THE RICE INSTITUTE

THE SCATTERING OF ALPHA PARTICLES FROM HELIUM

by

Charles Miller Jones

A THESIS

SUBMITTED TO THE FACULTY

IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS FOR THE DEGREE OF
MASTER OF ARTS

Houston, Texas

April, 1959

TABLE OF CONTENTS

I. INTRODUCTION	1
II. EXPERIMENTAL PROCEDURE	
A. Description of Equipment	3
B. Operation of the Van de Graaff Accelerator with a Doubly Charged Alpha Beam	5
C. Cross Section Measurements and Discussion	6
D. Bombarding Energy Determination	11
III. PRESENTATION OF THE DATA	14
IV. ANALYSIS OF THE DATA	
A. Phase Shift Analysis	16
B. Single Level Dispersion Theory	20
V. DISCUSSION AND CONCLUSIONS	25
VI. APPENDIX: IBM 650 Program for Phase Shift Analysis	
A. Description of the Program	27
B. Use of the Program	28
VIII. ACKNOWLEDGEMENTS	31

I. INTRODUCTION

The scattering of alpha particles by helium has been studied for many years. The first alpha-alpha scattering experiment was reported by Rutherford and Chadwick in 1927¹ and their work was repeated and extended by other investigators. This early work was summarized in 1941 by Wheeler². Unfortunately, the accuracy of these experiments was limited by the necessity of using natural sources. With the advent of charged particle accelerators it has become possible to study the scattering of alpha particles by helium with precision. The most recent and precise experiments have been performed by Heydenburg and Temmer³ at the Carnegie Institute in the energy range of .15 to 3 MEV, by Russell, Phillips and Reich^{4,5} at the Rice Institute in the energy range 2.5 to 5.5 MEV, and by Nilson, Jentschke, Briggs, Kerman and Snyder^{6,7} at the University of Illinois in the energy range 12 to 23 MEV.

-
1. E. Rutherford and J. Chadwick, Phil. Mag. 4 605 (1927).
 2. J.A. Wheeler, Phys. Rev. 59 16 (1941).
 3. N.P. Heydenburg and G.M. Temmer, Phys. Rev. 104 123 (1956).
 4. J.L. Russell, G. Phillips, and C. Reich, Phys. Rev. 104 135 (1956).
 5. J.L. Russell, Ph.D. Thesis, Rice Institute (1956).
 6. R. Nilson, W. Jentschke, G. Briggs, R. Kerman and J. Snyder, Phys. Rev. 104 1673 (1956).
 7. R. Nilson, W. Jentschke, G. Briggs, R. Kerman, Phys. Rev. 109 846 (1958).

Phase shift analyses have been performed on all of this recent data and it is found that the only non-zero phase shifts needed to fit the data to a laboratory energy of 20 MEV are those corresponding to $\ell = 0, 2, 4$. Furthermore, only three states are observed in the compound nucleus Be^8 in this energy range; a narrow 0^+ ground state at approximately 190 KEV bombarding energy, a broad 2^+ state at approximately 2.9 MEV in Be^8 , and an even broader 4^+ state at approximately 11 MEV in Be^8 . Since the alpha particle is a Bose-Einstein particle, it is impossible to observe states in Be^8 of either odd angular momentum or odd parity through alpha-alpha scattering. However, there is no evidence from the study of the reactions $\text{Li}^6(\text{He}^3, \text{p})\text{Be}^8$ ⁸, $\text{Li}^7(\text{d}, \text{n})\text{Be}^8$ ⁹, $\text{B}^{11}(\text{p}, \text{He}^4)\text{Be}^8$ ¹⁰, and $\text{B}^{10}(\text{d}, \text{He}^4)\text{Be}^8$ ¹⁰ for the presence of other states in Be^8 in this energy region.

In the present experiment the work of Russell and Phillips has been extended to 9 MEV using the He^{4++} beam from the Rice Institute 5.5 MEV Van de Graaff accelerator. Phase shifts have been extracted and the 2.9 MEV state has been studied using the single level dispersion theory. No new states in Be^8 were observed.

-
- 8. C.D. Moak and W.R. Wisseman, Phys. Rev. 101 1326 (1956).
 - 9. C.C. Trial and C.H. Johnson, Phys. Rev. 95 1363 (1954).
 - 10. R.E. Holland, D.R. Inglis, R.E. Malm and F.D. Mooring, Phys. Rev. 99 92 (1955).

II. EXPERIMENTAL PROCEDURE

A. Description of Equipment

The scattering experiment to be described was performed with the Rice Institute large volume gas scattering chamber. The source of energetic alpha particles used was the Rice Institute 5.5 MEV Van de Graaff accelerator, used in conjunction with a 90° analyzing magnet for energy determination.

The scattering chamber has been described in detail by Russell^{5,11} Reich¹², Henry¹³, and Miller¹⁴. Briefly, the scattering chamber consists of a large cylindrical aluminum tank of inside diameter 76 cm., which contains the target gas. The beam of incident particles enters the chamber through a reentrant differential pumping tube which extends into the chamber approximately 30 cm. The differential pumping tube makes it possible to maintain, without the use of a foil, a pressure of several cm. of Hg in the chamber without affecting the Van de Graaff vacuum. After emerging from the differential pumping tube the beam passes through the gas and then passes through a foil into an evacuated cylinder which contains a Faraday cup which is used for current integration. This Faraday cup assembly is reentrant into the chamber approximately 25. cm. The Faraday cup is equipped with electrostatic and magnetic secondary electron suppression.

