High Resolution OFDM Channel Estimation with Low Speed ADC using Compressive Sensing

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Abstract—Orthogonal frequency division multiplexing (OFDM) is a technique that will prevail in the next generation wireless communication. Channel estimation is one of the key challenges in an OFDM system. In this paper, we formulate OFDM channel estimation as a compressive sensing problem, which takes advantage of the sparsity of the channel impulse response and reduces the number of probing measurements, which in turn reduces the ADC speed needed for channel estimation. Specifically, we propose sending out pilots with random phases in order to “spread out” the sparse taps in the impulse response over the uniformly downsampled measurements at the low speed receiver ADC, so that the impulse response can still be recovered by sparse optimization. This contribution leads to high resolution channel estimation with low speed ADCs, distinguishing this paper from the existing attempts of OFDM channel estimation. We also propose a novel estimator that performs better than the commonly used ℓ_1 minimization. Specifically, it significantly reduces estimation error by combining ℓ_1 minimization with iterative support detection and limited-support least-squares. While letting the receiver ADC run at a speed as low as $1/16$ of the speed of the transmitter DAC, we simulated various numbers of multipaths and different measurement SNRs. The proposed system has channel estimation resolution as high as the system equipped with the high speed ADCs, and the proposed algorithm provides additional 6 dB gain for signal to noise ratio.

I. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) has been widely applied in wireless communication systems, because it transmits at a high rate, achieves high bandwidth efficiency, and is robust to multipath fading and delay [1]. OFDM applications can be found in digital television and audio broadcasting, wireless networking, and broadband internet access. Current OFDM based WLAN standards (such as IEEE802.11a/g) use variations of QAM schemes for sub-carrier modulations which require a coherent detection at the OFDM receiver and consequently requires an accurate (or near accurate) estimation of Channel State Information (CSI). The structure of OFDM signal makes it difficult to balance complexity and performance in channel estimation. The design principles for channel estimators are to reduce the computational complexity and bandwidth overhead while maintaining sufficient estimation accuracy.

Some channel estimation schemes proposed in literature are based on pilots, which form the reference signal used by both the transmitter and the receiver. This approach has two main challenges: (i) the design of pilots; and (ii) the design of an efficient estimation algorithm (i.e., the estimator).

There is a tradeoff between the spectrum efficiency and the channel estimation accuracy. Most of the existing pilot-assisted OFDM channel estimation schemes rely on the use of a large number of pilots to increase estimation accuracy; the spectral efficiency is therefore reduced. For example, there are approaches based on time-multiplexed pilot, frequency-multiplexed pilot, and scattered pilot [2], all achieving higher estimation accuracy at the price of using more pilots. There have been attempts to reduce the number of pilots, i.e. J. Byun et al. in [3]. However, the solutions generally require extra “test signal” for channel pre-estimation. By sending out “test signal”, they try to find out how many pilots are needed by firstly inserting a relatively small number of pilots and then, based on the results of the “test”, the number of pilots are decided. Therefore, there is no guaranteed overall reduction of pilots insertion.

As a sensing problem, OFDM channel estimation can benefit from the emerging technique of compressive sensing (CS), which acquires and reconstructs a signal from fewer samples than what is dictated by the Nyquist-Shannon sampling theorem, mainly by utilizing the signal’s sparse or compressible property. The field has exploded since the pioneering work by Donoho [4] and Candes, Romberg and Tao [5]. The main idea is to encode a sparse signal by taking its “incoherent” linear projections and recover the signal through algorithms such as ℓ_1 minimization. To maximize the benefits of CS for OFDM channel estimation, one shall skillfully perform the CS encoding and decoding steps, which are precisely the two focuses of this paper: the designs of pilots and estimator, respectively.

Contributions: CS has been applied to channel estimation in [13–16], which are reviewed in subsection III-E below. For OFDM channel estimation, there are papers [6–9], to which our work differs in various ways as follows. We skillfully design CS encoding and decoding strategies for OFDM channel estimation. Compared to existing work, we are able to obtain channel response in much higher resolutions and from much fewer pilots (thus taking much shorter time). This is achieved by designing pilots with uniform random phases and using a novel estimator. The pilot design preserves the information of high-resolution channel response during aggressive uniform down-sampling, which means that receiver ADC can run at a much lower speed. The estimator is tailored for OFDM channel response; in particular, instead of the generic ℓ_1 minimization, iterative support detection (ISD) [17]

and limited-support least-squares are adopted in order to take advantage of the characteristics of channel response. The resulting algorithm is very simple and performs better.

