator, starting from the fundamental meridian, the Greenwich meridian; θ is positive eastward; ϕ is the latitude, measured along the local meridian, starting from the equatorial plane; ϕ is positive northward; ψ is the heading angle, the angle between the local parallel and the projected velocity vector $\vec{V}_p$; $\vec{V}_p$ is obtained by projecting $\vec{V}$ on the local horizon, and ψ is positive if $\vec{V}_p$ is directed inward with respect to the local parallel; finally σ is the bank angle, the angle between the lift vector $\vec{L}$ and the $(\vec{r}, \vec{V})$ plane; σ is positive if the spacecraft is banked to the left.

Differential System. With the above assumptions, and upon normalizing the flight time to unity, the equations of motion are given by

$$\dot{h} = \tau [V \sin \gamma], \quad 0 \leq t \leq 1, \quad (1a)$$

$$\dot{\theta} = \tau [V \cos \gamma \cos \psi / r \cos \phi], \quad 0 \leq t \leq 1, \quad (1b)$$

$$\dot{\phi} = \tau [V \cos \gamma \sin \psi / r], \quad 0 \leq t \leq 1, \quad (1c)$$

$$\dot{V} = \tau [-D/m - g \sin \gamma], \quad 0 \leq t \leq 1, \quad (1d)$$

$$\dot{\gamma} = \tau [(L/mV) \cos \sigma + (V/r - g/V) \cos \gamma], \quad 0 \leq t \leq 1, \quad (1e)$$

$$\dot{\psi} = \tau [(L/mV) \sin \sigma / \cos \gamma - (V/r) \cos \gamma \cos \psi \tan \phi], \quad 0 \leq t \leq 1. \quad (1f)$$

In the above equations,

$$r = r_e + h, \quad g = \mu / r^2, \quad (2)$$

where μ denotes the Earth's gravitational constant. In addition,

$$D = (1/2) C_D \rho S V^2, \quad L = (1/2) C_L \rho S V^2, \quad (3)$$

where C_D is the drag coefficient, C_L is the lift coefficient, $\rho = \rho(h)$ is the air density and S is the reference surface area. Under extreme hypersonic conditions, the dependence of the aerodynamic coefficients on the Mach number and the Reynolds number is disregarded, and the following parabolic relation is postulated:

$$C_D = C_{D0} + K C_L^2, \quad (4)$$

where C_{D0} denotes the zero-lift drag coefficient, $K C_L^2$ denotes the induced drag coefficient, and K denotes the induced drag factor.

In the system (1), the independent variable is the normalized time t , $0 \leq t \leq 1$. The dependent variables include six state variables $(h, V, \gamma, \theta, \phi, \psi)$, two control variables (C_L, σ) , and one parameter (τ) .

Normalized Differential System. The following normalized variables are introduced:

$$\tilde{h} = h/h_R, \quad h_R = H, \quad (5a)$$

$$\tilde{r} = r/r_R, \quad r_R = r_a, \quad (5b)$$

$$\tilde{V} = V/V_R, \quad V_R = \sqrt{(\mu/r_a)}, \quad (5c)$$

$$\tilde{\tau} = \tau/\tau_R, \quad \tau_R = 2\pi r_a \sqrt{(r_a/\mu)}, \quad (5d)$$

$$\tilde{\rho} = \rho/\rho_R, \quad \rho_R = \rho_b, \quad (5e)$$

where the subscript R denotes a reference value. In (5), V_R is the circular velocity at $r=r_a$ and ρ_b is the air density at the altitude $h_b = 40$ km. Also, the following dimensionless constants are introduced:

$$k = r_a/H, \quad (6a)$$

$$C_{DR} = 2C_{D0}, \quad (6b)$$

$$C_{LR} = \sqrt{(C_{D0}/K)}, \quad (6c)$$

$$E_{\max} = (L/D)_{\max} = C_{LR}/C_{DR} = 1/\sqrt{(4C_{D0}K)}, \quad (6d)$$

$$A = (\rho_R SH/2m)C_{D0}, \quad (6e)$$

$$B = (\rho_R SH/2m)C_{LR}. \quad (6f)$$

Next, the normalized lift coefficient is given by

$$\lambda = C_L/C_{LR}. \quad (7)$$

Then, upon dropping the tilde, the differential system (1) is rewritten in the form

$$\dot{h} = (2\pi k \tau) [V \sin \gamma], \quad 0 \leq t \leq 1, \quad (8a)$$

$$\dot{\theta} = (2\pi k \tau) [V \cos \gamma \cos \psi / ((k-1+h) \cos \phi)], \quad 0 \leq t \leq 1, \quad (8b)$$

$$\dot{\phi} = (2\pi k \tau) [V \cos \gamma \sin \psi / (k-1+h)], \quad 0 \leq t \leq 1, \quad (8c)$$

$$\dot{V} = -(2\pi k \tau) [A \rho V^2 (1+\lambda^2) + k \sin \gamma / (k-1+h)^2], \quad 0 \leq t \leq 1, \quad (8d)$$

$$\dot{\gamma} = (2\pi k \tau) [B \rho V \lambda \cos \sigma + V \cos \gamma / (k-1+h) - k \cos \gamma / V (k-1+h)^2], \quad 0 \leq t \leq 1, \quad (8e)$$

$$\dot{\psi} = (2\pi k \tau) [B \rho V \lambda \sin \sigma / \cos \gamma - V \cos \gamma \cos \psi \tan \phi / (k-1+h)], \quad 0 \leq t \leq 1. \quad (8f)$$

In the system (8), the independent variable is the normalized time t , $0 \leq t \leq 1$. The dependent variables include six state variables ($h, V, \gamma, \theta, \phi, \psi$), two control variables (λ, σ), and one parameter (τ).

