THE PRECISE MEASUREMENT OF THREE RADIUM B BETA-PARTICLE ENERGIES

A thesis presented by

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to the Faculty of The Rice Institute, Houston,
Texas, in May, 1936, in partial satisfaction of
their requirements for the degree of Master of

Arts

CORRECTION

See page 10, lines 6 through 9. Instead of: Let ϵ_i , $i = 1$ thru 10, be the absolute errors which might possibly be present in single determinations of d , D , l , g , m , g , I , $\int H_0 F f' dy$, $H_0 \Delta l$, and $\int h(\theta) \sin \theta d\theta$ respectively; read, rather: Let ϵ_i , $i = 1$ thru 10, be the absolute errors which might possibly be present in single determinations of d , D , l , m , g , I , $\int H_0 F f' dy$, $H_0 \Delta l$, $\int h(x) dx$, and $\int h(\theta) \sin \theta d\theta$ respectively;

INTRODUCTION

In 1924 Ellis and Skinner¹ measured $H\rho$ (H is the magnetic field necessary to produce a radius ρ of curvature in the path of a β -particle) for many of the β -particle energies of Ra B; for the three most intense lines of Ra B they got, correctly to within what they thought was 1 part in 500,

$$H\rho = 1410, 1677, 1938 \text{ gauss cm.}$$

In 1934 Ellis² repeated this work with more precision and got

$$H\rho = 1400.4, 1665.9, 1925.5 \text{ gauss cm.}$$

(in which the last figures are not entirely significant) for these same lines. In 1934 also, Scott³ performed a precise experiment on the most intense of these lines using the Cotton balance method of measuring the magnetic field rather than any inductive method; he got for this most intense line

$$H\rho = 1931.8 \text{ gauss cm.}$$

and hoped it to be correct to 1 part in 10,000.

The present experiment was undertaken to check Scott's work and method and to extend the measurements to other lines in the Ra B β -particle spectrum. The values of

$$H\rho = 1406.0, 1671.1, 1931.5 \text{ gauss cm.}$$

were obtained, the last of which is in good agreement with Scott's value for the most intense line. The $H\rho$'s for the other two lines as found by this experiment yield slightly different relative values of $H\rho$ from those previously obtained.

A more thorough investigation of the theory of the Cotton

¹ Ellis and Skinner, Proc. Roy. Soc., A105, 165 (1924).

² Ellis, Proc. Roy. Soc., A143, 352 (1934).

³ Scott, Phys. Rev., 46, 633 (1934).

balance method of measuring H was carried out for this experiment. On the basis of it an analysis was made of the errors which might be present in the finally accepted values of $H\rho$. The conclusion was reached that in this experiment an accuracy of 1 part in 3,000 is attainable if the value of H used in the finally accepted $H\rho$'s is determined from the mean of at least forty separate, equally weighted determinations of the mass used with the Cotton balance and if the values of ρ used finally are the means of seven separate, equally weighted determinations of them.

The good agreement (within 1 part in 6,400 in $H\rho$) of the present experiment with that of Scott would seem to indicate that this type of measurement has at last established the absolute values of the $H\rho$'s for the three most intense lines in the Ra B β -particle spectrum to within 1 part in 3,000 except for possible systematic errors which may have been present and undetected in both experiments.

The three energies corresponding to the three $H\rho$'s we have determined are found to be

$$V = 1.512 \times 10^5, 2.044 \times 10^5, 2.610 \times 10^5 \text{ electron-volts}$$

and can be considered as accurate to not better than 1 part in 1,000.

APPARATUS

The apparatus used in this experiment was, except for a few small changes, that used by Scott in his work and described by him in his paper³ (q.v.). It consists of a large permanent magnet with cobalt steel poles and with a 1.52 cm. air gap faced on each side by soft iron disks 15 cm. in diameter and 7.5 cm. thick; the faces are quite nearly parallel and vertical. The flux density in the air gap can easily be adjusted to the desired value to within 1.5 gauss by means of a controlled current in two solenoids around the pole pieces. An investigation of the direction of the field in the gap showed that it deviates from the normal to the pole faces by less than $0^{\circ} 05'$. Computation by the series for $\cos 0^{\circ} 05'$ shows that the horizontal component of H differs from H by less than 1 part in 500,000. Thus the force which the Cotton balance measures, i.e., the horizontal component of H , is sensibly H .

The field in the air gap could be explored with search coils and galvanometers calibrated as fluxmeters. H as a function of the distance from the center of the field was ^{at center} done with a coil and galvanometer designed to give an accuracy of one per cent. Small variations of H near the center of the field were obtained with a 1,000 turn coil of diameter 0.9 cm. and an 18 ohm, high sensitivity galvanometer, the combination having a sensitivity of 0.0241 gauss per cm.

