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ULF MAGNETIC FLUCTUATIONS IN THE PLASMA
SHEET AS RECORDED BY THE
EXPLORER 34 SATELLITE

by

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ABSTRACT

From 1967 to 1969, the Explorer 34 satellite made approximately 20 passes (6 were used in this study) through the earth's plasma sheet at a distance of 20 to 34 R_E . On board was a tri-axial, fluxgate magnetometer which made possible the observation of the magnetic field vector every 2.556 seconds. This provided a platform for studying ULF (ultra-low-frequency) magnetic waves in the 0.01 to 0.2 Hz band. The signal was analyzed by calculating the power spectral density function. This function was proportional to the inverse square of the frequency in the plasma sheet and exhibited few, if any, significant spectral peaks. Further, the plasma sheet was characterized by higher power spectral density values than the high-latitude tail. The X_{sm} (sm: solar-magnetospheric coordinate) component exhibited the greatest power spectral density levels in comparison to the other two components. The most significant result, however, was that the power spectral density, calculated from the frequency range 0.012 to 0.1 Hz in the plasma sheet, increased from minimal levels during quiet-times to maximum levels during the later stages of the substorm. It is shown that this result is of importance in testing substorm theories.

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1. INTRODUCTION

The study of magnetospheric phenomena began when the first man thrilled to an auroral display. With the coming of the scientist, these studies took on a more analytic approach. Early work by Störmer [1912] lead to the connection between the earth's magnetic field and aurorae. Chapman and Ferraro [1931] introduced the concept of solar plasma interacting with the earth's magnetic field to produce geomagnetic storms. Data and theory slowly grew as the space age approached.

With the advent of satellites, space science grew rapidly, almost explosively, and with this growth came new knowledge of magnetospheric phenomena and the structure of the magnetosphere. Predictions by Parker [1958] and Piddington [1960] of a long magnetospheric tail drawn out by the solar wind were born out by actual experiment [Heppner et al., 1963; Ness, 1965]. In contrast to the discovery of the magnetospheric tail and, for that matter, the neutral sheet [see Dessler, 1964], the discovery of the plasma sheet [Bame et al., 1966; Cahill, 1966] came as a surprise. This discovery has occupied an important segment of the research efforts of the last few years [see, for instance, Ness et al., 1967 a, b; or Behannon, 1970]. Even so, the plasma sheet has not yet been adequately explained in a theoretical manner.

In 1971, two papers [Hill and Dessler, 1971; Hill, 1971] appeared which offered an alternative to previous models of the plasma sheet and made definite theoretical predictions. The central theme of the model was that the plasma sheet was contained on open (not closed) field lines. No specific mechanism was proposed for containing the plasma sheet, but wave-particle interactions leading to a diffusive mechanism were offered as one possibility.

In a plasma-sheet study by Garrett et al. [1971], experimental evidence for plasma flow away from the earth was suggested. This thesis is the next step in the study--investigation into actual wave-particle interactions in the plasma sheet. If such interactions are taking place, one frequency range where they might be observed would be near the proton cyclotron frequency which is about 0.16 Hz (for a 10 γ field) in the plasma sheet. If Doppler shifting resulting from the motion of the satellite relative to the plasma flow is considered, it might be possible to observe the effects of wave-particle interactions by looking for spectral features in the ultra-low-frequency (ULF) band.

Another hypothesis, that the Hill-Dessler theory proposed, suggested that the wave-particle interaction rate should go down with increasing magnetic activity. It was felt that this drop should be observed during the beginning of substorm onset and herald the sudden flow of plasma from the earthward plasma sheet [see Garrett et al., 1971]. Such

a drop in interactions implies a drop in fluctuation level in the ULF range which should be a measure of the ability of the magnetic field to scatter or interact with particles. One would expect, according to this proposal, to see the magnetic fluctuation level go down during substorms.

Although ULF magnetic fluctuations in the magnetosheath and solar wind have received increasing theoretical and experimental interest in the last few years [see reviews by Russell, 1971b, and McPherron et al., 1971], it is only recently that attention has been given to such fluctuations in the plasma sheet [Russell et al., 1971a]. This thesis will extend these plasma-sheet studies out to $34 R_E$ (earth radii) using the Explorer 34 satellite. The findings of the thesis will then be used to test the utility of the Hill-Dessler model of the plasma sheet. In this way, it is hoped to bring about better understanding of the new magnetospheric frontier--the plasma sheet.

2. THE EXPLORER 34 SATELLITE

The Explorer 34 satellite (IMP F) was the fifth to be launched in the IMP series of spin-stabilized spacecraft that have been used in studying the magnetosphere, bow shock, magnetosheath, and the solar wind. An extensive review of research accomplished with the Explorer 34 and other satellites in the series is contained in NSSDC 71-21.

Explorer 34 [Fairfield, 1969] was launched on May 24, 1967, and functioned satisfactorily until the spacecraft reentered the Earth's atmosphere on May 3, 1969. The satellite was placed in a highly inclined (67.4°), highly eccentric earth orbit. The orbital period was 4.3 days and had an initial perigee altitude of only 278 Km above the earth's surface, while apogee occurred at a geocentric distance of $34.1 R_E$ (R_E : earth radii). The apogee point lay very near the ecliptic plane which was extremely convenient for plasma-sheet studies. The orbit, as the earth revolved around the sun, passed through the earth's geomagnetic tail (see Figure 1). About 20 such passes through the plasma sheet were recorded, though only the 6 best passes (orbits 71-76) were used in this study.

An important feature of the satellite is that it is spin stabilized with the spin axis approximately perpendicular to the ecliptic plane. This spin period was about 2.59 to 2.63 seconds--the variations were caused by the slight heating or cooling of the satellite. Unfortunately,

as will be discussed later, the spin period is very close to that of the sampling frequency of the satellite magnetometers. However, the spin of the satellite makes it possible to determine magnetometer zero levels much more accurately than for an oriented satellite as the spin of the satellite can be used to remove any permanent magnetic field the satellite may have.