Boundary Conditions. At the entrance of the sensible atmosphere ($t = 0$) and at the exit of the sensible atmosphere ($t = 1$), certain static and dynamic boundary conditions must be satisfied. Specifically, the entrance conditions are represented by

$$h_0 = 1, \quad (9a)$$

$$(2-V_0^2)r_{00}^2-2r_{00}+V_0^2\cos^2\gamma_0 = 0, \quad (9b)$$

$$\theta_0 = 0, \quad (9c)$$

$$\phi_0 = 0, \quad (9d)$$

$$\psi_0 = 0, \quad (9e)$$

and the exit conditions are represented by

$$h_1 = 1, \quad (10a)$$

$$[2-(V_1+\xi)^2]r_{11}^2-2r_{11}+(V_1+\xi)^2\cos^2\gamma_1 = 0, \quad (10b)$$

$$\cos\phi_1\cos\psi_1 -\cos(\Delta i) = 0. \quad (10c)$$

Equations (9a) and (10a) are a consequence of the normalization scale for the altitude. Equations (9b) and (10b) are derived from energy conservation and angular momentum conservation applied to the HEO-to-entry elliptical transfer orbit and the exit-to-LEO elliptical transfer orbit, respectively; in particular, Eq. (10b) accounts for the fact that a tangential propulsive burn takes place at the exit of the atmosphere, incrementing the exit velocity from V_1 to $V_1 + \xi$. Equations (9c), (9d), (9e) assume a particular type of entry, with the entry velocity contained in the equatorial plane. Finally, Eq. (10c) constitutes a relationship between the exit latitude, the exit heading angle, and the prescribed plane change Δi under the assumption that the plane change is performed entirely in the atmospheric part of the trajectory. Note that Eq. (10c) is derived from the general relation

$$\cos\phi\cos\psi-\cos i = 0, \quad 0 \leq t \leq 1, \quad (11)$$

after observing that

$$\Delta i = i_{11} - i_{00} = i_1 - i_0 = i_1 - 0 = i_1. \quad (12)$$

To ensure that the vehicle enters into the atmosphere at point 0 and exits from the atmosphere at point 1, the path inclinations γ_0, γ_1 must satisfy the inequality constraints

$$\gamma_0 \leq 0, \quad (13a)$$

$$\gamma_1 \geq 0. \quad (13b)$$

These inequality constraints can be converted into equality constraints by means of the Valentine transformations

$$\gamma_0 + \eta^2 = 0, \quad (14a)$$

$$\gamma_1 - \xi^2 = 0, \quad (14b)$$

where η and ξ denote supplementary parameters.

Control Bounds. The presence of upper and lower bounds on the lift coefficient is necessary if realistic solutions are to be obtained. Therefore, the following assumption is made:

$$C_{La} \leq C_L \leq C_{Lb}, \quad 0 \leq t \leq 1, \quad (15)$$

with the implication that

$$\lambda_a \leq \lambda \leq \lambda_b, \quad 0 \leq t \leq 1. \quad (16)$$

The two-sided inequality constraints (16) can be converted into an equality constraint by means of a trigonometric transformation of the type

$$\lambda = (1/2)[(\lambda_a + \lambda_b) + (\lambda_b - \lambda_a)\sin\beta], \quad (17)$$

where $\beta(t)$ denotes an auxiliary control. Equation (17) can be employed to eliminate $\lambda(t)$ from (8). Therefore, the old control $\lambda(t)$ is replaced with the new control $\beta(t)$.

Remark. To sum up, the equations governing the atmospheric pass include the differential system (8), the boundary conditions (9)-(10), the

supplementary boundary conditions (14), and the two-sided inequality constraint (16), converted into an equality constraint by means of the trigonometric transformation (17). In this formulation, the independent variable is the time t , $0 \leq t \leq 1$. The dependent variables include six state variables $(h, V, \gamma, \theta, \phi, \psi)$, two control variables (β, σ) , and four parameters (τ, ξ, η, ζ) . After a solution is found for $\beta(t)$, then $\lambda(t)$ is computed with (17).

3. Performance Indexes

Subject to the previous constraints, different AOT optimization problems can be formulated, depending upon the performance index chosen. The resulting optimal control problems are either of the Bolza type [see Problems (P1) and (P2) below] or of the Chebyshev type [see Problem (Q1) below].

Problem (P1). It is required to minimize the energy required for orbital transfer. A measure of this energy is the total characteristic velocity ΔV , the sum of the initial characteristic velocity ΔV_{00} , associated with the propulsive burn from HEO, the final characteristic velocity ΔV_{11} , associated with the propulsive burn into LEO, and the intermediate characteristic velocity ΔV_1 , associated with the propulsive burn at the exit of the atmosphere. Clearly,

$$I = \Delta V = \Delta V_{00} + \Delta V_{11} + \Delta V_1. \quad (18a)$$

The characteristic velocities ΔV_{00} , ΔV_{11} , ΔV_1 are given by

$$\Delta V_{00} = \sqrt{(1/r_{00}) - (1/r_0)} V_0 \cos \gamma_0, \quad (18b)$$

$$\Delta V_{11} = \sqrt{(1/r_{11}) - (1/r_1)} (V_1 + \xi) \cos \gamma_1, \quad (18c)$$

$$\Delta V_1 = \xi. \quad (18d)$$

Problem (P2). It is required to minimize the time integral of the square of path inclination. Here, the performance index is given by

$$I = \int_0^1 \tau \gamma^2 dt. \quad (19)$$

The solution of this problem is called the nearly-grazing trajectory.