Supported above the air gap was a new Becker precision balance of which one pan had been removed. In place of the pan was hung a Pyrex glass plate about 30 cm. long and 3 cm. wide with its longer edges optically worked to be parallel to within $1/500$ mm. for 15 cm. from one end. This plate hung from the lower ends of two parallel

aluminum strips which hung in turn from a support-assembly attached to a knife edge of the balance. The plate-assembly was free to swing in two dimensions and was protected from external air currents by a surrounding brass case. Two straight silver strips about 1/10 mm. thick were cemented with very dilute white shellac around the lower end and the two long edges of the plate; they were joined by a flat fold at the middle of the lower edge and formed the conductor of current in the magnetic field. This lower edge was placed along the x-axis of Fig.1 (see page 7) while measurements of H were being made; the center line of the plate was along the y-axis.

Measurements of plate and strips could be made with a Brown and Sharpe gauge which was graduated in 10^{-4} 's of an inch and which could be reliably estimated to a 10^{-5} of an inch, with a Gaertner comparator and invar steel standard scale 10 cm. long, and with a new Brown and Sharpe 31 mm. standard disk. The invar scale was stated by the Bureau of Standards to be accurate to 1 part in 10^6 . All measurements of the plate, strips, and photographic plates are expressed in terms of this standard scale when used as absolute lengths.

When the current in the strips was reversed, the change in force on them was measured by balancing it with a mass m on the balance. m was nearly 400 mg. and was obtained in the main by combining the 200, 100, and 100 mg. weights of a new and previously unused set of platinum, assayers' weights by Becker; each of these weights is quoted by the manufacturer as being accurate to within 0.005 mg. To get m really accurately, small balance deflections (occurring when the current in the strips was reversed) could be measured by the motion of a spot of light reflected from a small mirror on the balance beam to a scale about two meters from the

balance. The location of the light spot could be read to within $1/5$ mm. with a magnifying glass. When fully loaded in the experiment, the balance had a sensitivity of 0.409 mg./cm., which was entirely sufficient to get m accurately.

Current was supplied to the strips around the plate by two 160 ampere-hour lead storage batteries connected in parallel. The current was sent thru two standard resistors of 4.0001 international ohms at 25° C and having negligible temperature corrections near 25° C. These resistors were connected in parallel by heavy copper bars and the current thru them adjusted until a galvanometer in series with a standard cell (the two being connected across the resistors) showed no deflection. A measurement of H using one resistor balanced by two standard cells yielded the same value of H as this arrangement, so the resistance between the resistors and the copper bars was entirely negligible. The resistors were calibrated to 0.005 per cent by the Bureau of Standards in February, 1934, and, except for Scott's work then, have not been used until the present experiment. The standard cell along with two others was calibrated to 0.01 per cent by the Bureau of Standards at the same time and was found to have an em.f. of 1.01884 international volts at 25° C; at the time of this experiment these cells had not changed their relative em.f.'s by more than $\frac{1}{2} \times 10^{-5}$ volts, so it was assumed that their absolute values had not altered appreciably. The galvanometer used to indicate the existence of the correct current was of sensitivity 1.84×10^{-5} amp./cm., readable to $1/5$ mm. easily. Current I could be adjusted by means of a variable resistance network to within $\frac{1}{2}$ mm. scale deflection, or to within 10^{-6} amp.

The Ra B β -particles (from the active deposit from the radon

of 23 mg. of radium) were emitted from the surface of a platinum wire placed at 0 (Fig.1); they travelled in opposite directions as allowed by slits in the camera and fell on photographic plates PP'. The distances between the outer edges of the three sets of clearly defined "images" on the plates could be measured with the Gaertner comparator and the invar scale. The camera and its enclosing box are of brass tested previously for magnetic effects and found to have none; Scott fully describes them in his paper³. The vacuum in the box was gotten by a Cenco Hyvac pump running continuously during the three and one half hour exposures.

All observations and measurements were made in two rooms which were maintained at temperatures between 23° and 25° C.

THEORY

Preliminary measurements have shown that when the glass plate is in position for field measurements, the center of the silver strip around the glass plate lies, in the large, along a curve C

FIG.1. PP' are photographic plates; O is the source of β -particles

of the form (exaggerated in the diagram) of abcdefgh in Fig.1; this

curve is sufficiently nearly symmetric to be treated for our purposes as symmetric. In Fig.1 the magnetic field H is perpendicular to the plane of the paper and directed into the paper; x - and y -axes perpendicular to H are chosen with their origin in line with the centers of the pole faces and half way between them, the x -axis horizontally; ds with x -component dx is an element of length along C . The width l of the current loop is measured at some convenient location (as bg) along the glass plate.