The Explorer 34 satellite was designed to study interplanetary magnetic fields, energetic particles, and plasma. Of particular interest to this study is the magnetic field experiment [N.F. Ness, Principal Investigator]. The magnetic-field experiment consisted of a triaxial fluxgate magnetometer--three sensors on two 6-foot booms. The sensor parallel to the spin axis is by itself on one of the booms so that it can be flipped occasionally to check its zero level. The zero levels of the magnetometer were determined with an accuracy of $\pm 0.3\gamma$ for the sensor parallel to the spin axis and $\pm 0.1\gamma$ for the sensors perpendicular to the spin axis [Fairfield, 1969]. The two sensitivity ranges were $\pm 32\gamma$ and $\pm 128\gamma$ with corresponding digitization errors of $\pm 0.16\gamma$ and $\pm 0.64\gamma$ (only the $\pm 32\gamma$ range is used in this study).

The high frequency cut-off of the magnetometers was set at 5 Hz. The sampling frequency was 0.39 Hz which implies that the highest observable frequency (called the Nyquist frequency) was half this frequency or 0.2 Hz. The disparity between the two frequency ranges developed because it had been originally planned to use an on-board computer system to calculate the autocorrelation function of the 0 to 3.1 Hz frequency band. Unfortunately, this computer system failed.

3. DATA ANALYSIS TECHNIQUES

In the analysis of periodicities in data, one of the most powerful mathematical tools used is the autocorrelation function. The function describes the dependence of the values of the data at one time on the values at another time. One equation that represents the stationary autocorrelation function is as follows [Bendat and Piersol, 1968; eq. 1.26]:

$$R_x(\delta t) = \lim_{T \rightarrow \infty} \frac{\int_0^T x(t)x(t + \delta t)dt}{T} \quad (1)$$

where $x(t)$ is the data as a function of time, t . The power spectral density (PSD) function is simply the Fourier transform of the correlation function, and, as such, describes the general frequency composition of the data in terms of the spectral density of its mean square value. In terms of the correlation function defined earlier [Bendat and Piersol, 1968; eq. 1.35]:

$$G_x(f) = 2 \int_{-\infty}^{\infty} R_x(\delta t) e^{-i2\pi f \delta t} d(\delta t) \quad (2)$$

where f is the frequency ($i = \sqrt{-1}$). In this paper we are concerned with studies of possible wave-particle interactions and the ULF spectrum, so we will concentrate on the PSD function.

So far, our discussion has explained the PSD function in a strict mathematical sense. A more physical interpretation of the PSD function can be had in terms of the Fourier transform or the Fourier series. For example, if we take the Fourier series representation of a periodic signal, it is well known that we obtain the signal in terms of its frequency components:

$$x(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos 2\pi n f_1 t + b_n \sin 2\pi n f_1 t) \quad (3)$$

where f_1 is the reciprocal of the duration of the function. Also, as is well known from simple algebra, we can write this function as:

$$x(t) = X_0 + \sum_{n=1}^{\infty} X_n \cos(2\pi n f_1 t - \theta_n) \quad (4)$$

where X_0 equals $\frac{a_0}{2}$, X_n is given by $(a_n^2 + b_n^2)^{\frac{1}{2}}$ and θ_n is $\tan^{-1}(b_n/a_n)$. For a given frequency (say, $n f_1 = f$), X_n^2 is a measure of the 'power' in a given frequency range between $f - \frac{1}{2}\delta f$ and $f + \frac{1}{2}\delta f$. A physical example is given by electric power--we can obtain the power of a signal at a given frequency by squaring the voltage and dividing by twice the resistance. If X_n^2 is divided by the frequency interval between Fourier components, the PSD is obtained. One needs to divide by the frequency interval so that the value given is standard for differing frequency intervals. In the specific case studied here, the value obtained is in

units of γ^2/Hz and (in arbitrary units) is proportional to the magnetic energy density (B^2/μ_0) in a given frequency interval. In Dragt [1961] or Kennel and Petchek [1966] [see also Cornwall, 1964] , it is shown that the ability of a magnetic perturbation to scatter a particle by changing its pitch angle is given by $\omega_c \delta B/B$ (where ω_c is the cyclotron frequency). $\omega_c \delta B/B$ is proportional to δB , so γ^2/Hz , which is essentially δB^2 , is a measure of the scattering ability of the magnetic field.

From studies of the data and by experimentation with computer techniques, it was determined that 21-minute segments provided reasonable information for PSD studies in the plasma sheet (as most phenomena were moderately stationary over this time interval) and an adequate set of data points (500) for statistical analysis. Taking longer intervals made uncertain whether the actual region under study was entirely plasma sheet or not, whereas taking a shorter interval led to too great a statistical uncertainty.

The computer procedures used to evaluate the PSD function involved approximating the complex Fourier transform of the input signal, calculating the absolute magnitude squared of the resulting Fourier components, and dividing these values by the bandwidth used:

Fourier Transform:

$$X_n = \frac{1}{N} \sum_{j=0}^{N-1} x'_j e^{i2\pi jn/N} \quad n = 0, 1, \dots, N-1 \quad (5)$$

where $i = \sqrt{-1}$; x'_j is the magnetic field measurement,
 $x'_j = x_j - \bar{x}_{avg.}$, and $N = 500$.

$$\text{PSD: } G_m(f, B) = \frac{f_m + B/4}{f_m - B/4} \left| \sum_{n=-B/4}^{+B/4} X_n \right|^2 / B \quad (6)$$

where B is the bandwidth. The frequency interval is summed over $B/2$ instead of B for it was found useful to "hann" the PSD function, G_m , a process which increases the band from $B/2$ to the desired bandwidth B . The best bandwidth was found to be 0.008 Hz.

Hanning is a standard technique used to smooth the PSD function, the process of hanning being very similar to the use of a running average. The function used for hanning the PSD function is:

Hanning:

$$G'_m = (1/4)G_{m-1} + (1/2)G_m + (1/4)G_{m+1} \quad (7)$$

$$m = 1, 2, \dots$$

To reiterate, the PSD function is the standard technique for displaying the frequency dependence of a signal. As the prime motivation behind this paper is to study such frequency variations so as to observe wave-particle interactions, the PSD function was used. G'_m , the hanned PSD value at a given frequency ($f = 0.004m$), was plotted versus f on a log-log graph to display the frequency dependence of the signal. One could have equally well have plotted the Fourier

components of the data segment studied. Essentially identical results will be obtained but the Fourier components are dependent on the frequency intervals involved and, further, phase information is included that is of little value. The PSD function avoids both the difficulties and presents the results in a clearer manner.