Problem (Q1). It is required to minimize the peak value of the heating rate at a particular point of the spacecraft, for instance, the stagnation point. Except for a proportionality constant, the performance index is given by

$$I = \max_t (\sqrt{\rho} V^{3.08}), \quad 0 \leq t \leq 1. \quad (20a)$$

This problem can be reformulated as that of minimizing the integral performance index (see Ref. 28)

$$J = \int_0^1 \tau [\rho V^{3.08}]^q dt, \quad (20b)$$

for large values of q (for example, $q = 5, 10$, or 20).

Remark. Problems (P1) and (P2) are standard problems of optimal control of the Bolza type. Problem (Q1) is a nonstandard problem of optimal control of the Chebyshev type, if treated in the format (20a); however, Problem (Q1) becomes a standard problem of optimal control of the Bolza type, if treated in the format (20b).

4. Data for the Examples

The data used in the numerical experiments are summarized in Tables 1-4.

Table 1 refers to the spacecraft data. It shows the mass per unit reference surface m/S , the zero-lift drag coefficient C_{D0} , the induced drag factor K , the lift coefficient for maximum lift-to-drag ratio C_{LR} , and the maximum lift-to-drag ratio E_{max} . Table 1 also shows the coefficients A , B appearing in the normalized differential system (8). In addition, Table 1 shows the bounds C_{La} , C_{Lb} on the lift coefficient C_L and the bounds λ_a , λ_b on the normalized lift coefficient λ , which is the one of the two control variables of Problems (P1), (P2), (Q1).

Table 2 supplies the major physical constants appearing in Problems (P1), (P2), (Q1). This includes the radius of the Earth r_e , the radius of the outer edge of the sensible atmosphere r_a , the height of the sensible atmosphere H , the Earth's gravitational constant μ , and the ratio $k = r_a/H$.

Table 3 supplies the reference values employed for the altitude h_R , the radius r_R , the velocity V_R , the flight time τ_R , and the density ρ_R .

Table 4 presents the radii r_{00} and r_{11} of the terminal orbits considered, in both dimensional form and dimensionless form. It presents also the terminal orbital inclinations i_{00} and i_{11} .

The atmospheric model assumed is that of the US Standard Atmosphere, 1976 (see Ref. 45). In this model, the values of the density are tabulated at discrete altitudes. For intermediate altitudes, the density is computed by assuming an exponential fit for the function $\rho(h)$.

5. Numerical Solutions

Problems (P1), (P2), (Q1) were solved employing the sequential gradient-restoration algorithm (SGRA). This algorithm was programmed in FORTRAN IV; a FORTRAN G1 compiler was used; the numerical results were obtained in double-precision arithmetic. Computations were performed at Rice University using an NAS-AS-9000 computer.

The interval of integration was divided into 100 steps. The differential systems were integrated using Hamming's modified predictor-corrector method, with a special Runge-Kutta starting procedure (Ref. 46). Definite integrals were computed using a modified Simpson's rule. Linear algebraic systems were solved using a standard Gaussian elimination routine. For both the gradient phase and the restoration phase of SGRA, the linear two-point boundary-value problem was solved using the method of particular solutions (Refs. 17-19).

For each problem, the functional being minimized was suitably scaled. The following stopping conditions were employed for the sequential gradient-restoration algorithm:

$$P \leq E-08, \quad Q \leq E-08, \quad (21)$$

where P is the norm squared of the constraint error and Q is the norm squared of the error in the optimality conditions. An upper bound was imposed on the total number of iterations, namely,

$$N \leq 100. \quad (22)$$

As noted in Section 1 and Ref. 15, SGRA is available in both the primal formulation (PSGRA) and the dual formulation (DSGRA). For the present problems, some test runs indicated that PSGRA is superior to DSGRA in terms of stability and ability to achieve a solution. Hence, the systematic runs were made with PSGRA.

The numerical solutions are shown in Figs. 2-4. Each figure refers to a different optimization problem and contains eight parts: the altitude history $h(t)$, the velocity history $V(t)$, the path inclination history $\gamma(t)$, the longitude history $\theta(t)$, the latitude history $\phi(t)$, the heading angle history $\psi(t)$, the lift coefficient history $\lambda(t) = C_L(t)/C_{LR}$, and the bank angle history $\sigma(t)$.

Summary data are presented in Tables 5-7. Each table refers to a different optimization problem and contains the following quantities: the initial characteristic velocity, the exit characteristic velocity, the final characteristic velocity, the total characteristic velocity; the integrated heating rate, the flight time; the peak heating rate, the peak dynamic pressure, the peak altitude drop, the minimum altitude; the peak tangential acceleration, the peak normal acceleration, the peak total acceleration; the plane change; the entry values of the altitude, the velocity, the path inclination, the longitude, the latitude, and the heading angle; and the exit values of the altitude, the velocity, the path inclination, the longitude, the latitude, and the heading angle.

A consistency check of the accuracy of the solutions obtained is presented in Table 8. This matrix table presents the values of all of the performance indexes (18)-(20) for all of the solutions of Problems (P1), (P2), (Q1) in the dimensionless form employed in the computation. Clearly, the consistency check requires the minimal values to occur along the principal diagonal of the square matrix table. This is precisely the case, as shown in Table 8.

Finally, the energy requirements are presented in Table 9 for the best outer space trajectory, the two-impulse Hohmann transfer (non-aeroassisted orbital transfer, NAOT).

With reference to Tables 5-9 and Figs. 2-4, the following comments are

pertinent.

