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**A Lumped Finite Element Method for
Cap Rock Heat Loss Calculations^{*}**

by

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Abstract

Numerical models of thermal recovery processes arising in petroleum engineering require accurate estimates of the interaction between a reservoir and the surrounding cap and base rock. It is important to obtain an accurate approximation of the energy flux at the interface between the reservoir and the surrounding rock, and the distribution of the temperature in the surrounding rock is of only secondary importance. A flux calculation procedure implemented as a post processing algorithm based upon a finite element method is used to calculate an accurate approximation to the energy losses.

Oscillatory behavior is observed in the computed temperature distribution using a standard computational scheme. If the time derivatives are lumped, the oscillatory behavior is avoided. Computational experiments performed on sample test cases possessing analytical solutions indicate an excellent agreement between the approximations obtained using the lumped procedure and the exact solution.

1. Introduction

Oil recovery by steam injection is an economically successful tertiary recovery mechanism. Accurate numerical simulation of candidate steam stimulation and drive projects is essential for proper optimization of process parameters. Numerical simulation of thermal recovery is a complex undertaking that requires concurrent models of various phenomena that are observed in the actual physical process. Examples of numerical models of thermal recovery by steam injection include Chu, Coats, George, and Marcum [1], Weinstein, Wheeler, and Woods [2], and Potempa [3].

An important phenomenon that must be treated in any model of thermal recovery is the energy flux from the reservoir to the surrounding cap and base rock by conductive heat flow. Many different methods are currently utilized for performing such energy loss calculations. Coats et. al. [1] use a finite difference superposition approach to calculate the energy flux from the reservoir to the overburden. Chase and O'Dell [4] introduce a two parameter variational approach for heat loss calculations. Weinstein [5] utilizes a semi-analytic approach for overburden temperature distribution calculations.

These approaches suffer from practical limitations of one form or another. The variational approach used by Chase and O'Dell [4] is unable to accurately treat flow reversal phenomena which are encountered in cyclic steam stimulation problems. The finite difference superposition approach used by Coats et. al. [1] does not perform enhanced energy flux calculations which are readily incorporated as a post processing of an approximate solution obtained using the finite element method [6]. It is difficult to decouple the overburden calculations in a backward Euler

fashion using the approach due to Weinstein [5]. Coats et. al. [1] implement a modified semi-analytic approach which allows an implicit decoupling of the reservoir and cap rock calculations. They report that the computational results are similar to the finite difference superposition results, but that the modified semi-analytic approach is more expensive than the finite difference superposition scheme, measured in terms of both storage and computer time. Another drawback of the procedure used by Weinstein [5] is that the flux is computed using a heuristic heat transfer coefficient. Computational experiments demonstrate that steam displacement calculations are extremely sensitive to the precise numerical value of the heat transfer coefficient.

The accuracy of some of these procedures degenerates for flow regimes exhibiting a low Peclet number. In particular, the methods due to Coats et. al. [1] and Chase and O'Dell [4] are inaccurate for combined reservoir and overburden systems exhibiting low Peclet numbers. The procedure due to Weinstein [5] can treat flow regimes at both low and high Peclet numbers..

A new procedure for calculating the energy losses to the cap rock is implemented in the steam displacement model due to Potempa [3]. This procedure uses a finite element method for calculating the overburden temperature distribution which is coupled with a procedure due to Wheeler [6] for obtaining accurate energy flux estimates. The algorithm of Wheeler obtains improved flux estimates by performing a straight forward post processing of the approximate temperature distribution obtained by the finite element method. Superconvergence properties of this algorithm are proved under certain assumptions by Douglas, Dupont, and Wheeler [7]. The superposition approach due to Coats et. al. [1] is used to decouple the

energy flux calculations from the reservoir material and energy balance calculations. Although the procedure outlined in this investigation is only accurate for flow regimes exhibiting high Peclet numbers, an extension is indicated for treating cases with a low Peclet number which typically arise in reservoir blowdown calculations.

Oscillatory behavior is observed in the computed temperature distribution using a standard computational scheme. The unphysical behavior is eliminated by lumping the time derivatives which arise in applying the finite element method to the parabolic differential equation governing the physical system. The lumped scheme is demonstrated to properly treat flow reversal.