The rest of this paper is organized as follows: Section II reviews the general OFDM system model and sets up the channel estimation formulation. Section III relates channel estimation to CS and presents the proposed pilot design. In Section IV, the estimator based on iterative support detection and limited-support least-squares are introduced. Section V gives the simulation results. Finally, Section VI concludes this work.

II. OFDM SYSTEM MODEL

A baseband OFDM system is shown in Figure 1. In this system, the modulated signal in the frequency domain, represented by $\mathbf{X}(k), k \in [1, N]$, is inserted with pilot signal and guard band, and then an N -point IDFT transforms the signal into the time domain, denoted by $x(n), n \in [1, N]$, where a cyclic extension of time length T_G is added to avoid inter-symbol and inter-subcarrier interferences. The resulting time series data is converted by a digital-to-analog converter (DAC) with a clock speed of $1/T_S$ Hz into an analog signal for transmission. We assume that the channel response comprises P propagation paths, which can be modeled by a time-domain complex-baseband vector with P taps:

$$h(n) = \sum_{p=1}^P \alpha_p \delta(n - \tau_p T_S), \quad n = 1, \dots, N, \quad (1)$$

where α_p is a complex multipath component and τ_p is the multipath delay ($0 \leq \tau_p T_S \leq T_G$). Since T_G is less than the OFDM symbol durations, the nonzero channel response concentrates at the beginning, which translates to $h = [h_1, h_2, \dots, h_{\tilde{N}}, 0, \dots, 0]$, i.e., only the first $\tilde{N}$ components of h can possibly take nonzero values and $\tilde{N} < N$. Assuming that interferences are eliminated, what arrives at the receiver is the convolution of the transmitted signal and the channel response plus noise, denoted by $z(n)$ given by

$$z = x \otimes h + \xi, \quad (2)$$

where $\otimes$ denotes convolution and $\xi(n), n \in [1, N]$ denotes the sampled AWGN noise. Passing through the analog-to-digital converter (ADC), $z(n), n \in [1, N]$ is sampled as $y(m), m \in [1, M]$, and the cyclic prefix (CP) is removed. Traditional OFDM channel estimation schemes assume $M = N$. If $M < N$, then y is a downsample of z . An M -point DFT converts y to $\mathbf{Y}(k), k \in [1, M]$, where the guard band and pilot signal will be removed.

For pilot assisted OFDM channel estimation, we shall design the pilots $\mathbf{X}$ (and thus x) and recover h from the measurements $\mathbf{Y}$ (or, equivalently y).

III. COMPRESSIVE SENSING OFDM CHANNEL ESTIMATION

A. Motivations

CS, which will be reviewed in the next subsection, allows sparse signals to be recovered from very few measurements,

Fig. 1. Baseband OFDM System

which translates to slower sampling rates and shorter sensing times. Because the channel impulse response h is very sparse, we are motivated to apply CS to recover h by using a reduced number of pilots so that the estimation becomes much faster. Furthermore, in a sharp contrast to conventional OFDM channel estimation in which ADC and DAC run at the same sampling rate, we can obtain a higher-resolution h by increasing the sampling rate of only the transmitter DAC, or we can reduce the receiver ADC speed which often defines the system cost. In other words, we have $N > M$. The rest of this section reviews CS and introduces our proposed approach for OFDM channel estimation.

B. CS Background

CS theories [4], [10], [11] state that a S -sparse signal¹ h can be stably recovered from linear measurements $y = \Phi h + \xi$, where Φ is a certain matrix with M rows and N columns, $M < N$, by minimizing the ℓ_1 -norm of h . Classical CS often assumes that Φ , after scaling, satisfies the restricted isometry property (RIP) $(1 - \delta)\|h\|_2^2 \leq \|\Phi h\|_2^2 \leq (1 + \delta)\|h\|_2^2$ for all S -sparse h , where $\delta > 0$ is the RIP parameter. The RIP is satisfied with high probability by a large class of random matrices, e.g., those with entries independently sampled from a subgaussian distribution. By minimizing the ℓ_1 -norm, one can stably recover h as long as $M \geq O(S \log N)$.