To get the field H_0 at the origin by the Cotton balance method, note that the force acting on the current I (em.u.) in the strip is, besides gravity, $I \int_C H dx$; when I is reversed, the change in force on the strip is measured by the weight of the mass m which will just counterbalance it; then

$$mg = 2I \int_C H dx.$$

Since H and C are necessarily continuous, we can split the integral into five parts:

$$\int_C = \int_a^c + \int_c^d + \int_d^e + \int_e^f + \int_f^h,$$

and since subsequent experiment shows that

$$\int_a^c, \int_c^d, \int_d^e, \int_e^f < 10^{-3} \int_d^e,$$

approximate evaluation of these integrals is satisfactory. Let

$$\begin{aligned} H &= H_0 + h(x) \text{ along } de, \\ H &= H_0 F(y) \text{ along } ac \text{ and } fh, \\ H &= H_0 \text{ along } cd \text{ and } ef; \end{aligned}$$

$$\text{then } mg = 2I \left\{ H_0 \int_a^c F(y) dx + H_0 \int_c^d dx + \int_d^e [H_0 + h(x)] dx + H_0 \int_e^f dx + H_0 \int_f^h F(y) dx \right\}.$$

Also let C be represented from a to c by $x = f(y)$, and let $dx = \frac{1}{2} \Delta l$; then in view of the symmetry of C with respect to the y -axis,

$$mg = 2I \left[2H_0 \int_a^c F(y) f'(y) dy + H_0 \Delta l + H_0 l + \int_{\frac{1}{2}l}^{\frac{1}{2}l} h(x) dx \right].$$

Solving for the coefficient of l ,

$$H_0 = \frac{1}{l} \left[\frac{mg}{2l} - 2H_0 \int_a^c F(y) f'(y) dy - H_0 \Delta l - \int_{-l/2}^{l/2} h(x) dx \right];$$

here m , g , l , and l are measureable with high accuracy; the last three terms in the right member are of the nature of corrections to the first and (as previously stated) are fairly small, so in them only the approximate value of H_0 need be used.

To this Cotton balance equation for H_0 must be added for this experiment the Hartree correction for non-uniformity of H along the β -particle paths. In Fig.1 two semi-circular paths for the same β -particle energy are Okj and Omn ; measuring θ along them as shown in this figure, let

$$\begin{aligned} H &= H_0 + h_1(\theta) \text{ along } Okj, \\ H &= H_0 + h_2(\theta) \text{ along } Omn. \end{aligned}$$

Then according to Hartree⁴, the first order correction to be applied to H_0 to give the constant value of H which, if it were acting all along the β -particle path Okj , would produce the observed deviation Oj is

$$\frac{1}{2} \int_0^{2\pi} h_1(\theta) \sin \theta d\theta.$$

Applying to the above equation for H_0 the mean of the two Hartree corrections for the two paths of the same energy of β -particle, we

$$\text{get } H_0 = \frac{1}{l} \left[\frac{mg}{2l} - 2H_0 \int_a^c F(y) f'(y) dy - H_0 \Delta l - \int_{-l/2}^{l/2} h(x) dx \right] + \frac{1}{4} \left(\int_0^{2\pi} h_1(\theta) \sin \theta d\theta + \int_0^{2\pi} h_2(\theta) \sin \theta d\theta \right), \quad (1)$$

which is the expression adopted for the H_0 to be used in H_p .

To get ρ , the distance d between the outer, clearly defined edges of corresponding images on the photographic plates is measured; from it is subtracted the diameter D of the platinum wire supporting the source at O . Then

$$\rho = \frac{d-D}{4}. \quad (2)$$

H_p for a β -particle energy is the product of the appropriate values of the expressions (1) and (2).

⁴Hartree, Proc. Camb. Phil. Soc., 21, 746 (1923).

The relative error to be expected in a determination of H_0 follows from two principles concerning errors⁵: the relative error in a product of quantities having known relative errors is the sum of the relative errors of the multiplicands, and the absolute error of a sum of quantities having known absolute errors is the sum of the absolute errors of the summands. Let ϵ_i , $i=1$ thru 10 , be the absolute errors which might possibly be present in single determinations of d , D , l , g , m , g , I , $\int H_0 F f' dy$, $H_0 \Delta l$, and $\int h(\theta) \sin \theta d\theta$ respectively; these must be estimated from the precisions of the observations on the several quantities entering into expressions (1) and (2). Then since (by the definition of relative error) there corresponds to the relative error in a quantity an absolute error whose magnitude is equal to the product of the magnitude of the quantity into the relative error,

$$\frac{mg}{2I} \left(\frac{\epsilon_4}{m} + \frac{\epsilon_5}{g} + \frac{\epsilon_6}{I} \right) + 2\epsilon_7 + \epsilon_8 + \epsilon_9$$

is the absolute error to be expected in the bracketed part of expression (1); to it corresponds quite nearly the relative error

$$\frac{\epsilon_4}{m} + \frac{\epsilon_5}{g} + \frac{\epsilon_6}{I} + \frac{2\epsilon_7 + \epsilon_8 + \epsilon_9}{mg/2I}.$$

