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**An Improved Implementation of the
McCracken and Yanosik Nine Point
Finite Difference Procedure***

by

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Abstract

The grid orientation phenomenon present in numerical models of fluid flow in a porous media can give rise to unrealistic predictions when simulating adverse mobility displacements. McCracken and Yanosik [1] proposed a nine point finite difference scheme for approximating the solution of the continuity equations that has the potential of eliminating many of the unrealistic predictions that are observed when using five point finite difference operators. Coats and Ramesh [2] have implemented this scheme in a steamflooding model, and have noted that serious grid effects are present in the simulation of an inverted seven spot pattern. Potempa [3] [4] has described a procedure which exhibits minimal grid effects for the problem described by Coats and Ramesh [2]. This paper describes modifications to the McCracken and Yanosik procedure which allow for realistic simulations of this inverted seven spot pattern under a steam drive. These modifications are based upon an approximation scheme that has been previously reported [3] [4], and affect the incorporation of upstream weighting in a simulator.

1. Introduction

The grid orientation effect in numerical simulation of petroleum reservoirs was first reported by Hirasaki, O'Dell and Todd [5], and was later reported for a steamflood model by Chu, Coats, George, and Marcum [6]. A large amount of research has been undertaken toward the development of methods that eliminate this phenomenon from reservoir models. One of the more successful methods for reducing the magnitude of the grid effects is a nine point finite difference procedure used by McCracken and Yanosik [1]. The grid effects that are present in this procedure were later studied by Coats and Ramesh for a steam displacement model [2].

The procedure described in [1] yields certain unrealistic predictions when utilized in models of adverse mobility ratio displacements. McCracken and Yanosik report "horned" and "bullet" shaped displacement fronts in their original paper [1]. The unrealistic behavior of the predicted fronts is strongly tied to the effective mobility ratio of the displacement mechanism. Coats and Ramesh find that the grid effects present in this scheme are almost as serious as the magnitude of grid orientation present in five point finite difference procedures when modeling an inverted seven spot pattern under a continuous steam drive [2].

Potempa [3] [4] has presented a numerical discretization procedure for approximating the solution of the continuity equations which model multiple component, multiple phase mass transfer. Finite element methods have attracted considerable attention as a means of eliminating the grid effects present in numerical discretization procedures [7] [8]

[9] [10] [11]. One drawback with these procedures is that they exhibit varying degrees of computational complexity which limit both their computational speed and their ability to be implemented in models of complex reservoir phenomenon. Young addressed these drawbacks, and made considerable headway with respect to dealing with many of the implementation problems [11]. The procedure that is described by Potempa [3] [4] is an attempt to place a finite element method in setting that is consistent with the "In minus out equals accumulation" philosophy. This scheme also incorporates upstream weighting as a means of regularizing the approximate solution.

There is very little grid orientation present in this scheme when it is used to model single phase, two component, miscible displacement problems [3]. Bell, Shubin, and Wheeler have proven convergence of this procedure to the true solution of the associated differential equations under certain simplifying assumptions regarding the distribution of the source terms [12]. The data structures arising in an implementation of this procedure are extremely similar to those which occur in an implementation of the procedure suggested by McCracken and Yanosik. Primarily because this scheme is conceptually similar to many of the state-of-the-art reservoir simulators, it is feasible to implement this procedure in numerical models of complex displacement mechanisms. In particular, this procedure has been implemented in a three dimensional steam displacement model [4]. This model produced realistic simulations of an inverted seven spot steam drive problem that had been previously described by Coats and Ramesh [2].

It has been stated previously [4] that the procedure of Potempa reduces to the McCracken and Yanosik scheme under certain simplifying assumptions. This topic is pursued in depth in this investigation. Using as a model problem the two phase immiscible displacement problem, the procedure of McCracken and Yanosik is reviewed. The method of Potempa, which is defined in terms of integral relationships [3] [4], is written in a difference form for the model problem. By making certain assumptions regarding the mass flux distribution, this difference formulation is perturbed to obtain a method very similar to that described by McCracken and Yanosik. The two procedures differ only in the manner in which upstream weighting is introduced. Computations involving an inverted seven spot pattern using both the original procedure of Potempa and the perturbed scheme are presented for both the single phase, two component, miscible displacement problem and the steam drive problem. The differences in the predicted recovery curves and either the computed isotherms or the computed compositional profiles are insignificant. The predicted steam breakthrough times differ by an insignificant amount for the case of steam injection. These results are unexpected in light of the previously published results [1] [2].