The classical random sensing matrices are not admissible in OFDM channel estimation because the channel response h is *not* directly multiplied by a random matrix; instead, as describe in Section II, h is first convolved with x , the noise is added, and then the received signal z is uniformly down-sampled to y . Because convolution is a circulant linear operator, we can present this process by $y = \downarrow_{\Omega} z = \downarrow_{\Omega} (Ch + \xi)$, where the sensing matrix C is the full circulant (convolution) matrix determined by x , and $\downarrow_{\Omega}$ denotes the uniform down sampling at points in $\Omega = \{1, 1 + N/M, \dots, N - N/M + 1\}$. As is widely perceived, CS favors fully random matrices, which admit stable recovery from fewest measurements (in terms of order of magnitude), but C is structured and thus much less “random”. This factor seemingly suggests that C would be not favored by CS. Nevertheless, carefully designed circulant matrices can deliver the same optimal CS performance.

¹In our case, S is equal to P , the number of non-zero taps in (1).

C. Pilot with Random Phases

To design the sensing matrix C , we propose to generate pilots $\mathbf{X}$ in either one of the following two ways: (i) the real and imaginary parts of $\mathbf{X}(k)$ are sampled independently from a Gaussian distribution, $k = 1, \dots, N$; (ii) (same as [15]) $\mathbf{X}(k)$, $k = 1, \dots, N$, has independent random phases but a uniform amplitude. Note that $\mathbf{X}(k)$ of type (i) also have independent random phases. Let F denote the discrete Fourier transform. Following from the convolution theorem $x \otimes h = F^{-1}(F(x) \cdot F(h))$ and $x = F^{-1}(\mathbf{X})$, we have $x \otimes h = F^{-1} \text{diag}(\mathbf{X}) Fh$, so the measurements y can be written as

$$y = \downarrow_{\Omega} (Ch + \xi) = \downarrow_{\Omega} (F^{-1} \text{diag}(\mathbf{X}) Fh + \xi). \quad (3)$$

Let us explain intuitively why (3) is an effective encoding scheme for a sparse vector h . First, it is commonly known that Fh is non-sparse and its mass is somewhat evenly spread over all its components. The random phases of $\mathbf{X}$ by design are of critical importance. They “scramble” Fh component wisely and break the delicate relationships among Fh ’s components; as a result, in contrast to the sparse $F^{-1}Fh = h$, $F^{-1} \text{diag}(\mathbf{X}) Fh$ is not sparse at all. Furthermore, $\mathbf{X}$ has a random-spreading effect. Due to a phenomenon called concentration of measure [12], the mass of Ch spreads over its components in a way that, with high probability, the information of h is preserved by down sampling of a size essentially linear in P — the sparsity of h (whether or not the down-sampling is equally spaced, i.e., uniform). Up to a log factor, the down sampled measurements permit stable ℓ_1 recovery. Both types (i) and (ii) of $\mathbf{X}$ have similar encoding strength, but $\mathbf{X}$ of type (ii) gives an orthogonal C , i.e., $C^*C = I$, so $x \otimes h$ transforms h into a random orthobasis. Such orthogonality results in multiple benefits such as faster convergence of our recovery algorithm. Due to the page limitation, we omit rigorous mathematical analysis of (3) and its guaranteed recovery.

Note that the proposed sampling $\downarrow_{\Omega} (F^{-1} \text{diag}(\mathbf{X}) F)$ is very different from partial Fourier sampling $\downarrow_{\Omega} F$. The latter requires a random Ω to avoid the aliasing artifacts in the recovery but the former, with randomly-phased $\mathbf{X}$, permits both random and uniform Ω . Below we numerically demonstrate its encoding efficiency.

D. Numerical Evidence of Effective Random Convolution

CS performance is evaluated by the number of measurements required for stable recovery. To compare the proposed sensing schemes with the well-established Gaussian random sensing, we conduct numerical simulations and show its results in Figure 2. We compare three types of CS encoding matrices: the i.i.d. Gaussian random complex matrix, and the circulant random complex matrices corresponding to $\mathbf{X}$ of types (i) and (ii) above, respectively. In addition, ℓ_1 minimization is compared to our proposed algorithm CS-OFDM, which is detailed in the next section. The simulations results show that the random convolutions of both types perform as well as the Gaussian random sensing matrix under ℓ_1 minimization, and

Fig. 2. MSE Vs. No. of Multipath for Different Cases (SNR=30dB).

our algorithm CS-OFDM further improves the performance by half of a magnitude.