From the energy viewpoint, we see that the total characteristic velocity of the solution (P1), $\Delta V = 1869 \text{ m sec}^{-1}$, is 62% of that of the two-impulse Hohmann transfer, $\Delta V = 3030 \text{ m sec}^{-1}$; also, the total characteristic velocity of the solution (P2), $\Delta V = 2343 \text{ m sec}^{-1}$, is 77% of that of the two-impulse Hohmann transfer. On the other hand, the total characteristic velocity of the solution (Q1), $\Delta V = 3621 \text{ m sec}^{-1}$, is 120% of that of the two-impulse Hohmann transfer; therefore, we must discard the solution (Q1).

As a consequence, we are left with two solutions: the minimum energy solution (P1) and the nearly-grazing solution (P2). While the solution (P2) requires 25% more energy than the solution (P1), at the same time we see that the integrated heating rate of the solution (P2), $IHR = 22470 \text{ W sec cm}^{-2}$, is 8% less than that of the solution (P1), $IHR = 24430 \text{ W sec cm}^{-2}$; the peak heating rate of the solution (P2), $PHR = 146.2 \text{ W cm}^{-2}$, is 33% less than that of the solution (P1), $PHR = 217.6 \text{ W cm}^{-2}$; the peak dynamic pressure of the solution (P2), $PDP = 2.284 \text{ N cm}^{-2}$, is 41% less than that of the solution (P1), $PDP = 3.893 \text{ N cm}^{-2}$. In addition, the solution (P2) requires less peak total acceleration (23%) than the solution (P1). Therefore, it is felt that both solutions (P1) and (P2) should be considered as candidates for flight operations, and hence candidates for spacecraft design.

6. Conclusions

This thesis considers both classical and minimax problems of optimal control arising in the study of noncoplanar, aeroassisted orbital transfer. The maneuver considered involves the transfer from a high planetary orbit to a low planetary orbit with a prescribed atmospheric plane change. An example is the HEO-to-LEO transfer of a spacecraft with a prescribed plane change, where HEO denotes high Earth orbit and LEO denotes low Earth orbit.

The basic idea is to employ the hybrid combination of propulsive maneuvers in space and aerodynamic maneuvers in the sensible atmosphere. Hence, this type of flight is also called synergetic space flight. With reference to the atmospheric part of the maneuver, trajectory control is achieved by modulating the lift coefficient (hence, the angle of attack) and the angle of bank. The presence of upper and lower bounds on the lift coefficient is considered.

Within the framework of classical optimal control, the performance indexes studied are the energy required for orbital transfer and the time integral of the square of the path inclination; these yield Bolza problems of optimal control. Within the framework of minimax optimal control, the performance studied is the peak heating rate; this yields a Chebyshev problem of optimal control, which can be converted into a Bolza problem by suitable transformations.

Numerical solutions for Problems (P1), (P2), (Q1) are obtained by means of the sequential gradient-restoration algorithm for optimal control problems. Numerical examples are presented, and their engineering implications are discussed. It appears that the minimum energy solution (P1) and the nearly-grazing solution (P2) are superior to the minimum peak heating rate solution

(Q1). While the nearly-grazing solution (P2) requires 25% more energy than the minimum energy solution (P1), at the same time it involves less integrated heating rate (8%), less peaking heating rate (33%), less peak dynamic pressure (41%), and less total acceleration (23%). Therefore, it is felt that both solutions (P1) and (P2) should be considered as candidates for flight operations, and hence candidate for spacecraft design.

Table 1. Spacecraft data.

Quantity	Value	Units
m/S	0.3000E+03	kg m ⁻²
C _{D0}	0.1000E+00	-
K	0.1110E+01	-
C _{LR}	0.3002E+00	-
E _{max}	0.1501E+01	-
A	0.0796E+00	-
B	0.2399E+00	-
C _{La}	-0.9000E+00	-
C _{Lb}	+0.9000E+00	-
λ _a	-0.3000E+01	-
λ _b	+0.3000E+01	-

Table 2. Physical constants.

Quantity	Value	Units
r _e	0.6378E+07	m
r _a	0.6498E+07	m
H	0.1200E+06	m
μ	0.3986E+15	m ³ sec ⁻²
k	0.5415E+02	-

Table 3. Reference values.

Quantity	Value	Units
h_R	0.1200E+06	m
r_R	0.6498E+07	m
V_R	0.7832E+04	m sec ⁻¹
τ_R	0.5212E+04	sec
ρ_R	0.3996E-02	kg m ⁻³

Table 4. Terminal orbits.

Quantity	Value	Units
r_{00}	0.12996E+08	m
r_{11}	0.65580E+07	m
r_{00}/r_R	0.20000E+01	-
r_{11}/r_R	0.10093E+01	-
i_{00}	0.00000E+00	deg
i_{11}	0.20000E+02	deg

Table 5A. Results for AOT, Problem (P1), minimum energy.

Quantity	Dimensionless value	Reference value	Dimensional value	Units
Initial characteristic velocity	0.1321E+00	0.7832E+04	0.1034E+04	m sec ⁻¹
Exit characteristic velocity	0.1043E+00	0.7832E+04	0.8166E+03	m sec ⁻¹
Final characteristic velocity	0.2314E-02	0.7832E+04	0.1812E+02	m sec ⁻¹
Total characteristic velocity	0.2386E+00	0.7832E+04	0.1869E+04	m sec ⁻¹
Integrated heating rate	0.1344E-01	0.1817E+11	0.2443E+09	W sec m ⁻²
Flight time	0.9090E-01	0.5212E+04	0.4738E+03	sec
Peak heating rate (t = 0.33)	0.6239E+00	0.3487E+07	0.2176E+07	W m ⁻²
Peak dynamic pressure (t = 0.35)	0.3175E+00	0.1226E+06	0.3893E+05	N m ⁻²
Peak altitude drop (t = 0.36)	0.5938E+00	0.1200E+06	0.7126E+05	m
Minimum altitude (t = 0.36)	0.4062E+00	0.1200E+06	0.4874E+05	m
Peak tangential acceleration (t = 0.35)	0.2602E+01	0.9810E+01	0.2553E+02	m sec ⁻²
Peak normal acceleration (t = 0.34)	0.3999E+01	0.9810E+01	0.3923E+02	m sec ⁻²
Peak total acceleration (t = 0.34)	0.4770E+01	0.9810E+01	0.4679E+02	m sec ⁻²
Plane change	0.3491E+00	0.5730E+02	0.2000E+02	deg

Table 5B. Results for AOT, Problem (P1), minimum energy.

