On the Global Convergence of a Modified
Augmented Lagrangian Linesearch Interior Point
Newton Method for Nonlinear Programming

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Abstract

In this work we consider a linesearch globalization of the local primal-dual interior-point Newton method for nonlinear programming recently introduced by El-Bakry, Tapia, Tsuchiya and Zhang. Our linesearch uses a merit function that is a modification of the standard augmented Lagrangian function and a weak notion of centrality. We establish a global convergence theory and present rather promising numerical experimentation.

Key words: Interior-Point, Primal-Dual, Nonlinear Programming Problem, Newton's Method.

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of El-Bakry et al [6]. For the sake of illustration it suffices to restrict our discussion to linear programming; however we do feel that these comments are particularly appropriate for nonconvex programming.

It is well-known that the logarithmic barrier function formulation promotes excellent global behavior at the expense of (theoretically) badly conditioned subproblems locally. Moreover, it is not difficult to see that the first-order conditions for the logarithmic barrier function subproblem and the perturbed KKT conditions for the nonlinear program are equivalent. However, Proposition 2.3 of El-Bakry et al [6] demonstrates that the Newton iterates obtained from the equivalent subproblems (perturbed KKT conditions and the first-order conditions for the logarithmic barrier function subproblem) never coincide. This is indeed surprising since many authors tacitly assumed that they were the same. It follows that if we continue taking Newton steps (with an effective globalization strategy) on the perturbed KKT conditions with fixed perturbation parameter we will arrive at the solution of the logarithmic barrier subproblem. The critical issue here is that we will have obtained this solution via not necessarily badly conditioned linear systems (Newton defining relations). Hence we have a way of implementing the logarithmic barrier function method and circumventing the inherent bad conditioning. Far from a solution of the nonlinear program one may need to solve the logarithmic barrier function more accurately than would be needed near the solution. This can be accomplished by taking a variable number of Newton iterations on the perturbed KKT conditions with the perturbation parameter held fixed. Moreover, a linesearch globalization strategy may also be added to the Newton iteration procedure that attempts to solve the perturbed KKT conditions with fixed perturbation parameter. This philosophical approach is what was proposed in Gonzalez-Lima, Tapia, and Potra [13] in an effort to effectively calculate the analytic center of the solution set. Moreover, it is exactly this philosophy that we carry over into our global algorithm described in Section 5. A linesearch strategy with our merit function is used to reach a point which approximately satisfies a new notion of centrality that is introduced in Section 4. We call this procedure the inner loop, and it can be viewed as an effective way of approximating the solution of the logarithmic barrier function subproblem. We point out that the El-Bakry et al [6] globalization strategy did not provide for the feature of performing a variable number

The Karush-Kuhn-Tucker (KKT) first-order necessary conditions for this problem are

$$F(x, y, z) \equiv \begin{pmatrix} \nabla f(x) + \nabla h(x)y - z \\ h(x) \\ XZe \end{pmatrix} = 0, \quad (x, z) \geq 0 \quad (2.3)$$

where $X = \text{diag}(x)$, $Z = \text{diag}(z)$ and $e = (1, \dots, 1)^T \in \mathbb{R}^n$. For a feasible point x of problem (2.1), we let $\mathcal{B}(x) = \{j : x_j = 0\}$. Clearly $\mathcal{B}(x)$ is the set of indices of active or binding inequality constraints. The set of gradients of active constraints is the set $\{e_j \in \mathbb{R}^n : j \in \mathcal{B}(x)\}$.

2.2 Standard assumptions

In the study of Newton's method, the standard assumptions for problem (2.1) are :

A2.1: (Existence) There exists x^* a solution to problem (2.1).

A2.2: (Smoothness) The Hessian operators $\nabla^2 f$, $\nabla^2 h_i$, $i = 1, \dots, m$ are Lipschitz continuous in a neighborhood of x^* .

A2.3: (Regularity) The set $\{\nabla h_1(x^*), \dots, \nabla h_m(x^*)\} \cup \{e_j : j \in \mathcal{B}(x^*)\}$ is linearly independent.

A2.4: (Second-order sufficiency) For all $\eta \neq 0$ satisfying $\nabla h_i(x^*)^T \eta = 0$, $i = 1, \dots, m$; $e_j^T \eta = 0$, $j \in \mathcal{B}(x^*)$ we have

$$\eta^T \nabla_x^2 \ell(x^*, y^*, z^*) \eta > 0.$$

A2.5: (Strict complementarity) For all j , $x_j^* + z_j^* > 0$.