11. J.L. Russell, M.A. Thesis, Rice Institute (1954).

12. C.W. Reich, M.A. Thesis, Rice Institute (1954).

13. R. Henry, M.A. Thesis, Rice Institute (1956).

14. P.D. Miller, Ph.D. Thesis, Rice Institute (1957).

Current integration is accomplished by first charging a known capacitor to a known voltage and then discharging the capacitor with the charge collected in the Faraday cup. The voltage on the capacitor is monitored by a D.C. amplifier which closes a relay when the voltage reaches zero. This relay is then used to control the pulse height analyzers. The details of this system have been described by Miller¹⁴. The charge Q collected is given by

$$Q = CV_0$$

where C is the capacitance and V_0 is the initial voltage on the capacitor.

The detectors consist of two scintillation counters equipped with slit systems which define their respective solid angles and employ CsI (Th) scintillation crystals of approximately .050 inch thickness. The detectors are located inside the chamber and are free to rotate about an axis perpendicular to the beam. The angles of rotation are read on vernier protractors which are attached to concentric tubes which in turn are attached to the detectors. These tubes also serve as conduits for the electrical cables which go to the detectors. The detector preamplifiers are operated at atmospheric pressure; the vacuum seals being made on the face of the photomultiplier tube in one counter and on a light pipe in the other counter.

The pressure of the target gas in the chamber and the pressure in the first stage of the differential pumping tube are measured with monometers filled with butyl phtalate oil of density 1.04578 gm/cm^3 . These monometers are backed with a diffusion pump and are read with Eberbach cathetometers which are accurate to 0.1 mm. The target gas flows from a high pressure storage tank through a standard demand valve

and then through a slow leak needle valve into the chamber.

A photograph of the interior of the chamber with the cover plate removed is shown in Figure 1. Proceeding from the lower left hand corner in a clockwise direction, one sees the differential pumping tube, counter number 2, the Faraday cup assembly, and counter number 1.

B. Operation of the Van de Graaff Accelerator with a Doubly Charged Alpha Beam

In order to achieve the desired bombarding energies it was necessary to use the doubly charged alpha beam from the Van de Graaff accelerator. This introduced several unusual problems. The first of these was the low intensity of the beam. The maximum beam current obtainable was approximately .01 micro-ampere and a more typical value was .005 micro-ampere. A concurrent difficulty was introduced by the presence of an intense singly charged alpha particle beam which was produced with the doubly charged beam. This singly charged beam caused severe "electron loading" which in turn necessitated the use of an unusually high belt charge. This high belt charge and "electron loading" limited the bombarding energy for the experiment to a maximum of 9.3 MEV.

The most serious problem, however, was the presence of the HH^{++} beam. Since the HH^+ ion has almost the same charge to mass ratio as the He^{++} ion, it is deflected by a magnetic field in almost exactly the same way. In particular, it was found that the two beams were separated by 2-3 mm. immediately after emerging from the 90° analyzing magnet. Since these beams were of roughly equal intensity, the Van de

FIG. 1

Graaff tended to operate with the He^{4+} beam completely stopped on the top regulation slit and the HH^+ beam completely stopped on the lower regulation slit. This problem was first solved by opening the regulation slits so that the Van de Graaff operated with the He^{4+} on the edge of the top slit and the HH^+ beam on the edge of the bottom slit. However, the Van de Graaff regulation still tended to be somewhat erratic. A more satisfactory solution was obtained by cutting a large hole in the top slit through which the He^{4+} beam passed and regulating the Van de Graaff on the HH^+ beam in the usual manner. Even this technique, however, did not completely eliminate the erratic regulation of the accelerator.

Another problem introduced by the HH^+ beam was its occasional presence in the scattering chamber. Since the Van de Graaff was regulating irregularly, the HH^+ beam was occasionally deflected into the chamber. This had two effects. The first was the observation of the protons elastically scattered from the target gas. A typical pulse height distribution with a large proton group is shown in Figure 2 and as can be seen from this figure, the protons are well resolved from the alpha particles. A more serious problem was the charge error introduced by the HH^+ beam. This error was held to less than 2% by discarding the data where there were large proton groups present.

C. Cross Section Measurements and Discussion of Errors.

The laboratory differential scattering cross section for one target nucleus, $\sigma_{lab}(\theta)$, is defined by

FIG. 2

$$\sigma_{lab}(\theta) \Delta \Omega = \frac{\# \text{ particles into solid angle } \Delta \Omega \text{ per unit time}}{\# \text{ particles per unit area per unit time in beam}}$$

Let N be the number of particles counted in a given time, n the number of target nuclei, Q the number of incident particles, and α the area of the beam. Then

$$N = \sigma_{lab}(\theta) \Delta \Omega n \frac{Q}{\alpha}$$

Now if d is the number of target nuclei per cm^3 and T is the target thickness in cm :

$$n = \alpha T d$$

and

$$N = \sigma_{lab}(\theta) \Delta \Omega T Q d$$

Reference to Figure 3 shows that $\Delta \Omega$ and T are given approximately by

$$\Delta \Omega = A/R^2$$

and

$$T = RW/S \sin \theta$$

where R is the distance from the center of the chamber to the rear slit; W is the width of the front slit; A is the area of the rear slit, and S is the distance between the front and rear slits. These approximations are in error .2% at 15° . Then defining $G = \frac{AW}{RS}$, we have

$$\sigma_{lab}(\theta) = N \sin \theta / d Q G$$

The number of counts, N , was measured with the scintillation counters in conjunction with two pulse height analyzers; a 20 channel differential pulse height analyzer and a 5 channel integral pulse height analyzer. As can be seen in Figure 2, there was no appreciable background during the experiment. The error in N then arises from two

SLIT SYSTEM

FIG. 3

sources; the statistical errors and the efficiency of the counting system. On the basis of previous experiments performed during the development of the scattering chamber wherein the counting rate for a given reaction at a fixed energy and angle was studied as a function of beam current, this efficiency is estimated to be between 99 and 100%.

