2D and 2.5D Kirchhoff inversion using upwind finite difference amplitudes

Kidane Araya* and William W. Symes, *The Rice Inversion Project*
Dept. of Computational and Applied Mathematics, Rice University

Key words: high frequency asymptotics, inversion, finite difference eikonal and transport solvers

Summary

Finite difference solution of the transport equation provides an efficient and accurate method for computation of 2D and 2.5D geometric acoustics amplitudes. These amplitudes can be used in simulation, migration and inversion formulas. Remodeled data based on high frequency asymptotic inversion using these amplitudes shows excellent agreement with both synthetic and field input data.

Introduction

High order essentially nonoscillatory (ENO) finite difference methods give very accurate traveltimes without tracing rays, Osher and Sethian (1988). The objective of this paper is to show that both 2D and 2.5D amplitudes may be computed using similar finite difference methods. The transport equation for the 2.5D wave propagation splits into a 2D transport equation and another advection equation (with no approximation). The solution of the 2D transport equation gives the in-plane component whereas the solution of the advection equation is the out-of-plane component of the amplitude. All advection equations can be approximated efficiently by finite difference methods. We tested this method on synthetic and field data examples. Elsewhere we have shown that the simulations resulting from employment of finite difference 2D amplitudes in a volumetric formulation of high frequency asymptotic linearized modeling give remarkably accurate approximations to full waveform finite difference simulations, Symes et. al., (1994). In this report we show that in both 2D and 2.5D the corresponding inversion formulae are also quite accurate, in the sense that resimulations (forward modeling using the inversion output as input) of the data come quite close to fitting it.

Methodology

In high frequency inversion methods, for constant density acoustics, the amplitude of the wavefield is computed by solving the transport equation,

$$2\nabla\tau \cdot \nabla a + a\nabla^2\tau = 0, \quad (1)$$

where τ and a are the traveltime and amplitudes of the wave field, respectively. For 2D amplitude spreading, the transport equation can be written as

$$\nabla\tau \cdot \nabla \log a_i = -\frac{(\tau_{xx} + \tau_{zz})}{2} \quad (2)$$

where a_i is to indicate that we have only in-plane spreading. For the 2.5D amplitude spreading, the corresponding transport equation is given by

$$\nabla\tau \cdot \nabla \log a = -\frac{(\tau_{xx} + \tau_{yy} + \tau_{zz})}{2}. \quad (3)$$

In a 2D or in-plane 2.5D data, the y component of $\nabla\tau$ is always equal to zero. Hence the difference between 2D and 2.5D amplitude computation is that, in the 2.5D case, the out-of-plane component of the Laplacian of the traveltime, (τ_{yy}) , is different from zero. Hence rewriting equation (3), we get two pairs of equations given by (2) and

$$\nabla\tau \cdot \nabla \log a_o = -\frac{\tau_{yy}}{2}. \quad (4)$$

In equation (4) a_o satisfies $a_o = a/a_i$. To compute a_o in equation (4), we need to know τ_{yy} . To get τ_{yy} , start with the eikonal equation

$$\left(\frac{\partial\tau}{\partial x}\right)^2 + \left(\frac{\partial\tau}{\partial y}\right)^2 + \left(\frac{\partial\tau}{\partial z}\right)^2 = \mathbf{p}^2, \quad (5)$$

where $\mathbf{p}$ is the slowness. Differentiating equation (5) twice with respect to y gives (after some algebra) to

$$\nabla\tau \cdot \nabla(1/\tau_{yy}) = 1, \quad (6)$$

which is an advection equation for τ_{yy} . For the 2.5D, $1/\tau_{yy} \equiv \sigma$, where σ is the running parameter along the ray for which

$$\frac{d\mathbf{x}}{d\sigma} = \nabla\tau(\mathbf{x}), \quad (7)$$

Lewis and Keller (1964). Hence equation (6) may be written as

Finite Difference 2.5D Kirchhoff Inversion

$$\nabla \tau \cdot \nabla \sigma = 1. \quad (8)$$

Solving equation (8) for τ_{yy} and using this in equation (4), leads to the out-of-plane component of the amplitude.

Asymptotic inversion formula, Beylkin (1985) involve the rate of change of ray take-off angle with respect to source and receiver positions. These quantities are constant along rays, hence obey yet other advection equations, e.g.

$$\nabla \tau \cdot \nabla (\partial \tau_s / \partial x_s) = 0. \quad (9)$$

Here τ_s is the traveltime of the ray from the source and/or the receiver at x_s to the output point in the subsurface.

