- Technical Report 83-20 -

Mobility Weighting in
Numerical Reservoir Simulation *

by

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September 30, 1983

* This research was supported in part by the International Business Machines Corporation, Palo Alto Scientific Center, Palo Alto, California.

Abstract

The sensitivity of a numerical steamflooding model with respect to mobility weighting is examined in depth. Three numerical discretization procedures are used in this investigation: a new numerical scheme, a five point finite difference method, and a procedure which, under certain assumptions, is equivalent to that introduced by McCracken and Yanosik. Three mobility weighting schemes are investigated. The first approach studied is upstream mobility weighting. The second method investigated uses harmonic total mobility weighting and upstream weighting of fractional flow terms. The approach introduced in this investigation uses the kinematic viscosity in the total mobility and the fractional flow terms. Computational results for a simulated steam drive indicate that this new mobility weighting approach is superior to the other two mobility weighting schemes. In particular, the steam displacement model formed from the combination of this new mobility weighting approach and the McCracken and Yanosik discretization procedure is shown to produce realistic simulations of an inverted seven spot pattern under a continuous steam drive.

1. Introduction

There are many unsolved problems in the area of numerical simulation of fluid flow in a porous media. One of the difficulties that is often encountered is known as the grid orientation effect. This phenomenon was first reported by Hirasaki, O'Dell and Todd [1], and was later reported for a steamflood model by Cnu, Coats, George, and Marcum [2]. Although reservoir models that exhibit grid effects are useful for studying the mechanisms involved in fluid displacement, the presence of this phenomenon raises doubts to the validity of these simulators for predicting future reservoir performance. Consequently, there is a large research effort directed toward the development of numerical reservoir models which do not exhibit this phenomenon. It is well known that the magnitude of the grid effects present in reservoir simulators is sensitive to both the numerical discretization procedure and the mobility weighting approach. This investigation studies the sensitivity with respect to the latter of these two parameters.

By way of introduction, the sensitivity of various numerical discretization schemes to grid effects is briefly discussed at this time. A significant portion of the literature regarding the grid orientation phenomenon is concerned with the development and analysis of numerical procedures for solving the differential equations associated with single phase, two component, two dimensional miscible displacement at adverse mobility conditions [3] [4] [5] [6] [7] [8]. Although this model problem has no particular physical significance, it does provide an adequate indication of the sensitivity of a particular numerical discretization procedure with respect to grid effects. Two conclusions

are clear from these investigations. First, these investigations find that it is necessary to add real or artificial dispersion to the differential system to "regularize" the nearly hyperbolic material balance equations. This conclusion also holds for numerical methods used to approximate the solution of the linear waterflood problem [9] [10]. Secondly, these studies demonstrate that it is necessary to add the numerical dispersion in a rotationally invariant manner to eliminate the grid effects.

Five point finite difference procedures generally add dispersion to "regularize" the associated differential equations by the method of upstream weighting [10]. A difference analysis of upwind five point finite difference procedures reveals that the implicit numerical dispersion is not rotationally invariant. This observation explains the seriousness of the grid effects that are present in the upstream weighted five point difference schemes. Five point finite difference methods are still widely used in physically realistic simulators, however, since they are both easy to implement and are also computationally inexpensive. Robertson and Woo [11] report that the utilization of a curvilinear coordinate system can significantly reduce the magnitude of the grid effects. The transmissibility modifications associated with the curvilinear schemes, however, require both an accurate initial estimate of the fluid streamlines and considerable user interaction.

Many of the procedures referenced above are not implemented in realistic reservoir simulators. For a significant percentage of these methods, the computational overhead associated with the discretization

procedure is prohibitive. One exception to this rule is a nine point finite difference scheme that is due to McCracken and Yanosik [12]. Coats and Ramesh have implemented this procedure in a steamflood model [13] and have studied the grid effects that are associated with this method. The reduction in grid orientation was observed to be quite dramatic for simulated steam displacements over five or nine spot patterns. The magnitude of the grid effects present in this scheme when modeling an inverted seven pattern under a continuous steam drive was shown to be significant.