4. INTRODUCTION TO THE EXPERIMENT

The next part of the paper will be concerned with the actual data and the results. Error analysis will be deferred to the end of the study. Here it will suffice to mention the major errors. By far the worst is the result of statistical uncertainty. This error is about $\pm 10\%$ to $\pm 20\%$ of the PSD values observed. False spectral peaks occur between 0.005 and 0.011 Hz as the result of spin modulation brought about by the near equality of the spin and sampling frequencies.

The mathematical analysis processes themselves introduce problems in studying the data. The worst of these is called "trending" and is the result of taking only a finite data segment. If there is a slow constant change in the field average during the data segment, it will appear to the computer that it is analyzing a sawtooth wave. A sawtooth wave has a frequency spectrum similar to that actually observed. Further, it will be shown that trending also introduces interference terms. Sudden discontinuities also introduce artificial spectra that resemble those actually observed. Keeping these problems in mind, let us now turn to the results.

5. OBSERVATIONS IN THE PLASMA SHEET

The plasma sheet will be defined to be that region in the earth's geomagnetic tail characterized by a magnetic depression (due to the diamagnetic effect of the plasma particles in the plasma sheet) which lies above and below the neutral sheet. It was found that this region was usually characterized by a $|B_{xsm}|$ (magnetic field in the X_{sm} direction) of from about 3γ to 10γ . Occasionally, plasma-sheet field magnitudes were larger than this while the high-latitude tail sometimes had lower values. In any case, when an obvious depression was observed in the field magnitude followed, perhaps, by a reversal in $|B_{xsm}|$, the region inbetween was termed plasma sheet.

When the data segments are converted to PSD functions and plotted on a log-log plot versus frequency, certain similarities between PSD functions from different intervals become apparent. First, the PSD values invariably seem to fall on a straight line. Since this is on a log-log plot, this means that the PSD function is proportional to the frequency (f) to the inverse power X . Figure 2 is an excellent example of such a function. Secondly, X was found to have values from 1.29 to 3.00 (mean of 2.1) when a least squares fitting was made. In general, the linear correlation coefficient (called r) indicates extremely good fitting to f^{-X} (typical values of r show that if the data were random, there is less than a 0.1% probability of the

observed correlation occurring). It should be emphasized here that we have avoided all discontinuities such as occur near the neutral sheet and at the plasma sheet/high-latitude tail boundary. These will be discussed in a later section.

The results discussed thus far are consistent with those reported by Russell et al. [1971a] at closer distances ($12-24 R_E$) to the earth. His work with the OGO 5 satellite is similar to the work presented here in some respects. We will be comparing and contrasting his results with the present analysis later.

The PSD functions studied were essentially featureless. The only deviations from a straight line on the log-log plots were the result of spin and sampling frequency modulation and of the white noise level at about $0.1\gamma^2/\text{Hz}$. As can be seen in Figure 2, there is a spectral peak at 0.16 Hz which is where cyclotron resonance phenomena are expected to occur for a cold plasma. About one-third of all the spectra studied in the plasma sheet exhibit such peaks in the range from 0.1 to 0.18 Hz. Unfortunately, almost all of these PSD values are very close to the uncertainty levels inherent in making a PSD calculation. Further, when the peak is observed in one component, it is not usually found in the other components for the same segment. These two observations indicate that the spectral peaks observed were only statistical fluctuations. One is strongly tempted to attribute such peaks to the sought-after cyclotron resonances,

but the only reasonable conclusion is that if such peaks are real, the upper limit for them is $0.5\gamma^2/\text{Hz}$ and the frequency band is about 0.05 Hz in width. With this stipulation, it is concluded that the PSD function is essentially devoid of features.

A new PSD value was calculated for the frequency interval from 0.012 to 0.1 Hz range for the purpose of increasing the statistical confidence in the PSD function. When these new values are compared for each of the components in each segment, the X_{sm} component is found to have a consistently higher fluctuation value than the other two components (see Figure 3a, b, and c). This observation is also reflected in the average fluctuation levels: $X_{sm}: 1.70\gamma^2/\text{Hz}$; $Y_{sm}: 1.18\gamma^2/\text{Hz}$ and $Z_{sm}: 0.74\gamma^2/\text{Hz}$. The importance of this observation will become apparent later.

The magnitude of a magnetic field component at a given frequency would be squared to give the PSD value. Therefore, it seems natural to define a quantity, which will be called the total fluctuation level (γ_T^2/Hz), by summing the individual component fluctuation levels. In equation form:

$$\gamma_T^2/\text{Hz} = \gamma^2(X_{sm})/\text{Hz} + \gamma^2(Y_{sm})/\text{Hz} + \gamma^2(Z_{sm})/\text{Hz} \quad (8)$$

When the total fluctuation level is plotted versus $|B|$, X_{sm} , Y_{sm} , and Z_{sm} , little, if any correlation is seen. The total fluctuation level does show a tendency to increase

with increasing Kp index and $|B_{zsm}|$. The Chi-square test was applied but was inconclusive for both. A least-squares fit was tried using first one and then the other variable as the independent variable for both cases. This produced two lines for each graph (see Figures 4 and 5). The r-values for these lines were indicative of reasonable fitting--for both graphs, only 5% of a random sample would yield as good a fit as observed. It is probably better to refer to the increase in γ_T^2/Hz with Kp and $|B_{zsm}|$ as "tendencies". An important point is that low values of γ_T^2/Hz do occur for high values of $|B_{zsm}|$ and Kp. This point will be used shortly.

Kp index may not be the best indicator of fluctuation levels in the ULF range as it is a 3 hour index of geomagnetic activity, whereas the generating process for the ULF fluctuations may be much shorter. A more meaningful measure is given by the AE index, an index of geomagnetic activity over a period of only $2\frac{1}{2}$ minutes. The AE index is calculated by superimposing magnetograms from the auroral zone stations at College, Sitka, Meanook, Great Whale River, and Leirvogur. The index is the deviation, in gammas, of the resulting envelope.