E. Relationship to Existing Results

1) *Random Convolution CS*: Random convolution has been used and proved to be an effective way of taking CS measurements that allow the signal to be recovered using ℓ_1 minimization. In [13], Toeplitz² measurement matrices are constructed with i.i.d random row 1 (the same as type (i)) but with only ± 1 or $\{-1, 0, 1\}$; their down sampling effectively takes the first M rows; and the number of measurements needed for stable ℓ_1 recovery is shown as $M \geq O(S^3 \cdot \log N/S)$. [14] uses a “partial” Toeplitz matrix, with i.i.d. Bernoulli or Gaussian row 1, for sparse channel estimation where the down sampling effectively also takes the first M rows. Their scheme requires $M \geq O(S^2 \cdot \log N)$ for stable ℓ_1 recovery. In [15], random convolution of type (ii) above with either random downsampling or random demodulation is proposed and studied. It is shown that the resulting measurement matrix is incoherent with any given sparse basis with high probability and ℓ_1 recovery is stable given $M \geq O(S \cdot \log N + \log^3 N)$. Our proposed type (ii) is motivated by [15]. On the other hand, no existing work proposes uniform down-sampling or shows its recovery guarantees. In addition, most existing analysis is limited to real-valued matrices and signals.

2) *CS-based Channel Estimation*: Our work is closely related to [14] and [16]. In [14], i.i.d. Bernoulli or Gaussian vector is used as training sequence, and downsample is carried out by taking only the first M rows, while channel estimation is obtained as a solution to the Dantzig selector. In [16], MIMO channels are estimated by activating all sources simultaneously. The receivers measure the cumulative response, which consists of random convolutions between multiple pairs of source signals and channel responses. Their goal is to reduce the channel estimation time. ℓ_1 minimization is used to recover channel response.

Our current work is limited to estimating a signal h -vector. While our work is based on similar random convolution techniques, we have proposed to use a pair of high-speed

²which is slightly more general than circulant.

source and low-speed receiver for the novel goal of high resolution channel estimation. Furthermore, we apply a novel algorithm for the channel response recovery based on iterative support detection and limited-support least-squares, which is described in details in Section IV below.

IV. NUMERICAL ALGORITHM

A. Problem Formulation

As a result of rapid decaying of wireless channels, P — the number of significant multipaths — is small, so the channel response h is a highly sparse signal. Recall that the non-zero components of h only appear in the first $\tilde{N}$ components. We shall recover a sparse high-resolution signal h with a constraint from the measurements y at a lower resolution of M . We define operation $|\cdot|$ as the amplitude of a complex number, $\|h\|_0$ as the total number of nonzero entries in $|h|$ and $\|h\|_1 = \sum_i |h_i|$. The corresponding model is

$$\min_{h \in \mathbb{R}^N} \|h\|_0, \text{ s.t. } \phi h = y \text{ and } h_i = 0 \ \forall i > \tilde{N}, \quad (4)$$

where ϕ denotes $\downarrow_{\Omega} C$ in (3), the submatrix of C formed by its rows corresponding to the down-sampling points in Ω . Generally speaking, problem (4) is NP-hard and is impossible to solve even for moderate N . A common alternative is its ℓ_1 relaxation model with the same constraints.

$$\min_{h \in \mathbb{R}^N} \|h\|_1, \text{ s.t. } \phi h = y \text{ and } h_i = 0 \ \forall i > \tilde{N}, \quad (5)$$

which is convex and has polynomial time algorithms. If y has no noise, both (4) and (5) can recover h exactly given enough measurements, but (5) requires more measurements than (4).

B. Algorithm

Instead of using a generic algorithm for (5), we design an algorithm to exploit the OFDM system features, including the special structure of h and noisy measurements y . At the same time, we maintain its simplicity to achieve low complexity and match with easy hardware implementation.

First of all, we can simply collaborate two constraints into one by letting the variables be $\tilde{h} = [h_1, h_2, \dots, h_{\tilde{N}}]$ and dropping the rest components of h . Let $\tilde{\phi}$ be the matrix formed by first $\tilde{N}$ columns of ϕ . Hence, the only constraint is $\tilde{\phi}\tilde{h} = y$, which reduces the size of our problem.