In two earlier papers [14] [15], a new numerical scheme for modeling fluid flow in a porous media is presented. This new procedure reduces to a finite element method for approximating the solution to the pressure equation and an upstream weighted nine point finite difference procedure for approximating the solution of the continuity equations. Realistic simulated solutions are obtained when using this procedure to approximate the solution of the single phase miscible displacement equations at high mobility ratios. The approximate solutions compare favorably with results that have been previously obtained by other investigators. Bell, Shubin, and Wheeler [16] rigorously prove convergence of this procedure to the true solution of the associated differential equations under assumptions regarding the distribution of the source terms. They show that this procedure obeys a maximum principle. They also demonstrate that the implicit numerical dispersion that is added by the upstream weighting is "nearly" rotationally invariant. This analysis is extended to cover compressible problems by Wheeler [17].

One computational advantage enjoyed by this new numerical procedure over other schemes that are based upon finite element procedures is that the method is defined in terms of material balances on computational molecules. Five and nine point finite difference schemes can also be viewed as material balances on computational molecules. Hence reservoir simulators that are based upon nine point finite difference procedures are readily modified to incorporate this new procedure. In order to determine the usefulness of this scheme as a numerical technique for general reservoir simulation, a three dimensional steam displacement model was developed [18]. A significant reduction in grid effects was obtained in comparison with the results of Coats and Ramesh [13], especially in the simulation of an inverted seven spot pattern under a steam drive.

Sensitivity of numerical procedures with respect to mobility weighting had been reported previously by both Hirasaki et.al. [1] and Au and Vinson [19]. In the course of the development of the thermal simulator mentioned above, the magnitude of the grid effects was likewise found to be sensitive with respect to the handling of the mobility terms. Unrealistic effects were observed in the simulated solutions using two standard mobility weighting approaches. A new mobility weighting procedure that circumvented these difficulties is briefly introduced in previous work [18].

This investigation explores the relationship between mobility weighting and grid effects for three different numerical discretization procedures. First, finite difference forms of three numerical methods that can be used to approximate the continuity equations are stated. In

particular, a five point finite difference scheme, a procedure similar to the approach utilized by McCracken and Yanosik, and a method introduced in earlier investigations [14] [15] [18] are used. The mobility weighting schemes studied in this investigation are introduced. The techniques of upstream mobility weighting and of harmonic mobility weighting with upstream weighted fractional flow terms are considered, as well as the kinematic fractional flow approach that was briefly introduced in an earlier paper [18]. Computational results using an improved version of a three dimensional steam displacement simulator [18] are presented. These results demonstrate that unrealistic effects can be observed using either of the two standard mobility weighting techniques. These results also demonstrate that the magnitude of the grid effects present in a scheme similar to that utilized by McCracken and Yanosik are significantly reduced using the kinematic fractional flow approach.

2. Discretization Procedures

In this investigation, a numerical model of the displacement of a dead oil by steam is studied. The hydrocarbons are assumed to consist of a single nonvolatile component present in only the oleic phase, and the aqueous component is assumed to be present in either a liquid or a vapor phase. Capillary pressure and gravity effects are ignored. A further description of the assumptions made in the model, as well as a description of the primary unknowns that are utilized, is found in an earlier paper [18].

Consider only the material balance on the hydrocarbon component. The extensions covering the aqueous and the energy material balances are

straight forward. Using the coefficient functions that are found in a previous paper [18], the differential equation associated with the hydrocarbon material balance is written as

$$\frac{\partial \phi_p^h}{\partial t} - \nabla K \cdot \frac{k_{ro}^p}{\mu_o} \nabla p - \sum_k Q_k^h = 0 \quad . . . (1)$$

Illustrated in Figure 1 is a rectangular domain of definition over which the hydrocarbon material balance (1) and the corresponding aqueous and energy balances are assumed to hold. This region is triangulated, that is to say subdivided, as illustrated. Following the finite element nomenclature, the rectangles formed by the discretization are known as the elements of the triangulation, and the intersection of the lines forming the triangulation are called the nodes of the discretization. For the purposes of discussion, the elements of the triangulation are labeled with capital alphabetic characters, and the nodes n_i are subscripted with numerals characters. Associated with each node n_i is a computational molecule Ω_i over which discretized versions of the material balance equations are assumed to hold. Values of the primary unknowns are computed for every computational molecule by forcing the discrete material balance equations to be satisfied. The thermodynamic system determined by the values of the primary unknowns is assumed to be in equilibrium within each computational molecule.