The following interesting relationship between conditions A2.4-A2.5 and the invertibility of the Jacobian matrix can be found in Section 4 of El-Bakry et al [6].

Proposition 2.1 *Let conditions A2.1 and A2.2 hold. Then the following statements are equivalent:*

1. *Conditions A2.3-A2.5 also hold.*

Hence this modification tends to keep the iterates away from the boundaries.

Therefore, the perturbed KKT conditions are

$$F_\mu(x, y, z) \equiv \begin{pmatrix} \nabla f(x) + \nabla h(x)y - z \\ h(x) \\ XZe - \mu e \end{pmatrix} = 0, \quad \mu > 0 \quad (2.5)$$

$$(x, z) > 0$$

where $X = \text{diag}(x)$, $Z = \text{diag}(z)$ and $e = (1, \dots, 1)^T \in \mathbb{R}^n$.

Since the perturbed KKT conditions are equivalent to the first-order KKT conditions for the logarithmic barrier function [6], the former can be used to promote the global convergence of the Newton interior-point method and need not employ ill-conditioned linear system of equations.

3 A new merit function for the general NLP

3.1 A generalized augmented Lagrangian function

One of the objectives of this research is to construct an appropriate merit function that couples the objective function with the constraint error in such a way that progress in the merit function effectively means progress in solving problem (2.1). Our strategy is to modify the augmented Lagrangian function associated with the equality constrained optimization problem by adding to its penalty term the potential reduction function utilized in some linear programming applications.

In line with the objective stated above, we present the following generalized augmented Lagrangian function.

Definition 3.1 *For any $\mu > 0$, we define our generalized augmented Lagrangian function by*

$$M_\mu : \mathbb{R}^{n+m+n} \rightarrow \mathbb{R}$$

$$M_\mu(x, y, z; \rho) = \ell(x, y, z) + \rho \Phi_\mu(x, z) \quad (3.1)$$

where $\ell(x, y, z)$ is the Lagrangian function associated with problem (2.1), i.e.,

$$\ell(x, y, z) = f(x) + h(x)^T y - x^T z,$$

Corollary 3.1 *Consider any $\mu > 0$. If $v_\mu^* = (x_\mu^*, y_\mu^*, z_\mu^*)$ satisfies the perturbed KKT conditions (2.5), then there exists a $\tilde{\rho} > 0$ such that*

$$x_\mu^* = \operatorname{argmin} M_\mu(x, y_\mu^*, z_\mu^*; \rho)$$

for all $\rho \geq \tilde{\rho}$.

4 The Newton direction as a descent direction

4.1 Fundamental definitions

In this section, we present the fundamental notions of interior and central points. In addition, we consider the primary variables associated with problem (2.1) and describe the manner in which we deal with these variables.

Definition 4.1 *A point (x, y, z) is said to be an interior-point for problem (2.1) if $(x, z) > 0$.*

Definition 4.2 *An interior-point (x, y, z) is said to be a quasi-central point for problem (2.1) for a given $\mu > 0$ if*

$$h(x) = 0 \text{ and } (XZ)e = \mu e. \quad (4.1)$$

The quasi-central path associated with problem (2.1) is defined as the collection of quasi-central points (4.1) and is parameterized by μ .

This notion will play an important role in the design of our global algorithm. We find it convenient to denote the triple (x, y, z) by v and $(\Delta x, \Delta y, \Delta z)$ by Δv . Recently Martinez, Parada, and Tapia [16] quite effectively demonstrated that in interior-point applications the variables (x, z) play a primary role and the variable y plays a secondary role. Observe that y does not enter into any of the constraints and at a solution $(x_\mu^*, y_\mu^*, z_\mu^*)$ one can readily obtain y_μ^* from x_μ^* and z_μ^* . This philosophical point of view is in strong alignment with the globalization strategy we are about to describe. We will treat the variable y essentially as a parameter, i.e., we will not differentiate our merit function with respect to y and we will exclude y from our descent considerations. This latter consideration will employ only the

Proposition 4.1 *For $\mu > 0$, the penalty term $\Phi_\mu(x, z)$ is bounded below by $n\mu(1 - \ln(\mu))$ in the class of all interior points. Moreover, it will be positive for $0 < \mu < \hat{e}$ (where $\hat{e}$ is the Euler constant).*

Proof. The proof follows directly from the observation that for $\mu > 0$ the function $g(w) = w - \mu \ln(w)$ has $w = \mu$ as its global minimizer. $\diamond$

Theorem 4.1 *Consider $\mu > 0$. Let $v = (x, y, z)$ be an interior-point. Then the Newton step $\Delta \tilde{v}$ at $\tilde{v} = (x, z)$ is a descent direction for the penalty term Φ_μ , i.e.,*

$$\nabla \Phi_\mu^T(\tilde{v}) \Delta \tilde{v} < 0,$$

if and only if v is not a quasi-central point.