We also develop our algorithm CS-OFDM for the purpose of handling noisy measurements. The iterative support detection (ISD) scheme proposed in [17] has a very good performance for solving (5) even with noisy measurements. Our algorithm uses the ISD, as well as a final denoising step. In the main iterative loop, it estimates a support set I from the current reconstruction and reconstructs a new candidate solution by solving the minimization problem $\min\{\sum_{i \in I^c} |\tilde{h}_i| : \tilde{\phi}\tilde{h} = y\}$, and it iterates these two steps for a small number of iterations. The idea of iteratively updating the index set I helps catch missing spikes and erase fake spikes. This is an ℓ_1 -based method but outperforms ℓ_1 . Analysis and numerical performance of ISD can be found in [18]. Because the measurements have noise, reconstruction is never exact. Our algorithm uses a

Algorithm 1 CS-OFDM

Input: ϕ, y ;

Initialize:

$\tilde{\phi}$ as the first $\tilde{N}$ columns of ϕ .

$I^0 \leftarrow \emptyset$ and $w_i^0 = 1, \forall i \in \{1, 2, \dots, \tilde{N}\}$

while the stopping condition is not met, **do**

Subproblem:

$$\tilde{h} \leftarrow \arg \min_{i \notin I^j} \sum |\tilde{h}_i|, \text{ s.t. } \tilde{\phi}\tilde{h} = y. \quad (6)$$

Support detection: $I^{j+1} \leftarrow \{i : |\tilde{h}_i^j| \geq 2^{-j} \|\tilde{h}^j\|_{\infty}\}$, where $\|\tilde{h}^j\|_{\infty} = \max_i \{|\tilde{h}_i^j|\}$.

Weights update:

$w_i^{j+1} \leftarrow 0, \forall i \in I^{j+1}$; otherwise $w_i^{j+1} \leftarrow 1$.

$j \leftarrow j + 1$

end while

Final least-squares:

let $T = \{i : |\tilde{h}_i| > \text{threshold}\}$, then:

$$\tilde{h}_T \leftarrow \arg \min_{\tilde{h}} \|\tilde{\phi}_T \tilde{h} - y\|_2^2, \text{ and } \tilde{h}_{T^c} \leftarrow 0. \quad (7)$$

Return $\tilde{h}$

final denoising step, which solves least-squares over the final support T , to eliminate tiny spikes likely due to noise. Our pseudocode is listed in Algorithm 1.

In Algorithm 1, at each iteration j , (6) solves a weighted ℓ_1 problem, and the solution $\tilde{h}^j$ is used for support detection to generate a new I^{j+1} . After the main loop is done, a support T is estimated above a threshold, which is selected based on empirical experiences. If the support detection is executed successfully, T would be the set of all channel multipath delay. Finally, $\tilde{h}$ is constructed by solving a small least-squares problem, and $\tilde{h}_i, \forall i \notin T$ fall to zero.

C. Complexity Analysis

This algorithm is efficient since every step is simple and the total number of iterations needed is small. The subproblem is a standard weighted ℓ_1 minimization problem, which can be solved by various ℓ_1 solvers. Since ϕ is a convolution operator, we choose YALL1 [19] since (i) it allows us to customize the operators involving $\tilde{\phi}$ and its adjoint to take advantages of DFTs, making it easier to implement the algorithm on hardware, (ii) YALL1 is asymptotically geometrically convergent and efficient even when the measurements are noisy. With our customization, all YALL1 operations are either an DFT/IDFT or one dimensional vector operations, so the overall complexity is $O(N \log N)$. Moreover, for support detection, we run YALL1 with a more forgiving stopping tolerance and always restart it from the last step solution. Furthermore, YALL1 converges faster as the index I^j gets closer to the true support. The total number of YALL1 calls is also small since the detect support threshold decays exponentially and bounded below by a positive number. Numerical experience shows that the total number of YALL1 calls never exceeds P .

Fig. 3. Multipath Delay Profile.

The computational cost of the final least-squares step is negligible because the associated matrix $\hat{\phi}_T$ has its number of columns approximately equal to the number of spikes in h , which is far less than its rows. For example, if the system has P multipaths, the associated matrix for least-squares has size $M \times P$. Generally speaking, the complexity for this least-squares is $O(MP + P^3)$. Since P and M are much smaller than N , the complexity of the entire algorithm is dominated by that of YALL1, which is $O(N \log N)$.