The angle θ was determined by the vernier protractors previously described and has an estimated inaccuracy of 5'. This leads to the following errors in $\sin \theta$ for the laboratory angles used:

Laboratory Angle	Error in $\sin \theta$
15°16.5'	0.6%
27°22'	0.3%
35°3.5'	0.2%

The number of target nuclei per cm^3 , d , was found by assuming the perfect gas law and measuring the pressure and temperature of the target gas. The pressure was measured with the oil monometers previously described. These monometers were read to an accuracy of .01 cm which for a typical pressure of 10 cm gives an error of .1%. The temperature of the target gas was measured by placing a thermometer on the top of the chamber and assuming that the gas was in thermal equilibrium with the chamber. The thermometer is estimated to have a maximum error of 1° which yields an error of .3%. An additional error is introduced into this determination by the heating of the gas due to the beam. This and other errors in d are estimated to be no greater than 1%. The estimated RMS (root mean square) error in d is then 1.05%. It might be noted that the target gas used in the experiment was obtained from the U.S. Navy and has a stated purity of at least 99.98%.

The number of incident particles Q was determined by the current integration technique described previously. It was assumed that the charge on the alpha particles was exactly $2e$.

This assumption is based on the results of a number of experiments summarized by Allison and Warshaw¹⁵. In these experiments the charge state of He ions emerging from a thin foil into a vacuum was studied as a function of energy and foil material. Let

$$\Phi^+ = \frac{\# \text{He}^+}{\# \text{He}^0 + \# \text{He}^+ + \# \text{He}^{++}}$$

$$\Phi^0 = \frac{\# \text{He}^0}{\# \text{He}^0 + \# \text{He}^+ + \# \text{He}^{++}}$$

These experiments indicate the following facts.

- (a) Φ^0 is negligibly small above 3 MEV.
- (b) Φ^+ is not a function of foil thickness (since equilibrium is established in a distance of several atomic diameters.)
- (c) Φ^+ is essentially independent of foil material.
- (d) Φ^+ decreases with increasing energy.
- (e) Φ^+ was approximately .0150 at 4.4 MEV and decreased to approximately .0080 at 6.03 MEV.

Since the alpha particle energy after passing through the foil was at least 4.2 MEV in the present experiment, these experiments indicate that the error introduced by the assumption of a charge of $2e$ is less than 1% for the energy range used in this experiment.

The capacitor used was calibrated in the following way with the same apparatus as was used for the actual current integration. The D.C. amplifier was adjusted so as to close the sensitive relay at a voltage slightly above zero. This voltage was measured with a

15. S.K. Allison and S.D. Warshaw, Rev. Mod. Phys. 25 779 (1953).

precision potentiometer. The capacitor was charged to the known original voltage V_0 and discharged through an accurately known resistor to the final measured voltage. The time required for discharge was measured with a 60 cycle scaler operated by the relay. The capacitance was then calculated using the simple relation for a decaying R-C circuit. This calibration is thought to be accurate to within .5%. The error in the charging voltage is estimated to be less than .1%. Including the 2% error introduced by the HH^+ beam, this then gives a RMS error of 2.29% in Q.

The quantities Δ and W were measured on a traveling stage microscope equipped with a dial indicator. The slit systems used in this experiment had been in use for several years prior to this experiment. Consequently, the quantities Δ and W had been measured several times for each counter. It is felt that a good estimate of the error in Δ and W is afforded by the RMS deviation in these measurements. These deviations are given by Miller¹⁴ and are given below.

Counter Number	RMS Deviation in ΔW
1	0.1%
2	1.6%

The quantities S and R were measured with a plum bob in conjunction with a meter stick taped to the top of the chamber. The absolute error in this technique is thought to be less than .05cm which gives an error of .5% in S and .2% in R. Including the .2% error introduced by the assumptions made in calculating T and ΔS , the estimated RMS errors in G are then .60% for counter number 1 and 1.70% for

counter number 2.

Now in the course of the experiment, counter number 2 was always used at a laboratory angle of $15^{\circ}16.5'$ while counter number 1 was used at laboratory angles of $27^{\circ}22'$ and $35^{\circ}3.5'$. The estimated RMS errors in $\sigma_{lab}(\theta)$ are given below.

Laboratory Angle	Estimated RMS Error
$15^{\circ}16.5'$	3.3%
$27^{\circ}22'$	2.9%
$35^{\circ}3.5'$	2.9%

It should be noted that these RMS errors do not include the statistical error.