2. Physical Problem

The areal extent of the reservoir is assumed to be a bounded domain Ω contained in R^2 . The cap rock is considered to be the semi-infinite three dimensional domain $\Omega_c = \Omega \times (0, \infty)$. Let k_c denote the thermal conductivity in the cap rock, ρ_c denote the cap rock density, and C_c denote the cap rock heat capacity. Let the temperature distribution T_c in the overburden Ω_c be governed by

$$\nabla \cdot k_c \nabla T_c = (\rho_c C_c T_c)_t \quad . . . (1)$$

Let T_r denote the reservoir temperature distribution, and k_r denote the reservoir thermal conductivity. Two compatibility conditions hold at the boundary separating the reservoir from the cap rock. The temperature distribution at the interface between the reservoir and the cap rock is continuous, i.e.,

$$T_c(x, y, 0^+, t) = T_r(x, y, 0^-, t) \quad . . . (2)$$

The flux at the interface is also continuous, i.e.,

$$k_c \frac{\partial T_c}{\partial z}(x, y, 0^+, t) = k_r \frac{\partial T_r}{\partial z}(x, y, 0^-, t) \quad . . . (3)$$

The energy flux vanishes at infinity, i.e.,

$$\lim_{z \rightarrow \infty} k_c \frac{\partial T_c}{\partial z}(x, y, z, t) = 0 \quad . . . (4)$$

The boundary condition at infinity (4) can be specified as either a Dirichlet or a flux condition. Flux conditions are preferred in practical computations, as zero flux conditions insure conservation of energy in the discretized system. Zero energy flux conditions are applied at the boundary of the region $\partial\Omega \times (0, \infty)$. The corresponding differential systems governing energy and material conservation in the reservoir system are not important to this investigation.

In many physical applications exhibiting a high Peclet number, such as continuous steam drive operations, the areal energy transfer terms appearing in (1) are inconsequential at practical discretization sizes. This conclusion is demonstrated experimentally by both Chase and O'Dell [4] and Coats et. al. [1]. The treatment of areal heat conduction is likewise omitted in this investigation. For flow regimes exhibiting low Peclet numbers, an explicit treatment of areal energy transfer is readily incorporated in the framework developed below. Preliminary computational results indicate that an explicit treatment in time of areal energy transport is sufficient to insure convergence for systems exhibiting low

Peclet numbers. Such systems arise in computations involving reservoir blowdown. Details regarding the explicit treatment of the areal energy transfer terms are omitted for purposes of brevity.

3. Discretization

Under the assumption that the effect of areal energy transfer is small, the cap rock is effectively partitioned into manifold semi-infinite systems of a single space variable. These one dimensional systems, which are considered computational molecules Ω_c^i , are in one to one correspondence with computational molecules Ω_{ru}^i contained in the upper areal plane of the discretized reservoir. The computational molecules associated with the reservoir system are discussed in greater length in an earlier report [8], and an illustration is forthcoming. The approximate differential equation governing energy conservation in each computational molecule Ω_c^i is

$$\frac{\partial}{\partial z} k_c^i \frac{\partial T_c^i}{\partial z} = (\rho_c^i C_c^i T_c^i)_t \quad . . . (5)$$

A schematic representation of a typical reservoir and cap rock system associated with block centered finite difference techniques is illustrated in Figure 1 for the x-z plane. A schematic representation of the base rock is omitted in Figure 1, although the underburden is treated in practical calculations. Typical couplings between the computational elements is indicated in Figure 1. Single bars indicate typical couplings between neighboring reservoir computational molecules. Double bars indicate typical couplings between the cap rock and the reservoir. Triple bars indicate typical well couplings.

Assume that the cap rock calculations can be decoupled from the reservoir computations, an assumption which is validated below. Under this assumption, two sets of boundary conditions completely determine the solution of (5). The boundary condition at infinity is readily applied. A second boundary condition is not readily available, as neither the boundary temperature nor the energy flux at the boundary are known in practice. For a procedure which employs a finite difference technique to model vertical reservoir variations, the temperature is assumed to be a discontinuous constant within each individual computational molecule. It is consistent with the discontinuous constant interpretation to enforce Dirichlet boundary conditions at the reservoir boundary using the temperature T_{ru}^i associated with the computational molecule Ω_{ru}^i coupled with the cap rock system Ω_c^i under consideration. A similar set of boundary conditions are utilized in the Lauwerier theory [9]. The energy flux is post calculated using the resulting approximate solution to (5).