In Figure 6, γ_T^2/Hz is plotted versus the average of the AE index for a 35 minute period covering the plasma-sheet time segment used to obtain the PSD function. The results are as before--no meaningful Chi-square results but less than a 5% chance of a random distribution causing the

same distribution. There is again the tendency for increasing AE index to correlate with increasing γ_T^2/Hz , but still only a tendency as low values of γ_T^2/Hz occur for high values of AE index. One possible explanation of these results is that the process generating the ULF fluctuations is shorter than 35 minutes. Shortly it will be shown that there is a better answer than this, though.

One further attempt, that was not completely successful, to find the generating source deserves mentioning before the process that is actually believed to be responsible is discussed. This attempt involved calculating $(\gamma_T^2/\text{Hz})/B_T^2$ (where B_T is the total field magnitude). This function should remove any residual variation with B_T (the average value of B for each component was subtracted out before the PSD was calculated and, as mentioned earlier, there did not appear to be any variation of γ_T^2/Hz with B , but there could still be a hidden variation with B_T that was overlooked). When K_p index, $|B_{zsm}|$, and AE index were plotted versus γ_T^2/Hz (see Figure 7), the data fell into apparently two distinct groups--one characterized by high γ_T^2/Hz values and high K_p , $|B_{zsm}|$, and AE values, and the other by low γ_T^2/Hz values and arbitrary values of K_p , $|B_{zsm}|$, and AE. The implication from these results is that the process generating the ULF fluctuations must not be taking place when just $|B_{zsm}|$ or K_p or AE are large, but when some other condition is also met. This added factor may be related to the tail structure and one of the other quantities as would be implied

by the way $(\gamma_T^2/\text{Hz})/B_T^2$ is observed to vary. One such source is the substorm.

The results to this point indicated that the ULF fluctuation generating process was more complicated than just a simple K_p , AE, or $|B_{zsm}|$ increase. In Figure 8, the segments for which data were obtained are superimposed on the corresponding plots of AE index. Also shown are the corresponding values of γ_T^2/Hz and $(\gamma_T^2/\text{Hz})/B_T^2$. In Figure 9, the signature of a substorm is shown in terms of AE index. The three intervals it has been divided into will be referred to as the "first phase" (marked A on the diagram), the "second phase" (marked B), and the "final phase" or "quiet time" (marked C). The first phase corresponds roughly to the so-called expansion phase of a substorm while the second phase is similar to the recovery phase. When the AE index plots are divided into these intervals and the average for the segments under each type of interval is calculated, the following results: A: $3.6\gamma_T^2/\text{Hz}$; B: $6.52\gamma_T^2/\text{Hz}$; and C: $1.81\gamma_T^2/\text{Hz}$. Likewise, the values for $(\gamma_T^2/\text{Hz})/B_T^2$ are: A: 0.06 Hz^{-1} ; B: 0.11 Hz^{-1} , and C: 0.024 Hz^{-1} . The conclusion that comes from these results is that the ULF fluctuation levels increase from the quiet time values through the first phase of the substorm and are maximum in the second phase, quiet times yielding the minimum results.

This concludes the discussion of observations in the plasma sheet. The following summarizes the major results

found in the plasma sheet:

1.) The plasma sheet is characterized by a PSD function proportional to f^{-2} and exhibits few, if any, spectral features.

2.) The values of γ_T^2/Hz and $(\gamma_T^2/\text{Hz})/B_T^2$ increase as the substorm passes from the A interval (first phase) to the B interval (second phase), being minimal during the C interval (quiet times).

3.) The X_{sm} component, corresponding to the major field direction, is increased in fluctuation level over the other two components.

Before closing this section, the above conclusions will be compared to observations made by Russell, Coleman, and McPherron with the OGO 5 at -12 to $-24 R_E$ (X_{sm}). They observed featureless PSD functions (0.03 to 3.4 Hz) proportional to the frequency to the -2 to -2.5 power in and near the plasma sheet. They did not find significant differences between the 3 components in contrast to the above results.

Russell [1972] may have observed the same results with OGO 5 as described in conclusion "2" above. He reported observing the magnetic fluctuations to be absent during quiet times and during plasma-sheet thinning (the total field increases) but increased during plasma-sheet expansion (the total field decreases). If plasma-sheet thinning corresponds to the first phase of the substorm and plasma-sheet expansion to the second phase, then the results are in agreement. As

will be described in the "Conclusions" section, Fairfield and Ness [1970] observed the total field to increase during the first phase of substorms and decrease during the final phase. This would imply that Russell's thinning phase is the same as the 'first phase' used in this study and his expansion phase is the same as the 'second phase'. Therefore, his observations are similar to those reported above.

6. OBSERVATIONS IN THE BOUNDARY REGIONS OF THE PLASMA SHEET

We will now treat the two boundaries of the plasma sheet. The first of these, called the neutral-sheet/plasma-sheet boundary, is characterized by sudden "crossings" (defined as a sign change in B_{xsm}) and an average $|B_{xsm}|$ less than 3γ . The other boundary is that with $|B_{xsm}|$ greater than 10γ (on some occasions, however, the average value of $|B_{xsm}|$ was a little lower than 10γ , in which case the distance from the predicted position of neutral sheet was used).

Neutral-sheet crossings were observed by Russell to have low fluctuation levels compared to the inner plasma-sheet/high-latitude tail boundary. As the field is fairly dipolar in the 10 to 24 R_E region studied by the OGO 5, such crossings do not exhibit the sudden, very large field reversals that occur farther down the tail in and near the neutral sheet. The result of these large, rapid field reversals, as will be discussed momentarily, is that the Explorer 34 satellite observes high fluctuation levels in every component in the neutral-sheet/plasma-sheet boundary. This is especially true of the $|B_{xsm}|$ component which is just what would be expected as the field reversals show up in this component.

Figure 10 is an excellent example of the rapid, almost discontinuous crossings that lead to the increased fluctuation levels observed in X_{sm} and the PSD function that re-

sults from such an interval. These rapid crossings could be due to sudden movements of the entire tail, rapid reconnection across the neutral sheet, or even by crossing the tearing mode "bubbles" postulated by Coppi et al. [1966], Biskamp et al. [1970], and Schindler and Ness [1972]. In any event, such sudden crossings do not appear to be simply the result of satellite movement but a real feature of the neutral-sheet/plasma-sheet boundary.