D. Bombarding Energy Determination

The energy of the incident alpha particles was determined with the 90° analyzing magnet which is associated with the Van de Graaff accelerator. The field of this magnet was measured with a Li nuclear magnet moment magnetometer used in conjunction with a Hewlett-Packard precision frequency meter. Since the regulation slits could not be operated in their usual position, it was necessary to recalibrate the magnet for the new slit settings. This was accomplished by performing a $\text{Li}^7(p,n)\text{Be}^7$ neutron threshold experiment using the HH^+ beam with the slits in the same position as was used for the scattering experiment. The threshold energy for this reaction was taken as 1.8814 MEV.¹⁶

16. Jones, Douglas, McEllistrem, and Richards, Phys. Rev. 94 947 (1954).

The bombarding energy was corrected for the inhomogenous magnetic field present at high magnet currents using the data of Chapman.¹⁷ The energy was also corrected for energy loss in the target gas. This was accomplished by using a simplified Bethe-Block expression for $\frac{dE}{dX}$:

$$\frac{dE}{dX} \approx \frac{C}{E} \ln \frac{4E}{I}$$

where

C is a constant

E is the bombarding energy

I is the average ionization potential

$\frac{dE}{dX}$ is the energy loss per unit length at energy E.

I was taken as 35.2 ev. and C was determined by using an experimental point from the stopping power vs. energy curves of Fuchs and Whaling.¹⁸ The amount of gas which the alpha particles passed through before reaching the target area was determined by measuring the pressure in the chamber and the first stage of the differential pumping tube with the oil monometers described above, using the known dimensions of the chamber and differential pumping tube, and assuming that the pressure drop in the first and second stages of the differential pumping tube was linear with distance.

17. R.A. Chapman, Private Report, Rice Institute Physics Department.

18. R. Fuchs and W. Whaling, Stopping Cross Sections, Kellogg Radiation Laboratory, California Institute of Technology.

There is also a correction in the energy determination due to the relativistic mass increase of the alpha particles at high energies. In particular, this correction is 13 KEV at 10 MEV. However, the protons in the $\text{Li}^7(\text{p},\text{n})\text{Be}^7$ calibration reaction also have a relativistic mass increase and this reduced the alpha particle relativistic correction to less than 3 KEV for all of the bombarding energies used. This was felt to be not significant and the energy was not corrected for this effect. In general, the bombarding energy is estimated to be accurate to within 20 KEV.

III. PRESENTATION OF THE DATA

Due to the low beam intensity, the number of data points which could be taken in a reasonable amount of time was limited. The phase shift analyses performed on the data of Heydenburg and Temmer³, Russell, Phillips and Reich^{4,5}, and Nilson, Jentschke, Briggs, Kerman and Snyder⁷ had shown that the experimental data could be fitted with only three non-zero phase shifts below 20 MEV bombarding energy. Since the phase shifts must be continuous it is possible to find uniquely three phase shifts from a three point angular distribution. Therefore, it was decided to measure three excitation functions beginning at 5 MEV and continuing to as high an energy as possible. This highest possible energy proved to be approximately 9.0 MEV. The laboratory angles chosen were $15^{\circ}16.5'$, $27^{\circ}22'$, and $35^{\circ}3.5'$. For identical particles, these correspond to center of mass angles of $30^{\circ}33'$, $54^{\circ}44'$, and $70^{\circ}7'$ respectively. These angles were chosen because the first and third angles are zeros of the fourth Legendre polynomial while the second is a zero of the second Legendre polynomial.

These excitation curves are shown in Figures 4 and 5. Also shown in these figures are points which indicate the fit from the derived phase shifts. Since the center of mass cross section is the quantity of interest, it is plotted in Figures 4 and 5. In the case of identical particles, the center mass cross section $\sigma_{cm}(\theta)$ is derived from the laboratory cross section by the following transformation:

$$\sigma_{cm}(\theta_{cm}) = \frac{\sigma_{lab}(\theta_{cm})}{[2]^{\frac{3}{2}} [1 + \cos \theta_{cm}]^{\frac{1}{2}}}$$

where $\Theta_{cm} = 2 \Theta_{lab}$

It might also be noted that for identical particles the center of mass energy, E_{cm} , is one half of the laboratory energy.

The statistical errors in this data are rather large. In particular, the lowest point in the minimum of the 70° curve near 5 MEV has a statistical error of 17%. However, with the exception of the seven points near this minimum the 70° data has a statistical error of less than 3%. The 54° data has a statistical error which ranges from 2% at 5 MEV to 7% at 9 MEV. The statistical error in the 30° data varies from 0.8% at 5MEV to 0.3% at 7 MEV.

Excitation curves at the center of mass angles $30^\circ 33'$, $54^\circ 44'$, and $70^\circ 7'$ in the energy range 1 to 20 MEV are shown in Figure 6. The smooth curves drawn through the data points in Figure 6 have no significance other than as an aid in following the curves.