Proof. The components of the gradient of Φ_μ with respect to x and z are

$$\nabla_x \Phi_\mu(\tilde{v}) = \nabla h(x)h(x) + z - \mu x^{-1}$$

and

$$\nabla_z \Phi_\mu(\tilde{v}) = x - \mu z^{-1}.$$

The directional derivative of $\Phi_\mu(\tilde{v})$ in the direction $\Delta \tilde{v}$ is

$$\nabla \Phi_\mu(\tilde{v})^T \Delta \tilde{v} = \nabla_x \Phi_\mu(\tilde{v})^T \Delta x + \nabla_z \Phi_\mu(\tilde{v})^T \Delta z.$$

If we set $w = (XZ)^{1/2}e$, then using the equality constraints and the complementary equation that comes from (4.3) we obtain

$$\nabla \Phi_\mu(\tilde{v})^T \Delta \tilde{v} = -(\|h(x)\|^2 + \|w - \mu w^{-1}\|^2) \leq 0. \quad (4.7)$$

This latter equation establishes the theorem. $\diamond$

Theorem 4.2 *Consider $\mu > 0$. Let $v = (x, y, x)$ be an interior-point. If v is not a quasi-central point, then there exists a real number $\hat{\rho}$ such that for any $\rho > \hat{\rho}$ the Newton step $\Delta \tilde{v}$ at $\tilde{v} = (x, z)$ is a descent direction for our generalized augmented Lagrangian function M_μ in the sense that*

$$\nabla_x M_\mu(x, y, z; \rho)^T \Delta x + \nabla_z M_\mu(x, y, z; \rho)^T \Delta z < 0.$$

perturbed KKT conditions (2.5) parameterized by μ . By the implicit function theorem we can guarantee that such a path exists locally (i.e. for $0 < \mu < \hat{\mu}$ for some $\hat{\mu}$) in a neighborhood of a solution $v = (x^*, y^*, z^*)$ of (2.1) which satisfies the standard Newton's method theory assumptions A2.1-A2.5 listed in Section 2. Related to the central path notion, we introduced a new notion of centrality called the quasi-central path given by Definition 4.2. It is worth mentioning that the quasi-central path is really a surface and as before under the latter assumptions, we can guarantee that this surface exists close to the solution of the problem. The use of the quasi-central path as opposed to the central path gives us a definite advantage. Specifically, far from the solution it may be the case that the central path point corresponding to a parameter μ does not exist. When we consider the quasi-central path we have relaxed the requirements, i.e., we do not require $\nabla_x l = 0$, and consequently the chance that a point on the quasi-central path corresponding to this μ exists are dramatically improved. Indeed, observe that a point x is on the quasi-central path, i.e.,

$$(x, z) \in S^* = \{(x, z) \in \mathbb{R}^{2n} : h(x) = 0, XZe = \mu e, z > 0\}$$

if and only if x is strictly feasible, i.e.,

$$x \in S = \{x \in \mathbb{R}^n : h(x) = 0, x > 0\}.$$

This notion plays a fundamental role in the formulation of our global algorithm. Now, we follow the lead given by Gonzalez-Lima, Tapia, and Potra [13] in their linear programming application. We are looking for a notion of an effective neighborhood of our quasi-central path corresponding to μ . Toward this end we offer the following notion of closeness to a point on the quasi-central path.

Definition 5.1 *We define a (μ, γ) -neighborhood of a point on the quasi-central path corresponding to μ by*

$$\mathcal{N}_\mu(\gamma) = \{v = (x, y, z) \in \mathbb{R}^{n+m+n} : x > 0, z > 0, \|h(x)\|^2 + \|w - \mu w^{-1}\|^2 \leq \gamma\mu\} \quad (5.1)$$

where $(\mu, \gamma) > 0$ and $w = (XZ)^{1/2}e$. We call the value $\gamma\mu$ the radius of this neighborhood.

The previous definition gives us a measure of how close an interior-point is to satisfying the perturbed KKT conditions for a corresponding $\mu > 0$. It is of value to observe that

5.3 Linesearch interior-point Newton algorithm

We propose the following global primal-dual interior-point Newton algorithm with a backtracking linesearch algorithm for the nonlinear optimization problem (2.1).