These results are explained by examining the effect of the upwinding on the discrete pressure equation obtained from a summation of the individual component material balances. In particular, the discrete pressure equation obtained by a summation of the material balance equations governing each component is upwinded for the scheme utilized by McCracken and Yanosik. The resulting discrete pressure equation is not upwinded for this new implementation. Upstream weighting is

introduced using fractional flow terms in this new implementation, rather than in the relative permeabilities, as is done in the McCracken and Yanosik scheme. Since the resulting difference operator for the pressure equation has a higher order of accuracy in this new implementation, the resulting approximation to the pressure distribution, and hence the computed flow field, exhibits a higher order of accuracy.

2. Definition of the Model Problem and Numerical Procedures

Consider the two phase, two component immiscible displacement problem. The components are assumed to be a hydrocarbon and water, and each component is assumed to form a distinct phase. Each component is therefore identified with a single phase, and the terms phase and component are henceforth used interchangeably. Assume further that the reservoir is of unit height, that the capillary pressure can be ignored, and that gravity can be neglected. A material balance on each component m is expressed by the differential equation

$$\frac{\partial \phi S_{\rho}^m}{\partial t} = \nabla \cdot \frac{K k_{\rho}^m}{\mu^m} \nabla P + q^m \quad . . . (1)$$

Some additional terms are defined so that the numerical procedures can be presented in a more compact fashion. Assuming that the porosity is a constant, the porosity and the time coefficients can be combined as

$$\tau = \frac{t}{\phi} \quad . . . (2)$$

The total density of a component is defined as

$$\frac{\partial \rho^m}{\partial \tau} = S^m \rho^m \quad . . . (3)$$

The kinematic viscosity of a phase is defined by

$$\nu^m = \frac{\mu^m}{\rho^m} \quad . . . (4)$$

The material balance equation (1) is hence written in the compact form

$$\frac{\partial \rho^m}{\partial \tau} = \nabla \cdot \frac{K k^{rm}}{\nu^m} \nabla P + q^m \quad . . . (5)$$

Illustrated in Figure 1 is a domain of definition Ω over which (5) holds corresponding to a rectangular element of symmetry in a petroleum reservoir. The mesh spacing is denoted by h . Define the diagonal transmissibility as

$$\tau^d = \frac{K}{6 h^2} \quad . . . (6)$$

In a similar manner, define the parallel transmissibility as

$$\tau^p = \frac{2 K}{3 h^2} \quad . . . (7)$$

Consider a computational molecule centered at the point labeled as "5" in Figure 1. This computational molecule is referred to as Ω_5 in the subsequent discussion. The neighboring cells that directly affect Ω_5 are also numbered per Figure 1. Assume for simplicity that no wells effect Ω_5 . A continuous time version of the material balance on the component m that is enforced upon Ω_5 corresponding to (5) that was

utilized by McCracken and Yanosik is

$$\begin{aligned}
 \frac{\partial \rho_5^m}{\partial \tau} = & \frac{\tau^d k_{51}^{rm}}{V_{51}^m} \{P_5 - P_1\} + \frac{\tau^d k_{53}^{rm}}{V_{53}^m} \{P_5 - P_3\} \\
 & + \frac{\tau^d k_{57}^{rm}}{V_{57}^m} \{P_5 - P_7\} + \frac{\tau^d k_{59}^{rm}}{V_{59}^m} \{P_5 - P_9\} \\
 & + \frac{\tau^p k_{52}^{rm}}{V_{52}^m} \{P_5 - P_2\} + \frac{\tau^p k_{54}^{rm}}{V_{54}^m} \{P_5 - P_4\} \\
 & + \frac{\tau^p k_{56}^{rm}}{V_{56}^m} \{P_5 - P_6\} + \frac{\tau^p k_{58}^{rm}}{V_{58}^m} \{P_5 - P_8\}
 \end{aligned}
 \quad . . . (8)$$

The relative permeability terms are upwinded in this procedure [1]. It is not possible to determine how the kinematic viscosity terms are handled from the text of the paper of McCracken and Yanosik.