Hence the absolute error in the first term of the right member of (1) is approximately

$$H_0 \left(\frac{\epsilon_3}{I} + \frac{\epsilon_4}{m} + \frac{\epsilon_5}{g} + \frac{\epsilon_6}{I} + \frac{2\epsilon_7 + \epsilon_8 + \epsilon_9}{mg/2I} \right);$$

added to the absolute error

$$\frac{1}{2} \left(\frac{1}{2} \epsilon_{10} + \frac{1}{2} \epsilon_{10} \right) = \frac{\epsilon_{10}}{2}$$

in the second term and divided by H_0 , we have as the relative error to be expected in a single determination of H_0 :

⁵See, e.g., Gibbs, Adjustment of Errors in Practical Science, Oxford Press, 1929, pp. 76-78.

$$e_1 = \frac{\epsilon_2}{1} + \frac{\epsilon_4}{m} + \frac{\epsilon_5}{g} + \frac{\epsilon_6}{1} + \frac{2\epsilon_1 + \epsilon_8 + \epsilon_9}{mg/21} + \frac{\epsilon_{10}}{2H_0}. \quad (3)$$

The relative error to be expected in a determination of ρ is obviously

$$e_2 = \frac{\epsilon_1 + \epsilon_3}{d - D}. \quad (4)$$

If ϵ_i , $i = 1$ thru 10, are estimated conservatively as the maximum absolute errors to be expected in their respective magnitudes, the corresponding maximum relative error to be expected in an accepted $H\rho$ is

$$e_3 = e_1 + e_2. \quad (5)$$

To get the energy E of a β -particle of known $H\rho$, combine with the relativistic expression

$$E = m_0 c^2 \left[\frac{1}{\left(1 - \frac{v^2}{c^2}\right)^{1/2}} - 1 \right]$$

for E the relativistic force equation

$$Hev = \frac{m_0 v^2}{\rho} = \frac{m_0 v^2}{\rho \left(1 - \frac{v^2}{c^2}\right)^{1/2}}$$

so as to eliminate v . Here v is the velocity of the β -particles, m_0 their rest mass, e their charge, and c the velocity of light. The resulting expression

$$E = m_0 c^2 \left[(A+1)^{1/2} - 1 \right], \quad A = \left(H\rho \frac{e}{m_0} \cdot \frac{1}{c} \right)^2,$$

is the one commonly used. If $H\rho$ is in gauss cm., if e/m_0 is in em.u. and if c is in es.u., then

$$m = \left(\frac{m_0}{e} \right) \frac{e}{c};$$

also if E is expressed (as V) in electron-volts, its value in ergs must be multiplied by $10^{-8} c/e$; thus the energy in electron-volts is

$$V = \frac{m_0}{e} c^2 \left[(A+1)^{1/2} - 1 \right] \times 10^{-8}. \quad (6)$$

This can be put more conveniently for computation if we define ϕ by

$$\phi = \tan^{-1} \left(H\rho \frac{e}{m_0} \cdot \frac{1}{c} \right) = \tan^{-1} \sqrt{A};$$

then
$$V = \frac{m_0}{e} c^2 (\sec \phi - 1) \times 10^{-8}. \quad (7)$$

If ϵ_u and ϵ_n are the absolute errors to be expected in e/m_0 and c respectively, the relative error to be expected in $(A+1)$ (see equation (6)) is

$$\frac{A}{A+1} \left(2e_3 + \frac{2\epsilon_u}{e/m_0} + \frac{2\epsilon_n}{c} \right).$$

Since the extraction of the square root of a quantity with a known relative error yields roots with relative errors only half that of their square⁶, the absolute error in $(A+1)^{1/2}$ is

$$\frac{A}{(A+1)^{1/2}} \left(e_3 + \frac{\epsilon_u}{e/m_0} + \frac{\epsilon_n}{c} \right),$$

which is also to be expected in $[(A+1)^{1/2} - 1]$. Hence by addition of the relative errors in m_0/c , c^2 , and $[(A+1)^{1/2} - 1]$, the relative error to be expected in V is

$$e_4 = \frac{\epsilon_u}{e/m_0} + \frac{2\epsilon_n}{c} + \frac{A}{(A+1) - (A+1)^{1/2}} \left(e_3 + \frac{\epsilon_u}{e/m_0} + \frac{\epsilon_n}{c} \right). \quad (8)$$

The values of ϵ_u and ϵ_n are preferably the maximum errors judged to be possible in the accepted values of their parent magnitudes; if they are not, but are the probable errors quoted with the accepted values, e_4 is of the nature of an hybrid which is only a lower bound of the maximum relative error to be expected in V .