Numerical analysis

Discretize the traveltime field on a uniform grid with steps $\Delta x, \Delta z$:

$$\tau_j^k \equiv \tau(k\Delta z, j\Delta x), \quad (10)$$

$k = K_-, \dots, K_+$ and $j = J_-, \dots, J_+$. Define the standard first difference operators

$$(D_x^\pm \tau)_j^k = \pm \frac{\tau_{j\pm 1}^k - \tau_j^k}{\Delta x}. \quad (11)$$

The ENO second order correction is defined by adding a multiple of the smallest second order difference oriented in the same direction:

$$(D_x^{\pm 2} \tau)_j^k = (D_x^\pm \tau)_j^k + \frac{\Delta x}{2} m[(D_x^\pm D_x^\pm \tau)_j^k, (D_x^\pm D_x^\mp \tau)_j^k], \quad (12)$$

where

$$m(x, y) = \begin{cases} x & \text{if } |x| \leq |y|, \quad xy > 0 \\ y & \text{if } |y| \leq |x|, \quad xy > 0 \\ 0 & \text{if } xy < 0. \end{cases} \quad (13)$$

Note that no second order correction is made if the centered and one-sided second differences differ in sign, as then higher-order corrections are presumably more important.

The depth marching scheme for the eikonal equation also gives an approximate z -derivative:

$$\begin{aligned} \left(\frac{\partial \tau}{\partial z} \right)_j^k &\cong \frac{\tau_j^{k+1} - \tau_j^k}{\Delta z} \\ &= \max\{(p_j^k)^2 \cos^2 \theta_{max}, (p_j^k)^2 \\ &\quad - [\max(D_x^{2,-} \tau_j^k, 0)]^2 \\ &\quad - [\min(D_x^{2,-} \tau_j^k, 0)]^2\}^{1/2} \end{aligned} \quad (14)$$

The choice of forward or backward difference inside the square root on the right hand side is according to the sign of the x -difference of τ , i.e., upwind (backwards along the rays). This choice is essential to maintain linear and nonlinear stability of the scheme. See Osher and Sethian (1988) for more details. The $p^2 \cos^2 \theta_{max}$ term under the square root limits the angle of rays computed accurately by this scheme, and enables us to treat the eikonal (and later the transport) problems as evolution problems in z (e.g. by preventing the argument of the square root from becoming negative). Travel-times and other quantities are computed accurately at points joined to the source by (first arrival) rays making at all times angles $< \theta_{max}$ with the vertical. This limitation is reasonable when most energy travels nearly vertically. Since the scheme just explained is explicit, it is conditionally stable, and a limit on $\Delta z / \Delta x$ must be enforced (CFL condition). Since the limit on Δz is unknown a priori and likely to be smaller than the grid Δz , we ensure stability through a substepping scheme, (see Symes, et. al., 1994 for details).

To make a similar two level depth marching scheme for the transport equation, it is useful to rewrite the traveltime Laplacian in a form involving only first z -derivatives. To do so we differentiate the 2D eikonal equation with respect to x and z and obtain (after some algebra)

$$\begin{aligned} \nabla^2 \tau &= p^2 \left(\frac{\partial \tau}{\partial z} \right)^{-2} \frac{\partial^2 \tau}{\partial x^2} + \frac{1}{2} \left(\frac{\partial \tau}{\partial z} \right)^{-1} \\ &\quad \cdot \left[\frac{\partial(p^2)}{\partial z} - \left(\frac{\partial \tau}{\partial z} \right)^{-1} \frac{\partial \tau}{\partial x} \frac{\partial(p^2)}{\partial x} \right]. \end{aligned} \quad (15)$$

Use of second order upwind x -differences approximations to the first x -derivative, the centered difference approximation to the second x -derivative, and the eikonal scheme just explained for $\partial \tau / \partial z$ gives a discrete approximation $(\nabla^2 \tau)_j^k$ to the above expression (equation (15)).

An upwind choice of derivative of $\log a_i$ guarantees that discretization error from discontinuities in $\partial \tau / \partial x$ (if such occur) does not pollute the solution, as such discontinuities only occur downwind of neighboring points. Also upwind differences for $\log a_i$ simplify the computation at the boundary, just as in the eikonal scheme, Symes et. al., (1994). Equation (2), (4), (8) and (9) all take the form

$$\begin{cases} u_z + a u_x = f \\ u = u_0, \end{cases} \quad (16)$$

Finite Difference 2.5D Kirchhoff Inversion

where $a = (\partial\tau/\partial x)/(\partial\tau/\partial z)$ (note that the ray angle limitation ensures that $\partial\tau/\partial z > 0$). We solve these using a second order upwind scheme of Sei and Symes (1995),

$$\begin{cases} \frac{u_i^{k+1} - u_i^k}{\Delta z} + (a^+)_j^k D_x^{2,-} u_j^k + (a^-)_j^k D_x^{2,+} u_j^k = f_j^k \\ u_j^k = u_{0,j}. \end{cases} \quad (17)$$

Here $a^+ = \max(a, 0)$ and $a^- = \min(a, 0)$. This scheme works if there is “outflow” at the boundaries, which translates here into: $(a^+)_j^k = 0$ and $(a^-)_j^k = 0$. This scheme is also explicit, hence conditionally stable at best. We have found by trial and error that the stability limit (CFL condition) for equation (17), is some what more restrictive than that for eikonal scheme. A substepping procedure ensures that the scheme remains stable while outputting the advected quantities on an apriori specified grid.