In order to apply the numerical procedure of Potempa to this problem as described previously [4], it is first necessary make some additional definitions. Let the mass mobility of a phase m be given as

$$\xi^m = \frac{k^{rm}}{V^m}
 \quad . . . (9)$$

Let the total mass mobility ξ be the sum of the individual phase mobilities. The mass fractional flow of a phase m is given by

$$f^m = \frac{\xi^m}{\xi}
 \quad . . . (10)$$

Let ξ_a denote the value of the total mass mobility evaluated at the

point which is indicated by an "a" in Figure 1. Furthermore, let ξ_{ab} denote the average of the total mass mobilities at the points which are denoted by "a" and "b" in Figure 1. Finally, using (9) and (10), (5) is rewritten as

$$\frac{\partial \rho^m}{\partial \tau} = \nabla \cdot K \xi^m \nabla P + q^m \quad . . . (11)$$

The fractional flow term is defined in terms of the kinematic viscosity rather than the viscosity for reasons beyond the scope of this text. In incompressible flow, such as that which is analyzed by Buckley and Leverett [13] and Welge [14], the total volumetric flux is conserved. This fact is exploited using fractional flows based upon volumetric quantities in order to obtain the characteristics [14]. In certain other displacement mechanisms, such as a steam drive, the total volumetric flux exhibits large variations, while the total mass flux is almost a constant quantity. Potempa [15] demonstrates that unrealistic phenomenon are observed in a steam displacement model when using standard volumetric fractional flow arguments as a model of multiple phase mass transfer. These unphysical effects are eliminated by using a model of multiple phase mass transfer that is based upon the mass fractional flow quantities defined above.

The procedure of Potempa has been previously defined in terms of integrals and tensor product linear basis functions [3] [4]. It has been stated that by applying a certain numerical approximation scheme in the evaluation of the integrals appearing in the definition of the procedure, the approximate mass transfer rates which are used to model the mass flux from a particular computational molecule to its neighbors

can be written as a linear combination of the nodal values of the pressure. Omitting the details, a continuous time version of the material balance which is enforced for each component m over the computational molecule Ω_5 is written in a difference form as

$$\begin{aligned}
 \frac{\partial \rho_5^m}{\partial \tau} = & \tau^d \xi_a f_{51}^m \{P_5 - P_1\} + \tau^d \xi_b f_{53}^m \{P_5 - P_3\} \\
 & + \tau^d \xi_c f_{57}^m \{P_5 - P_7\} + \tau^d \xi_d f_{59}^m \{P_5 - P_9\} \\
 & + \tau^p \xi_{ab} f_{52}^m \left\{ \frac{3}{4} P_5 + \frac{1}{8} P_4 + \frac{1}{8} P_6 - \frac{3}{4} P_2 - \frac{1}{8} P_1 - \frac{1}{8} P_3 \right\} \\
 & + \tau^p \xi_{ac} f_{54}^m \left\{ \frac{3}{4} P_5 + \frac{1}{8} P_2 + \frac{1}{8} P_8 - \frac{3}{4} P_4 - \frac{1}{8} P_1 - \frac{1}{8} P_7 \right\} \\
 & + \tau^p \xi_{bd} f_{56}^m \left\{ \frac{3}{4} P_5 + \frac{1}{8} P_2 + \frac{1}{8} P_8 - \frac{3}{4} P_6 - \frac{1}{8} P_3 - \frac{1}{8} P_9 \right\} \\
 & + \tau^p \xi_{cd} f_{58}^m \left\{ \frac{3}{4} P_5 + \frac{1}{8} P_4 + \frac{1}{8} P_6 - \frac{3}{4} P_8 - \frac{1}{8} P_7 - \frac{1}{8} P_9 \right\}
 \end{aligned}
 \tag{12}$$

This equation assumes that the region is uniformly discretized and that the permeability is an isotropic quantity. The differences are significantly more complex in a more general case. Upwinding is applied only to the mass fractional flow terms. This equation is quite similar to (8). The major differences occur in the approximation of the relative permeability / fractional flow terms and in the form of the pressure differences appearing in the parallel flow terms.