Consider the element of the triangulation that is labeled as "A" in Figure 1. Directional transmissibilities are computed as

$$\tau_A^x = \frac{K_A^x}{h_A^x h_A^x} \quad . . . (2)$$

$$\tau_A^y = \frac{K_A^y}{h_A^y h_A^y} \quad . . . (3)$$

Following Au and Vinson [19], harmonic mobility weighting is utilized over each element of the triangulation. Deferring possible definitions of the coefficient function λ until a later time, let λ_A be the harmonic average of the four nodal values of λ , e.g.,

$$\frac{4}{\lambda_A} = \frac{1}{\lambda_1} + \frac{1}{\lambda_2} + \frac{1}{\lambda_4} + \frac{1}{\lambda_5} \quad . . . (4)$$

The definitions (2), (3), and (4) are extended in the obvious fashion for other elements of the triangulation.

Consider for convenience a computational molecule which does not contain any sources or sinks. Also assume for simplicity that the triangulation is uniform in each coordinate direction; e.g., $h_A^x = h_B^x$. For the procedure described by earlier [18], a continuous time material balance on the hydrocarbon component for the computational molecule Ω_5 is

$$\frac{\partial \phi_{p_5}^h}{\partial t} + \sum_{i=1}^9 \Gamma_{i5} f_{i5}^o = 0 \quad . . . (5)$$

The coefficients denoted by Γ_{i5} represent a material transfer rate from the computational molecule Ω_5 to its neighbors. The term Γ_{i5} is defined

to be positive if material is leaving Ω_5 and entering Ω_1 . The quantities f_{15}^0 , which are as yet undefined, are upstream weighted based upon the sign of Γ_{15} .

Three different point centered numerical procedures are derived by using different approximations for the material transfer rates. For the new discretization procedure described earlier [18], the material transfer rates are in practice computed from a linear combination of the pressure values associated with each computational molecule. The resulting approximations to the material transfer rates are superscripted with an "I" to indicate that these formulas are the result of applying a quadrature rule to an integral formula. The equation used to approximate the material flux between two computational molecules having a diagonal connection is illustrated by Γ_{15}^I . Let

$$\Gamma_{15}^I = \frac{1}{12} \tau_A^x \lambda_A [p_5 + p_2 - p_4 - p_1] + \frac{1}{12} \tau_A^y \lambda_A [p_5 + p_4 - p_2 - p_1] \quad . . . (6)$$

For two computational molecules having a parallel connection, the approximation to the material flux is illustrated by Γ_{25}^I . Let

$$\Gamma_{25}^I = \frac{1}{12} \tau_A^x \lambda_A [3 p_5 + p_2 - 3 p_4 - p_1] + \frac{1}{12} \tau_C^x \lambda_C [3 p_5 + p_7 - 3 p_4 - p_6] \quad . . . (7)$$

A five point finite difference procedure is defined by setting the diagonal material transfer rates to zero, e.g.,

$$\Gamma_{15}^5 = 0 \quad . . . (8)$$

For this five point finite difference procedure, the approximation to the parallel mass transfer rates are illustrated by Γ_{25}^5 . Let

$$\Gamma_{25}^5 = \frac{1}{2} \tau_A^x \lambda_A [P_5 - P_4] + \frac{1}{2} \tau_C^x \lambda_C [P_5 - P_4] \quad \dots (9)$$

A nine point finite difference scheme is obtained from (6) and (7) under the assumptions that $\tau_A^x = \tau_A^y$ over every element of the triangulation and that the pressure gradient is approximately a constant over every element of the triangulation. This second assumption is an excellent approximation in the interior of the domain of definition. The approximation to the diagonal material transfer rates used in this procedure are illustrated by the approximation to Γ_{15}^9 . Let

$$\Gamma_{15}^9 = \frac{1}{12} \tau_A^x \lambda_A [P_5 - P_1] + \frac{1}{12} \tau_A^y \lambda_A [P_5 - P_1] \quad \dots (10)$$