ALPHA ALPHA EXCITATION CURVE
 $\theta = 30^{\circ}33'$ (CM)

FIG. 4

ALPHA ALPHA EXCITATION CURVES

FIG. 5

FIG. 6

IV ANALYSIS OF THE DATA

A. Phase Shift Analysis

The first step taken in the analysis of the data was the extraction of experimental nuclear phase shifts. The center of mass cross section $\sigma_{cm}(\theta)$ can be expressed¹⁹ in terms of the nuclear phase shifts δ_l in the following way.

$$\sigma_{cm}(\theta) = \left| -\left(\frac{\eta}{2k}\right) \csc^2\left[\frac{\theta}{2}\right] \exp\left\{i\eta \ln \csc^2\left(\frac{\theta}{2}\right)\right\} - \left(\frac{\eta}{2k}\right) \csc^2\left[\frac{\pi}{2} - \frac{\theta}{2}\right] \exp\left\{i\eta \ln \csc^2\left[\frac{\pi}{2} - \frac{\theta}{2}\right]\right\} + \frac{2}{k} \sum_{l=0,2,4,\dots}^{\infty} (2l+1) \sin \delta_l P_l(\cos \theta) \exp[i\alpha_l + i\delta_l] \right|^2$$

where

$$\eta = 4e^2/\hbar v, \quad \alpha_l = 2 \sum_{s=1}^l \tan^{-1}\left(\frac{\eta}{s}\right), \quad \alpha_0 = 0$$

k is the wave number

v is the velocity of the incident alpha particles

θ is the center-of-mass scattering angle

$P_l(\cos \theta)$ is the Legendre polynomial of order l .

The extraction of phase shifts was done in the following manner.

It was first assumed, on the basis of previous experiments, that the data could be fit with only $\delta_0, \delta_2, \delta_4$. An IBM 650 digital computer was then programmed to calculate $\sigma_{cm}(\theta)$ at a given energy for the three scattering angles used in the experiment from assumed values of

19. L.I. Schiff, Quantum Mechanics, McGraw Hill Book Co., New York (1955).

these three phase shifts. The computer then found an error function

ΣE defined by

$$\Sigma E = \sum_{\Theta} \frac{1}{\mathcal{V}_{\Theta}} \left(\frac{\sigma_{ex}(\Theta) - \sigma_{cal}(\Theta)}{\sigma_{ex}(\Theta)} \right)^2$$

where

$\sigma_{ex}(\Theta)$ is the center of mass experimental cross section,

$\sigma_{cal}(\Theta)$ is the center of mass calculated cross section,

$\mathcal{V}_{\Theta}$ is a confidence factor.

The computer then minimized this error function by successive computation of the $\sigma_{cal}(\Theta)$ with different values of δ_0 , δ_2 , and δ_4 . The details of this program are described in the appendix. The $\sigma(\Theta)$ computation part of the program was checked with an accurate hand calculation, while the minimizing logic was checked by means of several graphical solutions.

The program was first run with $\mathcal{V}_{\Theta} = 1$ for all Θ 's and energies. It was found that the phase shift fits to the excitation curves were in general good except near the maximum of the 30° and 70° curves. It was also found that the least squares fit produced a curve that tended to miss the 30° curve much more than it missed the 54° and 70° curves. This latter difficulty was rectified by adjusting the $\mathcal{V}$'s so as to produce a roughly equal miss in all of the curves. The cause of the miss is thought to be due to slight inaccuracies in the data. The phase shifts derived in this way are shown in Figure 7. It should be noted that in this energy range δ_4 is identically equal to zero. The theoretical fits to the excitation curves calculated from these phase shifts are shown in Figures 4 and 5.

EXPERIMENTAL PHASE SHIFTS

FIG. 7

An estimate of the errors to be associated with the phase shifts was made in the following way. By a series of cross section calculations at 1 MEV intervals, it was determined how much variation in the S wave phase shift was necessary to produce a $54^{\circ}44'$ cross section which deviated from the cross section predicted by the derived phase shift by more than the experimental error associated with the experimental cross section at that energy. This deviation was called the "basic" error in the S wave phase shift. This basic error was $\pm 1^{\circ}$ for the entire energy range. This procedure was then repeated for the D wave phase shift at 1 MEV intervals for each integral value of the S wave phase shift lying within the basic error. In this second series of calculations, however, the results were compared with the $30^{\circ}33'$ and $70^{\circ}7'$ cross section predictions of the derived phase shifts. In this way, a basic error for the D wave phase shift was established. This proved to be approximately $\pm 1.5^{\circ}$ at 5 and 6 MEV and $\pm 3^{\circ}$ at 7, 8, and 9 MEV. To obtain a final estimate of the phase shift errors, these basic errors were multiplied by a factor of 3 in consideration of the poor fits produced by the derived phases.

The phase shifts derived in the present work are shown in conjunction with those derived by Heydenburg and Temmer³, Russell and Phillips and Reich^{4,5}, and Nilson, Jentschke, Briggs, Kerman and Snyder⁷ in Figure 8. As can be seen, the D wave phase shift shows a typical resonant behavior near a laboratory energy of 6 MEV. Likewise, the beginning of a typical resonant behavior is also seen in the G wave phase shift. The S wave phase shift is presumed to rise very rapidly to 180° at an energy corresponding to the very narrow ground state,

FIG. 8

and as can be seen, decreases monotonically, passing through zero at approximately 20 MEV.