It should be remembered that expressions (3), (4), (5), and (8) do not take into account any systematic errors which might have been present and undetected in the determinations of the quantities appearing in expressions (1), (2), and (7).

⁶For if a is a quantity with a small known absolute error Δa , then
$$(a + \Delta a)^{1/2} = a^{1/2} + \Delta a / 2a^{1/2} + \dots \cong a^{1/2} + \Delta a / 2a^{1/2}.$$

Thus the relative error in $a^{1/2}$ is

$$\frac{1}{a^{1/2}} \cdot \frac{\Delta a}{2a^{1/2}} = \frac{1}{2} \left(\frac{\Delta a}{a} \right),$$

or is one half the relative error in a .

MEASUREMENTS

The experiment consisted in measuring the quantities appearing in expressions (1) and (2) so as to determine respectively H and the ρ 's for the three lines. H_0 was determined from forty determinations of m and each ρ was determined seven times in order to reduce the maximum relative error to be expected in $H\rho$ as far as is reasonably possible. The accepted $H\rho$ for an energy is the product of the H_0 value (from the forty determinations of m) as corrected for the proper path by the Hartree correction, into the appropriate radius of curvature of the path. The correction quantities were in general determined first. The error quantities ϵ_i were estimated as each quantity was measured. All measurements were repeated at least once, usually more than once. Data are usually recorded with more figures than are strictly significant, in order to prevent the entrance of errors in computation.

The functions $h(x)$ and $h(\theta)$ were determined from changes in H revealed by the 1,000 turn coil and the sensitive galvanometer. Table I is the data obtained for $\int h(x)dx$ at $H_0 = 1250$ gauss. Inte-

TABLE I. Data for $\int h(x)dx$

x (cm.)	$h(x)$ (gauss)
-1.5	-0.018
-1.0	-0.106
-0.5	-0.076
0.0	0.000
0.5	-0.113
1.0	-0.187
1.5	-0.263

gration was effected numerically by assuming that $h(x)$ is linear between adjacent experimentally determined points; the result is that

$$\int_{-1/2}^{1/2} h(x)dx = -0.319 \text{ gauss cm.}$$

It is estimated that $\epsilon_q = 0.01$. Table II is the data used in calculating $\int h(\theta) \sin \theta d\theta$ for $H_0 = 1250$ gauss; $h(\theta)$ are given in gauss. Integration is exactly as before and the results are tabulated in Table III; the Hartree correction as tabulated is the mean of the

TABLE II. Data for $\int h(\theta) \sin \theta d\theta$

θ	$\rho = 1.12$ cm.		$\rho = 1.34$ cm.		$\rho = 1.54$ cm.	
	$h_1(\theta)$	$h_2(\theta)$	$h_1(\theta)$	$h_2(\theta)$	$h_1(\theta)$	$h_2(\theta)$
0°	0.000	0.000	0.000	0.000	0.000	0.000
30°	-0.021	-0.219	-0.026	-0.246	-0.027	-0.263
60°	-0.084	-0.401	-0.107	-0.478	-0.115	-0.566
90°	-0.107	-0.468	-0.110	-0.547	-0.110	-0.667
120°	-0.108	-0.386	-0.211	-0.502	-0.234	-0.501
150°	-0.128	-0.297	-0.231	-0.300	-0.130	-0.314
180°	-0.108	-0.237	-0.152	-0.199	-0.189	-0.150

TABLE III. Hartree corrections

ρ (cm.)	$\frac{1}{2} \int h_1(\theta) \sin \theta d\theta$ (gauss)	$\frac{1}{2} \int h_2(\theta) \sin \theta d\theta$ (gauss)	Correction (gauss)
1.12	-0.098	-0.352	-0.225
1.34	-0.128	-0.418	-0.273
1.54	-0.122	-0.471	-0.296

two corrections for the two paths from 0. It is estimated from experience with these that $\epsilon_{10} = 0.01$.

The value of l was measured at the location bg (Fig.1), which is between $y = 8.0$ and $y = 9.0$. It was measured first with the comparator and standard scale and found to be 3.07918 cm. After the experiment was terminated, l was measured again, but with the 31 mm. disk and the Brown and Sharpe gauge. The value of 3.07891 cm. was obtained in this way. The finally accepted value of l is the mean of these two:

$$l = 3.07905 \text{ cm.}$$

These values of l are gotten ^{by} subtracting from the overall width of

the glass plate and silver strips at b_g , half the thicknesses of the silver strips at that location. To accompany 1 we assign $\epsilon_s = 0.00015$.