The above discussion assumes that the reservoir temperature T_{ru}^i is independent of the energy flux. This assumption is not in general true. In practical applications, the coupling of T_{ru}^i with the energy flux is treated using the superposition principle developed by Coats et. al. [1].

Let $\{0, z_0, z_1, \dots, z_n\}$ denote a partition of the z axis of the form $\{0, 1, 3, 7, \dots, 2^{n+1}-1\}$. In practical field calculations the partition is computed in units of feet. A geometric partition of this form is used since the physical solution at points near the interface between the reservoir and the cap rock is much more sensitive to perturbations in the boundary conditions than at points far from the interface. Coats et. al. [1] also use a partition which is finer near the interface than at

infinity, although they do not report the exact form of this partition.

The zero flux condition at infinity is imposed at a large, finite value of z , as computational methods do not treat the point at infinity in any meaningful fashion. The value of n which is sufficient to insure convergence in practical settings is determined by applying the flux calculation procedure (12) which is derived below for various values of n . Representative cap rock data, boundary data, and initial data used in this study is presented in Table 1. Figure 2 illustrates the computational results for various values of n . Table 2 presents the calculated value of the energy flux at five years. This study illustrates that applying a zero flux condition at 127 feet is adequate for practical calculations.

Associated with each node z_i is a function $w_i(z)$ given by

$$w_i(z) = \begin{cases} \frac{z - z_{i-1}}{z_i - z_{i-1}}, & z_{i-1} < z < z_i \\ \frac{z_{i+1} - z}{z_{i+1} - z_i}, & z_i < z < z_{i+1} \\ 0, & \text{otherwise} \end{cases} \quad . . . (6)$$

An additional linear spline function which is unity at the interface between the reservoir and the cap rock and vanishes at z_0 , which is denoted as $w_r(z)$, is also defined in a manner similar to that indicated by (6). These functions are defined on the interval $[0, z_n]$. An approximation T_a^i to the true temperature distribution T_c^i in each cap rock computational molecule Ω_c^i is written as a linear combination of these basis functions. This representation takes the form

$$T_a^i(z,t) = T_{ru}^i(t) w_r(z) + \sum_{j=0}^n T_j^i(t) w_j(z) \quad . . . (7)$$

The approximation to the temperature distribution is set to the initial temperature distribution if z is greater than z_n . The definite integrals that are used in the resulting procedure are signified by

$$(u,v) = \int_0^{z_n} u(z)v(z) dz \quad . . . (8)$$

An approximation to (5) obeying the Dirichlet boundary condition at zero and the zero flux boundary condition at a large value of z_n is obtained by enforcing a weak form of the partial differential equation to hold for all indices $j \in \{0,1,\dots,n\}$. The weak form of the differential equation is given by

$$\left(k_c^i \frac{\partial T_a^i}{\partial z}, \frac{\partial w_j}{\partial z} \right) + \left(\rho_c^i C_c^i T_a^i, w_j \right)_t = 0 \quad . . . (9)$$

The zero flux boundary condition at z_n is enforced weakly in (9). A backward Euler time discretization is applied to (9) to insure stability. The resulting linear system has full rank and is a tridiagonal system with $n+1$ equations and $n+1$ unknowns.

The resulting numerical method is, in practice, a disaster. Consider the sample problem which obeys the data presented in Table 1. After a simulated time of sixty days, the computed solution exhibits severe oscillations. Both the nodal values of the computed solution and the exact solution at these locations after sixty simulated days are presented in Table 3. The exact solution of this sample problem is monotonically decreasing with respect to distance from the reservoir and cap rock

interface. Figure 3 illustrates the variation of temperature with respect to distance from the interface separating the reservoir and the cap rock at a time of five years. Figure 3 clearly indicates unphysical behavior in the approximate solution generated using (9), as the maximum physical temperature encountered using these boundary conditions is 400 °F.

The oscillatory behavior is damped by lumping the time derivative integrals. Using lumping, the temperature distribution in the cap rock is calculated using

$$\left(k_c^i \frac{\partial T_a^i}{\partial z}, \frac{\partial w_j}{\partial z} \right) + \left(\rho_c^i C_c^i T_j^i w_j, w_j \right)_t = 0 \quad . . . (10)$$

A backward Euler discretization in time is also applied to (10) to insure stability. The linear system determined by (10) has full rank and is a tridiagonal system in $n+1$ equations and $n+1$ unknowns.