Algorithm 5.1 (Linesearch interior-point algorithm)

Step 0. Consider an initial interior-point $v_o = (x_o, y_o, z_o)$ (i.e. $(x_o, z_o) > 0$).

Choose $\beta, p, \gamma, \sigma \in (0, 1)$, and $\hat{c} > 0$.

Step 1. For $k=0, 1, 2, \dots$ until convergence do

1.1 Choose $\mu_k > 0$.

Step 2. Repeat (INNER LOOP)

2.1 Solve the linear system

$$F'(v_k)\Delta v_k = -F_{\mu_k}(v_k).$$

2.2 (Maintain x and z positive). Choose $\tau_k \in (0, 1)$ and calculate $\hat{\alpha}_k$ according to (4.6). Let $\tilde{\alpha}_k = \min(1, \tau_k \hat{\alpha}_k)$.

2.3 (Force a descent direction). Calculate c_k and ρ_k by (5.2) to ensure a Newton descent direction for M_μ .

2.4 (Armijo's condition of sufficient decrease). Find $\alpha_k = p^t \tilde{\sigma}_k$ where t is the smallest positive integer such that α_k satisfies

$$M_{\mu_k}(x_k + \alpha_k \Delta x_k, y_k, z_k + \alpha_k \Delta z_k; \rho_k) \leq M_{\mu_k}(v_k; \rho_k) + c_k \alpha_k \beta \nabla \Phi_{\mu_k}(\tilde{v}_k)^T \Delta \tilde{v}_k \quad (5.3)$$

2.5 Set $v_k = (x_k + \alpha_k \Delta x_k, y_k + \alpha_k \Delta y_k, z_k + \alpha_k \Delta z_k)$.

Step 3. (Proximity to the quasi-central path)

3.1 If $v_k \notin \mathcal{N}_{\mu_k}(\gamma)$ (see(5.1))

go to step 2

3.2 Else

go to step 1 (END OF INNER LOOP)

Remark 5.1 According to the material described in Equations 4.9-4.11, we know that our choice of ρ_k promotes descent in M_{μ_k} in the variables x and z . Moreover, Equation 4.11 explains the use of $\nabla \Phi_{\mu_k}$ in (5.3).

Lemma 6.1 Consider $\mu > 0$. Under Assumptions A.6.1 and A.6.2 the matrix $F'_\mu(x, y, z)$ is nonsingular for any interior-point (x, y, z) .

Proof. The matrix $F'_\mu(x, y, z)$ can be written

$$F'_\mu(x, y, z) = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

where

$$A = \begin{pmatrix} \nabla_x^2 \ell(x, y, z) & \nabla h(x) \\ \nabla h(x)^T & 0 \end{pmatrix}, \quad B = \begin{pmatrix} -I \\ 0 \end{pmatrix}, \quad C = (Z \ 0), \text{ and } D = X.$$

Since (x, y, z) is an interior-point, the diagonal matrix D is nonsingular. Therefore the matrix $F'_\mu(x, y, z)$ is nonsingular if $K = A - BD^{-1}C$ is a nonsingular matrix (see [9]). In our particular case K is given by

$$K = \begin{pmatrix} \nabla_x^2 \ell(x, y, z) + X^{-1}Z & \nabla h(x) \\ \nabla h(x)^T & 0 \end{pmatrix}.$$

From Assumptions A6.1 and A6.2, in a rather standard manner, it is possible to show that K is a nonsingular matrix. $\diamond$

6.2 Global convergence theory

In order to state our global convergence theory, we start by proving that for a $\mu > 0$ (fixed) any limit point of the sequence generated by the subproblem inner loop is a quasi-central point.

Theorem 6.1 (Subproblem inner loop). Consider $\mu > 0$ (fixed). Let $v_\mu^* = (x_\mu^*, y_\mu^*, z_\mu^*)$ be a limit point of the sequence $\{v_k = (x_k, y_k, z_k)\}$ generated by the inner loop of Algorithm 5.1, with the stopping criterion deactivated. Assume that F' is continuous at v_μ^* and $F'(v_\mu^*)$ is nonsingular, then $(x_\mu^*, y_\mu^*, z_\mu^*)$ is on the quasi-central path. i.e.,

$$h(x_\mu^*) = 0, \quad X_\mu^* Z_\mu^* e = \mu e,$$

and

$$x_\mu^* > 0, \quad z_\mu^* > 0.$$

First, the sequence $\{\hat{\rho}_k, k \in K\}$ can be unbounded. Then from Equation (4.9) and the fact that $\tilde{\nabla} \ell(v_k)^T \Delta \tilde{v}_k$ is bounded we have

$$\left| \nabla \Phi_\mu(\tilde{v}_k)^T \nabla \tilde{v}_k \right| \rightarrow 0, k \in K.$$