The levels observed in the neutral-sheet/plasma-sheet boundary are found to be: $X_{sm}: 3.5\gamma^2/\text{Hz}$; $Y_{sm}: 1.5\gamma^2/\text{Hz}$; and $Z_{sm}: 1.6\gamma^2/\text{Hz}$. The total fluctuation level is, therefore, $6.6\gamma_T^2/\text{Hz}$. It is seen that the X_{sm} component is very large compared to the other components as has already been mentioned. The average value of the slope X appears to be greater than in the plasma sheet; $X \approx 2.5$.

The plasma-sheet/high-latitude tail boundary actually consists of two regions--one slightly inside the plasma-sheet border and the other slightly outside of it. The inner region is chosen to be the transition zone between the high-latitude tail and the plasma sheet. An example of the outer region is shown in Figure 11 (note the spin and sampling frequency modulation that appears in the data) and shows how quiet this region is compared to the plasma sheet and the inner region.

Figure 12 contains a feature that typifies the inner part of the plasma-sheet/high-latitude tail boundary. In this figure, one observes a sudden decrease in $|B_{xsm}|$ and

accompanying oscillations in $|B_{ysm}|$ and $|B_{zsm}|$. A dot diagram in Figure 13 shows the rotational, wave-like nature of this feature while PSD plots for segments immediately before and during the feature are shown in Figure 12d. Clearly, the feature contains an unusually high fluctuation level (note how the PSD plot turns up near 0.2 Hz). As will be discussed in the section on error analysis, it is difficult, with the Explorer 34 satellite, at least, to decide whether such discontinuous features produce PSD plots that are the result of actual wave phenomena or are artifacts produced by the Fourier analysis and the smoothing process.

If we ignore the inner boundary of the plasma-sheet/high-latitude tail region, the average fluctuation levels observed are: $X_{sm}: 0.6\gamma^2/\text{Hz}$; $Y_{sm}: 0.32\gamma^2/\text{Hz}$; $Z_{sm}: 0.27\gamma^2/\text{Hz}$ (the total fluctuation level being $1.19\gamma_T^2/\text{Hz}$). It is apparent that the tail region exhibits much lower fluctuation levels than the plasma sheet or the neutral-sheet/plasma-sheet. The value of X in this region is about 1.9.

The preceding discussion has shown how the plasma sheet differs in its spectral characteristics from other regions in the geomagnetic tail. It is unusual, though, that the X_{sm} component has about two to three times the fluctuation level of the other two components in all the regions studied. This might be attributable to the higher average field in the X_{sm} component in these regions but the average value of the field is always subtracted out when calculating PSD functions (see Eq. 5). A fourth result can now be added to

those of the previous section:

4.) The level of magnetic fluctuations, γ_T^2/Hz , increases in the plasma sheet and at the inner part of the plasma-sheet/high-latitude tail boundary over the average level recorded in the high-latitude tail (the neutral-sheet/plasma-sheet boundary has the highest fluctuation level followed by the inner boundary between the plasma sheet and the high-latitude tail).

Russell, McPherron, and Coleman found that the maximum fluctuation levels occurred near the plasma-sheet/high-latitude tail boundary (or what was taken to be that boundary in this study). They did not find significant PSD levels at neutral-sheet crossings. Apparently this difference can be attributed to the greater distance down the geomagnetic tail of the Explorer 34 satellite ($34 R_E$) than the OGO 5 (less than $24 R_E$). The neutral sheet does not appear to be as pronounced closer to the earth as it is at greater distances. Since a sharp discontinuity should produce a higher PSD level than a weak one, it follows that the observations farther down the tail should exhibit higher levels than the closer OGO 5 observations.

7. ERROR ANALYSIS

There are two sets of errors present when analyzing Explorer 34 data. The first set consists of errors introduced by the instrument itself--digitization errors, aliasing errors, and errors due to the spin and sampling period. The second set is that set of errors introduced by analysis techniques--finite sample length, trends, and discontinuities.

Whenever a signal is digitized, one must choose how to determine the "average" value during the sampling interval. The fact that this can not usually be done introduces an uncertainty in the measurement called the digitization error. For the high sensitivity range used ($\pm 30\gamma$), the error is calculated as follows:

$$\begin{aligned} \text{Digitization Error} &\approx \pm \frac{1}{2} \text{ (field magnitude interval)} = \\ &\pm \frac{1}{2} (70\gamma/223 \text{ steps}) = \pm 0.16\gamma \end{aligned} \quad (9)$$

A more accurate estimate of this error is given in Bendat and Piersol [1968], eq. 7.3, as ± 0.29 scale units instead of the cruder estimate of ± 0.5 scale units used in Eq. 9. This gives an error of $\pm 0.096\gamma$. We will use the higher estimate throughout as this is the error quoted by Fairfield and Ness [1970a]. The error is random and will show up at all frequencies and constitute a "white noise" background.

The effect that this "white noise" will have on the PSD function can be estimated in the following manner:

1.) Since the error is considered to be random and usually assumed to be of a Poissonian nature, we can estimate the average error in a specific bandwidth by remembering that the error due to a collection of measurements goes down as the square root of the number of measurements. Since we use 500 components to form a given Fourier component, the error due to digitization is roughly:

$$\text{Error in one Bandwidth} = \frac{\pm 0.16}{\sqrt{500}} \quad (10)$$

2.) If we consider this value to be the amplitude of the white noise at a given frequency and in a given frequency interval, i.e., the Fourier component amplitude, then we can square the value and divide by the frequency interval to get an estimate of the white noise level on the PSD plot. Our basic (or smallest) frequency window is given by dividing the highest frequency observable (0.4 Hz) by the number of segments it can be broken up into (500). Carrying out these calculations, we arrive at an estimate of the white noise level:

$$\text{Error in PSD} \approx \frac{\left(\frac{0.16}{\sqrt{500}}\right)^2}{(0.4/500)} = (0.07)^2/\text{Hz} \quad (11)$$

The actual white noise level varies from about $0.06\gamma^2/\text{Hz}$ to about $0.13\gamma^2/\text{Hz}$, the mean value being around $0.1\gamma^2/\text{Hz}$. This is in good agreement with our estimates of the error. The white noise level corresponds to the lowest level detectable by the magnetometers.