Table 3 illustrates that the approximate solution obtained using (10) exhibits the proper monotonic behavior. Illustrated in Figure 3 is the computed solution using (10) for the sample system after five simulated years. Excellent agreement is observed between the approximate and exact solutions. The calculated solution possesses a maximum value of 400 °F at the boundary separating the reservoir and the cap rock regions.

4. Flux Calculations

Let $\Gamma^i(t)$ denote the energy flux from the cap rock computational molecule Ω_c^i to the reservoir computational molecule Ω_{ru}^i . Wheeler [6] develops a procedure for the accurate determination of the energy flux at the boundary of a domain which is based upon a post processing of the

computed temperature distribution. This approximation to the flux vector is demonstrated to exhibit superconvergence properties under certain assumptions by Douglas, Dupont, Wheeler [7]. Under certain conditions a superconvergent approximation to the energy flux is calculated using the approximate solution from (9) and

$$\Gamma^i(t) = \left(k_c^i \frac{\partial T_a^i}{\partial z}, \frac{\partial w_r}{\partial z} \right) + \left(\rho_c^i C_c^i T_a^i, w_r \right)_t \quad . . . (11)$$

As the temperature distribution generated by (9) is not accurate, it is essential to modify (11) to incorporate the lumping which is used to dampen the oscillatory behavior. Lumping is incorporated into the flux calculations by

$$\Gamma^i(t) = \left(k_c^i \frac{\partial T_a^i}{\partial z}, \frac{\partial w_r}{\partial z} \right) + \left(\rho_c^i C_c^i T_{ru}^i w_r, w_r \right)_t \quad . . . (12)$$

Although both (11) and (12) are expressed as continuous time approximations, stable backward Euler in time schemes are obtained in a straight forward fashion.

The energy flux calculated using a backward Euler in time version of (12) is a linear function of the boundary temperature T_{ru}^i . The slope $\Delta \Gamma^i(t)$ and the intercept $\Gamma_0^i(t)$ are in practice calculated using the principle of superposition in the manner described by Coats et. al. [1]. The coefficients $\Delta \Gamma^i(t)$ and $\Gamma_0^i(t)$ are constant for a given time step length and the current state of the cap rock temperature distribution. The resulting flux estimate is a linear function of the form

$$\Gamma^i(t) = \Gamma_0^i(t) + T_{ru}^i \Delta \Gamma^i(t) \quad . . . (13)$$

The flux estimate (13) appears in the discrete energy balance equation governing the corresponding computational molecule of the reservoir.

The flux calculation procedure indicated by (12) is tested by considering a sample flow reversal problem which possesses an analytical solution. Data for this experiment is similar to that found in Table 1. The boundary condition at the interface between the reservoir and the cap rock is varied at six month intervals. T_{ru}^i is set to 400 °F for the first six months of every year and to 100 °F for the final six months. Interface temperature oscillations of this form are encountered in cyclic steam stimulation. The approximate and exact total energy flux into the cap rock are compared in Figure 4. The agreement between the numerical and the exact solution is excellent. The maximum relative error in this example is two percent.

5. Conclusions

The computational results presented in this investigation demonstrate the necessity of applying lumping to integrals involving time derivatives in a practical setting.

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Table 1

Sample Data

$\rho_c C_c$	30 btu / cu ft / °F
k_c^i	30 btu / ft / day / °F
T_0^i	100 °F
T_{ru}^i	400 °F
dt	1 day
t	[0,5] years

Table 2

Computed Energy Flux after Five Years for Various Values of n

n	Γ (btu / day / sq ft)
5	4.85
6	89.00
7	121.88
8	122.30
exact	118.69

Table 3

Nodal Values of the Solution
After Sixty Simulated Days

z	Equation 9	Equation 10	Exact
0	400	400	400
1	440	377	378
3	386	323	335
7	289	250	257
15	154	141	151
31	95	102	101
63	101	100	100
127	99	100	100

FIGURE 1
 APPROXIMATE INTERACTIONS
 RESERVOIR-CAP ROCK-WELLS

-RESERVOIR COUPLINGS
 =CAP ROCK COUPLINGS
 =WELL COUPLINGS

FIGURE 2
COMPUTED FLUX VERSUS
TIME FOR VARIOUS
BOUNDARY POINTS

FIGURE 3
COMPUTED TEMPERATURE
DISTRIBUTION VERSUS
LOCATION

FIGURE 4
TOTAL HEAT CONTENT
VERSUS TIME