The parallel flow terms appearing in (12) are modified by assuming that the pressure differences with small coefficients can be adsorbed into those which have large coefficients. This approximation is

equivalent to an assumption that the mass flux vector is constant over a grid block. This approximation is in general valid over the interior of the domain of interest. This modification of (12) is written as

$$\begin{aligned}
 \frac{\partial \rho_5^m}{\partial \tau} = & \tau^d \xi_a f_{51}^m \{P_5 - P_1\} + \tau^d \xi_b f_{53}^m \{P_5 - P_3\} \\
 & + \tau^d \xi_c f_{57}^m \{P_5 - P_7\} + \tau^d \xi_d f_{59}^m \{P_5 - P_9\} \\
 & + \tau^p \xi_{ab} f_{52}^m \{P_5 - P_2\} + \tau^p \xi_{ac} f_{54}^m \{P_5 - P_4\} \\
 & + \tau^p \xi_{bd} f_{56}^m \{P_5 - P_6\} + \tau^p \xi_{cd} f_{58}^m \{P_5 - P_8\}
 \end{aligned}
 \quad . . . (13)$$

Since this equation only differs from (8) in the treatment of the upstream weighting, the resulting procedure is designated as the modified McCracken and Yanosik scheme.

3. Computational Results

The procedure indicated by (13) has been implemented in two different models of fluid flow in a porous media. These computational experiments are undertaken for an inverted seven spot pattern. The transmissibility terms appearing in (12) and (13) are modified in a straight forward manner to account for the slightly unequal partitions of the x and the y coordinate axis.

A two component, two dimensional, single phase, compressible miscible displacement code [3] can utilize this scheme through the specification of an input option. The single phase miscible displacement problem, although physically unrealistic, adequately

determines the sensitivity of a numerical discretization procedure to grid effects. A version of a steam displacement code [4] has also been modified to use this procedure. The code is further developed than that which had been described previously, including such improvements as a fully implicit time stepping procedure and the inclusion of solution gas effects. Many of the techniques used for these improvements are based upon the work of Coats [16] and Weinstein, Wheeler, and Woods [17], and do not represent any substantial new work of interest.

The data for the miscible displacement computational experiment is found in Table 1. The mobility ratio is taken to be 100. Well data is given on a full pattern basis. The simulated displacement occurs on an element of symmetry of the inverted seven spot pattern which is exhibited in Figure 2 using a 33×19 grid. Compositional profiles at 7.5×10^{-1} pore volumes injected are illustrated in Figures 3 and 4 using both the numerical scheme indicated by (12) and that which is indicated by (13), respectively. Corresponding recovery curves are illustrated in Figure 5. There is no significant difference between the two procedures when modeling miscible displacement at high mobility ratios.

The data for the computational experiment modeling a steam drive over the same element of symmetry of an inverted seven spot pattern is found in Table 2. A 12×7 grid is used in these computations. Two phase relative permeability data is found in Tables 3 and 4. This data has been previously used by Coats and Ramesh [2] and Potempa [4]. The simulated isotherms of 200 °F, 300 °F, and 400 °F at 1400 days are presented in Figure 6 for the procedure suggested by (12) and in Figure 7 for the procedure suggested by (13). The oil recovery curves can be

found in Figure 8. Predicted oil production rates are presented in Figure 9. Simulated steam breakthrough occurred at 1440 days using the original procedure, and at 1420 days using the perturbed scheme. The results concerning the original procedure differ from those reported previously [4] due to the fully implicit time stepping procedure. There is no significant difference between the two procedures when simulating this particular steam drive.

4. Discussion of Results

These results are not anticipated in light of those that have been previously reported [1] [2]. Possible reasons for this difference are now discussed. Letting q denote the total mass injection rate, and letting $\hat{\rho}$ denote the sum of the total densities, a pressure equation corresponding to either (5) or (11) is given by

$$\frac{\partial \hat{\rho}}{\partial \tau} = \nabla \cdot K \nabla P + q \quad . . . (14)$$

Adding together the material balances given by (12) for the oil and water components yields a set of equations which are equivalent to those obtained from applying the finite element method to (14) using bilinear splines as basis functions and lumping [11] for the time derivatives. A summation of the material balances indicated by (13) results in a set of equations that is equivalent to the system obtained from applying a point centered finite difference method to (14) which uses a difference operator for a weighted Laplacian which is similar to that which is utilized by McCracken and Yanosik. The two difference operators are identical for the case of the Laplacian. Both the methods indicated by

(12) and (13) do not exhibit upwinding in the discretized pressure equation. Adding together the material balances given by (8), on the other hand, yields a discrete approximation to the pressure equation which contains upstream weighted relative permeability terms.