The spin period of the satellite is very close (2.59 to 2.63 seconds) to the digital sampling period (2.556 seconds). This introduces peaks at 0.011 Hz and at 0.005 Hz [see Fairfield, 1970a].

A crude example of how such an error could be introduced will now be given. Assume that the input signal is a vector magnetic field:

$$\vec{B} = \begin{bmatrix} X_O \\ Y_O \end{bmatrix} \quad (\text{Consider only a 2-dimensional problem.}) \quad (12)$$

Let the satellite be spinning at frequency f_s . The sensors see the following vector components:

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} \cos(2\pi f_s t) & \sin(2\pi f_s t) \\ -\sin(2\pi f_s t) & \cos(2\pi f_s t) \end{bmatrix} \begin{bmatrix} X_O \\ Y_O \end{bmatrix} \quad (13)$$

Each sensor outputs a signal dependent on its gain. Assume these gains are G_x and G_y , respectively. Then the new sensor outputs are:

$$\begin{bmatrix} X' \\ Y' \end{bmatrix} = \begin{bmatrix} G_x & 0 \\ 0 & G_y \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} \quad (14)$$

Let us assume that: $G_x \approx 1$

$$G_y \approx 1 + \delta G; \quad \delta G \ll 1$$

The signal is later converted back to the original reference frame:

$$\begin{vmatrix} X'_O \\ Y'_O \end{vmatrix} = \begin{vmatrix} \cos(2\pi f_s t) & -\sin(2\pi f_s t) \\ \sin(2\pi f_s t) & \cos(2\pi f_s t) \end{vmatrix} \begin{vmatrix} X'_O \\ Y'_O \end{vmatrix} \quad (15)$$

Combining equations 13, 14, and 15 yields the magnetic field values seen on the earth for the constant field described in Eq. 12. The X component, for example, will reduce to:

$$X'_O = X_O \frac{+ \delta G X_O}{2} - \frac{\delta G}{2} (X_O \cos(4\pi f_s t) + Y_O \sin(4\pi f_s t)) \quad (16)$$

Thus, a small error in the gain factor introduces a small constant error and an error varying at twice the spin frequency. The sampling frequency, as has already been pointed out, is approximately equal to the spin frequency. As will be explained shortly, aliasing will cause the error varying at twice the spin frequency to show up at several lower frequencies:

$$f_O = 2f_C - 2f_s \approx 0.01 \text{ Hz (for } f_s = 0.386 \text{ Hz} \quad (17)$$

$$f_C = 0.391 \text{ Hz)}$$

Figure 14 shows such oscillations taking place. For want of a more descriptive term, we will call such oscillations "beating". Such beating does not appear to produce a signal greater than about $\pm 0.3\gamma$. Further, beating appears to be a sporadic occurrence--as if the gain, for example occasionally varied by a small amount.

The final instrument problem to be discussed is that introduced by the frequency cut-off of the magnetometer being 5 Hz while the Nyquist frequency (the Nyquist frequency is half the sampling frequency and is the highest frequency that the instrument can observe) is 0.2 Hz. Frequency components higher than the Nyquist frequency will appear at frequencies lower than the Nyquist frequency due to a process called aliasing. As there are frequency components between 0.2 Hz and 5 Hz that could be aliased, aliasing could introduce false spectral components. Figure 15a shows two sine signals, one with a frequency greater and one with a frequency lower than the Nyquist frequency. It is easily seen from this figure how, with aliasing, a frequency higher than the Nyquist frequency can be observed at a lower frequency.

Figure 15b shows a linear plot of the frequencies in a signal and shows which higher frequencies are folded back into a given frequency. The formula for determining such values can be derived from this figure [Behannon and Ness, 1966]:

$$f_1 = 2kf_c \pm f_o \quad (18)$$

where f_c is half the sampling frequency (Nyquist frequency), f_o is the lower frequency, f_1 is the higher frequency, and k is an integer.

It turns out that the PSD function is proportional to f^{-2} . Thus, a signal at 0.5 Hz appearing at 0.1 Hz is less in magnitude than the 0.1 Hz value by a factor of 0.04 ($0.1^2/0.5^2 = 0.04$). So aliasing, as far as most of our data is concerned, plays only a minor role.

Instrument errors that show up as noise are generally easy to pick out on a PSD plot. More subtle errors, which are almost impossible to separate from the PSD plots, are those introduced by the mathematical processes themselves. A good example is the error introduced by the finite length of the sample record. The finite length of the sample record defines a "standard deviation" for a given estimate of a PSD value. It is typical to call this value the "normalized standard error" and it is numerically equal to $\frac{1}{\sqrt{B_e T}}$ (B_e = resolution bandwidth, 0.008 Hz; T = duration of sample). For a record 21 minutes long and having a resolution of 0.008 Hz, this error is $\pm 30\%$ of the PSD value at a given point. The error is not as important as it appears as the data typically varies over 2 to 3 orders of magnitude from 0.01 to 0.2 Hz. However, to lower this value, the bandwidth was lengthened so that one PSD value was computed over the interval from 0.012 to 0.1 Hz (the center frequency becomes 0.056 Hz and the bandwidth 0.088 Hz). This will always be

the value plotted whenever γ^2/Hz is used (it was found that there was little disagreement between plots based on this value and plots based on the higher resolution at 0.008 Hz). The greater bandwidth (and consequent lower resolution) gives a normalized standard error of $\pm 10\%$ of a given PSD value (this means that for a PSD value of 100, there is approximately a 70% probability of finding the exact value between 90 and 110), which is quite reasonable.

Hanning, described earlier, introduces difficulties (not precisely errors) in the actual study of PSD plots. The hanning function is designed to act as a low pass filter and, as such, tends to smooth out spectral peaks in the PSD plots. This feature, while making the determination of specific spectral components difficult, has its advantages in that the basic shapes of the spectra are more easily calculated.