It will also be noticed from Figure 8 that the S and D wave phase shifts of Nilson and Jentschke at the laboratory energies 12.3 MEV and 15.2 MEV do not seem to agree with the other S and D wave phase shifts. In an attempt to understand this discrepancy, the angular distributions of Nilson and Jentschke at these two energies were re-analyzed with the technique described above. The re-analysis yielded essentially the same phase shifts as reported by Nilson and Jentschke. It was then decided to determine the difference between the experimental angular distributions actually measured and the angular distributions which would have been measured if the two phases of Nilson and Jentschke were in agreement with the present work or if the present work were in agreement with Nilson and Jentschke. Two angular distributions were calculated from assumed phases. The first of these angular distributions was calculated for a laboratory energy of 8 MEV with phase shifts which are compatible with the work of Nilson and Jentschke. This angular distribution is shown with experimental points from the present work in Figure 9. The second angular distribution was calculated for a laboratory energy of 12.3 MEV and phase shifts compatible with the present work. This angular distribution is shown in Figure 10 with the experimental points of Nilson and Jentschke. Examination of these two figures shows that there is not much difference between the angular distributions calculated from the assumed phases and the experimental points. This in turn indicates that the cross

ALPHA-ALPHA ANGULAR DISTRIBUTION
AT LAB ENERGY 8 MEV

FIG. 9

ALPHA-ALPHA ANGULAR DISTRIBUTION
AT LAB ENERGY 12.3 MEV

FIG. 10

section is not a strongly varying function of the phase shifts in this energy region. It thus appears that more accurate cross section measurements will have to be made before it can be decided whether the phase shifts extracted in the present work or those of Nilson and Jentschke more accurately represent the true phases.

B. Single Level Dispersion Theory.

In order to extract level parameters for the D state, it was decided to attempt to fit the D wave resonance with the single level dispersion theory. In this formalism²⁰ the scattering is considered to be the sum of scattering from a hard sphere of radius R and a resonance scattering which is of importance only near the resonance energy. Since the calculation techniques are somewhat easier, it was decided to fit the D wave phase shifts instead of the excitation curves.

It has been shown²¹ that the nuclear phase shift δ_L can be expressed in terms of the dispersion theory in the following form.

$$\delta_L = \delta_{\lambda,R} - \phi_L \quad , \quad \phi_L = \tan^{-1} [F_L/G_L]_{p=kR}$$

$$\delta_{\lambda,R} = \tan^{-1} \frac{\frac{1}{2} \sqrt{\lambda_L}}{E_{\lambda L} + \Delta_{\lambda L} - E_{cm}}$$

$$\frac{1}{2} \sqrt{\lambda_L} = \left[\frac{p \gamma \lambda^2}{A_L^2} \right]_{p=kR} \quad , \quad A_L^2 = F_L^2 + G_L^2$$

20. J.M. Blatt and V.F. Weisskopf, Theoretical Nuclear Physics, John Wiley and Son's, New York (1952).

21. R.K. Adair, Phys. Rev. 86 155 (1952).

$$\Delta_{\lambda,L} = -\tau_{\lambda}^2 [g_L + L] \rho = kR$$

$$g_L = \rho \left[\frac{1}{F_L} \cdot \frac{\partial F_L}{\partial \rho} - \frac{1}{A_L^2} \cdot \frac{G_L}{F_L} \right] \rho = kR$$

where

E_{cm} = Energy of the center of mass system, MEV

$E_{\lambda,L}$ = Constant expansion parameter, MEV

F_L = Regular Coulomb Wave Function

G_L = Irregular Coulomb Wave Function

$$\rho = kn$$

$$k = \text{wave number} = \sqrt{\frac{M E_{cm}}{\hbar^2}}$$

R = Nuclear radius, cm.

τ_{λ}^2 = Center of mass reduced width (constant), MEV

$\Gamma_{\lambda,L}$ = Center of mass laboratory width, MEV

Φ_L = hard sphere phase shift

$\delta_{\lambda,R}$ = Resonant phase shift.

The resonance energy E_S is defined as that energy where

$\delta_{\lambda,R} = 90^\circ$. An examination of these expressions shows that a fit is determined by three parameters. For example, E_S , τ_{λ}^2 , and R . To reduce this number to two, it was decided to require that $\left. \frac{d\delta_L}{dE_{lab}} \right|_{E_S}$ as determined from the experimental phase shifts be equal to $\left. \frac{d\delta_L}{dE_{lab}} \right|_{E_S}$ as determined from the dispersion theory fit.

This imposes the additional condition:

$$\frac{1}{2} \frac{1}{\Gamma_{\lambda,L}} = \left. \frac{d\delta_L}{dE_{lab}} \right|_{ex.} - \frac{1}{\frac{dE_{lab}}{d\rho}} \left[\left(\frac{1}{\rho A_L^2} \right) \cdot \frac{d}{d\rho} [g_L + L] - \frac{1}{A_L^2} \right]$$

where all the variable quantities are evaluated at E_s . This technique greatly facilitates the analysis.

The D wave phase shift was calculated as a function of energy for various sets of parameters with the use of the Coulomb Wave Function tables of Bloch, Hull, Broules, Bouricius, Freeman, and Breit²². Some of the poorer fits are shown in Figure 11 and the best fit is shown in Figure 12. It should be noted that the points representing the experimental phase shifts are points taken from a smooth line drawn through the actual experimental phase shifts.

The parameters used in the best fit and the Wigner Limit²³ for these parameters are given below.

Excitation in Be^8 :	3.1 MEV (cm)
Reduced Width; :	3.5 MEV (cm)
Hard Sphere Radius:	3.5F
Wigner Limit $= \frac{3/2 \hbar^2}{\mu a^2}$:	2.65 MEV (cm)

$$\Theta_\lambda^2 = r_\lambda^2 / \frac{3/2 \hbar^2}{\mu a^2} = 1.32$$

The extreme width of the state in comparison with the Wigner Limit indicates that the D state in Be^8 is of almost wholly alpha-alpha parentage.