The approximation to the parallel material transfer rates are illustrated by Γ_{25}^9 . Let

$$\Gamma_{25}^9 = \frac{1}{3} \tau_A^x \lambda_A [P_5 - P_4] + \frac{1}{3} \tau_C^x \lambda_C [P_5 - P_4] \quad \dots (11)$$

The scheme defined by the difference equations (10) and (11) yields a discrete approximation to a Laplacian operator that is equivalent to that which was used by McCracken and Yanosik.

3. Mobility Weighting Schemes

The coefficients functions denoted by λ and f are as of yet undefined. In this section, three possible definitions of the coefficients functions are given. The mobility weighting schemes studied in this investigation do not form an exhaustive list of possibilities. For example, two point upstream weighting, suggested by Hirasaki et. al. [1], is not investigated.

The first mobility weighting procedure that is studied uses upstream mobility weighting. In particular, let $\lambda = 1$, and let the upstream weighted term for the oleic phase be given by

$$f^o = \frac{\rho^o k^{ro}}{\mu^o} \quad . . . (12)$$

This approach is upstream mobility weighting.

The second mobility weighting procedure studied uses the total mobility and fractional flows. The total mobility is

$$\lambda = \frac{k^{ro}}{\mu^o} + \frac{k^{rl}}{\mu^l} + \frac{k^{rv}}{\mu^v} \quad . . . (13)$$

The upstream weighted quantity that is associated with the oleic phase is given by

$$f^o = \rho^o \frac{1}{\lambda} \frac{k^{ro}}{\mu^o} \quad . . . (14)$$

The density term appearing in (14) does not necessarily need to be upwinded. The computational results are observed to be insensitive

toward the approximation of the density term. The product of the last two terms on the right hand side of (14) is the fractional flow of the oleic phase. For this reason, the mobility weighting scheme defined by (13) and (14) is referred to as the fractional flow approach.

Finally, a new mobility weighting approach introduced earlier [18] is studied. This approach modifies the definition of the fractional flow terms using the kinematic viscosity. The kinematic viscosity is the quotient

$$\nu = \frac{\mu}{\rho} \quad . . . (15)$$

Let a total mass mobility be given by

$$\tilde{\lambda} = \frac{k^{ro}}{\nu^o} + \frac{k^{rl}}{\nu^l} + \frac{k^{rv}}{\nu^v} \quad . . . (16)$$

A mass fractional flow becomes the upstream weighted quantity appearing in (5). In particular, let

$$\tilde{f}^o = \frac{1}{\tilde{\lambda}} \frac{k^{ro}}{\nu^o} \quad . . . (17)$$

Notice that $\tilde{f}^o$ is a mass fractional flow that is defined using the kinematic viscosity. Hence the mobility weighting approach that is defined by (16) and (17) is referred to as kinematic mobility weighting.

4. Computational Results

The computational results presented below are obtained using an improved version of a three dimensional steam displacement simulator

[18]. Improvements in the model include the incorporation of a highly implicit time stepping procedure, the ability to simulate reduced elements of symmetry, and the modeling of solution gas effects. The results reported below differ from those reported earlier [18] due to the implicit time stepping procedure. Data is taken from Coats and Ramesh [13]. Reservoir and fluid property data is presented in Table 1. Two phase relative permeability curves are given in Tables 2 and 3. Stone's method [20] is used to obtain an approximation to three phase relative permeabilities from the two phase data.

The first set of computational results document the "wiggle" in the computed pressure solution at early times using various mobility weighting schemes in conjunction with the new discretization procedure. Consider upstream mobility weighting. At the start of the simulated displacement over a repeated five spot pattern, the computed pressure solution exhibits unrealistic phenomenon for simulations employing a parallel grid. This effect manifests itself at the start of simulated steam injection, before steam has formed in the computational molecules containing the injection wells. As soon as simulated steam forms in the computational molecules containing injection wells, this phenomenon disappears. This effect is illustrated for a $3 \times 3 \times 1$ parallel grid in Table 4. The injection wells are located at (0,0) and (231,231), while the production wells are situated at (0,231) and (231,0). This phenomenon is also present for parallel grid simulations which are performed on grids finer than the $3 \times 3 \times 1$ grid illustrated.