To evaluate $f'dy$ and A_1 , the total width of the glass plate and of the silver strips in place was measured at many locations along the plate with the 10^{-4} inch gauge; as above, from these widths were subtracted half the thicknesses at the same locations of the silver strips. The data is given in Table IV. As before it is assumed that $f(y)$ is linear between points determined experimentally. Since 1 was

TABLE IV. Width of current loop

y (cm.)	Width of strips and plate (cm.)	$\frac{1}{2}$ sum of strip thicknesses (cm.)	$2f(y)$ width of current loop (cm.)
0.0	3.09195	0.00944	3.08251
0.1	3.09107	0.00944	3.08163
0.2	3.09028	0.00944	3.08084
0.3	3.09009	0.00945	3.08064
0.4	3.08936	0.00945	3.07991
0.5	3.08914	0.00945	3.07969
1.0	3.08901	0.00945	3.07956
1.5	3.08870	0.00945	3.07925
2.0	3.08853	0.00947	3.07906
2.3	3.08835	0.00948	3.07887
2.4	3.08876	0.00949	3.07927
2.5	3.08893	0.00950	3.07943
2.8	3.08905	0.00950	3.07955
2.9	3.08862	0.00951	3.07911
3.0	3.08849	0.00952	3.07897
3.5	3.08867	0.00954	3.07913
3.7	3.08883	0.00954	3.07929
3.8	3.08898	0.00953	3.07945
4.0	3.08962	0.00953	3.08009
4.3	3.08966	0.00952	3.08014
4.5	3.08953	0.00953	3.08000
4.7	3.08929	0.00952	3.07977
4.9	3.08903	0.00952	3.07951
5.0	3.08886	0.00952	3.07934
5.5	3.08878	0.00952	3.07926
6.0	3.08877	0.00951	3.07926
6.5	3.08883	0.00949	3.07934
6.7	3.08903	0.00949	3.07954
7.0	3.08911	0.00950	3.07961
7.2	3.08893	0.00950	3.07941

TABLE IV. (Cont'd.)

7.3	3.08065	0.00951	3.07914
7.5	3.08856	0.00951	3.07905
8.0	3.08854	0.00949	3.07905
8.5	3.08853	0.00948	3.07905
9.0	3.08853	0.00947	3.07906
9.5	3.08837	0.00946	3.07891
10.0	3.08838	0.00944	3.07894
10.5	3.08844	0.00942	3.07902
11.0	3.08861	0.00955	3.07906
11.5	3.08861	0.00950	3.07911
12.0	3.08873	0.00949	3.07924
12.2	3.08869	0.00949	3.07920
12.4	3.08854	0.00948	3.07906
12.5	3.08867	0.00947	3.07920
12.6	3.08876	0.00946	3.07932
13.0	3.08870	0.00945	3.07927
13.2	3.08905	0.00944	3.07963
13.5	3.08861	0.00944	3.07917
14.0	3.08868	0.00945	3.07923
14.5	3.08861	0.00948	3.07913
15.0	3.08861	0.00945	3.07916
17.0	3.08869	0.00945	3.07924
17.5	3.08903	0.00943	3.07960
18.0	3.08854	0.00943	3.07911
20.0	3.08847	0.00942	3.07905
25.0	3.08872	0.00944	3.07928

TABLE V. Function $F(y)$

y (cm.)	$F(y)$
0.0	1.00
5.6	1.00
7.5	0.91
8.5	0.50
9.5	0.26
10.5	0.17
11.5	0.13
12.5	0.11
15.0	0.07
17.5	0.04
22.5	0.02

chosen as the width of the current loop between $y=0.0$ and $y=9.0$, Δl is obtained by subtracting the width of the current loop at $y=8.5$ from that at $y=0.0$; from Table IV we have

$$\Delta l = 3.08251 - 3.07905 = 0.00346 \text{ cm.}$$

The function $F(y)$ was obtained with the less sensitive flux-meter arrangement; its values are given in Table V. For purposes of integration $F(y)$ was also assumed to be linear between observed points. It is found that

$$2 \int Ff' dy = -0.00354 \text{ cm.}$$

The actual values of the corrections $H_0 \Delta l$ and $2H_0 \int Ff' dy$ are obtained directly from Tables IV and V; they are

$$H_0 \Delta l = 1250 \times 0.00346 = 4.33 \text{ gauss cm.,}$$

$$2H_0 \int Ff' dy = 1250 \times (-0.00354) = -4.43 \text{ gauss cm.}$$

It is estimated that $\epsilon_7 = \epsilon_8 = 0.05$.