Therefore, Equation (4.7) directly implies that

$$\lim_{k \rightarrow \infty} h(x_k) = 0, k \in K$$

and

$$\lim_{k \rightarrow \infty} X_k z_k = \mu e, k \in K.$$

Our second possibility is that, the sequence $\{\hat{\rho}_k, k \in K\}$ is bounded above. Let $\rho^* = \sup\{\hat{\rho}_k, k \in K\}$. Since $\{\hat{\rho}_k; k \in K\}$ is bounded, then for every $c > 0$ there exists a $k_o \in K$ such that $\hat{\rho}_k \leq \hat{\rho}_{k_o} + c$ and $\rho^* \leq \hat{\rho}_{k_o} + c$ for $k \geq k_o, k \in K$. We can assume that $\rho_{k_o} = \hat{\rho}_{k_o} + c_{k_o}$, and by (5.2), it follows that $\rho_k = \rho_{k_o}$, for $k \geq k_o, k \in K$. Now, we can define the merit function as

$$M_\mu(\tilde{v}_k; \rho_{k_o}) = \ell(\tilde{v}_k) + \rho_{k_o} \Phi_\mu(\tilde{v}_k), k \geq k_o, k \in K$$

Since $M_\mu(\tilde{v}_k; y, \rho_{k_o})$ is a continuously differentiable function on $\mathbb{R}^{2n}$, bounded below, and we are considering an iterative scheme

$$\tilde{v}_{k+1} = \tilde{v}_k + \alpha_k \Delta \tilde{v}_k > 0$$

where $\nabla M_\mu(\tilde{v}_k; \rho_{k_o})^T \Delta \tilde{v}_k < 0$, $\alpha_k \in (0, \tilde{\alpha}_k]$ satisfies the sufficient decrease condition given by Substep 2.4 of the Algorithm 5.1 with $\tilde{\alpha}_k$ bounded away from zero, then on the sequence $\{\tilde{v}_k, k \geq k_o, k \in K\}$ we have, see [20],

$$\frac{\nabla M_\mu(\tilde{v}_k; \rho_{k_o})^T \Delta \tilde{v}_k}{\|\Delta \tilde{v}_k\|} \rightarrow 0.$$

Since $\{\Delta \tilde{v}_k\}$ is bounded and by (4.11), we obtain

$$\nabla M_\mu(\tilde{v}_k; \rho_{k_o})^T \Delta \tilde{v}_k = c_k \nabla \Phi_\mu(\tilde{v}_k)^T \Delta \tilde{v}_k \rightarrow 0$$

where $c_k = \rho_{k_o} - \hat{\rho}_k \geq \rho_{k_o} - \rho^* > 0$, $k \geq k_o, k \in K$. Since c_k becomes a constant greater than zero for $k \geq k_o$, we have

$$\nabla \Phi_\mu(\tilde{v}_k)^T \Delta \tilde{v}_k \rightarrow 0, k \geq k_o, k \in K.$$

Now, since $v_{kj} = (x_{kj}, y_{kj}, z_{kj}) \in \mathcal{N}_{\mu_k}(\gamma)$, then its associated $\tilde{v}_{kj} = (x_{kj}, z_{kj})$ satisfies the inequality

$$\|h(x_{kj})\|^2 + \|w_{kj} - \mu_k w_{kj}^{-1}\|^2 \leq \gamma \mu_k.$$

Again, taking the limit as $k \rightarrow \infty$, we obtain

$$h(x^*) = 0, \tag{6.2}$$

$$X^* Z^* e = 0, \tag{6.3}$$

and

$$(x^*, z^*) \geq 0. \tag{6.4}$$

From (6.1)- (6.4), $F(x, y, z) = 0$ and $(x, z) \geq 0$ are satisfied by $v^* = (x^*, y^*, z^*)$. Therefore, v^* is a KKT point of problem (2.1). $\diamond$

7 Numerical results

7.1 Implementation of the algorithm

The numerical experiments were done on a SPARC station 5 running the SunOS Operating-System Release 4.1 with 64 Megabytes of memory. The programs were written in MATLAB version 4.2a.