It is well known that parabolic differential equations, such as (14), possess well behaved solutions. Numerical discretization procedures for approximating the solutions of such equations do not need any regularization, such as upstream weighting, to converge to the correct solution. For example, this result has been proved for a large class of finite element methods by Wheeler [18]. If upwinding appears in a discretized pressure equation, a corresponding loss of accuracy is expected in the approximate solution. Without being rigorous, this loss of accuracy is quantified below.

Although the approximation to the Laplacian used by McCracken and Yanosik is $O(h^4)$, the modification indicated by a summation of (13) for a weighted Laplacian is only $O(h^2)$. The pressure solution is only expected to be $O(h^2)$, and expecting to lose an order of accuracy when "differentiating" the pressure solution to obtain an approximation to the mass transfer rates between two computational molecules, this approximation is expected to be only $O(h)$. An accuracy of $O(h)$, however, is enough to guarantee asymptotic convergence to the true mass flux distribution.

On the other hand, the approximation to the weighted Laplacian indicated by a summation of (8) is only $O(h)$ due to the upstream weighting of the relative permeability terms. The approximate pressure solution is only expected to be $O(h)$. "Differentiation" of the pressure

solution results in approximations to the mass transfer rates between computational molecules that are $O(1)$. This does not guarantee asymptotic convergence to the true mass flux distribution. If the underlying flow field does not converge asymptotically, the component balances are not expected to converge, since the solution of the continuity equations is strongly dependent upon the distribution of the flow field.

The above remarks indicate that any approximation scheme to the weighted Laplacian appearing in (14) that is $O(h^2)$ should be adequate to insure the asymptotic convergence of a numerical procedure. The procedure described above evaluates the total mass mobility at the points denoted in Figure 1 as "a", "b", "c", and "d" simply because the scheme indicated by (12) originally evaluated the mobility at these points. When modifying existing code, there are two considerations which should be used in choosing the points at which the mass mobility is to be evaluated. The first consideration is that it is desirable to use points at which mobility terms are currently evaluated. Given these points, it is then only important to find an approximation to the weighted Laplacian that appears in the discrete pressure equation which is $O(h^2)$.

The above arguments do not imply that upwinding is not desirable in any phase of the simulation. The continuity equations (5) and (11) are first order hyperbolic equations. Numerical procedures perform poorly on such hyperbolic equations without adequate regularization. Dupont [19] presented an interesting example of divergence from the true solution when using the finite element method to approximate the

solution of a first order hyperbolic equation. Upstream weighting is a particularly simple method for obtaining the necessary regularization of the continuity equations. The schemes represented by (8) and (13) both implement upstream weighting, but the manner in which the upwinding is incorporated is significantly different. The upstream weighting of the relative permeability curves used in (8) appears in the resulting discrete approximation to the pressure equation. The upwinding of the fractional flow terms that is used in (13), however, does not affect the resulting discrete pressure equation.

No mention is made above of irregular domains and triangulations. It is known that upstream weighting is mesh dependent dispersion [20]. When the mesh is irregular, upwinding is related to a dispersion matrix which is significantly asymmetric with respect to each coordinate direction. For example, if the aspect ratio of a grid block is 2, the amount of numerical dispersion in the direction of the longer axis of the block is twice as large as the amount of dispersion in the other direction. Although some procedures which do not utilize upwinding can produce realistic simulations over triangulations which do not have an aspect ratio of unity [21]*, realistic simulations using a procedure based upon upstream weighting for irregular triangulations are not expected due to the high degree of rotational asymmetry in the dispersion matrix that is introduced by the upwinding.

*Although an experiment involving a code which can realistically simulate displacements on irregular domains is not reported, an example of such a procedure can be found in Darlow, Ewing, and Wheeler [10].

5. Conclusions

A modification of the McCracken and Yanosik scheme performs adequately on some representative simulated displacements at an unfavorable mobility ratio. This modification performs upstream weighting on mass fractional flow terms rather than performing the regularization on the relative permeabilities. The discrete pressure equation that arises from this implementation does not contain upwinded terms, unlike the discretized pressure equation that is obtained from the original procedure of McCracken and Yanosik.

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8. Nomenclature

f^m Mass fractional flow of component m.

h Mesh spacing, ft.

k^m Relative permeability of phase m.

q Total mass injection rate, lb / day.

q^m Mass injection rate of component m, lb / day.

t Time, days.