Since the resistors used in getting I are calibrated to only 0.005 per cent and since the standard cells, to only 0.01 per cent, the maximum error to be expected in I is at least 0.015 per cent. The error inherent in adjusting for I with the resistance network is at most 10^{-7} em.u. By Ohm's law I is 0.509403 international amperes; the conversion⁷ to em.u. gives

$$I = 0.0509433 \text{ em.u.}$$

Thus $\epsilon_6 = 0.0000076 + 0.0000025 = 0.00001$; this second term is added because of the uncertainty in the factor of conversion from international to em. units.

Because three weights are required to produce the 400 mg. of m , ϵ_4 must contain 0.015 mg. Because of errors possible in locating the ends of the balance deflections when I is reversed, which come to $2 \times 0.02 \times 0.41 = 0.0164$ mg., ϵ_4 is larger than 0.015. Since m was

⁷The Smithsonian Physical Tables, eighth edition (1933), p.81, gives one international ampere equal to (0.99995 ± 0.00005) absolute amperes.

measured nine times in each of the forty determinations of it, we take⁸

$$\epsilon_4 = (0.015 + 0.0164/9^{1/2})/40^{1/2} = 0.0032 \text{ mg.}$$

The value of g in the locality of the experiment is $979.28 \text{ cm./sec.}^2$ and $\epsilon_5 = 0.005 \text{ cm./sec.}^2$

By observing the balance deflections when I was reversed and when the 400 mg. were placed on the balance, the correction to be applied to the 400 mg. to give m accurately could be found. These deflections were observed nine times for each determination of m . The forty mean observations of the balance deflection are recorded with their mean in Table VI. The evaluation of H_0 by taking for m the mean of a large number of observations which extended over several days is valid because Scott³ showed in his experiment that over a period of weeks the value of H_0 remained sensibly constant.

TABLE VI. Observations of balance deflections for m

Trial	Defl. (cm.)	Trial	Defl. (cm.)	Trial	Defl. (cm.)	Trial	Defl. (cm.)
1	1.53	11	1.52	21	1.52	31	1.52
2	1.52	12	1.54	22	1.50	32	1.52
3	1.52	13	1.53	23	1.52	33	1.52
4	1.53	14	1.54	24	1.51	34	1.52
5	1.52	15	1.54	25	1.51	35	1.51
6	1.53	16	1.50	26	1.51	36	1.52
7	1.51	17	1.51	27	1.52	37	1.54
8	1.52	18	1.51	28	1.53	38	1.51
9	1.53	19	1.52	29	1.52	39	1.52
10	1.54	20	1.52	30	1.52	40	1.50

Mean = 1.520

The value of m to be used is thus $0.400000 + 1.52 \times 0.000409 = 0.400622$

⁸Analogously to the customary procedure used in the operation of classical error computations, we assume that the relative error of the mean of n determinations of a quantity varies inversely as $n^{1/2}$. See, e.g., Bartlett, The Method of Least Squares, third edition (1915), p.44, equation (46).

gm. By use of equation (1) and the previously evaluated correction quantities it is seen that, except for the Hartree corrections,

$$H_0 = \frac{1}{3.07905} \left(\frac{0.400622 \times 279.28}{2 \times 0.0509433} + 4.43 - 4.33 + 0.32 \right) = 1250.70 \text{ gauss.}$$

Values of d were read from the photographic plates with the Gaertner comparator corrected by the standard scale; they could be repeated to within 0.0001 cm. usually. Since each was done seven separate times, $\epsilon_1 = 0.0001/7^{\frac{1}{2}} = 0.000038$. The value of D was found with a direct-reading 10^{-4} -inch Brown and Sharpe micrometer caliper and is 0.02405 cm. to within $\epsilon_2 = 0.0001$ cm. In Table VII are presented the seven values of each ρ gotten by equation (2) and their three means. Since H_0 is constant, these should be constant within

TABLE VII. ρ

Trial	ρ (cm.)	Mean ρ (cm.)
1	1.12442	1.12440
2	1.12446	
3	1.12442	
4	1.12440	
5	1.12442	
6	1.12440	
7	1.12426	
1	1.33642	1.33644
2	1.33642	
3	1.33650	
4	1.33645	
5	1.33657	
6	1.33635	
7	1.33642	
1	1.54471	1.54472
2	1.54476	
3	1.54468	
4	1.54462	
5	1.54478	
6	1.54470	
7	1.54471	

the proper groups in the Table.

Knowledge of the H_0 from Table VI, of the mean ρ 's of Table VII, of the Hartree corrections of Table III, of the various correction terms, and of the estimates of the possible errors is sufficient to provide the desired values of $H\rho$ and of the possible errors present.

RESULTS

Tables VIII, IX, X, and XI contain the results of the experiment. The accepted H_p 's of Table VIII are from the value of H_0 from Table VI and corrected by Table III and from the ρ 's of Table VII.