Quantity	Dimensionless value	Reference value	Dimensional value	Units
Entry altitude	0.1000E+01	0.1200E+06	0.1200E+06	m
Entry velocity	0.1154E+01	0.7832E+04	0.9035E+04	m sec ⁻¹
Entry path inclination	-0.7740E-01	0.5730E+02	-0.4435E+01	deg
Entry longitude	0.0000E+00	0.5730E+02	0.0000E+00	deg
Entry latitude	0.0000E+00	0.5730E+02	0.0000E+00	deg
Entry heading angle	0.0000E+00	0.5730E+02	0.0000E+00	deg
Exit altitude	0.1000E+01	0.1200E+06	0.1200E+06	m
Exit velocity	0.8981E+00	0.7832E+04	0.7034E+04	m sec ⁻¹
Exit path inclination	0.4359E-06	0.5730E+02	0.2497E-04	deg
Exit longitude	0.5509E+00	0.5730E+02	0.3156E+02	deg
Exit latitude	0.1156E+00	0.5730E+02	0.6626E+01	deg
Exit heading angle	0.3301E+00	0.5730E+02	0.1891E+02	deg

Table 6A. Results for AOT, Problem (P2), minimum time integral of the square of the path inclination.

Quantity	Dimensionless value	Reference value	Dimensional value	Units
Initial characteristic velocity	0.1311E+00	0.7832E+04	0.1027E+04	m sec ⁻¹
Exit characteristic velocity	0.1657E+00	0.7832E+04	0.1298E+04	m sec ⁻¹
Final characteristic velocity	0.2314E-02	0.7832E+04	0.1812E+02	m sec ⁻¹
Total characteristic velocity	0.2992E+00	0.7832E+04	0.2343E+04	m sec ⁻¹
Integrated heating rate	0.1236E-01	0.1817E+11	0.2247E+09	W sec m ⁻²
Flight time	0.1021E+00	0.5212E+04	0.5321E+03	sec
Peak heating rate (t = 0.51)	0.4192E+00	0.3487E+07	0.1462E+07	W m ⁻²
Peak dynamic pressure (t = 0.52)	0.1863E+00	0.1226E+06	0.2284E+05	N m ⁻²
Peak altitude drop (t = 0.54)	0.5656E+00	0.1200E+06	0.6787E+05	m
Minimum altitude (t = 0.54)	0.4344E+00	0.1200E+06	0.5213E+05	m
Peak tangential acceleration (t = 0.52)	0.2114E+01	0.9810E+01	0.2074E+02	m sec ⁻²
Peak normal acceleration (t = 0.52)	0.3025E+01	0.9810E+01	0.2967E+02	m sec ⁻²
Peak total acceleration (t = 0.52)	0.3690E+01	0.9810E+01	0.3620E+02	m sec ⁻²
Plane change	0.3491E+00	0.5730E+02	0.2000E+02	deg

Table 6B. Results for AOT, Problem (P2), minimum time integral of the square of the path inclination.

Quantity	Dimensionless value	Reference value	Dimensional value	Units
Entry altitude	0.1000E+01	0.1200E+06	0.1200E+06	m
Entry velocity	0.1154E+01	0.7832E+04	0.9038E+04	m sec ⁻¹
Entry path inclination	-0.5943E-01	0.5730E+02	-0.3405E+01	deg
Entry longitude	0.0000E+00	0.5730E+02	0.0000E+00	deg
Entry latitude	0.0000E+00	0.5730E+02	0.0000E+00	deg
Entry heading angle	0.0000E+00	0.5730E+02	0.0000E+00	deg
Exit altitude	0.1000E+01	0.1200E+06	0.1200E+06	m
Exit velocity	0.8366E+00	0.7832E+04	0.6552E+04	m sec ⁻¹
Exit path inclination	0.0000E+00	0.5730E+02	0.0000E+00	deg
Exit longitude	0.6243E+00	0.5730E+02	0.3577E+02	deg
Exit latitude	0.9866E-01	0.5730E+02	0.5653E+01	deg
Exit heading angle	0.3354E+00	0.5730E+02	0.1922E+02	deg

Table 7A. Results for AOT, Problem (Q1), minimum peak heating rate.

Quantity	Dimensionless value	Reference value	Dimensional value	Units
Initial characteristic velocity	0.1323E+00	0.7832E+04	0.1036E+04	m sec ⁻¹
Exit characteristic velocity	0.3277E+00	0.7832E+04	0.2567E+04	m sec ⁻¹
Final characteristic velocity	0.2314E-02	0.7832E+04	0.1812E+02	m sec ⁻¹
Total characteristic velocity	0.4623E+00	0.7832E+04	0.3621E+04	m sec ⁻¹
Integrated heating rate	0.7667E-02	0.1817E+11	0.1394E+09	W sec m ⁻²
Flight time	0.7084E-01	0.5212E+04	0.3692E+03	sec
Peak heating rate (t = 0.50)	0.2978E+00	0.3487E+07	0.1038E+07	W m ⁻²
Peak dynamic pressure (t = 0.51)	0.1865E+00	0.1226E+06	0.2286E+05	N m ⁻²
Peak altitude drop (t = 0.52)	0.5872E+00	0.1200E+06	0.7047E+05	m
Minimum altitude (t = 0.52)	0.4128E+00	0.1200E+06	0.4953E+05	m
Peak tangential acceleration (t = 0.54)	0.5853E+01	0.9810E+01	0.5742E+02	m sec ⁻²
Peak normal acceleration (t = 0.54)	0.4854E+01	0.9810E+01	0.4762E+02	m sec ⁻²
Peak total acceleration (t = 0.54)	0.7604E+01	0.9810E+01	0.7459E+02	m sec ⁻²
Plane change	0.3491E+00	0.5730E+02	0.2000E+02	deg

Table 7B. Results for AOT, Problem (Q1), minimum peak heating rate.