K Permeability, 6.336×10^{-3} md.

P Pressure, psi.

S^m Saturation of phase m.

μ^m Viscosity of phase s, cp.

ν^m Kinematic viscosity of phase m, cp - cu ft / lb.

ρ^m Density of phase m, lb / cu ft.

$\rho^\wedge$ Total density, lb / cu ft.

$\rho^\wedge_m$ Total density of component m, lb / cu ft.

ϕ Porosity.

τ Time divided by porosity, days.

- τ^d Diagonal transmissibility, 6.336×10^{-3} md / sq ft.
- τ^p Parallel transmissibility, 6.336×10^{-3} md / sq ft.
- ξ Total mass mobility, cp - cu ft / lb.
- ξ^m Mass mobility of component m, cp - cu ft / lb.
- ξ_a Total mass mobility evaluated at the point a, cp - cu ft / lb.

Table 1	
Miscible Displacement Data	
Pattern Size (acres)	91.82
Fluid Properties	
Stock Tank Density (lbs / cu ft)	60
Compressibility (psi ⁻¹)	0
Viscosity (cp)	1
μ_r	.01
μ_i	60
Mass Diffusivity (lbs / ft / day)	60
Rock Properties	
Permeability (md)	100
Porosity	.1
Initial Conditions	
Pressure (psi)	100
Concentration	0
Well Data	
Injection Rate (pv / day)	.001
Bottom Hole Pressure (psi)	100

Table 2

Steam Drive Data

Pattern Size (acres)	2.5
Pattern Height (feet)	50
Oil Properties	
Stock Tank Density ($\frac{\text{lbs}}{\text{cu ft}}$)	60.3
Compressibility (psi^{-1})	.00001
Thermal Expansion ($^{\circ}\text{F}^{-1}$)	.0004
Specific Heat (btu / lbs)	.35
Viscosity (cp)	
@ 100 $^{\circ}\text{F}$	1380
@ 400 $^{\circ}\text{F}$	3
Rock Properties	
Permeability (md)	2000
Porosity	.3
Compressibility (psi^{-1})	.0005
Heat Capacity ($\text{btu} / ^{\circ}\text{F} / \text{cu ft}$)	30
Conductivity ($\text{btu} / ^{\circ}\text{F} / \text{ft} / \text{day}$)	30
Initial Conditions	
Pressure (psi)	100
Temperature ($^{\circ}\text{F}$)	100
Oil Saturation	.73
Well Data	
Injection Rate (bbls / day)	187.5
Steam Quality	.8
Temperature ($^{\circ}\text{F}$)	400
Bottom Hole Pressure (psi)	100
Wellbore Radius (ft)	.5

Table 3

Water / Oil Relative Permeability Data

S^w	k^{rw}	k^{row}
.250	.00000	.50000
.305	.00028	.39240
.360	.00224	.29928
.415	.00756	.22014
.470	.01792	.15443
.525	.03500	.10153
.580	.06048	.06077
.635	.09604	.03136
.690	.14336	.01234
.745	.20412	.00251
.800	.28000	.00000

Table 4

Gas / Oil Relative Permeability Data

S^g	k^{rg}	k^{rog}
.00	.00000	1.00000
.07	.00998	.71380
.14	.03241	.48965
.21	.06458	.31938
.28	.10531	.19502
.35	.15389	.10882
.42	.20981	.05338
.49	.27267	.02122
.56	.34216	.00580
.63	.41801	.00063
.70	.50000	.00000

Figure 1
Discretization of a Typical
Rectangular Symmetry Element

Figure 2
Discretization of the
Inverted Seven Spot

Figure 3
Miscible Displacement
Compositional Profiles
at .75 PVI, Original
Procedure

Figure 4
Miscible Displacement
Compositional Profiles
at .75 PVI, Modified
Procedure

Figure 5
Predicted Recovery for
Miscible Displacement
Using Both Procedures

Figure 6
Steam Displacement
Simulated Isotherms
at 1400 Days, Original
Procedure

Figure 7
Steam Displacement
Simulated Isotherms
at 1400 Days, Modified
Procedure

Figure 8
Predicted Oil Recovery
for an Inverted Seven
Spot Under Steam Drive

Figure 9
Predicted Oil Production
Rates for an Inverted Seven
Spot Under Steam Drive