Consider the fractional flow approach. The computed pressure solution exhibits unrealistic phenomenon near the injection wells. This

effect is present in calculations using the diagonal grid. This phenomenon manifests itself at simulated times between that at which steam forms in cells containing injection wells and that at which steam forms in the surrounding cells. Although this effect is present for any diagonal grid finer than a $2 \times 2 \times 1$ grid, the effect is illustrated using the coarsest possible grid in Table 5. The injection well is situated at (0,0), while the production well is located at (165,165). Not only is the "wiggle" evident in these results, but the computed pressure associated with the computational molecule containing the injection well is unrealistic.

The kinematic mobility weighting approach produces realistic simulations of the test cases. Computational results using this new scheme are given in Table 6 corresponding to those presented in Table 4 using the upstream mobility weighting approach. Likewise, computational results which correspond to those illustrated in Table 5 for the fractional flow scheme are presented in Table 7 for the kinematic mobility weighting approach.

The second set of computational experiments deal with the simulation of an inverted seven spot pattern under a continuous steam drive. Coats and Ramesh [13] report that the procedure due to McCracken and Yanosik exhibits serious grid effects for this problem. Steam displacement over this pattern is simulated in this investigation using all three discretization procedures and all three mobility weighting schemes. The $12 \times 7 \times 1$ rectangular element of symmetry illustrated in Figure 2 is employed in these calculations. The displacement is also simulated using a triangulation that is based upon equilateral triangles

over one-sixth of the pattern. A discussion of this procedure, although beyond the scope of this work, is found in [17]. The experiments using the triangular discretization employ a grid with $n = 8$. This number is based upon the consideration that the volume of representative interior computational molecules should be equal for meaningful comparisons. A list of the cases that are studied is presented in Table 8, as well as the computed steam breakthrough times. Simulated isotherms of 200°, 300°, and 400° within 100 days of simulated steam breakthrough are plotted in Figure 3 for all cases studied. For simulations employing a rectangular grid, the results are plotted using the element of symmetry involving one-sixth of the pattern. Injection wells are located at the lower left vertex of each plot, and production wells are located at the other two vertices. For the rectangular grids, the asymmetry present in Figure 3 is a measure of amount of grid orientation associated with a particular combination of a discretization procedure and a mobility weighting scheme.

Figure 3 indicates that the asymmetry in the computed isotherms is minimal for the McCracken and Yanosik procedure when the kinematic fractional flow mobility weighting approach is used. The computational results obtained using this procedure and those that are obtained using the new numerical scheme are extremely similar for this choice of mobility weighting. These results indicate that the kinematic fractional flow approach is a valid option for reducing the amount of grid orientation present in the simulation of an inverted seven spot pattern under a steam drive. The computational results using the upstream mobility weighting approach exhibit a large amount of grid

orientation for all three numerical discretization schemes that are defined over rectangles. Although the computational results obtained using the volumetric fractional flow approach with harmonic mobility weighting also exhibit little grid effects for any of the three numerical discretization procedures studied, the computed isotherms illustrate the large amount of numerical dispersion that is added by this mobility weighting scheme.

5. Discussion

Two points are addressed in this section. First, the "wiggle" documented in the previous section is not observed in models employing finite difference procedures. This fact can lead to the conclusion that this new discretization procedure is errant. Secondly, for the linear waterflood problem, a numerical model using the kinematic mobility weighting scheme should approach the Buckley-Leverett solution as the mesh is refined.