In the implementation of Algorithm 5.1 the parameters are chosen as follows. The initial perturbation parameter μ_o is $10^{-2} x_o^T z_o$. In Substep 1.1, we define μ_k by (5.5), with $\sigma = 10^{-2}$. In Substep 2.2, we choose the parameter τ_k (percentage of movement to the boundary) as

$$\tau_k = \max (.8, 1 - 100 * x_k^T z_k).$$

In Substep 2.3, the critical value for $\hat{c}$ in order to obtain a descent direction for the generalized augmented Lagrangian function is 2. Moreover, in Substep 2.4 we choose $\beta = 10^{-4}$ and set the backtracking factor $p = 0.5$. In Substep 3.1, we take $\gamma = .8$. We used a finite difference approximation to the Hessian of the Lagrangian function.

7.2 Numerical results

Our computations are directed at two main objectives. The first is to evaluate our generalized augmented Lagrangian function, $\mathbf{M}_\mu$, as a new merit function. The second objective

2. In problem 13, where strict complementarity does not hold at the solution, the approach ℓ_2 -NCP reported slow convergence (after 100 iterations the norm of the residual was $3.21 \cdot 10^{-2}$). Yamashita [28], states that his algorithm takes 197 iterations to solve this problem in the sense that he obtains a good approximation to the solutions of the primal variables, but the norm of the Karush-Kuhn-Tucker conditions is not small. Using our merit function M_μ with the *NCP* and the *CP* strategies, we obtain the solution of the problem. Moreover, with M_μ -CP strategy we obtain convergence in only 26 iterations.

7.3 Table notation

The abbreviations used in Tables 1-3 are collected in this section. The first three columns contain information about the problems:

n Number of variables

m Number of equality constraints

p Number of inequality constraints

The second set of three columns denotes the number of Newton iterations taken by the following algorithms:

ℓ_2 -NCP Algorithm using the ℓ_2 norm of the residual function as a merit function without our centrality strategy. This is the algorithm presented by El-Bakry et al [6].

M_μ -NCP Algorithm using the generalized augmented Lagrangian function as a merit function with the strategy given by El-Bakry et al [6].

M_μ -CP Algorithm using the generalized augmented Lagrangian merit function with the quasi-central path as a notion of centrality condition.

8 Summary and Concluding Remarks

In this work, we have presented a new interior-point Newton algorithm for solving nonlinear programming problems. The algorithm utilizes the perturbed KKT conditions to promote global convergence. In order to obtain a good strategy of globalization, we have presented a generalization of the augmented Lagrangian function to be used as a new merit function,

Numerical Results						
Problem	n	m	p	ℓ_2 -NCP	M_μ -NCP	M_μ -CP
1	2	0	1	70	33	23
2	2	0	1	9	6	7
3	2	0	1	6	7	2
4	2	0	2	6	8	3
5	2	0	4	7	11	6
10	2	0	1	10	10	9
11	2	0	1	9	15	7
12	2	0	1	10	11	9
13*	2	0	3	>100	76	26
14	2	1	1	7	5	5
15	2	0	3	15	23	8
16	2	0	5	19	25	17
17	2	0	5	34	32	14
18	2	0	6	18	27	35
19	2	0	6	15	24	8
20	2	0	5	13	38	14
21	2	0	5	13	14	10
22	2	0	2	7	4	3
23	2	0	9	21	>100	20
24	2	0	5	8	15	10
25	3	0	6	9	17	8
26	3	1	0	26	28	8

Numerical Results (cont.)						
Problem	n	m	p	ℓ_2 -NCP	M_μ -NCP	M_μ -CP
80	5	3	10	6	8	13
81	9	13	13	13	11	13
83	5	0	16	23	25	16
84	5	0	16	17	33	22
86	5	0	15	18	16	14
93	6	0	8	10	22	8
100	7	0	4	10	17	8
104	8	0	22	12	16	10
226	2	0	4	7	12	8
227	2	0	2	7	8	5
231	2	0	2	57	16	21
233	2	0	1	6	16	8
250	3	0	8	8	14	6
251	3	0	7	9	14	5
262	4	1	7	8	12	16
263	4	2	2	19	15	16
325	2	1	2	7	9	8
339	3	0	4	8	12	21
340	3	0	2	8	5	4
341	3	0	4	9	11	5
342	3	0	4	14	25	12
353	4	1	6	10	16	17
354	4	0	5	11	10	6

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