It is of interest to consider that this calculation in the light

22. Bloch, Hull, Broyles, Bouricius, Freeman, and Breit, Coulomb Functions for Reactions of Protons and Alpha Particles with the Lighter Nuclei, Yale University.

23. T. Teichmann and E. Wigner, Phys. Rev. 87 123 (1952).

D WAVE PHASE SHIFT EXPERIMENT:

xRUJSELL

•PRESENT WORK

+NILSON & JENTSCHE

FIG. II

D WAVE PHASE SHIFT

EXPERIMENT:

x RUSSELL

• PRESENT WORK

+ NILSON & JENTSCHKE

— DISPERSION THEORY

EXCITATION 3.1 MEV (CM)

REDUCED WIDTH 3.5 MEV (CM)

HARD SPHERE RADIUS 3.5 F.

WIGNER LIMIT 2.65 MEV (CM)

LAB ENERGY (MEV)

-160°

-140°

-120°

-100°

-80°

-60°

-40°

-20°

x

5

10

15

20

FIG. 12

of what is known about the alpha-alpha potential. Russell⁵ has shown that if the interaction between two alpha particles can be described by a scalar potential and if the Be⁸ nucleus is considered to be composed of two alpha particles at low excitation energies, this scalar potential will have the qualitative shape shown in Figure 13. Since we are considering an interaction with non-zero angular momentum, a centrifugal potential term, $\frac{l(l+1)\hbar^2}{2\mu r^2}$, must be added to this potential. The sum of the scalar and centrifugal potentials, for $l=2$, is indicated by the dotted line. Now it will be noticed that the dispersion theory predicts a radius which is too small. In particular, the dispersion theory radius is 3.5 F, while Russell's arguments would indicate a hard sphere radius of approximately 5.5 F. A possible explanation for this discrepancy is that at higher energies some of the hard sphere scattering does not occur at point A but rather at Point B, and thus the dispersion theory predicts a radius which approaches the radius of the hard core. This argument is supported by the fact that the low energy D wave phase shift (less than $E_{lab} = 6\text{MEV}$) is fitted very well with a radius of 5.2 F (see Figure 11). As the energy increases, a smaller radius must be used to fit the phase shift (Figure 12). These arguments indicate that the dispersion theory calculations are compatible with a repulsive core scalar alpha-alpha potential.

It should be noted that the Wigner Limit is also altered from the usual definition by this potential. The Wigner Limit may be thought of as the uncertainty in energy due to the confinement of the nuclear

FIG. 13

system within a specific volume (defined by R). Since there is a hard core to this potential, the system is really, more or less, confined to a spherical shell of outer radius A and inner radius B so that the uncertainty in energy is greater than if the system were confined in a sphere of radius A .

V. DISCUSSION AND CONCLUSIONS

As has been indicated before, the experimental phase shifts indicate only three states in Be^8 at excitation energies below 12 MEV; a 0^+ ground state at approximately 190 KEV bombarding energy, a 2^+ state at approximately 3 MEV in Be^8 and a 4^+ state at approximately 11 MEV in Be^8 . Unfortunately, this level structure is predicted by both the central force²⁴ model and the alpha particle model²⁵. However, the extreme width of these states indicates that they are alpha particle states and lends credence to the latter model.

With regard to the alpha-alpha potential, very interesting calculations have been performed by Humphrey²⁶ in which he has attempted to fit the experimental phase shifts with a Margenau potential²⁷ given by

$$V(r) = \begin{cases} \infty & , r < r_1 \\ -V_0 & , r_1 < r < r_0 \\ 4e^2/r & , r_0 < r \end{cases}$$

24. D. Kurath, Phys. Rev. 101 216 (1956).

25. Haefner, Rev. Mod. Phys. 23 228 (1951).

26. C. Humphrey, Private communication.

27. H. Margenau, Phys. Rev. 59 37 (1941).

and a Haefner potential²⁵, given by

$$V(\eta) = \begin{cases} -D + \hbar^2 q^2 / 2\mu \eta^2 & , \eta < \eta_0 \\ 4e^2 / \eta & , \eta > \eta_0 \end{cases}$$

Quite good fits were obtained with the Haefner potential but, unfortunately, these fits were obtained by using different values of D for different values of q . He was unable to find one potential of this form which would fit all the phase shifts. It is hoped that work of this nature can be continued with more sophisticated potentials.

As indicated in the discussion of the phase shift analysis, there is a need for a more precise and extensive study of this problem. In order to facilitate this study and also make possible the study of other problems using high energy alpha particles, plans are now being formulated to modify the Van de Graaff accelerator so as to produce a more intense beam of doubly charged alpha particles. It will then be possible to perform precise scattering and reaction experiments using alpha particles with energies up to 11 MEV.

There also appears to be a need for a type of dispersion theory which would be more suitable for hard core potentials. In terms of Figure 13, such a theory might consider the hard sphere scattering as composed of two parts. A certain fraction, f , of the hard sphere scattering would occur at radius B , while the rest of the hard sphere scattering would occur at radius A . This viewpoint, however, presents two difficulties. The first is that the fraction f would be a sensitive and probably complicated function of energy. The second problem is the effect of the potential between A and B on the hard sphere scattering at B . A simple solution to these problems is not evident.

VI. APPENDIX

IBM 650 Program for Phase Shift Analysis.

A. Description of the Program.

The program to be described was written in the Bell Laboratories Interpretative System, commonly called "Bell".

It was designed to perform the following functions.