TABLE VIII. H_p 's of three intense Ra B β -ray lines

Line	H_p (gauss cm.)
Least intense	1406.0
Of medium intensity	1671.1
Most intense	1931.5

In view of the good agreement (1 part in 6,400) of the present value of H_p for the most intense β -particles and that of Scott, it seems safe to consider the H_p 's listed in Table VIII as sufficiently adequately determined in absolute value (within the possible limits of relative error given below) to serve as rather reliable standards of β -particle H_p 's.

In Table IX are the possible relative errors necessary to the computations based on equations (3), (4), and (5) for the least H_p ;

TABLE IX. Maximum relative errors to be expected in least H_p

Error	Magnitude	Magnitude (parts per 10,000)
ϵ_3/l	0.00015/3.079	0.49
ϵ_4/m	0.00002/.4007	0.08
ϵ_5/g	0.005/979.3	0.05
ϵ_6/I	.00001/.05094	1.97
$2I(2\epsilon_7 + \epsilon_8 + \epsilon_9)/mg$	0.16/3850	0.41
$\epsilon_{10}/2H_0$	0.01/2500	0.04
$\epsilon_1/(d - D)$	.000038/4.496	0.08
$\epsilon_2/(d - D)$	.0001/4.496	0.22
Sum = ϵ_3		3.34

they are collected from the section on MEASUREMENTS. Thus the values of H_p in Table VIII cannot be considered as any more accurate than 1 part in 3,000, even though each measurement used in getting H_p is more accurate than 1 part in 3,000. Evidently some 60 per cent of the uncertainty is accounted for by the uncertainty in the calibrations of the standard resistors and standard cells, and in the uncertainty in the conversion factor for international and em. units of current. Within this limit of 1 part to 3,000 the present experiment yields the same value of H_p for the most intense Ra B β -particles as Scott's experiment, though it is seen that Scott's hopes for an accuracy of 1 part in 10,000 are rather too optimistic.

The energies corresponding to the H_p 's we have established are obtained from equation (7) using⁹

$$\begin{aligned} e/m_0 &= 1.7562 \times 10^7 \text{ em.u./gm.}, \\ c &= 2.99796 \times 10^{10} \text{ cm./sec.} \end{aligned}$$

with $\epsilon_{\infty} = 0.00027 \times 10^7$ and $\epsilon_{12} = 0.00004 \times 10^{10}$. They are in Table X.

TABLE X. Energies of three intense Ra B β -rays

H (gauss cm.)	V (electron-volts)
1406.0	1.512×10^5
1671.1	2.044×10^5
1931.5	2.610×10^5

In Table XI are the numbers necessary for the calculation of the lower bound e_4 of the relative error to be expected in the least V according to equation (8). They follow from the preceding two

⁹The value of e/m_0 is that given by Birge, Phys. Rev., 49, 204 (1936). c is taken from The Smithsonian Physical Tables, eighth edition (1933), p.74.

paragraphs. Equation (8) becomes by substitution of these numbers

TABLE XI. Quantities for calculation of e_4 for least V

Quantity	Magnitude	Magnitude (parts per 10,000)
$\epsilon_n / (e/m_0)$	.00027/1.7562	1.54
ϵ_n / c	0.00004/2.998	0.12
e_3	3.34/10,000	3.34
A	$\frac{1.406 \ 1.756}{2.998} = 0.679$	

$$e_4 = \frac{1.54}{10,000} + \frac{0.24}{10,000} + \frac{0.679}{0.383} \left(\frac{3.34}{10,000} + \frac{1.54}{10,000} + \frac{0.24}{10,000} \right) = \frac{1.1}{1,000}.$$

Thus the energies V cannot be considered as more accurate than 1 part in 1,000. In fact, since ϵ_n and ϵ_n are only classical probable errors, e_4 may be even larger than 1 part in 1,000, the increase depending only upon how much larger the maximum absolute errors possible in e/m_0 and c are than the corresponding probable errors.

It is also noteworthy that even though H_p is accurate to within 1 part in 3,000, the major part of the 1 part in 1,000 accuracy allowed for V is accounted for by possible errors in H_p .

Finally, it should be noted that the improvements of this experiment over that of Scott lie in the facts that: a new balance of higher sensitivity, a new set of weights for m , straight silver strips, the standard scale, and a new gauge graduated in 10^{-4} 's of an inch were available; twice as large a current, determined by a far simpler potentiometer, was used in the Cotton balance; it includes measurements for the three most intense Ra B lines rather than for only one; and it provides a conservative, explicit estimate of the

precision of which it is capable, 1 part in 3,000 in H_p , 1 in 1,000 in V.

We also wish in conclusion to record our appreciation both for the suggesting of this experiment and for the valuable aid given us during the experiment, by Dr. H.A.Wilson.