Numerical Experiments

Elsewhere Symes et. al., (1994) we have given direct evidence for the accuracy of the 2D amplitudes produced by the procedure just outlined, by comparison of high frequency asymptotic modeling results using these amplitudes with full wave finite difference simulation. The reader can also consult this reference for the precise continuum and discretized high frequency asymptotic (Kirchhoff, Ray-Born) modeling and migration operators used in our work; these formulae are quite conventional. In this section we illustrate the use of 2D and 2.5D amplitudes in high-frequency asymptotic inversion operators. Since our modeling operators accept a volumetric (gridded) reflectivity input, (as opposed to an interfacial description, say), as is output by the migration or inversion operators, we can judge the success of the inversion operator in a very simple way: we resimulate the data, i.e., model using the inversion output as input, and compare with the data. Example 1: This example is typical of many small synthetic tests. The velocity model is smooth and layered except for a localized low-velocity anomaly embedded near surface. The target reflectivity is layered (depends only on depth). Maximum receiver offset is 1728 m, receiver spacing is 24m. The source is isotropic point with 15Hz center frequency Ricker time dependence. Resimulation based on inversion of single gather shows excellent agreement with the input data, Figure (1), even in the details

of individual traces Figures (2)-(4). Example 2: Figure (5) shows a single shot gather extracted from a seismic line from the North Sea, donated by Mobil Oil Company. This data has been preprocessed using ProMAXTM to remove the receiver ghost and source signature and compensate for intrinsic attenuation. The velocities in the inversion and resimulation of the data were obtained from a well log near the seismic line. In Figure (5), we show the input and resimulated data using 2.5D amplitudes. All of the major and many minor features agree.

Figure 1. Input data (left) and resimulated (right) data

Figure 2. Input and resimulated near offset trace

Finite Difference 2.5D Kirchhoff Inversion

Conclusions

We have described efficient finite difference methods for computing 2D and 2.5D geometric acoustics amplitudes, compatible with high-order finite difference traveltimes solvers. When energy responsible for dominant events in the reflection seismogram travels within a subhorizontal aperture, these methods provide accurate components for effective asymptotic linearized modeling, migration and inversion operators.

Acknowledgement

This work was partially supported by the National Science Foundation, the Office of Naval Research, the Air Force Office of Scientific Research, the Schlumberger Foundation, and The Rice Inversion Project. TRIP Sponsors for 1996 are Advance Geophysical, Amoco Production Co., Conoco Inc., Cray Research Inc., Discovery Bay, Exxon Production Research Co., Interactive Network Technologies, and Mobil Research and Development Corp.

References

- Beylkin, G., 1982, Generalized radon transform and its applications: Ph.D. Thesis, New York University.
- , 1985, Imaging of discontinuities in the inverse scattering problem by inversion of causal Generalized Radon Transform: Jour. of Math. Phys., **26**, 99-108.
- Lambaré, G., Virieux, J., Madariaga, R., and Jin, S., 1992, Iterative asymptotic inversion in the acoustic approximation: Geophysics, **57**, 1138-1154.
- Lewis R.M., and Keller J.B., 1964, Asymptotic methods for partial differential equations: The reduced wave equation and Maxwell's equations: Res. Rep. EM-194, Div. of Electromagnetic Research, Courant Inst. Math. Sci., New York University.
- Osher S., and Sethian, J.A., 1988, Fronts propagating with curvature dependent speed: Algorithm based Hamilton-Jacobi formulations, J. Comp. Phys., **79**, pp 12-49.
- Sei, A., Symes, W. W., 1995, Convergent finite-difference traveltimes gradient for tomography: Extended Abstracts, 1995 International Meeting of the Society of Exploration Geophysicists, Houston, 1258-1261.
- Symes, W. W., Versteeg, R., Sei, A., and Tran, Q. H., 1994, Kirchhoff simulation migration and inversion using finite-difference traveltimes and amplitudes: Technical Report, The Rice Inversion Project (<http://www.caam.rice.edu/~trip>).
- Thierry, P., and Lambaré, G., 1995, 2.5-D true-amplitude migration on a workstation: Extended Abstracts, 1995 International Meeting of the Society of Exploration Geophysicists, Houston, 156-159.

Figure 3. Input and resimulated middle offset trace

Figure 4. Input and resimulated far offset trace

Figure 5. Field and resimulated data from the North Sea