By far the most questionable aspect introduced by hanning, or for that matter, any digital study of an analog signal, comes from analyzing sudden changes (sudden in the sense of times comparable to the sampling frequency) in the signal. First, one must decide whether the change is actually a wave phenomenon or a discontinuity. Suppose a satellite lies slightly above or in the neutral sheet such that one sees a magnetic field component directed toward the earth. A small movement of the satellite or the tail may move the satellite below the neutral sheet causing a very sudden 180° flip in the X_{sm} component. Figure 10 presents an excellent

example of this. A similar movement elsewhere would not produce nearly the magnitude change in the X_{sm} component as observed in the neutral sheet. Hanning smooths out the shape of the PSD function in such a way that such artificial discontinuities are impossible to distinguish from actual wave-like phenomena. Thus it was necessary to exercise extreme caution in analyzing such features.

In Figure 16, a random collection of various discontinuities of various forms and amplitudes are shown along with the resulting PSD function for the interval. As can be seen, the resulting PSD plot is proportional to f^{-2} (as always, f is the frequency) after it is hanned (note, however, how clearly the peaks due to the triangular wave show up). That such a random signal consisting of the very low frequency pulses shown could yield an inverse square spectrum can be explained as follows. If a low frequency sine wave is analyzed, for example, it will produce a Fourier transform (as we are using a finite interval) proportional to $\frac{\sin(2\pi fT)}{f}$. Squaring such a function will give a sine-squared envelope decreasing as f^{-2} . Smoothing the Fourier transform with a technique such as hanning would smooth out the sine-squared envelope and leave the variation with f^{-2} . Similarly, the PSD spectrum characteristic of random, low frequency step functions yields a f^{-2} PSD when smoothed. In no way is there an intent to imply here that our data are the result of some type of gross discontinuity (in fact, in the plasma sheet, care was taken to avoid such obvious discontinuities). The important point is that the spectra observed are of a

form consistent with assuming that the plasma sheet magnetic fluctuations are of a random nature with, perhaps, a low frequency peak outside the range of our data. That this is precisely the case can not be proven with the data presented here, however, other observations of lower frequencies (see review by Russell [1971b], for example) show this to be a possible explanation.

Apparently, the analysis of magnetic field data is fraught with difficulties and pitfalls. Possibly the most insidious of these is that due to "trends". "Trends" and the errors they introduce are very similar to the aforementioned discontinuities. In fact, one can think of discontinuities as a zeroth order "trend". "Trend", however, as discussed here, will mean the tendency for the mean value of the data to slowly change during the observation segment--referred to mathematically as "non-stationarity". In the data studied, a slow, linear drop in the background field would appear mathematically as a sawtooth function of 21 minute period for we are calculating the PSD function from finite, 21 minute segments.

To make the argument clearer, let us assume the magnetic field for a given component is given by:

$$B_j(t) = bt + B_{oj} \quad (19)$$

where b is the slope, t is the time, and B_{oj} is a constant

field. The Fourier series representation that would be calculated is given approximately by:

$$X_n = \frac{2}{T} \int_0^T (bt + B_{oj}) e^{-i2\pi n f_o t} dt = \frac{i2b}{2\pi n f_o} \quad (20)$$

$$X_o = B_{oj} + \frac{bt}{2}$$

If the PSD value is computed for these components, the spectrum is found to be proportional to f^{-2} . Usually this new PSD value is only a small fraction of the total PSD, however, a more serious problem is also introduced. When the total PSD function is calculated, the new term introduces interference:

$$\text{PSD}_{\text{actual}} = \frac{(A + iB)(A - iB)}{\delta f} = \frac{A^2 + B^2}{\delta f}$$

$$\text{PSD}_{\text{trend}} = \frac{(A + i(B + C))(A - i(B + C))}{\delta f} \quad (21)$$

$$= \frac{A^2 + B^2}{\delta f} + \frac{C^2}{\delta f} + \frac{2BC}{\delta f}$$

where A is the Fourier component of the cosine series representation of the original signal, B is the sine component (or imaginary part of the complex Fourier amplitude), C is the trend Fourier component, and δf is the appropriate frequency interval. The sign of C is known but that of B is not. Using this fact, an estimate of C (a least-squares

fit was made to each data segment for this purpose), and letting $A^2 \approx B^2$, it was possible to estimate a maximum value for the interference term and the trend term. The equation that was used was:

$$\text{PSD}_{\text{actual}} = P \pm \left(\frac{C^2}{\delta f} \right)^{\frac{1}{2}} \left(2P - \frac{C^2}{\delta f} \right)^{\frac{1}{2}} \quad (22)$$

where P is the observed PSD function. The results of these calculations show that seven intervals could have had total interference terms greater than 50%, eight intervals had interference terms in the range from 35% to 50% of the total PSD function, and the nine remaining intervals had interference terms less than 35%. It should be emphasized that these values are for the worst possible situations (i.e., assuming that the three interference components add together). Significantly, when only those intervals with trends less than 35% were retained (fortunately there were nine such intervals for which AE were known), the A:B:C values were: 3.66:7.61:1.49. Thus, our findings do not appear to be noticeably affected by trending.

The technique that is normally used to correct for "trending" is to subtract out a linear fit to the data. In more complicated situations, programs have been devised that not only do this, but continually recompute the major field direction and define the field vectors in terms of

this direction. Once the process of correcting the data starts, it is hard to justify not subtracting out higher order "trends". The "artifacts" that could result are many from such correction techniques. In view of the complications one gets into when trying to correct for "trends" and on the basis of the estimates of the possible interference terms, it was felt justified to ignore "trending" in calculating the PSD function.

This concludes the discussion of errors and difficulties that crop up in the study of Explorer 34 data. Since most of the errors were sufficiently well understood it was found unnecessary to take elaborate precautions in preparing the data for study. The only major precaution was to avoid the obvious discontinuities in the data that occurred in the plasma-sheet boundary regions. Beating was not removed for it was found to not contribute significantly to the total PSD function.

8. CONCLUSIONS

In this final section, an attempt will be made to explain the four observations summarized at the conclusions of sections 5 and 6, and at the same time, discuss some of their implications. The ideas will be only superficially discussed as it is not the purpose of this thesis to propose an elaborate theory.