Quantity	Dimensionless value	Reference value	Dimensional value	Units
Entry altitude	0.1000E+01	0.1200E+06	0.1200E+06	m
Entry velocity	0.1153E+01	0.7832E+04	0.9034E+04	m sec ⁻¹
Entry path inclination	-0.8073E-01	0.5730E+02	-0.4625E+01	deg
Entry longitude	0.0000E+00	0.5730E+02	0.0000E+00	deg
Entry latitude	0.0000E+00	0.5730E+02	0.0000E+00	deg
Entry heading angle	0.0000E+00	0.5730E+02	0.0000E+00	deg
Exit altitude	0.1000E+01	0.1200E+06	0.1200E+06	m
Exit velocity	0.6746E+00	0.7832E+04	0.5284E+04	m sec ⁻¹
Exit path inclination	0.0000E+00	0.5730E+02	0.0000E+00	deg
Exit longitude	0.3856E+00	0.5730E+02	0.2209E+02	deg
Exit latitude	0.6561E-01	0.5730E+02	0.3759E+01	deg
Exit heading angle	0.3431E+00	0.5730E+02	0.1966E+02	deg

Table 8. Consistency check (dimensionless performance indexes).

	(P1)	(P2)	(Q1)
I(P1)	0.2386E+00	0.2992E+00	0.4623E+00
I(P2)	0.1650E-03	0.1563E-03	0.3273E-03
I(Q1)	0.6239E+00	0.4192E+00	0.2978E+00

Table 9. Results for NAOT, two-impulse Hohmann transfer.

Quantity	Dimensionless value	Reference value	Dimensional value	Units
Initial characteristic velocity	0.2144E+00	0.7832E+04	0.1679E+04	m sec ⁻¹
Final characteristic velocity	0.1725E+00	0.7832E+04	0.1351E+04	m sec ⁻¹
Total characteristic velocity	0.3869E+00	0.7832E+04	0.3030E+04	m sec ⁻¹

References

1. HESTENES, M. R., Calculus of Variations and Optimal Control Theory, John Wiley and Sons, New York, New York, 1966.
2. BRYSON, A. E., JR., and HO, Y. C., Applied Optimal Control, Blaisdell Publishing Company, Waltham, Massachusetts, 1969.
3. LEITMANN, G., The Calculus of Variations and Optimal Control, Plenum Publishing Corporation, New York, New York, 1981.
4. MIELE, A., Flight Mechanics, Vol. 1, Addison-Wesley Publishing Company, Reading, Massachusetts, 1962.
5. VINH, N. X., BUSEMANN, A., and CULP, R. D., Hypersonic and Planetary Entry Flight Mechanics, The University of Michigan Press, Ann Arbor, Michigan, 1980.
6. VINH, N. X., Optimal Trajectories in Atmospheric Flight, Elsevier Scientific Publishing Company, New York, New York, 1981.
7. MAREC, J. P., Optimal Space Trajectories, Elsevier Scientific Publishing Company, Amsterdam, Holland, 1979.
8. MIELE, A., PRITCHARD, R. E., and DAMOULAKIS, J. N., Sequential Gradient-Restoration Algorithm for Optimal Control Problems, Journal of Optimization Theory and Applications, Vol. 5, No. 4, pp. 235-282, 1970.
9. MIELE, A., DAMOULAKIS, J. N., CLOUTIER, J. R., and TIETZE, J. L., Sequential Gradient-Restoration Algorithm for Optimal Control Problems with Nondifferential Constraints, Journal of Optimization Theory and Applications, Vol. 13, No. 2, pp. 218-255, 1974.
10. MIELE, A., Recent Advances in Gradient Algorithms for Optimal Control Problems, Journal of Optimization Theory and Applications, Vol. 17, Nos. 5/6, pp. 361-430, 1975.

11. GONZALEZ, S., and MIELE, A., Sequential Gradient-Restoration Algorithm for Optimal Control Problems with General Boundary Conditions, Journal of Optimization Theory and Applications, Vol. 26, No. 3, pp. 395-425, 1978.,
12. MIELE, A., Gradient Algorithms for the Optimization of Dynamic Systems, Control and Dynamic Systems, Advances in Theory and Applications, Edited by C. T. Leondes, Academic Press, New York, New York, Vol. 16, pp. 1-52, 1980.
13. MIELE, A., BASAPUR, V. K., and COKER, E. M., Combined Gradient-Restoration Algorithm for Optimal Control Problems with Nondifferential Constraints and General Boundary Conditions, Parts 1 and 2, Rice University, Aero-Astronautics Reports Nos. 163 and 164, 1983.
14. MIELE, A., and WANG, T., Primal-Dual Properties of Sequential Gradient-Restoration Algorithms for Optimal Control Problems, Parts 1 and 2, Rice University, Aero-Astronautics Reports Nos. 183 and 184, 1985.
15. MIELE, A., WANG, T., and BASAPUR, V. K., Primal and Dual Formulation of Sequential Gradient-Restoration Algorithms for Trajectory Optimization Problems, Rice University, Aero-Astronautics Report No. 199, 1985.
16. MIELE, A., KUO, Y. M., and COKER, E. M., Modified Quasilinearization Algorithm for Optimal Control Problems with Nondifferential Constraints and General Boundary Conditions, Parts 1 and 2, Rice University, Aero-Astronautics Reports Nos. 161 and 162, 1982.
17. MIELE, A., Method of Particular Solutions for Linear, Two-Point Boundary-Value Problems, Journal of Optimization Theory and Applications, Vol. 2, No. 4, pp. 260-273, 1968.
18. MIELE, A., and IYER, R. R., Modified Quasilinearization Method for Solving Nonlinear, Two-Point Boundary-Value Problems, Journal of Mathematical