- (a) Calculations of ten point angular distributions at given energies from given nuclear phase shifts.
- (b) Extraction of Nuclear Phase shifts from experimental angular distributions.

The program performs the first of these functions by substitution of given nuclear phase shifts into the expression given on page 16 . In the extraction of nuclear phase shifts, the computer is first directed to compute an angular distribution from given phase shifts. It then calculates an error function ΣE defined by

$$\Sigma E = \sum_{\theta} \frac{1}{2\theta} \left(\frac{\sigma_{ex}(\theta) - \sigma_{cal}(\theta)}{\sigma_{ex}(\theta)} \right)^2$$

where

$\sigma_{ex}(\theta)$ is the center of mass experimental cross section

$\sigma_{cal}(\theta)$ is the center of mass calculated cross section

2θ is a confidence factor.

The computer then minimizes this error function by varying the phase shifts one at a time by an amount $\Delta \delta$ in a pre-set order.

The program is limited to the problem of spin zero identical particles. It is also limited in that it is designed to handle phase shifts only to $l = 4$. However, only relatively simple modifications

would be needed to generalize the program to the general spin zero on spin zero case and to higher l .

A flow diagram of the program is shown in Figure 14. The symbols used in the flow diagram are defined below.

$$R(R) = -\frac{\eta}{2} \csc^2\left(\frac{\Theta}{2}\right) \left[\cos\left(\eta \ln \csc^2 \frac{\Theta}{2}\right) \right. \\ \left. - \frac{\eta}{2} \sec^2\left(\frac{\Theta}{2}\right) \left[\cos\left(\eta \ln \sec^2 \frac{\Theta}{2}\right) \right] \right]$$

$$I(R) = -\frac{\eta}{2} \csc^2\left(\frac{\Theta}{2}\right) \left[\sin\left(\eta \ln \csc^2 \frac{\Theta}{2}\right) \right. \\ \left. - \frac{\eta}{2} \sec^2\left(\frac{\Theta}{2}\right) \left[\sin\left(\eta \ln \sec^2 \frac{\Theta}{2}\right) \right] \right]$$

$$R(\Sigma) = 2 \sum_{l=0,2,4} (2l+1) \delta_l P_l(\cos \Theta) \cos \beta_l$$

$$I(\Sigma) = 2 \sum_{l=0,2,4} (2l+1) \delta_l P_l(\cos \Theta) \sin \beta_l$$

$$\sigma_{cm}(\Theta) = \frac{1}{k^2} \left\{ [R(R) + R(\Sigma)]^2 + [I(R) + I(\Sigma)]^2 \right\}$$

$$\beta_l = \alpha_l + \delta_l$$

$$\alpha_l = 2 \sum_{s=1}^l \tan^{-1} \left(\frac{\eta}{s} \right), \quad \alpha_0 = 0$$

B. Use of the Program.

The input format is divided into five sections. The locations and contents of this format are given below in the order in which they are read by the computer.

FIG. 14

Input Format

Location	Contents
900	.1574540049
901	.2090585049
902	Ξ_1 (charge of incident particle)
903	Ξ_2 (charge of target particle)
904	M_1 (amu)
905	M_2 (amu)
906	$\Delta \delta$ (degrees)
800	Θ_1 (degrees; cm)
801	Θ_2
:	:
809	Θ_9
820	ν_0
821	ν_1
:	:
829	ν_9
840	experimental σ_0 (Mb), (C.M.)
841	experimental σ_1
:	:
849	experimental σ_9
699	E (bombarding laboratory energy in MEV)
700	δ_0 (degrees)
701	χ (code word)
702	δ_2 (degrees)
703	$\frac{z}{\Xi}$ (code word)
704	δ_4 (degrees)
705	η (code word)

x, y , and z are code words with the following values and meanings.

Code Word	Value	Meaning
x	5	compute σ
x	1	compute δ
y	5	treat up to $l = 4$
y	1	treat up to $l = 2$
z	5	stop
z	1	continue

The program is presently set up to calculate three point angular distributions. If ten point angular distributions are desired, the sub deck entitled "10 pt. Angular Distributions" should be placed behind

the program deck.

The output format is given below.

Location	Contents
699	E
700	calculated δ_0
701	x
702	calculated δ_2
703	z
704	calculated δ_4
705	y
750	calculated σ_0 (barns)
751	calculated σ_1
:	:
759	calculated σ_q
760	<hr/>
761	ΣE

It will be noticed that the computer prints out calculated cross sections simultaneously with derived phase shifts. These cross sections are not, however, calculated from the final δ_l 's but rather from the final δ_l 's - $\Delta\delta$. Thus, in order to obtain a fit to the experimental cross sections from the derived phase shifts, it is necessary to run the program again in the "sigma computation" mode with the derived phase shifts. To enable the direct use of the output cards, a card has been prepared which performs the function of the code word x. The words "sigma computation" have been written on this card and for the "sigma computation" model, the card is placed behind the program.

VII. ACKNOWLEDGEMENTS

It is a pleasure to express my appreciation to those with whom I have been associated in working with the large volume gas scattering chamber. In particular, I would like to thank Dr. P.D. Miller who helped in taking the data and especially, Dr. G.C. Phillips whose guidance and encouragement have made this work possible.

I would like also to express my appreciation to The Shell Development Corporation who generously made available the use of their IBM 650 computer.

Finally, I wish to thank my wife who not only typed the thesis, but was also a source of understanding and encouragement throughout the course of this work.