According to Fairfield and Ness [1970b], during the early phases of a substorm (roughly, the first phase), the total field increases while B_{zsm} decreases. During the later stages (the second phase), there is a reversal of this trend as the total field decreases and B_{zsm} increases. Further, they find increased B_{zsm} during quiet times. These observations are in agreement with the findings reported here. It will be remembered that the fluctuation levels increased with increasing B_{zsm} but that there were still low values associated with high B_{zsm} (the low values now being seen to be associated with quiet times or the first phase). Further, when we divided by the total field, the data fell into two groups (see Figure 7). It should be clear that dividing by the total field emphasized the differences between the two substorm phases (the high field and lower fluctuation levels during the first phase making these values much lower than those of the second phase).

A further confirmation of the findings comes from a paper by McPherron and Coleman [1970]. In this brief paper,

it was reported that at the ATS 1 satellite, maximum fluctuation levels (0.002 to 0.6 Hz) were found to be along the direction parallel to the earth's spin axis and that these high levels occurred during the late phases of substorms. The parallel component to the spin axis was also the maximum field direction. This lends important support to our observation of the X_{sm} component (which is also the maximum field direction) having the maximum fluctuation levels (Hruška and Hrušková [1969] also observed maximal fluctuations in the X_{sm} direction using Imp 1 5.46-min data) in contrast to OGO 5 results. The increase in fluctuation level in the later stages of the substorm fits our results, but, at ATS 1, these fluctuations appear to last several hours (perhaps the shorter duration observed by Explorer 34 is the result of wave-particle interactions damping the fluctuations in the distant plasma sheet).

McPherron and Coleman also present a simple description of how the magnetic fluctuations might be caused. In this explanation, much like in Fairfield and Ness' paper, substorms are caused by the extended field lines in the tail suddenly collapsing and rushing earthward with plasma. Differences in pressure are likely to occur in a more or less random fashion. In attempting to eliminate these pressure fluctuations, it is proposed that the plasma generates magnetosonic waves. The important facet of such a theory is that it could explain the production of compressional or

magnetosonic waves. The observation that the X_{sm} component has the highest fluctuation level, leads to the proposal of waves transverse to the major field direction--magnetosonic waves being a likely explanation. The inward movement of a dipolar field would certainly produce such waves in the major field component as observed by ATS 1. Further, such a generation mechanism would, as mentioned, produce a random set of fluctuations. McPherron and Coleman go on to point out that, as bigger disturbances take longer, they would expect to see more power in the magnetosonic waves at lower frequencies. The observation that more or less random fluctuations could produce an f^{-2} PSD spectrum fits this interpretation quite well.

The interpretation mentioned here is not a unique solution and various other theories involving, for instance, plasma instabilities, certainly could be evoked to explain the essential features. In fact, there are obvious difficulties involved in such a simple interpretation. Chief among these is the answer to why the fluctuation level is higher in the plasma sheet than that in the high-latitude tail. The high- β characteristic of the plasma sheet perhaps causes fluctuations in the plasma sheet by coupling the turbulent auroral oval and its particles with the distant plasma sheet during substorms [C.S. Rao, 1969], the high tail not being connected as directly because of its lower- β characteristics. At this point, such theories are still pure conjecture.

All the observations reported thus far indicate that the increase in magnetic fluctuations during substorms is real. This is in contradiction to the results predicted by the Hill and Dessler model outlined earlier. When the PSD function is integrated over frequency and divided by 8π , an estimate of the average energy density per cubic centimeter is obtained for the fluctuation levels observed. If the PSD function is $l\gamma_T^2/\text{Hz}$ (about the lowest value observed) in the 0.012 to 0.1 Hz band, then the lowest energy densities observed were $3.5 \times 10^{-13} \text{ ergs/cm}^3$. Since the plasma sheet has a β of about 1, multiplying $(\gamma_T^2/\text{Hz})/B_T^2$ by the frequency interval (0.088 Hz) yields the ratio of the fluctuation energy density to the particle energy density. The values for the three phases are: A: 0.0053; B: 0.0098; and C: 0.0021. The ratio increases during substorms and again implies that wave-particle interactions probably increase in contrast to the expectations of the Hill and Dessler model.

The specific case for significant wave-particle interactions is difficult to decide. Very crude estimates of the levels of fluctuation needed indicate values of $l\gamma_T^2/\text{Hz}$. As our lowest levels are about $l\gamma_T^2/\text{Hz}$, it is possible that wave-particle interactions could take place. Similarly, the energy density ratios are not inconsistent with possible wave-particle interactions. Spectral peaks, as discussed earlier, occur, but are so close to statistical uncertainty levels that they are of doubtful value. However, an upper

limit was placed on these features (about 0.05 Hz in bandwidth and an amplitude of $0.5\gamma_T^2/\text{Hz}$). This yields an energy density of $10^{-13}\text{ ergs/cm}^3$ that is to be compared to the lowest energy densities resulting from γ_T^2/Hz in the 0.012 to 0.1 Hz band. It would seem from this that wave-particle interactions are more likely in the 0.012 to 0.1 Hz band than at a 0.16 Hz spectral peak.

This completes our discussion of ULF fluctuations in the plasma sheet. More detailed analysis, of course, still remains to be done. Specifically, particle data could be used to better determine the location of the plasma sheet and to help in the search for wave-particle interactions. This paper, however, does serve to point the way for future studies and provides a pleasing completeness to similar studies by Russell, McPherron, Coleman, and others at closer distances to the earth.

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FIGURE CAPTIONS

- Figure 1a. View of Explorer 34 orbits from above the ecliptic plane.
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- Figure 2. An excellent example of a plasma-sheet PSD plot. The arrow indicates a possible spectral peak (see text).
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- Figure 4. Graph of the magnetic fluctuation level versus $|B_{zsm}|$.
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FIGURE 1a

FIGURE 1b

FIGURE 2

FIGURE 3a

FIGURE 3b

FIGURE 3c

FIGURE 4

FIGURE 5

FIGURE 6

FIGURE 7a

FIGURE 7b

FIGURE 7c

FIGURE 8

FIGURE 9

FIGURE 10b

FIGURE 10a

FIGURE 11b

FIGURE 11a

FIGURE 12a

FIGURE 12b

FIGURE 12c

FIGURE 12d

FIGURE 14a

FIGURE 14b

FIGURE 15b

FIGURE 15a

FIGURE 16b

FIGURE 16a