- Analysis and Applications, Vol. 36, No. 3, pp. 674-692, 1971.
19. MIELE, A., and IYER, R. R., General Technique for Solving Nonlinear, Two-Point Boundary-Value Problems via the Method of Particular Solutions, Journal of Optimization Theory and Applications, Vol. 5, No. 5, pp. 382-399, 1970.
 20. WARGA, J., Minimax Problems and Unilateral Curves in the Calculus of Variations, SIAM Journal on Control, Vol. 3, No. 1, pp. 91-105, 1965.
 21. JOHNSON, C. D., Optimal Control with Chebyshev Minimax Performance Index, Journal of Basic Engineering, Vol. 89, No. 2, pp. 251-262, 1967.
 22. MICHAEL, G. J., Computation of Chebyshev Optimal Control, AIAA Journal, Vol. 9, No. 5, pp. 973-975, 1971.
 23. POWERS, W. F., A Chebyshev Minimax Technique Oriented to Aerospace Trajectory Optimization Problems, AIAA Journal, Vol. 10, No. 10, pp. 1291-1296, 1972.
 24. HOLMAKER, K., A Minimax Optimal Control Problem, Journal of Optimization Theory and Applications, Vol. 28, No. 3, pp. 391-410, 1979.
 25. HOLMAKER, K., A Property of an Autonomous Minimax Optimal Control Problem, Journal of Optimization Theory and Applications, Vol. 32, No. 1, pp. 81-87, 1980.
 26. MIELE, A., MOHANTY, B. P., VENKATARAMAN, P., and KUO, Y. M., Numerical Solution of Minimax Problems of Optimal Control, Part 1, Journal of Optimization Theory and Applications, Vol. 38, No. 1, pp. 97-109, 1982.
 27. MIELE, A., MOHANTY, B. P., VENKATARAMAN, P., and KUO, Y. M., Numerical Solution of Minimax Problems of Optimal Control, Part 2, Journal of Optimization Theory and Applications, Vol. 38, No. 1, pp.

- 111-135, 1982.
28. MIELE, A., and WANG, T., *An Elementary Proof of a Functional Analysis Result Having Interest for Minimax Optimal Control of Aeroassisted Orbital Transfer Vehicles*, Rice University, Aero-Astronautics Report No. 182, 1985.
 29. MIELE, A., and VENKATARMAN, P., *Minimax Optimal Control and Its Application to the Reentry of a Space Glider*, Recent Advances in the Aerospace Sciences, Edited by L. Casci, Plenum Publishing Corporation, New York, New York, pp. 21-40, 1985.
 30. MIELE, A., and VENKATARAMAN, P., *Optimal Trajectories for Aeroassisted Orbital Transfer*, Acta Astronautica, Vol. 11, Nos. 7/8, pp. 423-433, 1984.
 31. MIELE, A., and BASAPUR, V. K., *Approximate Solutions to Minimax Optimal Control Problems for Aeroassisted Orbital Transfer*, Acta Astronautica, Vol. 12, No. 10, pp. 809-818, 1985.
 32. MIELE, A., BASAPUR, V. K., and MEASE, K. D., *Nearly-Grazing Optimal Trajectories for Aeroassisted Orbital Transfer*, Journal of the Astronautical Sciences, Vol. 34, No. 1, pp. 3-18, 1986.
 33. MIELE, A., BASAPUR, V. K., and LEE, W. Y., *Optimal Trajectories for Aeroassisted, Coplanar Orbital Transfer*, Rice University, Aero-Astronautics Report No. 196, 1985.
 34. LONDON, H. S., *Change of Satellite Orbit Plane by Aerodynamic Maneuvering*, Journal of the Aerospace Sciences, Vol. 29, No. 3, pp. 323-332, 1962.
 35. PAINE, J. P., *Some Considerations on the Use of Lifting Reentry Vehicles for Synergetic Maneuvers*, Journal of Spacecraft and Rockets, Vol. 4, No. 5, pp. 698-700, 1967.