For finite difference procedures, the material transfer rates between neighboring computational molecules are directly proportional to the pressure difference between these two elements of volume. In order for a material transfer rate Γ_{ij} to be positive in a finite difference procedure, one must have that $p_j > p_i$. This observation explains the monotonic behavior of finite difference pressure solutions. Hence the "wiggle" effects documented above cannot be present in finite difference procedures. For the new discretization procedure used in this investigation, or indeed any procedure based upon a finite element method, the material transfer rates Γ_{ij} are based upon integrals involving pressure gradients. Since the approximation to the material

transfer rates is no longer exactly proportional to the pressure difference between the two elements of volume, a material transfer rate Γ_{ij} can, under some conditions, be positive although $p_j < p_i$.

The "wiggle" present in the simulated solutions almost certainly indicates a flaw in the mobility weighting scheme rather than one in the discretization procedure. A mathematically rigorous proof showing the asymptotic convergence of this new procedure has been undertaken for the single phase miscible displacement problem [16] [17]. Note that mobility weighting is not an issue in single phase problems.

Numerical results obtained using the kinematic mobility weighting approach should be equivalent to those obtained from the fractional flow approach in the Buckley-Leverett problem. To maintain the equivalence of the approaches for modeling incompressible flow, the equations must be scaled so that appropriate formation volume factors replace the density coefficients. The resulting mobility weighting scheme using formation volume factors differs from the Buckley-Leverett approach only in a physical model where a component is present in two or more phases. For the purposes of simplicity, however, this complexity is avoided. For a hot waterflood problem, the computational results obtained using fractional flow mobility weighting are almost identical to those obtained using the kinematic approach.

6. Conclusions

The computational results presented above support the conclusions of Hirasaki et. al. [1] and of Au and Vinson [19] that mobility weighting in a numerical reservoir simulator has a large effect upon the

magnitude of the grid orientation inherent in the model. Mobility weighting schemes based upon the upwinding of either fractional flows or mobility terms can lead to unphysical computational results when applying procedures based upon the finite element method. Upstream mobility weighting results in severe grid effects in reservoir simulations. The isotherms computed using the fractional flow approach with harmonic mobility weighting exhibit a large amount of numerical dispersion. A mobility weighting scheme based upon the kinematic viscosity reduces the magnitude of grid effects without adding a significant amount of numerical dispersion. This mobility weighting scheme does not yield the unrealistic results obtained using other mobility weighting methods. Lastly, the utilization of the kinematic approach in conjunction with the procedure due to McCracken and Yanosik yields realistic simulations of an inverted seven spot pattern under a continuous steam drive.

7. Acknowledgements

I would like to thank the International Business Machines Corporation for supporting this project and for permission to publish this paper. I would like to thank Mary Wheeler, Trond Steihaug, and John Dennis of Rice for their continued support and suggestions.

8. Nomenclature

$f, f^{\wedge}, \tilde{f}$	Upstream weighted coefficient functions appearing in the discretized material balances, variable units.
n	Mesh spacing in ft.
k^r	Relative permeability.

P	Pressure in lbs / sq in.
t	Time in days.
K	Absolute permeability in 6.336×10^{-3} md.
Q	Mass injection rate in lbs / day.
$\lambda, \lambda^{\wedge}, \tilde{\lambda}$	Mobility coefficient appearing in the discretized material balances, variable units.
μ	Viscosity in cp.
ν	Kinematic viscosity in cp - lbs / cu ft.
$\rho^{\wedge}$	Phase density in lbs / cu ft.
ρ	Total component density in lbs / cu ft.
ϕ	Porosity.
τ	Transmissibility.
T	Material transfer rate appearing in the discretized material balances, variable units.
Ω	Symbol denoting a computational molecule.

Superscripts

5	Five point difference scheme.
9	Nine point difference scheme.
h	Hydrocarbon component.
o	Oleic phase.
v	Vapor water phase.
x	X direction.
y	Y direction.
I ..	Difference scheme based upon integrals.
L	Liquid water phase.

Subscripts

1,2,3,4,5,6,7,8,9	Computational molecules associated with the triangulation.
i,j	Computational molecules associated with the triangulation.
k	Wells.
A,E,C,D	Elements of the triangulation.