V. NUMERICAL SIMULATIONS

In this section, we perform numerical simulations to illustrate the performance of the proposed CS-OFDM algorithm for high resolution OFDM channel estimation. Over varying channel profiles and SNR, we evaluate the the mean square errors (MSE) of channel estimation, as well as the performance of the multipath delay detection.

A. Simulation Settings

We consider an OFDM system with 1k-point IDFT ($N = 1024$) at the transmitter and 64-point DFT ($M = 64$) at the receiver, where we have a compression ratio of 16. The number of silent sub-carrier which acts as guard band is 256 among 1024 sub-carriers. The channel is estimated based on 768 pilot tones with uniformly random phases and a unit amplitude, with the Gaussian noise level ranging from 10 dB to 30 dB. We assume the usage of cyclic prefix and the impulse response of the channel is shorter than cyclic prefix which means there is no inter-symbol interference. For all simulations, we test the total numbers of multipath from 5 to 15. Moreover, we use only one OFDM symbol, i.e. use all non-silent subcarriers only once to carry pilot signals. All reported performances will substantially improve if more pilots are inserted.

B. MSE Performance

Figure 3 is a snapshot of one channel estimation simulation. It suggests that the proposed pilot arrangement and CS-OFDM successfully detect an OFDM channel with 7 multipaths when the signal to noise ratio is 30 dB. Our method not only exactly

Fig. 4. MSE Performance.

Fig. 5. Reconstructed SNR.

estimates the multipath delays but also correctly estimates the values of the corresponding multipath components.

Figure 4 depicts the MSE performance on OFDM channels with the number of multipaths ranging from 5 to 15 and noise level ranging from 10 dB to 30 dB. When there are only a moderate number of multipaths on the OFDM channel, e.g. 10 multipaths, even when SNR is 20 dB, MSE is as low as -17 dB.

Figure 5 shows the reconstructed SNR vs. the number of multipaths when the input SNR changes. We can see that CS-OFDM achieves a gain in SNR. For example, when the input SNR is 10 dB, we obtain a reconstructed SNR higher than 20 dB when there are 5 multipaths. As the number of multipaths increases, the SNR gain from the reconstructed signal to the input signal decreases. However, even when the number of multipaths is 10, we still have a 5 dB gain, e.g. reconstructed SNR is 15 dB when the input signal SNR is 10 dB. The similar SNR gain appears for input SNR= 20 dB and SNR= 30 dB cases. From the entire input SNR and the number of multipath range we have tested, there is an average gain of 6 dB from the input SNR to the recovered SNR.

Fig. 6. Probability of Support Detection.

Fig. 7. Probability of False Support Detection.

C. Multipath Delay Detection Performance

Figure 6 and Figure 7 depict the probability of correct detection (POD) of the multipath delay and the false alarm rate (FAR) while we change the SNR and the number of multipaths. When the SNR is above 10 dB, simulation shows almost 100% POD when the number of multipaths changing from 5 to 12. When there is a relatively large number of multipaths, e.g. 15, the probability of correct multipath delay detection is higher than 95% as $\text{SNR} \geq 10$ dB. Even when SNR is low, as long as the number of multipaths does not exceed 10, we still have a POD of greater than 95%. The FAR performance shows the similar results, as the SNR decreases and the number of multipaths increases, performance decreases. But, in a large range, e.g. $\text{SNR} \geq 10$ dB, the number of multipath ≤ 10 , we have almost zero FAR.

VI. CONCLUSIONS

Efficient OFDM channel estimation will drive OFDM to carry the future of wireless networking. An opportunity of high-efficiency OFDM channel estimation is given by the sparse nature of channel response. Riding on the recent development of sparse optimization and CS, we propose a design of probing pilots with random phases, which preserves the information of channel response during convolution and down-sampling, and a sparse recovery algorithm, which returns the channel response in high SNR. These benefits translate to the high resolution of channel estimation, the lower speed of the receiver ADC, as well as shorter probing times. In this paper, the presentation is limited to an idealized OFDM model, intuitive explanations, and simulated experiments. In the future, we will formalize the work with rigorous theorems and fuse it into more realistic OFDM frameworks. The results presented here hint a noticeable improvement of OFDM performance in practice.

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