36. ROSSLER, M., Optimal Aerodynamic-Propulsive Maneuvering for the Orbital Plane Change of a Space Vehicle, Journal of Spacecraft and Rockets, Vol. 4, No. 12, pp. 1678-1680, 1967.
37. WALBERG, G. D., A Survey of Aeroassisted Orbit Transfer, Journal of Spacecraft and Rockets, Vol. 22, No. 1, pp. 3-18, 1985.
38. JOOSTEN, B. K., and PIERSON, B. L., Minimum Fuel Aerodynamic Orbital Plane Change Maneuvers, Paper No. AIAA-81-0167, AIAA 19th Aerospace Sciences Meeting, St. Louis, Missouri, 1981.
39. KECHICHIAN, J. A., CRUZ, M. I., VINH, N. X., and RINDERLE, E. A., Optimization and Closed-Loop Guidance of Drag Modulated Aeroassisted Orbital Transfer, Paper No. AIAA-83-2093, AIAA Flight Mechanics Conference, Gatlinburg, Tennessee, 1983.
40. CRUZ, M. I., Trajectory Optimization and Closed-Loop Guidance of Aeroassisted Orbital Transfer, Paper No. AAS-83-413, AAS/AIAA Astrodynamics Specialist Conference, Lake Placid, New York, 1983.
41. VINH, N. X., and HANSON, J. M., Optimal Aeroassisted Return from High Earth Orbit with Plane Change, Paper No. IAF-83-330, 34th Congress of the International Astronautical Federation, Budapest, Hungary, 1983.
42. VINH, N. X., and HANSON, J. M., Optimal Aeroassisted Return from High Earth Orbit with Plane Change, Acta Astronautica, Vol. 12, No. 1, pp. 11-25, 1985.
43. MEASE, K. D., and VINH, N. X., Minimum-Fuel Aeroassisted Coplanar Orbit Transfer Using Lift Modulation, Journal of Guidance, Control, and Dynamics, Vol. 8, No. 1, pp. 134-141, 1985.
44. HULL, D. G., GILTNER, J. M., SPEYER, J. L., and MAPAR, J., Minimum Energy-Loss Guidance for Aeroassisted Orbital Plane Change, Journal of

Guidance, Control, and Dynamics, Vol. 8, No. 4, pp. 487-493, 1985.

45. NOAA, NASA, and USAF, US Standard Atmosphere, 1976, US Government Printing Office, Washington, DC, 1976.
46. RALSTON, A., Numerical Integration Methods for the Solution of Ordinary Differential Equations, Mathematical Methods for Digital Computers, Vol. 1, Edited by A. Ralston and H. S. Wilf, John Wiley and Sons, New York, New York, pp. 95-109, 1960.

List of Captions

Fig. 1A. Aeorassisted, noncoplanar orbital transfer.

Fig. 1B. Aeroassisted, coplanar orbital transfer.

Fig. 2A. Problem (P1), altitude versus time.

Fig. 2B. Problem (P1), velocity versus time.

Fig. 2C. Problem (P1), path inclination versus time.

Fig. 2D. Problem (P1), longitude versus time.

Fig. 2E. Problem (P1), latitude versus time.

Fig. 2F. Problem (P1), heading angle versus time.

Fig. 2G. Problem (P1), lift coefficient versus time.

Fig. 2H. Problem (P1), bank angle versus time.

Fig. 3A. Problem (P2), altitude versus time.

Fig. 3B. Problem (P2), velocity versus time.

Fig. 3C. Problem (P2), path inclination versus time.

Fig. 3D. Problem (P2), longitude versus time.

Fig. 3E. Problem (P2), latitude versus time.

Fig. 3F. Problem (P2), heading angle versus time.

Fig. 3G. Problem (P2), lift coefficient versus time.

Fig. 3H. Problem (P2), bank angle versus time.

Fig. 4A. Problem (Q1), altitude versus time.

Fig. 4B. Problem (Q1), velocity versus time.

Fig. 4C. Problem (Q1), path inclination versus time.

Fig. 4D. Problem (Q1), longitude versus time.

Fig. 4E. Problem (Q1), latitude versus time.

Fig. 4F. Problem (Q1), heading angle versus time.

Fig. 4G. Problem (Q1), lift coefficient versus time.

Fig. 4H. Problem (Q1), bank angle versus time.

Fig. 1A. Aeorassisted, noncoplanar orbital transfer.

Fig. 1B. Aeroassisted, coplanar orbital transfer.

FIG. 2A: ALTITUDE VS TIME,
PROBLEM (P1).

FIG. 2B: VELOCITY VS TIME,
PROBLEM (P1).

FIG. 2C: PATH INCLINATION VS TIME,
PROBLEM (P1).

FIG. 2D: LONGITUDE VS TIME,
PROBLEM (P1).

FIG. 2E: LATITUDE VS TIME,
PROBLEM (P1).

FIG. 2F: HEADING ANGLE VS TIME,
PROBLEM (P1).

FIG. 2G: LIFT COEFFICIENT VS TIME,
PROBLEM (P1).

FIG. 2H: BANK ANGLE VS TIME,
PROBLEM (P1).

FIG. 3A: ALTITUDE VS TIME,
PROBLEM (P2).

FIG. 3B: VELOCITY VS TIME,
PROBLEM (P2).

FIG. 3C: PATH INCLINATION VS TIME,
PROBLEM (P2).

FIG. 3D: LONGITUDE VS TIME,
PROBLEM (P2).

FIG. 3E: LATITUDE VS TIME,
PROBLEM (P2).

FIG. 3F: HEADING ANGLE VS TIME,
PROBLEM (P2).

FIG. 3G: LIFT COEFFICIENT VS TIME,
PROBLEM (P2).

FIG. 3H: BANK ANGLE VS TIME,
PROBLEM (P2).

FIG. 4A: ALTITUDE VS TIME,
PROBLEM (Q1).

FIG. 4B: VELOCITY VS TIME,
PROBLEM (Q1).

FIG. 4C: PATH INCLINATION VS TIME,
PROBLEM (Q1).

FIG. 4D: LONGITUDE VS TIME,
PROBLEM (Q1).

FIG. 4E: LATITUDE VS TIME,
PROBLEM (Q1).

FIG. 4F: HEADING ANGLE VS TIME,
PROBLEM (Q1).

FIG. 4G: LIFT COEFFICIENT VS TIME,
PROBLEM (Q1).

FIG. 4H: BANK ANGLE VS TIME,
PROBLEM (Q1).