9. References

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Table 1

Steam Drive Data

Pattern Size (acres)	2.5
Pattern Height (feet)	50
Oil Properties	
Stock Tank Density (lbs / cu ft)	60.3
Compressibility (1 / psi)	.00001
Thermal Expansion (1 / °F)	.0004
Specific Heat (btu / lbs)	.35
Viscosity (cp)	
@ 100 °F	1380
@ 400 °F	3
Rock Properties	
Permeability (md)	2000
Porosity	.3
Compressibility (1 / psi)	.0005
Heat Capacity (btu / °F / cu ft)	30
Conductivity (btu / °F / ft / day)	30
Initial Conditions	
Pressure (psi)	100
Temperature (°F)	100
Oil Saturation	.73
Well Data	
Injection Rate (bbls / day)	187.5
Steam Quality	.8
Temperature (°F)	400
Bottom Hole Pressure (psi)	100
Wellbore Radius (ft)	.5

Table 2

Liquid / Oil Relative Permeability Data

S^L	k^{rL}	k^{roL}
.250	.00000	.50000
.305	.00028	.39240
.360	.00224	.29928
.415	.00756	.22014
.470	.01792	.15443
.525	.03500	.10153
.580	.06048	.06077
.635	.09604	.03136
.690	.14336	.01234
.745	.20412	.00251
.800	.28000	.00000

Table 3

Vapor / Oil Relative Permeability Data

S^V	k^{rV}	k^{roV}
.00	.00000	1.00000
.07	.00998	.71380
.14	.03241	.48965
.21	.06458	.31938
.28	.10531	.19502
.35	.15389	.10882
.42	.20981	.05338
.49	.27267	.02122
.56	.34216	.00580
.63	.41801	.00063
.70	.50000	.00000

Table 4

Computed Pressure Solution at 50
Days Using Upstream Mobility
Weighting, 3x3x1 Grid

231 ft	89	123	251	248
154 ft	123	129	215	251
77 ft	251	215	129	123
0 ft	248	251	123	89
	0 ft	77 ft	154 ft	231 ft

Table 5

Computed Pressure Solution at 200
Days Using Fractional Flow Mobility
Weighting, 2x2x1 Grid

165.0 ft	364	326	249
82.5 ft	337	464	326
0.0 ft	1367	337	364
	0.0 ft	82.5 ft	165.0 ft

Table 6

Computed Pressure Solution at 50
Days Using Kinematic Mobility
Weighting, 3x3x1 Grid

231 ft	85	108	170	540
154 ft	108	110	168	170
77 ft	170	168	110	108
0 ft	540	170	108	85
	0 ft	77 ft	154 ft	231 ft

Table 7

Computed Pressure Solution at 200
Days Using Kinematic Mobility
Weighting, 2x2x1 Grid

165.0 ft	405	373	264
82.5 ft	462	449	373
0.0 ft	556	462	405
	0.0 ft	82.5 ft	165.0 ft

Table 8

Cases Studied for a Continuous Steam
Drive on an Inverted Seven Spot Pattern

Case Number	Mobility Weighting	Numerical Procedure	Steam Breakthrough
1	Upstream	New Method	1020
2	Upstream	Nine Point	1036
3	Upstream	Five Point	427
4	Upstream	Triangles	1020
5	Fractional Flow	New Method	1648
6	Fractional Flow	Nine Point	1657
7	Fractional Flow	Five Point	1576
8	Fractional Flow	Triangles	1679
9	Kinematic	New Method	1406
10	Kinematic	Nine Point	1396
11	Kinematic	Five Point	1085
12	Kinematic	Triangles	1396

Figure 1
Discretization of a Typical
Rectangular Symmetry Element

Figure 2
Discretization of the
Inverted Seven Spot

Figure 3
 Computed Isotherms for an
 Inverted Seven Spot Pattern
 Under a Steam Drive

----- Upstream Mobility Weighting -----

New Method
 1005 Days

Nine Point
 1002 Days

Five Point
 408 Days

Triangles
 1016 Days

----- Fractional Flow Mobility Weighting -----

New Method
 1601 Days

Nine Point
 1601 Days

Five Point
 1506 Days

Triangles
 1610 Days

----- Kinematic Mobility Weighting -----

New Method
 1401 Days

Nine Point
 1403 Days

Five Point
 1008 Days

Triangles
 1403 Days